ECE 4554 / ECE 5554: Computer Vision: Homework 1

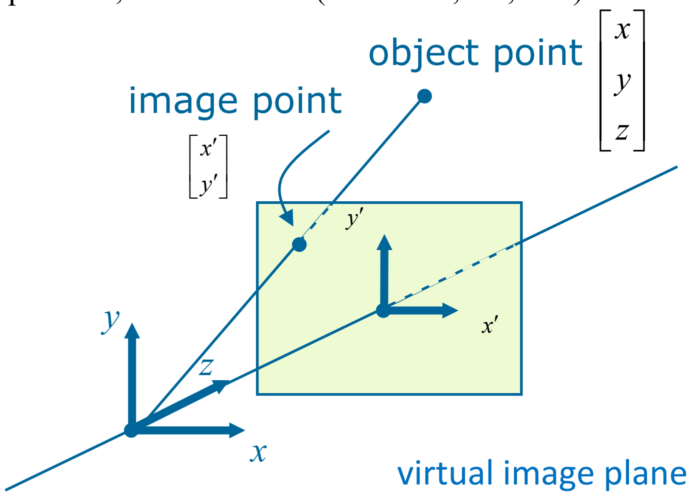
Fall 2023

Instructions

- This assignment is due at Canvas on Sept. 3 before 11:59 PM. As described in the syllabus, late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after the deadline will cost 1 token.
- Please review the Honor Code statement in the syllabus. For this assignment, you may discuss general approaches to solving the problems with other students. You may discuss software libraries and syntax. Beyond that point, you must work independently. The work that you submit for a grade must be your own.
- The assignment consists of 6 problems. Problems 1 through 4 are analytical in nature, and are presented here. Problems 5 and 6 require work using Colab. Each problem is worth 10 points.
- Notice that one of the problems is required for 5554 students, but is optional (extra credit) for 4554 students.
- Prepare an answer sheet that contains all of your written answers in a single file named `Homework1_Problems1-4_USERNAME.pdf`. (Use your own VT Username.) Handwritten solutions are permitted, but they must be easily legible to the grader. In addition, 2 more files related to Python coding must be uploaded to Canvas. Details are provided at the end of this assignment.
- For problems 5 and 6 (the coding problems), the notebook file that you submit must be compatible with Google Colab. Your code should execute after the grader makes only 1 change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of 0 for problems 5 and 6.
- After you have submitted to Canvas, it is your responsibility to download the files that you submitted and verify that they are correct and complete. *The files that you submit to Canvas are the files that will be graded.*

Problem 1. The diagram below represents a camera with a left-handed x-y-z coordinate system that is centered at the point of projection. The z axis lies on the optical axis, and a virtual image plane is located at a distance f (the focal length) from the point of projection. For this problem, locations within the image are indicated using metric units (i.e., meters, not pixels) with respect to coordinate system x'-y' as shown. Assuming ideal perspective projection, find the image locations that correspond to the following 3-dimensional scene points (given in meters). Assume f = 28 mm. For full credit in this problem, state the units (such as m, cm, mm).

a) $(x, y, z) = (3, 4, 5)$
b) $(x, y, z) = (9, 12, 15)$
c) $(x, y, z) = (-1, -2, 7)$
d) $(x, y, z) = (0, 0, 7)$



e) Assume that the sensor array for this camera is of size 36 mm × 24 mm (width × height). Determine the horizontal field of view for this camera. (The answer should be an angle, in degrees, that corresponds to the width of the sensor array.)

Problem 2. Two lines in the *x-y* plane can be described by the following pair of equations. The symbols a_i, b_i, and c_i represent constant terms. For full credit, your answers must accommodate 2D lines in any orientations.

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

a) Assume that these 2 lines are not parallel. Solve for the point of intersection of these 2 lines. Express your answer in terms of the quantities a_1, b_1, c_1, a_2, b_2, and c_2.

b) For your answer in (a), give a numerical solution for the following case:

$a_1 = 2$, $b_1 = 3$, $c_1 = 4$, $a_2 = 5$, $b_2 = 6$, $c_2 = 7$

c) Mathematically, state the condition(s) that must be true when the 2 lines are parallel. Express your answer in terms of the quantities a_1, b_1, c_1, a_2, b_2, and c_2.

Problem 3.

a) Write the Taylor series expansion up to 2nd order (i.e., up to and including the 2nd-derivative term) for an arbitrary function $f(x)$ of 1 variable about the value $x = x_0$.

b) Using your solution from (a), write an expression for the 2nd-order Taylor series approximation for function $f(x) = 3 + 4x + 5\cos(x)$.

c) Refer to a textbook or online resources as needed, and write the Taylor series expansion up to 2nd order for an arbitrary function $h(x, y)$ of 2 variables about the values $(x, y) = (x_0, y_0)$.

d) Using your solution from (c), write an expression for the 2nd-order Taylor series approximation for function $h(x, y) = 3 + 4xy + 5\cos(y)$.

Problem 4. (For 5554 students, this problem is required. For 4554 students, this problem is optional and can be submitted for extra credit.)

Consider a 3-dimensional scene in which 3 points *A*, *B*, and *C* are known to be collinear. Assume that 3D point *A* projects onto 2D point *a* in the image; point *B* projects onto point *b* in the image; and point *C* projects onto point *c* in the image.

a) Show that the projected points *a*, *b*, and *c* are collinear in the image plane. (Provide a convincing geometric or mathematical argument.)

b) If possible, show that the *ordering* of the projected points *a*, *b*, and *c* (along the projected line) is the same as for points *A*, *B*, and *C*. (In other words, if the scene points are encountered in *A–B–C* order along the line that they share in $\mathbb{R}^3$, then the projected points in $\mathbb{R}^2$ will be encountered in *a–b–c* order and never, for example, in *a–c–b* order. If possible, provide a convincing geometric or mathematical argument. If not, provide a counterexample.)

c) Suppose that the Euclidean distance separating *A* from *B* is the same as the distance separating *B* from *C*. Will the distance separating *a* from *b* be the same as the distance separating *b* from *c*? (Again, if yes, provide a convincing geometric or mathematical argument. If not, provide a counterexample.)

Problems 5 and 6.
You have been given a Jupyter Notebook file `Homework1_USERNAME.ipynb` and an image file `stripes.png`. Replace "`USERNAME`" with your Virginia Tech Username. Then upload both files to Google Drive. Open the `ipynb` file in Google Colab. Follow the instructions that you will find inside the notebook file.

What to hand in: After you have finished, you will have created the following 3 files. Upload these 3 files to Canvas before the deadline. Do not combine them in a single ZIP file.

`Homework1_Problems1-4_USERNAME.pdf` ← Your solutions to problems 1 through 4

`Homework1_Code_USERNAME.zip` ← Your zipped Jupyter notebook file

`Homework1_Notebook_USERNAME.pdf` ← A PDF version of your Colab session
