Quiz-2, Quantum Computing

Q : What will be the output of the following quantum circuit given input $|010\rangle$.

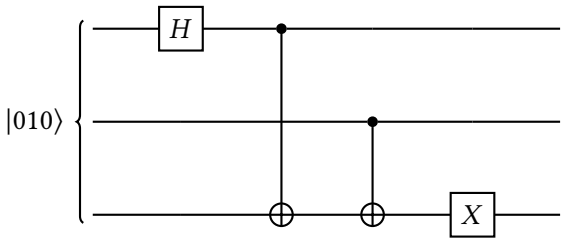

Figure 0.1: Quantum circuit

Solution: The question can be solved using matrices but below solution uses Bra-Ket notation.

$$|\psi_1\rangle = |010\rangle$$
$$|\psi_2\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} |10\rangle$$
$$|\psi_2\rangle = \frac{1}{\sqrt{2}} |010\rangle + |110\rangle$$
$$|\psi_3\rangle = \frac{1}{\sqrt{2}} |010\rangle + |111\rangle$$
$$|\psi_4\rangle = \frac{1}{\sqrt{2}} |011\rangle + |110\rangle$$
$$|\psi_5\rangle = \frac{1}{\sqrt{2}} |010\rangle + |111\rangle$$

Quiz-2, Quantum Computing

Q : What will be the output of the following quantum circuit given input $|110\rangle$.

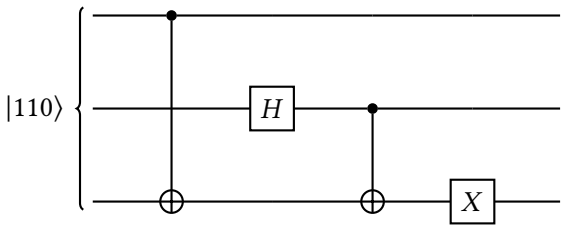

Figure 0.2: Quantum circuit

Solution:

$$
\begin{aligned}
|\psi_1\rangle &= |110\rangle \\
|\psi_2\rangle &= |111\rangle \\
|\psi_3\rangle &= |1\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} |1\rangle \\
|\psi_3\rangle &= \frac{1}{\sqrt{2}}(|101\rangle - |111\rangle) \\
|\psi_4\rangle &= \frac{1}{\sqrt{2}}(|101\rangle - |110\rangle) \\
|\psi_5\rangle &= \frac{1}{\sqrt{2}}(|100\rangle - |111\rangle)
\end{aligned}
$$

Quiz-2, Quantum Computing

Q : What will be the output of the following quantum circuit given input $|111\rangle$.

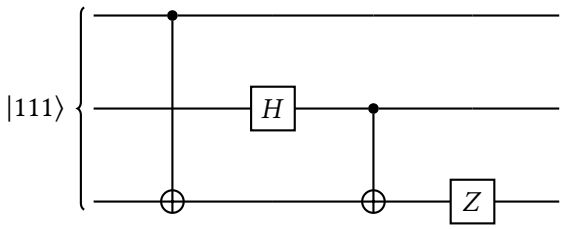

Figure 0.3: Quantum circuit

Solution

$$
\begin{aligned}
|\psi_1\rangle &= |111\rangle \\
|\psi_2\rangle &= |110\rangle \\
|\psi_3\rangle &= |1\rangle\, H\, |1\rangle\, |0\rangle \\
|\psi_3\rangle &= |1\rangle\, \frac{|0\rangle - |1\rangle}{\sqrt{2}}\, |0\rangle \\
|\psi_3\rangle &= \frac{1}{\sqrt{2}}(|100\rangle - |110\rangle) \\
|\psi_4\rangle &= \frac{1}{\sqrt{2}}(|100\rangle - |111\rangle) \\
|\psi_5\rangle &= \frac{1}{\sqrt{2}}(|100\rangle + |111\rangle)
\end{aligned}
$$

Quiz-2, Quantum Computing

Q : What will be the output of the following quantum circuit given input $|00\rangle$.

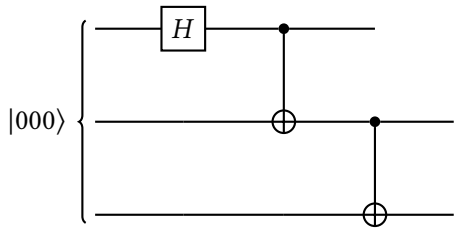

Figure 0.4: Quantum circuit

Solution:

$$
\begin{aligned}
|\psi_1\rangle &= |000\rangle \\
|\psi_2\rangle &= H\,|0\rangle\,|00\rangle \\
|\psi_2\rangle &= \frac{|0\rangle + |1\rangle}{\sqrt{2}}\,|00\rangle \\
|\psi_2\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |100\rangle) \\
|\psi_3\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |110\rangle) \\
|\psi_4\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)
\end{aligned}
$$

Quiz-2, Quantum Computing

Q : What will be the output of the following quantum circuit given input $|111\rangle$.

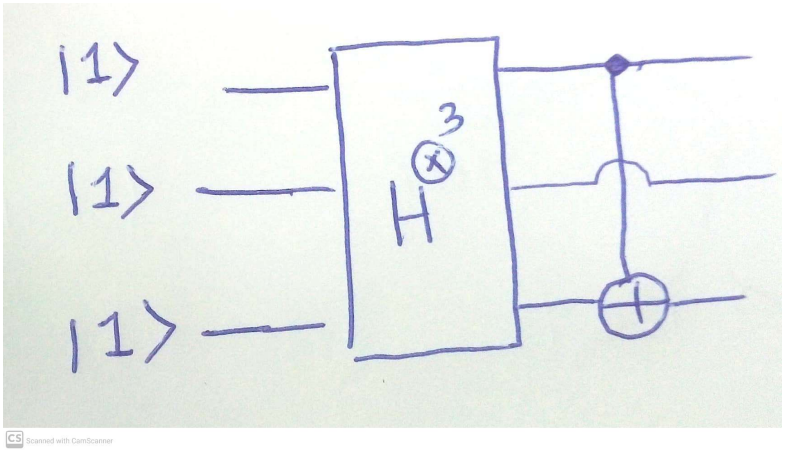

Figure 0.5: Quantum circuit

Solution

$$|\psi_1\rangle = |111\rangle$$

$$|\psi_2\rangle = H^{\otimes 3}\,|111\rangle$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2^3}}(|000\rangle - |001\rangle - |010\rangle + |011\rangle - |100\rangle + |101\rangle + |110\rangle - |111\rangle)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2^3}}(|000\rangle - |001\rangle - |010\rangle + |011\rangle - |101\rangle + |100\rangle + |111\rangle - |110\rangle)$$