Math 645 Problem Set 2

Tucker DiNapoli

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1. (a)

	Noodles	Pigs	Sausage
Noodles	10%	80%	10%
Pigs	50%	20%	30%
Sausage	20%	40%	40%

(b) N = 0.1S + 0.8P + 0.1N
P = 0.5S + 0.2P + 0.3N
S = 0.2S + 0.4P + 0.4N

(c) I'm not really sure what you mean by this. Even if we set S to 100 this is an over determined system. If we plug in S = 100 and solve we get N = P = 100, as is to be expected. The math to show this is easy enough so I'm not going to do it out.

2. (a) i. False, Ax = 0 has the trivial solution for any A, the columns of A are linearly independent if A has only the trivial solution

ii. False, if S is a linearly dependent set, then at least one vector is a linear combination of the others

iii. True, Given a vector in R^4 there can be at most 3 other vectors (in $\mathbb{R}^4$) that are linearly independent from it. A 4x5 matrix has 5 vectors in $\mathbb{R}^4$ meaning at least one must be a linear combination of the others.

iv. True, This is true by definition.

(b) i. True, If u and v are linearly independent vectors then au + bv = 0 or u = cv, which is the equation of a line through the origin.

ii. False, Any set containing the zero vector is linearly dependent, so a set could have fewer members than dimensions but, if it had the zero vector in it, it would still be linearly dependent.

iii. True, This is true by definition.

iv. False, See the answer to part b.

3. (a) u,v are linearly independent vector in $\mathbb{R}^3$, P is a plane through u,v and 0, with the equation x = st + uv. Show a linear transformation T: $\mathbb{R}^3$->$\mathbb{R}^3$ maps P to either a plane, line or point containing the origin.

(b) $f(x) = mx + b$
Show that f is only a linear transformation when b == 0, why is f called a linear function

f is linear if $\forall c \in \mathbb{R}$ $f(cu) = cf(u)$
$f(cu) = mcu + b$, $cf(u) = c*(mu + b) = cmu + c*b$;
$mcu + b = cmu + cb$ -> $b = cb$
$b = cb$ only when c or b is 0, since c can be any real number in order for f(cu) to equal cf(u) b must be 0. so f is only a linear transformation when b is 0. It is called a linear function because it is the function of a line (duh).

(c) Show an affine transformation ($T(x)=Ax + b$) is a linear transformation only when $b = 0$.

As with the last problem when we simplify $T(cx) = cT(x)$ we get $A(cx) + b = A(cx) + cb$ -> $b = cb$, which is only true (for nonzero c) when b is zero. We can make this a linear transformation by adding another dimension (i.e using homogeneous coordinates).

4. 1.9 #4,6,11,15,19,23

(a) The 2x2 rotation matrix is: [[cos(θ),-sin(θ)],[sin(θ),cos(θ)]]. So the matrix for a -π/4 radian rotation is:
A = [[cos(-π/4),-sin(π/4)], [sin(π/4),cos(π/4)]]
or [[sqrt(2)/2,-sqrt(2)/2],[sqrt(2)/2,sqrt(2)/2]].

(b) the 2x2 shear matrix is [[a,1],[1,b]], where a is the horizontal shear and b is the vertical shear. Which means to shear y by 3x you would use the following, [[3,1],[1,0]].

(c) Any reflection is just a rotation, in this case by π radians, this would be given by the matrix: A = [[cos(π),-sin(π)],[sin(π),cos(π)]] = [[-1,0],[0,-1]]. A reflection about some line is equivalent to a 3d rotation about that line by pi radians. Despite being a 3d rotation the fact the angle is π means the result will still be expressable as a 2d vector.

(d) [[3,0,-2],[4,0,0],[1,-1,1]]

(e) [[1,-5,4],[0,1,-6]]

(f) i. True, the columns of an nxn identity matrix form a set of orthogonal basis vectors in $\mathbb{R}^n$, so any column vector in $\mathbb{R}^n$ can be expressed as a linear combination of the columns of an identity matrix. Since T is a linear transformation we can transform a column vector into a linear combination of the unit vectors and then add them together and we get the same result as if we applied T to the original vector.

ii. True, a rotation of by angle φ about the origin can be expressed by the 2x2 matrix [[cos(φ),-sin(φ)],[sin(φ),cos(φ)]]

iii. False, assume f: $\mathbb{R}^m$->$\mathbb{R}^n$ and g:$\mathbb{R}^n$->$\mathbb{R}^l$ are linear transformations, then $h = g \circ f$: $\mathbb{R}^m$->$\mathbb{R}^l$ must also be linear. If we let A and B be the transformation matrices of f and g then $f(x) = Ax$ and $g(y) = By$, thus $h(x) = B(Ax) = (BA)x$. Since we can express h as a matrix transformation it must be a linear transformation.

iv. False, T:$\mathbb{R}^n$->$\mathbb{R}^m$ is onto if $\forall y \in \mathbb{R}^m$ $\exists x \in \mathbb{R}^n$ such that $T(x) = y$. By the wording in the question T could just map every x onto 1 y.

v. True, Assume $f(x) = Ax: R^3 \to R^2$ was one to one, then there must exist an inverse function $g(x) = Bx: R^2 \to R^3$. This would imply that we could span R^3 with only vectors from R^2, which is impossible, thus f cannot be one to one.