

Cellular Automata Simulation of Heterogeneous Freeway Traffic Flow in a Smart City

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Abstract—The paper proposes a simple simulation model of a heterogeneous transport system consisting of human-driven and automated vehicles. The time evolution of the system is based on a modified version of Wolfram's rule 184 for an elementary cellular automaton. Different rules for each type of pair of following vehicles are proposed (car-following modes). It is taken into account that a group of sequentially moving connected automated vehicles can form a platoon, which moves with a given velocity. The influence of the ratio of the types of vehicles (human-driven and automated) in the system on the characteristics of the traffic flow has been studied. It is shown that such a simplified model can qualitatively correctly describe the main features of a heterogeneous traffic flow.

Keywords—traffic flow, cellular automata, automated vehicles, heterogeneous traffic, smart city

I. INTRODUCTION

Progress in the development of individual transport has been characterized by an increase in the number of developments in the field of unmanned (or automated) vehicles designed for automatic movement on highways. Automated vehicles may significantly lower the frequency of car accidents and reduce traffic congestion, as they can provide more accuracy in coordinated control compared to human drivers. The widespread introduction of automated vehicles may occur in an interval of several years to several decades. According to modern concepts of the transport systems of a smart city, automated vehicles can interact with road infrastructure through vehicle-to-infrastructure (V2I) interfaces or with each other through vehicle-to-vehicle (V2V) interfaces [1]. Thus, an automated vehicle can not only assess the dynamic characteristics of other vehicles through a built-in sensor system, but also exchange information about its intentions with these other vehicles. These interfaces allow a group of connected automated vehicles (CAVs) to create synchronized platoons made up of several vehicles moving with the same velocity.

Despite active research and development in the field of automated vehicles, it is obvious that heterogeneous traffic, in which both automated vehicles (AVs) and human-driven vehicles (HDVs) are present, will be present in the transport systems of the world for a long time to come. Although there is empirical information about the traffic of HDVs, information about the traffic of automated vehicles so far can only be obtained using modeling, which in this case is the main research tool [2]–[5].

All traffic flow models can be conditionally divided into macro-, meso- and micro-level models [6], [7]. In macro-level models, traffic is likened to the movement of a continuous

medium; therefore, these models are mainly suitable for describing homogeneous transport systems. Micro-level models can describe the motion of individual vehicles at the kinematic scale. Therefore, they are suitable for describing heterogeneous traffic. Mesoscale models occupy an intermediate position between macro- and micro-level models and are usually based on kinetic equations.

Micro-level models are usually divided into car-following models and models based on cellular automata (CA), which have recently received major research attention. This is due to both the simplicity of the models themselves and their high computational performance due to natural parallelization. At the same time, models based on CA provide a realistic picture of traffic. The influential model of Nagel and Schreckenberg (NaSch) [8], [9] introduced a fundamental basis for the development of future CA traffic models. Over the past 30 years, a large number of traffic flow models based on CA have been proposed. Mostly, they have become more complicated: micro-level models are becoming more and more accurate. However, they have moved away from the original philosophy of the CA approach, which implies simplicity and even some schematism of the micro-level model; this allows one to obtain relatively realistic characteristics of the system at the macroscale. For example, in recent studies by Jiang et al. [10] and Vranken et al. [3], cellular automaton models were proposed for heterogeneous traffic consisting of human-driven and automated vehicles. In these models, in order to provide a time step equal to 0.1 s, a single vehicle occupied 500 cells. One can say that this approach already has little in common with cellular automata, but rather represents a high-resolution finite-difference discretization of a process that is continuous in space and time. Yang et al. [11] utilized the modified NaSch model, and a single car occupied 15 cells in their study. Lo and Hsu [12] studied heterogeneous traffic with AVs and HDVs based on a modified NaSch model, but there was no V2V interface in their study, so they were unable to consider platoon formation. At the same time, Kerner, in his recent work [13] dedicated to the development of a three-phase AV model, argues that a time step equal to 1 s is enough to simulate accident-free traffic that includes AVs. A time step equal to 1 s is used in the original NaSch model and its multiple modified versions, as it is the average human driver reaction time.

It is interesting to note that in parallel with the intensive development of models of the Nagel-Schreckenberg type, models of rather complex transport systems were proposed (for example, the model of the transport system of the city of Geneva [14]) that were based on the simplest rule of the evolution of cellular automata—Wolfram's rule 184 [15], which is completely deterministic.

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It may be useful in some situations to simplify the underlying rules to observe some processes more clearly. The aim of this paper is to develop a simple cellular automata model of a heterogeneous traffic flow that allows one to reproduce the main characteristics of a traffic flow and to evaluate the effect of the ratio of vehicles of various types on the traffic flow and average vehicle velocity.

II. SIMULATION MODEL DESCRIPTION

A. Rules of motion for different types of vehicles

In classical cellular automata models of traffic, which are suitable for describing HDV traffic, the following approach is applied. The roadway is divided into a finite number of cells. Each cell can contain only one vehicle. Hence, the state of the cell can be described with one bit—either the cell is occupied by the vehicle, or it is not. The proposed model is based on the simplest time evolution rule of a cellular automaton—Wolfram's rule 184, which is as follows. If the next cell in the direction of motion of the vehicle is not occupied by another vehicle, then the current vehicle moves one cell forward; otherwise, it remains in the current cell. The described rule is illustrated in Fig. 1.

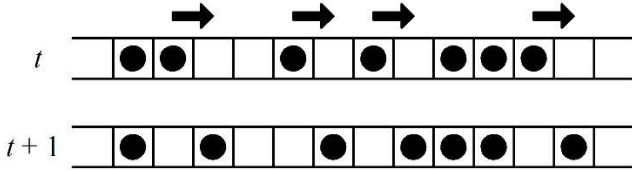


Fig. 1. Illustration of Wolfram's rule 184. The arrows indicate the vehicles that move when time evolves from step t to step $t + 1$

Let us denote the current position (cell number) of the i -th vehicle as x_i , and the distance to the next vehicle (the number of unoccupied cells between the i -th vehicle and the $(i + 1)$ -th vehicle) as g_i ; then, the described rule can be written as follows:

$$x_i(t+1) = \begin{cases} x_i(t) + 1, & \text{if } g_i(t) > 0, \\ x_i(t), & \text{if } g_i(t) = 0. \end{cases} \quad (1)$$

As a result of applying this rule, a fundamental diagram (or the flow-density diagram, which is the normalized vehicle flow versus normalized vehicle density; see subsection B of section 2 for more details) can be plotted; it has a symmetric triangular shape [16]. Although such a diagram is qualitatively correct, it should be noted that it is overly schematic—real-world data show a much more complex and rich behavior than this curve [6], [7].

In order to make the model of HDV motion more realistic, we will introduce elements of stochasticity into it. In this case, step (1) will be performed with a certain probability P , which will depend on g_i :

$$P = \begin{cases} p_1, & \text{if } g_i(t) = 1, \\ p_2, & \text{if } g_i(t) = 2, \\ p_3, & \text{if } g_i(t) > 2, \end{cases} \quad (2)$$

where it is assumed that $p_1 \leq p_2 \leq p_3$. Hence, the greater the probability of random braking for a vehicle, the smaller the distance to the next vehicle in the direction of its motion will be. It should also be noted that this model can show a different phase transition type if p_3 is set equal to 1. In this case, random braking is applied only if there is a vehicle no more than 2

unoccupied cells forward; otherwise, the motion is completely deterministic. For such a situation, the order parameter strictly equals zero for a free flow regime (see [17] for more information on the order parameter for a traffic model). This approach can be considered a generalization of Takayasu and Takayasu's model [18], in which only vehicles with more than one free cell in front of them can freely and deterministically move forward, while the vehicles with only one free cell only move with a certain probability. Unlike the classical Nagel-Schreckenberg model, which is reduced to the classical Wolfram model for a maximum velocity of 1 and a probability of accidental braking equal to zero, the proposed model takes into account the slow-to-start effect. If $p_1 = p_2 = p_3$, model (2) is reduced to the classical NaSch model with a maximum velocity equal to 1 [8].

Fig. 2 shows typical space-time diagrams (x - t diagrams) for the classic Wolfram's rule 184 (1) and the proposed rule (2) for the case of a periodic boundary condition. It can be qualitatively seen that the introduction of stochasticity to the simple cellular automata rule allows one to produce x - t patterns which look more realistic even when the maximum vehicle velocity is set to 1.

The reaction of the following vehicle to a change in the driving behavior of the leading vehicle is determined by the types of these vehicles. In the considered heterogeneous

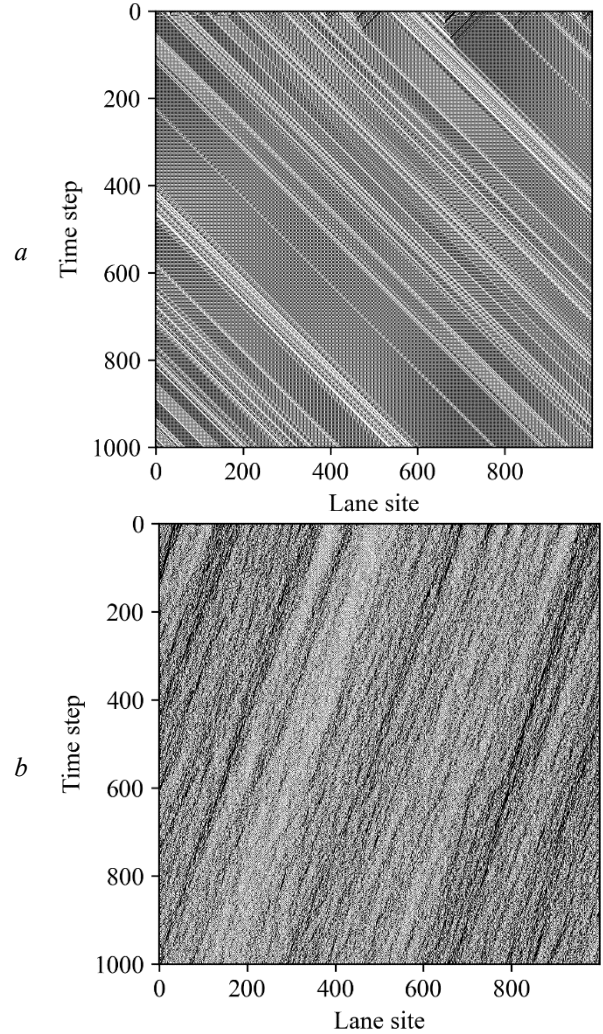


Fig. 2. Typical space-time diagrams for a periodic road with 1000 cells and 450 vehicles (vehicle density is 0.45): a shows a scenario in which rule (1) is used and b shows a scenario in which rule (2) is used

system, there can be four car-following types, which are illustrated in Fig. 3:

- 1) HDV follows HDV.
- 2) HDV follows AV.
- 3) AV follows HDV.
- 4) AV follows AV.

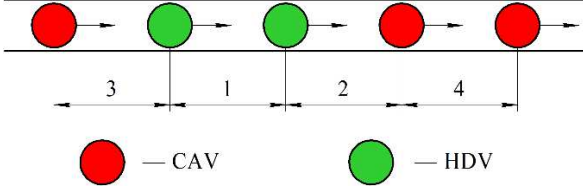


Fig. 3. Car-following types for heterogeneous traffic

We assume that the motion of a following HDV does not depend on how the leading vehicle is controlled (by a human or automatically), since the human reaction time to the change in the leading vehicle's driving behavior will be the same in both cases: it is determined by the driver only. Therefore, in cases 1 and 2, the rules of motion will be the same and will be described by rule (2).

The response time of the AV in case 3 is determined by the response time of the adaptive cruise control system, and in case 4, by the communication system between the two AVs. It is assumed that for AVs there are no random braking effects (which are inherent for HDVs); therefore, the difference between case 3 and cases 1 and 2 is that there is no random braking. Hence, the motion is described by the classical Wolfram's rule 184 (1).

The main difference between case 4 and the cases above is that a group of sequentially moving CAVs can form a platoon of several vehicles, which moves as a whole due to the exchange of information via the V2V interface. Therefore, in order to describe the motion in case 4, a different rule must be applied. First, if there is a free cell in front of the CAV, then it moves into that cell. Second, if the cell in front of the CAV is occupied by another CAV, then it checks to see if there is a free cell in front of the leading CAV, or if there is another CAV in this cell. If there is a set of sequentially located cells with a maximum length of S (here S is the maximum allowable platoon size that can be formed by CAVs) occupied by CAVs, and there is a free cell in front of these cells, then all these vehicles are shifted by one cell as a whole. The described rule is illustrated in Fig. 4.

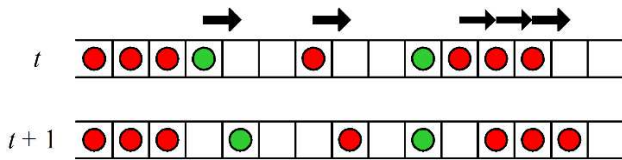


Fig. 4. Rules of motion for the platoon of CAVs. The arrows indicate the vehicles that move when time evolves from step t to step $t + 1$. HDVs are denoted by green circles, and CAVs are denoted by red circles

B. Traffic simulation model and macroparameters

We will consider the classical topological structure of a transport network—a single-lane road with periodic boundary

conditions that is equivalent to a closed circular road. There can be two types of agents in the system—HDVs and AVs.

The transportation system can be characterized by a pair of macroparameters: the normalized vehicle density ρ and normalized flow q .

Since the system is closed due to its topology, the total number of vehicles remains constant in a single simulation episode. Hence, the vehicle density can be calculated as follows:

$$\rho = \frac{N}{n_L}, \quad (3)$$

where N is the total number of vehicles in the system and n_L is the total number of cells.

The average normalized flow is determined by the following formula:

$$q = \frac{1}{T_p} \sum_{t=0}^{T_p-1} \left[\frac{1}{n_L} \sum_{i=0}^{n_L-1} v_i(t) \right], \quad (4)$$

where T_p is the number of time steps during which the simulation results are evaluated and $v_i(t)$ is the vehicle velocity at step t (its values can be 0 or 1; therefore this value can be interpreted as a logical flag indicating that the vehicle moves during the current time step), which can be calculated as

$$v_i(t) = x_i(t) - x_i(t-1). \quad (5)$$

At the initial moment of the simulation, a given number of vehicles (in accordance with a given vehicle density and the fraction of HDVs in the system) are randomly and uniformly distributed in the system. The velocity of all vehicles at the initial moment is equal to zero. The system then evolves for a run-up period of $T_{rup} = 5000$ time steps to reach a steady state. After that, for $T_{op} = 4000$ time steps, the observation period continues; during this period of time, the values of the normalized flow (4) are evaluated. This means that $T_p = T_{op}$. Thus, the total number of time steps for a single simulation of the system is 9000.

The total number of spatial cells is $n_L = 1000$. In the calculations, the following values of the probabilistic parameters of the model were taken: $p_1 = 0.3$, $p_2 = 0.7$ and $p_3 = 0.99$.

III. RESULTS AND DISCUSSION

Fig. 5 shows the fundamental diagrams for various fractions of HDVs in the system. The maximum platoon size for AVs in this case was set to 6.

These diagrams show a well-known tendency: an increase in the fraction of CAVs can increase the flow by a factor of 4–5 [10]. Remarkably, there is a significant gap even for cases in which the fraction of HDVs in the system is 25% and 0%—the flux differs by almost 2 times in these two cases.

Fig. 6 shows the distribution of the average velocity depending on the vehicle density for the same cases shown in Fig. 5. A decrease in the average vehicle velocity is associated with increasing vehicle density values and with an increase in the proportion of HDVs in the system.

The distribution of the order parameter for the cases discussed above is shown in Fig. 7.

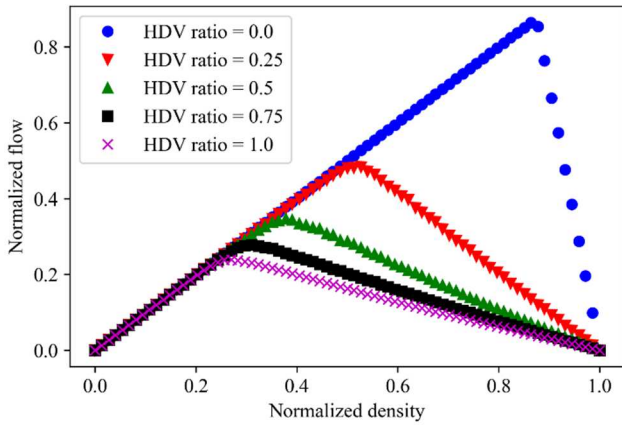


Fig. 5. Fundamental diagram (normalized flow vs normalized density) for different fractions of HDVs in the system

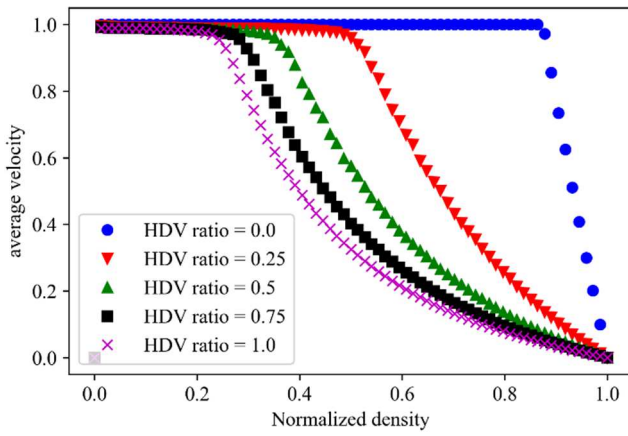


Fig. 6. Average vehicle velocity vs normalized vehicle density for different fractions of HDVs in the system

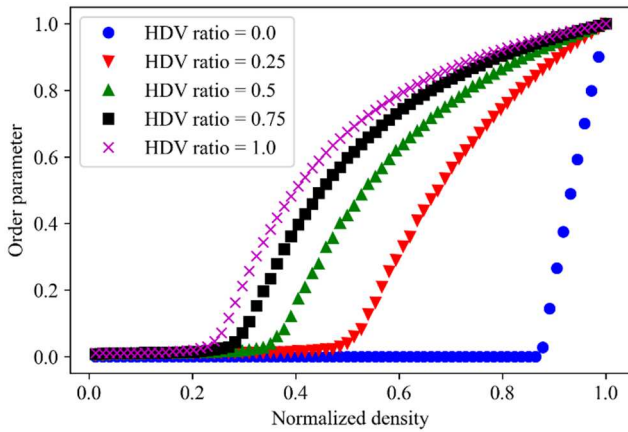


Fig. 7. Order parameter vs normalized vehicle density for different fractions of HDVs in the system

Fig. 8, Fig. 9 and Fig. 10 show fundamental diagrams, the average vehicle velocities and the values of the order parameter, respectively, in the system for the case in which all vehicles in the system are automated (which means that the HDV fraction is equal to zero), and the maximum allowable platoon size that CAVs can form varies.

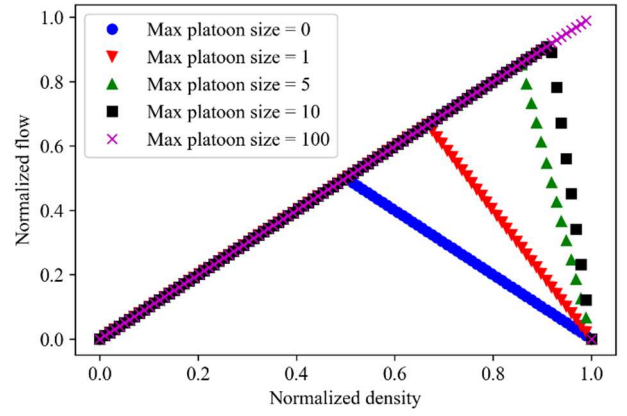


Fig. 8. Fundamental diagrams of the system of CAVs at various values of the maximum platoon size

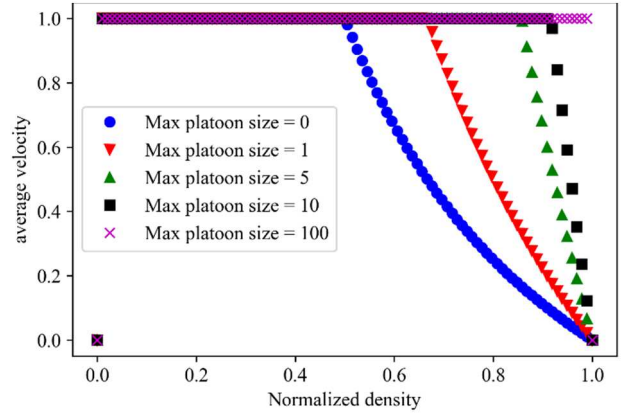


Fig. 9. Average vehicle velocities vs normalized vehicle density for the system of CAVs at various values of the maximum platoon size

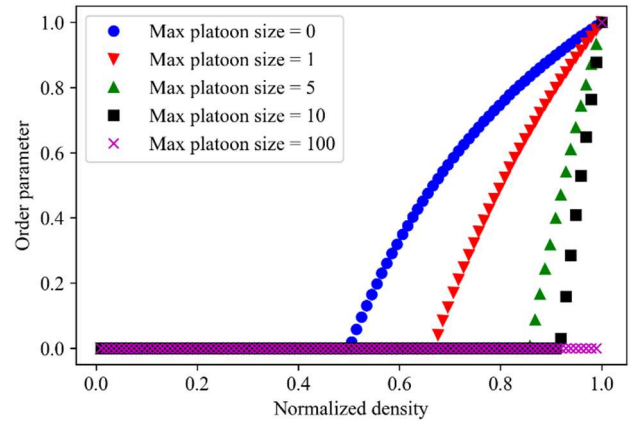


Fig. 10. Order parameter vs normalized vehicle density for the system of CAVs at various values of the maximum platoon size

A maximum platoon size equal to zero corresponds to the case of Wolfram's rule 184 since there is no random braking in the rules of motion for CAVs. As the maximum platoon size increases, the flow grows linearly—at the limit, with an infinite maximum platoon size (or, more precisely, a maximum platoon size equal to $n_L - 1$), the flow will be maximum if there is at least one free cell in the system. Once there is not, the motion will stop, and the flux drops down to zero.

Taking into account the idealization of the motion of the platoon of CAVs, an optimistic result was obtained, according

to which the increase in flow is practically unlimited with an increase in the fraction of CAVs. This result, however, is also supported by the results of more complex modeling [10]. Like many classical models, the proposed model correctly describes only two main phases of traffic flow, the free phase and the jammed phase, while the synchronized flow phase is not described by this model. Since the classical approach of cellular automata, like any modeling approach, should reflect only the aspects of the studied phenomenon that are essential from the point of view of the problem being solved, in this case it is permissible. Since from the point of view of calculating macroparameters and determining the influence of the presence of unmanned vehicles on the flow, the presence of a synchronized phase in the flow of human-driven vehicles can be neglected as a first approximation.

IV. CONCLUSION

The proposed approach for describing heterogeneous traffic flows containing human-driven and connected automated vehicles allows one to study the effect of the fraction of connected automated vehicles in the total traffic flow on the flow and average velocity of the vehicles. As a result of the study, it was shown that an increase in the fraction of connected automated vehicles in the transport systems of smart cities dramatically increases the flow and average velocity of vehicles. The main advantage of the proposed approach is the simplicity of the microscopic model of vehicle motion. Since the purpose of this approach is not predictive but explanatory, it can be further developed and made more complex through the introduction of a discrete set of vehicle velocities, for example.

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