Questions	Q1	Q2	Q3	Q4	Q5	Q6	**Total**
Marks	4	5	5	4	5	21	**44**
Marks Obtained							

Questions

1. Use order finding algorithm to find order r such that: $\alpha^r \equiv 1 \mod N$ given $N = 7$ and $\alpha = 2$. Must show the complete quantum circuit and results of all the stages of circuit, otherwise no marks will be given. [**10 Marks**]

2. You are given a simple unitary matrix $U = \begin{pmatrix} e^{\frac{i\pi}{2}} & 0 \\ 0 & e^{-\frac{i\pi}{2}} \end{pmatrix}$, and its one eigenvector $|v\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, you are asked to use phase estimation to estimate θ for **m=2** of the corresponding eigenvalue $\lambda = e^{2\pi i\theta}$. Must create quantum circuit , and show each stage result clearly. [2+10=12 Marks]

3. Given our data: $\frac{|00\rangle+|01\rangle+i|10\rangle-|11\rangle}{2}$, undergoes a linear shift of 2, what will be the corresponding phase-shift upon application of the Quantum Fourier Transformation (QFT)? Show both results before and after the phase shift. [**5 Marks**]

4. What is $QFT^{-1}\frac{|000\rangle-|010\rangle+|011\rangle}{\sqrt{3}}$ in simplified terms? Here QFT^{-1} refers to the inverse Quantum Fourier Transformation. Noted: You don't need to write the entire matrix, but you may use a more efficient (clever) approach. [**4 Marks**]

5. We have applied Simon's algorithm to a 4-bit input. Given that we have $|\psi_5\rangle = \frac{|0101\rangle-|1001\rangle+|1100\rangle+|0000\rangle+|0010\rangle-|1110\rangle-|1011\rangle+|0111\rangle}{\sqrt{8}}$. Assuming that you have the same $|\psi_5\rangle$ repeatedly, what is the secret message s? [**5 Marks**]

6. Use Deutsch Jozsa algorithm to find if the function $f(n) = 1$ where $n \in \{00, 10, 10, 11\}$ is constant or balanced. [**10 Marks**]

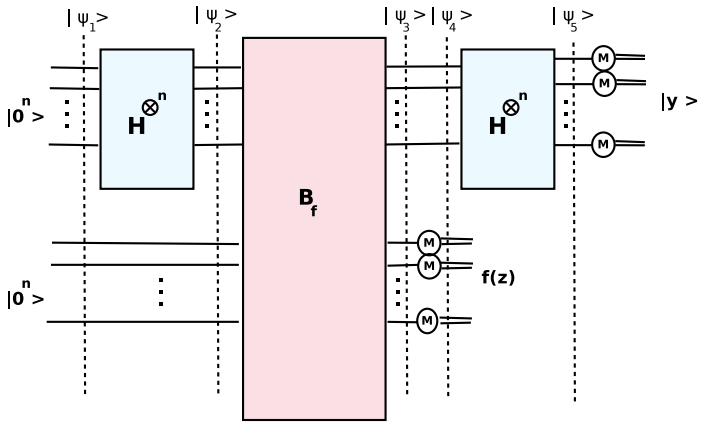

Figure 1: The circuit for the Simon's algorithm

7. Use Grover algorithm to search where $f(n) = 1$ where $n \in \{00, 10, 10, 11\}$. (PS: The function returns 1 for only 11)

8. What is the probability of success in Grover's algorithm given that $\cdots$

9. Prove that the following qubits not entangled $\cdots$

10. Use Superdense coding (explaining/calculating every stage) to sent two classical bits 1 and 0 from Alice to Bob.

11. Use quantum teleportation (explaining/calculating every stage) to send qubit $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$ from Alice to Bob.

12. Use Shor's prime factorization algorithm to find factor $N = 247$. Show each step clearly choose random $x = 3$.

13. Find eigenvector and eigenvalues of the following matrix $\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$

14. Use Euclidean algorithm to calculate GCD of 184 and 28. In case you do not know Euclidean algorithm then use the following fact $GCD(a, b)$ where $a > b$ is same as $GCD(b, a \mod b)$. Furthermore $GCD(c, 1) = c$.

15. Short questions [$3 \times 7 = 21$ Marks]:

    (a) Compute tensor product of the following: $\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 2 & 3 & 4 \end{pmatrix}$

(b) Given the $\frac{|00\rangle}{4} + \alpha |11\rangle$ is a valid qubits register what is the value of α.

(c) With what probability we will measure the last qubit as 0, given the following three qubits register?

$$|\phi\rangle = \frac{1}{\sqrt{7}} |000\rangle + \sqrt{\frac{2}{7}} |001\rangle + \sqrt{\frac{3}{7}} |101\rangle + \frac{1}{\sqrt{7}} |111\rangle$$

(d) What will be the resultant state after measuring the second qubit as 0, given the following three qubits register?

$$|\phi\rangle = \frac{1}{\sqrt{7}} |000\rangle + \sqrt{\frac{2}{7}} |001\rangle + \sqrt{\frac{3}{7}} |101\rangle + \frac{1}{\sqrt{7}} |111\rangle$$

(e) Write matrix $C = \begin{pmatrix} 1 & 2 & 0 \\ 4 & 0 & 1 \\ 0 & 0 & 99 \end{pmatrix}$ using Bra-Ket notation.

(f) In Period finding algorithm, if an output function period is 8 and size of your input function was 5-bits. What is the period of the input function?

(g) In Deutsch Jozsa algorithm what measurement at the end tells us if a given function is constant or balanced?

(h) Write the following vector using Bra-Ket notation $\begin{pmatrix} 0 \\ 1 \\ 5 \\ 8 \end{pmatrix}$

(i) What is $\langle\psi|\phi\rangle$ given $\cdots$.

(j) What is $|\psi\rangle |\phi\rangle$ given $\cdots$.

(k) What is $\langle\psi| \langle\phi|$ given $\cdots$.