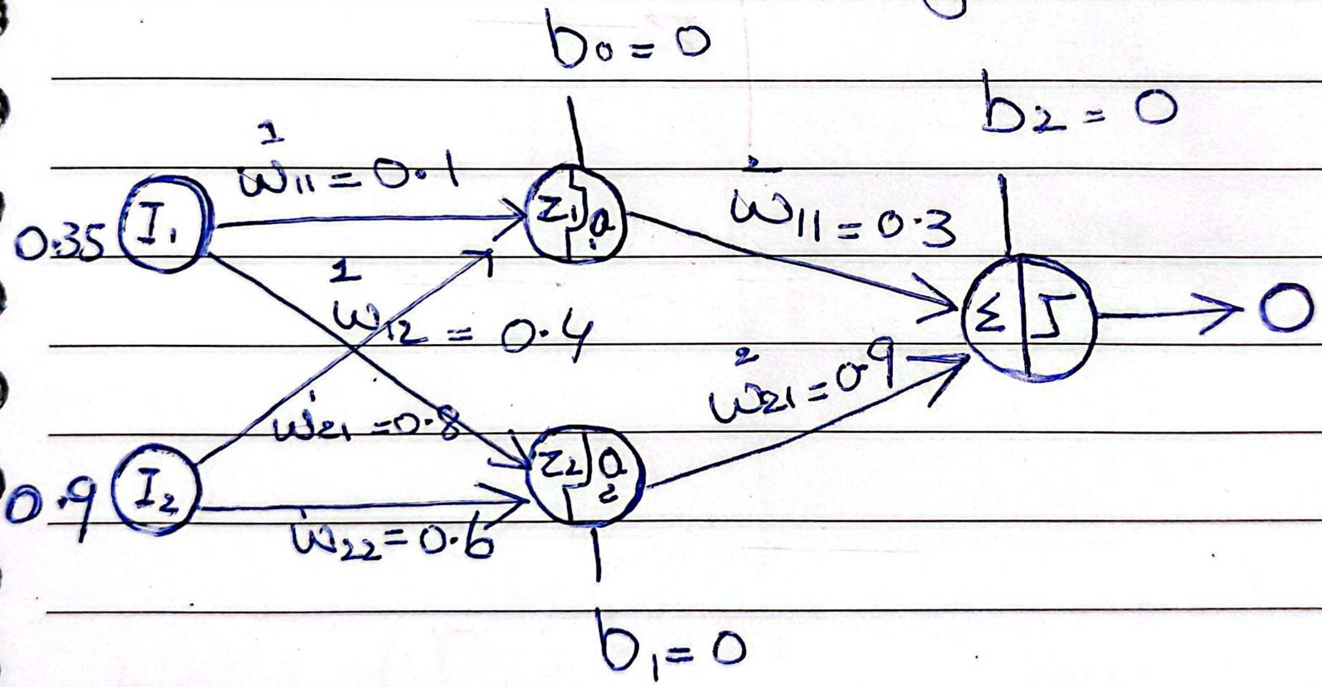
Feed Forward Neural Network with Back Propagation (2 - Pass)

$$I = \begin{bmatrix} 0.35 = I_1 \\ 0.9 = I_2 \end{bmatrix} \qquad \vec{\omega}^{1} = \begin{bmatrix} \overset{\omega^1_{11}}{0.1} & \overset{\omega^1_{12}}{0.4} \\ \overset{\omega^1_{21}}{0.8} & \overset{\omega^1_{22}}{0.6} \end{bmatrix}$$

$$\vec{\omega}^{2} = \begin{bmatrix} 0.3 & \omega^2_{11} \\ 0.9 & \omega^2_{21} \end{bmatrix} \qquad \alpha = \text{or } \eta = 0.5$$

$$y = 1$$



Forward Pass:-

$$z_1 = \omega^1_{11}(I_1) + \omega^1_{21}(I_2) + b_0$$
$$= (0.1)(0.35) + (0.8)(0.9) + 0$$
$$= 0.035 + 0.72 = \boxed{0.755}$$

$a_1 = ?$

$$a_1 = \sigma(z_1) = \frac{1}{1+e^{-z}} = \frac{1}{1+e^{-0.755}}$$

$$= \frac{1}{1+0.47} = \frac{1}{1.47}$$

$$\boxed{a_1 = 0.68}$$

$$z_2 = w_{12}(I_1) + w_{22}(I_2) + b_1$$

$$= (0.4)(0.35) + (0.6)(0.9) + 0$$

$$= 0.14 + 0.54$$

$$z_2 = 0.68$$

$$a_2 = \sigma(z_2) = \frac{1}{1+e^{-z}}$$

$$= \frac{1}{1+e^{-(0.68)}} = \frac{1}{1+0.506}$$

$$= \frac{1}{1.506} = 0.664$$

$$\boxed{a_2 = 0.664}$$

$$Val = \overset{2}{w}_{11} a_1 + \overset{2}{w}_{22} a_2 + b_2$$

$$= (0.3)(0.68) + (0.9)(0.664) + 0$$

$$= 0.204 + 0.5976$$

$$= 0.8016$$

$$O \Rightarrow \sigma(Val) = \frac{1}{1+e^{-val}} = \frac{1}{1+e^{-0.8016}}$$

$$O = \frac{1}{1+0.448} = \frac{1}{1.448} = \boxed{0.6902}$$

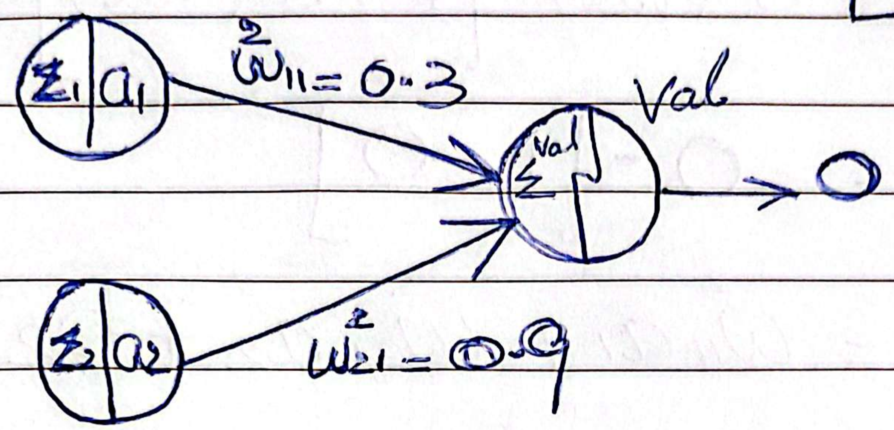
Back Propagation:-

Cost/Loss function:-

$$L = \frac{1}{2}(y-O)^2 = \frac{1}{2}(1-0.6902)^2$$

$$\boxed{L = 0.04799}$$

Loss Functi



Output Layer :-

update w_{11}^2, w_{21}^2 :-

$$\frac{\partial L}{\partial w_{11}^2} = w_{11}^2 \to Val \to O \to L$$

$$= \frac{\partial L}{\partial O} \cdot \frac{\partial O}{\partial Val} \cdot \frac{\partial Val}{\partial w_{11}^2}$$

$$\therefore \frac{\partial L}{\partial O} = \frac{1}{2}(y-O)^2$$

$$= \frac{1}{\cancel{2}} * \cancel{2}(y-O)\frac{\partial L}{\partial O}(y-O)$$

$$= (y-O)(-1)$$

$$= -(y-O)$$

$$\therefore \quad O = \sigma(val)$$

$$\frac{\partial O}{\partial val} = \sigma(val)\cdot\left[1-\sigma(val)\right]$$

$$= O\cdot\left[1-O\right]$$

$$\frac{\partial val}{\partial w_{11}^{2}} = w_{11}^{2}a_1 + w_{21}^{2}a_2 + b_2$$

$$= a_1 + 0 + 0$$

Put all Values:-

$$\frac{\partial L}{\partial w_{11}^{2}} = -(y-O)\cdot O\left[1-O\right]\cdot a_1$$

$$= -(1-0.6902)(0.6902)\left[1-0.6902\right]\cdot(0.68)$$

$$= -(0.3098)(0.6902)(0.3098)(0.68)$$

$$\boxed{\frac{\partial L}{\partial w_{11}^{2}} = -0.04505}$$

$$\frac{\partial L}{\partial w_{21}^2} - w_{21}^2 \rightarrow Val \rightarrow O \rightarrow L$$

$$\frac{\partial L}{\partial w_{21}^2} = \frac{\partial L}{\partial O} \cdot \frac{\partial O}{\partial Val} \cdot \frac{\partial Val}{\partial w_{21}^2}$$

$$\therefore \frac{\partial L}{\partial O} = -(y-O)$$

$$\therefore \frac{\partial O}{\partial Val} = O(1-O)$$

$$\therefore \frac{\partial Val}{\partial w_{21}^2} = w_{21}^2 a_2 + w_{11}^2 a_1 + b_2$$

$$= a_2 + 0 + 0$$

$$\frac{\partial L}{\partial w_{21}^2} = -(y-O) O(1-O) \cdot a_2$$

$$= -[1-0.6902](0.6902)[1-0.6902](0.664)$$

$$= -(0.3098)(0.6902)(0.3098)(0.664)$$

$$\boxed{\frac{\partial L}{\partial w_{21}^{2}} = -0.0439}$$

New weights:

$$\therefore w_{11}^{2} = w_{11}^{2} - \eta \frac{\partial L}{\partial w_{11}^{2}}$$

$$= (0.3) - (0.5)(-0.04505)$$

$$= 0.3 + 0.022525$$

$$\cancel{\partial L} \quad \boxed{w_{11}^{2} = 0.3225} \Rightarrow \text{new}$$

$$\therefore \underset{(new)}{w_{21}^{2}} = w_{21}^{2} - \eta \frac{\partial L}{\partial w_{21}^{2}}$$

$$= (0.9) - (0.5)(-0.0439)$$

$$= (0.9) + (0.02195)$$

$$w_{21}^{2} = \boxed{0.92195}$$

$$\frac{\partial L}{\partial w^1_{11}} = w^1_{11} \rightarrow z_1 \rightarrow a_1 \rightarrow val \rightarrow O \rightarrow L$$

$$= \frac{\partial L}{\partial O} \cdot \frac{\partial O}{\partial val} \cdot \frac{\partial val}{\partial a_1} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z_1}{\partial w^1_{11}}$$

$$\therefore \frac{\partial L}{\partial O} = -(y-O)$$

$$\therefore \frac{\partial O}{\partial val} = O(1-O)$$

$$\therefore \frac{\partial val}{\partial a_1} = w^2_{11}a_1 + w^2_{22}a_2 + b_2$$

$$= w^2_{11} + 0 + 0$$

$$\therefore \frac{\partial val}{\partial a_1} = w^2_{11}$$

$$\therefore \frac{\partial a_1}{\partial z_1} = a_1(1-a_1)$$

$$\therefore \frac{\partial z_1}{\partial w^1_{11}} = w^1_{11}(I_1) + w^1_{21}(I_2) + b_0$$

$$= I_1 + 0 + 0$$

$$\frac{\partial L}{\partial w_{11}^{'}} = -(y-O)\cdot O[1-O]\cdot w_{11}^{2}\cdot a_1(1-a_1)\cdot I_1$$

$$= -[1-0.6902](0.6902)[1-0.6902](0.3)\cdot(0.68)[1-0.68]\cdot(0.35)$$

$$= -[0.3098](0.6902)[0.3098](0.3)(0.68)[0.32](0.35)$$

$$= \boxed{-0.0015135}$$

$$\frac{\partial L}{\partial w_{12}^{'}} = w_{12} \rightarrow z_2 \rightarrow a_2 \rightarrow w_{21} \rightarrow O \rightarrow L$$

$$= \frac{\partial L}{\partial O}\cdot\frac{\partial O}{\partial w_{21}}\cdot\frac{\partial w_{21}}{\partial a_2}\cdot\frac{\partial a_2}{\partial z_2}\cdot\frac{\partial z_2}{\partial w_{12}^{'}}$$

$$\therefore \frac{\partial L}{\partial O} = -(y-O)$$

$$\frac{\partial L}{\partial w'_{11}} = -(y-O)\cdot O[1-O]\cdot w^2_{11}\cdot a_1(1-a_1)\cdot I_1$$

$$= -[1-0.6902](0.6902)[1-0.6902](0.3)\cdot(0.68)[1-0.68]\cdot(0.35)$$

$$= -[0.3098](0.6902)[0.3098](0.3)(0.68)[0.32](0.35)$$

$$= \boxed{-0.0015135}$$

$$\frac{\partial L}{\partial w'_{12}} = w'_{12} \rightarrow z_2 \rightarrow a_2 \rightarrow val \rightarrow O \rightarrow L$$

$$= \frac{\partial L}{\partial O}\cdot\frac{\partial O}{\partial val}\cdot\frac{\partial val}{\partial a_2}\cdot\frac{\partial a_2}{\partial z_2}\cdot\frac{\partial z_2}{\partial w'_{12}}$$

$$\therefore \frac{\partial L}{\partial O} = -(y-O)$$

$$\therefore \frac{\partial O}{\partial val} = O(1-O)$$

$$\therefore \frac{\partial val}{\partial a_2} = w^2_{11} a_1 + w^2_{22} a_2 + b_2$$

$$= 0 + w^2_{22} + 0$$

$$\frac{\partial val}{\partial a_2} = w^2_{22}$$

$$\therefore \frac{\partial a_2}{\partial Z_2} = a_2(1-a_2)$$

$$\therefore \frac{\partial Z_2}{\partial w^1_{12}} = w^1_{12}(I_1) + w^1_{22}(I_2) + b_0$$

$$= I_1 + 0 + 0$$

$$\frac{\partial L}{\partial w^1_{12}} = -(y-O)O(1-O)w^2_{22}\,a_2(1-a_2)I_1$$

$$= -(1-0.6902)0.6902(1-0.6902)(0.9)(0.664)(1-0.664)(0.9)$$

$$= -(0.3098)(0.6902)(0.3098)(0.9)(0.664)(0.336)(0.35)$$

$$= \boxed{-0.01197}$$

$$\boxed{\frac{\partial L}{\partial w_{12}} - 0.0046355}$$

Similarly we calculated

$$\frac{\partial L}{\partial w_{21}} = -0.00389$$

$$\frac{\partial L}{\partial w_{22}} = -0.01199$$

update Weights :-

$$w'_{11\,new} = w'_{11\,old} - \eta \frac{\partial L}{\partial w_{11}}$$

$$= 0.1 - (0.5)(-0.0015135)$$

$$\boxed{w'_{11\,new} = 0.1007}$$

$$w'_{12\,(new)} = w'_{12\,(old)} - \eta \frac{\partial L}{\partial w'_{12}}$$

$$= 0.4 - (0.5)(-0.00465)$$

$$w'_{12\,(new)} = \boxed{0.4023}$$

Similarly for others

$$w'_{21\,new} = \cancel{0.1}\, w_{21\,old} - \eta\left(\frac{\partial L}{\partial w_{21}}\right)$$

$$= 0.8 - 0.5(-0.00389)$$

$$= \boxed{0.8015}$$

$$w'_{22\,new} = w_{22\,old} - \eta \frac{\partial L}{\partial w'_{22}}$$

$$= 0.6 - 0.5(-0.01199)$$

$$= \boxed{0.605995}$$

$$\vec{w}'_1 = \Big[\, 0$$