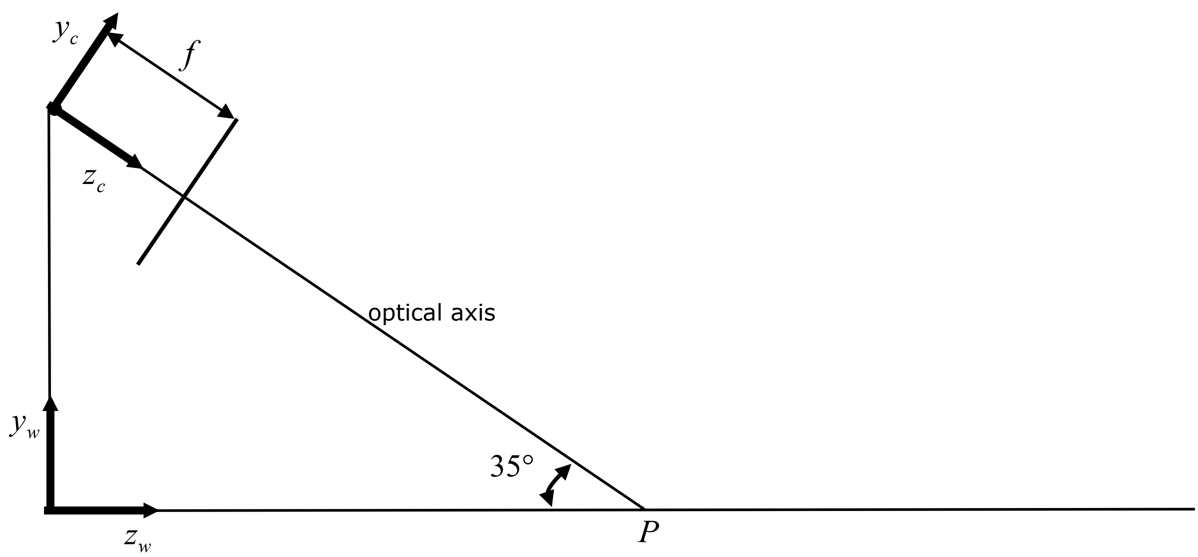
ECE 4554 / ECE 5554: Computer Vision: Homework 3

Fall 2023

Instructions

- This assignment is due at Canvas on October 1 before 11:59 PM. As described in the syllabus, late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after the deadline will cost 1 token.
- Please review the Honor Code statement in the syllabus. For this assignment, you may discuss general approaches to solving the problems with other students. You may discuss software libraries and syntax. Beyond that point, you must work independently. The work that you submit for a grade must be your own.
- The assignment consists of 6 problems. Problems 1 through 3 are analytical in nature, and are presented here. Problems 4 through 6 require work using Colab. Each problem is worth 10 points.
- Prepare an answer sheet that contains all of your written answers in a single file named `Homework3_Problems1-3_USERNAME.pdf`. (Use your own VT Username.) Handwritten solutions are permitted, but they must be easily legible to the grader. In addition, more files related to Python coding must be uploaded to Canvas. Details are provided at the end of this assignment.
- For problems 4 through 6 (the coding problems), the notebook file that you submit must be compatible with Google Colab. Your code should execute after the grader makes only 1 change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of 0 for those problems.
- After you have submitted to Canvas, it is your responsibility to download the files that you submitted and verify that they are correct and complete. *The files that you submit to Canvas are the files that will be graded.*

Problem 1. Consider the figure below, which represents a camera that is mounted above the ground with its optical axis at 35° with respect to the horizontal. (The figure is not drawn to scale.) The camera is viewed from the side. The camera-coordinate system is represented by (x_c, y_c, z_c) and is centered at the point of projection, but the x_c axis is perpendicular to this drawing. Similarly, a world-coordinate system is represented by (x_w, y_w, z_w), and the x_w axis is also perpendicular to this drawing. You may assume that the ground coincides with the x_w-z_w plane. The camera is mounted on a stand, so that the camera center is located at $(x_w, y_w, z_w) =$ (0, h, 0), where h = 3 meters. The camera's z_c axis is aligned with the optical axis, and it intersects the ground plane at point P.



a) For this camera arrangement, any point A can be represented using coordinates relative to the world-coordinate system, and the same point can also be represented using coordinates relative to the camera-coordinate system. Find a homogeneous transformation matrix that will map any point A from world-coordinate representation to camera-coordinate representation.

b) Find the location of P relative to the world-coordinate system. You may use any geometric method.

c) Using your answers from (a) and (b), solve for the location of P relative to the camera-coordinate system.

Problem 2. A 2D linear filter $h(x, y)$ is sometimes called *separable* if it can be decomposed into the convolution of two 1D filters: $h(x, y) = h_1(x) * h_2(y)$, where * represents convolution. In the discrete domain, an example is the following "box" filter:

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} * \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

a) Show that both Sobel operators are separable, using the definition given above.

b) Assume that 2D Gaussian filters are separable into 1D Gaussian filters. The following is an approximation to a 1D Gaussian filter in the horizontal direction: $g = \frac{1}{16}\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix}$. Construct a 2D Gaussian filter using the filter g and its transpose, g^{T}.

Problem 3. In a recent lecture, it was stated that the following template is a popular approximation of the Laplacian operation:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Provide an analytical derivation to explain why this kernel is a reasonable approximation of the Laplacian.

Problems 4 through 6.
You have been given a Jupyter Notebook file `Homework3_USERNAME.ipynb` and an image file. Replace "`USERNAME`" with your Virginia Tech Username. Then upload those files to Google Drive. Open the `ipynb` file in Google Colab. Follow the instructions that you will find inside the notebook file.

What to hand in: After you have finished, you will have created the following 4 files. Upload these 4 files to Canvas before the deadline. Do not combine them in a single ZIP file.

`Homework3_Problems1-3_USERNAME.pdf` ← Your solutions to problems 1 through 3.

`Homework3_USERNAME.ipynb` ← Your Jupyter Notebook file. (Don't zip it.)

`Homework3_Notebook_USERNAME.pdf` ← A pdf version of your Colab session.

`newimage.png` ← An image file that you provide.
(Image file extensions other than png are acceptable.)
