9 Assignment 3: QFT and Quantum Algorithms

Please solve the following tasks manually and ensure timely submission well before the deadline. Kindly refrain from requesting a late submission, as such requests will not be accommodated.

x	f(x)
00	1
01	1
10	1
11	1

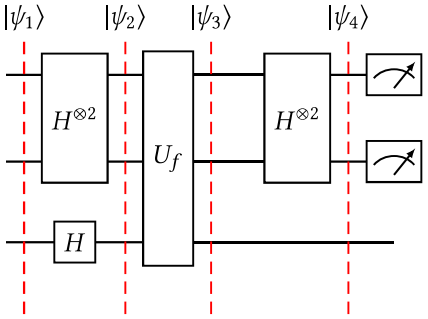

Figure 9.1: **(a)** Classical function inputs/outputs, **(b)** Deutsch-Jozsa Quantum circuit

1. A classical function $f : \{0,1\}^n \rightarrow \{0,1\}$ for the 2-bits case is described in a table in Figure 9.1-a. The quantum gate U_f implements the classical circuit such that $U_f |x\rangle |y\rangle = |x\rangle |y \oplus f(x)\rangle$. What will be the output of the Deutsch-Jozsa circuit of Figure 9.1-b given its input is $|00\rangle |1\rangle$? You must clearly show output of $|\psi_1\rangle$, $|\psi_2\rangle$, $|\psi_3\rangle$, and $|\psi_4\rangle$ (Otherwise no marks). [**1+2+2+2 Marks**]

2. Given a function f that takes n-bits as input and produces a single bit output, what is the exact probability of finding an input x such that $f(x) = 1$ using Grover's algorithm? Please provide your answer for n=4, n=5, n=6, and n=8. In each cases, it is known that exactly 4 inputs return 1. You must clearly show your working to derive the correct equation. [**5 Marks**]

3. Apply Simon's algorithm on 2-bits input and secret message $s = 11$.
   a) Create it quantum circuit, [**2 Marks**]
   b) Show output of each of possible five stages clearly. [**1+2+2+2+2 Marks**].
   c) Must show post output calculations including matrix transformation.[**4 Marks**]

4. What will be the outcome of the following operation. Use tablular approach discuss in class to quickly solve it. $H^{\otimes 4}(\frac{-|0000\rangle+|0101\rangle-|1101\rangle+|1110\rangle-|1111\rangle}{\sqrt{5}})$

5. Given a function $f(x) = 2x + 1 \mod 7$ where x is of 4-bits. Find its period using period

finding algorithm. Show every step clearly as there are no mark of writing period which is 4. [**10 Marks**]

6. Using an input $\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\0\\-1\end{pmatrix}$, show that quantum Fourier transform convert linear shift to phase shift. Must show use linear shift of 1 and 3. [**5 Marks**]

7. Using an input $\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\0\\-1\end{pmatrix}$, show that quantum Fourier transform convert phase shift to linear shift. Must show use two different phase shifts. [**5 Marks**]

8. Given a function $f(x) = 2x - 1 \mod 7$ where x is of 4-bits. Show that quantum Fourier transform changes its period. Drive and show what will be the changed period. (No mark for answer, all marks are for showing your work). [**7 Marks**]

9. Prove that quantum Fourier transform is unitary. [**5 Marks**]

10. Write inverse of quantum Fourier transform 8×8 bit matrix. Write each term as simplified as possible. [**5 Marks**]