National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Quantum Computing	**Course Code:**	CS-4084
Degree Program:	BS Computer Science	**Semester:**	Fall 2023
Exam Duration:	3 Hours	**Total Marks:**	91
Paper Date:	26/12/2023	**Weight:**	50%
Section:	All	**No. of Page(s):**	18
Exam Type:	Final		

Student Name: _____, **Roll Number:** _____, **Section:** _____

Instruction/Notes:

- You are permitted to use a self-prepared A4-sized cheatsheet written in your own handwriting. Boldly write your roll number on this sheet.
- Sharing calculators is strictly prohibited.
- Do not use additional sheets for rough work; make use of the provided ample space.

Questions	Q1	Q2	Q3	Q4	Q5	Q6	Q7	**Total**
Marks	10	10	5	10	10	10	36	**91**
Marks Obtained								

Questions

1. You are given a simple unitary matrix $U = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$, and its one eigenvector $|v\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, you are asked to use the Phase estimation algorithm to estimate θ for **m=2** bits. Must create quantum circuit, and show each stage's result clearly. [**10 Marks**]

2. Use Deutsch Jozsa algorithm to show that the function $f(x) = 1$ where $x \in \{00, 10, 01, 11\}$ is constant. Must clearly show all the steps. [**10 Marks**]

3. Prove that the following qubits are NOT entangled $\frac{\sqrt{3}}{2\sqrt{2}}|000\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|001\rangle + \frac{1}{2\sqrt{2}}|100\rangle + \frac{1}{2\sqrt{2}}|101\rangle$ [**5 Marks**]

4. Use quantum teleportation to send qubit $\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$ from Alice to Bob. Create quantum circuit and show all the steps. [**10 Marks**]

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5. Use Shor's prime factorization algorithm to find factors of $N = 119$. Show each step clearly. Please choose random number $x = 4$ for your calculations. [**10 Marks**]

6. Given a black-box of function $f : \{0,1\}^3 \rightarrow \{0,1\}^3, f(x) = x \mod 2$. You have to find its period r using Period finding Algorithm. Calculate all the stages of its quantum circuit. [**10 Marks**]

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7. Short questions

(a) Compute tensor product of the following: $\begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 & 1 \\ 2 & 3 & 4 \end{pmatrix}$ [**3 Marks**]

(b) Given the $\frac{|00\rangle}{4} + \alpha\,|11\rangle$ is a valid qubits register what is the value of α. [**2 Marks**]

(c) With what probability we will measure the last qubit as 0, given the following register? Furthermore, what will be the resultant state after the measurement. [**2+2 Marks**]

$$|\phi\rangle = \frac{1}{\sqrt{7}}|000\rangle + \sqrt{\frac{2}{7}}|001\rangle + \sqrt{\frac{3}{7}}|101\rangle + \frac{1}{\sqrt{7}}|110\rangle$$

(d) Write vector $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 9 \end{pmatrix}$ in Bra-ket notation. [**2 Marks**]

(e) Given $|\psi\rangle = \left(\frac{i\sqrt{3}}{4}|0\rangle + \frac{\sqrt{13}}{4}|1\rangle\right)$, and $|\phi\rangle = \left(\frac{1}{\sqrt{3}}|0\rangle - \frac{i\sqrt{2}}{\sqrt{3}}|1\rangle\right)$ Calculate following: [**2+2+2 Marks**]

- $\langle\psi|\phi\rangle$
- $|\psi\rangle\,|\phi\rangle$
- $\langle\psi|\,\langle\phi|$

(f) Is that the matrix $A = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$ is unitary? Must clearly show your steps. [**3 Marks**]

(g) Is that the matrix $A = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$ is Hermitian? Must clearly show your steps. [**3 Marks**]

(h) Prove the $\frac{|\rho_0\rangle+|\rho_1\rangle+|\rho_2\rangle+|\rho_3\rangle}{2} = |1\rangle$, where $|\rho_j\rangle = \frac{1}{2}(|1\rangle + \omega_4^j |\alpha\rangle + \omega_4^{2j} |\alpha^2\rangle + \omega_4^{3j} |\alpha^3\rangle)$ [**5 Marks**]

(i) Create quantum circuit to entangle the given qubits $|10\rangle$. What will the the resultant Bell state? [**3 Marks**]

(j) Given our data: $\frac{|00\rangle+|01\rangle+i|10\rangle-|11\rangle}{2}$, undergoes a linear shift of 3, what will be the corresponding phase-shift upon application of the Quantum Fourier Transformation (QFT)? Show both results before and after the phase shift. [**5 Marks**]

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