deVinci Camera Calibration Report

1. Problem Formulation

deVinci Camera Calibration is to find the several quantities internal to camera which will affect the imaging process, and the intrinsic unknowns which should be found are listed below:

1. Uc: central pixel in u direction of image plane, Vc: central pixel in v direction of image plane
2. Base line
3. focal length
4. the position of camera frame origin w/rt (with respect to) world frame
5. the orientation of camera frame w/rt world frame (known aligned with world frame)

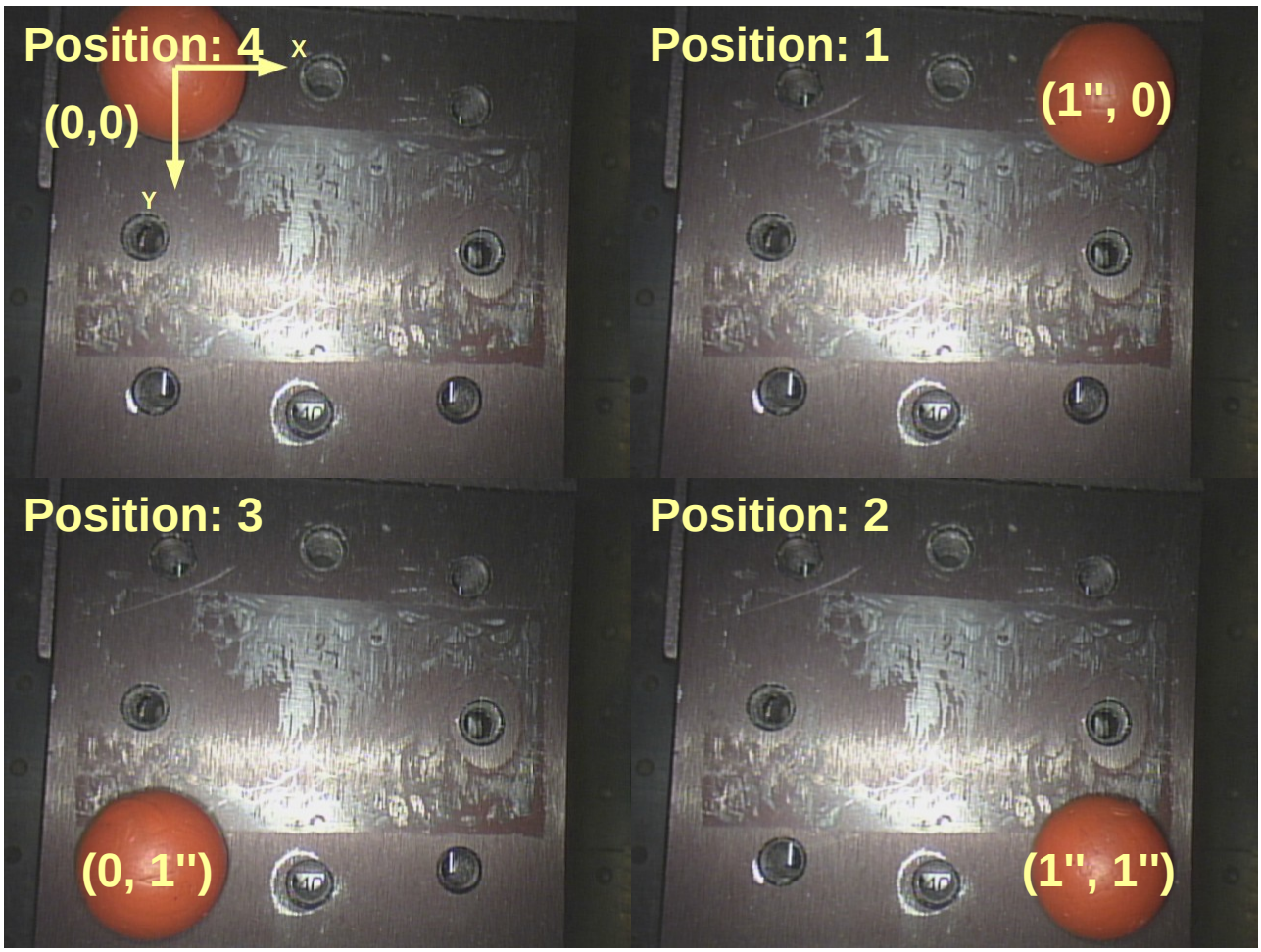

Figure 1.1 Fiducials positions and world coordinate frame

By taking 24 snapshots of four fiducials in 5 elevations, each elevation height = 0.005m starting from 0 to 0.025m, both left and right cameras captured 24 pixels of fiducials centroid, as listed in Table 1.1. Also, a world coordinate frame was built on the height of 0.0025 as shown in figure 1.1 and 1/2. The sequence of taking snapshots was from position 1 to position 4 in each elevation. Based on the built world frame, the fiducials centroid coordinates with

respect to (w/rt) world frame are knowns and listed in Table 1.2. The orientation of camera frame was assumed to be aligned with world frame.

Table 1.1 24 points pixels in left and right cameras

Left Camera Pixel		Right Camera Pixel	
U pixel	V pixel	U pixel	V pixel
445.324	144.365	497.049	142.883
441.884	388.443	493.499	385.394
193.438	376.058	248.114	373.613
209.934	131.556	264.212	130.028
453.546	139.663	508.27	138.364
451.986	388.364	506.431	385.311
192.678	379.228	250.592	376.69
203.851	121.679	261.276	120.243
461.462	129.613	519.196	128.964
451.995	397.915	509.829	394.438
179.349	382.101	241.013	379.502
193.447	106.608	254.73	105.634
477.192	109.642	538.088	109.223
464.873	386.816	526.427	383.706
164.489	384.872	230.586	382.458
182.47	89.2744	248.294	88.5892
488.179	95.9831	552.613	96.1822
478.568	397.432	543.748	393.622
161.856	387.238	232.445	384.379
183.753	78.3656	253.627	77.4641
485.126	87.3354	554.039	87.5052
480.265	398.193	550.607	394.68
150.345	395.706	226.242	392.551
173.021	64.0857	247.877	63.3438

Table 1.2 24 points coordinates w/rt world frame

X coordinate	Y coordinate	Z coordinate
0.0254	0	0.025
0.0254	0.0254	0.025
0	0.0254	0.025
0	0	0.025
0.0254	0	0.02
0.0254	0.0254	0.02
0	0.0254	0.02
0	0	0.02
0.0254	0	0.015
0.0254	0.0254	0.015
0	0.0254	0.015
0	0	0.015
0.0254	0	0.01
0.0254	0.0254	0.01
0	0.0254	0.01
0	0	0.01
0.0254	0	0.005
0.0254	0.0254	0.005
0	0.0254	0.005
0	0	0.005
0.0254	0	0
0.0254	0.0254	0
0	0.0254	0
0	0	0

The geometric relationship between world frame and two camera frames are shown in figure 1.2.

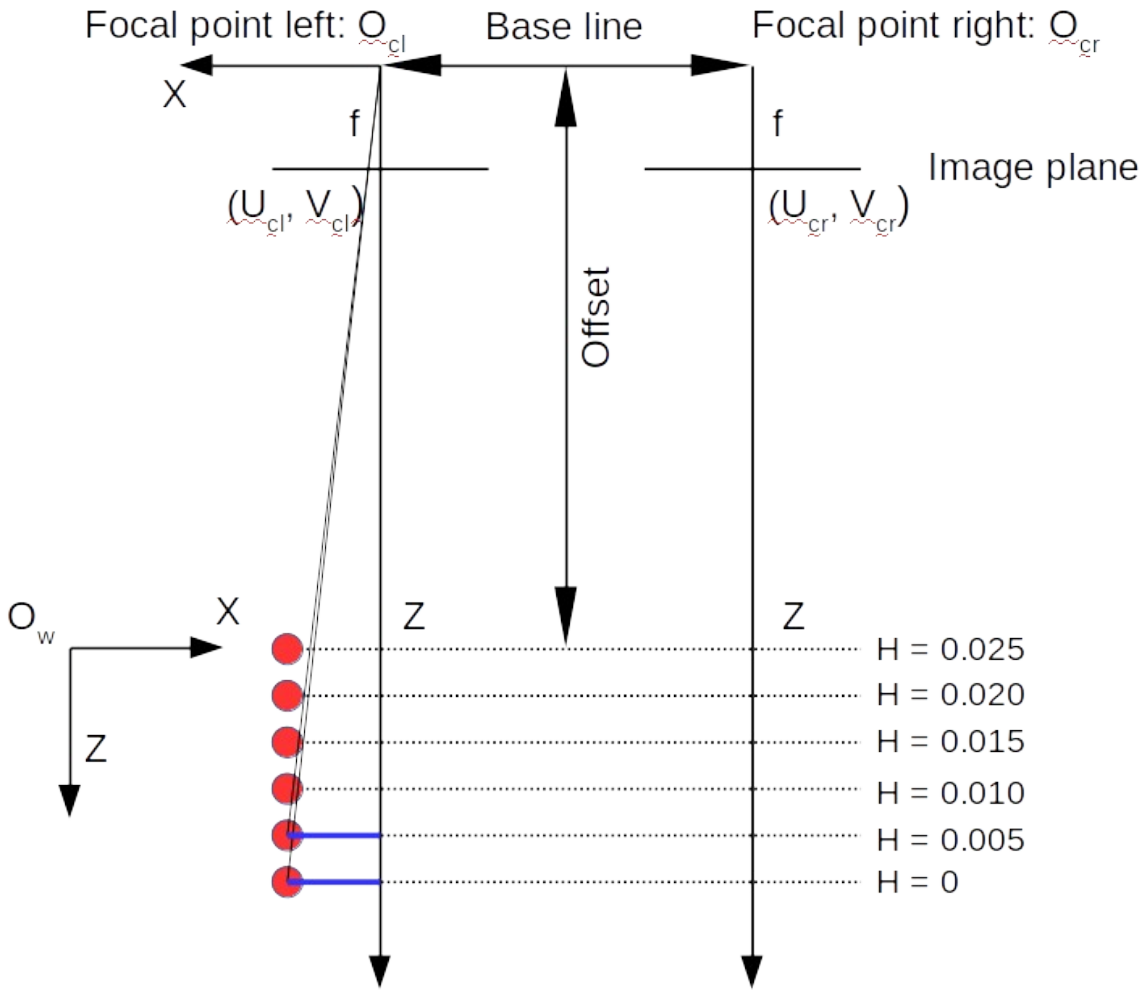

Figure 1.2 Geometric relationship between unknowns

2. Equations to solve focal length, offset and delta theta

Since each points Z coordinate in world frame are knowns, so that each point Z coordinate in camera frame can be found by relationship as shown in Eqn 2.1.

$$P_{ij,z/w} + offset \approx P_{ij,z/c} \left(i = 1,2,..,6,\ j = 1,2,3,4\right) \qquad \text{(Eqn 2.1)}$$

In Eqn 2.1

i: number of layers

j: number of points on each layer

From there each point coordinates in world frame have the following relationship with coordinates in camera frame.

$$P_{ij/w} = {}^{w}R_c \begin{bmatrix} (U_{ij} - U_c)/f_x \\ (V_{ij} - V_c)/f_x \\ 1 \end{bmatrix} (P_{ij,z/w} + offset) + O_{c/w} \quad \text{(Eqn 2.2)}$$

In Eqn 2.2

$O_{c/w}$: Origin of camer frame expressed in world frame

${}^{w}R_c$: Rotaion matrix from camera frame to world frame

Since in each layer the z coordinate of each point either in camera frame of world frame is same, in each layer using position 1 and 2 coordinates (pixels) minus position 4 and 3 coordinates (pixels), respectively, consequently two delta_x and two delta_u values were obtained. In the same way, using position 2 and 3 coordinates (pixels) minus position 1 and 4 coordinates (pixels), respectively, two delta_y and two delta_v values were also obtained. By now, Eqn 2.2 evolved to following format.

$$\begin{bmatrix} \Delta P_{ij,x} \\ \Delta P_{ij,y} \end{bmatrix} = \begin{bmatrix} 1 & -\delta\theta_z \\ \delta\theta_z & 1 \end{bmatrix} \begin{bmatrix} \Delta U_{ij} \\ \Delta V_{ij} \end{bmatrix} ((P_{ij,z/w} + offset)/f) \ (i = 1,2,\ldots,6 \ j = 1,2) \quad \text{(Eqn 2.3)}$$

In Eqn 2.3

i: number of layers

j: number of points coordinates (pixels) difference

By assuming camera frame is aligned with respect to world frame, the rotation matrix is identity, so by expanding Eqn 2.3, the unknowns offset and f is able to be found.

3. Results

For the first case, the focal length is set to two different values between left and right side cameras, one is fx and the other is fy. Also, the delta theta was assumed to be zero at first, thus 48 linear equations were found to solve 3 unknowns, fx, fy and offset. The result is list in Table 2.1

Table 2.1 Focal length and offset

Fx (unit: pixel)	Fy (unit: pixel)	Offset (unit: mm)
883	877	68.3

Base on the found Fx, Fy, and offset, putting those values back to Eqn 2.3, four delta theta was obtained, corresponding to left camera X/Y and right camera X/Y. Using RMS(root mean square) method, $\delta\theta_z = 0.006$.

Another way to have delta theta estimation was to use $\Delta U_{ij}/\Delta V_{ij}$, so that there will be 12 such ratios for each camera, and the best fit delta theta for left camera is $\delta\theta_z = 0.053$ and $\delta\theta_z = 0.05$ for right camera. Because of $\delta\theta_z$ is so small, so that $\theta_z \approx 0.05$. Again, using this estimated $\delta\theta_z$ to recalculate focal length and offset, and the estimated values were

found as list in Table 2.2.

Table 2.2 Focal length and offset, when $\delta\theta_z = 0.05$

Fx (unit: pixel)	Fy (unit: pixel)	Offset (unit: mm)
837	920	68.3

Compared with the value list Table 2.1, large difference was found except offset. In same way, focal length and offset values in Table 2.2 were again replaced in Eqn 2.3 and this time, rotation matrix was replaced as, and the unknowns became to be $\cos(\theta_z)$ and $\sin(\theta_z)$.

$$ {}^{w}R_c = \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) \\ \cos(\theta_z) & \sin(\theta_z) \end{bmatrix} \qquad \text{(Eqn 3.1)} $$

The result was listed in Table 2.3.

Table 2.3 $\delta\theta_z$ *values*

	Left Camera	Right Camera
$\cos(\theta_z)$	0.9986	0.0522
$\sin(\theta_z)$	0.9988	0.0486
Average	0.9987	0.0504

Thus the system is self consistent. Further, the assumption that camera frame is aligned with world frame is acceptable.

4. Solving central pixels

Finding central pixels is basically to solve geometric relationships found by similar triangles, as shown in figure 4.1

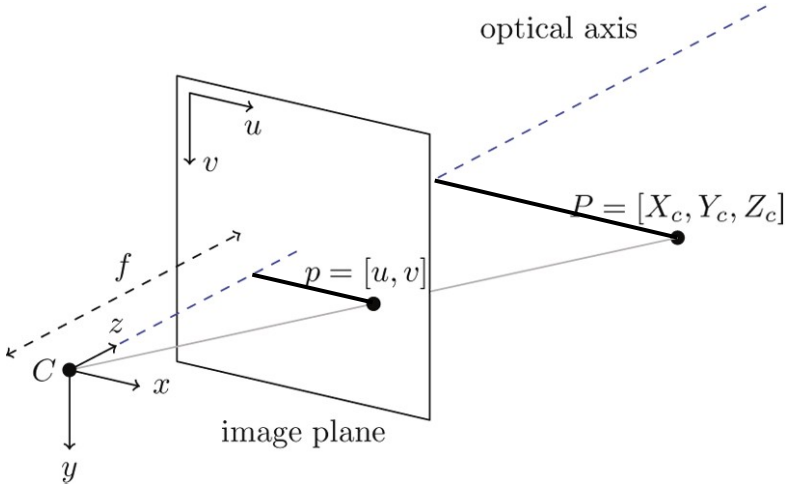

Figure 4.1 Geometric relationship of similar triangle

Based on figure 4.1, and figure 1.1 and 1.2, the following relationship can be deduced as expressed by Eqn 4.1

$$\begin{aligned}(U_{i1}-U_c)/f_x &= P_{11,x/c}/P_{11,z/c}\\ (U_{i2}-U_c)/f_x &= P_{i2,x/c}/P_{i2,z/c} = P_{11,x/c}/P_{12,z/c}\\ (U_{i3}-U_c)/f_x &= P_{i3,x/c}/P_{i3,z/c} = (P_{11,x/c}-\Delta x)/P_{i3,z/c}\\ (U_{i4}-U_c)/f_x &= P_{i4,x/c}/P_{i4,z/c} = (P_{11,x/c}-\Delta x)/P_{i4,z/c}\end{aligned} \quad i=1,2,\ldots,6 \qquad \text{(Eqn 4.1)}$$

In Eqn 4.1, the approximation is that $P_{11,x/c}=P_{i1,x/c}=P_{i2,x/c}$ and $P_{i4,x/c}=P_{i3,x/c}$ for each elevation, also $P_{i1,x/c}-P_{i4,x/c}=1$ and $P_{i2,x/c}-P_{i3,x/c}=1$, thus 24 linear equations were built, two unknowns need to be found, $P_{11,x/c}$ and U_c . Same method applied to Y and V direction, Eqn 4.2 were built.

$$\begin{aligned}(V_{i1}-V_c)/f_y &= P_{11,y/c}/P_{11,z/c}\\ (V_{i2}-V_c)/f_y &= P_{i2,y/c}/P_{i2,z/c} = (P_{11,y/c}+\Delta y)/P_{12,z/c}\\ (V_{i3}-V_c)/f_y &= P_{i3,y/c}/P_{i3,z/c} = (P_{11,y/c}+\Delta y)/P_{i3,z/c}\\ (V_{i4}-V_c)/f_y &= P_{i4,y/c}/P_{i4,z/c} = P_{11,y/c}/P_{i4,z/c}\end{aligned} \quad i=1,2,\ldots,6 \qquad \text{(Eqn 4.2)}$$

In Eqn 4.1, the approximation is that $P_{11,y/c}=P_{i1,y/c}=P_{i4,y/c}$ and $P_{i2,y/c}=P_{i3,y/c}$ for each elevation, also $P_{i2,y/c}-P_{i1,y/c}=1$ and $P_{i3,y/c}-P_{i4,y/c}=1$, thus 24 linear equations were built, two unknowns need to be found, $P_{11,y/c}$ and V_c . After applying these to equations to left and right camera, central pixels and $P_{11,x/c}$ and $P_{11,y/c}$ were found. According to $P_{11,x/c}$ and $P_{11,y/c}$, the origin of left camera frame and right camera frame w/rt world frame can also be determined. The results in Table 4.1 was based on results from Table 2.1. Another set of results was also listed in Table 4.2 which is based on focal length and offset from Table 2.2.

Table 4.1 Central pixels and point coordinates

	Left Camera	Right Camera
Uc	321	321
Vc	333	330
$P_{11,x/c}$	0.013	0.0186
$P_{11,y/c}$	-0.0204	-0.0203
Origin's X coordinate wrt World frame	0.0124	0.0068
Origin's Y coordinate wrt World frame	0.0204	0.0203

Table 4.2 Central pixels and point coordinates

	Left Camera	Right Camera
Uc	321	321
Vc	333	330
$P_{11,x/c}$	0.013	0.0189
$P_{11,y/c}$	-0.02	-0.0199

Origin's X coordinate wrt World frame	0.0124	0.0065
Origin's Y coordinate wrt World frame	0.02	0.0199

According to Table 4.1 and Table 4.2, the results were very close.

5. Solving base line

To get base line, 24 disparities were found in both cameras, so that base line need to fit 24 equations by using least squares method.

$$baseline = P_{ij,z/c} * [(U_{ij/L} - U_{cl}) - (U_{ij/R} - U_{cr})]/f \quad i = 1,2,\ldots,6 \quad j = 1,2,\ldots,4 \qquad \text{(Eqn 5.1)}$$

After solving linear equations, $baseline = -0.0056$, base on either Table 2.1 or Table 2.2.

To check the validity of baseline, 24 points x coordinates in both left camera frame and right camera frame were obtained, as shown in Table 5.1 and Table 5.2. Then using x coordinates in left camera minus baseline, the estimated x coordinates in right camera were gained. Finally comparing this estimated x coordinates with x coordinates in right camera obtained at first step, the errors was small.

Table 5.1 24 points coordinates w/rt camera frame and error (based on results from Table 2.1 and Table 4.1)

0.0132	0.0186	0.000170906
0.0128	0.0183	0.0001825304
-0.0134	-0.0077	-0.0001409451
-0.0117	-0.006	-9.89E-005
0.0133	0.0188	0.0001636617
0.0131	0.0186	1.92E-004
-0.0128	-0.007	-0.0001553824
-0.0117	-0.0059	-0.0001064756
0.0133	0.0187	1.89E-004
0.0124	0.0178	0.0001799087
-0.0133	-0.0075	-0.0001814565
-0.012	-0.0062	-0.0001455087
0.0139	0.0193	2.36E-004
0.0128	0.0182	0.000177277
-0.0139	-0.008	-0.0002256354
-0.0123	-0.0064	-0.0002014234
0.0139	0.0193	2.87E-004
0.0131	0.0185	2.25E-004
-0.0132	-0.0073	-0.0002245083
-0.0114	-0.0056	-0.0001651447
0.0127	0.018	3.05E-004
0.0123	0.0178	1.94E-004
-0.0132	-0.0073	-0.0002356368
-0.0114	-0.0056	-0.0001551015

Table 5.3 24 points coordinates w/rt camera frame and error (based on results from Table 2.2 and Table 4.2)

X coordinates in left camera frame	X coordinates in right camera frame	Error
0.0139	0.0196	-0.0001184714
0.0135	0.0192	-0.000106218
-0.0142	-0.0081	-0.0004471966
-0.0123	-0.0063	-4.03E-004
0.014	0.0198	-0.0001261077
0.0138	0.0196	-9.67E-005
-0.0135	-0.0074	-0.0004624151
-0.0123	-0.0063	-0.000410862
0.014	0.0197	-9.90E-005
0.0131	0.0188	-0.0001089816
-0.0141	-0.0079	-0.0004899001
-0.0127	-0.0066	-0.0004520071
0.0146	0.0203	-5.02E-005
0.0135	0.0192	-0.0001117557
-0.0146	-0.0084	-0.0005364695
-0.0129	-0.0068	-0.0005109474
0.0147	0.0203	3.40E-006
0.0138	0.0195	-6.19E-005
-0.0139	-0.0077	-0.0005352814
-0.012	-0.0059	-0.0004727056
0.0134	0.019	2.25E-005
0.013	0.0187	-9.40E-005
-0.0139	-0.0077	-0.0005470121
-0.012	-0.0059	-0.000462119

6.Validation

By now, all intrinsic values were got, so that validation was carried out by comparing the transformed points coordinates in camera frame to world frame between points coordinates in world frame as shown in Table 1.2. This comparison was performed between X and Y coordinate. And the results is list below. From the tables below, the errors is acceptable.

Table 6.1 Points coordinates comparison btw left camera frame with world frame based on results from Table 2.1

X coord from left cf to wf	X coord in world frame	Y coord in world frame	Y coord from left cf to wf	Error in X dir	Error in Y dir
0.0254	0.0256	0	0.0003	-0.0002	-0.0003
0.0254	0.0252	0.0254	0.0263	0.0002	-0.0009
0	-0.001	0.0254	0.025	0.001	0.0004
0	0.0007	0	-0.001	-0.0007	0.001
0.0254	0.0257	0	0.0009	-0.0003	-0.0009
0.0254	0.0256	0.0254	0.026	-0.0002	-0.0006
0	-0.0004	0.0254	0.0251	0.0004	0.0003
0	0.0007	0	-0.0009	-0.0007	0.0009
0.0254	0.0257	0	0.0011	-0.0003	-0.0011
0.0254	0.0248	0.0254	0.0266	0.0006	-0.0012
0	-0.0009	0.0254	0.0251	0.0009	0.0003
0	0.0004	0	-0.0011	-0.0004	0.0011
0.0254	0.0263	0	0.0005	-0.0009	-0.0005
0.0254	0.0252	0.0254	0.0252	0.0002	0.0002
0	-0.0014	0.0254	0.025	0.0014	0.0004
0	0.0002	0	-0.0014	-0.0002	0.0014
0.0254	0.0263	0	0.0006	-0.0009	-0.0006
0.0254	0.0255	0.0254	0.0258	-0.0001	-0.0004
0	-0.0008	0.0254	0.0249	0.0008	0.0005
0	0.0011	0	-0.0009	-0.0011	0.0009
0.0254	0.0251	0	0.0013	0.0003	-0.0013
0.0254	0.0248	0.0254	0.0255	0.0006	-0.0001
0	-0.0008	0.0254	0.0253	0.0008	0.0001
0	0.001	0	-0.0005	-0.001	0.0005

Table 6.2 Points coordinates comparison btw right camera frame with world frame based on results from Table 2.1

X coord from left cf to wf	X coord in world frame	Y coord in world frame	Y coord from left cf to wf	Error in X dir	Error in Y dir
0.0254	0.0254	0	0.0004	0	-0.0004
0.0254	0.0251	0.0254	0.0262	0.0003	-0.0008
0	-0.0009	0.0254	0.025	0.0009	0.0004
0	0.0008	0	-0.001	-0.0008	0.001
0.0254	0.0256	0	0.001	-0.0002	-0.001
0.0254	0.0254	0.0254	0.0259	0	-0.0005
0	-0.0002	0.0254	0.025	0.0002	0.0004
0	0.0009	0	-0.0008	-0.0009	0.0008
0.0254	0.0255	0	0.0012	-0.0001	-0.0012
0.0254	0.0247	0.0254	0.0264	0.0007	-0.001
0	-0.0007	0.0254	0.025	0.0007	0.0004
0	0.0006	0	-0.001	-0.0006	0.001
0.0254	0.0261	0	0.0006	-0.0007	-0.0006
0.0254	0.0251	0.0254	0.0251	0.0003	0.0003
0	-0.0012	0.0254	0.025	0.0012	0.0004
0	0.0004	0	-0.0012	-0.0004	0.0012
0.0254	0.0261	0	0.0008	-0.0007	-0.0008
0.0254	0.0253	0.0254	0.0256	0.0001	-0.0002
0	-0.0005	0.0254	0.0249	0.0005	0.0005
0	0.0012	0	-0.0008	-0.0012	0.0008
0.0254	0.0249	0	0.0014	0.0005	-0.0014
0.0254	0.0246	0.0254	0.0253	0.0008	0.0001
0	-0.0005	0.0254	0.0252	0.0005	0.0002
0	0.0012	0	-0.0005	-0.0012	0.0005

Table 6.3 Points coordinates comparison btw right camera frame with world frame based on results from Table 2.2

X coord from left cf to wf	X coord in world frame	Y coord in world frame	Y coord from left cf to wf	Error in X dir	Error in Y dir
0.0254	0.0263	0	0.0009	-0.0009	-0.0009
0.0254	0.0259	0.0254	0.0257	-0.0005	-0.0003
0	-0.0018	0.0254	0.0244	0.0018	0.001
0	0.0001	0	-0.0004	-0.0001	0.0004
0.0254	0.0264	0	0.0015	-0.001	-0.0015
0.0254	0.0263	0.0254	0.0254	-0.0009	0
0	-0.0011	0.0254	0.0245	0.0011	0.0009
0	0.0001	0	-0.0002	-0.0001	0.0002
0.0254	0.0264	0	0.0016	-0.001	-0.0016
0.0254	0.0255	0.0254	0.0259	-0.0001	-0.0005
0	-0.0016	0.0254	0.0245	0.0016	0.0009
0	-0.0002	0	-0.0004	0.0002	0.0004
0.0254	0.027	0	0.001	-0.0016	-0.001
0.0254	0.0259	0.0254	0.0246	-0.0005	0.0008
0	-0.0022	0.0254	0.0245	0.0022	0.0009
0	-0.0005	0	-0.0007	0.0005	0.0007
0.0254	0.0271	0	0.0012	-0.0017	-0.0012
0.0254	0.0262	0.0254	0.0252	-0.0008	0.0002
0	-0.0015	0.0254	0.0244	0.0015	0.001
0	0.0004	0	-0.0002	-0.0004	0.0002
0.0254	0.0258	0	0.0018	-0.0004	-0.0018
0.0254	0.0254	0.0254	0.0249	0	0.0005
0	-0.0015	0.0254	0.0247	0.0015	0.0007
0	0.0004	0	0.0001	-0.0004	-0.0001

Table 64 Points coordinates comparison btw right camera frame with world frame based on results from Table 2.2

X coord from left cf to wf	X coord in world frame	Y coord in world frame	Y coord from left cf to wf	Error in X dir	Error in Y dir
0.0254	0.0261	0	0.001	-0.0007	-0.001
0.0254	0.0257	0.0254	0.0256	-0.0003	-0.0002
0	-0.0016	0.0254	0.0244	0.0016	0.001
0	0.0002	0	-0.0003	-0.0002	0.0003
0.0254	0.0263	0	0.0016	-0.0009	-0.0016
0.0254	0.0261	0.0254	0.0253	-0.0007	0.0001
0	-0.0009	0.0254	0.0244	0.0009	0.001
0	0.0002	0	-0.0002	-0.0002	0.0002
0.0254	0.0262	0	0.0018	-0.0008	-0.0018
0.0254	0.0253	0.0254	0.0258	0.0001	-0.0004
0	-0.0014	0.0254	0.0244	0.0014	0.001
0	-0.0001	0	-0.0004	0.0001	0.0004
0.0254	0.0268	0	0.0012	-0.0014	-0.0012
0.0254	0.0257	0.0254	0.0245	-0.0003	0.0009
0	-0.0019	0.0254	0.0244	0.0019	0.001
0	-0.0003	0	-0.0006	0.0003	0.0006
0.0254	0.0268	0	0.0013	-0.0014	-0.0013
0.0254	0.026	0.0254	0.025	-0.0006	0.0004
0	-0.0012	0.0254	0.0243	0.0012	0.0011
0	0.0006	0	-0.0002	-0.0006	0.0002
0.0254	0.0255	0	0.0019	-0.0001	-0.0019
0.0254	0.0252	0.0254	0.0247	0.0002	0.0007
0	-0.0012	0.0254	0.0246	0.0012	0.0008
0	0.0006	0	0.0002	-0.0006	-0.0002

7. Conclusion

Based on the errors estimation in chapter 6, the intrinsic properties from Table 2.1 and Table 4.1, also Table 2.2 and Table 4.2 are both acceptable.