ECE 4554 / ECE 5554: Computer Vision: Homework 2

Fall 2023

Instructions

- This assignment is due at Canvas on Sept. 17 before 11:59 PM. As described in the syllabus, late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after the deadline will cost 1 token.
- Please review the Honor Code statement in the syllabus. For this assignment, you may discuss general approaches to solving the problems with other students. You may discuss software libraries and syntax. Beyond that point, you must work independently. The work that you submit for a grade must be your own.
- The assignment consists of 6 problems. Problems 1 through 4 are analytical in nature, and are presented here. Problems 5 and 6 require work using Colab. Each problem is worth 10 points.
- Prepare an answer sheet that contains all of your written answers in a single file named `Homework2_Problems1-4_USERNAME.pdf`. (Use your own VT Username.) Handwritten solutions are permitted, but they must be easily legible to the grader. In addition, 2 more files related to Python coding must be uploaded to Canvas. Details are provided at the end of this assignment.
- For problems 5 and 6 (the coding problems), the notebook file that you submit must be compatible with Google Colab. Your code should execute after the grader makes only 1 change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of 0 for problems 5 and 6.
- After you have submitted to Canvas, it is your responsibility to download the files that you submitted and verify that they are correct and complete. *The files that you submit to Canvas are the files that will be graded.*

Problem 1. Consider two signals g and h, which are shown below. Let the symbol $\otimes$ represent 2D cross-correlation, and let * represent 2D convolution. For the following parts, find answers and indicate to the grader how you arrived at those answers. (For full credit, don't simply show the results.) If needed for this problem, you may enlarge templates for both g and h by simply appending rows or columns that contain the value 0 only.

$$g = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad h = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

a) Solve for $g \otimes h$.

b) Solve for $h \otimes g$. Briefly state how this answer compares to your answer for part (a).

c) Solve for $g * h$.

d) Solve for $h * g$. Briefly state how this answer compares to your answer for part (c).

Problem 2. Recall that rotation of a 2D point (x, y) about the origin can be described by

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = R \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \tag{1}$$

where

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

and where (x', y') is the location of the point after rotation. A positive value of θ causes rotation in the counterclockwise direction. Similarly, translation by an amount $(\Delta x, \Delta y)$ can be expressed using a matrix of the following form:

$$\begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$$

a) Suppose that you want to rotate a point about an arbitrary reference location (x_0, y_0), instead of rotating about the origin. Write an equation that shows a <u>sequence of matrix-multiplication operations</u> that performs this desired rotation, using homogeneous coordinate representation.

b) Compute some numerical results using your solution to part (a). Assume that $\theta = 15$ degrees and $(x_0, y_0) = (250, 150)$. Find (x', y') for the following cases:

$(x, y) = (0, 0)$
$(x, y) = (200, 150)$
$(x, y) = (400, 300)$

Problem 3. (10 points) This is an example of the "homogeneous linear least squares problem." Consider the matrix equation $\mathbf{Ax} = \mathbf{0}$, where $\mathbf{A} = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}$, and $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Assume that all components of $\mathbf{A}$ are known, and you want to solve for the unknown vector $\mathbf{x}$. When $\mathbf{A}$ satisfies certain conditions, the solution is the eigenvector associated with the smallest eigenvalue of $\mathbf{M} = (\mathbf{A}^T)\mathbf{A}$. (The superscript T represents the transpose operation.) To avoid the trivial solution that all $x_i = 0$, it is common to impose the condition that $\mathbf{x}$ is a unit vector.

Solve for $\mathbf{x}$, a unit vector, for the case $\mathbf{A} = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ using this eigenvector-based approach. As part of your answer, clearly show all eigenvalues of $\mathbf{M} = \mathbf{A}^T\mathbf{A}$. (You may use any matrix solver to find the numerical values. If you use a matrix solver, cut and paste your code as part of your solution. For example, the NumPy functions `np.linalg.eig()` or `np.linalg.eigh()` might be used. The latter function will prevent warnings due to small imaginary values if you are working with matrices that are real and symmetric.)

Problem 4. This problem required by all students (<u>not</u> optional for ECE4554). Recall that a 2D homography can be written using homogeneous coordinate representation as follows:

$$\tilde{w} \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = \begin{bmatrix} H_{00} & H_{01} & H_{02} \\ H_{10} & H_{11} & H_{12} \\ H_{20} & H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \tag{3}$$

The equation represents a mapping from source point (x_i, y_i) to destination point (x_i', y_i'). As discussed in the textbook near equations (2.20)-(2.21), there are only 8 degrees of freedom in this equation. For this reason, many formulations set H_{22} to 1. It may help you when working on coding problems later if you do <u>not</u> constrain H_{22} to be 1. The term $\tilde{w}$ is a scalar value that can be eliminated by writing the following pair of equations:

$$x_i' = \frac{H_{00}x_i + H_{01}y_i + H_{02}}{H_{20}x_i + H_{21}y_i + H_{22}} \tag{4a}$$

$$y_i' = \frac{H_{10}x_i + H_{11}y_i + H_{12}}{H_{20}x_i + H_{21}y_i + H_{22}} \tag{4b}$$

Next, suppose that you are given $N \geq 4$ pairs of corresponding points:

source point (x_0, y_0) maps to destination point $(x_0{}', y_0{}')$;
source point (x_1, y_1) maps to destination point $(x_1{}', y_1{}')$;
$\dots$;
and source point (x_{N-1}, y_{N-1}) maps to destination point $(x_{N-1}{}', y_{N-1}{}')$.

a) Using these corresponding point pairs, show that you can convert equations (4a-b) to a matrix expression of the form $\mathbf{Ah} = \mathbf{0}$, where $\mathbf{A}$ is a matrix of size $2N \times 9$, $\mathbf{h}$ is a 9×1 vector containing all of the terms $\{H_{00}, \dots, H_{22}\}$, and $\mathbf{0}$ is a zero vector of size $2N \times 1$.

b) Continuing from part (a), a least-squares solution to **h** is the eigenvector associated with the smallest eigenvalue of the matrix $\mathbf{A}^{\mathrm{T}}\mathbf{A}$. Use this approach to find numerical solutions for all of the homography parameters $\{H_{00}, \dots, H_{22}\}$, for the following point correspondences:

$$
\begin{array}{ccc}
(x_i', y_i') & \leftrightarrow & (x_i, y_i) \\
(5,4) & \leftrightarrow & (0,0) \\
(7,4) & \leftrightarrow & (1,0) \\
(7,5) & \leftrightarrow & (1,1) \\
(6,6) & \leftrightarrow & (0,1)
\end{array}
$$

To help with the grading, please <u>normalize your numerical solution</u> by dividing all parameters $\{H_{00}, \dots, H_{22}\}$ by the value of H_{22}.

As for the previous problem, you may use any matrix solver to find the numerical values. If you use a computational methods, cut and paste your code as part of your solution.

Problems 5 and 6.
You have been given a Jupyter Notebook file `Homework2_USERNAME.ipynb` and several image files. Replace "`USERNAME`" with your Virginia Tech Username. Then upload those files to Google Drive. Open the `ipynb` file in Google Colab. Follow the instructions that you will find inside the notebook file.

What to hand in: After you have finished, you will have created the following 3 files. Upload these 3 files to Canvas before the deadline. Do not combine them in a single ZIP file.

`Homework2_Problems1-4_USERNAME.pdf` ← Your solutions to problems 1 through 4.

`Homework2_USERNAME.ipynb` ← Your Jupyter Notebook file. (Don't zip it.)

`Homework2_Notebook_USERNAME.pdf` ← A PDF version of your Colab session.
