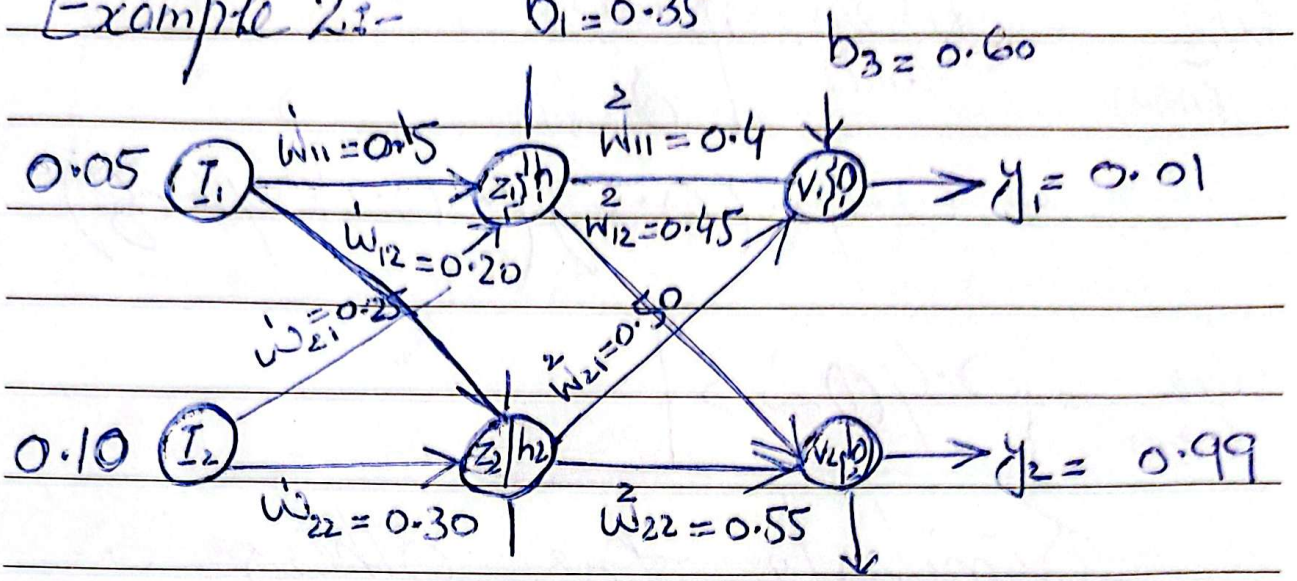
Example 2:-



$$W_1 = \begin{bmatrix} 0.15 & 0.20 \\ 0.25 & 0.30 \end{bmatrix} \quad W_2 = \begin{bmatrix} 0.4 & 0.45 \\ 0.50 & 0.55 \end{bmatrix}$$

$$I = \begin{bmatrix} 0.05 \\ 0.10 \end{bmatrix}$$

η = learning rate = 0.5

$$y_{t(1)} = 0.01$$

$$y_{t(2)} = 0.99$$

Forward Pass:-

$$z_1 = w_{11} I_1 + w_{21} I_2 + b_1$$

$$= (0.15)(0.05) + (0.25)(0.10) + (0.35)$$

$$(0.15)(0.05) + 0.375$$

$$\boxed{z_1 = 0.3825}$$

$$h_1 = \sigma(z_1) = \frac{1}{1+e^{z_1}} = \frac{1}{1+e^{-0.3825}}$$

$$= 1/1.6822 = \boxed{0.5945} \Rightarrow h_1$$

$$z_2 = w_{12}(I_1) + w_{22} I_2 + b_2$$

$$= (0.20)(0.05) + (0.30)(0.10) + (0.35)$$

$$= (0.01) + (0.03) + (0.35)$$

$$\boxed{z_2 = 0.39}$$

$$h_2 = \sigma(z_2) = \frac{1}{1+e^{0.39}} = \frac{1}{1.677}$$

$$\boxed{h_2 = 0.5963}$$

$$V_1 = w_{11}(h_1) + w_{21}(h_2) + b_3$$

$$= (0.4)(0.5945) + (0.50)(0.5963) + (0.60)$$

$$= (0.2378) + (0.29815) + (0.60)$$

$$\boxed{V_1 = 1.135}$$

$$O_1 = \sigma(V_1) = \frac{1}{1+e^{-1.135}} = 1/1.3214$$

$$\boxed{O_1 = 0.756}$$

$$V_2 = W_{12}^2(h_1) + W_{22}^2(h_2) + b_4$$

$$= (0.45)(0.5945) + (0.55)(0.5963) + (0.60)$$

$$V_2 = (0.2675) + (0.3279) + (0.60)$$

$$V_2 = 1.1954$$

$$O_2 = \sigma(V_2) = \frac{1}{1+e^{-1.1954}} = \frac{1}{1.3025}$$

$$\boxed{O_2 = 0.7677}$$

Cost/Loss function:-

$$L = \frac{1}{2}\sum_{i=1}^{2}(y_t - y_p)^2$$

$$= \frac{1}{2}\left\{(0.01 - 0.756)^2 + (0.99 - 0.767)^2\right\}$$

$$= \frac{1}{2}\left\{(0.5565) + (0.0494)\right\}$$

$$\boxed{L = 0.3029}$$

Update Weights:

$w_{11}^2, w_{12}^2, w_{21}^2, w_{22}^2$

w_{11}^2 :

$$\frac{\partial L}{\partial w_{11}^2} = (w_{11}^2) \to (V_1) \to (O_1) \to (L)$$

$$= \frac{\partial L}{\partial O_1} \cdot \frac{\partial O_1}{\partial V_1} \cdot \frac{\partial V_1}{\partial w_{11}^2}$$

$$\therefore \frac{\partial L}{\partial O_1} = \frac{1}{2}(y_1 - O_1)^2$$

$$= \frac{1}{2} \times (y_1 - O_1) \frac{\partial L}{\partial O_1}(y_1 - O_1)$$

$$= -(y_1 - O_1)$$

$$\therefore \frac{\partial O_1}{\partial V_1} = O_1(1 - O_1)$$

$$\therefore \frac{\partial V_1}{\partial w_{11}^2} = w_{11}^2(h_1) + w_{21}^2(h_2) + b_3$$

$$= h_1 + 0 + 0$$

$$\frac{\partial L}{\partial w_{11}^2} = -(y_1 - a) \cdot o_1(1 - o_1) \cdot h_1$$

$$= -[0.01 - 0.756] \cdot (0.756)[1 - 0.756] \cdot (0.5945)$$

$$= -(-0.746) \cdot (0.1844) \cdot (0.5945)$$

$$\boxed{\frac{\partial L}{\partial w_{11}^2} = 0.0817}$$

Similarly:-

$$\frac{\partial L}{\partial w_{21}^2} = \frac{\partial L}{\partial o_1} \cdot \frac{\partial o_1}{\partial v_1} \cdot \frac{\partial v_1}{\partial w_{21}^2}$$

$$= -(y_1 - o_1) \cdot o_1(1 - o_1) \cdot h_2$$

$$\boxed{\frac{\partial L}{\partial w_{21}^2} = 0.08195}$$

$$\frac{\partial L}{\partial w_{12}^2} = \frac{\partial L}{\partial O_2} \cdot \frac{\partial O_2}{\partial V_2} \cdot \frac{\partial V_2}{\partial w_{12}^2}$$

$$= -(y_2 - O_2) \cdot O_2(1 - O_2) \cdot h_1$$

$$\boxed{\frac{\partial L}{\partial w_{12}^2} = -0.02357}$$

$$\frac{\partial L}{\partial w_{22}^2} = \frac{\partial L}{\partial O_2} \cdot \frac{\partial O_2}{\partial V_2} \cdot \frac{\partial V_2}{\partial w_{22}^2}$$

$$= -(y_2 - O_2) \cdot O_2(1 - O_2) \cdot h_2$$

$$\boxed{\frac{\partial L}{\partial w_{22}^2} = -0.02364}$$

New Weights :-

$$w_{11}^{2\,(new)} = w_{11}^{2\,(old)} - \eta \frac{\partial L}{\partial w_{11}^2}$$

$$= 0.4 - (0.5)(0.0817)$$

$$= 0.4 - 0.0408$$

$$\boxed{w_{11}^{2\,(new)} = 0.3592}$$

$$W_{12}^{(2)}_{(new)} = 0.4618$$

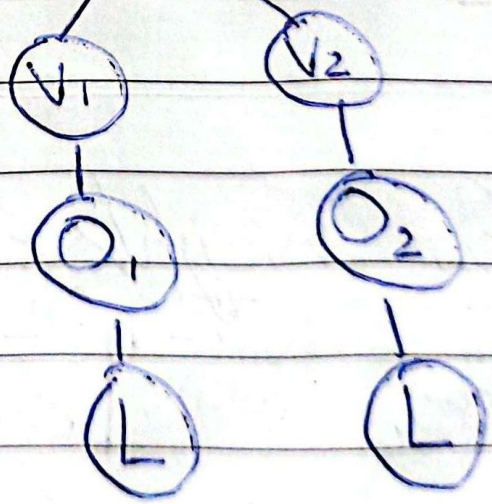
$$W_{21}^{(2)}_{(new)} = 0.459$$

$$W_{22}^{(2)}_{(new)} = 0.5618$$

Now update weights:

$w_{11}', w_{12}', w_{21}', w_{22}'$

$$\frac{\partial L}{\partial w_{11}'} = (w_{11}') \rightarrow (z_1) \rightarrow (h_1)$$



$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{11}}$$

↳ two parts i.e.

$$\left\{ \left[\frac{\partial L}{\partial o_1} \cdot \frac{\partial o_1}{\partial v_1} \cdot \frac{\partial v_1}{\partial h_1} \right] + \left[\frac{\partial L}{\partial o_2} \cdot \frac{\partial o}{\partial v_2} \cdot \frac{\partial v_2}{\partial h_1} \right] \right\}$$

$$= \left[\frac{\partial L}{\partial o_1} \cdot \frac{\partial o_1}{\partial v_1} \cdot \frac{\partial v_1}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{11}} \right] + \left[\frac{\partial L}{\partial o_2} \cdot \frac{\partial o}{\partial v_2} \cdot \frac{\partial v_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{11}} \right]$$

As we know

$$= \left[-(y_1 - o_1) \cdot o_1(1-o_1) \cdot w_{11}^2 \cdot h_1(1-h_1) I_1 \right] + \left[-(y_2 - o_2) \cdot o_2(1-o_2) \cdot w_{12} \cdot h_1(1-h_1) \cdot I_1 \right]$$

$$= \left[-(-0.764) \cdot (0.1844) \cdot (0.4) \right] \cdot (0.5945)(1-0.5945) \cdot (0.05) \Big] +$$

$$\left[-(0.222)(0.1783)\cdot(0.45)\cdot(0.5945)\cdot(1-0.5945)\cdot(0.05)\right]$$

$$= \left[0.0006792 + (-0.0002146)\right]$$

$$= (0.0006792 - 0.0002146)$$

$$\boxed{\frac{\partial L}{\partial w_{11}} = 0.0004646}$$

$$w_{11}^{(new)} = w_{11}^{(old)} - \eta \frac{\partial L}{\partial w_{11}}$$

$$= 0.15 - (0.5)(0.0004646)$$

$$= (0.15 - 0.0002323)$$

$$\boxed{w_{11}^{(new)} = 0.1497}$$

∴ Similarly Complete all Calculations :-

$$w'_{12\,(new)} = 0.1997$$

$$w'_{21\,(new)} = 0.2496$$

$$w'_{22\,(new)} = 0.2994$$