Numerical Computations - Ex 2, Oct 22, 2020

5. Implement Hermite interpolation using divided differences: find $p \in P^{2n+1}$ such that

$$p(x_i) = f(x_i) \qquad \text{and} \qquad p'(x_i) = f'(x_i) \qquad \text{for } i = 0 \dots n$$

   Test your implementation for $f(x) = \sin(x)$, and $x_0 = 0, x_1 = \pi/2, x_2 = \pi$.

   You can start from the notebook *NewtonInterpolation.ipynb* provided in the lecture.

6. Compare the interpolation errors (convergence plots) of polynomial interpolation and spline interplation with classical cubic splines for the functions

   - $f(x) = \sqrt{x}$ on $[0, 1]$
   - $f(x) = \arctan(10x)$ on $[-1, 1]$

   You can start from the notebook *splines.ipynb* using the interpolate module from scipy. Try to find good knot distributions.

7. Find a good approximation to

$$\sum_{j=1}^{\infty} \frac{6}{i^2}$$

   by extrapolation. Hint: define the function

$$f : \{1/n : n \in \mathbb{N}\} \to \mathbb{R} : x \mapsto \sum_{j=1}^{1/x} \frac{6}{j^2},$$

   apply polynomial interpolation with knots $x_i = 2^{-i}$, and evaluate the interpolation polynomial at $x = 0$.

8. Let

$$\begin{aligned} I_n : C([a, b]) &\to C([a, b]) \\ f &\mapsto \sum_{i=0}^{n} f(x_i) l_i(\cdot) \end{aligned}$$

   be the Lagrange interpolation operator. Prove that its operator norm defined by

$$\|I_n\|_\infty := \sup_{f \in C([a,b]), f \neq 0} \frac{\|I_n f\|_\infty}{\|f\|_\infty}$$

   evaluates to

$$\|I_n\|_\infty = \sup_{x \in [a,b]} \sum_{i=0}^{n} |l_i(x)|.$$

   Hint: part of the work was already done in class