An Improved Code Acquisition Scheme for Band-limited DSSS Systems with Sampling Offset

Sheng Ding, Zhengrong Yi, Huijie Liu, and Xuwen Liang

***Abstract*—In this paper, we investigate the problem of code acquisition for band-limited direct-sequence spread-spectrum (DSSS) systems in the presence of sampling offset. By analyzing the detection output property of the traditional noncoherent detection scheme with and without the existence of sampling offset, an improved acquisition scheme that utilizes two decision variables for joint detection is proposed. Numerical results show that the proposed scheme can offer better performance and is more robust to the variation of the sampling offset than the traditional scheme.**

***Index Terms*—Acquisition, direct-sequence spread-spectrum (DSSS), sampling offset, noncoherent detection.**

I. Introduction

CODE acquisition in direct-sequence spread-spectrum (DSSS) systems is the process of estimating the delay offset between the spreading code in the received signal and the locally generated replica of the code. Successful code acquisition is very essential and important since it precedes despreading and demodulation.

The most common approach for acquisition is to search, sequentially or in parallel, all the possible code phases over the uncertainty region through noncoherent detection [1]. In real applications, DSSS systems often utilize band-limited chip waveforms, such as the raised-cosine pulse [2]. In this case, however, when the sampled signal for detection is not optimal (i.e., sampling offset exists), there may be substantial degradation of the acquisition performance.

To alleviate the influence caused by the sampling offset, one plausible way is to decrease the cell spacing by over-sampling, which may nonetheless lead to an increase in complexity and acquisition time. The technique based on parabolic interpolation proposed in [3] provides some immunity to the sampling offset but has a downside in the increase of system complexity and power consumption. Recently, a new detection method called twin-cell detection has been proposed to cope with problems such as the fractional Doppler frequency offset, the residual code phase offset and the frequency-selective channel fading encountered in the acquisition process [4]–[7]. Twin-cell detection can also be used to alleviate the influence of the sampling offset, but it can not provide better performance when the sampling offset is small.

In this paper, we propose an improved code acquisition scheme that utilizes two decision variables for joint detection. The system model and the effects incurred by the sampling offset are presented in Section II. In Section III, the proposed scheme is presented and analyzed. Section IV presents simulation results of the performance for the proposed scheme. Finally, conclusion remarks are summarized in Section V.

Manuscript received March 28, 2012. The associate editor coordinating the review of this letter and approving it for publication was C. Sacchi.

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Digital Object Identifier 10.1109/LCOMM.2012.052112.120688

II. System Model and the Effect of Sampling Offset

The received signal after downconverting is filtered with a chip pulse-shaping matched filter. When the data modulation and the carrier frequency offset are not concerned, the signal after being sampled at the chip rate can be represented as

$$x_n = \sqrt{P} \sum_{i=-\infty}^{\infty} c_i p_{T_c}(n-i-\tau) e^{j\phi} + \eta_n \tag{1}$$

where P is the signal power; τ is the time delay normalized to the pseudo-noise (PN) code chip duration T_c; ϕ is the relative carrier phase; $c_i \in \{-1,+1\}$ is the i-th chip of the code sequence with length L, and $p_{T_c}(n)$ is the sample of the PN code waveform which is assumed to be band-limited (such as the pulse waveform with raised-cosine spectrum). Moreover, η_n is modeled as a complex additive white Gaussian noise (AWGN). We can rewrite $\tau = p + \delta$, where p is an integer, and δ is the fractional time delay that incurs the sampling offset.

Fig. 1 shows a typical noncoherent detector for PN code acquisition. x_n is correlated with a locally generated PN code over the a correlation time NT_c. The detector output z_k is expressed as

$$\begin{aligned} z_k = |y_k|^2 &= |\frac{1}{N}\sum_{n=0}^{N-1} x_n c_{n-k}|^2 \\ &= |\sqrt{P} D(\tau-k) e^{j\phi} + w_k|^2 \end{aligned} \tag{2}$$

where w_k is a complex Gaussian random variable with mean zero and variance $2\sigma^2$; we have $D(x) = \frac{\mathrm{sinc}(x)\cos(\alpha\pi x)}{1-4(\alpha x)^2}$, where $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, α is the roll-off parameter. The normalized outputs of the detector in the absence of noise are shown in Fig. 2. In this figure, an arrow represents an output instant corresponding to the k-th code phase. Note that even if the current test cell is H_1 ($k = p$), the correlation peak would suffer different degrees of degradation if $\delta \neq 0$.

III. The Proposed Scheme

A. System Description

Referring again to Fig. 2, it is observed that when $\delta = 0$, there is a single correlation peak at which all the signal power is concentrated and the output for the H_1 cell is

$$z_p = P. \tag{3}$$

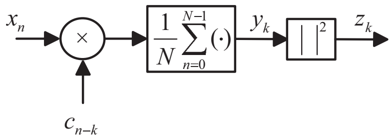

Fig. 1. Structure of a typical noncoherent detector.

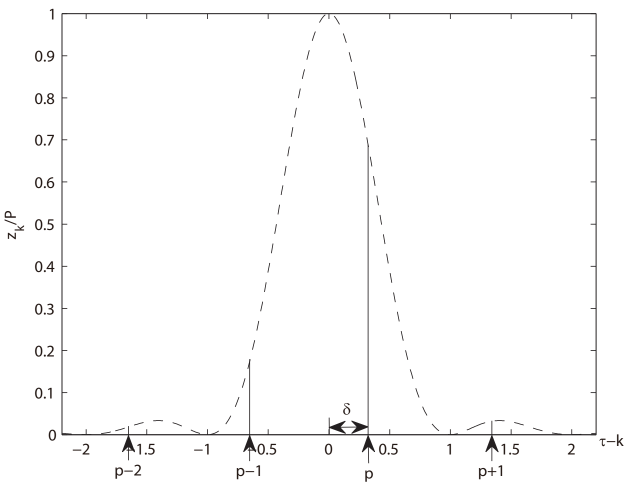

Fig. 2. Normalized output of the noncoherent detector when the noise is absent.

This is the ideal case of the traditional noncoherent detection method. When $\delta \neq 0$, the correlation peak is mainly split into two adjacent smaller peaks

$$\left.\begin{aligned} z_p &= PD^2(\delta) \\ z_{p-1} &= PD^2(1-\delta) \end{aligned}\right\} \tag{4}$$

In this case, if two consecutive cells are jointly used for detection, most of the signal power can be combined and then efficiently used.

The proposed acquisition scheme is motivated by the implication of (3) and (4). A structure of the proposed acquisition scheme is depicted in Fig. 3. In the structure, there are two detection branches. The operation through the upper branch is the same as that of the traditional noncoherent detector shown in Fig. 1. At the lower branch, two consecutive correlator outputs are summed and squared for detection. Thus, the two decision variables are given by

$$u_{1k} = z_k \tag{5}$$

$$u_{2k} = |y_k + y_{k-1}|^2 \tag{6}$$

Clearly, the first variable u_{1k} suffers little signal-to-noise (SNR) loss when δ is near zero. On the other hand, due to the combination of the current and previous desired samples, the second variable u_{2k} is expected to result in better detection performance when δ is around 0.5. The two variables are used for joint detection. If either u_{1k} or u_{2k} exceeds its corresponding threshold β_1 or β_2, H_1 is declared and the synchronization process is transferred to the tracking process; otherwise, the acquisition process resumes with the next candidate of the code phase.

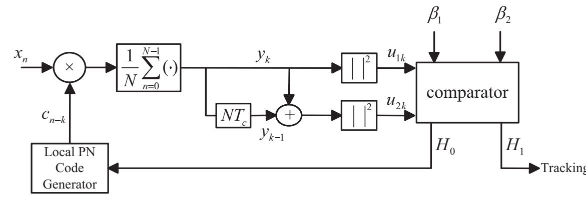

Fig. 3. Structure of the proposed acquisition scheme.

B. Performance Analysis

To evaluate the performance of the proposed scheme, we introduce the quantities $f_1 = \Re(y_k)$, $f_2 = \Re(y_k + y_{k-1})$, $g_1 = \Im(y_k)$, and $g_2 = \Im(y_k + y_{k-1})$, with mean values $[m, m+n, 0, 0]$ and variance $[\sigma^2, 2\sigma^2, \sigma^2, 2\sigma^2]$. Since u_{1k} and u_{2k} are the sum of squares of two Gaussian random variables of arbitrary but equal variances, they obey the chi-square distribution with 2 degrees of freedom (DOF). Thus, the probability density functions (pdfs) $p(u_{1k}; m)$ and $p(u_{2k}; m, n)$ of u_{1k} and u_{2k} are given, respectively, by

$$p(u_{1k}; m) = \frac{1}{2\sigma^2} e^{-\frac{u_{1k}+m^2}{2\sigma^2}} I_0\left(\frac{m\sqrt{u_{1k}}}{\sigma^2}\right) \tag{7}$$

$$p(u_{2k}; m, n) = \frac{1}{4\sigma^2} e^{-\frac{u_{2k}+(m+n)^2}{4\sigma^2}} I_0\left(\frac{(m+n)\sqrt{u_{2k}}}{2\sigma^2}\right) \tag{8}$$

where $I_0(\cdot)$ is the first kind modified Bessel function of order zero.

The proposed method utilizes u_{1k} and u_{2k} for joint detection. Thus, the joint pdf of the two decision variables should be evaluated due to the strict correlation between them. According to the definition of the variables $[f_1, f_2, g_1, g_2]$, their covariance matrix $\mathbf{K}$ is given by

$$\mathbf{K} = \begin{pmatrix} \sigma^2 & \sigma^2 & 0 & 0 \\ \sigma^2 & 2\sigma^2 & 0 & 0 \\ 0 & 0 & \sigma^2 & \sigma^2 \\ 0 & 0 & \sigma^2 & 2\sigma^2 \end{pmatrix} \tag{9}$$

Thus, by computing the inverse matrix from (9), we can get the expression of the joint pdf

$$\begin{aligned} p(f_1, f_2, g_1, g_2; m, n) &= \frac{1}{\sigma^4 \cdot (2\pi)^2} \\ &\times \exp\Big(-\frac{1}{2\sigma^2}[2(f_1-m)^2 - 2(f_1-m)(f_2-m-n) \\ &\quad + (f_2-m-n)^2 + 2g_1^2 - 2g_1g_2 + g_2^2]\Big) \end{aligned} \tag{10}$$

Substituting $f_1 = \sqrt{u_{1k}}\cos\theta_1$, $f_2 = \sqrt{u_{2k}}\cos\theta_2$, $g_1 = \sqrt{u_{1k}}\sin\theta_1$, and $g_2 = \sqrt{u_{2k}}\sin\theta_2$ into (10), after some algebra manipulations we arrive at the following expression for the joint pdf of the two decision variables

$$\begin{aligned} p(u_{1k}, u_{2k}; m, n) &= \frac{1}{4\sigma^4 \cdot (2\pi)^2} e^{-\frac{1}{2\sigma^2}(2u_{1k}+u_{2k}+m^2+n^2)} \\ &\times \int_0^{2\pi}\int_0^{2\pi} \exp\Big[\frac{(m-n)\sqrt{u_{1k}}}{\sigma^2}\cos\theta_1 + \frac{n\sqrt{u_{2k}}}{\sigma^2}\cos\theta_2 \\ &\quad + \frac{\sqrt{u_{1k}u_{2k}}}{\sigma^2}\cos(\theta_1-\theta_2)\Big] d\theta_1 d\theta_2 \end{aligned} \tag{11}$$

Under zero hypothesis H_0, $m = n = 0$, the false alarm probabilities P_{fa}^u, P_{fa}^l and P_{fa} of the upper branch, the lower branch and the proposed scheme can be obtained, respectively, as

$$P^u_{fa} = \int_{\beta_1}^{+\infty} p(u_{1k};0)du_{1k} = \exp(-\frac{\beta_1}{2\sigma^2}) \quad (12)$$

$$P^l_{fa} = \int_{\beta_2}^{+\infty} p(u_{2k};0,0)du_{2k} = \exp(-\frac{\beta_2}{4\sigma^2}) \quad (13)$$

$$\begin{aligned} P_{fa} &= 1 - \Pr(u_{1k} < \beta_1, u_{2k} < \beta_2 | H_0) \\ &= 1 - \int_0^{\beta_2}\int_0^{\beta_1} p(u_{1k}, u_{2k}; 0, 0)\mathrm{d}u_{1k}\mathrm{d}u_{2k} \\ &= 1 - \int_0^{\frac{\beta_2}{\sigma^2}}\int_0^{\frac{\beta_1}{\sigma^2}} \frac{1}{4} e^{-(x+\frac{y}{2})} I_0(\sqrt{xy})dxdy \quad (14) \end{aligned}$$

Under hypothesis H_1, $m^2 = z_p$ and $n^2 = z_{p-1}$, the detection probabilities of the upper branch, the lower branch and the proposed scheme are given, respectively, by

$$\begin{aligned} P^u_d &= \int_{\beta_1}^{+\infty} p(u_{1k}; \sqrt{z_p})du_{1k} \\ &= Q(\sqrt{\frac{PD^2(\delta)}{\sigma^2}}, \sqrt{\frac{\beta_1}{\sigma^2}}) \quad (15) \end{aligned}$$

$$\begin{aligned} P^l_d &= \int_{\beta_2}^{+\infty} p(u_{2k}; \sqrt{z_p}, \sqrt{z_{p-1}})du_{2k} \\ &= Q(\sqrt{\frac{P[D(\delta) + D(1-\delta)]^2}{2\sigma^2}}, \sqrt{\frac{\beta_2}{2\sigma^2}}) \quad (16) \end{aligned}$$

$$\begin{aligned} P_d &= 1 - \Pr(u_{1k} < \beta_1, u_{2k} < \beta_2 | H_1) \\ &= 1 - \int_0^{\beta_2}\int_0^{\beta_1} p(u_{1k}, u_{2k}; \sqrt{z_p}, \sqrt{z_{p-1}})du_{1k}du_{2k} \quad (17) \end{aligned}$$

where $Q(a,b) = \int_b^{+\infty} ue^{-(u^2+a^2)/2} I_0(au)du$ is the Marcums Q function.

C. Weight of the Two Decision Variables

In the proposed acquisition scheme, it is noteworthy that the relation between the two thresholds β_1 and β_2 determines the contribution of the upper branch and the lower branch to the whole detection performance. Here, we define the weight factor

$$\gamma = \frac{P^u_{fa}}{P^l_{fa}} \quad (18)$$

which can implicate the dominance of the decision variables u_{1k} and u_{2k} in the acquisition process. When $\gamma = +\infty$, only u_{1k} is used for detection and the proposed scheme is converted to the traditional noncoherent detection which is preferred when δ is near zero. On the other hand, when $\gamma = 0$, only u_{2k} is utilized for detection and this is the expected case when δ is around 0.5. In real applications, since δ is uniformly distributed, γ is expected to be around one so that good performance can be obtained when the sampling offset changes.

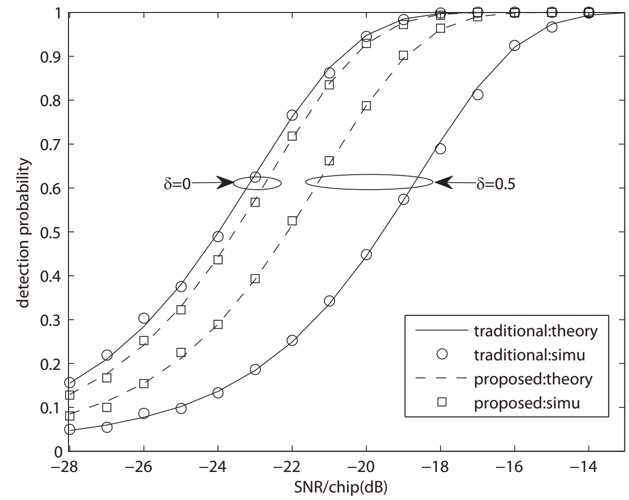

Fig. 4. Detection probability of the traditional and proposed schemes for $\delta = 0$ and $\delta = 0.5$.

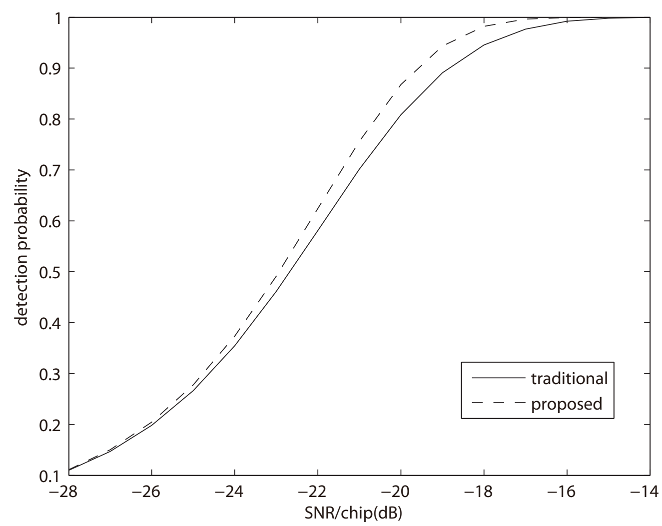

Fig. 5. Average detection probability of the traditional and proposed schemes when δ is uniformly distributed.

IV. Simulation Results and Discussion

In our numerical analysis, we consider a spread spectrum system with a PN code of length $L = 1024$, chip rate $= 1.2288$MHz, $N = 1024$ (full period correlation), $\alpha = 0.5$. Threshold setting is based on constant false alarm rate (CFAR) with $P_{fa} = 0.01$.

Fig. 4 shows the results of the simulations for the detection probability of the traditional and proposed schemes against SNR per chip in dB for $\delta = 0$ (best case) and $\delta = 0.5$ (worst case) with $\gamma = 1$. When $\delta = 0$, only a single correlation peak exists and the sum of two consecutive correlator outputs at the lower branch of the proposed scheme naturally incurs SNR loss of u_{2k}. Consequently, the detection probability of the proposed scheme is slightly inferior to that of the traditional scheme. When $\delta = 0.5$, on the other hand, due to assistance of the lower branch which combines the power spread by the sampling offset, the proposed scheme outperforms the traditional scheme and the performance gain is about 2.6dB.

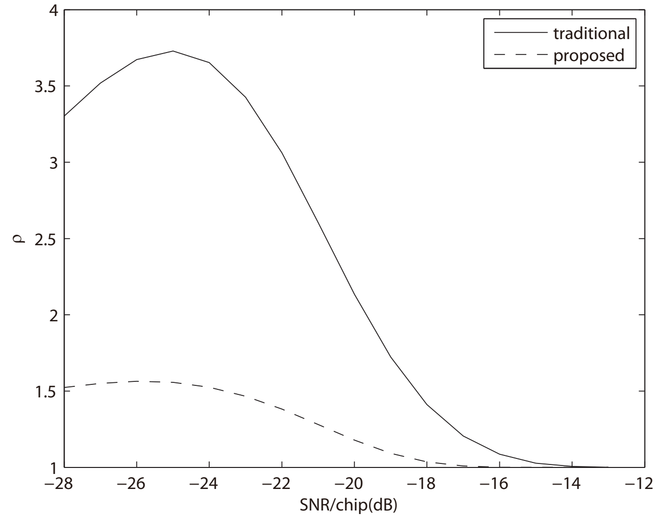

Fig. 6. Sensitivity of the traditional and proposed schemes to the sampling offset.

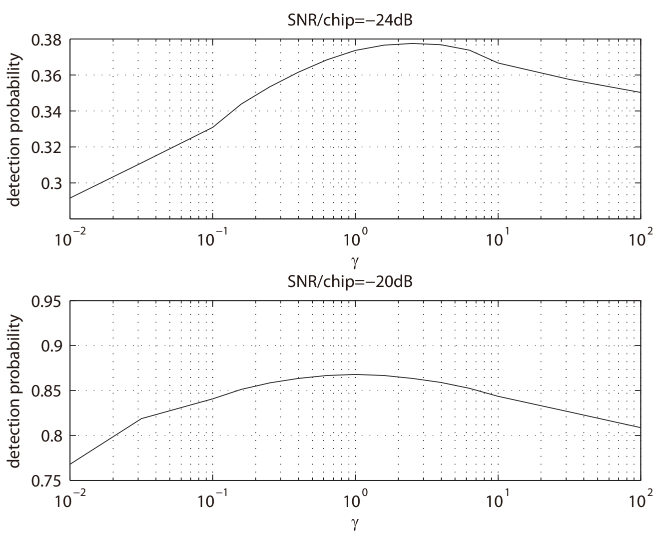

Fig. 7. Influence of the weight factor on the average detection probability.

Fig. 5 shows the average detection probability of the traditional and proposed schemes when δ is uniformly distributed over $[0, 1)$ and $\gamma = 1$. From this figure, it is observed that the proposed scheme can, on average, performs better than the traditional scheme.

Fig. 6 shows the sensitivity of the detection probability of the traditional and proposed schemes to the sampling offset. Here, the sensitivity factor is defied as

$$\rho = \frac{(P_d)_{\delta=0}}{(P_d)_{\delta=0.5}} \tag{19}$$

which is a measure of performance robustness. It is clear that the proposed scheme is more robust to the variation of the sampling offset than the traditional scheme.

Fig. 7 shows the influence of the weight factor γ on the average detection probability of the proposed scheme for SNR/chip $= -24$ and -20dB. As expected, for both SNR condition, the proposed scheme can offer better performance when γ is around one, where u_{1k} and u_{2k} has the same weight in the acquisition process. As γ increases or decreases, the detection performance degrades and gradually approaches that of the upper branch or the lower branch.

V. Conclusion

In this paper, an improved code acquisition scheme is proposed for band-limited DSSS systems in the presence of sampling offset. The proposed scheme utilizes two decision variables for joint detection so that the influence of the sampling offset can be alleviated. Simulation results approved the validity of the proposed scheme.

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