Quiz-5 Quantum Computing

Time: 20 minutes

Marks: 7+12=19

1. Find all eigenvectors and eigenvalues of the matrix $\begin{pmatrix} -6 & 3 \\ 4 & 5 \end{pmatrix}$

   [**3+2+2=7 Marks**]

   Solution:

   First we find eigenvalues using characteristics equation:

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} -6-\lambda & 3 \\ 4 & 5-\lambda \end{vmatrix} = (-6-\lambda)(5-\lambda) - 12 = 0$$

$$-30 + 6\lambda - 5\lambda + \lambda^2 - 12 = \lambda^2 + \lambda - 42 = 0$$

$$\lambda^2 + 7\lambda - 6\lambda - 42 = \lambda(\lambda+7) - 6(\lambda+7) = 0$$

$$(\lambda - 6)(\lambda + 7) = 0$$

$$\lambda = 6, -7$$

   Now we find eigenvectors. First for $\lambda = 6$

$$\begin{pmatrix} -6-6 & 3 \\ 4 & 5-6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -12 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

   Now, we convert it into Gauss Jordan form:

$$\begin{pmatrix} -12 & 3 & 0 \\ 4 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 4 & -1 & 0 \\ -12 & 3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

   So x_2 is a free variable. Let's take it 4. Then $4x_1 - x_2 = 0$ $x_1 = 1$. Our eigenvector is: $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

   Now we find eigenvectors. First for $\lambda = -7$

$$\begin{pmatrix} -6+7 & 3 \\ 4 & 5+7 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 4 & 12 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Now, we convert it into Gauss Jordan form:

$$\begin{pmatrix} 1 & 3 & 0 \\ 4 & 12 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Here x_2 is free variable. Lets take it equal to $x_2 = 1$. Then $x_1 = -3$. Thus, our second eigenvector is $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

2. You are given a simple unitary matrix $U = \begin{pmatrix} e^{\frac{i\pi}{2}} & 0 \\ 0 & e^{-\frac{i\pi}{2}} \end{pmatrix}$, and its one eigenvector $|v\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, you are asked to use phase estimation to estimate θ for **m=2** of the corresponding eigenvalue $\lambda = e^{2\pi i\theta}$. Must create quantum circuit , and show each stage result clearly. [2+10=12 Marks]

Solution: We need to calculate eigenvalue so that the oracle produce it. That is, $\lambda = e^{\frac{i\pi}{2}} = i$

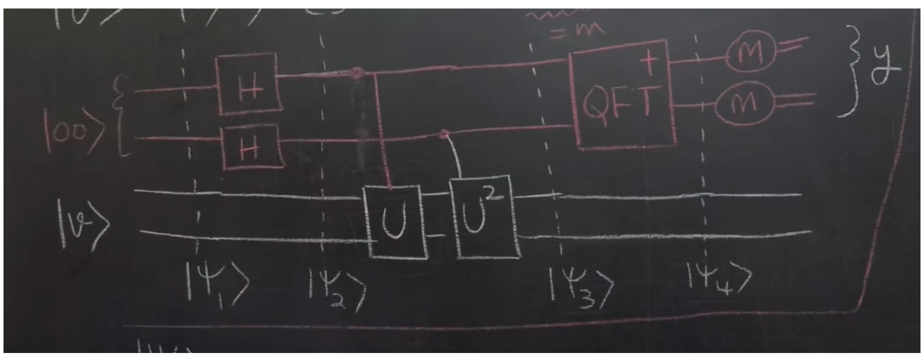

Figure 1: Quantum circuit of Phase Estimation

$$
\begin{aligned}
|\psi_1\rangle &= |00\rangle\,|\lambda\rangle \\
|\psi_2\rangle &= \frac{1}{2}\sum_{x=0}^{3}|x\rangle\,|\lambda\rangle \\
|\psi_3\rangle &= \frac{1}{2}\sum_{x=0}^{3}|x\rangle\,U^x\,|\lambda\rangle \\
|\psi_3\rangle &= \frac{1}{2}\sum_{x=0}^{3}|x\rangle\,i^x\,|\lambda\rangle \\
|\psi_3\rangle &= \frac{1}{2}\sum_{x=0}^{3}i^x\,|x\rangle \\
|\psi_3\rangle &= \frac{1}{2}\Big[i^0\,|0\rangle + i^1\,|1\rangle + i^2\,|2\rangle + i^3\,|3\rangle\Big]+ \\
|\psi_3\rangle &= \frac{1}{2}\Big[\,|0\rangle + i\,|1\rangle - |2\rangle - i\,|3\rangle\,\Big] \\
QFT_4 &= \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & \omega & \omega^2 & \omega^3\\ 1 & \omega^2 & \omega^4 & \omega^6\\ 1 & \omega^3 & \omega^6 & \omega^9\end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & \omega & \omega^2 & \omega^3\\ 1 & \omega^2 & 1 & \omega^2\\ 1 & \omega^3 & \omega^2 & \omega\end{pmatrix} \\
QFT_4^\dagger &= \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & \omega^3 & \omega^2 & \omega\\ 1 & \omega^2 & 1 & \omega^2\\ 1 & \omega & \omega^2 & \omega^3\end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -i & -1 & i\\ 1 & -1 & 1 & -1\\ 1 & i & -1 & -i\end{pmatrix} \\
|\psi_4\rangle &= QFT_4^\dagger\frac{1}{2}\Big[\,|0\rangle + i\,|1\rangle - |2\rangle - i\,|3\rangle\,\Big] \\
|\psi_4\rangle &= \frac{1}{4}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -i & -1 & i\\ 1 & -1 & 1 & -1\\ 1 & i & -1 & -i\end{pmatrix}\begin{pmatrix}1\\ i\\ -1\\ -i\end{pmatrix} \\
&= \begin{pmatrix}0\\ 1\\ 0\\ 0\end{pmatrix} = |1\rangle
\end{aligned}
$$

Based on above we will measure y=1. Thus our $\theta = \frac{y}{2^m} = \frac{1}{4}$. This will give us our eigenvalue $e^{\frac{2\pi i}{4}} = e^{\frac{\pi i}{2}} = i$

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1. Find all eigenvectors and eigenvalues of the matrix $\begin{pmatrix} 2 & 2 \\ 5 & 1 \end{pmatrix}$

   [**3+2+2=7 Marks**]

   Solution

   Step 1: We will find eigenvalues first using the characteristic equation:

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 2 \\ 5 & 1-\lambda \end{vmatrix}$$
$$= 0 = (2-\lambda)(1-\lambda) - 10 = 0$$
$$= \lambda^2 - 3\lambda - 8 = 0$$

   Solving above equation we get result of

$$\lambda_1 = \frac{1}{2}(3 + \sqrt{41})$$
$$\lambda_2 = \frac{1}{2}(3 - \sqrt{41})$$

   To find eigenvector $\vec{x}$ we solve the equation:

$$(A - \lambda I)\vec{x} = \vec{0}$$

   We have to solve them for both values of both lambdas one after another. Solving it will give use answer:

$$v_1 = \begin{pmatrix} 1+\sqrt{41} \\ 10 \end{pmatrix}$$
$$v_2 = \begin{pmatrix} 1-\sqrt{41} \\ 10 \end{pmatrix}$$

2. You are given a simple unitary matrix $U = \begin{pmatrix} e^{\frac{i\pi}{8}} & 0 \\ 0 & e^{-\frac{i\pi}{8}} \end{pmatrix}$, and its one eigenvector $|v\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, you are asked to use phase estimation to estimate θ for **m=2** of the corresponding eigenvalue $\lambda = e^{2\pi i\theta}$. Must create quantum circuit , and show each stage result clearly. [2+10=12 Marks]

   Solution See the solution above.

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   Now, we convert it into Gauss Jordan form:

$$\begin{pmatrix} -12 & 3 & 0 \\ 4 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 4 & -1 & 0 \\ -12 & 3 & 0 \end{pmatrix}$$

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