Computer Vision

18AI742

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Dr. D. Antony Louis Piriyakumar, Dean (Research & Development) Registered Indian patent agent (IN/PA 3041)





Contents

- 1) Binocular camera geometry and the epipolar constraints
- 2) Binocular reconstruction
- 3) Human stereopsis
- 4) Local methods for binocular fusion
- 5) Global methods for binocular fusion
- 6) Using more cameras
- 7) Application: robot navigation
- 8) Conclusion
- 9) Q&A

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{j}_c$$
where
$$\nabla = \hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial y} + \hat{\mathbf{k}} \frac{\partial}{\partial z}$$



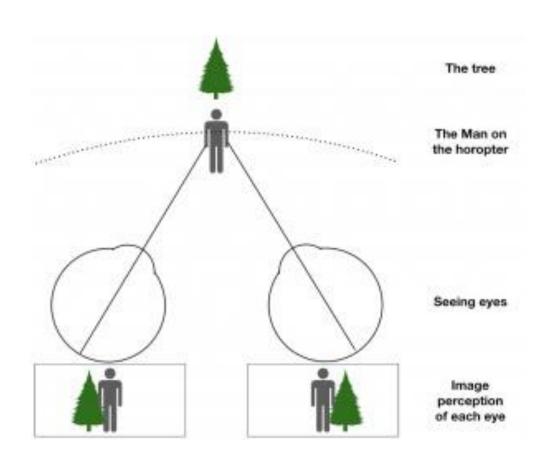
Fusing the pictures recorded by our two eyes

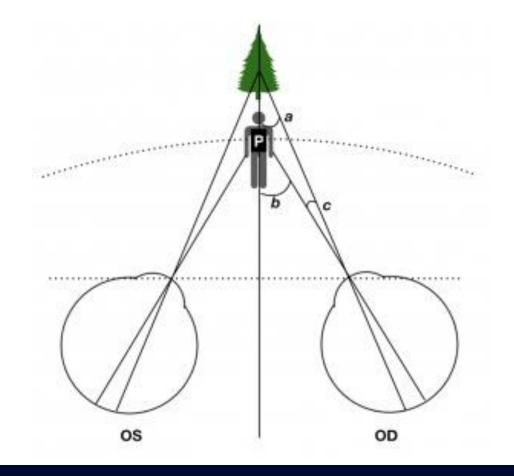
Exploiting the difference (or disparity) between them

Allows us to gain a strong sense of depth

Mimicing our ability to perform this task, known as stereopsis









Robocup finals 2023



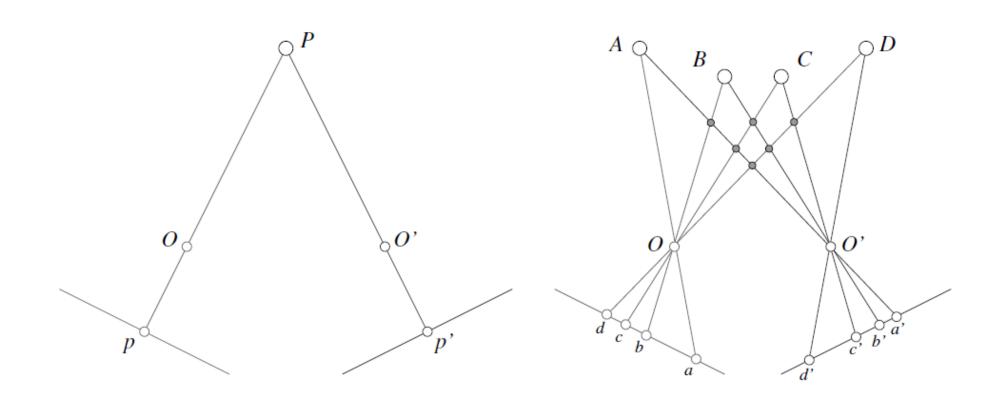


Humanoid robot playing tennis with human being

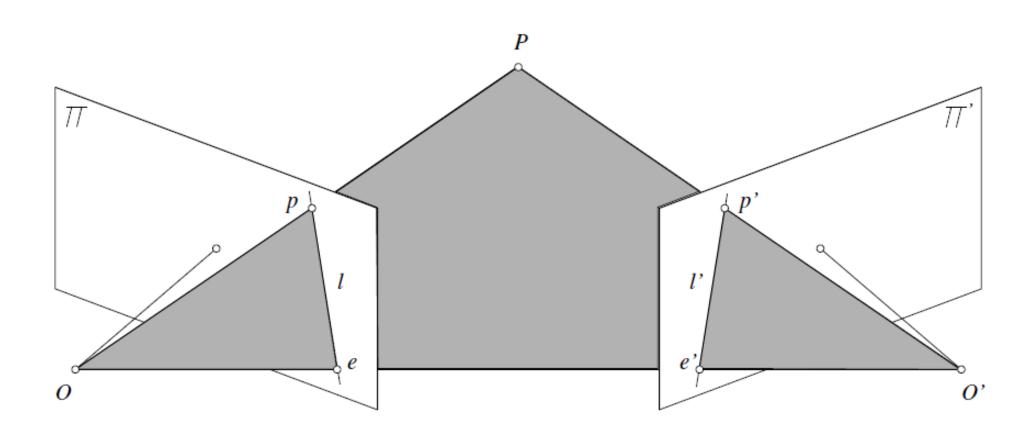




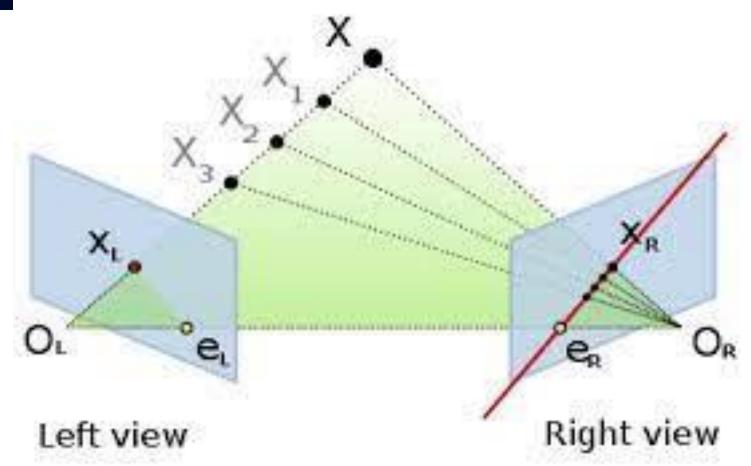
The binocular fusion problem



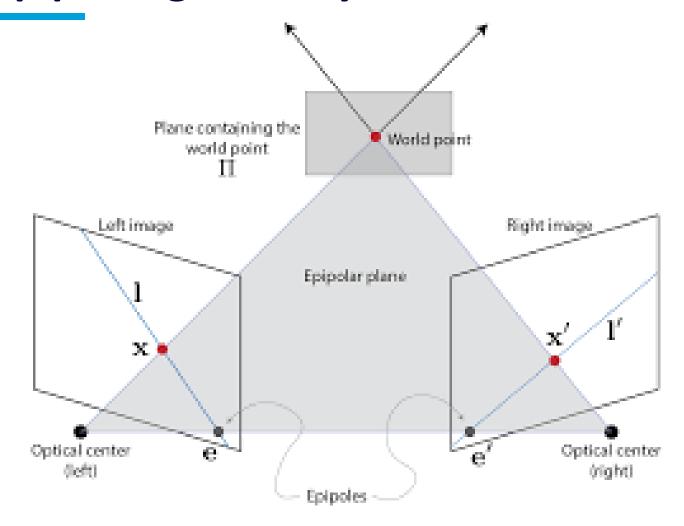




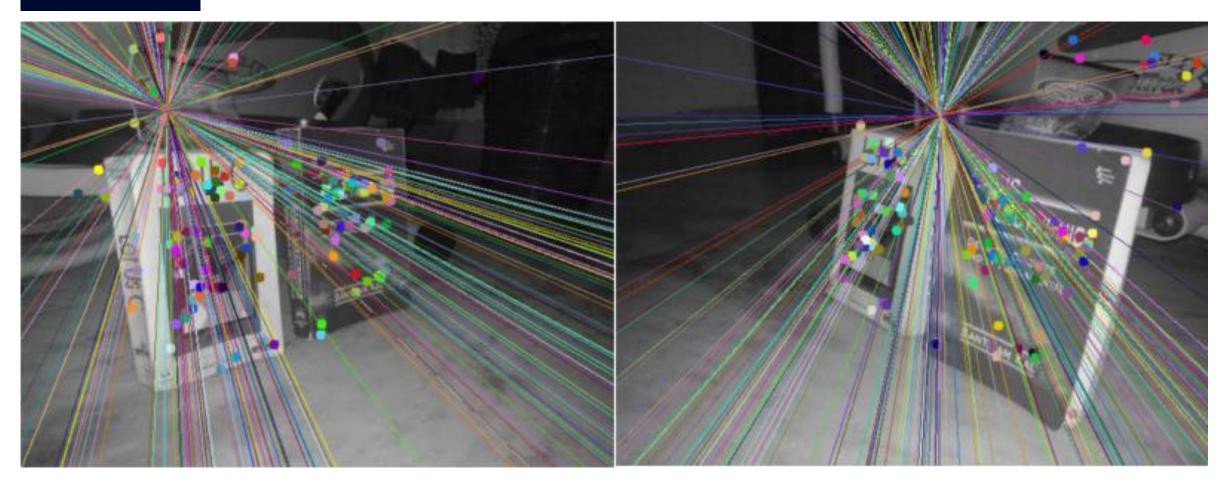








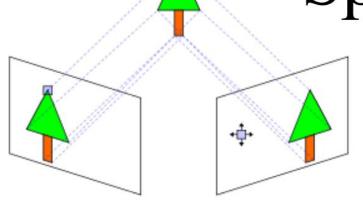


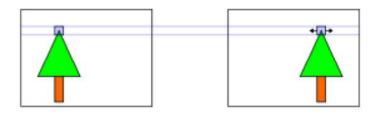


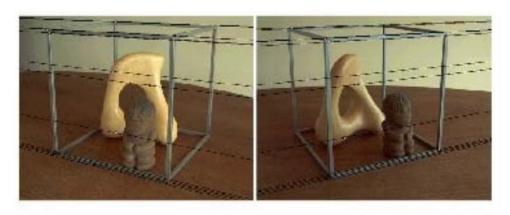


Stereo Pair 2nd Rectified Stereo Pair

Special case



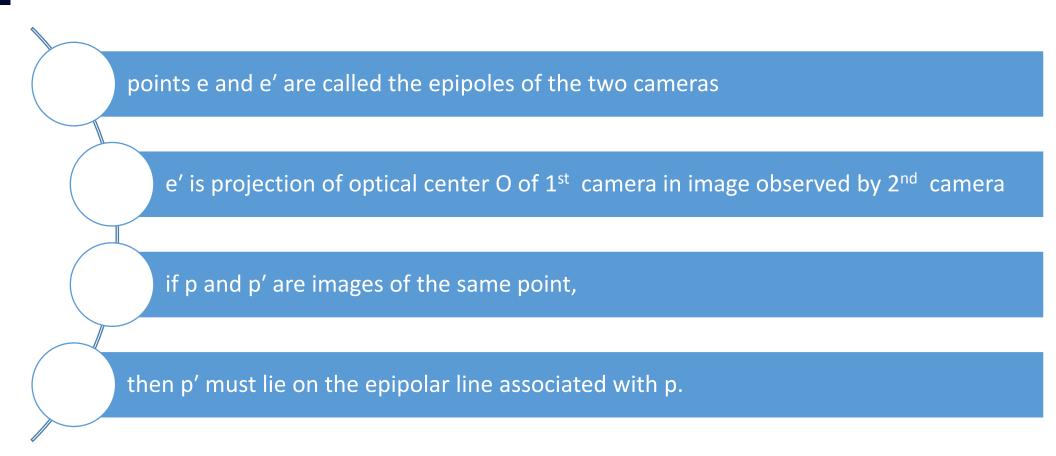








Binocular camera geometry and the epipolar constraint





The Essential Matrix

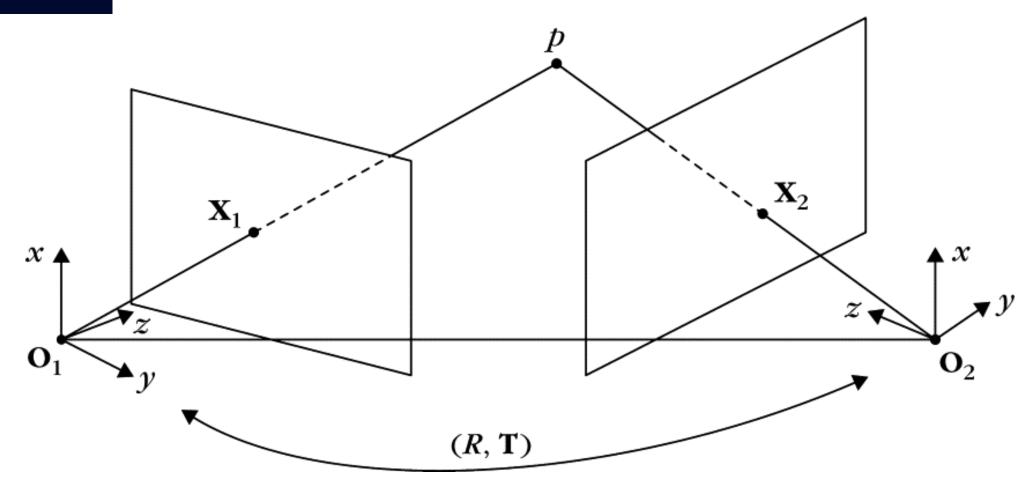
normalized image coordinates— take p = p.

three vectors \overrightarrow{Op} , $\overrightarrow{O'p'}$, and $\overrightarrow{OO'}$ must be coplanar.

one of them must lie in the plane spanned by the other two

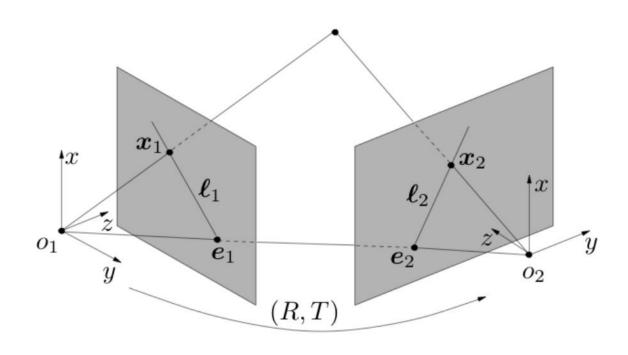
$$\overrightarrow{Op} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'p'}] = 0.$$







Calibrated 2 view geometry



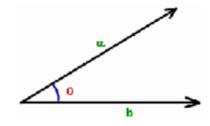
$$\boldsymbol{X}_2 = R\boldsymbol{X}_1 + \boldsymbol{T}$$

$$oldsymbol{X}_1 = \lambda_1 oldsymbol{x}_1, \quad oldsymbol{X}_2 = \lambda_2 oldsymbol{x}_2$$

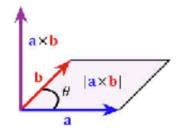


Recall

Dot product: $\mathbf{a} \cdot \mathbf{b} = ||\mathbf{a}|| \ ||\mathbf{b}|| \cos \theta$



Cross product: $\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \sin \theta \mathbf{n}$



Cross product matrix:
$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \equiv \mathbf{\hat{a}b}$$

Important property (skew symmetric): $\mathbf{\hat{a}}^T = -\mathbf{\hat{a}}$



The Essential Matrix

$$\overrightarrow{Op} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'p'}] = 0.$$

$$p \cdot [t \times (\mathcal{R}p')] = 0,$$

$$p^T \mathcal{E}p' = 0,$$

$$\mathcal{E} = [t_{\times}]\mathcal{R},$$

[a \times] denotes the skew-symmetric matrix such that [a \times]x = a \times x is the cross-product of the vectors a and x.



Derivation of essential matrix equation/ epipolar geometry constraint

$$\boldsymbol{X}_2 = R\boldsymbol{X}_1 + \boldsymbol{T}$$

$$oldsymbol{X}_1 = \lambda_1 oldsymbol{x}_1, \quad oldsymbol{X}_2 = \lambda_2 oldsymbol{x}_2$$

$$\lambda_2 \boldsymbol{x}_2 = R\lambda_1 \boldsymbol{x}_1 + \boldsymbol{T}$$

Take (left) cross product of both sides with T

$$\lambda_2 \widehat{T} \boldsymbol{x}_2 = \widehat{T} R \lambda_1 \boldsymbol{x}_1 + \underbrace{\widehat{T} T}_{=\boldsymbol{0}}$$

Take (left) dot product of both sides with x_2

$$\lambda_2 \underbrace{\boldsymbol{x}_2^{\top} \widehat{T} \boldsymbol{x}_2}_{=0} = \boldsymbol{x}_2^{\top} \widehat{T} R \lambda_1 \boldsymbol{x}_1$$

$$\boldsymbol{x}_2^{\top} \widehat{T} R \boldsymbol{x}_1 = 0$$

$$\mathcal{E} = [t_{\times}]\mathcal{R},$$



Key points in essential matrix 3*3

 $I = \xi p'$ coordinate vector of epipolarline I associated with p' in the first image

 $p \cdot l = 0$, expressing the fact that the point p lies on l.

Essential matrices are singular

because t is parallel to the coordinate vector e of the first epipole,

$$\mathcal{E}^T e = -\mathcal{R}^T [t_{\times}] e = 0.$$



The fundamental matrix

Native image coordinates

$$p = \mathcal{K}\hat{p}$$
 and $p' = \mathcal{K}'\hat{p}'$,

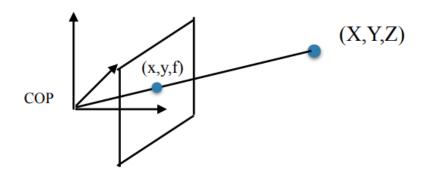
K and K' are the 3×3 calibration matrices associated with the two cameras

$$p^T \mathcal{F} p' = 0,$$
 $\mathcal{F} = \mathcal{K}^{-T} \mathcal{E} \mathcal{K}'^{-1}$

$$\widetilde{m{l}}' = \mathcal{F} m{p}' \; (\mathrm{resp.} \; \; m{l} = \mathcal{F}^T m{p})$$



Projecting from camera coordinate system to image coordinates

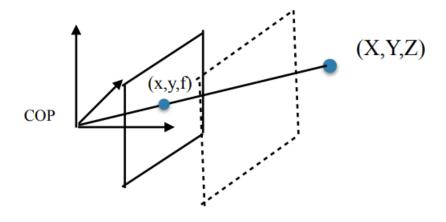


$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\lambda \mathbf{x} = K\mathbf{X}$$



Projecting from camera coordinate system to normalized image coordinates



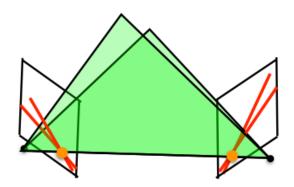
If K is known, work with warped image

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = K^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\lambda \mathbf{x}' = \mathbf{X}$$



The fundamental matrix

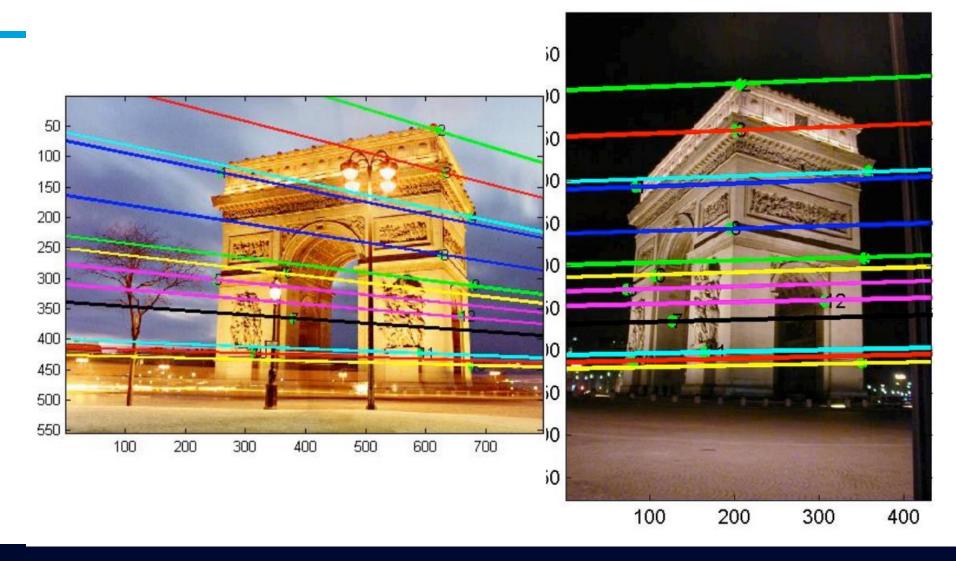


In uncalibrated case, we need to account for camera intrinsics:

$$\lambda \mathbf{x} = K\mathbf{X}$$

$$E = \hat{T}R$$
$$F = K_2^{-T}EK_1^{-1}$$







$$F = \begin{pmatrix} -0.00310695 & -0.0025646 & 2.96584 \\ -0.028094 & -0.00771621 & 56.3813 \\ 13.1905 & -29.2007 & -9999.79 & 1.0 \end{pmatrix}$$



$$x = 343.5300 y = 221.7005$$

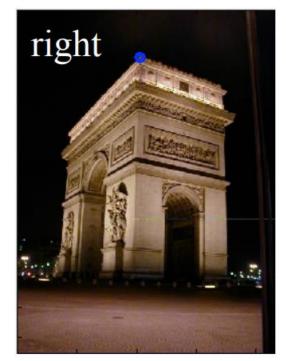
0.0001 0.0295 $0.0045 \rightarrow 0.9996$ -1.1942 -265.1531

normalize so sum of squares of first two terms is 1 (optional)



$$L = (0.0010 -0.0030 -0.4851)$$

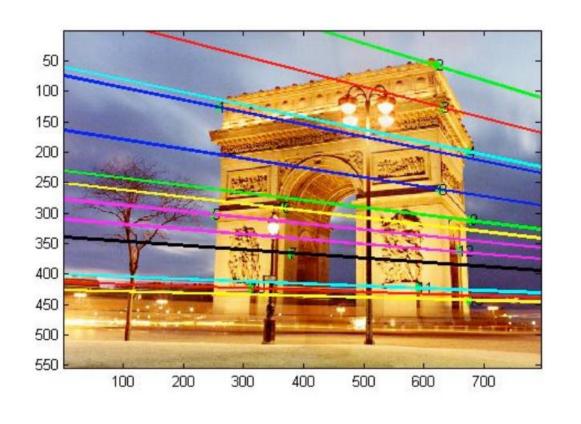
$$\rightarrow$$
 (0.3211 -0.9470 -151.39)



x = 205.5526 y = 80.5000



where is the epipole?



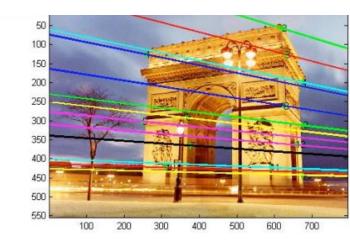
$$F * e_L = 0$$

vector in the right nullspace of matrix F

However, due to noise, F may not be singular. So instead, next best thing is eigenvector associated with smallest eigenvalue of F



$$>> [u,d] = eigs(F' * F)$$



eigenvector associated with smallest eigenvalue

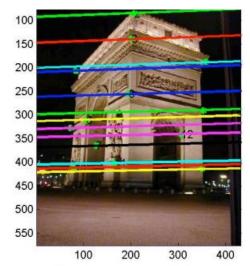
>>
$$uu = u(:,3)$$

 $uu = (-0.9660 -0.2586 -0.0005)$

>> uu / uu(3) : to get pixel coords (1861.02 498.21 1.0)



$$\gg$$
 [u,d] = eigs(F * F')



eigenvector associated with smallest eigenvalue

>>
$$uu = u(:,3)$$

 $uu = (-0.9981 \ 0.0618 \ 0.0001)$

>> uu / uu(3) : to get pixel coords (-19021.8 1177.97 1.0)



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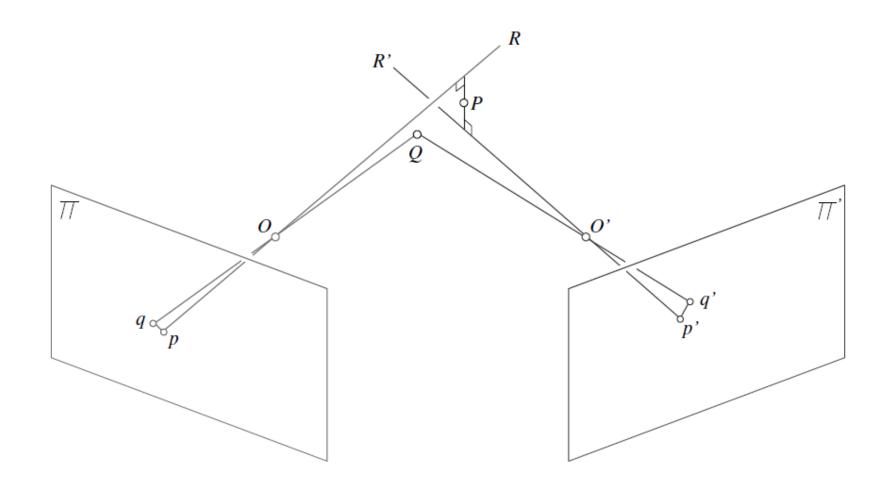
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$$where$$

$$\nabla = \hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial v} + \hat{\mathbf{k}} \frac{\partial}{\partial z}$$



Triangulation in the presence of measurement errors





Binocular reconstruction

Reconstruct the corresponding scene point by intersecting R = Op and R' = O'p'

the rays R and R' never actually intersect in practice,

consider the line segment perpendicular to R and R' that intersects both rays

mid-point P is the closest point to two rays and preimage of p and p'.



Alternatively, one can reconstruct a scene point using a purely algebraic approach

proach: given the projection matrices \mathcal{M} and \mathcal{M}' and the matching points p and p', we can rewrite the constraints $Zp = \mathcal{M}P$ and $Z'p' = \mathcal{M}P$ as

$$\begin{cases} \mathbf{p} \times \mathcal{M} \mathbf{P} = 0 \\ \mathbf{p}' \times \mathcal{M}' \mathbf{P} = 0 \end{cases} \iff \begin{pmatrix} [\mathbf{p}_{\times}] \mathcal{M} \\ [\mathbf{p}'_{\times}] \mathcal{M}' \end{pmatrix} \mathbf{P} = 0.$$

This is an overconstrained system of four independent linear equations in the homogeneous coordinates of P that is easily solved using the linear least-squares tech-



A rectified stereo pair

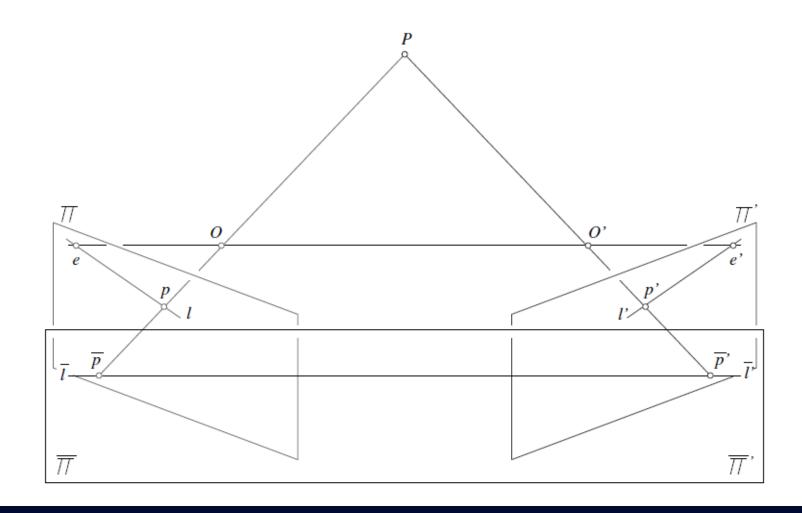




Image Rectification

replaced by two equivalent pictures with a common image plane parallel

to the baseline joining the two optical centers

by projecting the original pictures onto the new image plane

two degrees of freedom involved in the choice of the rectified image plane



Disparity and depth

left and right images, with coordinates (x, y) and (x', y),

disparity is defined as the difference d = x' - x.

B denotes the distance between the optical centers,

depth of P in the (normalized) coordinate system attached to the first camera

$$Z = -B/d$$
.

Coordinate vector of the point P in the frame attached to the first camera

$$P = -(B/d)p$$
, where $p = (x, y, 1)T$



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Human stereopsis

I and r denote the (counterclockwise) angles

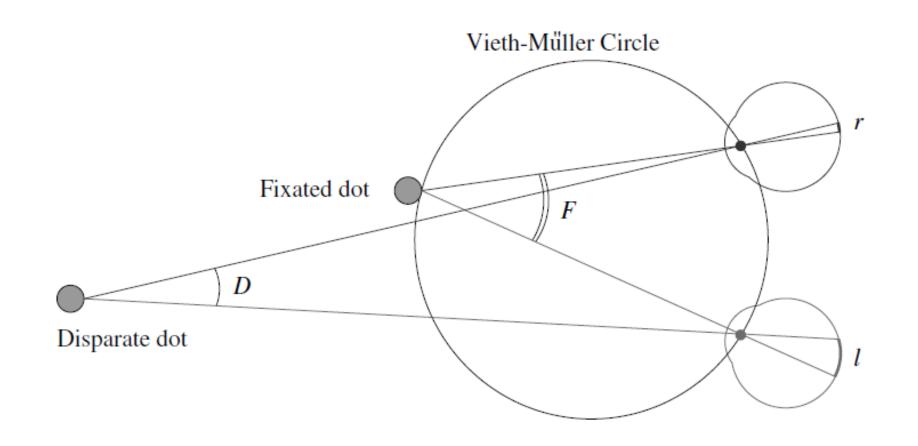
disparity as d = r - l. Also, d = D - F,

Points lying inside this circle have a positive disparity,

to rank order dots that are near the fixation point according to their depth.

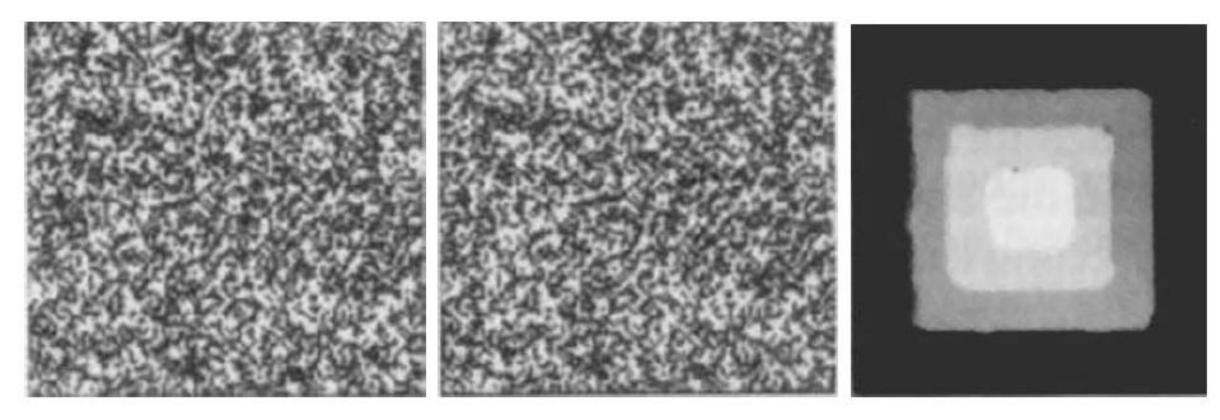


Vieth-M"uller circle





two pictures forming a random dot stereogram





a random dot stereogram

a pair of synthetic images obtained by randomly spraying black dots on white objects When viewed monocularly, the images appear completely random. when viewed stereoscopically, image pair gives impression of a square markedly Human binocular fusion cannot be explained by peripheral processes directly must involve the central nervous system and an imaginary cyclopean retina combines the left and right image stimuli as a single unit.



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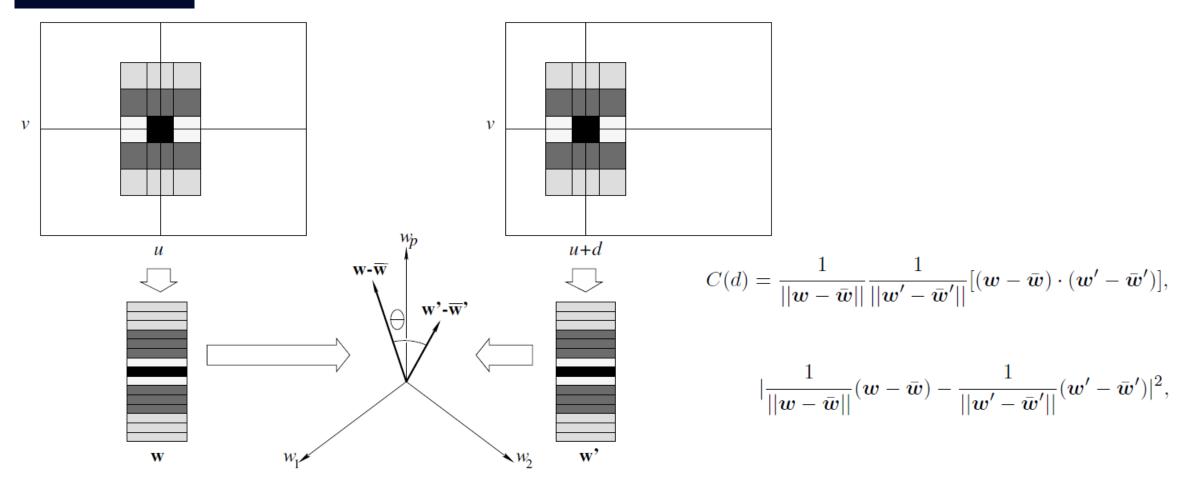
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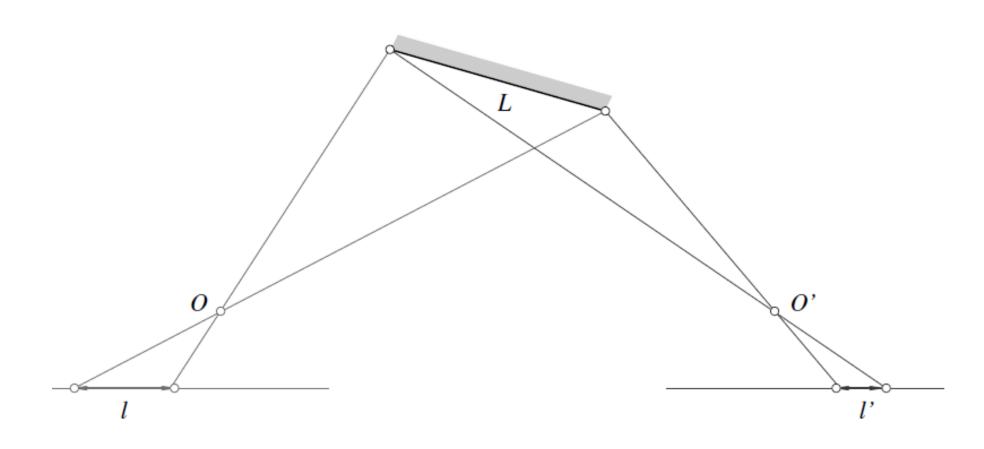


Correlation of two 3×5 windows along corresponding epipolar lines.





The foreshortening of an oblique plane is not the same for the left and right cameras: I/L != I'/L.





Correlation-based stereo matching

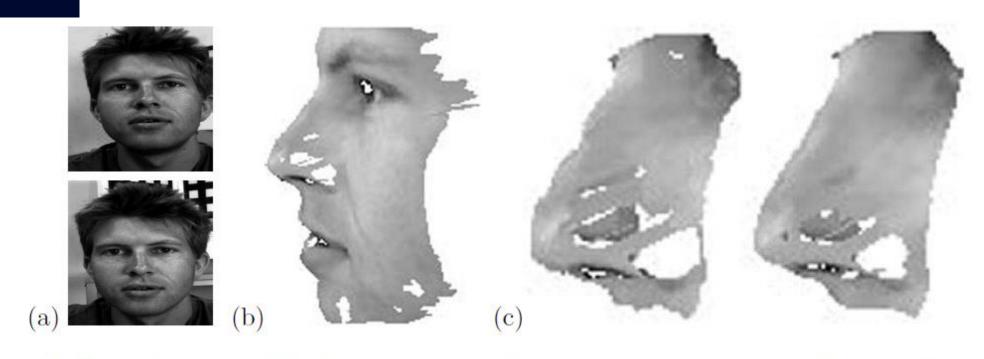


FIGURE 7.11: Correlation-based stereo matching: (a) a pair of stereo pictures; (b) a texture-mapped view of the reconstructed surface; (c) comparison of the regular (left) and refined (right) correlation methods in the nose region. The latter clearly gives better results. Reprinted from "Computing Differential Properties of 3D Shapes from Stereopsis

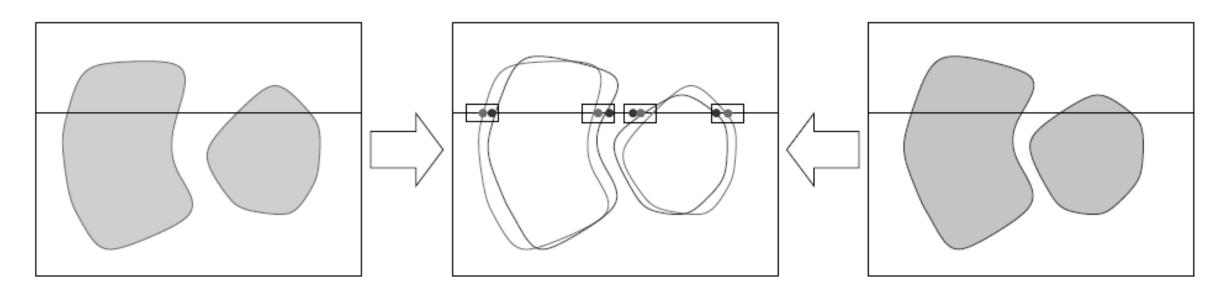


Algorithm 7.1: The Marr-Poggio (1979) Multi-Scale Binocular Fusion Algorithm.

- 1. Convolve the two (rectified) images with $\nabla^2 G_{\sigma}$ filters of increasing standard deviations $\sigma_1 < \sigma_2 < \sigma_3 < \sigma_4$.
- 2. Find zero crossings of the Laplacian along horizontal scanlines of the filtered images.
- 3. For each filter scale σ , match zero crossings with the same parity and roughly equal orientations in a $[-w_{\sigma}, +w_{\sigma}]$ disparity range, with $w_{\sigma} = 2\sqrt{2}\sigma$.
- 4. Use the disparities found at larger scales to offset the images in the neighborhood of matches and cause unmatched regions at smaller scales to come into correspondence.

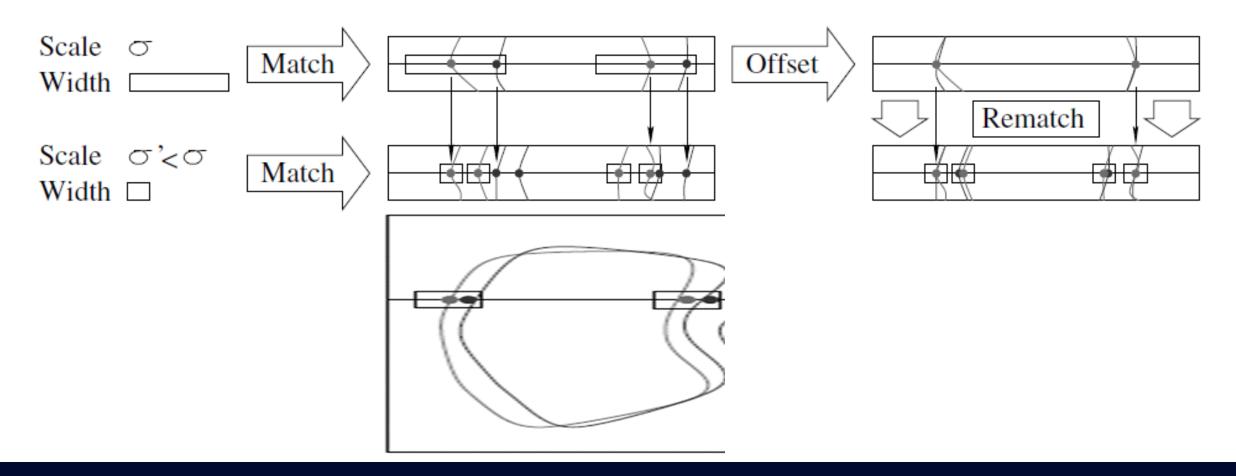


Matching zero crossings at a single scale



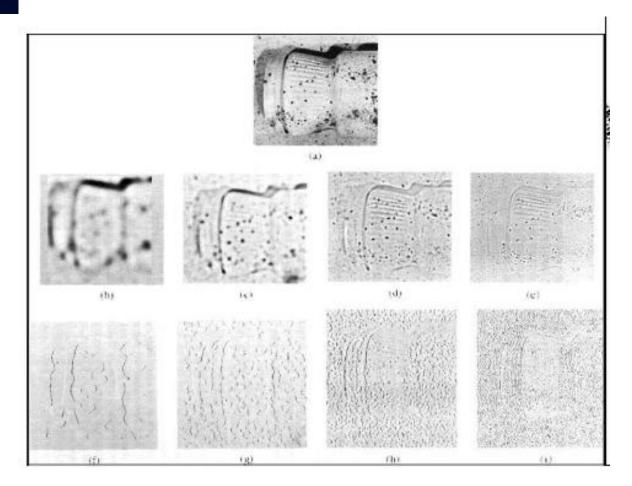


Matching zero crossings at multiple scales



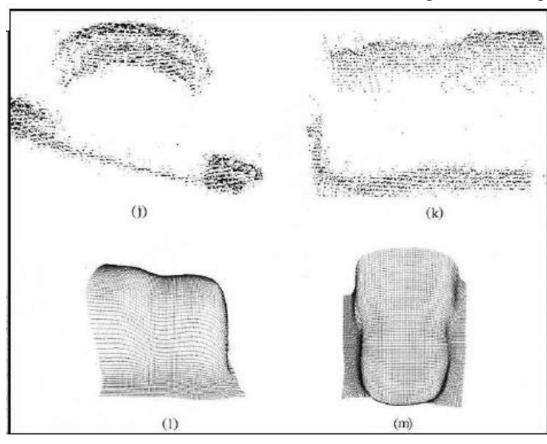


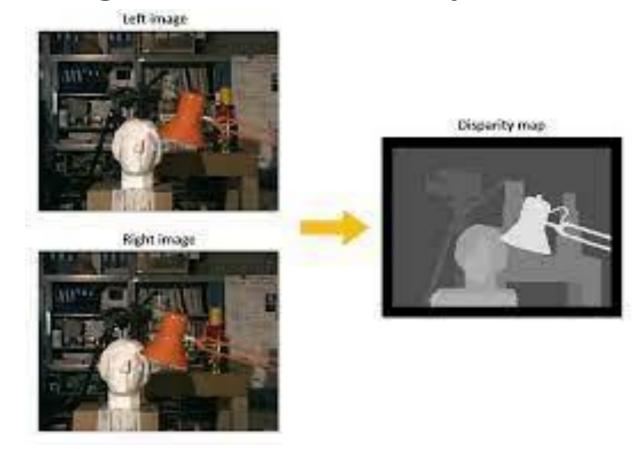
Output of four $\nabla 2G\sigma$ filters, and the corresponding zero crossings





Two views of the disparity map constructed by the matching process and two views of the surface obtained by interpolating the reconstructed points







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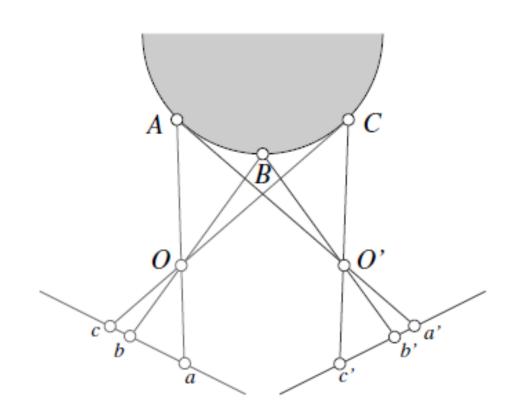
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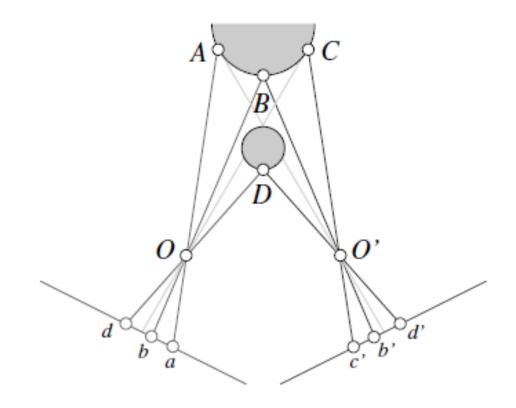
$$where$$

$$\nabla = \hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial v} + \hat{\mathbf{k}} \frac{\partial}{\partial z}$$



Ordering Constraints





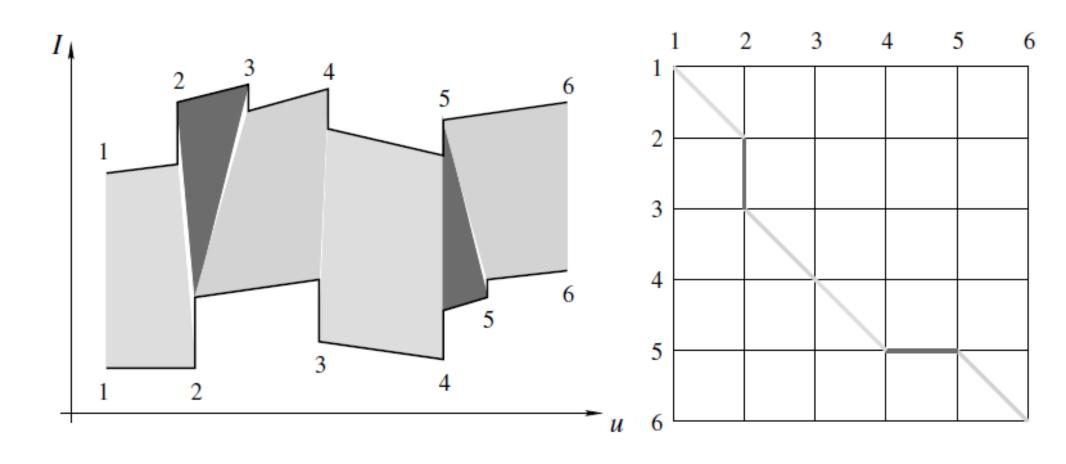


Ordering Constraints

Order of matching image features along a pair of epipolar lines is The inverse of the order of the corresponding surface attributes along the curve the epipolar plane intersects the observed object's boundary corresponding to missing correspondences associated with occlusion and/or noise.



Dynamic programming and stereopsis





A Dynamic-Programming Algorithm for Establishing Stereo Correspondences Between Two Corresponding Scanlines.

We assume the scanlines have m and n edge points, respectively (the endpoints of the scanlines are included for convenience). Two auxiliary functions are used: Inferior-Neighbors(k,l) returns the list of neighbors (i,j) of the node (k,l) such that $i \leq k$ and $j \leq l$, and Arc-Cost(i,j,k,l) evaluates and returns the cost of matching the intervals (i,k) and (j,l). For correctness, C(1,1) should be initialized with a value of zero.

```
% Loop over all nodes (k, l) in ascending order.
for k = 1 to m do
  for l = 1 to n do
    % Initialize optimal cost C(k,l) and backward pointer B(k,l).
    C(k,l) \leftarrow +\infty; B(k,l) \leftarrow \text{nil};
    % Loop over all inferior neighbors (i, j) of (k, l).
    for (i, j) \in \text{Inferior-Neighbors}(k, l) do
       % Compute new path cost and update backward pointer if necessary.
      d \leftarrow C(i, j) + \text{Arc-Cost}(i, j, k, l);
      if d < C(k, l) then C(k, l) \leftarrow d; B(k, l) \leftarrow (i, j) endif;
      endfor:
    endfor;
  endfor:
% Construct optimal path by following backward pointers from (m, n).
P \leftarrow \{(m,n)\}; (i,j) \leftarrow (m,n);
while B(i,j) \neq \text{nil do } (i,j) \leftarrow B(i,j); P \leftarrow \{(i,j)\} \cup P \text{ endwhile.}
```



Energy function $E: D^n \rightarrow R$

Let us assume as usual that the two input images have been rectified, and define a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ whose n nodes are the pixels of the first image and whose edges link pairs of adjacent pixels on the image grid (not necessarily on the same scanline). Given some allowed disparity range $\mathcal{D} = \{-K, \ldots, K\} \subset \mathbb{Z}$, we can define an energy function $E : \mathcal{D}^n \to \mathbb{R}$ by

$$E(d) = \sum_{p \in \mathcal{V}} U_p(d_p) + \sum_{(p,q) \in \mathcal{E}} B_{pq}(d_p, d_q), \tag{7.4}$$

where d is a vector of n integer disparities d_p associated with pixels p, $U_p(d_p)$ (unary term) measures the discrepancy between pixel p in the left image and pixel $p+d_p$ in the second one, and $B_{pq}(d_p,d_q)$ (binary term) measures the discrepancy between the pair of assignments $p \to p+d_p$ and $q \to q+d_q$. The first of these terms records the similarity between p and p+dp. It may be, for example, the sum of squared differences $U_p(d_p) = \sum_{q \in \mathcal{N}(p)} [I(q) - I'(q+dp)]^2$, where $\mathcal{N}(p)$ is some neighborhood of p. The second one is used to regularize the optimization process, making sure that the disparity function is smooth enough. For example, a sensible choice may be $B_{pq}(d_p,d_q) = \gamma_{pq}|d_p - d_q|$ for some $\gamma_{pq} > 0$.



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$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

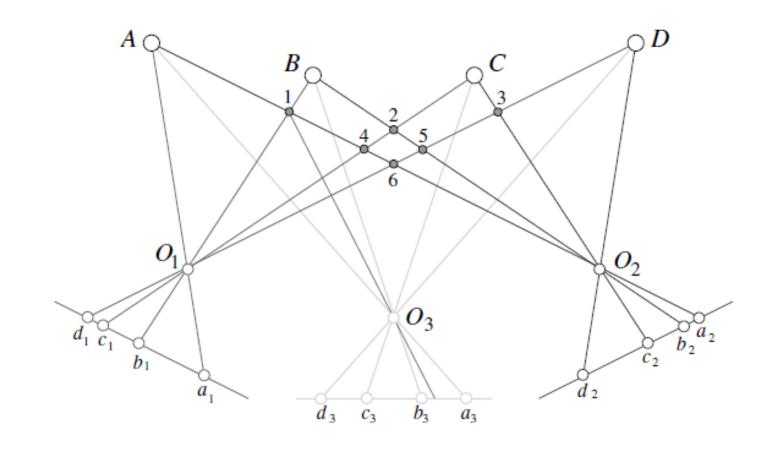
$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{j}_c$$

$$where$$

$$\nabla = \hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial v} + \hat{\mathbf{k}} \frac{\partial}{\partial z}$$

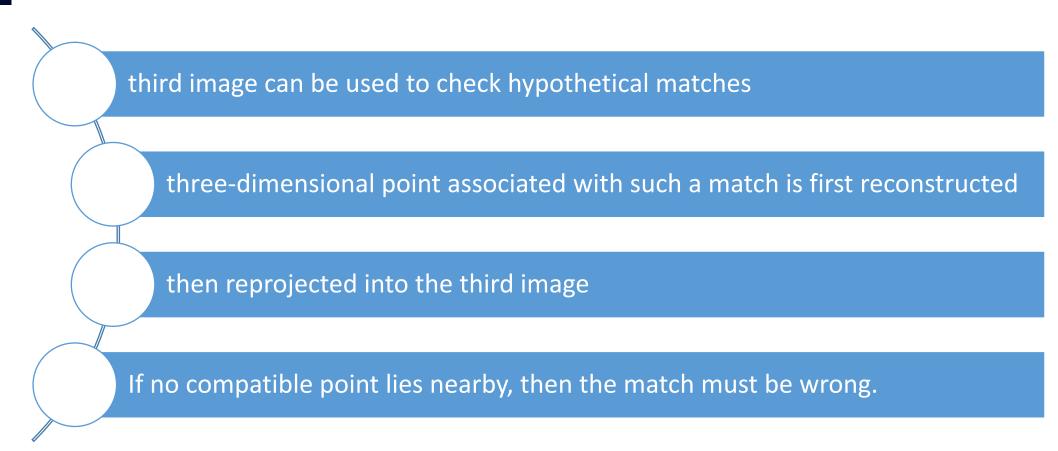


Using more cameras





Adding a third camera eliminates ambiguity inherent in two view point matching





A series of 10 images and the corresponding reconstruction.













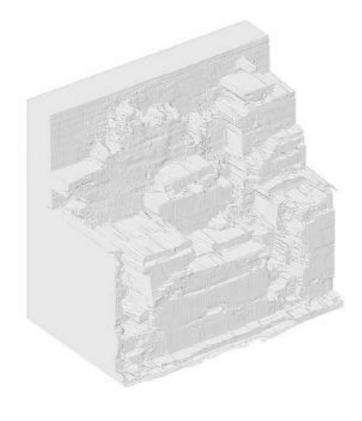














Stereopsis

Contents

- 1) Binocular camera geometry and the epipolar constraints
- 2) Binocular reconstruction
- 3) Human stereopsis
- 4) Local methods for binocular fusion
- 5) Global methods for binocular fusion
- 6) Using more cameras
- 7) Application: robot navigation
- 8) Conclusion
- 9) Q&A

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{j}_c$$

$$where$$

$$\nabla = \hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial v} + \hat{\mathbf{k}} \frac{\partial}{\partial z}$$

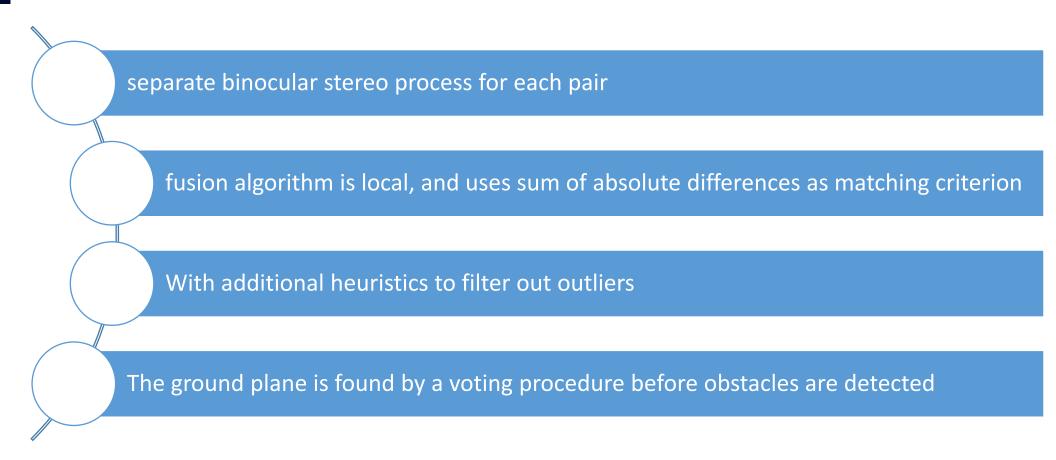


Robot navigation using the approach proposed in Hadsell et al. (2009)



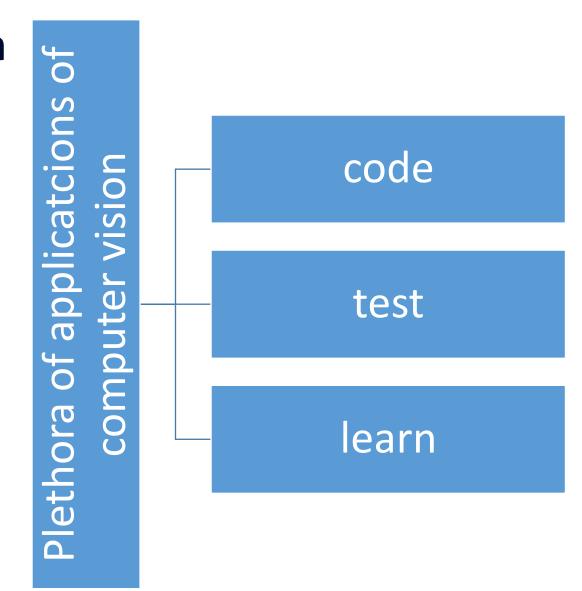


Robot navigation





Conclusion







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