SW Competency Advanced Exam

Tunnel Construction 28th Sept'16

Problem Statement

- * There are V number of tunnels in parallel. A tunnel is a combination of blocks in horizontal direction. Number of blocks is given as H. So total tunnels will be V and total blocks is V*H.
- * There are two construction machines, one at each end of the tunnel under construction.
- * Each machine will work for a day and will construct one block of the tunnel but only one of the machines will work for a day, both can't work on the same day.
- * A cost is associated with each machine, for working for one day.(C1 and C2 for machine 1 and 2 respectively)
- * For every block there is a factor given as S, which will be multiplied by the cost of one day's work of a machine. So the cost of constructing one block will be S*C1 or S*C2.
- * Additional cost is there if a machine is working for 2 or more consecutive days and is given as R1 and R2.
- * Once we get to know the construction cost associated with all the tunnels(V), we have to select N out of them such that the cost is minimum keeping in mind that there should be at least one tunnel between the chosen tunnels. Distance must at least be 2.
- * A cost is associated with the movement of machines(M1 and M2) based on the distance between the construction sites(Tunnel distance) and is given as (M1*M1 + M2*M2)*D. D is the distance between chosen tunnels.

Output: The minimum cost associated with this construction of N Tunnels.

Inputs:

T as number of test cases, followed by test cases.

Each test case consists of:

N,H,V (in the first line)

V lines follow the first line, with factor S associated with each Block in one horizontal line. (H entries in each of the V lines).

C1,R1,M1(for machine 1)

C2,R2,M2(for machine 2)

Constraints:

1<=N<=5, 3<=H<=500, (N*2-1)<=V<=15, 1<=S<=300, 1<=C<=200, 1<=R<=500, 1<=M<=300

Approach

- Calculate normal construction cost for every possible case by both the machines. By going through every possible case. starting with 0 working days for machine 1 and H for machine 2. Then keep on incrementing number of days for machine 1 and reduce for machine 2. Each time calculate normal cost.
- For every possible case, add the additional cost for a machine working on consecutive days.
- Calculating normal cost is easy, additional cost should be such that the consecutive days for the machines should be minimum. Can be achieved by making them work on alternate days for maximum times.

Note:- The example they gave for minimum cost as:

2->1->2->2 means that machine 1 will work on second day and machine 2 will work on 1^{st} day, 3^{rd} day, 4^{th} day and the 5^{th} day.

This does not mean that machine 1 will construct the second block and machine 2 will construct rest of the blocks.

The machines will construct the blocks in sequence, one machine from one end and the other machine from the other end. They will never cross each other.

This was the most important observation.

So according to given example, minimum additional cost will come for 2 consecutive days. le. Machine 2 working on 3rd 4th and 5th day consecutively.

So we will add the cost for 2 days(4th and 5th).

One more possible case to get minimum additional cost could be

2->2->1->2->2 or 2->2->1->2. All will give same additional cost as the number of consecutive days for machine 2 is same. Each will give additional cost of 2 days.

We can analyze other cases also and can come to a general equation to get the minimum cost.

(D2-D1-1)*R2 or (D1-D2-1)*R1, depending upon which machine has worked for more days.

D1=> days machine 1 has worked, D2=> days machine 2 has worked.

Adding the above calculated cost to the normal cost and checking the minimum gives us the minimum cost to construct one tunnel.

Same way, calculate for all the possible tunnels and finally select N out of them.

Complexity

Requirement: Construct N tunnels

Max length of a tunnel is 500.

To calculate normal cost(S*C), for all the scenarios, total cases = 500*500.

A machine can construct all the bocks, or one machine can construct one block and other can construct the rest, or 2 blocks by machine 1 and rest by machine 2 and so on.....

- 1) So total number cases for one tunnel = 500*500. For Maximum of 15 Tunnels= 15*500*500.
- 2) Now to select N which can at max be 5, we have to calculate every possible case.

Gives us the worst case complexity less than 7^5. (seven possible places for single tunnel)

3) So total complexity can at worst be $< 15*500*500 + 7^5$.





