National University of Computer and Emerging Sciences, Lahore

Invigilator Signatures:

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Course:	Quantum Computing	**Course Code:**	CS-4084
Program:	BS (Computer Science)	**Semester:**	Fall 2023
Duration:	60 Minutes	**Total Marks:**	57
Paper Date:	01-Oct-2023	**Weightage:**	15%
Exam:	First Midterm	**Page(s):**	10
Instructor Name:	Dr. Faisal Aslam	**Section:**	
		Roll Number:	

Questions	Q1	Q2	Q3	Q4	Q5	Q6	**Total**
Marks	10	7	7	8	5	15	**52**
Marks Obtained							

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Questions

1. Consider the quantum circuit shown in Figure 1. If the input to the circuit is $|011\rangle$, determine its output. Please show your calculations at each stage of the circuit. [10 Marks]

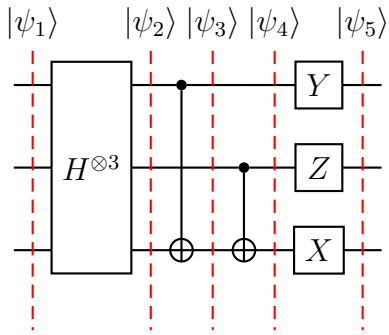

Figure 1: Quantum circuit

x	f(x)
00	1
01	1
10	0
11	0

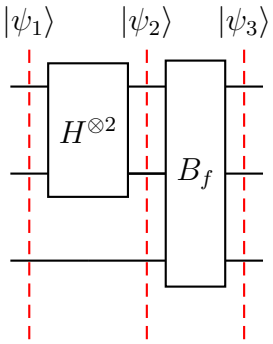

Figure 2: **(a)** Classical function inputs/outputs, **(b)** Quantum circuit

2. A classical function $f : \{0,1\}^n \to \{0,1\}$ for the 2-bits case is described in a table in Figure 2-a:

   A quantum circuit B_f implements the classical circuit such that $B_f |x\rangle |y\rangle = |x\rangle |y \oplus f(x)\rangle$. What will be the output of the circuit of Figure 2-b given its input is $|10\rangle |+\rangle$? [7 Marks]

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3. Construct a **single** unitary matrix representing the entire quantum circuit of Figure 3. [7 Marks]

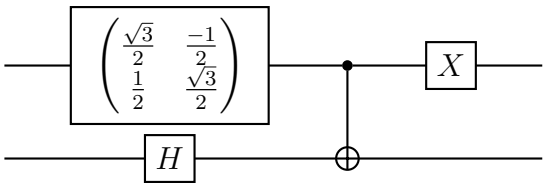

Figure 3: Quantum circuit

4. **a)** Draw the reverse quantum circuit of Figure 3 and **b)** determine the unitary matrix of the entire reverse circuit. [5+3=8 Marks]

5. Demonstrate that the qubits

$$|\psi\rangle = \frac{1}{2\sqrt{2}}|000\rangle - \frac{i}{2\sqrt{2}}|010\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|100\rangle - \frac{i\sqrt{3}}{2\sqrt{2}}|110\rangle$$

are not entangled by expressing individual qubits separately. Please clearly display their amplitudes. [5 Marks]

6. Short questions [$3 \times 5 = 15$ Marks]:

   (a) Express

$$|\psi\rangle = \frac{1}{4}|000\rangle + \frac{1}{4}|010\rangle + \frac{1}{\sqrt{2}}|100\rangle + \frac{1}{2}|110\rangle$$

   as a vector.

   (b) Calculate $\langle\psi|\phi\rangle$, given

$$|\psi\rangle = 2|00\rangle + 3|01\rangle,$$

   and

$$|\phi\rangle = 3|00\rangle + 4|01\rangle + 10|11\rangle$$

(c) Calculate $|\psi\rangle \langle\phi|$ given

$$|\psi\rangle = 2|0\rangle + 3|1\rangle,$$

and

$$|\phi\rangle = 3|0\rangle + 4|1\rangle$$

(d) Calculate $|\psi\rangle |\phi\rangle$ given

$$|\psi\rangle = 2|0\rangle + 3|1\rangle,$$

and

$$|\phi\rangle = 3|0\rangle + 4|1\rangle$$

(e) Calculate $\langle\psi|$ given

$$|\psi\rangle = (2i + 3)|0\rangle + 3|1\rangle$$

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