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NUMBER PROPERTIES

Number Systems:
A Number is an abstract entity that represents a count or measurement.
All numbers fall in 2 categories Real Number, Complex Number
A Real Number can either be a Rational number or an irrational number.
Rational number can either be Natural numbers or Negative integers or Fractions

Natural Numbers
The natural numbers start off as follows: 1, 2, 3, 4, and 5 ... The "..." means that the list goes on forever. We give this set the name N.

If a number is in N, then its successor is also in N.
Thus, there is no greatest number, because we can always add one to get a larger one. N is an infinite set and hence can never be exhausted by removing its members one at a time.

Whole Numbers
If we add zero to our above list then we have the set of whole numbers.
i.e Whole numbers 0,1,2,3...

Negative numbers
Negative numbers are numbers which are less than zero. They are used to indicate a number that is opposite to the corresponding positive number (the absolute value), but equal in magnitude.
Example: -1, -2, -3, . . .

Remember -(n + 1)is always smaller than -n where n is a positive number.

Integer
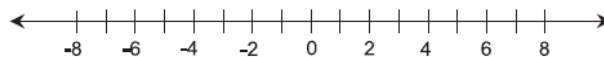Integers are the whole numbers, negative whole numbers, and zero.
One of the numbers ..., -2, -1, 0, 1, 2, . . .
But numbers like 1/2, 4.00032, 2.5, Pi, and -9.90 are not integers.

Note that zero is neither positive nor negative.
It may help you to think of numbers as occurring along a line that stretches infinitely in both directions.
Numbers to the left of the 0 point are negative, numbers to the right are positive.

Along the number line there are a series of dots that correspond to whole numbers (integers).
The spaces between the whole numbers are occupied by the numbers that are not whole (they contain fractions, and are called real numbers ex.1/2, 4.00032, 2.5, Pi, and -9.90).

Even and Odd
The terms even and odd only apply to integers. A number is said to be an even number if it is divisible by 2 or else it is an odd number.

Even numbers are: 2, 4, 6, 8, 10. . . . .40, 42, 44,. . . 312, 314, ... 1008,1010, ...686860....
Odd numbers are: . . 5, 7, 9. . . . .41, 43, 45,. . . 311, 313, ... 1007,1009, ...686861....

Note:
- 2.5 is neither even nor odd.
- Zero, on the other hand, is even since it is 2 times some integer: it's 2 times 0.

To check whether a number is odd, see whether it's one more than some even number:
Example:
7 is odd since it's one more than 6, which is even.
Another way to say this is that zero is even since it can be written in the form 2*n, where n is an integer. Odd numbers can be written in the form 2*n + 1.

Again, this lets us talk about whether negative numbers are even and odd: -9 is odd since it's one more than -10, which is even.
Every positive integer can be factored into the product of prime numbers, and there's only one way to do it for every number.
Example:
280 = 2x2x2x5x7, and there's only one way to factor 280 into prime numbers

Rational Number
A rational number is a number that can be expressed as a fraction p/q where p and q are integers and q ≠ 0.
i.e., Rational numbers are simply defined as ratios of integers. 1/2 is a rational number. 2/3 is also a rational number.

Note that all the integers are rational numbers, because you can think of them as the ratio of themselves to 1, as in 2 = 2/1 which is certainly the ratio of two integers, and so 2 is a rational number.

The decimal form of a rational number is either a terminating or repeating decimal.

Representation of rational numbers in decimal form

Any positive rational number p/q, after actual division, if necessary can be expressed as,

$p / q = m + r/q$

where m is non-negative integer and $0 \leq r < q$

Example:

$31/5 = 6 + 1/5 = 6.2$

There are few fractions for which the right most digit (or set of right most digit) recurs endlessly.

Example:

$1/3 = 0.33333. . . .$ and $5/11 = 0.45454 . . . .$

Note that the dots .... represent endless recurrence of digits.

The above examples are decimal numbers of the "non-terminating type".

In case of "non-terminating type" we have decimal fractions having an infinite number of digits. Some decimal fractions from this group have digits repeating infinitely. They are called "repeating or recurring " decimals.

In "endless recurring or infinite repeating" decimal fractions we can see that when p is actually divided by q the possible remainders are 1, 2, 3, ..., q -1. So one of them has to repeat itself in q steps. Thereafter the earlier numeral or group of numerals must repeat itself.

All the rational numbers thus can be represented as a finite decimal (terminating type) or as a recurring decimal.

Irrational Numbers

In mathematics, an irrational number is any real number that is not a rational number

i.e., one that cannot be written as a ratio of two integers

i.e., it is not of the form a/b where a and b are integers and b is not zero.

It can readily be shown that the irrational numbers are precisely those numbers whose expansion in any given base (decimal, binary, etc) never ends and never enters a periodic pattern.

The square root of 2 is a classic example of an irrational number: you cannot write it as the ratio of ANY two integers.

Prime Number

A prime number is a whole number that is not the product of two smaller numbers.

Note that the definition of a prime number doesn't allow 1 to be a prime number : 1 only has one factor, namely 1.

Prime numbers have exactly two factors, not "at most two" or anything like that. When a number has more than two factors it is called a composite number.

Here are the first few prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, etc.

PRIME FACTORS

Suppose n is a natural number, then there exists a unique sequence of prime numbers p1, p2, p3, . . . , pm, such that both of the following statements are true:

p1 ≤ p2 ≤ p3 ≤ . . . ≤ pm

p1 x p2 x p3 . . . x pm

The numbers p1, p2, p3, . . . , pm are called the prime factors of the natural number.

Every natural number n has one, but only one, set of prime factors.

This is an important principle known as the Fundamental Theorem of Arithmetic.

Number of Prime Factors

A number N of the form am x bn x cp

where a, b, c are all prime factors of number N has (m + 1)(n + 1)(p + 1) no. of prime factors

What is the fastest way to determine if a number is Prime?

The easiest & simplest method is to divide the number up to the closet square root of that number.

Example:

Lets consider 53. Number close to 53 having a perfect square is 64 and its square root is 8. Now start dividing 53 from 2 to 8.

There is no such number between 2 to 8 which divides 53 so 53 is a prime number.

Composite Numbers

A composite number is a positive integer which is not prime (i.e., which has factors other than 1 and itself).

The first few composite numbers (sometimes called "composites" for short) are 4, 6, 8, 9, 10, 12, 14, 15, 16, . . .

Note that the number 1 is a special case which is considered to be neither composite nor prime.

Numeric Operations

1. $A + 0 = A$ $A - 0 = A$
   $A \times 0 = 0$ $A/0 =$ Value Does not exist
2. $A \times 1 = 1$ $A + (-A) = 0$
   $A \times (1/A) = 1$
3. $A + B = B + A$
4. $A \times B = B \times A$
5. $A(B + C) = AB + AC$

Integer Roots

Suppose that 'a' is a positive real number. Also suppose that 'n' is a positive integer. Then n^{th} root of 'a' can also be expressed as the $(1/n)$ power of 'a'.

Thus, the second root (or square root) is the same thing as the 1/2 power

the third root (or cube root) is the same thing as the 1/3 power

the fourth root is the same thing as the 1/4 power; and so on.

Irrational-Number Powers

Suppose that 'a' is a real number. Also suppose that 'b' is a rational number such that $b = m/n$,

where 'm' and 'n' are integers and $\neq 0$.

Then the following formula holds true:
$a^b = a^{(m/n)} = a^{m(1/n)} = a^{(1/n)m}$
and $(1/a)^b = 1/(a^b)$
In case of a negative power $a^{-b} = (1/a^b)$

Important Formula
$A^{(b+c)} = A^b A^c$ and $A^{(b-c)} = A^b / A^c$

Let A be a real number. Let b and c be rational numbers.
Then the following formula holds good:
$A^{(bc)} = (A^b)^c = (A^c)^b$

Dividing by 3

Add up the digits: if the sum is divisible by three, then the number is as well.

Examples:
111111: the digits add to 6 so the whole number is divisible by three. 87687687. The digits add up to 57, and 5 plus seven is 12, so the original number is divisible by three.

Dividing by 4

Look at the last two digits. If the number formed by its last two digits is divisible by 4, the original number is as well.
Examples:
100 is divisible by 4.
17327829892648648264821834612 is divisible by four also, because 12 is divisible by four.

Dividing by 5

If the last digit is a five or a zero, then the number is divisible by 5.

Dividing by 6

Check 3 and 2. If the number is divisible by both 3 and 2, it is divisible by 6 as well.

Dividing by 7

To find out if a number is divisible by seven, take the last digit, double it, and subtract it from the rest of the number.
Example:
If you had 203, you would double the last digit to get six, and subtract that from 20 to get 14. If you get an answer divisible by 7 (including zero), then the original number is divisible by seven. If you don't know the new number's divisibility, you can apply the rule again.

TEST - Take the number and multiply each digit beginning on the right hand side (ones) by 1, 3, 2, 6, 4, 5.
Repeat this sequence as necessary and Add the products.
If the sum is divisible by 7 - so is your number
Example:
Is 2016 divisible by 7?
$6(1) + 1(3) + 0(2) + 2(6) = 21$. 21 is divisible by 7 and we can now say that 2016 is also divisible by 7.

Dividing by 8

Check the last three digits. Since 1000 is divisible by 8, if the last three digits of a number are divisible by 8, then so is the whole number.
Example:
33333888 is divisible by 8; 33333886 isn't.

Dividing by 9
Add the digits. If that sum is divisible by nine, then the original number is as well.
Example:
12348 is divisible by 9; as the sum is 18
If the number ends in 0, it is divisible by 10.
Example: 20, 345ABCV80

PRIME FACTORISATION
A prime factorisation of a natural number can be expressed in the exponential form.
For example:
(i) $48 = 2 \times 2 \times 2 \times 2 \times 3 = 24 \times 3$
(ii) $420 = 2 \times 2 \times 3 \times 5 \times 7 = 22 \times 3 \times 5 \times 7$

LEAST COMMON MULTIPLE (L.C.M.)
A common multiple is a number that is a multiple of two or more numbers. The common multiples of 3 and 4 are 0, 12, 24, ...
The least common multiple (LCM) of two numbers is the smallest number (not zero) that is a multiple of both.

Method 1
Simply list the multiples of each number (multiply by 2, 3, 4, etc.) then look for the smallest number that appears in each list.
Example:
Find least common multiple for 5, 6, and 15.
Multiples of 5 are 10, 15, 20, 25, 30, 35, 40,...
Multiples of 6 are 12, 18, 24, 30, 36, 42, 48,...
Multiples of 15 are 30, 45, 60, 75, 90,....
Now, when you look at the list of multiples, you can see that 30 is the smallest number that appears in each list. Therefore, the least common multiple of 5, 6 and 15 is 30.

Method 2
To use this method factor each of the numbers into primes. Then for each different prime number in all of the factorizations, do the following...
1. Count the number of times each prime number appears in each of the factorizations.
2. For each prime number, take the largest of these counts.
3. Write down that prime number as many times as you counted for it in step 2.

The least common multiple is the product of all the prime numbers written down.
Example:
Find the least common multiple of 5, 6 and 15.
Factor into primes

Prime factorization of 5 is 5
Prime factorization of 6 is 2 x 3
Prime factorization of 15 is 3 x 5
* Notice the different primes are 2, 3 and 5.

Now, we do
Step #1 - Count number of times each prime number appears in each of the factorizations...
The count of primes in 5 is one 5
The count of primes in 6 is one 2 and one 3
count of primes in 15 is one 3 and one 5

Step #2 - For each prime number, take the largest of these counts. So we have...
The largest count of 2s is one
The largest count of 3s is one
The largest count of 5s is one

Step #3 - Since we now know the count of each prime number, you simply - write down that prime number as many times as you counted for it in step 2.
Here they are...2, 3, 5

Step #4 - The least common multiple is product of all prime numbers written down.
2 x 3 x 5 = 30
Therefore, the least common multiple of 5, 6 and 15 is 30.
So there you have it. *A quick and easy method for finding least common multiples.*

HIGHEST COMMON FACTOR (HCF)
H.C.F. of two natural numbers is the largest common factor (or divisor) of the given natural numbers.
In other words, H.C.F. is the greatest element of set of common factors of given numbers.
H.C.F. is also called Greatest Common Divisor (abbreviated G.C.D.)
Example:
Find the H.C.F. of 72, 126 and 270.
Solution:
Using Prime factorisation method
$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$
$126 = 2 \times 3 \times 3 \times 7 = 2^1 \times 3^2 \times 7^1$
$270 = 2 \times 3 \times 3 \times 3 \times 5 = 2^1 \times 3^3 \times 5^1$
H.C.F. of the given numbers
= product of common factors with least index
= 21 x 32

Using Division method find H.C.F. 72 & 126

```
72|126|1
   72
54| 72|1
   54
18| 54|3
   54
    0
```

H.C.F. of 72 and 126 = 18
Similarly calculate H.C.F. of 18 and 270 as 18
Hence H.C.F. of the given three numbers = 18

Guard Against the Probable Errors

1. 1 is **not** a prime prime number. 119 is also **not** a prime number.
2. Sum of two prime numbers can also be a prime number – this is possible only if one of the numbers is 2.
3. Sum of k integers is odd does **not** necessarily mean all the integers are odd – it is sufficient if an odd number of integers are odd.
4. If a number is divisible by n and m, it does **not** necessarily mean that the number is divisible by nm – it is so only if n and m are prime to each other.
5. HCF of a set of numbers **cannot** be greater than the smallest of the numbers.
6. LCM of a set of numbers **canno**t be less than the largest of the numbers.
7. The remainder is not always less than the dividend – it can also be equal to the dividend if the dividend is less than the divisor.
8. The conjugate of $(\sqrt{3} + 1)$ is **not** $(\sqrt{3} - 1)$ – it is $(1 - \sqrt{3})$.
9. The conjugate of $(i - 2)$ is **not** $(i + 2)$ – the correct conjugate is $(- 2 - i)$.

Example 1:
If x and y are odd integers which of the following cannot be odd?

 a. x + y   b. xy   c. 2x + y   d. x + 2y

Explanation:
Since sum of two odd integers is always even, $x + y$ cannot be odd.
Product of 2 odd integers is always odd
So xy is odd.
For any values,
$2x$ and $2y$ are even and even plus odd is odd.
Hence $(2x + y)$ and $(2y + x)$ are odd.

Example 2:
If product of k integers is odd, which of the following must be true?
Solution:

I. k is odd.
II. All the k integers are odd.
III. At least one of the k integers is odd.

Product of an even number of odd integers is also odd. So, I is not necessary.
Product of an even integer with any number of odd or even integers is always even. So, if the product is odd all the integers must be odd.
=> II must be true.
II also implies III is not sufficient to get the product as odd. Hence III is not true. Therefore, only II is true.

Example 3:
If $pq = 60$, what is the maximum number of values p can take?
Solution:
Given condition implies that p and q are divisors of 60.
Now, $60 = 2^2.3^1.5^1$ and hence the number of divisors of $60 = (2 + 1)(1 + 1)(1 + 1) = 12$.
So, p can take any of these 12 values.

Example 4:
Find the HCF and LCM of 42, 91 and 154.
Solution:
$42 = 2 \times 3 \times 7$; $91 = 7 \times 13$; $154 = 2 \times 7 \times 11$.
The only common factor is 7 and hence the HCF = 7.

Example 5:
What is the value of p if q is 119, LCM and HCF of p and q are 595 and 17 respectively?
Solution:
LCM $\times$ HCF $= 595 \times 17 = pq$
Given q = 119, $p = (595 \times 17)/119 = 85$.

Example 6:
A P. T. Master wants to make a formation with 299 students – 39 from Primary, 65 from Secondary and 195 from Higher Secondary sections, fulfilling all the following conditions:
1. Number of students in each row should be the same.
2. The front rows can have students from Primary section only.
3. The last rows can have students from Higher Secondary section only.

4. Students of Primary section cannot be accomodated in any row other than the front rows and the Higher Secondary students can be only in the last rows.
5. No student should be left behind.

If the P. T. Master wishes to have the formation with as few rows as possible, what should be the number of students per row?

Solution:

Conditions (2), (3) and (4) imply that each row can have students from one section only.

This in conjunction with (1) and (5) means that the number of students in each section should be a multiple of the number of students per row.

Or in other words, the number of students per row should be a factor of 39, 65 and 195.

More the number of students per row, less the number of rows. Thus the last condition requires the number of students per row to be as large as possible. Combining this with the earlier implication, the number of students per row should be the largest factor of 39, 65 and 195, which is nothing but the HCF.

Now, $39 = 3 \times 13$; $65 = 5 \times 13$; $195 = 15 \times 13$. The HCF is 13.

Example 7:

When a number is divided by 9 the remainder is 3 and when the same number is divided by 5 the remainder is 3. What is the largest 3-digit number which satisfies this condition?

Solution:

Let n be the number.

Then by rule (2) above, $(n - 3)$ is a multiple of 9 and also 5. So, the least possible value of $(n - 3)$ is the LCM of 9 and 5, which is 45 and all other values $(n - 3)$ can take must be multiples of 45.

Now, the largest 3-digit number is 999. On dividing 999 by 45, the remainder is 9 and again applying rule (2) above, $(999 - 9) = 990$ is a multiple of 45.

Thus, the largest possible 3-digit value for $(n - 3)$ is 990 or the largest possible value for n is 993.

Example 8:

What is the largest 4-digit number that leaves a remainder of 6 when divided by 15?

Solution:

The largest 4-digit number is 9999. On dividing 9999 by 15, the quotient is 666with a remainder of 9. So, $9999 = (15 \times 666) + 9$,

which in turn implies $(9999 – 9) = 9990$ is the largest 4-digit multiple of 15.

Hence $(9990 + 6) = 9996$ must be the largest 4-digit number yielding a remainder of 6 when divided by 15.

Example 9:

A number leaves a remainder 5 when divided by 7 and 7 when divided by 9. What is the least possible number that has this property?

Solution:

Noting $(7 – 5) = 2 = (9 – 7)$, if n is the least possible number, then $(n + 2)$ is a multiple of both 7 and 9. Hence least possible value for $(n + 2)$ is the LCM of 7 and 9, viz. 63. Therefore, the least possible value for n is 61.

5 divided by 3 leaves a remainder of 2, 4 divided by 3 leaves a remainder of 1 and 7 divided by 3 leaves a remainder of 1.

Now, 11 divided by 3 leaves a remainder of 2 and 20 divided by 3 leaves a remainder of 2.

$11 = 4 + 7$

and remainder of $11/3$ = (remainder of $4/3$ + remainder of $7/3$)

Also, $20 = 4 \times 5$ and the remainder of $20/3$ = (remainder of $4/3 \times$ remainder of $5/3$)

Or in general, if $R(n/k)$ denotes the remainder of n when divided by k,

$R\{(n + m)/k\} = R(n/k) + R(m/k)$

$R\{(n \times m)/k\} = R(n/k) \times R(m/k)$

And extending the second rule,

$R\{(nm)/k\} = \{R(n/k)\}m$

Example 10:

What is the remainder when 429 is divided by 63?

Solution:

43 is 64 which is just 1 more than 63 and hence $R(43/63) = 1$.

Now, $429 = (43)9 \times 42$ and so

$R(429/63) = R[\{(43)9 \times 42\}/63]$

$= R[\{(43)9\}/63] \times R[(42)/63]$

$= [R(43/63)]9 \times 16 = 19 \times 16 = 16$.

In general, to find the remainder when nm is divided by k, a step by step rule would be

Step 1: Identify a power t of n so that nt is very close to k.

Step 2: Divide m by t and get the quotient and remainder, say q and r respectively.

Step 3: Determine $R(nt/k)$ and $R(nr/k)$.

Step 4: Raise the first quantity of Step 3 to power q and multiply this by the second quantity to get the final answer.

Example 11:
If the remainder is 75 when a number is divided by 85, what is the remainder when the same number is divided by 17?
Solution:
Let the number be n. Then $n = 85k + 75$.
Now, R($n/17$) = R{$(85k + 75)/17$}
= R($85k/17$) + R($75/17$) = 0 + 7 = 7.

Example 12:
What is the maximum power of 7 in 1024!?
Solution:
7)1024
7) 146
7) 20
   2
Since $2 < 7$, the process is stopped and the maximum power = 146 + 20 + 2 = 168.

Example 13:
Sum of $i^{101} + i^{102} + i^{103} + \ldots\ldots\ldots i^{200}$ = ?
Solution:
$i^{101} = I; i^{102} = -1; i^{103} = -I; i^{104} = 1$.
So, $i^{101} + i^{102} + i^{103} + i^{104} = 0$.
Similarly,
$i^{105} + i^{106} + i^{107} + i^{108} = 0$
$i^{109} + i^{110} + i^{111} + i^{112} = 0$
$\ldots\ldots\ldots\ldots i^{197} + i^{198} + i^{199} + i^{200} = 0$.
Thus $i^{101} + i^{102} + i^{103} + \ldots\ldots\ldots i^{200} = 0$
To simplify a complex number with i in the denominator, multiply both the numerator and the denominator by the conjugate of the denominator.

Example 14:
Reduce $(14 - 5i)/(7 - 9i)$ to standard form.
Solution:
The conjugate of $(7 - 9i) = (7 + 9i)$.
Multiplying both the numerator and the denominator by the conjugate,
$(14 - 5i)/(7 - 9i)$
= {$(14 - 5i) \times (7 + 9i)$}/{$(7 - 9i) \times (7 + 9i)$}
= $(98 - 35i + 126i - 45i^2)/(49 - 81i^2)$
= $(98 + 91i + 45)/(49 + 81)$
= $(143 + 91i)/130$
= $(143/130) + (91/130)i$.
The principles in finding the square root of a complex number is the same as the one for irrational numbers.

Example 15:
What is the square root of $(32 - 126i)$?
Solution:
Let $\sqrt{(32 - 126i)} = (a + bi)$.

Squaring both sides,
$(32 - 126i) = a^2 - b^2 + 2abi$.
Or, $a^2 - b^2 = 32$ and $ab = -63$.
Clearly, $a = 9$ and $b = -7$.
So, $\sqrt{(32 - 126i)} = +(9 - 7i)$.

Questionnaire for Practice

1. $77^3 + 13^3 - 90^3$ is divisible by
   a. Both 13 and 17   b. Both 1 and 17
   c. Both 11 and 13   d. Both 3 and 19

2. If P is the set of prime numbers from 1 to 100, and all elements of P are multiplied, the product will be exactly divisible by
   a.10   b. 100   c. 1000   d. None of these

3. If n = 1 + x, where x is the product of 4 consecutive positive integers, then which of the following is/are true?
   I. n is odd   II. n is prime
   III. n is a perfect square
   a. I and III only   b. I and II only
   c. I only   d. None of these

4. In the middle of a round pool lies a beautiful water-lily. The water-lily doubles in size every day. After exactly 20 days the complete pool will be covered by the lily. After how many days will half of the pool be covered by the water-lily?
   a. 10   b. 15   c. 17.5   d. 19

5. A fraction is first multiplied by itself and product so obtained is divided by the reciprocal of the original fraction. If the final result is the fraction $42\frac{7}{8}$, what is the original fraction?
   a. $\frac{2}{7}$ b. $\frac{3}{8}$ c. $\frac{11}{16}$ 4. None of these

6. If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321}$ = ?
   a. 2.23   b. 4.33   c. 11.37   d. 27.64

7. There are 8 bags of rice looking alike, 7 of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, minimum number of weighings required to identify heavier bag is
   a. 2   b. 3   c. 4   d. 8

8. A number when divided by 187 leaves a reminder 62. What will be the reminder when 17 divide that number?
   a. 8   b. 9   c. 10   d. 11

9. $n^3 + 2n$, for any natural number n. is always a multiple of
   a. 3   b. 4   c.5   d. 6

10. If x = 20^4 and y = 17 x 19 x 21 x 23 then
    a. x > y   b. x < y
    c. x = y   d. None of these

11. If 4 X 56 is divisible by 33 then X is
    a. 3   b. 4   c. 5   d. 6

12. $2^{61} + 2^{62} + 2^{63} + 2^{64} + 2^{65}$ is divisible by
    a. 3   b. 31   c. 11   d. 17

13. If x is a positive integer such that 3x+12 is perfectly divisible by x, then the number of possible values of x is
    a. 2   b. 5   c. 6   d. 12

14. Two alarm clocks ring their alarms at regular intervals of 50 seconds and 48 seconds. If they first beep together at 12 noon, at what time will they beep again for the first time?
    a. 2:10pm   b.12:12pm   c.12:11pm   d.None

15. The remainder when 2^{31} is divided by 5
    a. 2   b. 4   c. 8   d. 3

16. 434758X0 is divisible by 4, Find the number of possible values of X
    a. 2   b. 4   c. 5   d. 3

17. A man earns Rs 20 on the first day and spends Rs 15 on the next day. He again earns Rs 20 on the third day spends Rs 15 on the fourth day. If he continues to save like this, how soon will he have Rs 60?
    a. On 24th day   b. On 19th day
    c. On 17th day   d. On 20th day

18. Unit digit in the sum $(264)^{102} + (264)^{103}$ is
    a. 0   b. 4   c. 6   d.8

19. 100 oranges were distributed among friends equally. Had there been 5 more friends each would have received one orange less. How many friends were there?
    a. 20   b. 25   c. 30   d. None

20. A number when divided by D leaves a remainder of 8 and if divided by 3D leaves 21. What is the remainder twice the number is divided by 3D?
    a. 3   b. 8   c. 5   d. None

21. What is the highest power of 7 that divides 77!
    a. 11   b. 12   c. 1   d. None

22. Which is greater among 4^{300} or 3^{400} ?
    a. 4^{300} b. 3^{400}
    c. Both are equal   d. None

23. For a Natural Number n, $n^4 + n^2 + 1$ is
    a. Odd   b. Even
    c. Either even or odd   d. None

24. By what smallest number must 21600 be multiplied or divided in order to make it a perfect square?
    a. 6   b. 5   c. 8   d. 10

25. Dividing by 3/8, then multiplying by 5/6 is equivalent to dividing by
    a. 5/16   b. 16/40
    c. 9/20   d. 40/18

26. If $32^{x-2} = 64/8^x$. Find the value of x
    a. -2   b. 3   c. 2   d. -3

27. A call center agent has a list of 305 phone numbers of people in alphabetic order of names (but she does not have any of the names). She needs to quickly contact Deepak Sharma to convey a message to him. If each call takes 2 minutes to complete, and every call is answered, what is the minimum amount of time in which she can guarantee to deliver the message to Mr Sharma.
    a. 610 minutes   b. 18 minutes
    c. 206 minutes   d. 34 minutes

28. n and p are integers greater than 1, 5n is the square of a number, 75np is the cube of a number. The smallest value for n + p is
    a. 14   b. 18   c. 20   d. 30   e. 50

29. For how many integer values of n will the value of the expression 4n + 7 be an integer greater than 1 and less than 200?
    a. 48   b. 49   c. 50   d. 51   e. 52

ALGEBRA

AP AND GP

Concept

In an AP or GP, the first term of the sequence is denoted by a, the nth term by tn and the sum to n terms by Sn. Sn may also be referred to as 'sum upto n terms' or 'sum of first n terms.

AP

An AP is a sequence in which the difference between any two consecutive terms is a constant. This constant difference is called the *Common Difference* denoted by d.

The nth term $tn = a + (n − 1)d$.

The sum to n terms $Sn = (n/2)\{2a + (n − 1)d\}$
$= (n/2)(a + l)$, l being the last term, i.e. tn.

Example 1:

What is the 27th term of an AP: 7, 10, 13, 16, ….?

Solution:

The difference between the second and the third terms is 3 and same is the difference between the fourth and the third terms. So, $d = 3$.

The first term $a = 7$.

So, the 27th term $t27 = 7 + (27 − 1) \times 3$
$= 7 + (26 \times 3)$
$= 7 + 78 = 85.$

Example 2:

What is the 52nd Term of an AP whose 15th and 20th terms are respectively 76 and 101 ?

Solution:

Given $t15 = 76$, $a + 14d = 76$ ……………(1)
$t20 = 101$, $a + 19d = 101$…………………(2)
(2) - (1): $5d = 25$ or
$d = 5$……………………………………(3)
(3) in (1): $a + (14 \times 5) = 76$ or
$a = 76 − 70 = 6$ …………………………(4)
Now, $t52 = a + 51d$
$= 6 + (51 \times 5)$ [substituting from (3) and (4)]
$= 6 + 255 = 261.$

Example 3:

In an AP with 4 as the common difference, the 100th term is 403. What is the sum to 100 terms of this sequence?

Solution:

Given $d = 4$ and $t100 = 403$, $a + (99 \times 4) = 403$ or
$a = 403 − 396 = 7.$
Now, $s100 = (100/2)(7 + 403)$
$= 50 \times 410 = 20500.$

Properties of AP

1. Equidistant terms of an AP are also in AP.
2. (1) implies that $(tn - tn\text{-}k) = (tn\text{+}k - tn)$.
3. The sum of any two terms equidistant from either end is a constant and the constant is equal to twice the middle term.
4. The average of terms in an AP is always the middle term.
5. The sum to n terms $= n \times$ middle term.
6. The average of any two terms of an AP = the term exactly in the middle of the two terms. i.e. $(tn + tm)/2 = t(n+m)/2$.
7. The difference between any two terms of an AP is the product of the number of in-between terms and the common difference.
8. i.e. $(tn − tm) = (n − m)d$.
9. If a, b and c are in AP, $b = (a + c)/2$.

Example 4:

The 17th and the 27th term of an AP are 100 and 150. What is the 37th term?

Solution:

Conventional Method

Given $t17 = 100$ and $t27 = 150$,
$a + 16d = 100$……………………………….(1)
and $a + 26d = 150$………..…..…….…..…(2)
(2) - (1): $\quad 10d = 50$ or
$\quad\quad\quad d = 5$…...…………. (3)
(3) in (1): $\quad a = 100 − (16 \times 5) = 20.$
Now, $t37 \quad = a + 36d$
$\quad\quad\quad = 20 + (36 \times 5) = 200.$

Short-cut Method

37th term and 17th terms are equidistant from 27th term and so by Property (2),
$(t27 − t17) = (t37 − t27)$. Or
$(150 − 100) = (t37 − 150)$ or
$t37 = 150 + 50 = 200.$

Example 5:

The 5th and the 25th term of an AP are 115 and 250. What is the sum of the 17th term and the 13th term?

Solution:

Conventional Method

Given $t5 = 115$ and $t25 = 250$,
$a + 4d = 115$…………….…..(1)
and $a + 24d = 250$……………(2)
(2) - (1): $20d = 135$ or
$d = 6.75$…………………………(3)
(3) in (1): $a = 115 − (4 \times 6.75) = 88.$
Now, $t17 = a + 16d$
$= 88 + (16 \times 6.75) = 196$
$t_{13} = a + 12d$

= 88 + (12 × 6.75) = 169.
So, the sum of the 17th term and the 13th term
= 196 + 169 = 365. [Answer]
Short-cut Method
The 5th term and 25th term are equidistant
from either end of the AP of 30
terms and so are the 13th term and 17th term.
So, by Property (3),
$(t5 + t25) = (t13 + t17)$. Or
$(t13 + t17) = (115 + 250) = 365$.

Example 6:
The average of 27 consecutive odd integers is
39. What is the average of the next 27
consecutive odd integers?
Solution:
Let the 27 odd integers be
$(a − 26)$, $(a − 24)$, ……$(a − 2)$, a, $(a + 2)$, $(a + 4)$…. $(a + 24)$ $(a + 26)$.
Then, their total = $27a$ and hence their average
= a = 39 (given).
Now, the last term of the set of first 27 odd
numbers is $(a + 26)$ and so the next 27 odd
integers are $(a + 28)$, $(a + 30)$, ……$(a + 80)$.
Their total = $27a + (28 + 30 +……..+ 80)$
= $27a + (27/2)(28 + 80)$
= $27a + (27/2)(108)$
= $27a + (27)(54)$.
Hence their average = $a + 54$
= 39 + 54 = 93. [Answer]

Short-cut Method
The set of odd integers form an AP with
common difference 2 and hence by Property 4,
the average is the middle term, i.e. the 14th
term. Now, every number in the set of next 27
consecutive odd integers is 54 more than the
corresponding term in the initial set of 27
consecutive odd integers. So, the middle term
of the next 27 consecutive integers must also
be 54 more than the middle term of the initial
set of 27 odd integers and it is also the average
of the next 27 odd integers. Thus, the required
average = 39 + 54 = 93.

GP
A GP is a sequence in which the ratio of any
two consecutive terms is a constant. This
constant ratio is called the *Common Ratio*
denoted by r.
The nth term $tn = ar$n - 1.
Sum to n terms $Sn = a(rn – 1)/(r – 1)$, if $r > 1$
$= a(1 – rn)/(1 – r)$
if $r < 1$.
The sum of an infinite GP = $a/(1 – r)$, if $r < 1$.

If a, b and c are in GP, $b = \sqrt{(ac)}$

Example 7:
Find the 9th term of the GP: 4, 12. 36,……..
Solution:
The ratio of the second term to the first term =
12/4 = 3, which is also the same as the ratio of
the third term to the second term.
So, $r = 3$. The first term $a = 4$. So, $t9 = 4 × 38$
$= 4 × 6561$
$= 26244$.

Example 8:
The 7th term and the 10th term of a GP are
192 and 1536. What is the sum to 11 terms of
the GP?
Solution:
Given $t7 = 192$ and $t10 = 1536$,
ar^7 - 1 = 192 …………………………(1)
and ar^{10} - 1 = 1536…………….……(2)
(2) ÷ (1): $r^3 = 8$ or $r = 2$……………... (3)
(3) in (1): 26a = 192 or
$a = 192/64 = 3$……………………… .(4)
Sum to 11 terms = $a(r^{11} – 1)/(r – 1)$…(5)
(3), (4) and (5): S11 = 3(211 − 1)/(2 − 1)
= 3(2048 − 1)
= 6141.

Example 9:
If a, b and c are in GP, what can be inferred
about the sequence {log$a$, log$b$, log$c$}?
Solution:
If a, b and c are in GP, b = H (ac) or $b2 = ac$.
Taking log, log$b2$ = log(ac) or
$2\log b = \log a + \log c$ [since log$x$k = $k$log$x$ and
logxy = log$x$ + log$y$]
Or, log$b$ = (log$a$ + log$c$)/2 => {log$a$, log$b$,
log$c$} is in AP

Computational Tips
1. In problems involving 3 terms of an AP.,
   take the terms as $(a − d)$, a and $(a + d)$.
2. In problems involving 4 terms of an AP.,
   take the terms as $(a$ - $3d)$, $(a$ ñ $d)$, $(a + d)$
   and $(a + 3d)$. In this case the common
   difference is not d, but $2d$.
3. In problems involving 3 terms of a GP.,
   take the terms as (a/r), a and (ar).
4. In problems involving 4 terms of a GP.,
   take the terms as $(a/r3)$, (a/r), (ar) and
   $(ar3)$. In this case the common ratio is not
   r, but r 2.
5. These simple tips greatly reduce calculation
   efforts.

Example 10:
The sum of three terms of an AP is 351 and the difference between the largest and the smallest terms is 80. Identify the AP?
Solution:
Let the three terms be $(a - d)$, a and $(a + d)$.
Then, their sum = $3a$ = 351 (given)
or a = 117.
The difference between the extreme terms
= $(a + d) - (a - d) = 2d = 80$ or $d = 40$.
So, the AP is $(117 - 40)$, 117 and $(117 + 40)$ or 77, 117, 157.

Example 11:
Sum and product of three terms of a GP are respectively 438 and 110592. Identify the GP?
Solution:
Let the three terms be (a/r), a and (ar).
Then, their product = $a3 = 110592 = 212. 33$
or $a = 24.3 = 48$.
Substituting the value of a, the three terms are: $(48/r)$, 48 and $(48r)$.
Their sum = $(48r2 + 48r + 48)/r = 438$ (given)
or $48r2 + 48r + 48 = 438r$ or $48r2 + 390r + 48 = 0$ or $8r2 - 65r + 8 = 0$ or $8r2 - 64r - r + 8 = 0$
or $8r(r - 8) - 1(r - 8) = 0$
or $(8r - 1)(r - 8) = 0 => r = 8$ or $1/8$.
So, the terms are 6, 48 and 384 or 384, 48, 6.

[**Note:** With $r = 1/8$, the terms would be the same but in the reverse order.]

Arithmetic Mean and Geometric Mean
As stated earlier, if a, b and c are in AP, $b = (a + c)/2$ and if a, b and c are in GP, b = H (ac). In the former case b is called the *Arithmetic Mean* of a and c and in the latter case b is called the *Geometric Mean* of a and c.
If A1, A2,......., Ak are k AMs between two numbers a and b, then a, A1, A2,.......,Ak and b form an AP and consequently,
$b = a + (k + 1)d$ or $d = (b - a)/(k + 1)$
Similarly,
If G1, G2,.......,Gk are k GMs between two numbers a and b, then a, G1, G2,.......,Gk and b form a GP and consequently,
$b = (ar) (k + 1)$ or $r = (b / a)1/(k + 1)$
The above two results enables one to insert any number of AMs and GMs between any two given numbers as illustrated below.

Example 12:
Insert 4 AMs between 2.5 and 15?
Solution:
Let A1, A2, A3 and A4 be the four AMs.
Here, $a = 2.5$, $b = 15$ and $k = 4$. So, $d = (15 - 2.5)/5 = 2.5$.
Since a, A1, A2, A3, A4 and b are in AP, A1 = $a + d = 2.5 + 2.5 = 5$.
A2 = A1 + $d = 5 + 2.5 = 7.5$, A3 = A2 + d = $7.5 + 2.5 = 10$,
A4 = A3+ $d = 10 + 2.5 = 12.5$
So, the AMs are 5, 7.5, 10 and 12.5. [Answer]
[Verification: Since a, A1, A2, A3, A4 and b are in AP, b should also be equal to A4 + d
A4 = 12.5 and d = 2.5 and so A4 + d = 15 which is the same as b confirming the answer]

Example 13:
Insert 3 GMs between 2.5 and 40?
Solution:
Let G1, G2, and G3 be the three GMs.
Here, $a = 2.5$, $b = 40$ and $k = 3$. So, $r = (40/2.5)1/4 = 161/4 = 2$.
Since a, G1, G2, G3 and b are in GP,
G1 = $ar = 2.5 \times 2 = 5$.
G2 = G1 $\times r = 5 \times 2 = 10$,
G3 = G2 $\times r = 10 \times 2 = 20$.
So, the GMs are 5, 10 and 20. [Answer]
[Verification: Since a, G1, G2, G3, and b are in GP, b should also be equal to A3 $\times r$. A3 = 20 and r = 2 and so A3 $\times r$ = 40 which is the same as b confirming the answer]

Some Useful Results on Natural Numbers
1. Sum of first n natural numbers = $(1 + 2 +....+ n) = n(n + 1)/2$.
2. Sum of squares of first n natural numbers = $(12 + 22 +....+ n2) = \{n(n + 1)(2n + 1)/6$.
3. Sum of cubes of first n natural numbers = $(13 + 23 +....+ n3) = \{n(n + 1)/2\}2$.

Example 14:
What is the ratio of the sum of cubes of first 4 natural numbers, sum of squares of first 6 natural numbers and the sum of first 12 natural numbers?
Solution:
Sum of cubes of first 4 natural numbers
= $\{4(4 + 1)/2\}2 = 100$.
Sum of squares of first 6 natural numbers
= $\{6(6 + 1)(12 + 1)/6\} = 91$.
Sum of first 12 natural numbers
= $\{12(12 + 1)/2\} = 78$.
So, the required ratio = 100 : 91 : 78.

AGE PROBLEMS

CONCEPT BASE

If the father is 25 years older than the son in 2008, he will be 25 years older than the son in 2015 and he must have also been 25 years older than the son in 2005 as well. This is the basic principle covering age problems.

Thus, the age difference between any two persons remains the same at all points of time. Suppose A is 40 years old and B is 30 years old. When A was B's age, he was 10 years younger and so at that time B also was 10 years younger,

i.e., B was 20 years old.

Or in general,

if A is *a* years old and B is *b* years old, when A was B's age he was (*a* - *b*) years younger and so B's age at that time would be *b* - (*a* - *b*) or (2*b* – *a*).

Guard Against Probable Errors

1. A is 200% of B => A = 2B. But A is 200% more of B => A = 3B.
2. Similarly, P is twice as old as Q
   => P's age = 2 × Q's age,
   but A is twice older than B
   => A's age = 3 × B's age.

Example:

If A = 40% of 872 and B = 900% of 42, then which of the following MUST be true?
(a) A = B   (b) A > B
(c) A < B   (d) Cannot say

Since *x*% of *y* = *y*% of *x*, A = 40% of 872 = 872% of 40 which is clearly smaller than 900% of 42.
So, A < B.

Example 1:

If A is twice as old as B but was thrice as old as B ten years back, what are their present ages?

Solution:

Let *a* and *b* be their respective present age. Then,

$a = 2b$………………..………......……. (1)

$(a – 10) = 3(b – 10)$………................... (2)

(1) in (2): $2b – 10 = 3b – 30$ or

$b = 20$ and hence $a = 40$ years.

So, their present ages are 40 years and 20 years.

Example 2:

A father tells his eldest son, "I am thrice as old as your youngest sister and when I was as old as you, you were as old as your youngest sister." If the youngest sister is 17 years old, how many years ago was her eldest brother born?

Solution:

Let the present age of father, the eldest son and the youngest daughter be *f*, *e* and *y*.

Then, $f = 3y$………………..(1)

When father was as old as the elder son, his age was *e*.

This implies that he was (*f* - *e*) years younger which in turn implies that the son was also younger by (*f* - *e*) years.

So, the son's age at that time must have been $\{e - (f - e)\} = (2e – f)$ years. This is given to be *y*.

So, $(2e - f) = y$……………………………..(2)

(1) and (2) => $(2e – 3y) = y$ or

$2e = 4y$ or

$e = 2y = 2 × 17 = 34$, since *y* =17(given).

The son's age is 34 years or he was born 34 years ago.

Example 3:

10 years back A's age was 5 more than what B's age would be 5 years hence, but 5 years hence his age would be 5 more than twice B's age 10 years back. If A's age is 20% more than C's age, what are their ages?

Solution:

Let *a*, *b* and *c* be their respective ages. Then,

$(a - 10) = (b + 5) + 5$ or

$a – b = 20$ ………….…………….............… (1)

$(a + 5) = 2(b - 10) + 5$ or

$a – 2b = - 20$ ………………….................… (2)

(1) - (2): $b = 40$ years ………………..……. (3)

(3) in (1): $a = 60$ years ……………….….... (4)

Given A's age is 20% more than C's age,

$a = 1.2c$ ….............…................................ (5)

(4) and (5): $c = 60/1.2 = 50$ years.

So, A's age = 60 years, B's age = 40 years and C's age = 50 years

Questionnaire for Practice

1. Mid-points of equilateral triangle (side = 24cm) are joined to form another triangle. Another triangle is formed by joining the mid points of this triangle, the process continues infinitely. Sum of perimeters of all triangles is:
   a. 146 cm   b. 148 cm
   c. 144 cm   d. 154 cm

2. Doctor prescribed an old man to go regularly for a walk. He goes for some distance an first day but being old next day he goes half the distance. Similarly on each subsequent day he continuous but goes only half the distance of previous day. Find the total distance traveled by him if on the first day he walks 2 km.
a. 4 km b. 5 km
c. 6 km d. 3 km

3. If twice the son's age in years is added to the father's age, the sum of their ages is 70 years. But if twice the father's age is added to the son's age, the sum is 95years. The age of father is
a. 50 years b. 45 years
c. 40 years d. 30 years

4. If p and q are two positive real numbers whose sum is unity, then pq
a. $\leq \dfrac{1}{4}$ b. $\leq \dfrac{1}{5}$
c. $\leq \dfrac{1}{6}$ d. None of these

5. Ram is twice as old as Rahim was when Ram was as old as Rahim is now. And he combined age of Ram and Rahim is 210 years. What is Ram's age now
a. 120 b. 126
c.130 d. 80

6. 10 years ago, the sum of the ages of 2 sons was half of their father's age. The ratio of the present ages of the sons is 4:3 and the sum of the present ages of all the three is 117 years. Find the present age of the father.
a. 48 b. 68
c. 28 d. 21

7. A piece of string is 40 cm long is cut into three pieces. The longest piece is 3 times the middle sized piece and shortest piece is 23 cm shorter than longest piece. Length of middle piece is
a. 9 cm b. 5 cm
c. 6 cm d. 3 cm

8. If $x = 2 + \sqrt{2}$ then what is the value of $x^2 + \dfrac{4}{x^2}$?
a. $20\sqrt{2}$ c. 12
c. $15\sqrt{2}$ d. 16

9. If $\sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}}}} = 3^n$, the value of n is
a. $\dfrac{63}{64}$ b. $\dfrac{15}{16}$
c. $\dfrac{31}{32}$ d. $\dfrac{127}{128}$

10. $\sqrt{2 + \sqrt{3}} + \sqrt{2 - \sqrt{3}}$ is:
a. $\sqrt{6}$ b. $\sqrt{3}$
c. $\sqrt{3}\sqrt{2}$ d. $\sqrt{3} - \sqrt{2}$

11. What is the value of
$$6 + \cfrac{1}{6 + \cfrac{1}{6 + \cfrac{1}{6 + \cfrac{1}{6 + \ldots}}}}$$
a. $6\dfrac{2}{3}$ b. $3 + \sqrt{10}$
c. $6 + \sqrt{6}$ d. 7

12. A trader quotes Rs. 45 for an article whose cost price is Rs. 30. The customer agrees to buy and pays him with a fifty rupee note. The trader does not have change to give back Rs. 5 to the customer. He therefore goes to the neighbouring shop and gets change for this fifty rupee note. The customer collects Rs. 5 as change and returns with the article. The next day the neighbouring shop owner realizes that the fifty rupee note received from the trader was fake and demanded 50 rupees back from the trader. The trader had no option but to give him fifty rupees. What is the total loss to the trader?
a. Rs. 80 b. Rs. 85
c. Rs. 35 d. Rs. 30

13. Middle – earth is a fictional land inhabited by Hobbits, Elves, dwarves and men. The Hobbits and the Elves are peaceful creatures who prefer slow, silent lives and appreciate nature and art. The dwarves and the men engage in physical games. The game is as follows . A tournol is one where out of the two teams that play a match, the one that loses get eliminated. The matches are played in different rounds where in every round , half of the teams get eliminated from the tournament. If there are 8 rounds played in a knock-out tournol how many matches were played?
   a. 257　　　　　　b. 256
   c. 72　　　　　　d. 255

14. Alok and Bhanu play the following coins in a circle game. 99 coins are arranged in a circle with each coin touching two other coin. Two of the coins are special and the rest are ordinary. Alok starts and the players take turns removing an ordinary coin of their choice from the circle and bringing the other coins closer until they again form a (smaller) circle. The goal is to bring the special coins adjacent to each other and the first player to do so wins the game. Initially the special coins are separated by two ordinary coins O1 and O2. Which of the following is true?
   a. In order to win, Alok should remove O1 on his first turn
   b. In order to win, Alok should remove one of the coins different from O1 and O2 on his first turn.
   c. In order to win, Alok should remove O2 on his first turn.
   d. Alok has no winning strategy.

15. A women with dollar bills go to the shopping he spent half of the money she had for shopping as she was so kind she gave one dollar to the beggar. she went to the hotel and spent half of the remaining and she gave 2 dollars to the waiter, the she buy some goods with half of the remaining and she gave 3 dollars to the receptionist. how much money she had in the beginning if she left with 1 dollor?
   a. 42　　　　　　b. 64
   c. 22　　　　　　d. 44

16. Find the number, such that its five sixths exceeds its three fifths by 28 ?
   a. 100　　　　　　b. 120
   c. 140　　　　　　d. 160

17. If 65% of x = 13% of y and y = z% of x, then find the value of z.
   a. 400　　　b. 500
   c. 100　　　d. insufficient data

18. Out of a group of swans, seven times half of the square root were seen going away from the bank of a river and one pair remained spotting the water. How many swans are there in the group?
   a. 9　　　　　　b. 16
   c. 25　　　　　　d. 36

19. Fifty minutes ago if it was four times as many minutes past 3'O clock' as it is from now to 6'O clock, how many minutes is it to 6'O clock now?
   a. 36　　　b. 44
   c. 26　　　d. None of these

20. The ages of two sisters are 10 & 15 years. In how many years will the product of their ages be 594?
   a. 10 years　　　b. 12 years
   c. 14 years　　　4. None of these

21. Out of the three consecutive positive integers, the middle number is p. If three times the square of the largest is greater than the sum of the square of the other two numbers by 240, calculate the value of p.
   a. 8　　　　　　b. 10
   c. 11　　　　　　d. 12

22. The sum of two integers is 10 and the sum of their reciprocals is $\frac{5}{12}$. The larger of these integers is
   a. 2　　　　　　b. 4
   c. 6　　　　　　d. 8

23. A says to B `I am twice as old as you were when I was as old as you are'. The sum of their present ages is 49 years. Age of A is
a. 21   b. 28
c. 30   d. 32

24. A number consists of 2 digits such that the square of the digit in the 10th place exceeds the digit in the units place by 3. If the number is 4 times the sum of the digits, find the number.
a. 54   b. 43
c. 45   d. 3

Directions for questions 25 – 26: Refer the following data
- 'm' and 'n' are real numbers
- h(m, n) = |m + n|
- H(h(m, n)) = - h(m, n)
- K(h(m, n)) = - H(h(m, n))

25. What is the value of h(K(h(0, 1)), h(H(h(2, 1)), K(h(1, 2)))) ?
a. 0   b. 1
c. -2   d. -3

26. Which of the following statements evaluates to zero?
a. H(h(5, 5)) + K(h(5, -5))
b. K(h(5, 5)) + H(h(-5, 5))
c. K(h(5, 5)) + H(h(5, 5))
d. All the above

PERCENTAGES

Concept Base
Percent:
The term per cent means for every hundred. A fraction whose denominator is 100 is called a percentage and the numerator of the fraction is called the rate per cent. It is denoted by the symbol %.

I. Any percentage can be expressed as a decimal fraction by dividing the figure by 100.
1. **To express X% as a fraction:** We have, x% = x/100.
   Thus, 20% = 20/100 = 1/5; 48% = 48/100 = 12/25, etc.
2. **To express a/b as a percent: We have, a/b = (a/b * 100) %.**
   Thus, 1/4 = (1/4 * 100) % = 25%; 0.6 = 3/5 = (3/5 * 100) % = 60%.

II. PERCENTAGE INCREASE or DECREASE of a quantity is the ratio expressed in percentage of the actual increase or decrease of the quantity to the original amount of the quantity.
   PERCENTAGE INCREASE
   = (Actual increase/original quantity)*100
   PERCENTAGE DECREASE
   = (Actual decrease/original quantity)*100

III. If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is $(\frac{R}{(100+R)} * 100)\%$
   If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is $(\frac{R}{(100-R)} * 100)\%$

IV. **Results on Population**:
   Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:
   1. Population after n years = P $(1+\frac{R}{100})^n$.
   2. Population n years ago = P/ $(1+\frac{R}{100})^n$.b

V. **Results on Depreciation**:
   Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum.
   Then:
   1. Value of the machine after n years
      = P $(1-\frac{R}{100})^n$.
   2. Value of the machine after n years ago
      = P/ $(1-\frac{R}{100})^n$.

VI. If A is R% more than B, then B is less than A by $(\frac{R}{(100+R)} * 100)\%$
   If A is R% less than B, then B is more than A by $(\frac{R}{(100-R)} * 100)\%$.

Example 1

In a class, 60% of the students in Section A are girls, while in Section B only 30% of the students are girls. The number of students in Section B is 50% more than that in Section A. If Section A has 80 students, the number of girl students of Section B is what percentage of the number of girl students in Section A?

Solutions:

Number of students in Section A = 80.

Number of students in Section B

= 80 + (80 × 0.5) = 120.

Number of girl students in Section A

= 80 × 0.6= 48.

Number of girl students in Section B

= 120 × 0.3= 36.

Number of girl students in Section B as a percentage of the number of girl

Students in Section A = (36/48) × 100 = 75%.

70% of a number n is 350 => 0.7n

= 350 or n = 350/0.7 = 500.

So, if 70% of a quantity is 350, the quantity is 500. Or in general,

If x% of y is z, y = (z/x) × 100.

Example 2

A Person spends 40% of his salary on food and 20% on house rent. If he is left with Rs. 8000, what is his salary?

Solution:

Total spent is 60% and left over is 40%

Therefore 40% of salary is 8000

Salary = (8000/40) x 100= Rs. 20000

Example 3:

A buys a shirt from a shop offering three successive discounts of 20%, 10% and 5%. B purchases from a shop giving 25%, 5% and 5% successively. Who gets better deal A or B?

Solution:

Let m be the marked price.

For A: First discounted price = 0.8m.

Second discounted price = 0.9 × 0.8m= 0.72m.

Final discounted price = 0.95 × 0.72m

= 0.684m.

The total discount = 1 – 0.684

= 0.316 or 31.6%.

For B:

First discounted price = 0.75m.

Second discounted price = 0.95 × 0.75m

= 0.7125m.

Final discounted price = 0.95 × 0.7125m

= 0.676875m.

The total discount = 1 – 0.676875

= 0.323125 or 32.31%.

Clearly B gets a better deal.

Example 4:

When the price on Tuesday is 60% more than the Monday price and the Wednesday price is 40% more than the Tuesday price, then the Wednesday price is

(a) 100% more than the Monday price

(b) More than double the Monday price

(c) Less than double the Monday price

(d) none of the above

Solution:

The most popular wrong answer would be (a). But the correct answer is (b).

60% over 40% = 1.6 × 1.4 = 2.24.

So, the Wednesday price is 124% more than the Monday price => more than double.

Example 5:

A fruit vendor buys some apples at 6 for Rs.100 and an equal number of a different variety at 4 for Rs.50. If she sells the entire stock at 15 for Rs.200, what is her profit or loss percent?

Solution:

Let the number of apples of each variety be n.

Then, total cost price = {(100n/6) + (50n/4)}

= (200n + 150n)/12 = 350n/12.

Noting that the entire stock is 2n apples, the total selling price = 400n/15.

Since 350/12 > 400/15, the net is loss.

The loss = (350n/12) – (400n/15)

   = (1750n - 1600n)/60

   = 150n/60 = 5n/2.

Loss percent

   = {(5n/2)/ (350n/12)} × 100

   = 60/7% = 84/7%.

Alternatively, by *Speed Maths* method:

When equal quantities are bought and prices are expressed as certain number for certain amount, calculations can be done faster by taking the common quantity as the LCM of those numbers.

In the present case, the numbers are 6, 4 and 15, LCM of which is 60.

Cost price of 60 apples at 6 for Rs.100 = 1000.

Cost price of 60 apples at 4 for Rs.50 = 750.

Total cost price of 120 apples = 1750.

Selling price of 120 apples at 15 for Rs.200

= 1600.

Loss = 1750 – 1600 = 150.

Loss percentage = (150/1750) × 100 = 84/7%.

Successive discounts are given on discounted price only.

Guard Against the Probable Errors
1. *x* as a percentage of *y* is **not** the same as *y* as a percentage of *x*.
2. An annual increase of 60% in a year followed by an annual fall of 10% in the next year does **not** give annual growth rate of {60 + (- 10)}/2 = 25%.

The actual growth rate is 20% per annum.
To offset a fall of 20%, an increase of 20% is **not** sufficient – 25% is required

PROFIT AND LOSS

Concept Base

In any business commercial environment the most important concern is about the profit/loss of the transaction conducted.

1. Cost Price:
   The price at which an article is purchased, is called its cost price, abbreviated as C.P.
2. Selling Price:
   The price at which an article is sold, is called its selling price, abbreviated as S.P
3. Profit or Gain
   If S.P. is greater than C.P., the seller is said to have a profit or gain.
4. Loss
   If S.P. is less than C.P., the seller is said to have incurred a loss.

FORMULAE

1. Gain = (S.P.) – (C.P.)
2. Loss = (C.P.) – (S.P.)
3. Loss or gain is always reckoned on C.P.
4. Gain % = $\left(\frac{\text{Gain}*100}{\text{C.P.}}\right)$
5. Loss % = $\left(\frac{\text{Loss}*100}{\text{C.P.}}\right)$
6. S.P. = $\frac{(100+Gain\%)}{100}*\text{C.P.}$
   Ex: If an article is sold at a gain of say, 35%, then S.P. = 135% of C.P.
7. S.P. = $\frac{(100-Loss\%)}{100}*\text{C.P.}$
   Ex: If an article is sold at a loss of say, 35%, then S.P. = 65% of C.P.
8. C.P. = $\frac{100}{(100+Gain\%)}*\text{S.P.}$
9. C.P. = $\frac{100}{(100-Loss\%)}*\text{S.P.}$
10. When a person sells two similar items, one at a gain of say, x%, and the other at a loss of x%, then the seller always incurs a loss given by:
    Loss% = $\left(\frac{\text{Common Loss and Gain \%}}{10}\right)^2$
    = $(x/10)^2$
11. If a trader Professes to sell his goods at cost price, but uses false weights, then
    Gain% = $\left(\frac{\text{Error}}{(\text{True value})-(\text{Error})}*100\right)\%$

Example 1:
If a merchant offers a discount of 40% on the marked price of his goods and thus ends up selling at cost price, what was the % mark up?
Solution:
If the merchant offers a discount of 40% on the marked price, then the goods are sold at 60% of the marked price.
The question further states that when the discount offered is 40%, the merchant sells at cost price.
Therefore, selling @ 40% discount = 60% of marked price (M) = cost price (C)
i.e., a mark up 66.66%

Example 2:
A shopkeeper bought an almirah from a wholesale dealer for Rs 4500 and sold it for Rs 6000. Find his profit or loss percent.
Solution:
Here C.P. of the almirah = Rs 4500
S.P. of the almirah = Rs 6000
Since S.P. > C.P., there is a profit
Profit = S.P. – C.P
= Rs 6000 – Rs 4500
= Rs 1500
Profit % = 1500/4500 = 33.33% .

Example 3:
A trader professes to sell his goods at a loss of 8% but weights 900 grams in place of a kg weight. Find his real loss or gain per cent?
Solution:
The trader professes to sell his goods at a loss of 8%.
Therefore, Selling Price
= (100 - 8) % of Cost Price or SP = 0.92CP
But, when he uses weights that measure only 900 grams while he claims to measure 1 kg.
Hence, CP of 900gms = 0.90 * Original CP
So, he is selling goods worth 0.90CP at 0.92CP
Therefore, he makes a profit of 0.02 CP on his cost of 0.9 CP profit % = 2.22%

Example 4:
A retailer buys a cooler for Rs 3800 and overhead expenses on it are Rs 50. If he sells the cooler for Rs 4400, determine his profit percent?

Solution:
Here, C.P. of the cooler = Rs (3800 + 50)
= Rs 3850
S.P. of the cooler = Rs 4400
Since S.P. > C.P., there is a profit
Profit = Rs 4400 – Rs 3850
= Rs 550
Profit % = 14. 27%.

Example 5:
The cost price of 40 articles is equal to the selling price of 35 articles. What is the profit /loss percent?

Solution:
Let the cost price of one article be $1, then the cost price of 40 articles is $40
The question states that the selling price of 35 articles = cost price of 40 articles = $40
Therefore, now we know the selling price for 35 articles
Now, let us find the cost price for 35 articles. We have assumed the cost of 1 article = $1. Therefore, cost price of 35 articles = $35
Therefore, the profit/loss made on 35 articles
= S.P of 35 articles - C.P of 35 articles
i.e., $40 - $35= $5, or a profit of $5
Hence, % Profit = 14.28%

Guard Against the Probable Errors
1. A mark-up of 40% followed by a 10% discount does **not** yield a profit of 30% - the actual profit is only 26%.
2. Selling two articles at the same price, incurring 10% loss in one and making 10% profit on the other does **not** lead to no-profit-no-loss situation – reality is 1% loss. [No profit - no - loss is the right answer had the two articles been bought, and **not sold**, at the same price.]
3. Doubling the price and then reducing it by 50% does **not** yield 50% profit – the net effect is no-profit-no-loss.
4. Successive discounts of 10%, 20% and 30% does **not** yield an overall 60% discount – the actual total is only 49.6%.
5. Successive discounts of 25%, 10% and 5% is **not** the same as successive discounts of 20%, 15% and 5% although both add up to 40%. The actual total discounts are 35.875% and 35.4% respectively.

Questionnaire for Practice

PERCENTAGES
1. Mr. Abhimanyu Banerjee is worried about the balance of his monthly budget. The price of petrol has increased by 40%. By what percent should he reduce the consumption of petrol so that he is able to balance his budget?
   a. 33.33   b. 28.56   c. 25   d. 14.28

2. A number is mistakenly divided by 5 instead of being multiplied by 5. Find the percentage change in the result due to this mistake.
   a. 96%   b. 95%
   c. 2400%   d. 200%

3. The length, breadth and height of a room in the shape of a cuboid are increased by 10%, 20% and 50% respectively. Find the percentage change in the volume of the cuboid.
   a. 77%   b. 75%
   c. 88%   d. 98%

4. $4/5^{th}$ of the voters in Bellary promised to vote for Sonia and the rest promised to vote for Sushma. Of these voters, 10% of the voters who had promised to vote for Sonia, did not vote on the election day, while 20% of the voters who had promised to vote for Sushma did not vote on the election day. What is the total number of votes polled if Sonia got 216 votes?
   a. 200   b. 300   c. 264   d. 100

5. In an examination, 48% students failed in Hindi and 32% students in History, 20% students failed in both the subjects. If the number of students who passed the examination was 880, how many students appeared in the examination if the examination consisted only of these two subjects?
   a. 2000   b. 2200   c. 2500   d. 1800

6. The salary of Anuj is 20% lower than Bhuwan's salary and the salary of Chauhan is 56.25% greater than Anuj salary. By how much percent the salary of Bhuwan is less than the salary of Chauhan?
   a. 20%   b. -20%
   c. 25%   d. Data Insufficient

7. The price of a certain article was raised by 10% in India. The consumption of the same article was increased from 200 tons to 225 tons. By how much percent will the expenditure on the article rise in the Indian economy?
   a. 24.25%   b. 22.5%
   c. 23.75%   d. 26%

8. In a class, 25% of the students were absent for an exam. 30% failed by 20 marks and 10% just passed because of grace marks of 5. Find the average score of the class if the remaining students scored an average of 60 marks and the pass marks are (counting the final scores of the candidates).
   a. 37.26   b. 37.6
   c. 37.8   d. 36.93

9. After receiving two successive raises, Hursh's salary became equal to 15/8 times of his initial salary. By how much percent was the salary raised the first time if the second raise was twice as high (in percent) as the first?
   a. 15%   b. 20%
   c. 25%   d. 30%

10. The hourly wages of a female labor was increased by 12.5%, whereas the weekly working hours are reduced by 8%. Find the percentage change in the weekly wages if she was getting Rs. 1200 per week for 50 hours previously.
    a. 3.5%   b. 4%
    c. 4.5%   d. None of these

11. According to a recent survey report issued by commerce Ministry, Govt of India, 30% of the total FDI goes to Gujarat and 20% of this goes to rural areas. If Gujarat FDI, which goes to urban areas, is $72m, find FDI in rural AP, which gets 50% of AP FDI, which accounts for 20% of the total FDI?
    a. $30m   b. $60m
    c. $9m   d. $40m

12. A person saves 6% of his income. Two years later, his income shoots up by 15% but his savings remains the same. Find the hike in his expenditure.
    a. 15.95%   b. 15%
    c. 14.8%   d. 15.5%

13. In order to maximize his gain, a theatre owner decides to reduce the price of tickets by 20% and as a result of this, the sales of tickets increase by 40%. If, as a result of these changes, he is able to increase his weekly collection by Rs. 1,68,000, find by what value did the gross collection increase per day.
    a. 14000   b. 18000
    c. 24000   d. 20000

14. A shopkeeper announces a discount scheme as follows: for every purchase of Rs. 3000 to Rs. 6000, the customer gets a 15% discount or a ticket that entitles him to get a 7% discount on a further purchase of goods costing more than Rs. 6000. The customer, however, would have the option of reselling his right to the shopkeeper at 4% of his initial purchase value (as per the right refers to the 7% discount ticket). In an enthusiastic response to the scheme, 10 people purchase goods worth Rs. 4000 each. Find the maximum possible revenue for the shopkeeper.
    a. 38400   b. 38000
    c. 39400   d. 39000

15. For the above question, find the maximum possible discount that the shopkeeper would have to offer to the customer.
    a. 1600   b. 2000   c. 6000   d. 4000

16. A 14.4 kg gas cylinder runs for 104 hours when the smaller burner on the gas stove is fully opened while it runs for 80 hours when the larger burner on the gas stove is fully opened. Which of these values are the closest to the percentage difference in the usage of gas per hour, between the smaller and the larger burner?
    a. 26.23%   b. 30%   c. 32.23%   d. 23.07%

17. For the above question, assume that the rate of gas dispersal is directly proportional to the degree of opening of the aperture of the gas. If we are given that the smaller burner is open to 60% of its maximum and the larger burner is open to 50% of its maximum, the percentage decrease in the percentage difference between the smaller burner and the larger burner (in terms of hours per kg) is,
    a. 72.22%   b. 73.33%
    c. 66.66%   d. None

18. Of the adult population in Nagpur, 45% of men and 25% of women are married. What percentage of the total population of adults is married (assume that no man marries more than one woman and vice versa)?
    a. 33.33%   b. 32.14%
    c. 31.1%   d. None of these

19. Abhimanyu and Banerjee has 72% vision in his left eye and 68% vision in his right eye. On corrective therapy, he starts wearing contact lenses, which augment his vision by 15% in the left eye and 11% in the right eye. Find out the percentage of normal vision that he possesses after corrective therapy. (Assume that a person's eyesight is a multiplicative construct of the eyesight's of his left and right eyes)
    a. 52.5%   b. 62.5%
    c. 72.5%   d. 68.6%

PROFIT AND LOSS

1. A shopkeeper bought 240 chocolates at Rs. 9 per dozen. If he sold all of them at Rs. 1 each, what was his profit percent?
    a. 66(1/6)%   b. 33(1/3)%
    c. 24%   d. 27%

2. A cellular phone when sold for Rs. 4600 fetches a profit of 15%. Find the cost price of the cellular phone.
    a. 4300   b. 4150
    c. 4000   d. 4500

3. A manufacturer estimates that on inspection 12% of the articles he produces will be rejected. He accepts an order to supply 22,000 articles at Rs. 7.50 each. He estimates the profit of his outlay including the manufacturing of rejected articles, to be 20%. Find the cost of manufacturing each article.
    a. 6   b. 5.50
    c. 5   d. 4.50

4. A pet shop owner sells two puppies at the same price. On one he makes a profit of 20% and on the other he suffers a loss of 20%. Find the loss or gain percent on the whole transaction.
    a. Gain of 4%   b. No profit or loss
    c. Loss of 10%   d. Loss of 4%

5. The printed price of a calculator is Rs. 180. A retailer pays Rs. 137.7 for it by getting successive discounts of 10% and another rate which is illegible. What is the second discount rate?
    a. 12%   b. 12.5%
    c. 15%   d. 20%

6. The cost price of 50 mangoes is equal to the selling price of 40 mangoes. Find the percentage profit.
    a. 20%   b. 25%
    c. 30%   d. None of these

7. Advertising worth Rs. 50,000 is done for the sales promotion of A/C rooms (advertising as 20% reduction in the bill for A/C rooms). This leads to a doubling of the occupancy rate of A/C rooms. Besides, it also has an effect of increasing non-A/C room occupancy by 20%. Is this advised?
    a. Yes   b. No
    c. Indifferent   d. Cannot be Determined

8. 'A' sells a car priced at Rs. 36,000. He gives a discount of 8% on the first Rs. 20,000 and 5% on the remaining Rs. 16,000. His competitor B sells a car of the same make, priced at Rs. 36,000. If we wants to be competitive what percent discount should 'B' offer on the marked price?
    a. 5%   b. 5.5%
    c. 6.6%   d. 8.33%

9. A dishonest dealer professes to sell at cost price but uses a 900 gram weight instead of a 1 kilogram weight. Find the percent profit to the dealer.
    a. 10%   b. 11.11%
    c. 12.5%   d. None of these

10. After selling a watch, Shyam found that he had made a loss of 10%. He also found that had he sold it for Rs. 27 more, he would have made a profit of 5%. The actual initial loss was what percentage of the profit earned, had he sold the watch for a 5% profit?
    a. 23%   b. 150%
    c. 200%   d. 180%

PERMUTATIONS

1. If there are 4 different roads from Hyderabad to Mumbai and 3 different roads from Mumbai to Delhi, how many roads are there from Hyderabad to Delhi that go through Mumbai?
   a. 4!x3! b. 3^4 c. 4^3 d. 12

2. How many five digit numbers, which are divisible by 6, can be formed from the digits 1, 3, 2, 5, 7 (digits cannot be repeated)?
   a. 48 b. 36 c. 12 d. 24

3. Ways in which 3 balls be kept in 4 baskets?
   a. 128 b. 120 c. 64 d. 144

4. For a cricket team of 11 players, there are 17 probables, 2 players are to be selected from Jharkhand there are 5 players from Jharkand among the probables. In how many ways can the selection be made?
   a. 3400 b. 2200 c. 8800 d. 4400

5. How many numbers, greater than 52000 can be formed with the digits 0, 3, 4, 7, 5 without repetition?
   a. 96 b. 25 c. 42 d. 30

6. In how many ways can 5 pens be selected from 10 different pens such that one particular pen is never selected and one particular pen is always selected?
   a. 56 b. 126 c. 70 d. 100

7. There are 9 points in a plane of which 3 are collinear. How many different straight lines can be drawn by joining these points?
   a. 36 b. 35 c. 33 d. 34

8. Number of 4 digit numbers that can be formed from 4,5,6,7,0 without repetition.
   a. 96 b. 256 c. 120 d. 156

9. There is a question paper consisting of 8 questions. Each question has an internal choice of 2 questions where one can answer at most one of them. In how many ways can a student attempt one or more questions of 8 questions in the paper?
   a. 6561 b. 6578 c. 6560 d. 6256

10. In how many ways can 6 letters be posted into 5 letterboxes?
    a. 15625 b. 7776 c. 1296 d. 3125

11. Out of 4 boys and 6 girls of a class, a committee of 7 is to be selected. Find the number of ways in which this can be done when there is a majority of girls.
    a. 86 b. 112 c. 100 d. 64

12. Find the number of arrangements that can be made using all the letters of the word 'EDUCATION' such that all the vowels come together.
    a. 6! 5! b. 6! 6! c. 5! 5! d. 4! 5!

13. How many 5 digit numbers are there that have either 2 or 3 in them?

14. How many Eight letter words can you form using the letters UMBRELLA such that Each letter is used only once and R always comes before M?

15. In how many ways can 5 girls, 3 boys stand in a row wherein no 2 boys're together?

16. If there are 8 stations along a railway line, then how many different types of tickets must be printed so that the commuters can purchase a ticket from any one station to any other station?

17. In a meeting there were a total of 36 handshakes. If each participant shook hands with every other participant exactly once, then what is the total number of guests?

18. 30 chocolates have to be distributed among 3 kids, each kid getting minimum 3. In how many ways it could be done?

19. Find the rank of NITIN in the dictionary?

20. In a year, if a month is selected what is the probability that it has exactly 30 days?

PROBABILITY

1. Find the probability that the 3 digit number we have created using 0,1,2,3,4 & 5, is even (repetition not allowed)?

2. If six friends are to be seated in a row, then the probability that two particular friends never sit together is
   a. 2/3   b. 5/6   c. 1/3   d. 1/6

3. If 5 coins are tossed together, what is the probability of getting exactly 3 heads?
   a. 15/32   b. 5/16   c. 3/5   d. 3/32

4. A card is drawn from a well-shuffled pack of cards; find the probability that it is a red honor.
   a. 1/26   b. 1/13   c. 3/26   d. 2/13

5. A bag contains 3 green, 2 white and 4 black balls. If two balls are drawn simultaneously, what is the probability that both are of same colour?
   a. 5/18   b. 11/36   c. 3/18   d. 1/3

6. Harika picked a number from the set of two digit numbers and found it to be a perfect square. What is the probability that it ends with 4?
   a.1/3   b.1/6   c. 2/3   d. 1/7

7. When two dice are thrown together, find the probability of getting the same number on both the dice?
   a. 1/6   b. ½   c. ¾   d. 1

8. If two dice I and II are thrown together, what is the probability of getting 6 on the dice I and a prime number on the dice II?

   a. 1/9   b. ½   c. 5/36   d. 1/12

9. When a dice is rolled, what is the probability of getting a composite number or a number less than 5?
   a. 1   b. 1/6   c. 5/6   d. None of these

10. The probability that a leap year chosen at random has exactly 52 Tuesdays and 52 Wednesdays is
    a. 2/7   b. 4/7   c. 1/7   d. 3/7

11. If a number is chosen at random from the set {1, 2, ... 100}, then the probability that the chosen number is a perfect cube or a prime number is
    a. 7/50   b. 29/100   c. 3/10   d. 1/25

12. When 8 coins are tossed, what is the probability of getting no tail?
    a. 255/256   b. 1/256   c. 1/512   d. None

13. The chance that year 2016 has 52 Sundays and 53 Mondays is
    a. 3/7   b. 4/7   c. 1/7   d. 2/7

14. There are 10 Letter and correspondingly 10 different Address if the letter are put into envelope randomly then find the Probability that Exactly 9 letter will at the Correct Address??
    a. 1/10   b. 0   c. 2/10   d. 3/10

15. When a dice is rolled, what is the probability of getting a composite number or a number less than 5?
    a. 1   b. 1/6   c. 5/6   d. None of these

16. If 2 dice are tossed, what is the probability that the sum of the output is minimum 9?
    a. 5/36   b. 5/12   c. 7/36   d. 5/18

17. Here is 10 dots, out of which 4 are collinear. If you select 3 dots randomly, what is the probability that 3 dots make a triangle?

18. A man can hit the target once in four shots. If he fires 4 shots in succession, what is the probability that he will hit target?

19. 7 committee members sit around a table. What is the probability that the President and the Secretary are sitting together?

20. 5 boys & 3 girls are sitting in a linear arrangement. Find the probability that no 2 girls are together?

SPEED TIME DISTANCE

Concept Base

In this chapter, we will look at problems in the following different areas:

- General problems on Time, Speed and Distance
- Relative Speed
- Boats and Streams
- Races and Circular Tracks

Before we look at problems in various areas, Let us first look at some basic concepts pertaining to speed, time and distance.

I. SPEED

Distance covered per unit time is called speed.

i.e., Speed = Distance/Time

$$S = D/T$$

From this, we get

$$T = D/S$$
$$D = S \times T$$

Note:

a. D and S are directly proportional, but T and S are inversely proportional.

b. Distance is normally measured in Kilometres, metres or Miles; time in hours or seconds and Speed in Km/hour (also denoted by kmph), miles/hour (also denoted by mph) or metres/second (denoted by m/s).

Some Important Results

a. To convert speed in kmph to m/sec, multiply it with 5/18.

b. To convert speed in to m/sec to kmph, multiply it with 18/5.

Q1. A car and a bike start from the same point at 9 a.m. and reach the same destination at 10 a.m. and 10:30 a.m. respectively. If the average speed of the car is 60 kmph, what is the average speed of the bike?

Solution:

If *d* is the distance covered, *d* = 60 km

Since the car took 1 hour to cover the speed is 60 kmph

The bike took 1.5 hours to cover the same distance.

So, Speed = Distance/Time

Speed = 60/1.5 = 40kmph.

II. Average Speed

Average Speed = Total Distance/Total Time

If the time is constant, the average speed is the arithmetic mean of the speeds and if the distance is constant, the average speed is the harmonic mean of the speeds.

Given two numbers *a* and *b*, their

a. Arithmetic mean (AM) = $(a+b)/2$,

b. Harmonic mean (HM) = $2ab/(a + b)$

c. Also AM > HM

Q2. A car covered a certain distance at 90 kmph and return back at 60 kmph. Find its average speed for the entire journey?

Solution:

Let x km be the distance to be covered, each way.

Total time of travel (in hours)

x/90 + x/60 = 5x/180 = x/36

Average speed (in km/hr)

= Total distance travelled/Total time taken

= 2x/(x/36) = 72

Q3. Find the length of the platform which a train. 400 m long, travelling at 45 kmph can cross in 40 seconds ?

Solution:

Speed of the train = (45)(5/18) = 12.5 m/sec

Length of the platform

= Distance travelled by the train – Length of the train

= (12.5) (40) – 400 = 100 m

Q4. Ashok would reach this office 15 minutes early if he walked at 4 kmph from his house. He would reach it 45 minutes late if he walked at 3 kmph from his house. Find the distance between his house and office.

Solution:

Let the distance be x km.

Time taken by Ashok If he walked at 4 kmph = x/4 hours.

Time taken by Ashok if he walked at 3 kmph = x/3 hours.

In this case he would take one hour more to reach his office compared to the time taken if he had walked at 4 kmph.

Therefore,

x/3 – x/4 = 1

x = 12.

Note:
In general, if a person travelling between two points reaches p hours late travelling at a speed of u kmph and reaches q hours early travelling at v kmph, the distance between the two points is given by **vu/(v-u) *(p+q).**

III. Relative Speed

Relative speed is the rate at which the distance between two moving particles increases or decreases and depends on the direction in which the particles are moving.

To find relative speed,
- Sum of speeds if the particles move in opposite directions
- Difference in speeds if the particles move in the same direction

Steps to solve problems:
a. Determine the initial distance between the particles.
   i. If both particles move simultaneously, the distance between them is the initial distance
   ii. If they do not start at the same time
      1. Calculate the distance between them when the late starter begins to move
      2. Consider it as the initial distance.
b. Evaluate the relative speed.
c. Calculate the time taken for the two particles to meet or cross each other as the ratio of initial distance to the relative speed.

Note:
The relative speed concept can also be employed to determine the distance between two moving particles at any given time by the following rule:
Required distance = initial distance ± (time lapse × relative speed).

Q5. Two trains running at 36 kmph and 45 kmph cross each other in 20 seconds when they run in opposite directions. When the trains run in the same direction, a person in the faster train observed that he overtook the slower train in 48 seconds. Find the length of the trains.
Solution:
Let the lengths of the faster and slower train be f and s respectively.
Given that, $f+s/(36+45)(5/18) = 20$
$f+s = 450$…………..(1)

When the run the same direction, the time taken by the person in the faster train to cross the slower train
$= s/(45-36)(5/18) = 48$
$S = 120$
From (1), f = 330

Q6. Two trains have lengths of 300 m and 200 m. When they run in the same direction, the faster train will take 100 seconds to cross the slower train. When they run in the opposite directions, they will take 20 seconds to cross each other. Find the speeds of the trains.
Solution:
Let the speeds of the trains be x m/sec and y m/sec where x>y.
$(300+200)/(x-y) = 100$
$5 = x-y$………………………. (1)
$(300+200)/(x+y) = 20$
$25 = x+y$……………………..(2)
Solving (1) and (2)
$x = 15$ and $y = 10$

IV. Boat and Stream Problems
These problems are governed by the following results:
Downstream (along the current) speed (D)
= Boat-speed (B) + Stream speed (C)
$D = B + C$
Upstream (against the current) speed (U)
= Boat speed – current (stream) speed.
$U = B – C$
Speed of the boat = average of downstream and upstream speeds
$B = (D + U)/2$
Speed of the current = half the difference of downstream and upstream speeds
Or as formulae,
$C = (D – U)/2$

Q7. A boat takes 7 hours to go from A to B and 9 hours to return to A. If AB distance is 63 km, find the speed of
(a) the boat   (b) the current.
*Solution***:**
Since B to A takes more time, it is upstream and hence AB is downstream.
Downstream speed = 63/7= 9 kmph.
Upstream speed = 63/9= 7 kmph.
Boat speed = (9 + 7)/2= 8 kmph.
Current speed = (9 - 7)/2=1 kmph.

V. RACES

The understanding of the following terminology is essential to solve problems involving races.

1. A beats B by 10 seconds in100 m race
   ⇨ B takes 10 seconds more than A to cover 100 m.
2. A beats B by 10ms in100m race
   ⇨ B covers only 90 metres when A covers 100 m.
3. A gives a *head start* or simply *start* of 10 seconds to B
   ⇨ B can start 10seconds ahead of A.
4. A gives a *head start* or simply *start* of 10 metres to B
   ⇨ B can start from a point 10 metres ahead of the point from where A would start.
5. Dead *heat* implies all runners reach the finishing line at the same time.

Q8. In a 200 m race, A beats B by 10 m or 2 seconds. Find B's speed and A's speed ?

Solution:

A beat B by 10 m or 2 seconds.

When A reached the finishing line B was 10 m behind the finishing line and took 2 seconds to cover it.

B's speed =10/2=5m/sec

Time taken by B to complete the race= 200m/5m/s

=40 seconds

Therefore, Time taken by A to complete the race

=38 seconds

A's speed = 200/38 = 100 = 19 m/sec

Q9. In a 1200 m race, Ram beats Shyam by 300 m. In the same race, Shyam beats Tarun by 400 m. Find the distance by which Ram beats Tarun ?

Solution:

Let the speed of Ram, Shyam and Tarun be r m/sec, s m/sec and t m/sec respectively.

r/s = 1200/(1200-300) = 4/3

s/t = 1200/(1200-400) = 3/2

r/t = (r/s)(s/t) = 2

By the time Ram covers 1200 m, Tarun covers 600 m.

Therefore, Ram beats Tarun by (1200 - 600) i.e. by 600 m

VI. CIRCULAR TRACKS

Below things essential to solve problems,

a. If two runners start at the same time and from the same point, when they meet for the first time, the faster runner would have a lead of one complete round over the other.
   The time taken for the first meeting = track length/relative speed.
b. If two runners start at the same time and from the same point, the time taken for them to meet at the starting point for the first time is the LCM of the time taken by each to complete one round.
c. When three runners start from the same point at the same time, the time taken for their first meeting is the LCM of the time taken by the fastest runner to get a lead of one complete round over each of the other two.

[Result 'a' above can be used to determine these times.]

Q10. On a circular track of length 1800 m, X and Y start from the same point simultaneously with speeds of 36 kmph and 45 kmph respectively. Find the minimum time after which they will meet if they are running in
   a. The same direction.
   b. Opposite direction to each other.

Solution:

   a. Time taken to meet for the first time
      = 1800/(45-36)(5/18) = 720 seconds
   b. Time taken to meet for the first time
      =1800/(45+36)(5/18) = 80 seconds

Q11. In the previous example, find the time after which they will meet at the starting point for the first time?

Solution:

Time taken to meet at the starting point for the first time

= L.C.M(1800/(36)(5/18), 1800/(45)(5/18))

= LCM (180, 144)

= 720 seconds

Guard Against the Probable Errors

* Average speed is **not** always the average of the speeds − it is so only when the time is constant; if the distance is constant, it is the HM of the speeds.

Questionnaire for Practice

1. A train traveling continuously covers 200 kilometer in 3 hours 30 minutes. If it has covered 40 km each for the first and last half an hour, then at what speed should it travel so as to maintain the same time for the whole distance?
   a. 50 km/hr   b. 60 km/hr
   c. 64 km/hr   d. 48 km/hr

2. The Jammu and Kashmir Express from Delhi to Srinagar was delayed by snowfall for 16 minutes and made up for the delay on a section of 80 kilometer travelling with a speed 10 km per hour higher than its normal speed. Find the original speed of Jammu and Kashmir Express (according to the schedule)
   a. 60 km/h   b. 66.66 km/h
   c. 50 km/h   d. 40 km/h

3. Two ants start simultaneously from two ant holes towards each other. The first ant covers 8% of the distance between the two ant holes in 3 hours, the second ant covered 7/120 of the distance in 2 hours 30 minutes. Find the speed (feet/h) of the second ant if the first ant travelled 800 feet to the meeting point.
   a. 15 feet/h   b. 25 feet/h
   c. 45 feet/h   d. 35 feet/h

4. Ozair starts for Pune from Mumbai at 2 PM, after reaching Pune he takes 2 hours to finish his work. After finishing his work he starts his return journey at $4/5^{th}$ of his previous speed. He reached Mumbai at 08:30 PM on the same day. If the distance between Pune and Mumbai is 100 kilometers, then find the speed (in km/hr) with which he returns back to Mumbai
   a. 40 kmph   b. 50 kmph
   c. 35 kmph   d. 45 kmph

5. A man is 20 minutes late to office when he travels at a speed of 20 km/hr and 25 minutes early, when he travels at a speed of 80 km/hr. The distance to his office is
   a. 30 kmph   b. 20 kmph
   c. 25 kmph   d. 45 kmph

6. Two trains, 200 km apart, are moving toward each other at the speed of 50 km/hour each. A fly takes off from one train flying straight toward the other at the speed of 75 km/hour. Having reached the other train, the fly bounces off it and flies back to the first train. The fly repeats the trip until the trains collide and the bug is squashed. What distance has the fly travelled until its death?

7. A man jogs at 6 mph over a certain journey and walks over the same route at 4 mph. What is his average speed for the journey?
   a. 2.4mph   b. 4 mph
   c. 4.8 mph   d. 5 mph

8. A horse chases a dog 2 hours after the dog runs. Horse takes 3 hours to reach the dog. If the average speed of the horse is 81Kmph. Then what is the average speed of the dog?
   a. 48.6   b. 50
   c. 40   d. None

9. A horse can pull an empty cart at the speed of 18km per hour and the reduction in its speed is directly proportional to the square root of the number of boxes it can carry of equal weight of 10kg each. If the speed of the cart is 12km per hour when 9 boxes are loaded in the cart, find the maximum weight that can be carried if the speed of the cart is to be maintained at least 10km per hour.
   a. 16 kg   b. 160 kg
   c. 100 kg   d. 159

10. In Kerala, students go to school by boat. The distance between the school and house is 21 km. A boat leaving the students to school returns back in 270 minutes. In which 60 min is taken to drop the students near School. What is speed of boat in still water, if the speed of river is 2.5 km/hr?
    a. 10.4 km/hr   b. 12.5 km/hr
    c. 22.5 km/hr   d. 11.5 km/hr

11. Ferrari S.P.A is an Italian sports car manufacturer based in Maranello, Italy. Founded by Enzo Ferrari in 1928 as Scuderia Ferrari , the company sponsored drivers and manufactured race cars before moving into production of street-legal vehicles in 1947 as Feraari S.P.A.

Throughout its history, the company has been noted for its continued participation in racing, especially in Formula One where it has employed great success .Rohit once bought a Ferrari . It could go 4 times as fast as Mohan's old Mercedes. If the speed of Mohan's Mercedes is 46 km/hr and the distance traveled by the Ferrari is 953 km, find the total time taken for Rohit to drive that distance.

a. 20.72   b. 5.18

c. 238.25   d. 6.18

12. The pace length P is the distance between the rear of two consecutive footprints. For men, the formula, $n/P = 144$ gives an approximate relationship between n and P where, n = number of steps per minute and P = pace length in meters. Bernard knows his pace length is 164cm. The formula applies to Bernard's walking. Calculate Bernard's walking speed in kmph.

a. 23.62   b. 36.4

c. 28.5   d. None

13. Two rifles are fired from the same place at a difference of 11 minutes 45 seconds. But a man who is coming towards the place in a train hears the second sound after 11 minutes. Find the speed of train,

a. 72 kmph   b. 36 kmph

c. 81 kmph   d. 108 kmph

14. One bad day, at 7 am I started on my bike at the speed of 36 kmph to meet one of my relatives. After I had travelled some distance, my bike went out of order and I had to stop. After resting for 35 minutes, I returned home on foot at a speed of 14 kmph and reached home at 1 pm. Find the distance from my house at which my bike broke down.

a. 54 kmph   b. 63 kmph

c. 72 kmph   d. None of these

15. Two sea trawlers left a sea port simultaneously in two mutually perpendicular directions. Half an hour later, the shortest distance between them was 17 km, and another 15 minutes late, one sea trawler was 10.5 km farther from the origin than the other. Find the speed of each sea trawler.

a. 16kmph, 30kmph   b. 18kmph, 24 kmph

c. 20kmph, 22kmph d. 18kmph, 36 kmph

16. Jaideep travels from Alaska, which is on a highway, to Burgen, which is 60 km from highway. The distance between Alaska and Burgen along a straight line is 34 km. At what point should Jaideep turn from the highway to reach Burgen in the shortest possible time, if his speed along the highway is 10 km/h and 6km/h otherwise.

  a.   30 km away from A

  b.   20 km away from A

  c.   18 km away from A

  d.   15 km away from A

17. Two ships sail in a fog towards each other with the same speed. When they are 4 km apart, the captains decelerate the engines for 4 minutes with a deceleration rate of 0.1 m/s^2, and then the ships continue sailing with the speeds attained. For what range of values of the initial speed V_o will the ships avoid collision?

  a. $0 < V_o < 10$ m/s

  b. $0 < V_o < 20$ m/s

  c. $0 < V_o < 30$ m/s

  d.   None of these

18. A boat went down the river for a distance of 20km. It then turned back and returned to its starting point, having travelled a total of 7 hours. On its return trip, at a distance of 12 km from the starting point, it encountered a log, which had passed the starting point at the moment at which the boat had started downstream. The downstream speed of the boat is,

a. 7 kmph   b. 13 kmph

c. 16 kmph   d. 10 kmph

19. Two friends started walking simultaneously from point A and B towards each other. 144 minutes later the distance between them was 20% of the original distance. How many hours does it take the faster walker to cover the distance AB if he needs 8 hours less to travel the distance than his friend (assume all times to be in whole numbers and in hours)?

a. 3 hrs   b. 6 hrs

c. 12 hrs   d. 4 hrs

TIME AND WORK

Concept and Formulae

1. If A can do a piece of work in n days, then

   A's 1 day's work = $\dfrac{1}{n}$.

2. If A's 1 day's work $= \dfrac{1}{n}$, then A can finish

   the work in n days.

3. If A is thrice as good a workman as B, then:

   Ratio of work done by A and B = 3 : 1.
   Ratio of times taken by A and B to finish a work = 1 : 3.

Pipes And Cisterns
Important Facts And Formulae

1. **Inlet:** A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

   Outlet: A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

2.
   I. If a pipe can fill a tank in x hours, then : part filled in 1 hour = 1/x
   II. If a pipe can empty a full tank in *y* hours, then : part emptied in 1 hour = 1/y
   III. If a pipe can .fill a tank in x hours and another pipe can empty the full tank in y hours (where *y> x),* then on opening both the pipes, the net part filled in 1 hour = (1/x)-(1/y)
   IV. If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where x > *y),* then on opening both the pipes, the net part emptied in 1 hour = (1/y)-(1/x)

Example 1:

Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same job. How long should it take both A and B, working together but independently, to do the same job?

Solution:

A's 1 hour's work = $\dfrac{1}{8}$,

B's 1 hour's work = $\dfrac{1}{10}$.

(A + B)'s 1 hour's work = $\left[\dfrac{1}{8} + \dfrac{1}{10}\right] = \dfrac{9}{40}$.

∴ Both A and B will finish the work in $\dfrac{40}{9}$

$= 4\dfrac{4}{9}$ days.

Example 2:

A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work?

Solution:

(A + B)'s 1 day's work $= \dfrac{1}{4}$,

A's 1 Day's work $= \dfrac{1}{12}$.

∴ B's 1 day's work $= \left[\dfrac{1}{4} - \dfrac{1}{12}\right] = \dfrac{1}{6}$.

B alone can complete the work in 6 days.

Example 3:

A Can do a piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How long will they take to do it, working together $8\dfrac{2}{5}$ hours a day?

Solution:

A can complete the work in (7 x 9) = 63 hours.
B can complete the work in (6 x 7) = 42 hours.

∴ A's 1 hour's work $= \dfrac{1}{63}$

and B's 1 hour work $= \dfrac{1}{42}$.

(A + B)'s 1 hour's work $= \left[\dfrac{1}{63} + \dfrac{1}{42}\right] = \dfrac{5}{126}$.

∴ Both will finish the work in $\left[\dfrac{126}{5}\right]$ hrs.

Number of days of $8\dfrac{2}{5}$ hrs each

$= \left[\dfrac{126}{5} \text{ x } \dfrac{5}{42}\right] = 3$ days.

Example 4:

A and B can do piece of work in 18 days; B and C can do it in 24 days; A and C can do it in 36 days. In how many days will A, B and C finish it, working together and separately?

Solution:

(A + B)'s 1 days work $= \dfrac{1}{18}$

(B + C)'s 1 day's work $= \dfrac{1}{24}$

(A + C)'s 1 day's work = $\dfrac{1}{36}$

Adding, we get : 2 (A + B + C)'s 1 day's work

$= \dfrac{1}{18} + \dfrac{1}{24} + \dfrac{1}{36} = \dfrac{9}{72} = \dfrac{1}{8}$.

∴(A + B + C)'s 1 day's work = $\dfrac{1}{16}$

Thus, A, B and C together can finish the work in 16 days.

Now, A's 1 day's work = [(A + B + C)'s 1 day's work] – [(B + C)'s 1 day's work]

$= \left[\dfrac{1}{16} - \dfrac{1}{24}\right] = \dfrac{1}{48}$

∴ A alone can finish the work in 48 days.

Similarly, B's 1 day's work

$= \left[\dfrac{1}{16} - \dfrac{1}{36}\right] = \dfrac{5}{144}$.

B alone can finish the work in $\dfrac{144}{5}$

$= 28\dfrac{4}{5}$ days.

And, C's 1 day's work $= \left[\dfrac{1}{16} - \dfrac{1}{18}\right] = \dfrac{1}{144}$.

∴ C alone can finish the work in 144 days.

Example 5:

A is twice as good a workman as B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?

Solution:

(A's 1 day's work) : (B's 1 day's Work)= 2 : 1

(A + B)'s 1 day's work = $\dfrac{1}{18}$.

Divide $\dfrac{1}{18}$ in the ratio 2 : 1.

∴A's 1 day's work $= \left[\dfrac{1}{18} \times \dfrac{2}{3}\right] = \dfrac{1}{27}$.

Example 6:

A can do certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?

Solution:

Ratio of times taken by A and B = 160 :100 = 8 : 5.

Suppose B alone takes x days to do the job. Then,

8:5 :: 12 : x = 8x = 5 x 12 = x = $7\dfrac{1}{2}$ days.

Example 7:

A job can be completed by A in 10 hours while B takes only 8 hours to complete the same job. A and B start the work together but A leaves the job after 3 hours. B works alone for 1 hour and then C joins B. Together they complete the job in 1 hour. If C were to work alone, how long would he take to complete the job?

Solution:

Time taken by A to complete the job :10 hours

Work done by A in one hour: 1/10

Time taken by B to complete the job ;8 hours

Work done by B in 1 hr ; 1/8

Work done by A and B together in 1hr; (1/10)+(1/8) = 9/40

Work done by A and B together in 3 hr; 27/40

Work done by B in next 1 hr; 1/8

Work completed when C joins; (27/40)+(1/8)=32/40=4/5

Therefore, the left over work will be ;1-(4/5)=1/5

Work done by B and C together in 1 day; 1/5

Work done by C alone in 1 day ; (1/5)-(1/8)=3/40

Time taken by C to complete the job ; 40/3=13(1/3) hrs

Example 8:

5 men or 8 women can complete a work in 4 days. The same work can be completed the by 3 women and 4 boys in 8 days. How long would 1 man, 2 women and 8 boys take to complete the same work?

Solution:

Work done by 5 men in 4 days   : 1

Work done by 5 men in 1 day   : ¼

Work done by 1 man in 1 day : (1/5) (1/4)   : 1/20

Work done by 8 women in 4 days   :1

Work done by 8 women in 1 day :1/4

Work done by 1 women in 1 day : (1/8) x (1/4) :1/32

Work done by 3 woman & 4 boys in 8 days : 1

Work done by 3 woman and 4 boys in 1 days : 1/8

Work done by 3 woman in 1 days : 3/32

Work done by 4 boys in 1 days : 1/8 – 3/32=1/128

Work done by 1 man, 2 women and 8 boys (1/20)+(2 x 1/32)+(8 x 1/128)

(1/20)+(1/16)+(1/16)

(4+5+5)/80 14/80 = 7/40

Number of days required to complete the job: 40/7 days

Example 9:
Two pipes A and B can fill a tank in 36 hours and 46 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?
Solution:
Part filled by A in 1 hour = (1/36);
Part filled by B in 1 hour = (1/45);
Part filled by (A + B) In 1 hour =(1/36)+(1/45)=(9/180)=(1/20)
Hence, both the pipes together will fill the tank in 20 hours.

Example 10:
Two pipes can fill a tank in 10hours and 12 hours respectively while a third, pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled?
Solution:
Net part filled In 1 hour
=(1/10)+(1/12)-(1/20) = (8/60)=(2/15).
The tank will be full in 15/2 hrs = 7 hrs 30 min.

Example 11:
If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does it take the second pipe to fill the reservoir?
Solution:
Let the reservoir be filled by first pipe in x hours.
Then ,second pipe fill it in (x+10)hrs.
Therefore (1/x)+(1/x+10)=(1/12)
⇨ (x+10+x)/(x(x+10))=(1/12).
⇨ x^2 -14x-120=0 => (x-20)(x+6)=0
⇨ x=20 [neglecting the negative value of x]
So, the second pipe will take (20+10)hrs
(i.e) 30 hours to fill the reservoir

Example 12:
A cistern has two taps which fill it in 12 minutes and 15minutes respectively. There is also a waste pipe in the cistern. When all the 3 are opened , the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?
Solution:
Work done by the waste pipe in 1min
= (1/20)-(1/12) + (1/15)
= -1/10 [negative sign means emptying]
Therefore the waste pipe will empty the full cistern in 10min

Example 13:
An electric pump can fill a tank in 3 hours. Because of a leak in, the tank it took 3(1/2) hours to fill the tank. If the tank is full, how much time will the leak take to empty it ?
Solution:
Work done by the leak in 1 hour
= (1/3)-(1/(7/2))
= (1/3)-(2/7) = (1/21).
The leak will empty .the tank in 21 hours.

Example 14:
Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?
Solution:
Work done by the two pipes in 1 hour = (1/14)+(1/16)=(15/112).
Time taken by these pipes to fill the tank = (112/15) hrs = 7 hrs 28 min.
Due to leakage, time taken
= 7 hrs 28 min + 32 min = 8 hrs
Work done by (two pipes + leak) in 1 hour
= (1/8).
Work done by the leak m 1 hour
= (15/112)-(1/8) = (1/112).
Leak will empty the full cistern in 112 hours.

Example 15:
Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened. after 7 min, C is also opened. In how much time, the tank is full?
Solution:
Part filled in 7 min
= 7*((1/36)+(1/45))=(7/20).
Remaining part=(1-(7/20))=(13/20).
Net part filled in 1min. when A,B and C are opened=(1/36)+(1/45)-(1/30)=(1/60).
Now,(1/60) part is filled in one minute.
(13/20) part is filled in (60*(13/20))=39min

Questionnaire for Practice
1. A & B can do a piece of work in 48 days; B & C can do it in 60 days; A & C can do it in 80 days. In how many days will A, B & C finish it, all working together?
   a. 40   b. 35   c. 20   d. None of these

2. A can do a piece of work in 7 days of 10 hours each & B can do it in 5 days of 12 hours each. How many days will they take to do it, working together $\frac{70}{13}$ hours a day?
   a. 4   b. 6   c. 10   d. 12

3. 24 men can complete a work in 36 days. Twelve days after they started working, 8 men joined them. How many days will all of them take to finish the remaining work?
   a. 16   b. 20   c..18   d. None of these

4. If 20 men or 36 boys can do a piece of work in 15 days, then 50 men and 30 boys together will do twice the work in how many days?
   a. 6   b. 7   c. 8   d. 9

5. X can do a piece of work in 40 days. He works at it for 8 days and then Y finished it in 16 days. How many days will they together take to complete the work?
   a. 13 1/3   b.15   c. 20   d.56

6. Kim can do a work in 3 days while David can do the same work in 2 days. Both of them finish the work together and get Rs 150 what is the share of Kim?
   a. 50   b. 60   c. 90   d. 75

7. An engineer undertakes a project to build a road 15 km long in 300 days and employs 46 men for the purpose. After 100 days, he finds only 2.5 km of the road has been completed. Find the number of extra men he must employ to finish the work in time?
   a. 46   b. 23   c. 69   d. 115

8. Working independently, Tina can do a certain job in 12 hours. Working independently, Ann can do the same job in 9 hours. If Tina works independently at the job for 8 hours and then Ann works independently, how many hours will it take Ann to complete the remainder of the jobs?
   a. 2/3   b. ¾   c. 1   d. 3

9. Two pipes can fill a cistern in 14 and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage. 32 minutes extra is taken to fill the cistern. In what time will the leak empty the full tank?
   a. 114   b. 112   c. 100   d. 82

10. A can do a certain job in 120 days and B can do the same in 150 days. They together worked for 20 days. Then B leaves and A continues for 20 days. Thereafter C joins and the work and the work is completed in 32 days. In how many days will C alone completed work
    a. 80   b. 100   c. 90   d. 120

11. 50*35 m fishing pond was dug by 250 workers in 18 days. The numbers of days in which a 70 *40m pond having the same depth can be dug by 300 workers is?
    a. 20   b. 22   c. 24   d. 26

12. Ajay, Vijay and Sanjay are employed to do a piece of work for 529/- Ajay and Vijay together are supposed to do 19/23 of the work and Vijay and Sanjay together 8/23 of the work. How much Ajay should be paid?
    a. 245   b. 295   c. 300   d. 345

13. The tank full petrol in Ajay's motor cycle lasts for 10 days. If he starts using 25% more every day, how many days will the tank full petrol lasts?
    a. 6   b. 8   c. 10   d. 12

14. A and B completed a work together in 5 days. Had A worked at twice the speed and B at half the speed, it would have taken them 4 days to complete the job. A alone does the work in _____ days?
    a. 10   b. 20   c. 25   d. 15

15. The Bubna dam has 4 inlets. Through the first 3 inlets, the dam can be filled in 12 minutes. Through the second, the third and the fourth inlet, it can be filled in 15 minutes; and through the first and fourth inlet, in 20 minutes. How much time will it take all the four inlets to fill up the dam?
    a. 8 min   b. 10 min
    c. 12 min   d. None of these

16. A tank of 3600 m^3 capacity is being filled wherein the discharging tap is 20% more faster than the filling tap. hence its takes 12 minutes more to fill the tank than to empty it. Determine the capacity of discharging tap
    a. 40 m^3/min   b. 50 m^3/min
    c. 60 m^3/min   d. 80 m^3/min

SI CI AND PARTNERSHIP

Concept and Formulae

1. **Principal :** The money borrowed or lent out for a certain period is called the *principal* or the *sum.*
2. **Interest :** Extra money paid for using other's money is called **interest.**
3. **Simple Interest (S.I) :** If the interest on a sum borrowed for a certain period is reckoned uniformlyLet principal = P, Rate = R % per annum (p.a) and Time = T years. Then,

(i) S.I = $\dfrac{P \times R \times T}{100}$.

(ii) P = $\dfrac{100 \times S.I}{R \times T}$; R = $\dfrac{100 \times S.I}{P \times T}$; T = $\dfrac{100 \times S.I}{P \times R}$.

Example 1:

A student purchases a phone by obtaining a simple interest loan. The phone costs 1500 and the interest rate on the loan is 12%. If the loan is to be paid back in weekly instalments over 2 years, calculate:

1. The amount of interest paid over two years
2. The total amount to be paid back
3. The weekly payment amount

Solution:

Principal : 1500, interest: 'R' 12 % = 0.12, repayment time 'N' = 2 years

Interest: 'I' = PNR

= 1500 x 0.12 x 2 = 360

Total repayments = principal + interest

1500 + 360 = 1860

Total repayments weekly payment amount

= 1860/2 x 52 = 17.88per week

Compound Interest

Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say *yearly* or *half-yearly* or *quarterly* to settle the previous account.

In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

After a specified period, the *difference between the amount and the money borrowed is called the **Compound Interest** (abbreviated as **C.I**) for that period.*

Concept and Formulae

If a sum (Principal) P is invested for n years at r% (i in decimal form) per annum, the simple interest S, the compound interest C and the amount A are given by

$S = Pnr/100 = Pni$ and $A = P\{1 + (nr/100)\} = P(1 + ni)$

$C = P[\{1 + (r/100)\}n - 1] = P\{(1 + i)n - 1\}$ and

$A = P\{1 + (r/100)\}n = P\{(1 + i)n\}$

Example 2 :

A invests a sum of Rs. 4000 in a bank which gives simple interest of 10% p.a. B invests Rs. 3000 in a private financial company which gives him a compound interest of 12% p.a, compounded annually. If both keep the sum in the bank and with the company respectively for 3 years, who gets a higher interest?

Solution:

For A, interest : 4000x0.1= 1200

For B, interest : 3000 (1.2^3 -1)= 1214.784

Therefore, B gets a higher interest.

Example 3:

If A invested Rs. 3000 at 12% interest p.a simple interest, what is the excess interest B gets over A in the first year?

Solution:

Interest for A = 3000x 1 x 0.12= 360

Interest for B = 3000 x ($1.2^1 - 1$) = 360

Therefore, B does not get any excess interest over A.

Example 4:

What will Rs.1500 amount to in three years if it is invested in 20% p.a. compound interest, interest being compounded annually?

a. 2400 b. 2592 c. 2678 d. 2540

Solution:

The usual way to find the compound interest is given by the formula A = P(1+)^n

In this formula, A is the amount at the end of the period of investment

P is the principal that is invested

r is the rate of interest in % p.a

And n is the number of years for which the principal has been invested.

In this case, it would turn out to be A = 1500(1+)^3

Hence answer is (b)

Example 5:
How long will it take for a sum of money to grow from Rs.1250 to Rs.10,000, if it is invested at 12.5% p.a simple interest?
a. 8 years   b. 64 years   c. 72 years   d. 56 years

Solution:
Simple interest is given by the formula SI = (pnr/100), where p is the principal, n is the number of years for which it is invested, r is the rate of interest per annum
In this case, Rs. 1250 has become Rs.10,000.
Therefore, the interest earned = 10,000 – 1250 = 8750.
8750 = [(1250*n*12.5)/100]
=> n = 700 / 12.5 = 56 years.
Hence ans is (d).

Example 6:
Rs. 5887 is divided between Shyam and Ram, such that Shyam's share at the end of 9 years is equal to Ram's share at the end of 11 years, compounded annually at the rate of 5%. Find the share of Shyam.

Solution:
Shyam's share * (1+0.05)9 = Ram's share * (1 + 0.05)11
Shyam's share / Ram's share
= (1 + 0.05)11 / (1+ 0.05)9 = (1+ 0.05)2 = 441/400
Therefore Shyam's share = (441/841) * 5887 = 3087.

Example 7:
A man invests Rs.5000 for 3 years at 5% p.a. compound interest reckoned yearly. Income tax at the rate of 20% on the interest earned is deducted at the end of each year. Find the amount at the end of the third year.

Solution:
5% is the rate of interest. 20% of the interest amount is paid as tax. That is 80% of the interest amount stays back. Therefore, if we compute the rate of interest as 80% of 5% = 4% p.a., we will get the same value.
Interest accrued for 3 yrs in compound interest = 3*simple interest on principal + 3*interest on simple interest + 1*interest on interest on interest
= 3*(200) + 3*(8) + 1*0.32 = 600 + 24 + 0.32 = 624.32
The amount at the end of 3 years = 5000 + 624.32 = 5624.32

Example 8:
Shawn invested one half of his savings in a bond that paid simple interest for 2 years and received Rs.550 as interest. He invested the remaining in a bond that paid compound interest, interest being compounded annually, for the same 2 years at the same rate of interest and received Rs.605 as interest. What was the value of his total savings before investing in these two bonds?

Solution:
Shawn received an extra amount of (Rs.605 – Rs.550) Rs.55 on his compound interest paying bond as the interest that he received in the first year also earned interest in the second year.
The extra interest earned on the compound interest bond = Rs.55

The interest for the first year = $\frac{550}{2}$ = Rs.275

Therefore, the rate of interest =   = 20% p.a.
20% interest means that Shawn received 20% of the amount he invested in the bonds as interest.
If 20% of his investment in one of the bonds = Rs.275, then his total investment in each of the bonds
= (275/20)100 = 1375.
As he invested equal sums in both the bonds, his total savings before investing = 2*1375 = Rs.2750.

Questionnaire for Practice

1. Divide Rs. 3,650 into two parts such that, the SI on the first part at 6% for two years, is equal to the SI on the second part at 4% in 3 years.
   a. Rs.1,900; Rs.1,750   c.   Rs.1,825; Rs.1,825
   c. Rs.2,000; Rs.1,650   d.   Rs.1,800; Rs.1,850

2. How many years will it take for some amount to double itself if the rate of interest is 8% per annum?
   a. 10 years   b. 12.5 years
   c. 20 years   d. 15 years

3. Two equal sums of money were lent out at 7% and 5% per annum. The total interest earned for four years amounts to Rs. 960. What is the total amount lent?
   a. Rs. 3200   b. Rs. 4750
   c. Rs. 1980   d. Rs. 4000

4. A and B enter into a partnership. A puts in Rs. 3000/ and at the end of 4 months withdraws Rs. 1500 whereas B brought in additional capital of Rs. 1000 after 3 months. Out of the total profit of Rs. 390 at the end of the year, if A's share is Rs. 240. Find the initial capital of B.
   a. Rs. 1000   b. Rs. 500
   c. Rs. 2000   d. None of these

5. Three friends A, B, C started a joint venture by investing money in the ratio of 2:3:4 respectively. A withdrew half of his money after some months. A few months before the end of the year, C too withdrew one-fourth of his money. If they distributed profits in the ratio 2:4:5 respectively, then after how many months did C withdraw one-fourth of his money?
   a. 6   b. 9
   c. 4   d. Cannot be determined

6. A certain sum of money borrowed in SI triples itself in 10 years. What is the rate of interest?
   a. 30%   b. 25%
   c.20%   d. None

7. Find the compound interest on a sum of Rs.20,000 at the rate of 20% per annum for 2 years, interest being calculated after every 6 months?
   a. Rs.9282   b. Rs.21472
   c. Rs.41472   d. None

DIRECTION SENSE

Dev, Kumar, Nilesh, Ankur and Pintu are standing facing to the North in a playground
- Kumar is standing 40 m to the right of Ankur.
- Dev is 60 m to the south of Kumar.
- Nilesh is at a distance of 25 m to the west of Ankur.
- Pintu is at a distance of 90 m to the north of Dev.

1. Who is to the North-East of the person who is to the left of Kumar?
   a.Dev   b.Nilesh   c.Ankur   d.Pintu

2. If a boy starting from Nilesh, walked up to Ankur and then to Kumar and after this he went to Dev and then to Pintu and walked in a straight line from boy to boy, then how much total distance did he cover?
   a.215m   b. 155m   c.245m   d.185m

3. One morning, A and B were talking to each other face to face. If A's shadow was exactly to the left of B, which direction was B facing?
   a.North   b.South   c.East   d.West

4. If South-East becomes North, North-East becomes West and so on. What will West become?
   a.North-East   b. North-West
   c.South-East   d. South-West

5. Some boys are sitting in three rows all facing North such that A is in the middle row. P is just to the right of A but in the same row. Q is just behind of P while R is in the North of A. In which direction of R is Q?
   a. South   b. South-West
   c. North-East   d. South-East

6. If A x B means A is to the south of B; A + B means A is to the north of B; A % B means A is to the east of B; A - B means A is to the west of B; then in P % Q + R - S, S is in which direction with respect to Q?
   a. South-East   b. South-West
   c. North-East   d. North-West

7. Radha moves towards South-East a distance of 7 km, then she moves towards West and travels a distance of 14 km. From here she moves towards North-West a distance of 7 km and finally she moves a distance of 4 km towards east. How far is she now from the starting point?
   a. 3 km   b.4km   c.10km   d.11km

Each of the following questions is based on the following information:

A # B means B is at 1 metre to the right of A.
A $ B means B is at 1 metre to the North of A.
A * B means B is at 1 metre to the left of A
A @ B means B is at 1 metre to the south of A
In each question first person from the left is facing North

8. According to X @ B * P, P is in which direction with respect to X?
a.North   b.South
c.North-East   d. South-West

9. According to M # N $ T, T is in which direction with respect to M?
a.North-West   b. South-East
c.North-East   d. South-West

10. According to P # R $ A * U, in which direction is U with respect to P?
a.East   b.South
c.West   d.North

11. Ramesh starting from a fixed point goes 15 km towards North and then after turning to his right he goes 15 km. Then he goes 10, 15 and 15 metres after turning to his left each time. How far is he from his starting point?
a. 1.5m   b.10m   c.20m   d.15m

12. Sonalika goes 12 km towards North from a fixed point and then she goes 8 km towards South from there. In the end she goes 3 km towards east. How far and in what direction is she from her starting point?
a.7km East   b. 5 km West
c.7 km West   d. 5 km North-East

13. A man starts on his bike towards west from a certain point. After driving 5 km he turns towards north and drives another 4 km. From the point he turns towards east drives 8 km and stops at a coffee shop. How far is he from the starting point and in which direction from it?
a. 5 km Northeast   b. 4 km Northeast
c. 5 km Southeast   d. 4km Southwest

CODING AND DECODING

1. If EFGHIJK is coded as VUTSRQP, then LIMIT is coded as
1. KNRNC   2. ORNRG
3. JKOKG   4. RSTSG

2. If in a certain code, 'bir le nac' means 'green and tasty'; 'pic nac hor' means 'tomato is green' and 'coc bir hor' means 'food is tasty'. Which of the following means 'tomato is tasty' in that code?
1. bir le hor   2. pic hor nac
3. hor bir pic   4. None

3. If in a certain language , CALCUTTA is coded as GEPGYXXE, which word would be coded as FSQFCE?
1. BOMBYA   2.BOMBAY
3. BOMYAB   4.BOBAYM

4. In a certain code , RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code?
1. 318826   2. 776655
3. 786543   4. 156724

5. If JUNK is written as B5C7B7A11, which one among the following words can be written as B4C3B7B2?
1. BIND   2. BEND   3. HANG   4. HIND

6. GO AT ONCE is a coded message received as 'JB SM BQZY' and you are required to relay the answer in a code saying GO TO GATE. Select the code you will use.
1. HP BU PMDF   2. JB MK JSMY
3. IM CS QMDF   4. JB MB JSMY

7. If OVER is coded as QYIW and UP as WS, then STAR is coded as
1. UWEV   2. UWDV   3. UVBS   4. UWEW

8. In a certain code, COMPUTER is written as RFUVQNPC. How is MEDICINE written in the same code?
   1. EOJDJEFM   2.EOJDEJF
   3. MFEJDJOE   4. MFEDJJOE

In a certain code,'il be pee' means 'roses are blue', 'sik hee' means 'red flowers' and 'pee mit hee' means 'flowers are vegetables'

9. How is 'red' written in that code?
   1. hee   2. sik   3. be   4.None

10. How is 'roses' written in that code?
    1. il   2. pee   3. be   4. Cannot be determined

11. How is 'vegetables are red flowers' written in this code?
    1. pee sik mit hee   2. sik pee hee be
    3. il sik mit hee   4. None

12. If GO = 32, SHE = 49, then SOME will be equal to:
    1. 56   2. 58   3. 62   4. 64

13. If ROSE is coded as 6821, CHAIR is coded as 73456 and PREACH is coded as 961473, what will be the code for SEARCH?
    1. 246173   2. 214673   3. 214763   4. 216473

If in a certain language, ENTRY is coded as 12345 and STEADY is coded as 931785, then state the correct code for each of the given words.

14. TENANT
    1. 956169   2. 196247   3. 352123   4. 312723

15. SEDATE
    1. 918731   2. 954185   3. 814195   4. 614781

16. If '245' means — 'Art and Talent' in a certain code language, '316' means — 'Callous to Generous', '147' means — 'Callous and Polite' then what is the code used for 'to' ?
    1. Only 3   2. Only 1   3. 3 or 6   4. Only 6I am facing south. I turn right and walk 20 m. Then I turn right again and walk 10 m. Then I turn left and walk 10 m and then turning right walk 20 m. Then I turn right again and walk 60 m. In which direction am I from the starting point?
    a. North   b. Northwest
    c. East   d. Northeast

14. A child is looking for his father. He went 90 meters in the east before turning to his right. He went 20 meters before turning to his right again to look for his father at his uncle's place 30 meters from this point. His father was not there. From here he went 100 meters to the north before meeting his father in a street. How far did the son met his father from the starting point?
    a. 80 meters   b. 100 meters
    c. 140 meters   d. 260 meters

NUMBER SERIES

Type I
1. 16, 33, 65, 131, 261, ?
   1. 523   2. 521   3. 613   4. 721

2. 10, 5, 13, 10, 16, 20, 19, ?
   1. 22   2. 40   3. 38   4. 23

3. 2, 4, 12, 48, 240, ?
   1. 960   2. 1440   3.1080   4.1920

4. 8, 7, 11, 12, 14, 17, 17, 22, ?
   1. 27   2. 20   3. 22   4. 24

5. 8, 24, 12, 36, 18, 54, ?
   1. 27   2. 108   3. 68   4. 72

6. 4, -8, 16, -32, 64, ?
   1. 128   2. -128   3. 192   4. -192

7. 7, 26, 63, 124, 215, 342, ?
   1. 481   2. 511   3. 391   4. 421

8. 3, 7, 6, 5, 9, 3, 12, 1, 15, ?
   1. 18   2. 13   3. -1   4. 3

Type II

9. 8, 27, 64, 100, 125, 216, 343
    1. 27   2. 100   3. 125   4. 343

10. 396, 462, 572, 427, 671, 264
    1. 396   2. 427   3. 671   4. 264

11. 2, 5, 10, 17, 26, 37, 50, 64
    1. 50   2. 26   3. 37   4. 64

12. 8, 13, 21, 32, 47, 63, 83
    1. 47   2. 63   3. 32   4. 83

13. 36, 54, 18, 27, 9, 18.5, 4.5
    1. 18.5   2. 4.5   3. 18   4. 54

14. 56, 72, 90, 110, 132, 150
    1. 72   2. 110   3. 132   4. 150

15. 7, 8, 18, 57, 228, 1165, 6996
    1. 7   2. 6996   3. 228   4. 57

16. 445, 221, 109, 46, 25, 11, 4
    1. 221   2. 109   3. 46   4. 25

DATA INTERPRETATION

Direction for questions 1 - 5:
Study the following graph carefully and answer the questions given below it
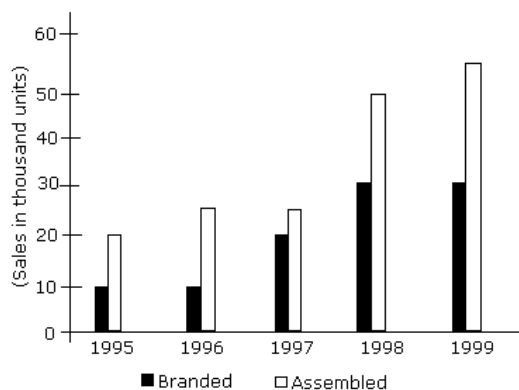
1. What is the approximate average percentage growth of sales of Assembled PCs for the given years?
    a. 30   b.20   c.40   d. 34

2. If the Branded PCs sold in 1996 were 1,00,000 how many Branded PCs were sold in 1999?
    a. 2,02,800   b. 3,00,000
    c. 2,34,000   d. None of these

3. What is the difference between total Branded the total Assembled PCs sold for the given years?
    a. 75,000   b.7,50,000
    c.1,75,000   d. None of these

4. In which year is the difference in the growth between Branded and Assembled the lowest?
    a. 1995   b. 1998   c. 1999   d. None

5. For Assembled PC sale, which year the per cent growth is the highest compared to previous year?
    a.1999   b.1996   c.1998   d. None

Directions for questions 6 - 10:
Refer the following and answer the questions

Stream	Male : Female
Arts	2:3
Medicine	1:1
Engineering	7:5
Science	4:5
Management	9:7
Commerce	3:5

Total number of students = 8000
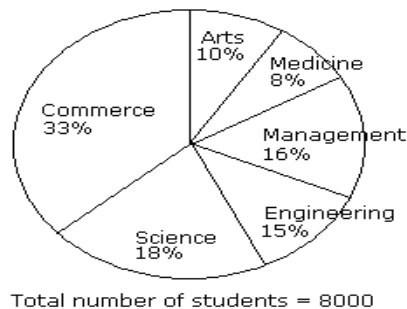
6. Total number of female students studying Engineering and Medicine is
    a. 1280   b. 5000   c. 820   d. 480

7. Number of Female students studying Management is what percentage of the total number of students in the University?
    a. 27   b. 12   c. 9   d. None

8. How many male students are there in the Arts stream?

    a. 320   b. 480   c. 800   d. 720

9. What is the total number of male students studying Commerce?

    a. 1280   b. 1440   c. 1650   d. None

10. The approximate central angle for male students studying science?

    a. 36^0 b. 28^0 c. 50^0 d. None

Direction for questions 11 - 15:

Refer the following and answer the questions

In a class of 120 students, each student studies at least one of the subjects from History, English & Maths. 59 study History, 67 study English and 73 Maths. 34 study Maths and History, 26 English & Maths and 33 History and English.

11. How many students study exactly two subjects?

    a. 54   b. 51   c. 48   d. 46

12. How many students study all the three subjects?

    a. 12   b. 51   c. 13   d. 14

13. How many students study more than one subject?

    a. 63   b. 65   c. 62   d. 66

14. How many study English and Maths but not History?

    a. 12   b. 13   c. 14   d. 11

15. How many study English and Maths & English and History?

    a. 41   b. 43   c. 47   d. 45

Direction for questions 16 - 20:

Refer the following and answer the questions

Distribution of Personal Income of Citizens of a country in 2000

Rs. 2499.4 Billion = 100%
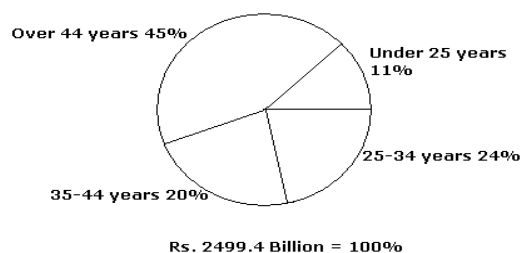
16. The personal income of the citizens in the age group 35-44 years is (in Rs. Billion) approximately

    a. 500   b. 600   c. 1125   d. 1375

17. The ratio of the personal income of the citizens in the 25-34 years age group to that of 35-44 years age group is

    a. 9 : 4   b. 5 : 6   c. 8 : 15   d. 6 : 5

18. If the total personal income in 2000 is 20% more than that of 1997, the total personal income of 1997 was (in Billion Rupees)

    a. 2200   b. 2100   c. 2000   d. 1900

19. The angle in the pie chart representing the income of citizens of the 35-44 years age group is

    a. 20^0 b. 40^0 c. 72^0 d. 86^0

20. If the total personal income in 2005 is one and a half times that in 2000, what is the average annual growth rate during the period 2000 – 2005?

    a. 15%   b. 12%   c. 10%   d. None

Direction for questions 21 - 24:

Prof. Singh has been tracking the number of visitors to his homepage? His service provider has provided him with the following data on the country of origin of the visitors and the university they belong to:

Number of visitors:

COUNTRY	DAY 1	DAY 2	DAY 3
Canada	2	0	0
Netherland	1	1	0
India	1	2	0
UK	2	0	2
USA	1	0	1

Number of visitors:

UNIVERSITY	DAY 1	DAY 2	DAY 3
University 1	1	0	0
University 2	2	0	0
University 3	0	1	0
University 4	0	0	2
University 5	1	0	0
University 6	1	0	1
University 7	2	0	0
University 8	0	2	0

21. To which country does University 5 belong?
a. India or Netherlands not USA
b. India or USA but not Netherlands
c. Netherlands or USA but not India
d. India or USA but not UK

22. University 1 can belong to
a. UK   b. Canada
c. Netherlands   d. USA

23. Visitors from how many universities from UK visited Prof. Singh's homepage?
a. 1   b. 2   c. 3   d. 4

24. Which among the listed countries can possibly host three of the eight listed universities?
a. None   b. Only UK
c. Only India   d. Both India and UK

DATA SUFFICIENCY

Type I - Each question is followed by two statements, I and II. Mark the answer
a. If the question can be answered by using one of the statements alone, but cannot be answered using the other statement alone.
b. If the question can be answered by using either statement alone.
c. If the question can be answered by using both statements together, but cannot be answered using either statement alone.
d. If the question cannot be answered even by using both statements together.

1. What is the sum of 2 numbers?
I. The LCM of the numbers is 51
II. One of the numbers is 17

2. What are the distinct integers A and B?
I. The product of A and B is 4
II. A and B are both positive.

3. LCM of 2 numbers is 630. What is the absolute difference between them?
I. HCF is 9
II. The sum of the 2 numbers is 153

4. Is $(a^2 - b^2)$ even?
I. $(a + b)$ is odd   II. $(a - b)$ is odd

5. If a, b and c are integers, is $3(a + b) + c$ divisible by 3?
I. $(a + b)$ is divisible by 3
II. c is divisible by 3.

6. If $d = 0.2f6$, then what is the value of d rounded off to one decimal place?
I. $f < 5$ II. $d < ¼$

7. A log of wood is 8m long. It is cut into three smaller pieces. How long is the longest of the three pieces?
I. The smallest piece is 1.5m long
II. One of the pieces is 4.1m.

8. How many children does M have?
I. H is only daughter of X who is wife of M.
II. K and J are brothers of M.

9. If $x \neq y$, then what is the value of x?
I. $7y/(3x+67) = 5$ II. $(x + y)/(y-x) = 1$

10. What is the value of $(c + d)$?
I. $c^2 - d^2 = 5$ II. $c-d = 1$

11. Is g greater than h?
I. $(g + 3)$ is greater than $(h + 2)$
II. The cube of g is greater than the cube of h.

12. What is the value of the ratio $(a + c): c$?
I. The ratio of a: b = 1: 5
II. The ratio of b:c = 3:2.

13. Ten children are sitting at a table. What is the average age of 10 children?
I. Average age of 6 youngest children is 12
II. Average age of 4 oldest children is 16

14. What is the sum of three real numbers?
I. Their product is 450   II. Their average is 40

15. What is the value of x?
I. $q - x = x - p$ II. $p + q = 20$

16. Is Arun taller than Sachin?
    I. Dinesh is of the same height as Arun and Sachin.
    II. Sachin is not shorter than Dinesh.

17. In a certain code language, '13' means 'stop smoking' and '59' means 'injurious habit'. What is meaning of '9' and '5' in that code?
    I. '157' means 'stop bad habit'
    II. '839' means 'smoking is injurious'.

18. Five persons --- A, B, C, D and E are sitting in a row. Who is sitting in the middle?
    I. B is in between E and C.
    II. B is to the right of E.
    III. D is in between A and E.

a. I and II together   b. II and III together
c. I and III together   d. I, II and III together
e. Data insufficient

19. Four Subjects --- Physics, Chemistry, Mathematics and Biology were taught in four consecutive periods of one hour each starting from 8.00 a.m. At what time was the Chemistry period scheduled?
    I. Mathematics period ended at 10.00 am which was preceded by Biology.
    II. Physics was scheduled in the last period.
    III. Mathematics period was immediately followed by Chemistry.
    a. Only I   b. Only I and II
    c. Only II   d. II and III together

PROBLEMS ON AVERAGE

Concept and Formulae:

1. Average = $\left[\dfrac{\text{Sum of observations}}{\text{Number of observations}}\right]$

2. Suppose a man covers a certain distance at x kmph and an equal distance at y kmph. Then, the average speed during the whole journey is $\dfrac{2xy}{x + y}$ kmph.

Example 1:
Find the average of all prime numbers between 30 and 50.
Solution:
There are 5 prime number between 30 and 50.
They are 31, 37, 41, 43 and 47.
∴ Reqd avg = $\dfrac{31+37+41+43+47}{5}$ = $\dfrac{199}{5}$ = 39.8.

Example 2:
Find the average of first 40 natural numbers.
Solution:
Sum of first n natural numbers = $\dfrac{n(n + 1)}{2}$
So, sum of first 40 natural numbers
= $\dfrac{40 \times 41}{2}$ = 820.
∴ Required average = $\dfrac{820}{40}$ = 20.5.

Example 3:
Find average of first 20 multiples of 7.
Solution:
Reqd avg = $\dfrac{7(1 + 2 + 3 + \ldots + 20)}{20}$
= $\dfrac{7 \times 20 \times 21}{20 \times 2}$ = $\dfrac{147}{2}$ = 73.5

Example 4:
Average of four consecutive even numbers is 27. Find the largest of these numbers.
Solution:
Let the numbers be x, x + 2, x + 4 and x +6.
Then, $\dfrac{x + (x + 2) + (x + 4) + (x + 6)}{4}$ = 27
$\dfrac{4x + 12}{4}$ = 27 => x + 3 = 27 => x = 24.
∴ Largest number = (x + 6) = 24 + 6 = 30.

Example 5:
There are two sections A and B of a class, consisting of 36 and 44 students respectively. If the average weight of section A is 40 kg and that of section B is 35 kg, find the average weight of the whole class.
Solution:
Total weight of (36 + 44) students
= (36 x 40 + 44 x 35) kg = 2980kg.
∴ Avg weight of the whole class
= $\dfrac{2980}{80}$ kg = 37.25 kg

Questionnaire for Practice

1. The average age of students in a class of 40 is 16 years. 12 more students of average age 19 years are admitted to the class. Find the average age of the class.
   a.12 years   b.14 years
   c.15.6 years   d.16.7 years

2. The average weight of patients on a particular day in a hospital is recorded as 50 kg. Of these there were 20 children of average weight 30 kg, 40 males of average weight 60 kgs and 50 female patients. Find the average weight of the female patients.
   a. 45 kg   b.50kg   c. 60 kg   d. None

3. The average runs scored by a batsman in 5 innings in 45. After his sixth innings the average falls to 43. How many runs did he score in the 6th innings?
   a.31   b.32   c.33   d.34

4. 6 persons standing in queue with different age group, after two years their average age will be 43 and seventh person joined with them. Hence the current average age has become 45. Find the age of seventh person?
   a. 45   b. 56   c. 69   d. 57

5. In a shopping mall with a staff of 5 members the average age is 45 years. After 5 years a person joined them and the average age is again 45 years. What's the age of 6th person?
   a.40   b. 30   c. 20   d. 25

BLOOD RELATIONS

1. If A + B means A is the mother of B; A - B means A is the brother B; A % B means A is the father of B and A x B means A is the sister of B, which of the following shows that P is the maternal uncle of Q?
   a. Q - N + M x P   b. P + S x N - Q   c. P - M + N x Q   d. Q - S % P

2. Pointing at a photograph Upendra says "He is the only son of the only son to the father of my father's wife". How is the person in the photograph related to the speaker?
   a. Brother   b. Father   c. Uncle   d. Cousin

3. A is the son of C; C and Q are sisters; Z is the mother of Q and P is the son of Z. Which of the following statements is true?
   a. P and A are cousins   b. P is the maternal uncle of A
   c. Q is the maternal grandfather of A   d. C and P are sisters

4. Pointing at a photograph Ramaprasad says "I have no brothers or sisters but his father is my father's son". How is the person in the photograph related to the speaker?
   a. Himself   b. Father   c. Son   d. Brother

5. B5D means B is the father of D. B9D means B is the sister of D. B4D means B is the brother of D. B3D means B is the wife of D. Which of the following means F is the mother of K?
   a. F3M5K   b. F5M3K   c. F9M4N3K   d. F3M5N3K

In a family of seven, three generations are living together.

> ➤ The family consists of two married couples having two children each.
> ➤ Gopal is lucky to have two grandchildren.
> ➤ There are two housewives and both are beautiful.
> ➤ Gopal who is Manoj's father, is a lawyer and earns the most.
> ➤ Jyotsna is the sister of a lecturer and herself is a nurse.
> ➤ Anuradha is married to a lecturer who is Nidhi's son.
> ➤ Jyothika is the granddaughter of one of the housewives and is a classical dancer.

6. What is Manoj's profession?
    1. Student   2. Lecturer   3. Lawyer   4. Cannot be determined
7. How many male members are there in the family?
    1.2   2.3   3. 4   4. Cannot be determined

Directions for questions 8 - 11: Read the following data and answer the questions.
Radha and Minnilal have two children, Simmi and Divya. Divya is married to Anuj who is the son of Madhu and Jabbar. Resham is the daughter of Anuj. Kiran, who is Anuj's sister is married to Subodh and has two sons Tarun and Aman. Tarun is the grandson of Madhu and Jabbar.

8. What is the relationship between Aman and Resham?
    A. Uncle-Niece   B. Father-Daughter
    C. Husband-wife   D. Cousins
9. How is Subodh related to Jabbar?
    A. Son-in-law   B. Son   C. Brother   D. Father-in-law
10. How is Resham related to Kiran?
    A. Niece   B. Daughter   C. Mother   D. Aunt
11. How is Kiran related to Divya?
    A. Aunt   B. Grandmother   C. Sister-in-law   D. None of these

LOGICAL REASONING

Directions for questions 1 - 5: Read the following data and answer the questions
There are four girls – Kavitha, Meera, Meena and Teju. One of them lives at Chennai and her hobbies are clay modeling and reading. Meera lives at Mumbai. Kavitha also lives at Mumbai and her hobby is dancing. Both girls staying at Mumbai are fond of gardening. Teju lives at Delhi and her hobbies are painting and photography. One girl staying at Mumbai enjoys traveling. If all the girls have two hobbies each, answer the following questions:

1. Who enjoys traveling?
    1. Kavitha   2. Meera   3. Meena   4. Teju
2. Who is the girl with the hobbies dancing and gardening?
    1. Kavitha   2. Teju   3. Meera   4. Meena
3. Who is staying at Chennai?
    1. Meena   2. Teju   3. Both 1 and 2   4. None of the above
4. Who among the following is fond of gardening?
    1. Meena   2. Meera   3. Both 1 and 2   4. None of the above
5. What are Meena's hobbies?
    1. Reading and Dancing   2. Gardening and painting
    3. Travelling and dancing   4. Clay modeling and reading

Directions for questions 6 - 11: Read the following data and answer the questions A genealogist has determined that M, N, P, Q, R, S and T are the father, the mother, the aunt, the brother, the sister, the wife and the daughter of X, but she has been unable to determine which person has which status. She does know :

1. P and Q are the same sex.   2. M and N are not of the same sex.

3. S was born before M.   4. Q is not the mother of X.

6. How many of seven people - M, N, P, Q, R, S and T - are female?
   1. 3   2. 4   3. 5   4.7

7. Which of the following must be true?
   1. M is a female.   2. N is a female.   3. P is a female.   4. T is a female

8. If T is the daughter of X, which of the following must be true?
   1. M and P are of the same sex   2. M and Q are of the same sex
   3. P is not of the same sex as N   4. R is not of the same sex as S

9. If M and Q are sisters, all of the following must be true EXCEPT
   1. N is a male   2. M is X's mother   3. Q is X's aunt   4. T is X's daughter

10. If S is N's grandfather, then which of the following must be true?
    1. R is N's aunt   2. X is P's son   3. M is X's brother   4. Q is S's husband

11. If M is X's wife, all of the following could be true EXCEPT
    1. S is X's daughter   2. P is X's sister   3. Q is X's sister   4. R is X's father

Directions for questions 12 – 16: Read the following data and answer the questions
All the roads of city Z are either perpendicular or parallel to one another. The roads are all straight
Roads. A, B, C, D, E are parallel to one another Roads G, H, I, J, K, L, M are parallel to one another.
Road A is 1 mile east of Road B   Road B is ½ mile West of C
Road D is 1 mile West of E   Road I is 1 mile north of J
Road K is ½ mile north of L   Road K is 1 mile South of M

12. If E is between B and C which is false

1. D is less than 1 mile from B   2. C is less than $1\frac{1}{2}$ mile from D

3. E is less than 1 mile from A   4. D is 2 miles West of A

13. Which of the following possibilities would make 2 roads coincide?

1. L is $\frac{1}{2}$ mile north of I   2. D is $\frac{1}{2}$ mile east of A

3. I is $\frac{1}{2}$ mile north of K   4. C is 1 mile West of D

14. If X is parallel to I and X is $\frac{1}{2}$ mile South of J and 1 mile north of G, which two roads would be $2\frac{1}{2}$ miles apart?
   1. I and X   2. X and H   3. J and G   4. I and G

15. If road E is between B and C then distance between A and D is
   1. $\frac{1}{2}$ mile   2. 1 mile   3. $1\frac{1}{2}$ miles   4. $1\frac{1}{2}$ - 2miles

16. Which is necessarily true?
   1. I is 1 mile north of L   2. D is 2 miles West of B
   3. E and B intersect   4. M is $1\frac{1}{2}$ miles north of L

Directions for questions 17 - 21: Read the following data and answer the questions

A group of six players P, Q, R, S, T and U are participating in a challenge tournament. All matches played are challenge matches and are governed by the following rules:

A player may challenge another player if and only if that player is ranked either one or two places above her.

If a player successfully challenges the player ranked immediately above her, the two players exchange ranks.

If a player successfully challenges the player two ranks above her, she moves up two ranks, and both the loser of the match and the player ranked below the loser move down one rank.

If a player is unsuccessful in her challenge, she and the player immediately below her exchange ranks, unless the unsuccessful challenger was already ranked last, in which case the rankings remain unchanged.

The initial rankings from the highest (first) to the lowest (sixth) are P, Q, R, S, T, U. Only one match is played at a time.

17.   Which of the following is possible as the first match of the tournament?
        1. P challenges Q   2. Q challenges R   3. R challenges P   4. S challenges P

18.   If S reaches first place after the first two matches of the tournament, which of the following must be ranked fourth at that point in play?
        1. P   2. Q   3. R   4. T

19.   All of the following are possible rankings, from highest to lowest, after exactly two matches EXCEPT
        1. P, R, Q, T, S, U   2. P, R, Q, S, U, T   3. R, P, Q, U, S, T   4. Q, P, S, R, U, T

20.   If exactly two matches have been played, what is the maximum number of players whose initial ranks could have been changed?
        1. 2   2. 3   3. 4   4. 6

21.   If after the initial two matches two players have improved their rankings and four players have each dropped in rank, which of the following could be the third match of the tournament?
        1. R challenges P   2. R challenges Q   3. Q challenges U   4. U challenges

Directions for questions 22 - 26: Read the following data and answer the questions

Three friends A. B and C have Rs. 155, Rs.180, Rs. 160. They started playing a game in which a person who has maximum money gives other two persons an amount equal to one-fifth of their own respective amount. The game stops when each of them has given money at least once.

22.   What is the approximate amount left with A at the end of the game?
        1. Rs. 155   2. Rs. 169   3. Rs.159   4. Rs. 162

23.   What is the approximate amount left with B at the end of the game?
        1. Rs. 175   2. Rs. 154   3. Rs. 168   4. Rs. 182

24.   What is approximate amount left with C at the end of the game?
        1. Rs. 158   2. Rs. 155   3. Rs. 151   4. Rs. 145

25.   In how many steps does the game end?
        1. 3   2. 4   3. 5   4. 6

26.   After the first step who among them has highest amount?
        1. A   2. B   3. C   4. A and B

Directions for questions 27 - 31: Read the following data and answer the questions

At the end of the soccer season, every player had scored a prime number of goals and the average of the 11 players was also a prime number. No player's individual tally was the same as anyone else's or as the average. Nobody had scored more than 45 goals.

27. What was the average of their goals scored?

     1. 27   2. 23   3. 29   4. 31

28. What was the maximum number of goals scored by a single player?

     1. 43   2. 41   3. 37   4. 29

29. What was the minimum number of goals scored by a single player?

     1. 5   2. 7   3. 11   4. 13

30. How many players had scored above 20 goals individually?

     1. 6   2. 5   3. 7   4. None of these

31. What was the second minimum number of goals scored by a single player?

     1. 2   2. 5   3. 3   4. 7

DEDUCTIVE REASONING

In each of the following questions two statements are given and these statements are followed by two conclusions numbered (1) and (2). You have to take the given two statements to be true even if they seem to be at variance from commonly known facts. Read the conclusions and then decide which of the given conclusions logically follows from the two given statements, disregarding commonly known facts. Give answer:

A. If only (1) conclusion follows

B. If only (2) conclusion follows

C. If both (1) and (2) follows.

D. If neither (1) nor (2) follows

Questionnaire for Practice

1. Statements:

Some cows are crows.

Some crows are elephants.

Conclusions:

    1. Some cows are elephants.

    2. All crows are elephants.

1. A 2.B   3.C   4. D

2. Statements:

Some dogs are bats. Some bats are cats.

Conclusions

1. Some dogs are cats.

2. Some cats are dogs.

1. A 2.B   3.C   4. D

3. Statements:

All the trucks are flies. Some scooters are flies.

Conclusions:

1. All the trucks are scooters.

2. Some scooters are trucks.

1. A 2.B   3.C   4. D

4. Statements

Some ants are parrots.

All the parrots are apples.

Conclusions:

1. All the apples are parrots.

2. Some ants are apples

1. A 2.B   3.C   4. D

5. Statements:

All the windows are doors. No door is a wall

Conclusions:

    1.   Some windows are walls

    2.   No wall is a door

1. A 2.B   3.C   4. D

Directions for questions 6 - 8: In each question below are given two or three statements by three or four conclusions numbered I, II, III and IV. You have to take the m statements to be true even they seem to be at variance with commonly known Read all the conclusions and then decide which of the conclusions can, disregarding commonly known facts, be logically derived

6. Statements:

1. All men are women.

2. All females are men.

3.No woman is a vampire.

Conclusions:

I.   Some women are not females.

II.   All females are vampires.

III.   Some women are not vampires.

IV.   IV. No man is a woman.

a. I&II follows   b. II&III follows

c. I&III follows   d. None of these

7. Statements:

    1. All buses are trucks

    2. No truck is a lorry.

    3. No truck is a jeep.

Conclusions:
I. All trucks are lorries.
II. Some lorries are not trucks.
III. All jeeps are trucks.
IV. Some jeeps are not buses.
a. Only I and II follow
b. Only II and IV follow
c. Only I and III follow   d. None of these

8. Statements:
1. All camels are donkeys.
2. Some horses are mules.
3. Some camels are horses.
Conclusions:
I. All horses are donkeys.
II. All camels are mules.
III. Some donkeys are not horses.
IV. Some camels are not horses.
a. Either I or II follows
b. Either II or IV follows
c. only I and II follows   d. None of these

Questions from 9 to 11: In each question below is given a statement followed by two conclusions numbered I and II. You have to assume everything in the statement to be true, then consider the two conclusions together and decide which of them logically follows beyond a reasonable doubt from the information given in the statement. Give answer:
(A) If only conclusion I follows.
(B) If only conclusion II follows.
(C) If either I or II follows
(D) If neither I nor II follows
(E) If both I and II follow.

3. 9. *Statements***:** In Japan, the incidence of stomach cancer is very high, while that of bowel cancer is very low. But Japanese immigrate to Hawaii, this is reversed - the rate of bowel cancer increases but the rate of stomach cancer is reduced in the next generation. All this is related to nutrition - the diets of Japanese in Hawaii are different than those in Japan.
Conclusions:
I. The same diet as in Hawaii should be propagated in Japan also.
II. Bowel cancer is less severe than stomach cancer.

4. 10. *Statements***:** Monitoring has become an integral part in the planning of social development programmes. It is recommended

that Management Information System be developed for all programmes. This is likely to give a feedback on the performance of the functionaries and the efficacy with which services are being delivered.
Conclusions:
I. All the social development programmes should be evaluated.
II. There is a need to monitor the performance of workers.

5. 11. *Statements***:** The T.V. programmes, telecast specially for women are packed with a variety of recipes and household hints. A major portion of magazines for women also contains the items mentioned above.
Conclusions:
I. Women are not interested in other things.
II. An average woman's primary interest lies in home and specially in the kitchen

Questions from 12 to 13: In each question below is given a statement followed by two assumptions numbered I and II. You have to consider the statement and the following assumptions and decide which of the assumptions is implicit in the statement. Give answer
(A) If only assumption I is implicit
(B) If only assumption II is implicit
(C) If either I or II is implicit.
(D) If neither I nor II is implicit
(E) If both I and II are implicit.

6. 12. *Statement***:** "In order to bring punctuality in our office, we must provide conveyance allowance to our employees." - In charge of a company tells Personnel Manager.
Assumptions:
I. Conveyance allowance will not help in bringing punctuality.
II. Discipline and reward should always go hand in hand.

13. *Statement***:** Unemployment allowance should be given to all unemployed Indian youth above 18 years of age.
Assumptions:
There are unemployed youth in India who needs monetary support.
The government has sufficient funds to provide allowance to all unemployed youth.

Questions from 14 to 15: Each question given below consists of a statement, followed by two arguments numbered I and II. You have to decide which of the arguments is a 'strong' argument and which is a 'weak' argument. Give answer:

(A) If only argument I is strong
(B) If only argument II is strong
(C) If either I or II is strong
(D) If neither I nor II is strong and
\(E) If both I and II are strong.

14. *Statement***:** Should the prestigious people who have committed crime unknowingly, be met with special treatment?
Arguments:
I. Yes. The prestigious people do not commit crime intentionally.
II. No. It is our policy that everybody is equal before the law.

15. *Statement***:** Should internal assessment in colleges be abolished?
Arguments:
I. Yes. This will help in reducing the possibility of favouritism.
II. No, teaching faculty will lose control over students.

Directions for Questions 16 - 18:
Each question contains six statements followed by four sets of combinations of three Choose the set in which the statements are logically related.

16 .
A. All crows are birds.
B. All birds are black.
C. All crows are black.
D. All crows have beaks.
E. All crows lay eggs.
F. All birds have claws.
a. ABC   b. BCD   c. ADE   d. DEF

17.
A. All roses are red.
B. Some roses have no thorns.
C. All red flowers are roses.
D. This flower has thorns.
E. This flower is red in colour.
F. This flower is a rose
a. DBA   b. DEF   c. ACB   d. CEF

18.
A. Some insects are multi-colored.
B. Some insects are blind.
C. All multi-colored are insects.
D. Some blind are not insects.
E. All insects are bees
F. Some multi-colored insects may be blind.
a. FCA   b. DCB   c. DEA   d. ABF

Directions for questions 19 - 21: Each question is followed by two statements, 1 and 2. Mark the answer as:
A – Conclusion 1 follows
B – Conclusion 2 follows
C – Both 1 and 2 follows
D – Either 1 or 2 follows
E – Neither 1 or 2 follows

19. Statement:
The British in India made some infrastructure in transportation, but its aim was to facilitate trade to their advantage.
Conclusion:
1. Foreign rule is always exploitative.
2. Transport infrastructure is necessary to facilitate trade.
1) A   2) B   3) C   4) D   5) E

20. Statement
Frequent increase in petroleum prices will lead to inflation.
Conclusion:
1. Increase in the price of petroleum products generally leads to increase in the price of other goods.
2. The government cannot help increasing the price of petroleum products because the crude prices have risen globally.
1) A   2) B   3) C   4) D   5) E

22. Statement
All that glitters is not gold
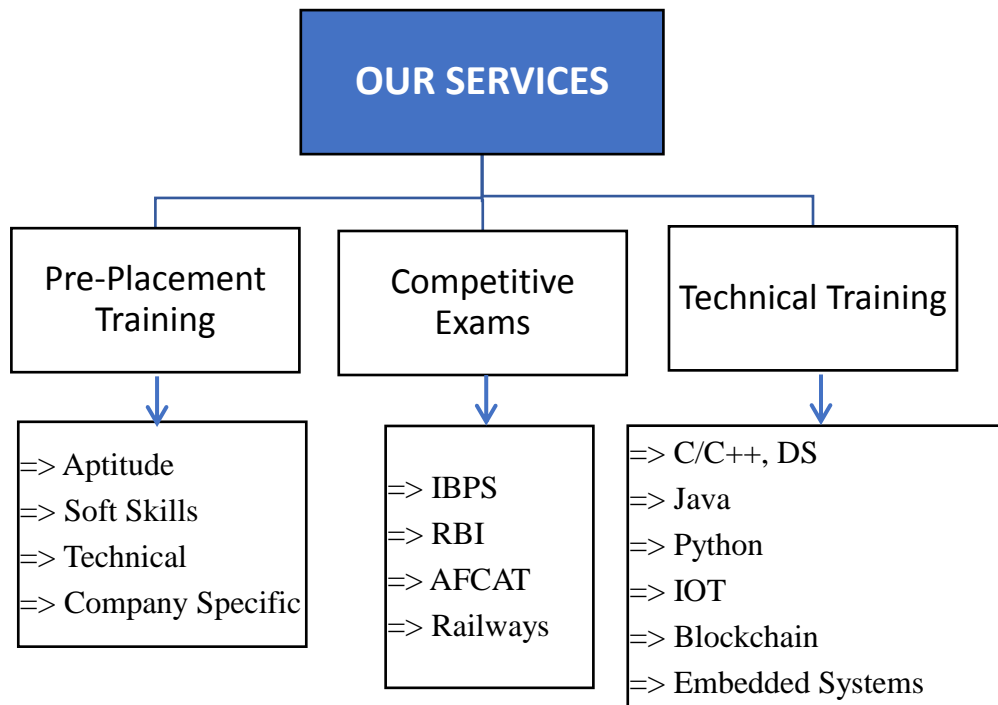Conclusion:
1. Gold does not always glitter
2. The worth of gold is gauged by glitter
1) a   2) b   3) c   4) d   5) e

23. Statement
Nutritious food is necessary to maintain good health
Conclusion:
1. All healthy people take nutritious diet
2. Nutritious food is expensive that only few can afford it, that is why few people stay healthy
1) a   2) b   3) c   4) d   5)e

OUR SERVICES

Pre-Placement Training

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=> Python
=> IOT
=> Blockchain
=> Embedded Systems

"A good trainer is the one who serves knowledge like never served before"

-   Anonymous

"There are no shortcuts to any place worth going"

-   Beverly Sills