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“Philosophy is written in this grand book – I mean the universe – which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering about in a dark labyrinth.”

Galileo Galilei (1564 – 1642)

1

Mathematical Preliminaries

Like the other sciences, physics depends heavily on the tools of mathematics to model the physical world. In this chapter, we will introduce vectors and single-variable calculus, which are the most important mathematical underpinnings to classical mechanics. Vectors are used throughout the book to represent quantities having some direction, such as velocity. Calculus is used to quantify processes which involve continuous change. In the following chapters, these two topics will serve as the language for all our discussions¹.

¹Vectors and calculus are both developed further in Appendix A.