Zadachi

Задача 1

$$\int \frac{1}{cosx} dx$$

Задача 2

$$\int dx$$

Задача 3

$$\int dx$$

Задача 4

$$\int dx$$

Задача 5 *Да се изследва функцията:*

$$f(x) = \sqrt[3]{x}\sqrt[3]{(1-x)^2}$$

Решение :

$$F(G(x))' = F'G'$$

$$f'(x) = [\sqrt[3]{x^3 - 2x^2 + x}]' = \frac{1}{3}\frac{3x^2 - 4x + 1}{\sqrt[3]{(x^3 - 2x^2 + x)^2}}$$

$$f'(x) = 0 \to x_1 = 1, x_2 = \frac{1}{3}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

$$P(x) = x^3 - 2x^2 + x$$

$$f''(x) = \frac{1}{3}\frac{(6x-4)(zn) - (zn)'(3x^2 - 4x + 1)}{zn^2} =$$

$$= \frac{1}{3}\frac{(6x-4)(P(x)^{\frac{2}{3}}) - (\frac{2}{3}P(x)^{-\frac{1}{3}}P'(x))(3x^2 - 4x + 1)}{P(x)^{\frac{4}{3}}} =$$

$$= \frac{1}{3}\frac{(6x-4)(P(x)) - (\frac{2}{3}P'(x))(3x^2 - 4x + 1)}{P(x)^{\frac{4}{3}}P(x)^{\frac{1}{3}}} =$$

$$= \frac{1}{3}\frac{(6x-4)(x^3-2x^2+x)-(\frac{2}{3}(3x^2-4x+1))(3x^2-4x+1)}{P(x)^{\frac{4}{3}}P(x)^{\frac{1}{3}}} =$$

$$= \frac{1}{3}\frac{(6x-4)(x(x-1)^2)-(\frac{2}{3}(3.3(x-1)^2(x-\frac{1}{3})^2))}{[x(x-1)(x-1)]^{\frac{5}{3}}} =$$

1 Algebra

Задача 6 *Да се диагонализира матрицата:* $\begin{vmatrix} 29 & 15 & -15 \\ -20 & -11 & 10 \\ 30 & 15 & -16 \end{vmatrix}$

$$\det(A-xE) = \begin{vmatrix} 29-x & 15 & -15 \\ -20 & -11-x & 10 \\ 30 & 15 & -16-x \end{vmatrix} =$$

$$= (29-x)(-11-x)(-16-x)+15.10.30+(-20).15.(-15)-30.(-11-x)(-15)-15.10.(29-x)$$

$$-(-20)15(-16-x) =$$

$$-x^3+(29-11-16)x^2+(29.11+29.16-11.16)x+29.11.16+$$

$$+15.10.60-30.15(11+x)-15.10(29-x)-20.15(16+x)$$

$$-x^3+(29-11-16)x^2+(29.11+29.16-11.16-30.15+15.10-20.15)x+$$

$$29.11.16+15.10.60-30.11.15-15.10.29-20.15.16 =$$

$$-x^3+2x^2+7x+4$$

схема на хорнер:
$-x^3+2x^2+7x+4$
-1 -2 -1 0
$-x^3+2x^2+7x+4 = (x-4)(-x^2-2x-1) = -(x-4)(x+1)^2$
собствени вектори: x=-1 -> $\begin{vmatrix} 29-x & 15 & -15 \\ -20 & -11-x & 10 \\ 30 & 15 & -16-x \end{vmatrix} =$

$\begin{vmatrix} 30 & 15 & -15 \\ -20 & -10 & 10 \\ 30 & 15 & -15 \end{vmatrix} = |2 \quad 1-1|$ общо решение $(p, q-2p, q)$

$\begin{vmatrix} 25 & 15 & -15 \\ -20 & -15 & 10 \\ 30 & 15 & -20 \end{vmatrix} = \begin{vmatrix} 5 & 3 & -3 \\ -4 & -3 & 2 \\ 6 & 3 & -4 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -1 \\ -4 & -3 & 2 \\ 2 & 0 & -2 \end{vmatrix}$ общо решение$(p, -\frac{2}{3}p, p)$
собствен вектор: $(3, -2, 3)$

Всички вектори са $a_1 = (3, -2, 3)$, $a_2 = (1, -2, 0)$, $a_3 = (0, 1, 1)$ Матрицата има вида: $\begin{vmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$

Ортогонаизация по Gram-Schmidt: Търсим вектори $b_2 b_3$, т.че $l(a_2, a_3) = l(b_2, b_3)$ и $b_2 \perp b_3$.
$-6, 4, 1$

$$b_3 = a_3$$
$$b_2 = a_2 + \lambda b_3$$

$\lambda = -\frac{(a_2,b_3)}{(b_3,b_3)} = -\frac{-2}{2} = 1$, $b_2 = a_2 + b_3 = (1, -1, 1)$ Намираме ортонормиран базис $c_1 = \frac{1}{\sqrt{22}}(3, -2, 3), c_2 = \frac{1}{\sqrt{3}}(1, -1, 1), c_3 = a_3 = \frac{1}{\sqrt{2}}(0, 1, 1)$

(A | E) $\approx (E|A^{-1})$
$A|B \approx (E|BA^{-1})$