Date: 01/04/21

Black Board

Data Structures

Lecture # 7

Topics:

- **Worst Case Analysis**
- **Asymptotic Notation**: Asymptotic Tight bound, Upper Bound and Lower Bound

Repeat n times:

- Take an input x
- Insert x into array A at its correct place in non-decreasing order.

Here is the code:

```
void insertNumberIntoArray(int A[], int n){

    int x;
    for(int i=0; i<n; i++){
        cin>>x;
        int k=i-1;
        while(k>=0 && A[k]>x){
            A[k+1]=A[k];
            k--;
        }

        A[k+1]=x;
    }
}
```

`int x;` → c_1

`for(int i=0; i<n; i++){` → $c_2 n$

`cin>>x;` `int k=i-1;` → $c_3 n$

`while(k>=0 && A[k]>x){` → $c_4 \sum_{i=0}^{n} i$

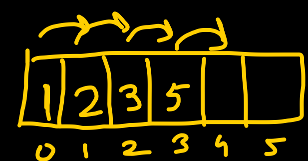
`A[k+1]=A[k];` `k--;` → $c_5 \sum_{i=0}^{n} i$

`A[k+1]=x;` → $c_6 n$

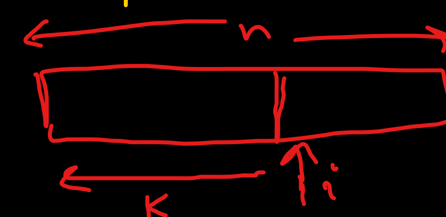
worst case analysis

shifting

Perform analysis for a worst case Scenario



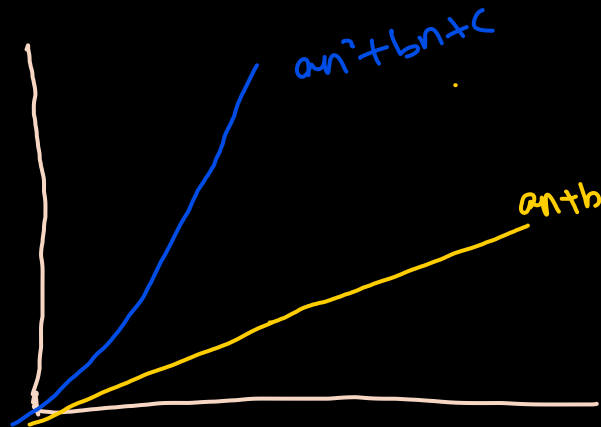
$x = 0$



$$T(n) = a\left(\sum_{i=1}^{n} i\right) + bn + c = a\frac{n(n+1)}{2} + bn + c$$

$$T(n) = a'n^2 + b'n + c'$$

Families of T(n) (or classes or sets of T(n))



$T(n) = 4n^2 + 5n + 6$

$T(n) = 2n^2 - 6n + 2$

Same family

$T(n) = 2n + 3$

$T(n) = 4n + 5$

linear

Same family when n is large.

Different family

How can we write in math that T(n) is in the linear family.

$T(n) = 2n + 7$

belongs to the linear family only if there is a line below and above $T(n)$.

membership in the n family

$2n \leq 2n+7 \leq 9n$

$an \leq T(n) \leq bn$

$T(n) = \Theta(n)$

$T(n) \in \Theta(n)$

tight bound

$\forall n \geq n_0$

$d n \le 3n^2+5 \le cn$ $\quad n \ge 1$

find c & d

$$\boxed{3n^2 \le 3n^2+5 \le 8n^2 \quad n \ge 1}$$

$\Theta(n^2)$

$3n^2+5 \le cn$

$\Rightarrow n^2 + \frac{5}{n} \le c$

$T(n) = 2n + 3\lg n + 5 = \Theta(n)$

Asymptotically belongs to $\Theta(n)$

$2n \le 2n + 3\lg n + 5 \le 10n$

$T(n) = 2n^2+5$

Is $T(n) = \Theta(n^3)$?

Ans: No

$cn^3 \le 2n^2+5 \nleq dn^3$

$2n^2+5 \le 7n^3$

$an+b$

$$T(n) = 5n\lg n + 2n + 6$$

$$\boxed{dn \le 5n\lg n + 2n + 6} \le cn$$

can't find

O

$$\boxed{5n \le 5n\lg n + 2n + 6}$$

$2n\lg n - \lg^2 n + 5$

$2n\lg n + 6n + 7$

$\Theta(n\lg n)$

everything in this family is the "same" for us.

$$6n\lg n \le 6n\lg n + 7n + 2$$

$$\le 15n\lg n$$

If $T(n) \le cf(n) \implies T(n) = O(f(n))$

$$3n^2 + 5 \le 8n^2 \implies T(n) = O(n^2)$$

$n \ge 1$

Big-Oh one-sided (upper bound)

$2n^2 + 5 = O(n^2) \longrightarrow$ true

$\leq 7n^2$

$2n^2 + 5 = O(n^3) \longrightarrow$ true

$2n^2 + 5 = \Theta(n^3) \longrightarrow$ false

$an^3 \leq 2n^2 + 5 \leq 7n^3$

$an^3 \leq 2n^2+5$ → not possible

$2n^2+5 \leq 7n^3$ → possible

may not be a tight bound.

$2n^2 + 5 = O(n^2)$

$= O(n^3)$

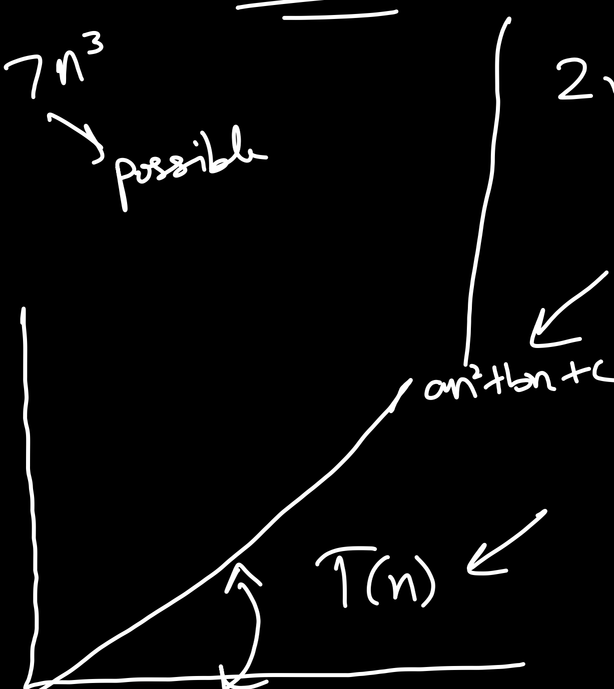
$= O(n^4)$

$\vdots$

$T(n) = O(n^2)$

$T(n) = \lg n = O(n^2)$] loose bound.

$= 2n + 3 = O(n^2)$



$T(n) = 3\lg n$

Big Omega is one-sided lower bound

$\cancel{3n^2 + 2n +} \; 3n^2+2n+5 = \Omega(n^2)$

$3n^2 \le 3n^2+2n+5$

IF $T(n) \geq c f(n) \implies T(n) = \Omega(f(n))$

Summary. IF $T(n) = O(f(n))$
AND $T(n) = \Omega(f(n))$ $\Big\} \implies T(n) = \Theta(f(n))$