

- Fully measure $|\alpha\rangle = \frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$ in basis $|+\rangle, |-\rangle$ [5 Marks]

Let's first write in Hadamard basis. Then,

$$\begin{aligned} |\alpha\rangle &= \langle\alpha|+\rangle|+\rangle + \langle\alpha|-\rangle|-\rangle \\ &= \left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)|+\rangle + \left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)|-\rangle \end{aligned}$$

- Probability we measure $|+\rangle$ is $\left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)^2 = 0.9$
 - After measuring $|+\rangle$ our resultant state will be $|+\rangle$
 - Probability we measure $|-\rangle$ is $\left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)^2 = 0.1$
 - After measuring $|-\rangle$ our resultant state will be $|-\rangle$
- With what probability we measure the first qubit of $|\beta\rangle = \frac{1}{\sqrt{7}}|01\rangle + \frac{2}{\sqrt{7}}|10\rangle + \sqrt{\frac{2}{7}}|11\rangle$ as $|0\rangle$? Furthermore, what will be the resultant state? [5 Marks]
 - Probability of measuring first qubit as zero is $|\frac{1}{\sqrt{7}}|^2 = \frac{1}{7} = 0.143$
 - The resultant state afterwards will be $|01\rangle$

- Fully measure $|\alpha\rangle = \frac{1}{\sqrt{5}}|+\rangle + \frac{2}{\sqrt{5}}|-\rangle$ in basis $|0\rangle, |1\rangle$ [5 Marks]

Let's first write in standard basis. Then,

$$\begin{aligned} |\alpha\rangle &= \langle\alpha|0\rangle|0\rangle + \langle\alpha|1\rangle|1\rangle \\ &= \left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)|0\rangle + \left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)|1\rangle \end{aligned}$$

- Probability we measure $|0\rangle$ is $\left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)^2 = 0.9$
 - After measuring $|0\rangle$ our resultant state will be $|0\rangle$
 - Probability we measure $|1\rangle$ is $\left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)^2 = 0.1$
 - After measuring $|1\rangle$ our resultant state will be $|1\rangle$
- With what probability we measure the first qubit of $|\beta\rangle = \frac{1}{\sqrt{7}}|01\rangle + \frac{2}{\sqrt{7}}|10\rangle + \sqrt{\frac{2}{7}}|11\rangle$ as $|1\rangle$? Furthermore, what will be the resultant state? [5 Marks]

- Probability of measure first qubit as $|1\rangle$ is $|\frac{2}{\sqrt{7}}|^2 + |\sqrt{\frac{2}{7}}|^2 = \frac{6}{7}$
- The resultant state after measuring it will be:

$$\begin{aligned} &= \sqrt{\frac{7}{6}} \left(\frac{2}{\sqrt{7}}|10\rangle + \sqrt{\frac{2}{7}}|11\rangle \right) \\ &= \sqrt{\frac{2}{3}}|10\rangle + \frac{1}{\sqrt{3}}|11\rangle \end{aligned}$$

- With what probability we measure the first qubit of $|\beta\rangle = \frac{1}{\sqrt{7}}|+-\rangle + \frac{2}{\sqrt{7}}|-+\rangle + \sqrt{\frac{2}{7}}|--\rangle$ as $|+\rangle$? Furthermore, what will be the resultant state? [5 Marks]

- We measure first qubit as $|+\rangle$ with probability $|\frac{1}{\sqrt{7}}|^2 = \frac{1}{7}$
- The resultant state will be $|+-\rangle$

- Fully measure $|\alpha\rangle = \frac{1}{\sqrt{5}}|+\rangle + \frac{2}{\sqrt{5}}|-\rangle$ in basis $|0\rangle, |1\rangle$ [5 Marks]
Let's first write in standard basis. Then,

$$\begin{aligned} |\alpha\rangle &= \langle\alpha|0\rangle|0\rangle + \langle\alpha|1\rangle|1\rangle \\ &= \left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)|0\rangle + \left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)|1\rangle \end{aligned}$$

- Probability we measure $|0\rangle$ is $\left(\frac{1}{\sqrt{10}} + \sqrt{\frac{2}{5}}\right)^2 = 0.9$
- After measuring $|0\rangle$ our resultant state will be $|0\rangle$
- Probability we measure $|1\rangle$ is $\left(\frac{1}{\sqrt{10}} - \sqrt{\frac{2}{5}}\right)^2 = 0.1$
- After measuring $|1\rangle$ our resultant state will be $|1\rangle$