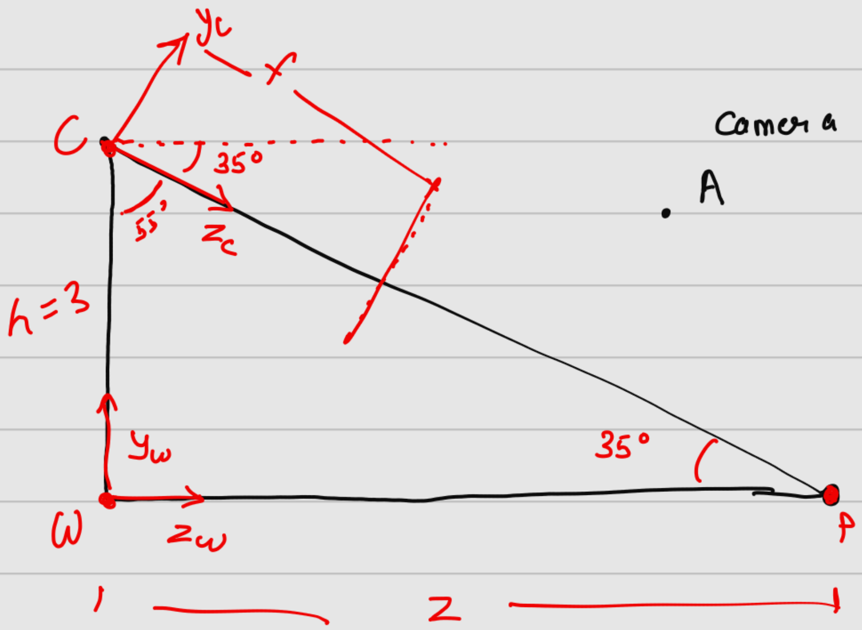
# ECE5554 : Homework 3 : Hiten Kothari

P1)




Camera centre = (0, 3, 0)

. A

(a) For world coordinate → camera coordinate.

1. Translation of $x_w\ y_w\ z_w \rightarrow x_c\ y_c\ z_c$
   3 metres in y direction.
2. Rotation of xz plane of 35° clockwise direction.
   = −35° counterclockwise.

$$W \rightarrow C = R(x) \times T(y) \times A$$

world → camera

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix} = \begin{pmatrix} \text{Rotate along } X \text{ axis} \end{pmatrix} \begin{pmatrix} \text{Translate by } \Delta y \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & \Delta y\cos\theta - \Delta z\sin\theta \\ 0 & \sin\theta & \cos\theta & \Delta y\sin\theta + \Delta z\cos\theta \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Homogeneous matrix

(b) $\because$ Since ground coincides with $x_w z_w$ plane & Point P lies on the ground, x & y coordinates of P should be 0.

Using trigonometry (looking at above diagram)

$$\tan 55^\circ = \frac{Z}{h}$$

$$\therefore Z = h\tan 55 = 4.2844 \text{ m}$$

$\therefore$ coordinate of P in world coordinate $= (0, 0, 4.2844)$ m.

(c) from (a), (b), $P = (0, 0, 4.2844)$m, $\theta = -35^\circ$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & \Delta y\cos\theta - \Delta z\sin\theta \\ 0 & \sin\theta & \cos\theta & \Delta y\sin\theta + \Delta z\cos\theta \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-35) & -\sin(-35) & -3\cos(-35) \\ 0 & \sin(-35) & \cos(-35) & -3\sin(-35) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 4.2844 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -\sin(-35)\times 4.2844 - 3\cos(-35) \\ \cos(-35)\times 4.2844 - 3\sin(-35) \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5.2303 \\ 1 \end{bmatrix}$$

$\therefore$ P in camera coordinates $= (0, 0, 5.2303)$ m

P2)

(a) To prove both Sobel operators are separable:

$$S_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} ; \quad S_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}_{S_{xx}} \underbrace{\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}}_{S_{xy}} \qquad \underbrace{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}_{S_{yx}} \underbrace{\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}}_{S_{yy}}$$

Multiplying these 1-D filters would generate 2D-Sobel Filter.

(b) $g = \frac{1}{16}\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix}$

$$2DGF = g^T * g = \frac{1}{256}\begin{bmatrix} 1 \\ 4 \\ 6 \\ 4 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

$$= \frac{1}{256}\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

P3.) Appx of Laplacian = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$L(x,y) = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$ : Laplacian definition

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

here pixel change by 1, $\therefore h = 1$

$\therefore f'(x) = f(x+1) - f(x)$

$f'(x+1) = f(x+2) - f(x+1)$

$$\begin{aligned} f''(x) &= f'(x+1) - f'(x) \\ &= f(x+2) - f(x+1) - f(x+1) + f(x) \\ &= f(x+2) - 2f(x+1) + f(x) = f(x+1) - 2f(x) + f(x-1) \\ &= [1 \quad -2 \quad 1] \end{aligned}$$

centering

Similarly $f''(y) = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$$\begin{aligned} \therefore L(x,y) &= f''(x) + f''(y) \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

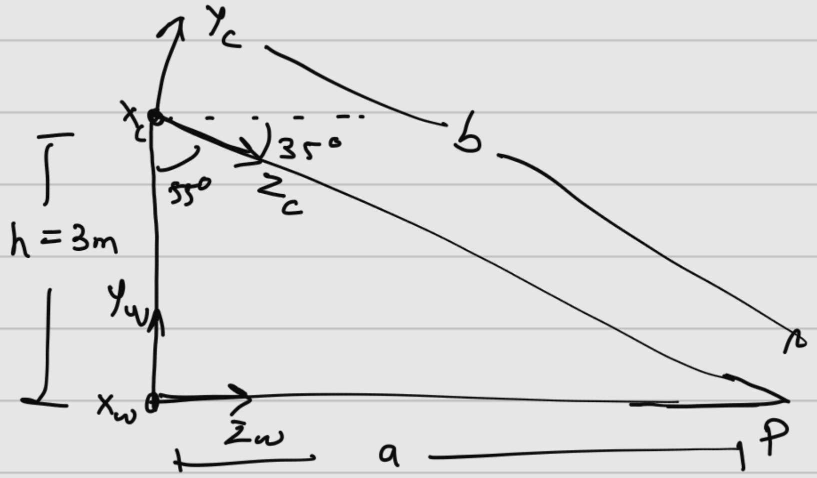
P1)(a)

P in world coordinate

$= (0, 0, a)$

P in camera coordinate

$= (0, 0, b)$




$\frac{a}{h} = \tan 55$ ; $\frac{h}{b} = \cos 55$

$a = h \tan 55$ $\therefore b = h/\cos 55$ (By geometry)

$= 4.2844m$ $= 5.2303m$