## **Instructions**

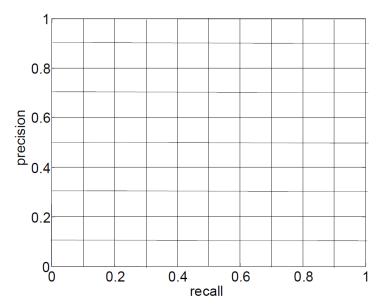
- This assignment is due at Canvas on October 29 before 11:59 PM. As described in the syllabus, late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after the deadline will cost 1 token. Do not allow your token count to fall below 0.
- Please review the Honor Code statement in the syllabus. For this assignment, you may discuss general approaches to solving the problems with other students. You may discuss software libraries and syntax. Beyond that point, you must work independently. The work that you submit for a grade must be your own.
- The assignment consists of 6 problems. Problems 1 through 3 are analytical in nature, and are presented here. Problems 4 through 6 require work using Colab. Each problem is worth 10 points.
- One of the problems is optional (extra credit) for all students, both 4554 and 5554.
- Prepare an answer sheet that contains all of your written answers in a single file named Homework4\_Problems1-3\_USERNAME.pdf. (Use your own VT Username.) Handwritten solutions are permitted, but they must be easily legible to the grader. In addition, more files related to Python coding must be uploaded to Canvas. Details are provided at the end of this assignment.
- For problems 4 through 6 (the coding problems), the notebook file that you submit must be compatible with Google Colab. Your code should execute after the grader makes only 1 change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of 0 for those problems.
- After you have submitted to Canvas, it is your responsibility to download the files that you submitted and verify that they are correct and complete. *The files that you submit to Canvas are the files that will be graded.*

**Problem 1.** Consider a system that detects interest points in images, and performs image-to-image matching using descriptors (such as SIFT). The system then uses those point correspondences to solve for a 2D homography that maps from one image to the other image. The system uses RANSAC to address the problem of outliers.

Suppose that p is the probability that any particular correspondence is an inlier. Assume that the main RANSAC loop repeats N times, and for each iteration of the loop the system randomly selects  $s \ge 4$  correspondences in order to solve for the 2D homography.

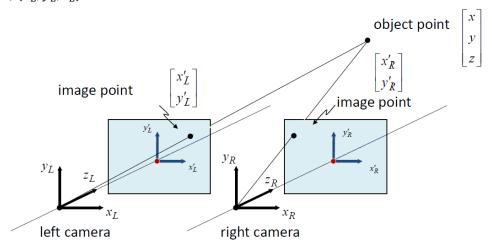
- a) Write an expression for the probability that <u>at least 1</u> selection of s correspondences within the RANSAC loop contains only inliers. (I.e., what is the probability that the RANSAC procedure will have at least one set of corresponding points that is free of outliers?)
- b) Assume that p = 0.6 and s = 4. What is the minimum value of N that will guarantee a 95% chance of success by the RANSAC procedure? (For this part, you may assume that a correct result will be generated if at least 1 RANSAC loop receives a set of s correspondences that are free from outliers.)
- **Problem 2.** Consider a system that performs image retrieval from a collection of images based on the content of the images. (This is the problem known as Content-Based Image Retrieval, or CBIR.) For example, if a user presents a photograph of a cat as a query image, we might expect the system to return all images of cats from the collection.
- a) Use the following information to create a *precision-recall curve* that characterizes the performance of this system. The entire data collection consists of 10 images. In response to a query image, the system returns images from the collection as shown below, ranked left-to-right in order of decreasing similarity. The abbreviations "TP" and "FP" refer to "True Positive" and "False Positive", respectively.

Determine the points on the diagram below that are needed to create a precision-recall curve. Show your work, and indicate the final precision-recall curve.



b) Based on your answer above, compute the *average precision* of this experiment with this CBIR system.

**Problem 3.** Consider the common binocular stereo geometry that is shown in the figure below. Both cameras have focal length f, and the two optical axes are parallel. The coordinate systems of the two cameras are aligned, except that the right camera has been translated to the right (along the  $x_L$  axis) by baseline distance B. The world-coordinate system, (x, y, z), coincides with the coordinate system of the left camera,  $(x_L, y_L, z_L)$ .



Suppose that the stereo-matching system reports that a particular point from the left image,  $(x'_L, y'_L)$ , corresponds to point  $(x'_R, y'_R)$  in the right image.

a) For the following case, with all distances provided in meters, solve numerically for the corresponding 3D location (x, y, z):

$$f = 0.025, B = 0.05, (x'_L, y'_L) = (0.006, 0.002), \text{ and } (x'_R, y'_R) = (0.005, 0.002)$$

b) Suppose that image point  $(x'_L, y'_L)$  has been detected with high accuracy, but the estimated location of the corresponding point in the right image contains a small error  $\Delta x'$  in the horizontal direction only. Solve for the resulting errors in each component of the estimated 3D location,

- (x, y, z). (Your answers should not be numerical, but should be a functions of the algebraic terms  $x'_L, y'_L, x'_R, y'_R, \Delta x'$ , etc. Try to write your answer in simplified form.)
- c) Repeat part (b), but now assume that the corresponding point in the right image contains a small error  $\Delta y'$  in the vertical direction only. (In other words, the error in the horizontal direction is  $\Delta x' = 0$ .)

## Problems 4 through 6.

You have been given a Jupyter Notebook file <code>Homework4\_USERNAME.ipynb</code>. Replace "USERNAME" with your Virginia Tech Username, and upload that file to Google Drive. You have also been given 2 zipped directories that contain images. Unzip them, and upload the entire directories to Google Drive.

Open the ipynb file in Google Colab. Follow the instructions that you will find inside the notebook file.

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*What to hand in:* After you have finished, you will have created the following files. Upload these files to Canvas before the deadline. Do not combine them in a single ZIP file.