The RAM model of Computation

Thursday, 19 March 2020 10:55 PM

The RAM (Random Access Machine) model of computation represents an abstract machine that describes a typical modern day computer.

It has the following key features:

- Any RAM based device can perform in constant time all arithmetic, logical, bitwise and data movement operations between operands that fit into the registers of the machine, i.e. typically 32 to 64 bit operands.
  - This means all operations such as +, -, /, *, =, <, =, ==, etc. are constant time operations on a RAM device.

  We will call any such operation a ***step,*** and each step will take the same constant amount of work in our analysis.
- Any RAM based device is also able to perform indirect addressing, for example, array indexing, in constant time. Hence, statements such as A[i], are also considered a single step.
- There is a single processing unit (core, etc.) in the RAM based device.
- We do not consider the complexity of memory hierarchy.

Hence, to make a theoretical estimate of the running time taken by a program we count the number of steps (as defined above) taken by this program and multiply that count with the constant amount of time taken by each step.

This kind of an analysis of the running time of a program is called **Step Count Analysis.**

Performing Step Count Analysis and the Time function

Saturday, 15 August 2020 10:26 PM

Constant: Something that does not change due to input.

What is the total **step-cost** of the following pieces of code?

\# steps over the lifetime of the prog.

```
int i = 0, sum=0;
for(; i<10; i++)
    sum=sum+i;
```

$\rightarrow c_1$

$\rightarrow 10c_2 = c_2'$

$\rightarrow 10c_3 = c_3'$

$T(n) = c_1 + c_2' + c_3'$

$\boxed{T(n) = c_4}$

```
int i = 0, sum=0;
for(; i<n; i++)
    sum=sum+i;
```

$\rightarrow c_1$

$\rightarrow nc_2$

$\rightarrow nc_3$

step count analysis

$T(n) = (c_2 + c_3)n + c_1$

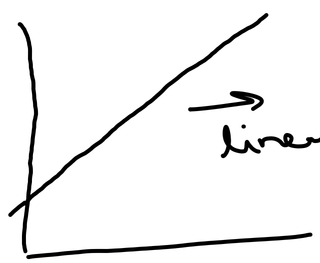
$= an + b$

$a = 2$

$b = 3$

Asymptotic Behavior

T(n)



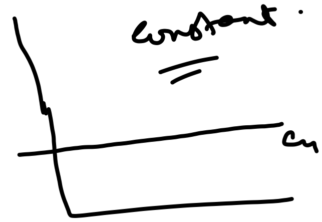
n →

→ linear growth.

2n+3

constant.

T(n)



c_4

n →

→ linear time

→ constant

flip stack

Math Review

Saturday, 15 August 2020 11:04 PM

(R.1) Arithmetic Series

Simplest form:

$$1 + 2 + \cdots n = \frac{n(n+1)}{2}$$

(R.1) Geometric Series

$$1 + r + r^2 + \cdots r^{k-1} = \frac{1 - r^{k+1}}{1 - r}$$

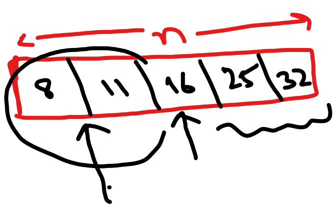
More generally,

$$a + ar + ar^2 + \cdots ar^{k-1} = \frac{a(1 - r^{k+1})}{1 - r}$$

Importantly, for an infinite geometric series with r<1

$$1 + r + r^2 + \cdots r^{k-1} = \frac{1}{1 - r}$$

(R.3) Logarithms



binary search

step 1 → n

step 2 → $n/2$

step 3 → $n/4$

⋮

how many steps.

2^{50}, 2^{100}

$\lg(2^{200}) = 200$

$n = 2^k$

$\Rightarrow \boxed{k = \lg n} \to \log_2 n$

{ $\lg_2 n$: # of times you divide n by 2 to reach 1.

estimated # of H atoms in the universe $< 2^{200}$.

Fast

c

$\lg n$

$an+b$

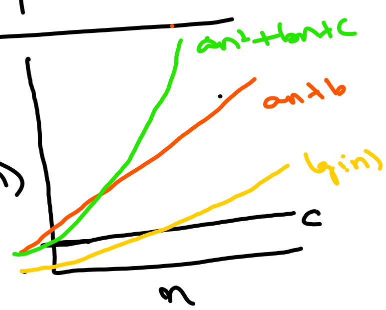
an^2+bn+c

⋮

2^n

slow

$T(n)$



Question 1: How many terms does the following series have?
1 + 2 + 4 + 8 + … + n?

Answer: ?

Question 2: what is n is not an exact power of 2? What if n is a number like 27 or 35, for example?

For any number n we have some k such that,

$2^{k-1} \leq n \leq 2^k, where\ n, k \geq 1$

Simply put:

For any n, the answer to lgn is either an integer k>=1, or a number k-1 or k+1.

Question 3: What is the sum of the following series?

1 + 2 + 4 + 8 + … + n?

Answer:

This is a geometric series with k = lgn, and r = 2, a=1

$$S = \frac{1 - 2^{lgn+1}}{1-2} = 2^{lgn+1} = 2n$$

Question 4: What is the sum of the following series?

$$n + \frac{n}{2} + \frac{n}{4} + \cdots + 1?$$

Answer:

Two more examples of Step Count Analysis

Sunday, 16 August 2020 12:37 PM

What is the total **step-cost** of the following pieces of code?

```
int i = 1, sum=0;
for(; i<n; i=i*2)
    sum=sum+i;
```

n

$i = i/2$

$\to c_1$

$\to c_2 \lg n$

$\to c_3 \lg n$

$\lg n +$

1, 2, 4, 8

$\boxed{T(n) = a\lg n + b}$

1, 2, 4, 8, 16, 32

$\ldots \frac{n}{8}, \frac{n}{4}, \frac{n}{2}, n$

```
int i = n, sum=0;
for(; i>0; i=i/5)
    sum=sum+i;
```

$\to c_1$

$\to c_2 \log_5 n$

$\to c_3 \log_5 n$

$\log_5 n$

$T(n) = a\log_5 n + b$

$i = n, \frac{n}{5}, \frac{n}{25}, \frac{n}{125}, \ldots$

$\log_5 n \le \log_2 n$