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Section BCS-4B
Course Design and Analysis of Algorithms.
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a) Find min
Minimum(A)
1 min = A[1]
2 for i = 2 to A.length
3 if min > A[i]
4 min = A[i]
5 return min

Loop Invariant: At the start of each iteration
'min' holds the value of smallest element
A[1...A.length]

Initialization: Before first iteration 'min'
is initialized to A[1] so loop invariant holds
first iteration.

Maintenance: At the end of each iteration, if
 $A[i] < \text{min}$ then min is updated with the
value of A[i]. Min then holds the smallest
element in A[1...i].

Termination: Loop terminates when i is equal
to A.length so min holds the smallest element
in A[1...A.length].

b) BubbleSort(A)

- 1 for $i = 1$ to $A.length - 1$
- 2 for $j = A.length$ down to $i + 1$
- 3 if $A[j] < A[j - 1]$
- 4 exchange $A[j]$ with $A[j - 1]$

loop invariant: At start of each iteration of outer loop elements in $A[1 \dots i]$ are sorted.

Initialization: Before the first iteration $i = 1$ so $A[1 \dots 0]$ is empty. Therefore loop invariant holds.

Maintenance: At the end of each outer loop iteration, largest element in $A[i \dots A.length]$ bubbles up to the end and $A[1 \dots i]$ is still sorted.

Termination: loop terminates when $i = A.length - 1$ hence $A[1 \dots A.length - 1]$ is sorted since inner loop insures the largest element is at the end of each subarray.

C) SelectionSort(A)

```
1 for j=1 to A.length-1
2   smallest_index = j
3   for i=j+1 to A.length
4     if A[i] < A[smallest_index]
5       smallest_index = i
6   exchange A[j] with A[smallest_index]
```

loop invariant: At the start of each iteration of outer loop elements in $A[1 \dots j-1]$ are sorted.

Initialization: Before first iteration $j=1$ so $A[1 \dots 0]$ is empty. therefore first iteration holds.

Maintenance: At the end of each iteration of outer loop ~~min~~ smallest_index stores the index of smallest element in $A[i \dots A.length]$. Then the $A[\text{smallest_index}]$ is swapped with $A[j]$. Hence array is sorted $A[1 \dots j-1]$.

Termination: When outer loop terminates $j == A.length-1$ and $A[j \dots A.length-1]$ is sorted, Since smallest_index gets the element of the smallest index in $A[1 \dots A.length]$ and ~~put in~~ ^{swaps with} $A[j]$. Hence A is sorted.