

# Lecture with Computer Exercises: Modelling and Simulating Social Systems with MATLAB

Project Report

Comparison of Two Modifications
of the Nagel-Schreckenberg Cellular Automaton
Model of Vehicular Traffic
with Respect to Traffic Flow and Fuel Consumption

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Zurich December 2013

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# **Declaration of Independence**

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# Contents

1	Abstract	5
<b>2</b>	Individual contributions	5
3	Introduction and Motivations	5
4	Description of the Model 4.1 The Nagel-Schreckenberg Model	6 6 7 7
5	Implementation5.1 Organisation of Code5.2 Implementation of Standard Nagel-Schreckenberg Model5.3 Implementation of the adaptive cruise control modification5.4 Implementation of the Flexible Speed Limits Modification5.5 Measurements	8 9 9 10 10
6	Simulation Results and Discussion  6.1 Basic Representation of the Model	
7	Summary and Outlook	15
8	References	15

# 1 Abstract

The aim of this investigation was to find out to what extent modifications to the Nagel Schreckenberg model such as flexible speed limits and adaptive cruise control influence traffic flux and fuel consumption. This was achieved by first implementing the basic Nagel Schreckenberg model consisting of a 1D cellular automata lattice representing the street where some basic rules for the behaviour of the agents throughout discrete time steps were introduce. These rules were then altered such that on one hand reduced speed limits were introduced in zones where the relative density of the zone in front was high. This model is later referred to as the flexible speed limits modification and on the other hand cars were forced to keep different minimal distances to the cars in front which is later called adaptive cruise control.

In the end, neither Flexible speed limits nor adaptive cruise control control had any positive effect on flow. The adaptive cruise control modification, however, reduced the fuel consumption.

# 2 Individual contributions

The main ideas for the project were developed in group discussions on a weekly basis. All group members contributed in equal parts to literature research and the formulation of research questions. The tasks were then split as follows:

• Code writing for simulations: Patrick Seewald

• Visualisation: Timo Hinzmann

• Report and additional research: Claudia Deuber and Jacqueline Mock

### 3 Introduction and Motivations

Traffic and especially traffic jams is a phenomenon often investigated on, since in industrial countries such as Switzerland mobility of people especially on the streets is increasing. For instance there is a high number of people covering large distances between their homes and their working places on a daily basis. As a consequence many people are concerned by the occurrence of traffic jams especially at rush hours. This leads to the need of finding ways for optimizing traffic flow. An inexpensive way to do this is to analyze different driving behaviours in a computer simulation. The drawbacks of traffic simulations such as the necessary reduction of complexity leading to a simplified picture and the difficulty to directly compare simulation results to reality are compensated by the possibility to control and to set parameters, which is something that cannot be done in real traffic experiments.

Traffic modelling can be done in different ways: either in discrete or in continuous simulation. The benefit of a cellular automaton model such as Nagel-Schreckenberg over models in continuous space and time is its simplicity and computational efficiency, allowing for a computation of large systems. Even though the model has very simple rules, it is shown in previous literature ([CSS00]) that most traffic phenomena can be realistically reproduced. This is the reason is why a discrete traffic model, the Nagel-Schreckenberg 1D cellular automata model was chosen.

The search for not already covered research questions concerning traffic phenomena led us to two modifications of the Nagel-Schreckenberg model: flexible speed limits and adaptive cruise control. Flexible speed limits are sometimes introduced in real life traffic in situations where there is a long jam in some road in order to slow down the oncoming traffic. The goal is to maintain flux instead of blockage, which would ideally help dissolve the jam and reduce fuel consumption due to motors running on stillstand in jams and the fuel consumption used for starting a car and for driving in very low gear. Adaptive Cruise Control is a mechanism that is currently being tested for real cars. The idea is that there is a device forcing cars to keep a minimal distance to the car in front and would thus influence the speed of the car such that this distance is kept [Dav04].

This leads to the research questions of how and to what extent can the modifications of the basic Nagel Schreckenberg influence flow and fuel consumption?

# 4 Description of the Model

# 4.1 The Nagel-Schreckenberg Model

In this investigation the Nagel-Schreckenberg Model for 1D cellular automata was used. In cellular automata time as well as space is discrete and therefore easy to compute. In traffic simulations this means that the lane is represented by a 1D lattice, where each site can either be occupied by one car or be empty. As space as well as time are discrete variables, speed is described by number of sites transcended at each time step.

The whole system is uptdated in discrete time steps  $t \mapsto t+1$ , where cars can either move forward at a constant speed, accelerate, decelerate or remain on the same site. The cars can only move forward at time step t+1 providing the site in front of them is empty at time step t in order to avoid collision (see Fig. 1). Unless a car has reached maximal velocity its speed is increased by 1 in each time step. This rule reflects the goal of each driver to drive at maximal possible speed. Moreover there is a random factor p that describes the probability of a driver to slow down at a certain time step t. This is one prerequisition for a traffic to occur and furthermore reflects individual driving behaviour of drivers as it is observed in real traffic. If the gap between the next vehicle and a vehicle is less than the velocity of the vehicle behind, the vehicle will decelerate to the velocity corresponding to the gap between the two cars [CSS00].

It is clear, that the random probability of one car slowing down is eventually the cause of traffic jams, as there are no further obstacles in the system. It is possible, however, to interpret this deceleration in different ways. On one hand it can be interpreted as the individual driving style of a driver, who decreases speed for no obvious reason. On the other hand it can represent a randomly appearing obstacle on the driveway forcing a driver to slow down. This is a further indicator for the broad validity of this model.

The basic parameters were set as follows [Nin00]:

- 1 cell=7.5m  $\longrightarrow$  1000 cells= 7.5km
- 1 time step = 1s (corresponds to the reaction time of a driver)
- velocity:  $1=27 \text{km/h} \longrightarrow 5=135 \text{km/h}$
- brake parameter = 0.25

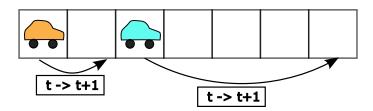


Figure 1. Nagel-Schreckenberg Model

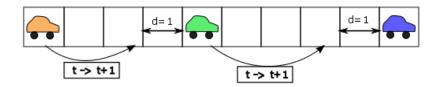


Figure 2. Adaptive Cruise Control

#### 4.2 Nagel-Schreckenberg Model and Modifications

In this project we study two modifications of the Nagel-Schreckenberg model. These two modifications were designed to answer the questions whether the traffic flow and the fuel consumption can be optimized by:

- 1. a driving behaviour maintaining a larger distance to the car in front which can smoothly compensate disturbances in the traffic flow (see Fig. 2)
- 2. different speed limits for different regions of the road that are automatically imposed based on the current traffic situation aiming for a density reduction in jam regions

For simplicity, the three models under discussion will be referred to as NaSch (standard Nagel-Schreckenberg model), ACC (Adaptive Cruise Control modification) and FSL (Flexible Speed Limits modification) in the following.

The ACC modification can be interpreted in two different ways. First, it can be seen as the behaviour of intelligent drivers who adjust their driving style to the purpose of a better global traffic flow and emission reduction (assuming that the proposed rule really does have these desired consequences). This is opposed to the more selfish deceleration rule of the NaSch model which can be paraphrased as "always drive as fast as possible". The problem of this interpretation is that in realistic traffic different agents stick to a different driving style where in our simulation only the consequences of a rule applying to all agents is investigated. It is not tested in our simulation whether a different driving behaviour of single agents lead to a globally better traffic situation, only the influence of a change of the behaviour of all agents is considered. In practice it is highly unlikely that the behaviour of all agents can be changed in a significant way corresponding to a change of a global model parameter. The second interpretation is that not the drivers act intelligently but the cars which are equipped with an adaptive cruise control system automatically maintaining the ideal distance to the next car. The ACC modification is then a realistic model for the case that each vehicle on the road is equipped with an adaptive cruise control system which may be the case in the near future.

The intention of the FSL modification is to reduce the inflow of vehicles to dense regions and thereby to dissolve jams by imposing different speed limits to different regions of the road. This is implemented by the following rule: if a certain region has a high density the region behind will get a low speed limit and if the density is low, a high speed limit is imposed in the region behind. The speed limits are valid for a defined time and are then recalculated based on the current density of vehicles in the different regions. It is hoped that the density of vehicles will be equalized over the entire road by this measure and jams that are characterized by a locally high density will be prevented or dissolved as soon as they emerge.

#### 4.3 Measured Quantities

The simulation results of the three models are investigated by measuring the following quantities:

- the flux and the velocity distribution
- the fuel consumption
- the average length of jams and the fraction of cars which are stuck in a jam

The average flux is calculated as  $\bar{J}=c\bar{v}$  where c is the global density of cars and  $\bar{v}$  is the average of the velocity over all cars. This relation is derived as follows: the flux J is locally defined as the number of cars passing a certain site per time and is equal to  $J=\rho v$ , where v is the local velocity and  $\rho$  is the local density. In our case, the density is defined as the number of cars per site. The global flux is an average over all sites:

$$\bar{J} = \frac{1}{L} \sum_{k=1}^{L} \rho_k v_k. \tag{1}$$

The density is  $\rho_k = 1$  if site k is occupied and  $\rho_k = 0$  if site k is empty. Thus the average flux is

$$\bar{J} = \frac{1}{L} \sum_{n=1}^{N} v_n = \frac{N}{L} \left( \frac{1}{N} \sum_{n=1}^{N} v_n \right) = c\bar{v} ,$$
 (2)

where N is the total number of cars.

It is difficult to estimate the fuel consumption realistically because this depends on a large number of empirical factors which are not available in our simple model. Therefore the calculation is restricted to a very rough estimation which assumes that the fuel consumption is proportional to the acceleration energy, neglecting friction and efficiency losses. The fuel consumption of a car n at a time t is thus defined as

$$\Delta f_n(t) = \max(v_n(t)^2 - v_n(t-1)^2, 0) , \qquad (3)$$

i. e. it is proportional to the difference in kinetic energy when the difference is positive and it is zero else. This approximation only considers fuel consumption due to frictionless acceleration and can not be directly compared to realistic values. Still, this approximation is sufficient for a general comparison of the three models in terms of fuel consumption, caused by a different pattern of acceleration and deceleration. Friction losses and efficiency details are not so important in the view of the above, if one is aware of the restriction that only a general tendency is investigated and not an exact representation of real fuel consumption.

In order to measure jam-related quantities we defined a jam as a chain of direct neighbours of vehicles with a total length of at least l (typically chosen as l = 2).

# 5 Implementation

#### 5.1 Organisation of Code

The three models NaSch, AAC and FSL are implemented using three independent functions which have a different set of input arguments,  $sim\_nasch$ ,  $sim\_aac$  and  $sim\_fsl$ . A simulation with graphical output showing the state of traffic for all positions and times is done by using the three functions  $plot\_nasch$ ,  $plot\_aac$  and  $plot\_fsl$ . The function  $SpeedLim\_its$  is called by the function  $sim\_fsl$  to determine the speed limits. The function fuel measures the average fuel consumption per distance and the function jamDetect evaluates the number of jams and their sizes for a given simulation result. The script par contains all parameter used for running the simulations. All other scripts contain all code related to specific measurements that we have done and the plotting of the data.

### 5.2 Implementation of Standard Nagel-Schreckenberg Model

The standard Nagel-Schreckenberg model is contained in the function  $sim_n asch$  which has the following input arguments: the number of sites L, the number of iterations representing the time t, the density of cars c, the random brake parameter p and the maximum velocity  $v_{\text{max}}$ . As initial condition the sites are randomly occupied by vehicles such that each site has probability c to be occupied. In order to maintain the traffic flow and the number of cars over time periodic boundary conditions have been applied, meaning that the road is circular. A vector containing the positions x and velocities v of all cars is created and updated in each iteration. Each iteration consists of a loop over all cars in which the positions and velocities are calculated based on the Nagel-Schreckenberg update rules. The update is done in parallel such that all cars are updated based on the position vector of the last iteration, which is stored in a separate variable  $x_0$ . It must be emphasized that updating the cars based on the current positions without storing the old positions would be parallel only in cases where no periodic boundary conditions are applied. Then each car is updated based on the car in front which has not already been updated. However this order of cars is not the case any more for periodic boundary conditions where the last car is updated based on the position of the first car. Therefore the cars have to be updated based on the positions  $x_0$  of the preceding iteration.

The steps of the Nagel-Schreckenberg algorithm are implemented as follows for the nth car if complications arising due to periodic boundary conditions are ignored for the time being [CSS00]:

- Acceleration:  $v_n = \min(v_n + 1, v_{\text{max}})$
- Deceleration:  $v_n = \min(v_n, d-1)$  where  $d = x_{0,n+1} x_n$  is the distance to the leading car
- Random braking:  $v_n = \max(v_n 1, 0)$  with probability p
- Vehicle movement:  $x_n = x_n + v_n$

For the calculation of the distance d in the second step and the position  $x_n$  in the last step the periodic boundary conditions have been considered by using the matlab function 'mod' in the following way:

$$d = \text{mod}(x_{0,n'} - x_n, L), \ n' = \text{mod}(n, n_{\text{tot}}) + 1 \ , \tag{4}$$

where  $n_{\rm tot}$  is the total number of cars and

$$x_n = \text{mod}(x_n + v_n - 1, L) + 1. \tag{5}$$

The output of the function  $sim_nasch$  consists of two matrizes x(t, n) and v(t, n) which contain the positions and velocities for all times t and cars n, respectively. Even though these two outputs contain all the information needed, an additional matrix k is returned which is used for a visualization of the data (see Fig. 3).

#### 5.3 Implementation of the adaptive cruise control modification

The adaptive cruise control modification is implemented in a simple way by a slight change of the deceleration rule of the NaSch model. In the NaSch model the deceleration rule ensures that a car does not drive further than to the site directly behind the next car (which may move forward in the same iteration t or not). In the simplistic description of the NaSch model in which only the distance to the next car is considered for the determination of the velocity, this rule corresponds to the maximally allowed velocity

when vehicles are not allowed to drive into each other. In the adaptive cruise control modification an additional gap of length  $d_0$  between two cars is introduced, i.e. vehicles accelerate to the velocity that maintains a distance  $d_0 + 1$  to the car in front. If a car has a distance smaller or equal than  $d_0 + 1$  to the next car, it is allowed to have a velocity v = 1 if d > 1 and v = 0 if d = 1. This rule is implemented using the following expression:

$$v_n = \min(v_n, \min(\max(d - (d_0 + 1), 1), d - 1)) \tag{6}$$

This rule thus has the effect that cars need more steps to move forward to the next car because they maintain a safe distance to the next car and only tailgate if not otherwise possible. The size of the gap  $d_0$  is an additional parameter compared to the standard Nagel-Schreckenberg model and the ACC modification is equal to the NaSch model for  $d_0 = 0$ . The ACC modification is implemented in the function  $sim_aac$  which has the same structure and the same output as the function  $sim_aac$  besides the changed deceleration rule.

# 5.4 Implementation of the Flexible Speed Limits Modification

The flexible speed limits modification is implemented by dividing the road into different zones each of which can have a different speed limit. There are two possible speed limits, a high speed limit  $v_{\text{max}}$  and a low speed limit  $v_{\text{min}}$ . The speed limits for each zone are chosen based on the density of cars in the next zone. If the density of cars is smaller than c (the global density of cars), the speed limit  $v_{\text{max}}$  is applied and if the density of cars is higher than c, the smaller speed limit  $v_{\text{min}}$  is applied. The additional parameters of the FSL modification compared to the NaSch model are: the two speed limits  $v_{\text{min}}$  and  $v_{\text{max}}$ , the number of zones with different speed limits and the time to let pass with a given distribution of speed limits before the speed limits are recalculated based on the current traffic situation. The FSL modification is contained in the two functions  $sim_{-}fsl$  and speedLimits. The function  $sim_{-}fsl$  is the main simulation similar to the function  $sim_{-}nasch$  but with speed limits that can be different for each site. Within this function, the function speedLimits is called which calculates the speed limit for each site based on the current traffic situation (i.e. the positions of all cars).

#### 5.5 Measurements

All quantities have been measured for a road consisting of 1000 sites. Each measured quantity has been averaged over 100 different simulations. For each simulation, the system has been let evolve for  $t_0 = 10$  iterations before any measurement was taken. Then the measured quantity was measured as a time average over  $1000 - t_0$  iterations. All quantities were measured for 100 different values of the density c between 0 and 1. The probability of a car to slow down was always set to p=0.25. The fuel consumption was always measured per distance covered.

# 6 Simulation Results and Discussion

# 6.1 Basic Representation of the Model

In Fig.s 3 and 4a the cars are represented by dots colored according to their speed. Therefore the red lines can be interpreted as traffic jams as this is where no cars are moving. It is important to notice that the traffic jams propagate backwards meaning that as time elapses, new cars are added at the back of the jam, while cars at the front are eventually driving away. Fig. 3 shows that the traffic jams in the ACC (Fig. 3b) are

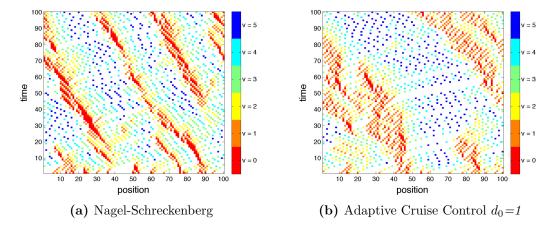
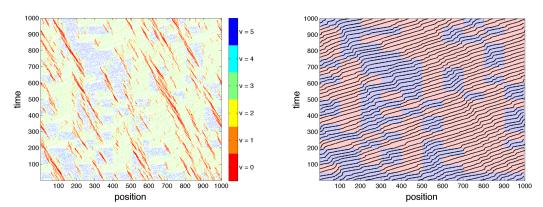


Figure 3. Distribution of Cars over time the cars are coloured according to their actual speed. length of lattice=100; nr. time steps=100;  $v_{max}$  =5; p=0.25; nr. of simulations=100

much broader than those in NaSch, but the gap between the cars are also bigger in the ACC.

The green patches in Fig. 4a show where  $v_{min}$  was introduced. As the time as well as the positional frame was set differently by a factor of 10, comparisons between the NaSch or ACC and FSL have to be done cautiously. Nevertheless Fig. 4a shows that FSL leads to small evenly distributed jams usually immediately followed by a green patch where  $v_{min}$  was introduced since the density in the zone in front is high. In Fig. 4b the trajectories of certain cars are represented as black lines (the red patches correspond to the green patches in Fig. 4a) and the slope of the lines corresponds to the speed of the car. Jams are therefore visible where there is an aggregation of vertical lines meaning that cars remain at the same position over a certain period of time.

Fig. 3 suggets that with ACC there are fewer jams than with NaSch. Since the cars keep a bigger distance to the car in front in ACC, the jams seem to be longer, even if the same number of vehicles is concerned by the jam.



(a) Distribution of Cars over Time for the Flexible Speed Limits Modification The green patches show where a reduced speed limit  $v_{min} = 3$  was introduced.

(b) Trajectories of Single Cars in the Flexible Speed Limits Modification The red patches show where a reduced speed limit  $v_{min} = 3$  was introduced, whereas for the blue zones  $v_{max} = 5$  was maintained.

**Figure 4.** length of lattice=1000; nr. of time steps=1000;  $v_{min}$  =3;  $v_{max}$  =5, 10 zones, change of speed limit after 50 time steps, length of zone=10

### 6.2 Velocity Distribution

Next, the relative velocity distribution with respect to density is shown (Fig. 5). As expected there is a general trend that as the density increases, there are more cars standing still. At a density of 1, where every field in the lattice is occupied by a car, all cars are of course standing still. Since this is not very interesting, the values obtained in this simulation are probably most relevant for medium or low densities for this is where the difference between the different models is biggest.

In NaSch the ratio of cars that can drive at maximal velocity is biggest, yet this is only true for very low densities (up to 0.12). The FSL model seems not to influence flux or even avoid traffic jams since the ratio of cars standing still is the same in Fig. 5b as in the one representing NaSch (Fig. 5a). On the other hand, the ACC leads to the smallest ratio of cars standing still at densities lower than 1, yet it has also a considerably low ratio of cars driving at speed 1, whereas its ratio of cars at maximal speed is nearly as big as the one of NaSch also only at low densities.

This can be explained by the fact that cars always keep a minimum distance to the car in front. As a consequence they are expected to drive at the same speed as the car in front. On the other hand they tend to slow down earlier than in NaSch because they have to maintain a minimal distance to the car in front. Apparently this leads to more cars driving at lower speed and to less cars standing still.

The fact that there are most cars driving at medium velocity in FSL could be explained by the fact that there is an induced  $v_{min}$  of 3 in certain zones, especially when the density in the zone in front is high, leading to cars there driving mostly at a velocity of two or three. This confirms the expectation that ACC could help avoid jams as the ratio of moving cars is biggest in Fig. 5c.

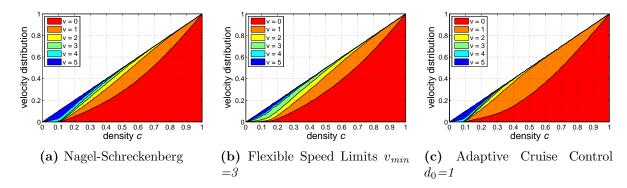


Figure 5. Velocity Distribution length of lattice=1000; nr. time steps=1000;  $v_{max}$  =5; p=0.25; nr. of simulations=100

### 6.3 Flux versus Density

Fig. 6 shows a plot of the flux versus the density for different values of  $v_{max}$ . As general trend there is one distinct peak for each  $v_{max}$  showing the optimal density allowing the highest flux. As one would expect, higher  $v_{max}$  allow higher flux, however, the optimum is shifted to the left as the  $v_{max}$  is increased, meaning that at higher  $v_{max}$  the optimal flow is only reached at lower densities. This makes sense since with a higher  $v_{max}$  the probability of a car driving too slow increases as there are more possible values for the speed. Moreover in order to drive at a high speed more space in terms of empty fields in front is required than for driving at a lower speed. As the density increases there are less empty fields thus leading to less cars being able to drive at high velocities. As a consequence for higher  $v_{max}$  a higher number of fields need to be unoccupied, which is only possible at low densities.

When studying Fig. 6b it is important to consider that  $v_{max}$  is always 5 and  $v_{min}$  is varied. Therefore it is to be expected that a curve for any value of  $v_{min}$  would lie between the corresponding curve in NaSch (Fig. 6a). Since this is not the case it can be concluded that FSL reduced the flux.

As  $d_0$  is increased in ACC the maximal flux decreases remarkably for densities above 0.1 in comparison with the other two models.

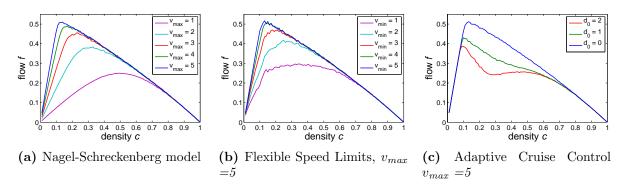


Figure 6. Flux vs Density length of lattice=1000; nr. time steps=1000;  $v_{max}$  =5; p=0.25; nr. of simulations=100

#### 6.4 Fuel Consumption

In addition, by comparing the average fuel consumption for different speed limits in the three models (Fig. 7), it is obvious that for medium and high densities the ACC helps saving fuel at densities between 0.3 and 0.9. This is probably due to the fact that the drivers are more evenly distributed on the streets when they keep an average minimal distance that leads to some extent to an avoidance of constant braking and accelerating, rather than in the NaSch where all cars try to drive at maximal velocity. The curves of the NaSch as well as of the FSL look similar. Yet the fuel consumption in the FSL is generally a higher for  $v_{min}$  of 1-3 and for all  $v_{min}$  at densities between 0 and 0.1. Most probably this is due to the fact that the speed of each zone is adapted depending on the relative density in the zone in front. As a consequence at low overall density speed reductions are introduced even though this would not be necessary since the absolute density in the zone in front is not high enough to justify a reduction. This leads to unecessary braking and accelerating in comparison with the NaSch.

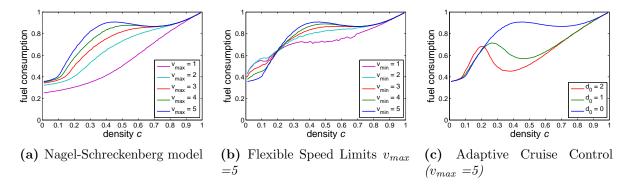


Figure 7. Fuel Consumption length of lattice=1000; nr. time steps=1000;  $v_{max}$  =5; p=0.25; nr. of simulations=100

#### 6.5 The Nature of Jams

Finally, the percentage of cars in a jam (Fig. 8) as well as the average length of a jam (Fig. 9) versus the density was plotted for the different velocities. Fig. 8c shows that the ACC helps to reduce traffic jams since for densities between 0.2 and 0.6 the curve is considerably below the other curves. Interestingly although ACC reduces traffic jams, it does not increase the flux as was shown in Fig. 6c. There is no big difference between NaSch and FSL.

Furthermore, Fig. 9 shows how robust the length of jams are with respect to density since the curves are low until they reach a certain density where the length of a jam increases almost explosively. This shows that jams are usually evenly distributed and the distance between two jams is just bigger at lower densities, else there would be no steady increase in the percentage of cars in a jam as shown in Fig. 8. As the density increases to about 0.8 probably many smaller jams begin to merge leading to the sudden increase in the length of jams.

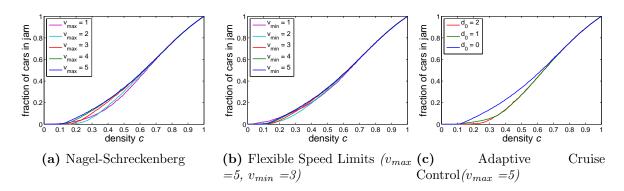


Figure 8. Percentage of Cars in a Jam length of lattice=1000; nr. time steps=1000;  $v_{max}$  =5; p=0.25; nr. of simulations=100

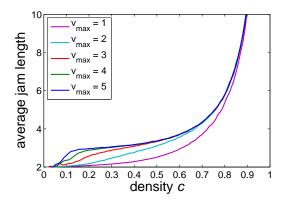


Figure 9. Average Length of a Jam (Nagel-Schreckenberg) length of lattice=1000; nr. time  $steps=1000; v_{max}=5; p=0.25; nr. of simulations=100$ 

# 7 Summary and Outlook

In conclusion, it was shown that FSL does not have any positive effects on neither flux nor fuel consumption. On the other hand ACC proved to reduce fuel consumption according to the estimates in this paper, however, it did not have any positive effect on flux neither. Where flux is concerned, NaSch is therefore already optimal. The only improvement that could be achieved in this project was the reduction of the fuel consumption by ACC.

There are many possible extentions to the project demonstrated in this paper. First of all it would be interesting to compare this to real life data, for instance comparing it to a certain street in Switzerland at different times of the day, where different densities appear and to check the reliability of this model.

In order to enhance this model, a better assumption for the fuel consumption could be worked out. This estimate would for example include the efficiency at different gears and thus the increase in fuel consumption when starting a car from the stop. This, however, would require extensive research in order to find a reasonable assumption, since this data differs from type of car to type of car. One possible approach would be to take the average of the most popular cars.

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