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A Method to Regulate the Torque of Flexible-joint Manipulators with Velocity Control Inputs

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Abstract— **Nowadays, robotic systems make use of compliant joint actuation to provide safe human-robot physical interactions, and in general, to improve the friendliness of the mechanism. However, note that with the introduction of these passive flexible elements, the actuators (which input the control actions) can not explicitly set the driving torque of the joints; for these types of systems, the difference between the position of the motor and the position of the joint determines the driving elastic torque. To cope with this problem, in this paper we present a control method to indirectly regulate the driving torque of flexible-joint manipulators. For that we first propose a new mathematical model for manipulators with velocity control inputs; next we derive a servo-controller which can asymptotically regulate the driving torque while online estimating unknown parameters. We prove the stability of the numerical algorithm with Ljapunov theory. To validate the proposed method, we conduct an experimental study with a simple 1-DOF manipulator in a compliant force regulation task.**

I. Introduction

Robot manipulators are no longer exclusively used in the well-structured and typically repetitive industrial scenarios. Nowadays, manipulators are required to safely perform physical interactions in the dynamically changing and unstructured human environments [1]. Some of these new applications are for example robotic surgery [2], elderly care [3], rehabilitation [4], or entertainment [5]. A common approach to improve the physical interactiveness of the manipulator's mechanical structure is to make use of a flexible joint actuation [6]. Flexible joints provide the manipulator with a valuable compliant behaviour, however, they also impose some complications to the system and its controller design.

The elastic driving torque of a flexible-joint manipulator is determined by the angular position of the actuator and the position of the joint itself. Therefore, when these passive elements are used in the mechanical structure, the actuators (which provide the control inputs to the system) can no longer *explicitly* impose/set the driving torque to the joints, as opposed to a traditional rigid-joint design. In this paper, we aim to mathematically model this situation and to develop stable control methods that can asymptotically regulate the driving torque of these types of robot manipulators.

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A. Related Work

In the last two decades, many research works have addressed the problem of the control of flexible joint manipulators. Comprehensive surveys about the different control methods for these systems are reported in [7], [8]. We remark that most works in the literature mathematically model and design the control law considering effort inputs from the actuator, i.e. the control variable represents the driving torque (or current in some cases) of the motor; see for example [9]–[20]. However, note that many—if not most—commercially available robot manipulators and its respective control systems do not provide an explicit torque control interface. In order to access to the torque/current loop of an industrial manipulator, requires to open the architecture of the low-level servo-controller; this task is typically difficult and not always available, see [21], [22]. Despite this situation, there are very few works that address this control problem using velocity inputs.

B. Contribution

To contribute to this problem, in this paper we develop a method to control flexible-joint manipulators but considering velocity inputs from the actuators; this feature contrast with most works in the literature. The control objective of our new method is to regulate the driving torque of the manipulator's joint. For that, we first propose a new mathematical model to represent the dynamics of a flexible-joint manipulator with velocity (kinematic) controls; we derive this model under the energy-motivated Hamiltonian framework [23]. Next, we develop a control approach to asymptotically regulate the driving elastic torque on the joints, which also has the capability to estimate unknown parameters of a model; we prove the stability of this method using Ljapunov theory. To validate the controller, we report experimental results of a one degree-of-freedom robot manipulator in a compliant force regulation task, similar to [24]–[26].

C. Organisation

The rest of this manuscript is organised as follows: section II presents the mathematical modelling; in section III we develop the proposed control method; section IV presents the conducted experimental study; in section V we give final conclusions and future work.

II. Modelling

This section presents the mathematical preliminaries of the control problem addressed along the paper.

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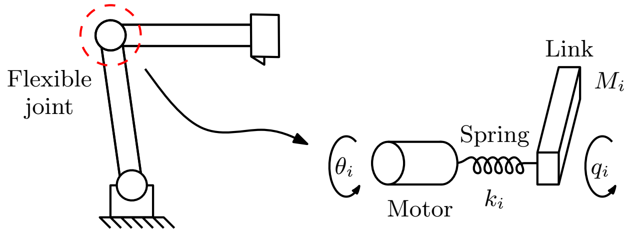

Fig. 1. Conceptual representation of a flexible-joint robot manipulator. In our model the flexible joint is composed of a motor, a rigid link, and an elastic element that connects these two.

A. Notation

In this work we employ the following notation: column vectors and matrices are denoted by bold small and bold capital letters, respectively; the identity matrix is denoted by $\mathbf{I}_P \in \mathbb{R}^{P\times P}$; the square null matrix and null vector are denoted by $\mathbf{0}_P \in \mathbb{R}^{P\times P}$ and $\mathbf{0} \in \mathbb{R}^P$, respectively; the weighted Euclidean norm is denoted by $\|\mathbf{z}\|_{\mathbf{A}} = \sqrt{\mathbf{z}^\mathsf{T}\mathbf{A}\mathbf{z}}$, for $\mathbf{A}$ as a symmetric matrix of appropriate dimensions.

B. Flexible Actuation

Consider a serial non-redundant manipulator with N revolute joints. The N links of this manipulator are mechanically connected to the actuator by a rotational elastic element. We denote the vectors of angular positions of the joints and the motors by $\mathbf{q} \in \mathbb{R}^N$ and $\boldsymbol{\theta} \in \mathbb{R}^N$, respectively. See Fig. 1 for a conceptual representation of this system.

The driving (elastic) torque $\boldsymbol{\tau} \in \mathbb{R}^N$ of the manipulator is given by

$$\boldsymbol{\tau} = -\mathbf{K}(\mathbf{q} - \boldsymbol{\theta}), \tag{1}$$

where the positive matrix $\mathbf{K} = \mathrm{diag}(k_1, \ldots, k_N) \in \mathbb{R}^{N\times N}$ represents the spring stiffness.

C. Manipulator Rigid Dynamics

In this section we derive the standard 'rigid' dynamic model of robot manipulators. We develop the dynamical model using the Hamiltonian framework, see [26], [27]. The energy storage function (i.e. the Hamiltonian) of the serial manipulator is given by

$$H = U(\mathbf{q}) + \frac{1}{2}\|\mathbf{p}\|^2_{\mathbf{M}^{-1}}, \tag{2}$$

where $U(\mathbf{q}) \in \mathbb{R}$ represents a positive definite potential energy function (modelling e.g. gravitational forces and/or physical interaction with a compliant environment); the matrix $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{N\times N}$ represents the symmetric and positive definite mass matrix; the vector $\mathbf{p} = \mathbf{M}(\mathbf{q})\dot{\mathbf{q}} \in \mathbb{R}^N$ denotes the manipulator's momenta. The Hamiltonian dynamic equations of the manipulator are given by

$$\begin{bmatrix}\dot{\mathbf{q}}\\ \dot{\mathbf{p}}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_N & \mathbf{I}_N\\ -\mathbf{I}_N & -\mathbf{B}\end{bmatrix}\begin{bmatrix}\frac{\partial H}{\partial \mathbf{q}}^\mathsf{T}\\ \frac{\partial H}{\partial \mathbf{p}}^\mathsf{T}\end{bmatrix} + \begin{bmatrix}\mathbf{0}\\ \boldsymbol{\tau}\end{bmatrix}, \tag{3}$$

where the symmetric and positive matrix $\mathbf{B} \in \mathbb{R}^{N\times N}$ models energy dissipation, introduced e.g. by joint bearings.

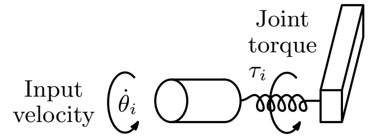

Fig. 2. Conceptual representation of the torque generated by the velocity control inputs. In contrast with rigid-joint manipulators, the driving torque is determined by the difference between the angular position of the joint and the position of the motor.

D. Velocity-controlled Manipulator

In contrast with most methods in the literature to control flexible-joint manipulators, in this paper we consider the angular velocity of the motor as the control input variable, which we denote by $\mathbf{v} = \dot{\boldsymbol{\theta}} \in \mathbb{R}^N$ (see Fig. 2). We model this system with the following third order dynamic equations

$$\begin{bmatrix}\dot{\boldsymbol{\tau}}\\ \dot{\mathbf{q}}\\ \dot{\mathbf{p}}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_N & \mathbf{0}_N & -\mathbf{K}\\ \mathbf{0}_N & \mathbf{0}_N & \mathbf{I}_N\\ \mathbf{K} & -\mathbf{I}_N & -\mathbf{B}\end{bmatrix}\begin{bmatrix}\frac{\partial G}{\partial \boldsymbol{\tau}}^\mathsf{T}\\ \frac{\partial G}{\partial \mathbf{q}}^\mathsf{T}\\ \frac{\partial G}{\partial \mathbf{p}}^\mathsf{T}\end{bmatrix} + \begin{bmatrix}\mathbf{K}\mathbf{v}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}, \tag{4}$$

with energy storage function $G \in \mathbb{R}$

$$G = \frac{1}{2}\|\boldsymbol{\tau}\|^2_{\mathbf{K}^{-1}} + U(\mathbf{q}) + \frac{1}{2}\|\mathbf{p}\|^2_{\mathbf{M}^{-1}}, \tag{5}$$

whose time derivative satisfies the power balance

$$\dot{G} = -\|\mathbf{p}\|^2_{\mathbf{B}} + \boldsymbol{\tau}^\mathsf{T}\mathbf{v}. \tag{6}$$

E. Parametrisation of a Feedback Signal

Consider a signal $\mathbf{y} \in \mathbb{R}^N$ of interest, which is directly measured by a feedback sensor. In this work, we assume that the analytical structure of this vector is exactly known and can be expressed as

$$\mathbf{y} = \mathbf{F}\mathbf{a}, \tag{7}$$

where $\mathbf{F}(\boldsymbol{\tau}, \mathbf{q}, \mathbf{p}) \in \mathbb{R}^{N\times P}$ denotes a known regression matrix, and $\mathbf{a} \in \mathbb{R}^P$ represents a vector of unknown parameters. The vector $\mathbf{y}$ can be used to represent, for example, the feedback from a force/moment transducer or the image (pixel) measurements of a vision sensor; in latter sections we provide a simple case of study to exemplify the utility of this linear parametrisation.

III. Controller Design

This section describes the proposed control method for flexible-joint manipulators. To clarify the problem being addressed in this work, consider the following statement:

Problem. Design a state feedback kinematic controller which can asymptotically regulate the elastic joint torque $\boldsymbol{\tau}$ while simultaneously estimate the vector of unknown parameters $\mathbf{a}$.

A. Low-level Adaptive Controller

We now present the design of a stable torque regulator; this method is intended for set-point tasks, i.e. for a constant or *slowly* varying target $\boldsymbol{\tau}_d \in \mathbb{R}^N$. To implement the feedback velocity control input, we assume that the stiffness matrix $\mathbf{K}$ is exactly known such that the elastic joint torque (1) can be computed from the angular position measurements $\mathbf{q}$ and $\boldsymbol{\theta}$. Consider the feedback velocity input

$$\mathbf{v} = -\gamma \Delta \boldsymbol{\tau}, \tag{8}$$

where $\Delta\boldsymbol{\tau} = \boldsymbol{\tau} - \boldsymbol{\tau}_d \in \mathbb{R}^N$ represents the torque error and $\gamma > 0$ denotes a feedback scalar.

In our method, we dynamically compute a vector of estimated parameters $\widehat{\mathbf{a}} \in \mathbb{R}^P$ with the following update rule:

$$\dot{\widehat{\mathbf{a}}} = -\varphi \mathbf{F}^\mathsf{T} \left(\mathbf{F}\widehat{\mathbf{a}} - \mathbf{y} \right), \tag{9}$$

where the scalar $\varphi > 0$ represents a positive learning gain; the estimated parameters $\widehat{\mathbf{a}}$ are used in the design of the high-level motion controller, to be presented shortly.

We now analyse the stability of the low-level adaptive control method.

Proposition 1. *For a stiff-enough potential function $U(\mathbf{q})$, the velocity control input* (8) *with adaptive vector parameters* (9) *enforce a passive closed-loop dynamical system which asymptotically minimises $\Delta\boldsymbol{\tau}$.*

Proof. Substitution of the input (8) into the plant system (4) along with the numerical state (9) yields the extended Hamiltonian system

$$\dot{\mathbf{x}} = \mathbf{Q}(\mathbf{x}) \frac{\partial J^\mathsf{T}}{\partial \mathbf{x}}(\mathbf{x}), \tag{10}$$

with an extended state vector $\mathbf{x} \in \mathbb{R}^{4N}$ defined as

$$\mathbf{x} = \begin{bmatrix} \widehat{\mathbf{a}}^\mathsf{T} & \boldsymbol{\tau}^\mathsf{T} & \mathbf{q}^\mathsf{T} & \mathbf{p}^\mathsf{T} \end{bmatrix}^\mathsf{T}, \tag{11}$$

a gyroscopic-dissipation matrix $\mathbf{Q}(\mathbf{x}) \in \mathbb{R}^{4N \times 4N}$ (see [28] for details) defined as

$$\mathbf{Q}(\mathbf{x}) = \begin{bmatrix} -\varphi\mathbf{F}^\mathsf{T}\mathbf{F} & \mathbf{0}_N & \mathbf{0}_N & \mathbf{0}_N \\ \mathbf{0}_N & -\gamma\mathbf{K} & \mathbf{0}_N & -\mathbf{K} \\ \mathbf{0}_N & \mathbf{0}_N & \mathbf{0}_N & \mathbf{I}_N \\ \mathbf{0}_N & \mathbf{K} & -\mathbf{I}_N & -\mathbf{B} \end{bmatrix}, \tag{12}$$

and a closed-loop Hamiltonian function $J \in \mathbb{R}$ defined as

$$J = \frac{1}{2}\|\Delta\mathbf{a}\|^2 + \frac{1}{2}\|\Delta\boldsymbol{\tau}\|^2_{\mathbf{K}^{-1}} + U(\mathbf{q}) - \mathbf{q}^\mathsf{T}\boldsymbol{\tau}_d + \epsilon + \frac{1}{2}\|\mathbf{p}\|^2_{\mathbf{M}^{-1}}, \tag{13}$$

for a parameter's error $\Delta\mathbf{a} = \widehat{\mathbf{a}} - \mathbf{a} \in \mathbb{R}^P$ and an arbitrary scalar ϵ. The positive definiteness of J depends on whether the natural potential energy forces are large-enough around the desired equilibrium point $\mathbf{x}^*$ such that the following is satisfied:

$$U(\mathbf{q}) - \mathbf{q}^\mathsf{T}\boldsymbol{\tau}_d + \epsilon \geq 0. \tag{14}$$

When this condition is fulfilled, the functional $J(\mathbf{x})$ qualifies as a Ljapunov function for the dynamical system (11) since

$$\begin{aligned} \dot{J} &= -\|\Delta\mathbf{a}\|^2_{\mathbf{F}_2} - \gamma\|\Delta\boldsymbol{\tau}\|^2 - \|\mathbf{p}\|^2_{\mathbf{B}}, \\ &\leq 0, \end{aligned} \tag{15}$$

for a matrix $\mathbf{F}_2 = \varphi\mathbf{F}^\mathsf{T}\mathbf{F} \in \mathbb{R}^{P \times P}$. This proves passivity of the total system; asymptotic stability of $\Delta\boldsymbol{\tau}$ directly follows by invoking the Krasovskii–LaSalle principle [29]. ■

Remark 1. From the previous stability analysis, we can see that the parameter adaptation rule (9) can not guarantee the identification of the true parameters (this because $\mathbf{F}_2$ is not necessary full-rank). The update rule (9) is designed with the purpose of minimising the instantaneous model matching error $\mathbf{e} = \mathbf{F}\widehat{\mathbf{a}} - \mathbf{y}$; note that this rule can be re-written as as a gradient descent estimator $\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\mathbf{a}}^\mathsf{T} = -\varphi\frac{\partial}{\partial \widehat{\mathbf{a}}}\mathbf{e}^\mathsf{T}\mathbf{e}$. This type of adaptation law is useful to estimate differential relations, e.g. the online computation of an unknown Jacobian matrix [30].

B. Case of Study: Contact Force Regulation

In most cases, the sole regulation of the elastic joint torque has no clear useful application. Therefore, the proposed method must be used along with a high-level algorithm which computes the desired torque $\boldsymbol{\tau}_d$ to be regulated by the low-level adaptive controller.

As our case of study, in this work we consider the situation where the end-effector of the robot manipulator physically interacts with a compliant environment. Without loss of generality, we model that this lossless environment imposes no dissipative (damping) forces into the system. We assume that the robotic system is instrumented with a force transducer conveniently located at the contact point. We model the vector of force measurements $\mathbf{f} \in \mathbb{R}^N$ with the expression

$$\mathbf{f} = \mathbf{T}\mathbf{r}, \tag{16}$$

where the constant and positive-define matrix $\mathbf{T} \in \mathbb{R}^{N \times N}$ models the environments stiffness; the vector $\mathbf{r}(\mathbf{q}) \in \mathbb{R}^N$ represents the relative (with respect to the undeformed configuration) displacements of the end-effector. See Fig. 3 for a conceptual representation of this constrained scenario.

We use the proposed method to indirectly control the force applied onto the compliant environment. To this end, by defining the feedback signal as $\mathbf{y} = \dot{\mathbf{r}}$ we can exploit the adaptive algorithm to iteratively estimate the force-to-displacement differential relation

$$\begin{aligned} \mathbf{y} &= \mathbf{C}\dot{\mathbf{f}}, \\ &= \mathbf{F}(\mathbf{f}(\mathbf{q}), \mathbf{p})\mathbf{a}, \end{aligned} \tag{17}$$

where the vector of parameters $\mathbf{a}$ is constructed with the elements of the compliance matrix $\mathbf{C} = \mathbf{T}^{-1}$ (note that this matrix provides a Jacobian-like differential relation).

Given a reference contact force $\mathbf{f}_d \in \mathbb{R}^M$, we compute the desired torque reference with the following expression:

$$\boldsymbol{\tau}_d = -\widehat{\mathbf{C}}\lambda \int_0^t \Delta\mathbf{f}(s)\mathrm{d}s, \tag{18}$$

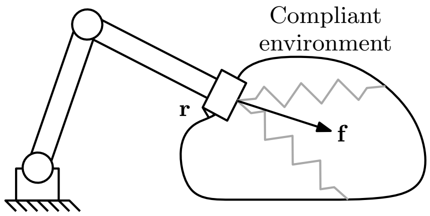

Fig. 3. Conceptual representation of the robot manipulator interacting with a compliant environment. We model this deformable object as a lossless body which has no internal damping, therefore, the reaction forces $\mathbf{f}$ only contribute to the natural potential energy $U(\mathbf{q})$.

where the adaptive matrix $\widehat{\mathbf{C}} \in \mathbb{R}^{N\times N}$ is constructed with the vector of parameters $\widehat{\mathbf{a}}$ as in [31], the scalar $\lambda > 0$ denotes a feedback gain, and $\Delta\mathbf{f} = \mathbf{f} - \mathbf{f}_d \in \mathbb{R}^N$ represents the contact force error. The computation of the torque reference (18) clearly follows a standard integral action approach, which results in smooth and *slowly* varying torque profiles to be regulated by the low-level servo-controller.

IV. Experiments

A. Setup

To test the performance of our control method, we conducted an experimental study with a flexible-joint one degree-of-freedom robot manipulator interacting with a compliant environment. In our study, the robot manipulator (whose rigid link has a length of 0.2 m) physically interacts with a piece of soft foam. To measure the interaction forces with the environment, our system is instrumented with an ATI Mini40 force/torque transducer. To command the angular velocity $\mathbf{v} = \dot{\boldsymbol{\theta}}$ of the motor, we use a Galil DMC-1842 motion controller (configured in speed mode) along with a Maxon 4Q DC motor driver. The control system measures the position of the motor and the joint with a 12 MHz encoder rate. Fig. 4 shows the details of the experimental setup.

To conduct the experiments, we first identified the stiffness gain of the joint by using standard linear regression techniques; the experimentally identified value is $\mathbf{K} = 0.05629$ Nm/deg (note that this value is used for the computation of the elastic joint torque (1)). We implemented the parameter estimator (9) with an initial state vector $\widehat{\mathbf{a}} = [0, \ldots, 0]^\intercal$ and gain $\varphi = 0.001$. The desired joint torque (18) is computed with a gain $\lambda = 50$. We implement the velocity control input with a proportional gain $\gamma = 13$. For this one degree-of-freedom experimental study, we only consider the magnitude of the contact forces, that is, we compute $\mathbf{f} = \|\mathbf{f}_s\| \in \mathbb{R}$, for $\mathbf{f}_s$ as the vector of measured forces in sensor space.

B. Results

For our compliant force regulation case of study, we set the desired force reference as $\mathbf{f}_d = 5$ N. The asymptotic minimisation of the measured force error $\Delta\mathbf{f}$ is shown in Fig. 5. The trajectories of the measured elastic joint torque $\boldsymbol{\tau}$ and its respective target $\boldsymbol{\tau}_d$ are shown in Fig. 6. The profiles shown in this figure experimentally corroborate that the proposed control method can asymptotically regulate with velocity inputs the driving torque of a flexible-joint manipulator. Note that our method is designed for set-point regulation tasks, i.e. for constant or slowly varying references. This, however, does not impose severe constraints to many practical applications that involve physical interactions with deformable environments, where slow motion is typically preferred for safety reasons, see e.g. [30]. Finally, the trajectory of the velocity control input $\mathbf{v}$ computed the expression (8) is shown in Fig. 7.

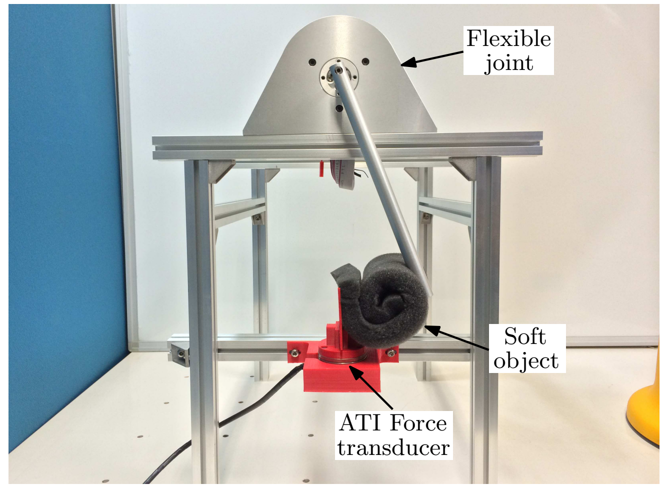

Fig. 4. Robotic setup used for the control experiments. For this study, we use a one degree-of-freedom manipulator with an embedded flexible joint, and to measure the force applied onto the soft object, we use an ATI force transducer.

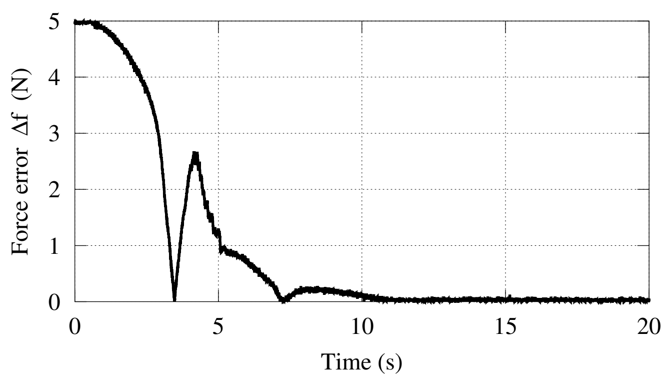

Fig. 5. Magnitude of the measured contact force error $\Delta\mathbf{f}$. In this one degree-of-freedom application, we compute the force $\mathbf{f}$ as the one dimensional norm of the sensor measurements, i.e. $\mathbf{f} = \|\mathbf{f}_s\|$, for $\mathbf{f}_s$ as the vector of forces in sensor space.

V. Conclusions

In this paper we presented a method to control the driving torque of flexible-joint manipulators. For that, we first developed a new mathematical model of the system that considers velocity control inputs; this model was derived using the energy-motivated Hamiltonian framework. Next, we designed a static-state feedback controller that asymptotically regulates the joint torque and adaptively estimates unknown parameters; these parameters can be used to online estimate Jacobian-like matrices. Finally, we experimentally validated the proposed method with a one degree-of-freedom

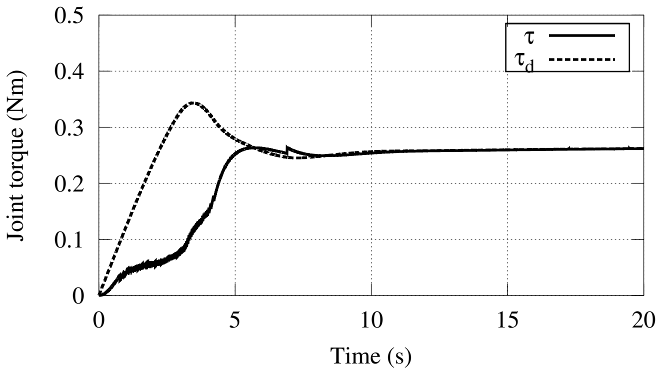

Fig. 6. Profiles of the measured and desired joint torques $\boldsymbol{\tau}$ and $\boldsymbol{\tau}_d$. Note that the integral action (18) compute a slowly varying reference $\boldsymbol{\tau}_d$, a feature needed with our set-point torque regulator.

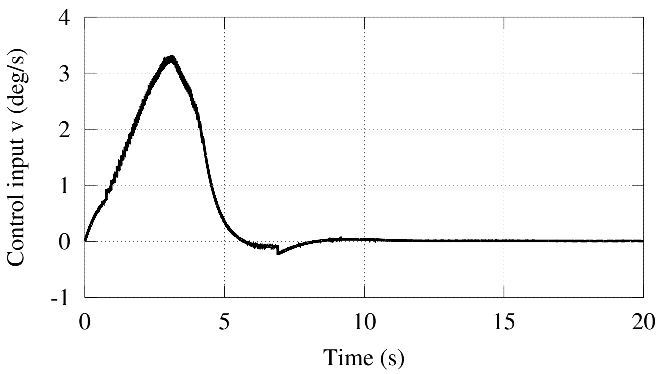

Fig. 7. The velocity control input $\mathbf{v}$ that is computed with the proposed torque regulator during the experiment.

manipulator in a compliant force control application.

Note that in contrast with most controllers for flexible-joint manipulators, our method designs the control strategy based on velocity inputs. We think that the proposed mathematical model provides a useful energy-based framework to design other types of kinematic motion controllers. In this paper we used the parameter update rule to estimate stiffness Jacobian matrix of a compliant environment; this algorithm can also be used, for example to coordinate the manipulator's motion in an image-guided application with uncalibrated cameras.

As future work, we want to test our control method with multi-joint robot manipulators; we are currently working on the development of a planar two degrees-of-freedom flexible-joint manipulator. Additionally, we would like to implement the torque regulator with different cases of study, for example in visual servoing applications or in multi-manipulator cooperative tasks.

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