Math 789

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1 Shattering

1.1 example 1

Irrespective their labels, a continuous interval can be found for any set of two input vectors that can separate them from each other. Whereas, for a set of three input vectors, consider a case where they are collinear and the two points on side have positive label and the one in the middle has negative label. A consistent interval cannot be found that shatters such a set of points.

1.2 example 2

Consider an example where the input vectors are alternately labeled as positive and negative. If there are three such points, with the given concept class, we can shatter such input space. However, if there are four such input vectors with alternate labels, with the given concept class, we cannot shatter the input vector space.

1.3 example 3

Given a 3-element set of input vectors, we can shatter it with a continuous interval of half input space such that the decision boundary separates positive labeled vectors from the negative ones. In this example, it doesn't matter how many vectors are labeled positive and how many are labeled as negative.

2 Values of VC dimensions for given examples

1. the cardinality of the largest set that is shattered by the concept class is 2. Therefore, the value of its VC dimension is 2
2. Similarly, since three element set is the largest set that is shattered by the concept clas, the VC dimension value is 3
3. Similar logic of 1,2 applies here hence, the VC dimension value is 3

4. We can use infinite convex sets from the concept class to shatter the input vector space no matter its size. Therefore, the cardinality of largest set could be infinite. Hence, in this case, the value of VC dimension is ∞

3 Mapping from $\beta \to C$

1. The given mapping of β forms a linear decision boundary. Such a linear decision boundary can cut the plane thus forming an open interval i.e. the given concept class C

2. The given mapping of β forms a parabola. A parabola can cut an axis on a plane in three ways. When the vertex of parabola is above the axis, it forms empty set. When the vertex of parabola is below the axis, it cuts the axis at two points. If we consider the outer sections of parabola, it forms two disjoint infinite open intervals. When the axis of parabola is rotated and it cuts the axis at its vertex, we get an open interval

3. The given mapping leads to a hyperplane in the $\mathbb{R}^3 \times \mathbb{R}^2$ coordinate plane. If we consider the section cut by this hyperplane, we get half-space