1. **A bag contains 4 balls.Two balls drawn at random without replacement and are found to be white. What is the probability that all balls are white?**

Solution Verified by Toppr

Let A be the event of drawing 2 white balls.
from the bag containing 4 balls.
The remaining 2 balls of the bag has three options.

Let E_1 be the event that the remaining 2 balls of the bag are not white

Let E_2 be the event that are remaining 2 balls of the bag are one white and one not white.

Let E_3 be the event that rae remaining 2 balls of the bag are white.

$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P(A/E_1) = P$ (drawing 2 white balls from the bag contains 2 white and 2 not white)

$= \frac{^2C_2}{^4C_2} = \frac{1}{6}$

$P(A/E_2) = P$ (drawing 2 white balls from the bag containing 2 white and 1 non-white)

$P(A/E_2) = P$ (drawing 2 white balls from the bag containing 2 white and 1 non-white)

$=^3 .C_2/^4.C_2 = \frac{3}{6} = \frac{1}{2}$

$P(A/E_3) = P$ (drawing 2 white ball from bag containing 4 whire balls)

$= \mathbf{1.}$

$P(E_2/A) = \frac{P(E_2)(P A/E_2)}{P(E_1)P(A/E_1)+P(E_2)+P(E_3)P(A/E_3)}$

$= \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{1}$

$= \frac{1}{3}(\frac{1}{6} + \frac{1}{2} + 1)$

$= \frac{1}{3}$

$P(E_2/A) = \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{5}{3}} = \frac{\frac{1}{6}}{\frac{5}{9}}$

$= \frac{1}{6} \times \frac{3}{5} = \frac{3}{10}$

2. **Three urns are there containing white and black balls; first urn has 3 white and 2 black balls, second urn has 2 white and 3 black balls and third urn has 4 white and 1 black balls. Without any biasing one urn is chosen from that one ball is chosen randomly which was white. What is probability that it came from the third urn?**

Let U_1, U_2, U_3 be an urn and E_1, E_2, E_3 be the events that a ball is chosen from an urn U_1, U_2, U_3 respectively.

Let A be event that white ball is drawn.

We have, $U_1 = \{2 \text{ White}, 3 \text{ Black balls}\}$

$U_2 = \{3 \text{ white}, 2 \text{ Black balls}\}$

$U_3 = \{4 \text{ white}, 1 \text{ Black ball}\}$

$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$\therefore \quad P(A/E_1) = \frac{2}{5}, \quad P(A/E_2) = \frac{3}{5}, \quad P(A/E_3) = \frac{4}{5}$

Now, Using Bayes' theorem,

$$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + P(E_3).P(A/E_3)}$$

$$\therefore \quad P(E_1/A) = \frac{1/3 \times 2/5}{(1/3 \times \frac{2}{5}) + (\frac{1}{3} \times \frac{3}{5}) + (\frac{1}{3} \times \frac{4}{5})}$$

$$\therefore \quad P(E_1/A) = \frac{2}{2+3+4} = \frac{2}{9}$$

$$\therefore \quad P(E_1/A) = \frac{2}{9}.$$

3. Given the full joint distribution shown in Figure 13.3, calculate the following:

a. P(toothache).
b. P(Cavity).
c. P(Toothache | cavity).
d. P(Cavity | toothache ∨ catch)

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Figure 13.3 A full joint distribution for the *Toothache*, *Cavity*, *Catch* world.

The main point of this exercise is to become completely familiar with the basic mechanics of answering queries by adding up entries in the joint distribution. It also helps you to understand the various notations of bold versus non-bold P, and uppercase versus lowercase variable names.

(a) This asks for the probability that *Toothache* is true.

$$P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$$

(b) This asks for the vector of probability values for the random variable *Cavity*. It has two values, which we list in the order $\langle true, false \rangle$. First add up $0.108 + 0.012 + 0.072 + 0.008 = 0.2$. Then we have

$$\mathbf{P}(Cavity) = \langle 0.2, 0.8 \rangle \ .$$

(c) This asks for the vector of probability values for *Toothache*, given that *Cavity* is true.

$$\mathbf{P}(Toothache|cavity) = \langle (.108 + .012)/0.2, (0.072 + 0.008)/0.2 \rangle = \langle 0.6, 0.4 \rangle$$

(d) This asks for the vector of probability values for *Cavity*, given that either *Toothache* or *Catch* is true. First compute $P(toothache \lor catch) = 0.108 + 0.012 + 0.016 + 0.064 + 0.072 + 0.144 = 0.416$. Then

$$\begin{aligned} &\mathbf{P}(Cavity|toothache \lor catch) = \\ &\quad \langle (0.108 + 0.012 + 0.072)/0.416, (0.016 + 0.064 + 0.144)/0.416 \rangle = \\ &\quad \langle 0.4615, 0.5384 \rangle \end{aligned}$$

4. Three persons A,B and C apply for a job of Manager in a Private company. Chance of their selection (A,B and C) are in the ratio 1:2:4. The probability that A,B and Ccan introduce changes to improve profits of company are 0.8,0.5 and 0.3 respectively, if the changes does not take place, find the probability that it is due to the appointment of C.

Solution

Verified by Toppr

Let the events be described as below :

A : No change takes place

E_1 : Person A gets appointed

E_2 : Person B gets appointed

E_3 : Person C gets appointed.

The chances of selection of A, B and C are in the ratio 1 : 2 : 4.

Hence, $P(E_1) = \frac{1}{7}$, $P(E_2) = \frac{2}{7}$, $P(E_3) = \frac{4}{7}$

Probabilities of A, B and C introducing changes to improve profits of company are 0.8, 0.5 and 0.3 respectively. Hence probability of no changes on appointment of A, B and C are 0.2, 0.5 and 0.7 respectively.

Hence, $P(A|E_1) = 0.2 = \frac{2}{10}$

$P(A|E_2) = 0.5 = \frac{5}{10}$

$P(A|E_3) = 0.7 = \frac{7}{10}$

$\therefore$ the required probability is

$$P(E_3|A) = \frac{P(A|E_3).P(E_3)}{P(A|E_1).P(E_1) + P(A|E_2).P(E_2) + P(A|E_3).P(E_3)}$$

$$= \frac{\frac{4}{7}.\frac{7}{10}}{\frac{1}{7}.\frac{2}{10} + \frac{2}{7}.\frac{5}{10} + \frac{4}{7}.\frac{7}{10}}$$

$$= \frac{\frac{28}{70}}{\frac{2}{70} + \frac{10}{70} + \frac{28}{70}}$$

$$= \frac{28}{40} = \frac{7}{10}$$

$\therefore$ if no change takes place, the probability that it is due to appointment of C is $\frac{7}{10}$

5. Show from first principles that P(a | b ∧ a)=1

1. **13.1**[5pts] Show from first principles that

$$P(A|B \wedge A) = 1$$

The "first principles" needed here are the definition of conditional probability, $P(X|Y) = P(X \wedge Y)/P(Y)$, and the definitions of the logical connectives. It is not enough to say that if $B \wedge A$ is "given" then A must be true! From the definition of conditional probability, and the fact that $A \wedge A \Leftrightarrow A$ and that conjunction is commutative and associative, we have

$$P(A|B \wedge A) = \frac{P(A \wedge (B \wedge A))}{P(B \wedge A)} = \frac{P(B \wedge A)}{P(B \wedge A)} = 1$$