Measuring Cosmological Parameters Using Type Ia Supernovae

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Abstract

Type Ia supernovae serve as precise standardizable candles and play a crucial role in modern observational cosmology. In this work, we analyze the Pantheon+SH0ES compilation of Type Ia supernovae to constrain key cosmological parameters within the flat ΛCDM framework. We construct the Hubble diagram, fit for the Hubble constant H_0 and matter density parameter Ω_m, estimate the age of the Universe, analyze residuals, and investigate the redshift dependence of H_0. Our results are compared across low- and high-redshift subsamples to assess possible systematic trends.

1 Introduction

The expansion of the Universe is one of the central discoveries of modern cosmology. Observations of Type Ia supernovae (SNe Ia) have provided compelling evidence for cosmic acceleration and the existence of dark energy. Due to their remarkably uniform peak luminosities, SNe Ia act as standardizable candles, enabling precise distance measurements across cosmological scales.

In this project, we utilize the Pantheon+SH0ES supernova dataset to constrain cosmological parameters within the flat ΛCDM model. Our analysis focuses on estimating the Hubble constant H_0, the matter density parameter Ω_m, and the age of the Universe.

2 Supernova Dataset

We use the Pantheon+SH0ES compilation of Type Ia supernovae, which provides calibrated distance moduli, redshifts corrected for observational effects, and associated uncertainties. From the dataset, we extract:

- z_{HD}: Redshift used in the Hubble diagram
- μ: Distance modulus
- σ_μ: Uncertainty in the distance modulus

Rows with missing values in these key quantities are removed to ensure a robust analysis.

3 Cosmological Model

We adopt the flat ΛCDM model, where the expansion history is governed by the Hubble constant H_0 and matter density parameter Ω_m.

The dimensionless Hubble parameter is given by:

$$E(z) = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}. \tag{1}$$

The luminosity distance is defined as:

$$d_L(z) = (1+z)\,\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}. \tag{2}$$

The corresponding distance modulus is:

$$\mu(z) = 5\log_{10}\left(\frac{d_L}{\mathrm{Mpc}}\right) + 25. \tag{3}$$

These relations allow direct comparison between theoretical predictions and observed supernova data.

4 Hubble Diagram

We construct the Hubble diagram by plotting the observed distance modulus μ as a function of redshift z. A logarithmic scale is used for the redshift axis to display both nearby and distant supernovae clearly.

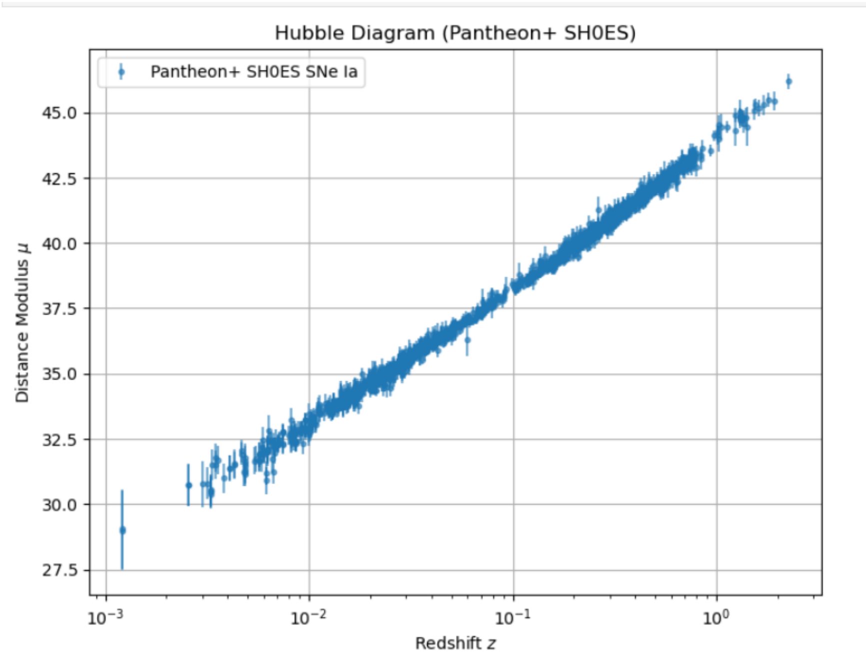

Figure 1: Hubble diagram constructed from the Pantheon+SH0ES supernova dataset.

5 Parameter Estimation

We perform a non-linear least squares fit using the `scipy.optimize.curve_fit` routine. The observed distance moduli are compared with the theoretical model, weighted by their uncertainties.

The free parameters are H_0 and Ω_m, with initial guesses:

$$H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}, \tag{4}$$
$$\Omega_m = 0.3. \tag{5}$$

Best-fit values and uncertainties are extracted from the covariance matrix.

6 Age of the Universe

Using the best-fit cosmological parameters, we estimate the age of the Universe by integrating the inverse of the Hubble parameter:

$$t_0 = \int_0^{\infty} \frac{1}{(1+z)\,H(z)}\,dz, \tag{6}$$

where $H(z) = H_0 E(z)$.

The Hubble constant is converted to SI units, and the final age is expressed in gigayears (Gyr). This provides an important consistency check against independent measurements such as those from the cosmic microwave background.

7 Residual Analysis

To evaluate the quality of the fit, we compute residuals defined as:

$$\text{Residual} = \mu_{\text{obs}} - \mu_{\text{model}}. \tag{7}$$

Residuals are plotted as a function of redshift to identify potential systematic trends or biases.

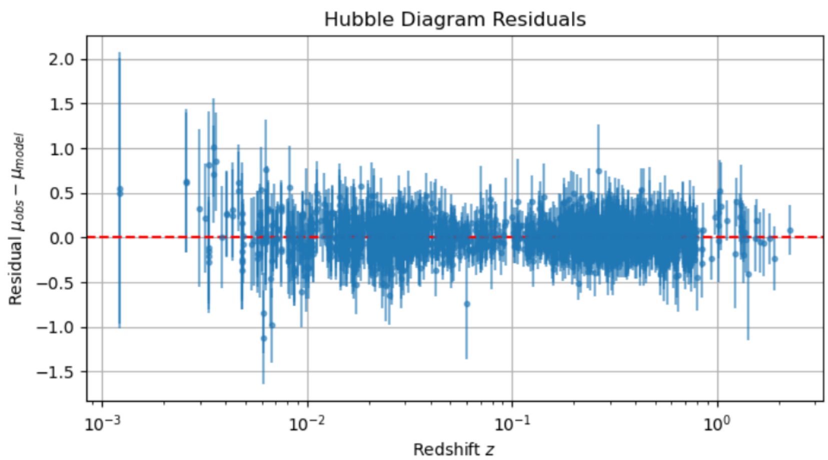

Figure 2: Residuals of the best-fit ΛCDM model as a function of redshift.

A good fit is characterized by residuals randomly scattered around zero.

8 Fit with Fixed Matter Density

To reduce parameter degeneracy, we repeat the fit while fixing the matter density parameter to $\Omega_m = 0.3$. In this case, only the Hubble constant H_0 is treated as a free parameter. This allows a direct comparison with external measurements and highlights the dependence of H_0 on cosmological assumptions.

9 Low- and High-Redshift Comparison

To explore potential redshift-dependent effects, the dataset is divided into:

- Low-redshift sample: $z < 0.1$

- High-redshift sample: $z \geq 0.1$

Each subsample is fitted independently with Ω_m fixed. Differences in the inferred values of H_0 may indicate systematic effects or cosmological tensions.

10 Conclusions

In this work, we analyzed the Pantheon+SH0ES Type Ia supernova dataset within the flat ΛCDM framework. We constructed the Hubble diagram, constrained cosmological parameters, estimated the age of the Universe, and examined residuals and redshift-dependent trends.

Our results are broadly consistent with standard cosmological expectations and highlight the power of Type Ia supernovae as probes of cosmic expansion. Future work may include joint analyses with other cosmological datasets to further investigate tensions in the value of the Hubble constant.

References

- Riess et al., *ApJ*, 2022 (SH0ES Collaboration)
- Scolnic et al., *ApJ*, 2018 (Pantheon Sample)
- Planck Collaboration, *A&A*, 2020