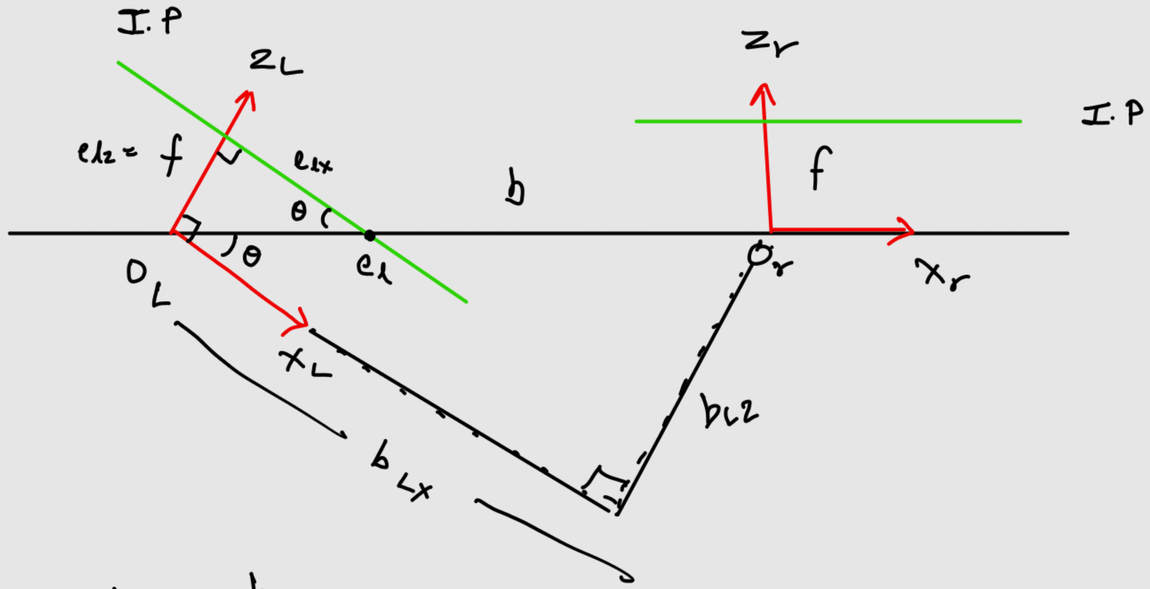
ECE 5554 – Homework 5

1)



tran in $x = b_{Lx}$
tran in $z = b_{Lz}$

$\sin\theta = \dfrac{b_{Lz}}{b}$ $\therefore b_{Lz} = b\sin\theta$

$\cos\theta = \dfrac{b_{Lx}}{b}$ $\therefore b_{Lx} = b\cos\theta$

$\therefore t = \begin{bmatrix} b\cos\theta \\ 0 \\ b\sin\theta \end{bmatrix}$; $[t]_\times = \begin{bmatrix} 0 & -b\sin\theta & 0 \\ b\sin\theta & 0 & -b\cos\theta \\ 0 & b\cos\theta & 0 \end{bmatrix}$

Rot along y-axis $R = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

$\therefore E = [t]_\times R$

$$= \begin{bmatrix} 0 & -b\sin\theta & 0 \\ b\sin\theta & 0 & -b\cos\theta \\ 0 & b\cos\theta & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & -b\sin\theta & 0 \\ 2b\sin\theta\cos\theta & 0 & b(\sin^2\theta - \cos^2\theta) \\ 0 & b\cos\theta & 0 \end{bmatrix}$$

For left epipole,

$$E^T e = 0$$

$$\therefore \begin{bmatrix} 0 & 2b\sin\theta\cos\theta & 0 \\ -b\sin\theta & 0 & b\cos\theta \\ 0 & b(\sin^2\theta - \cos^2\theta) & 0 \end{bmatrix} \begin{bmatrix} e_{lx} \\ e_{ly} \\ e_{lz} \end{bmatrix} = 0$$

$$\therefore \quad 2b\sin\theta\cos\theta \, e_{ly} = 0$$

$$\because b, \sin\theta, \cos\theta \neq 0, \quad e_{ly} = 0$$

$$-b\sin\theta \, e_{lx} + b\cos\theta \, e_{lz} = 0$$

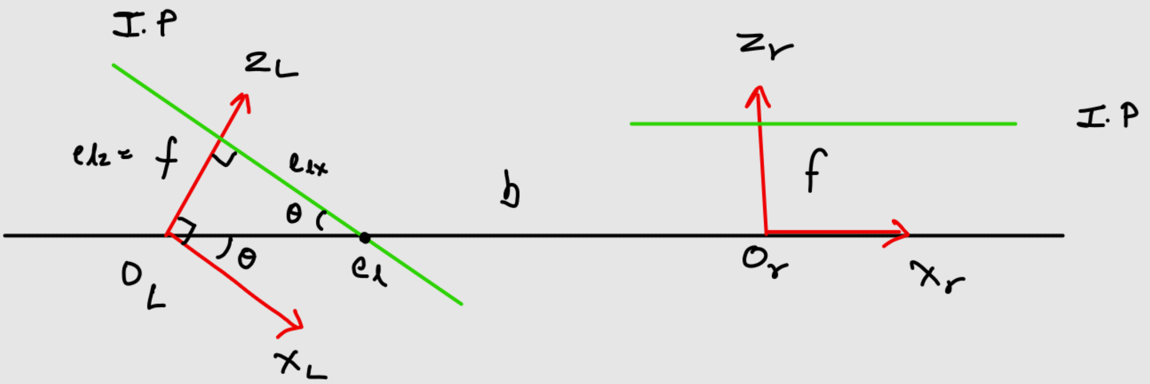
$$\therefore \quad \sin\theta \, e_{lx} = \cos\theta \, e_{lz}$$

$$\therefore \quad e_{lx} = \cot\theta \, e_{lz}$$

$$e_{lz} = f, \quad \therefore e_{lx} = f\cot\theta$$

$$\therefore \quad e = [\, f\cot\theta \quad 0 \quad f \,]$$

Using geometry,



e_l is location of left epipole

Using geometry, location of e_l w.r.t. O_L [considering O_L as world co-ordinate center]

$e_l = [e_{lx} \quad e_{ly} \quad e_{lz}]$

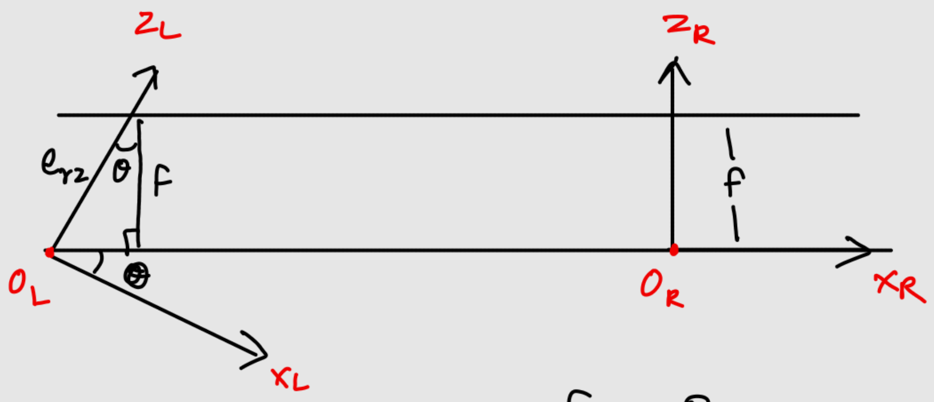
$e_{ly} = 0$ since on x-z plane

$e_{lz} = f$

$e_{lx} = f\cot\theta$

$\therefore e_l = [f\cot\theta \quad 0 \quad f] \,//\, [\cot\theta \quad 0 \quad 1]$

(c) For right epipole, By geometry.



$\therefore \frac{f}{e_{rz}} = \cos\theta$, $e_{rz} = f\sec\theta$

$\because$ I/p of right image is parallel to baseline, it won't meet baseline. Or it will meet baseline at $e_{rx} = \infty$

$e_{ry} = 0$ $\because$ it is on x-y plane.

$\therefore e' = [\infty \quad 0 \quad f\sec\theta] \,//\, [\infty \quad 0 \quad \sec\theta]$

$$Ee' = 0$$

$$\therefore \begin{bmatrix} 0 & -b\sin\theta & 0 \\ 2b\sin\theta\cos\theta & 0 & b(\sin^2\theta - \cos^2\theta) \\ 0 & b\cos\theta & 0 \end{bmatrix} \begin{bmatrix} e_{rx} \\ e_{ry} \\ e_{rz} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad -b\sin\theta \, e_{ry} = 0$$

$$\because b, \sin\theta \neq 0, \quad e_{ry} = 0$$

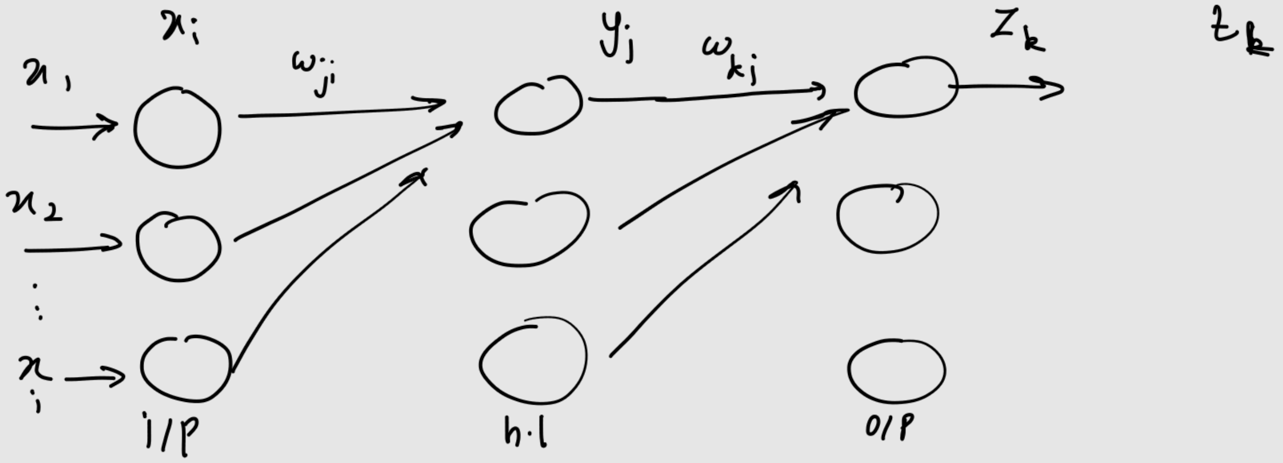
$s^2 + c^2 = 1$
$c^2 = 1 - s^2$

$$e_{rx}(2b\sin\theta\cos\theta) + b(\sin^2\theta - \cos^2\theta)e_{rz} = 0$$

$$\therefore \quad e_{rx} = \left(\frac{\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}\right)e_{rz}$$

$$= \frac{1}{2}(\cot\theta - \tan\theta)\, e_{rz} \quad ?$$

2)



$$J = \frac{1}{2} \sum (t_k - z_k)^2 \; ; \; \frac{\partial J}{\partial z_k} = \frac{-2}{2}(t_k - z_k) = (z_k - t_k) \quad \text{— 1}$$

$$z_k = f(net_2) \; ; \; \frac{\partial z_k}{\partial net_2} = z_k(1 - z_k) \quad \left[\text{signum funct derivative}\right] \quad \text{— 2}$$

$$net_2 = \sum w_{kj} y_j \; ; \; \frac{\partial net_2}{\partial w_{kj}} = y_j \quad \text{— 3},$$

$$\frac{\partial net_2}{\partial y_j} = w_{kj} \quad \text{— 4}$$

$$\frac{\partial J}{\partial w_{kj}} = \frac{\partial J}{\partial z_k} \; \frac{\partial z_k}{\partial net_2} \; \frac{\partial net_2}{\partial w_k}$$

$$\therefore \frac{\partial J}{\partial w_{kj}} = (z_k - t_k)(z_k)(1 - z_k)\, y_j \quad (\text{from 1, 2 \& 3})$$

Now, $y_j = f(net_1)$; $\dfrac{dy_i}{\partial net_1} = y_i(1 - y_i)$ — 5

$net_1 = \sum w_{ji} x_i$; $\dfrac{\partial net_1}{\partial w_{ji}} = x_i$ — 6

$$J = \frac{1}{2} \sum (t_k - z_k)^2$$

$$\frac{\partial J}{\partial \omega_{ji}} = \frac{\partial}{\partial \omega_{ji}} \left(\frac{1}{2} \sum (t_k - z_k)^2 \right)$$

$$= -\frac{2}{2} \sum (t_k - z_k) \frac{\partial z_k}{\partial \omega_{ji}}$$ (summation will remain since all i/p weight affects the o/p)

$$= - \sum (t_k - z_k) \frac{\partial z_k}{\partial net_2} \cdot \frac{\partial net_2}{\partial y_j} \frac{\partial y_j}{\partial net_1} \frac{\partial net_1}{\partial \omega_{ji}}$$

$$= - \sum_{k=0}^{k} (t_k - z_k)(z_k)(1 - z_k)(\omega_{jk})(y_j)(1 - y_j)(x_i)$$

from 2, 4, 5 & 6