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Course: Numerical Computing
Section: BCS-5B

$f(x) = x^2 \sin x$

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$
1	0.8414	2.7957	-5.1628	-5.8401	18.343	-7.0059	-24.28927	30.8723
2	3.6371	-2.3671	-11.0029	12.5029	11.33371	-31.2951	6.5831	
3	1.27	-13.37	1.5	23.84	-19.958	-24.712		
4	-12.1	-11.87	25.34	3.882	-44.67			
5	-23.97	13.47	29.222	-40.788				
6	-10.5	42.692	-11.566					
7	32.192	31.126						
8	63.318							

Newton Forward Interpolation

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \frac{p(p-1)(p-2)(p-3)(p-4)}{5!}\Delta^5 y_0$$
$$+ \frac{p(p-1)(p-2)(p-3)(p-4)(p-5)}{6!}\Delta^6 y_0 + \frac{p(p-1)(p-2)(p-3)(p-4)(p-5)(p-6)}{7!}\Delta^7 y_0$$

At $x = 1.5$ $\quad p = \frac{1.5-1}{1} = 0.5$

$$y_{0.5} = 0.8414 + \frac{(0.5)(2.7957)}{1} + \frac{(-0.25)(-5.1628)}{2!} + \frac{(0.375)(-5.8401)}{3!} + \frac{(-0.9375)(18.343)}{4!}$$
$$+ \frac{(3.28125)(-7.0059)}{5!} + \frac{(-24.2892)(-14.765625)}{6!} + \frac{(81.2109375)(30.8723)}{7!}$$

$$= 0.8414 + 1.3978 + 0.6453 + (-0.365) + (-0.7165) + (-0.1915) + 0.4979 + 0.4974$$

$$= 2.6068 \quad \text{accurate to 1 s.f}$$

original value: $(1.5)^2(\sin(1.5)) = 2.244$

Newton Backward Interpolation

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_n$$
$$+ \frac{p(p+1)(p+2)(p+3)(p+4)(p+5)}{6!}\nabla^6 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)(p+5)(p+6)}{7!}\nabla^7 y_n$$

At $x = 7.5$ $\quad p = \frac{7.5 - 8}{1} = -0.5$

$$y_{-0.5} = 63.318 + (-0.5)(31.126) + \frac{(-0.25)(-11.566)}{2!} + \frac{(-0.375)(-40.788)}{3!} + \frac{(-0.9375)(-44.67)}{4!}$$
$$+ \frac{(-24.712)(-3.28125)}{5!} + \frac{(-14.765625)(6.5831)}{6!} + \frac{(30.8723)(-81.2109375)}{7!}$$

$$= 63.318 + (-15.563) + 1.4457 + 2.5492 + 1.7449 + 0.6757 + (-0.135) + (-0.4974)$$

$= 53.5381$ accurate to 1 s.f

original value $(7.5)^2 \sin(7.5) = 52.7624$

At $x = 4.5$ Newton forward interpolation $p = \frac{4.5 - 1}{1} = 3.5$

$$y_{3.5} = 0.8414 + (-7.7957)(3.5) + \frac{(-5.1628)(8.75)}{2!} + \frac{(-5.8401)(13.125)}{3!} + \frac{(18.343)(6.5625)}{4!}$$
$$+ \frac{(-7.0059)(-3.28125)}{5!} + \frac{(-24.2892)(4.9218)}{6!} + \frac{(30.8723)(-12.3046)}{7!}$$

$$= 0.8414 + 9.7849 + (-22.5872) + (-12.7752) + 5.0156 + 0.195 + (-0.1660) + (-0.0755)$$

$= -19.7703$ accurate to 1 decimal place

original value $(4.5)^2 \sin(4.5) = -19.7949$

At $x = 4.5$ Newton Backward interpolation $p = \frac{4.5 - 8}{1} = -3.5$

$$y_{3.5} = 63.318 + (-3.5)(31.126) + \frac{(-11.566)(8.75)}{2!} + \frac{(-40.788)(-13.125)}{3!} + \frac{(-44.67)(6.5625)}{4!}$$
$$+ \frac{(-24.712)(3.28125)}{5!} + \frac{(6.5831)(4.9218)}{6!} + \frac{(30.8723)(12.3046)}{7!}$$

$$= 63.318 + (-108.941) + (-50.60125) + 89.22375 + (-12.214) - 0.6757 + (0.045)$$
$$+ 0.0753$$

$= -19.7699$ accurate to 1 decimal place

original value $= -19.7699$