

Project D Section B3

CS 101

AIM OF THE PROJECT:

Numerical approximation of 1D integrals is used to find the area under the curve. There are a wide range of methods available for computing numerical integration.

TASK AND APPROACH:

We were assigned the task to compute 1D integrals. We used many ways for integration

- i. Riemann sum(left, right and centre) ii. Trapezoidal rule iii. Monte-carlo integration(1D and 2D)
- iv. Simpson rule v. Boole's rule

We plotted the Riemann left, right and centre rectangles, trapezoids and we also plotted random points for monte-carlo on the given curve to make it clear what is going on.

References:

We took reference from T.M.Apostol and the following websites:

- i <http://www.intmath.com/integration/6-simpsons-rule.php>
- ii. http://en.wikipedia.org/wiki/Monte_Carlo_integration
- iii. http://en.wikipedia.org/wiki/Boole's_rule
- iv. http://matplotlib.org/mpl_toolkits/mplot3d/tutorial.html

Working of each function:

i. Riemann sum: We define `riemann_left`, `riemann_right` and `riemann_centre` which takes 4 arguments: `f` (function), `xmin`, `xmax` and `n` (number of rectangles). The entire region is divided into `n` rectangles and the area of the rectangles are found and then summed up to find the value of the integral.

ii. Monte-Carlo integration: We define the function `monte_carlo` which takes 4 arguments: `f` (function), `xmin`, `xmax` and `n` (number of random uniform points). In the function we define a `rectangle_area = (xmax-xmin)*(ymax-ymin)`. The function then takes `n` random points in this rectangle and checks how many of them are above and below the curve. If there are `k` points below the curve then the function computes the integral as `area of the rectangle * probability of getting a point below the curve`.

iv. Boole's rule: We define the function `boole` which takes 4 arguments: `f` (function), `a`, `b`, `k`. It approximates the integral by using the value of `f` at 5 equally spaced points.

v. Trapezoidal rule: We define the function `trapez` which takes 4 arguments: `f` (function), `a`, `b` and `n` (number of trapezoids). It approximates the region under the graph as a trapezoid and calculating its area.

vi. Simpson Rule: We define the function `simpson` which takes 4 arguments: `f` (function), `a`, `b`, `n` (number of intervals). The function approximates the integral using a polynomial of degree 2.

vii. Monte-Carlo 2D integration: : We define the function `monte_carlo_cuboid` which takes 6 arguments: `f` (function), `xmin`, `xmax`, `ymin`, `ymax` and `n` (number of random uniform points). In the function we define a `cuboid_volume = (xmax-xmin)*(ymax-ymin)*(zmax-zmin)`. The function then takes `n` random points in this cuboid and checks how many of them are above and below the curve. If there are `k` points below the curve then the function computes the integral as volume of the cuboid * probability of getting a point below the curve.

Plotting makes use of inbuilt function `pylab`.

viii. Plotting Riemann rectangles: We defined the functions `plot_riemann_left`, `plot_riemann_right` and `plot_riemann_centre` which take 5 arguments: `f` (function), `xmin`, `xmax`, `name` (name of the function) and `n` (no. of rectangles). The functions plot the Riemann left, right and centre rectangles on the curve of the function.

ix. Plotting Trapezoidal: We define the function `plot_trapezoid` which takes 5 arguments: `f` (function), `xmax`, `xmin`, `name` (name of the function) and `n` (no. of trapezoids). The function plots the trapezoids under the curve which is used for the approximation of the integral.

x. Plotting Monte-Carlo: We define the function `plot_monte_carlo` which takes 5 arguments: `f` (function), `xmax`, `xmin`, `name` (name of the function) and `n` (no. of random points). The function plots the random points in the rectangle that we have defined around the function.

xi. Plotting Monte-Carlo 2_D: We define the the function `plot_monte_carlo_cuboid`. We managed to make Monte carlo 2D graph by looking into website for plotting 3D curves.

Difficulties faced:

We faced difficulty in Monte carlo 2D and the plotting of its curve was even more difficult but after doing a bit research in websites relating to plotting of 3D curves we finally succeeded.