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$$\frac{dy}{dx} + xy = xy^2$$

$$\frac{dy}{dx} = xy^2 - xy$$

value of x at $0.25, 0.5, 0.75, 1$
which means $h = 0.25$

Euler Method

$x_0 = 0 \quad y_0 = \frac{1}{2}$

$$y_1 = y_0 + h f(x_0, y_0)$$

$$y_1 = \frac{1}{2} + 0.25\left[0\left[\frac{1}{2}\right]^2 - [0]\left[\frac{1}{2}\right]\right]$$

$$y_1 = \frac{1}{2} + 0.25[0]$$

$$y_1 = \frac{1}{2} \quad \text{at } x_1 = 0.25$$

$$y_1 = 0.5$$

$$y_2 = y_1 + h f(x_1, y_1)$$

$$y_2 = \frac{1}{2} + 0.25\left[0.25\left[\frac{1}{2}\right]^2 - 0.25\left[\frac{1}{2}\right]\right.$$

$$y_2 = \frac{1}{2} + 0.25\left[-\frac{1}{16}\right]$$

$$y_2 = \frac{1}{2} - \frac{1}{64}$$

$$y_2 = \frac{31}{64} \text{ at } x_2 = 0.5$$

$$y_2 = 0.48438$$

$$y_3 = y_2 + hf(x_2, y_2)$$

$$y_3 = \frac{31}{64} + 0.25\left[0.5\left[\frac{31}{64}\right]^2 - 0.5\left[\frac{31}{64}\right]\right]$$

$$y_3 = \frac{31}{64} + 0.25\left[\frac{-1023}{8192}\right]$$

$$y_3 = \frac{31}{64} - \frac{1023}{32768}$$

$$y_3 = 0.45316 \text{ at } x_3 = 0.75$$

$$y_4 = y_3 + hf(x_3, y_3)$$

$$y_4 = 0.45316 + 0.25\left((0.75)(0.45316)^2 - (0.75)(0.45316)\right)$$

$$y_4 = 0.45316 - 0.04646$$

$$y_4 = 0.40669 \text{ at } x_4 = 1$$

$$y_4 = 0.4067 + 0.25[1(0.4067)^2 - 4(0.4067)]$$

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Modified Euler Method

$$y_{m+1} = y_m + hf\left(x_m + \frac{h}{2}, y_m + \frac{h}{2} f(x_m, y_m)\right)$$

$$f(x_0, y_0) = f(0, 0.5) = 0$$

$$x_0 + \frac{h}{2} = 0.125$$

$$y_0 + \frac{1}{2} hf(x_0, y_0) = 0.5 + \frac{0.25}{2} \cdot 0 = 0.5$$

$$f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2} hf(x_0, y_0)\right)$$

$$= f(0.125, 0.5) = -0.03125$$

$$y_1 = y_0 + hf\left(x_0 + \frac{h}{2}, \frac{1}{2} hf(x_0, y_0)\right)$$

$$y_1 = 0.5 + 0.25 + 0.25 \cdot -0.03125$$

$$y(0.25) = 0.49219$$

$$y_2 = y_1 + hf\left(x_1 + \frac{h}{2}, y_1 + \frac{h}{2} f(x_1, y_1)\right)$$

$$y_2 = 0.49219 + 0.25 f\left(0.375, 0.49219 + \frac{0.25}{2} \cdot -0.06248\right)$$

$$y_2 = 0.49219 + 0.25 f(0.375, 0.48438)$$

$$y_2 = 0.49219 + 0.25 \cdot -0.0936$$

$$y(0.5) = 0.46877$$

$$y_3 = y_2 + h f\left(x_2 + \frac{h}{2}, y_2 + \frac{h}{2} f(x_2, y_2)\right)$$

$$y_3 = 0.46877 + 0.25 f\left(0.625, 0.46877 + \frac{0.25}{2} \cdot -0.12457\right)$$

$$y_3 = 0.46877 + 0.25 f(0.625, 0.45321)$$

$$y_3 = 0.46877 + 0.25 \cdot -0.15488$$

$$y(0.75) = 0.43005$$

$$y_4 = y_3 + h f\left(x_3 + \frac{h}{2}, y_3 + \frac{h}{2} f(x_3, y_3)\right)$$

$$y_4 = 0.43005 + 0.25\left(0.875, 0.43005 + \frac{0.25}{2} \cdot -0.18383\right)$$

$$y_4 = 0.43005 + 0.25 f(0.875, 0.4907)$$

$$y_4 = 0.43005 + 0.25 = 0.21119$$

$$y(1) = 0.37725$$

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Improved Euler Method

$$y_{m+1} = y_m + \frac{h}{2}\left[f(x_m, y_m) + f(x_m + h, y_m + hf(x_m, y_m))\right]$$

$$y_1 = 0.5 + \frac{0.25}{2}\left[0 + f(0.25, 0.5)\right]$$

$$y_1 = 0.5 + \frac{0.25}{2}\left[0 - 0.0625\right]$$

$$y_1 = 0.49219$$

$$y(0.25) = 0.49219$$

$$y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_1 + h, y_1 + hf(x_1, y_1))\right]$$

$$y_2 = 0.49219 + \frac{0.25}{2}\left[-0.06248 + f(0.5, 0.47657)\right]$$

$$y_2 = 0.49219 + \frac{0.25}{2}\left[-0.06248 - 0.12473\right]$$

$$y_2 = 0.46879$$

$$y(0.5) = 0.46879$$

$$y_3 = y_2 + \frac{h}{2}\left[f(x_2, y_2) + f(x_2 + h, y_2 + hf(x_2, y_2))\right]$$

$$y_3 = 0.46879 + \frac{0.25}{2}\left[-0.12451 + f(0.75, 0.43766)\right]$$

$$y_3 = 0.46879 + \frac{0.25}{2}\left[-0.12451 - 0.18459\right]$$

$$y(0.75) = 0.43015$$

$$y_4 = y_3 + \frac{h}{2}\left[f(x_3, y_3) + f(x_3 + h, y_3 + hf(x_3, y_3))\right]$$

$$y_4 = 0.43015 + \frac{0.25}{2}\left[-0.18384 + f(1, 0.38419)\right]$$

$$y_4 = 0.43015 + \frac{0.25}{2}\left[-0.18384 - 0.23659\right]$$

$$y(1) = 0.3776$$

	Euler	Modified	Improved	Exact
.25	0.5	0.49219	0.49219	0.49219
.5	0.48438	0.46877	0.46879	0.46879
.75	0.45316	0.43005	0.43015	0.43015
1	0.40669	0.37725	0.37760	0.37754