- Apply quantum Fourier transform on the **equal superposition** of all the inputs of function $f : \{0,1\}^2 \rightarrow \{0,1\}^2, f(x) = x \mod 2$. To that end, you could use quantum implementation of f as $U_f |x\rangle |0^n\rangle = |x\rangle |f(x)\rangle$ Clearly show your output? [**10 Marks**]

  TA: This question statement is not correct and is confusing. Please give marks to each student who has demonstrated effort and applied intelligent problem-solving strategies.

- Given our data: $\frac{|00\rangle+|01\rangle+i|10\rangle-|11\rangle}{2}$, undergoes a linear shift of 2, what will be the corresponding phase-shift upon application of the Quantum Fourier Transformation (QFT)? Show both results before and after the phase shift. [**5 Marks**]

  Before our calculation. Let's discuss it theoretically,

$$QFT_4 \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ i \\ -1 \end{pmatrix} = \begin{pmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{pmatrix}$$

  The by the properties of QFT:

$$QFT_4 \frac{1}{2}\begin{pmatrix} i \\ -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \omega_4^0 B_0 \\ \omega_4^2 B_1 \\ \omega_4^4 B_2 \\ \omega_4^6 B_3 \end{pmatrix} = \begin{pmatrix} B_0 \\ -B_1 \\ B_2 \\ -B_3 \end{pmatrix}$$

  Now, lets verify it with our calculation.

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ i \\ -1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+i \\ 1+i \\ 1+i \\ 1-3i \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ -1 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+i \\ -(1+i) \\ 1+i \\ -(1+3i) \end{pmatrix}$$

- What is $QFT^{-1}\frac{|000\rangle-|010\rangle+|011\rangle}{\sqrt{3}}$ in simplified terms? Here QFT^{-1} refers to the inverse Quantum Fourier Transformation. Noted: You don't need to write the entire matrix, but you may use a more efficient (clever) approach. [**5 Marks**]

$\frac{|000\rangle - |010\rangle + |011\rangle}{\sqrt{3}} = \frac{|0\rangle - |2\rangle + |3\rangle}{\sqrt{3}}$

Therefore,

$$QFT^{-1}\frac{|0\rangle - |2\rangle + |3\rangle}{\sqrt{3}} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1-1+1 \\ w_8^{-1} - w_8^{-2} + w_8^{-3} \\ w_8^{-2} - w_8^{-4} + w_8^{-6} \\ w_8^{-3} - w_8^{-6} + w_8^{-9} \\ w_8^{-4} - w_8^{-8} + w_8^{-12} \\ w_8^{-5} - w_8^{-10} + w_8^{-15} \\ w_8^{-6} - w_8^{-12} + w_8^{-18} \\ w_8^{-7} - w_8^{-14} + w_8^{-21} \end{pmatrix}$$

Rest of it does not matter. Give student full mark if he has done correctly this far.

- Given our data: $\frac{|00\rangle + |01\rangle + i|10\rangle - |11\rangle}{2}$, undergoes a linear shift of 3, what will be the corresponding phase-shift upon application of the Quantum Fourier Transformation (QFT)? Show both results before and after the phase shift. [**5 Marks**]

  Before our calculation. Let's discuss it theoretically,

$$QFT_4\ \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ i \\ -1 \end{pmatrix} = \begin{pmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{pmatrix}$$

  The by the properties of QFT:

$$QFT_4\ \frac{1}{2}\begin{pmatrix} 1 \\ i \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} \omega_4^0 B_0 \\ \omega_4^3 B_1 \\ \omega_4^6 B_2 \\ \omega_4^9 B_3 \end{pmatrix} = \begin{pmatrix} B_0 \\ -iB_1 \\ -B_2 \\ iB_3 \end{pmatrix}$$

  Now, lets verify it with our calculation.

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ i \\ -1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+i \\ 1+i \\ 1+i \\ 1-3i \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 \\ i \\ -1 \\ 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+i \\ -i(1+i) \\ -(1+i) \\ i(1-3i) \end{pmatrix}$$