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Course Design and Analysis of Algorithms.
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a) Find min

```
Minimum(A)
1   min = A[1]
2   for i = 2 to A.length
3       if min > A[i]
4           min = A[i]
5   return min
```

Loop Invariant: At the start of each iteration 'min' holds the value of smallest element A[1 ....A.length]

Initialization: Before first iteration 'min' is initialized to A[1] so loop invariant holds first iteration.

Maintenance: At the end of each iteration, if A[i] < min then min is updated with the value of A[i]. So Min then holds the smallest element in A[1 .... i].

Termination: Loop terminates when i is equal to A.length so min holds the smallest element in A[1.... A.length].

b)

```
BubbleSort(A)
1  for i = 1 to A.length-1
2      for j = A.length down to i+1
3          if A[j] < A[j-1]
4              exchange A[j] with A[j-1]
```

loop invariant: At start of each iteration of outer loop elements in $A[1 \ldots i-1]$ are sorted.

Initialization: Before the first iteration $i = 1$ so $A[1 \ldots 0]$ is empty. Therefore loop invariant holds.

Maintenance: At the end of each outer loop iteration, largest element in $A[i \ldots A.length]$ bubbles up to the end and $A[1 \ldots i]$ is still sorted.

Termination: loop terminates when $i = A.length - 1$ hence $A[1 \ldots A.length-1]$ is sorted since inner loop insures the largest element is at the end of each subarray.

C) SelectionSort(A)

```
1  for j=1 to A.length-1
2      smallest_index = j
3      for i=j+1 to A.length
4          if A[i]<A[smallest_index]
5              smallest_index=i
6      exchange A[j] with A[smallest_index]
```

loop invariant: At the start of each iteration of outerloop elements in A[1 ... Aj-1] are sorted.

Initialization: Before first iteration, j==1 so A[1.... 0] is emptly. therefor first iteration holds.

Maintenance: At the end of each iteration of outerloop ~~min~~ smallest_index stores the index of smallest element in A[i.... A.length]. ~~At~~ Then the A[smallest_index] is swapped with A[j]. Hence array is sorted A[1.... j-1].

Termination: When outer loop terminates j == A.length-1 and A[j.... A.length-1] is sorted, Since smallest_index gets the element of the smallest index in A[1.... A.length] and ~~put on~~ swaps with A[i]. Hence A is sorted.