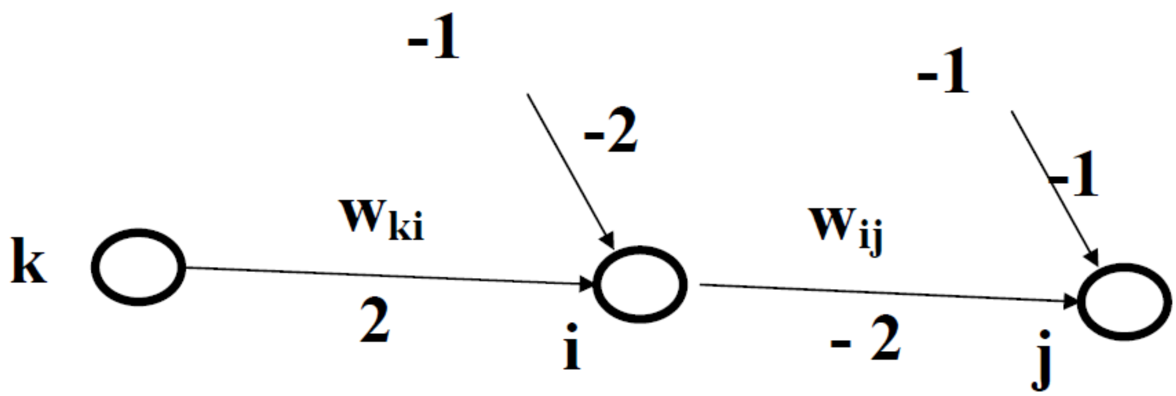
Practice Problems

1. For the following network, find the new weight w_{ki} (new) by the delta rule. Activation Functions of both neurons "i" and "j" are linear, i.e. y = activation. The learning rate is 0.1. The training pair is [2; 3]; i.e. input = 2 and output = 3. The current weights are shown on the links. There is only one input "k" and only one output "j". The hidden layer has one neuron "i". The node "k" is not a neuron and just passes on the input without any processing or modification. (50 marks)



Solution

According to the generalized delta rule, the hidden weight change is governed by:

$$\Delta w_{ki} = w_{ki}(new) - w_{ki}(old) = -c[-2\Sigma_j \{(y_j(desired) - y_j(actual))\, f'(act)_j\, w_{ij}\} f'(act)_i\, x_k]$$

For our problem, there is only one output neuron (i.e. "j" = 1). Hence the Equation becomes

$$w_{ki}(new) = w_{ki}(old) - c[-2\Sigma_j \{(y_j(desired) - y_j(actual))\, f'(act)_j\, w_{ij}\} f'(act)_i\, x_k]$$

We have the following given values:

$$w_{ki}(old) = 2,\ c = 0.1,\ y_j(desired) = 3,\ w_{ij} = -2,\ x_k = 2$$

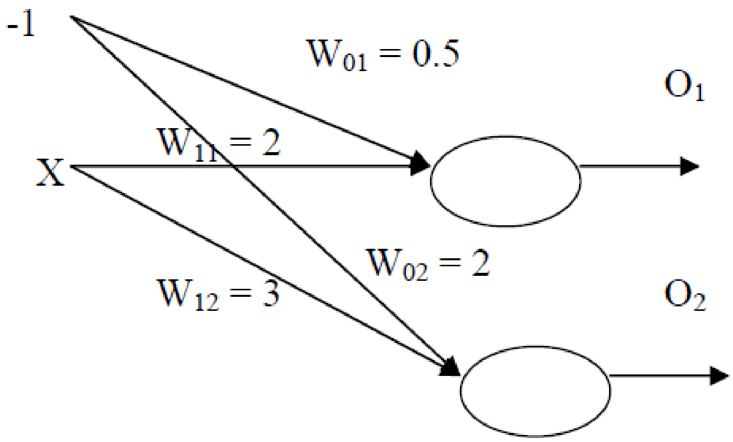
Since $f'(act)_j$ and $f'(act)_i$ are both linear functions, therefore $f'(act)_j = 1$, and $f'(act)_i = 1$

For $y_j(actual)$ we have to first calculate the output of neuron "i", which is the input of neuron "j".
Activation of neuron "i" is: (Input) (w_{ki})+ (-1) (-2) = (2 x 2) + 2 = 6
Since activation function is "linear function", hence this activation is also the output of the neuron "i".
Activation of neuron "j" is: (Output of "i") (w_{ij}) + (-1) x -1 = (6 x -2) + 1 = -11
Since activation function is "linear function", hence this activation is also the output of the neuron "j".
Hence $y_j(actual)$ = -11

Substituting the values in Generalized Delta Rule, we have

$$w_{ki}(new) = 2 - 0.1[-2 \{(3 - (-11)) (1) (-2) \} 1 (2)] = 2 - 0.1[112] = 2 - 11.2 = -9.2$$

For the following network, find the new weights by the generalized delta rule. Activation Function of the neuron is linear, i.e. y = activation. The learning rate (c) is 0.2. The training pair is [2; 3, 2]; i.e. input = 2 and output 1 = 3 and output 2 = 2. The current weights are shown on the link. There is only one input and two outputs.



Solution: The derivative of activation function is 1.

Hence, $W_{01(new)} = 0.5 - 0.2[-2(3 - 3.5).1.(-1)] = 0.5 - 0.2\{2(-0.5)\} = 0.7$

$W_{02(new)} = 02 - 0.2[-2(2 - 4).1.(-1)] = 2.8$

$W_{11(new)} = 2 - 0.2[-2(3 - 3.5).1.(2)] = 1.6$

$W_{12(new)} = 3 - 0.2[-2(2 - 4).1.(2)] = 1.4$

Show the mathematical working of Artificial Neural Network by taking the case in figure below. First two columns are the input values for X1 and X2 and the third column is the desired output.

0	0	0
0	1	0
1	0	1
1	1	1

Learning rate = 0.2
Threshold = 0.5
Actual output = W1X1+W2X2

Next weight adjustment = Wn+ΔWn

Change in weight = ΔWn = learning rate * (desired output- actual output) * Xn

Show two complete iterations for acquiring the desired output?

x1	x2	w1	w2	d	y	Δw1	Δw2

1. Suppose we want to implement a Boolean function $X_1 \wedge \neg X_2 \wedge X_3$ with a single neuron.
    a. Calculate the appropriate threshold assuming the weights to be $W_{X1} = 1$ $W_{X2} = -2$ & $W_{X3} = 1$. The truth table of the function is given below. (60 marks)
    b. What would be the bias weight for this problem, if bias (with a constant input of -1) is used instead of threshold? (40 marks)

Truth Table

X_1	X_2	X_3	$X_1 \wedge \neg X_2 \wedge X_3$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Solution

X_1	X_2	X_3	$X_1 \wedge \neg X_2 \wedge X_3$ (Output)	$X_1W_{X1} + X_2W_{X2} + X_3W_{X3}$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	-2
0	1	1	0	-1
1	0	0	0	1
1	0	1	1	2
1	1	0	0	-1
1	1	1	0	0

The threshold can be any value θ such that $1 < \theta \leq 2$ (e.g. 1.5)

If bias is used instead of threshold, it can be set at the same value as that of threshold.