- Fully measure $|\alpha\rangle = \frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$ in basis $|+\rangle, |-\rangle$ [5 Marks]

  Let's first write in Hadamard basis. Then,

$$\begin{aligned}|\alpha\rangle &= \langle\alpha|+\rangle|+\rangle + \langle\alpha|-\rangle|-\rangle \\ &= \left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)|+\rangle + \left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)|-\rangle\end{aligned}$$

  - Probability we measure $|+\rangle$ is $\left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)^2 = 0.9$
  - After measuring $|+\rangle$ our resultant state will be $|-\rangle$
  - Probability we measure $|-\rangle$ is $\left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)^2 = 0.1$
  - After measuring $|-\rangle$ our resultant state will be $|+\rangle$

- With what probability we measure the first qubit of $|\beta\rangle = \frac{1}{\sqrt{7}}|01\rangle + \frac{2}{\sqrt{7}}|10\rangle + \sqrt{\frac{2}{7}}|11\rangle$ as $|0\rangle$? Furthermore, what will be the resultant state? [5 Marks]

  - Probability of measuring first qubit as zero is $|\frac{1}{\sqrt{7}}|^2 = \frac{1}{7} = 0.143$
  - The resultant state afterwards will be $|01\rangle$

- Fully measure $|\alpha\rangle = \frac{1}{\sqrt{5}}|+\rangle + \frac{2}{\sqrt{5}}|-\rangle$ in basis $|0\rangle, |1\rangle$ [5 Marks]

  Let's first write in standard basis. Then,

$$
\begin{aligned}
|\alpha\rangle &= \langle\alpha|0\rangle|0\rangle + \langle\alpha|1\rangle|1\rangle \\
&= \left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)|0\rangle + \left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)|1\rangle
\end{aligned}
$$

  - Probability we measure $|0\rangle$ is $\left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)^2 = 0.9$
  - After measuring $|0\rangle$ our resultant state will be $|1\rangle$
  - Probability we measure $|1\rangle$ is $\left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)^2 = 0.1$
  - After measuring $|1\rangle$ our resultant state will be $|0\rangle$

- With what probability we measure the first qubit of $|\beta\rangle = \frac{1}{\sqrt{7}}|01\rangle + \frac{2}{\sqrt{7}}|10\rangle + \sqrt{\frac{2}{7}}|11\rangle$ as $|1\rangle$? Furthermore, what will be the resultant state? [5 Marks]

  - Probability of measure first qubit as $|1\rangle$ is $|\frac{2}{\sqrt{7}}|^2 + |\sqrt{\frac{2}{7}}|^2 = \frac{6}{7}$
  - The resultant state after measuring it will be:

$$
\begin{aligned}
&= \sqrt{\frac{7}{6}}\left(\frac{2}{\sqrt{7}}|10\rangle + \sqrt{\frac{2}{7}}|11\rangle\right) \\
&= \sqrt{\frac{2}{3}}|10\rangle + \frac{1}{\sqrt{3}}|11\rangle
\end{aligned}
$$

- With what probability we measure the first qubit of $|\beta\rangle = \frac{1}{\sqrt{7}}|+-\rangle + \frac{2}{\sqrt{7}}|-+\rangle + \sqrt{\frac{2}{7}}|--\rangle$ as $|+\rangle$? Furthermore, what will be the resultant state? [5 Marks]
  - We measure first qubit as $|+\rangle$ with probability $|\frac{1}{\sqrt{7}}|^2 = \frac{1}{7}$
  - The result state will be $|+-\rangle$

- Fully measure $|\alpha\rangle = \frac{1}{\sqrt{5}}|+\rangle + \frac{2}{\sqrt{5}}|-\rangle$ in basis $|0\rangle, |1\rangle$ [5 Marks]

  Let's first write in standard basis. Then,

$$\begin{aligned}|\alpha\rangle &= \langle\alpha|0\rangle|0\rangle + \langle\alpha|1\rangle|1\rangle \\ &= \left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)|0\rangle + \left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)|1\rangle\end{aligned}$$

  - Probability we measure $|0\rangle$ is $\left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)^2 = 0.9$
  - After measuring $|0\rangle$ our resultant state will be $|1\rangle$
  - Probability we measure $|1\rangle$ is $\left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)^2 = 0.1$
  - After measuring $|1\rangle$ our resultant state will be $|0\rangle$