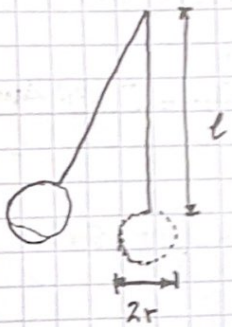
Vaja 15 Težno nihalo

Nihajni čas matematičnega nihala je:

$$t_0 = 2\pi\sqrt{\frac{l}{g}}$$

$$g = l\left(\frac{2\pi}{t_0}\right)^2$$



Matematičnemu nihalu se lahko približamo s številnimi popravki: (15.3)

1) $T = 2\pi\sqrt{\frac{l}{g}}\left[1+\left(\frac{1}{2}\right)^2\sin^2(\alpha/2)+\left(\frac{3}{8}\right)^2\sin\left(\frac{\alpha}{2}\right)\dots\right]$

2) $T = 2\pi\dfrac{m_k l_0^2 + \frac{2m_k r^2}{5} + \frac{m_z(l_0-r^2)}{3}}{\left[m_k l_0 + \frac{m_z(l_0-r)}{2}\right]\cdot g} \approx 2\pi\sqrt{\frac{l_0}{g}\left(1+\frac{2r}{5l_0}-\frac{1m_z}{6m_k}\right)}$ (15.4.)

3) $g' = g\cdot\left(1-\frac{\rho_{zr}}{\rho F_e}\right)$, ker vzgon deluje navpično gor, zmanjšuje težo.

4) $\Lambda = \ln\frac{\lambda_m}{\lambda_{m+1}}$ $\qquad \lambda_m = S_0 e^{-\beta m\cdot t_0}$

5) $t_0' = t_0\left(\sqrt{1+k\frac{\rho_{zr}}{\rho F_e}}\right.$

6) ostali popravki

$$g = l_0\left(\frac{2\pi}{t_0}\right)^2\left(1+\frac{1}{2}\sin^2\left(\frac{\alpha}{2}\right)+\frac{2}{5}\left(\frac{r}{l_0}\right)^2-\frac{1m_z}{6m_k}+(1+k)\frac{\rho_{zr}}{\rho F_e}+\left(\frac{\Lambda}{2\pi}\right)^2\right)$$

Naloga

- S merjenjem nihajnega časa nihala določi težni pospešek najmanj na 0,1 % natančno

Potrebščine

- ~ nihalo, obešeno na stropu
- ~ meter z vrvico, pritrjenim na zid
- ~ vrvica,
- ~ žoga,
- ~ stoparica,*
- ~ elektronsko merilo,*
- ~ vžigalice*

Vaja 15 Težno nihalo

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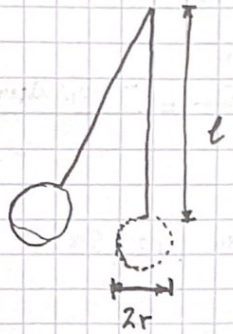

Matematičnemu nihalu se lahko približamo s številnimi popravki:

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Naloga

- S merjenjem nihajnega časa nihala določi težni pospešek vsaj na 0,1 % natančno

Potrebščine

- nihalo, obešeno na stropu
- merilo z zrakcem, pritrjenim na zid
- vrvica,
- žrela,
- štoparica, *
- kljunasto merilo, *
- vžigalice *

Meritve

$a = 4{,}3\ cm\ (\pm 0{,}05\ cm)$

$h = 6{,}8 - 1{,}5 = 5{,}3\ mm\ (\pm 0{,}01\ mm)$

$l = 219\ cm \pm 0{,}1\ cm$

$$R = \frac{-h^2 - \frac{a^2}{3}}{-2h} = \frac{-(5{,}3\ cm \pm 0{,}01\ cm)^2 - \frac{(4{,}3\ cm \pm 0{,}02\ cm)^2}{3}}{-2(5{,}3\ mm \pm 0{,}01\ mm} =$$

$$= 0{,}0607\ m\ (1 \pm 1{,}5\%) = 6{,}07\ cm\ (1 \pm 1{,}5\%)$$

- $\left(\frac{a}{\sqrt{3}}\right)^2 + (R-h)^2 = R^2$

$$\frac{a^2}{3} + R^2 - 2Rh + h^2 = R^2$$

$$-2Rh + h^2 + \frac{a^2}{3} = 0$$

$$R = \frac{-h^2 - \frac{a^2}{3}}{-2h}$$

$R =$

- $r_0 = 90\ mm \Rightarrow \alpha = 2{,}350 \pm 0{,}0$
- $r_{150} = 83{,}5\ mm \Rightarrow \alpha = 2{,}1850 \pm 0{,}0$
- $r_{100} = 86\ mm \Rightarrow \alpha = 2{,}250 \pm 0{,}0$

1. MERITEV

$t_5 = 14{,}84\ s$	$t_{70} = 14{,}81\ s$	$t_{135} = 15{,}05\ s$
$t_{10} = 14{,}48\ s$	$t_{75} = 15{,}42\ s$	$t_{140} = 14{,}81\ s$
$t_{15} = 15{,}28\ s$	$t_{80} = 14{,}35\ s$	$t_{145} = 14{,}89\ s$
$t_{20} = 14{,}81\ s$	$t_{85} = 14{,}74\ s$	$t_{150} = 14{,}88\ s$
$t_{25} = 14{,}75\ s$	$t_{90} = 15{,}08\ s$	
$t_{30} = 15{,}00\ s$	$t_{95} = 14{,}92\ s$	
$t_{35} = 14{,}94\ s$	$t_{100} = 14{,}65\ s$	
$t_{40} = 14{,}83\ s$	$t_{105} = 15{,}09\ s$	
$t_{45} = 14{,}87\ s$	$t_{110} = 14{,}95\ s$	
$t_{50} = 14{,}93\ s$	$t_{115} = 14{,}95\ s$	
$t_{55} = 14{,}59\ s$	$t_{120} = 14{,}69\ s$	
$t_{60} = 15{,}06\ s$	$t_{125} = 15{,}09\ s$	
$t_{65} = 14{,}90\ s$	$t_{130} = 14{,}73\ s$	

$\delta t \pm (0{,}1\ s + 0{,}225) \Rightarrow \delta t \pm 0{,}325\ s$

$\delta t_n = \frac{\delta t}{\sqrt{n}} = \frac{0{,}325}{\sqrt{150}} \pm 0{,}03\ s$

2. MERITEV

$\delta t \pm (0{,}1 + 225\,\mu s) \Rightarrow \delta t \pm 0{,}325\,s$

i	t [s]	i	t [s]	i	t [s]
1	$t_5 = 14{,}99\,s$	11	$t_{55} = 2'43{,}69\,s$	21	$t_{105} = 5'12{,}38\,s$
2	$t_{10} = 29{,}63\,s$	12	$t_{60} = 2'58{,}44\,s$	22	$t_{100} = 5'27{,}30\,s$
3	$t_{15} = 44{,}66\,s$	13	$t_{65} = 3'13{,}42\,s$	23	$t_{115} = 5'42{,}07\,s$
4	$t_{20} = 59{,}26\,s$	14	$t_{70} = 3'28{,}30\,s$	24	$t_{120} = 5'56{,}96\,s$
5	$t_{25} = 1'14{,}26\,s$	15	$t_{75} = 3'43{,}05\,s$	25	$t_{125} = 6'11{,}79\,s$
6	$t_{30} = 1'29{,}14\,s$	16	$t_{80} = 3'58{,}03\,s$	26	$t_{130} = 6'26{,}54\,s$
7	$t_{35} = 1'44{,}23\,s$	17	$t_{85} = 4'12{,}82\,s$	27	$t_{135} = 6'41{,}60\,s$
8	$t_{40} = 1'59{,}81\,s$	18	$t_{90} = 4'28{,}30\,s$	28	$t_{140} = 6'56{,}51\,s$
9	$t_{45} = 2'13{,}8\,s$	19	$t_{95} = 4'42{,}52\,s$	29	$t_{145} =$
10	$t_{50} = 2'28{,}81\,s$	20	$t_{100} = 4'57{,}45\,s$	30	$t_{150} =$

$\delta t_p = \frac{\delta t}{\sqrt{n}} = \frac{0{,}325}{\sqrt{140}} = \pm 0{,}03\,s$

$m_k = \frac{4\pi r^3}{3} \cdot \rho_{Fe} = \frac{4\pi (6{,}07\,cm\ (1 \pm 1{,}5\%))^3}{3} \cdot 7800\,\frac{kg}{m^3} = 7{,}3\,g\ (1 \pm 3\%)$

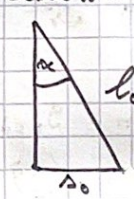
$m_z = \pi r_z^2 \cdot l \cdot \rho_{jekla} = 0{,}0308\,g\ (1 \pm 26\%)$

$r_k = 0{,}75\,mm \pm 0{,}05\,mm\ (1 \pm 13\%)$

Povprečne meritve 1) $\overline{T_1} = 2{,}97 \pm 0{,}11\,s$

2) $\overline{T_2} = 2{,}97 \pm 0{,}11\,s$

Odklon:



$\sin\alpha = \frac{s_0}{l_0}$

$\alpha = \arcsin\frac{s_0}{l_0} =$

$\alpha = \quad (1 \pm 1{,}6\%)$

$\alpha_0 = 2{,}35° \pm 0{,}001°$

$\alpha_{150} = 2{,}18° \pm 0{,}001°$

Logaritemski dekrement:

$\Lambda = \ln\frac{s_n}{s_{n+1}} =$

$s_n = s_0 \cdot e^{-\beta n T}$

$\beta = \frac{\ln\left(\frac{s_n}{s_0}\right)}{nT} =$

$\Lambda = \ln\frac{s_0 \cdot e^{-\beta n T}}{s_0 e^{-\beta(n+1)T}} = \ln e^{-\beta n T + \beta(n+1)T} = T\beta(n+1) - T\beta(n) =$

$= -0{,}49005\,(1 \pm 0{,}08\,\%)$

Gravitacijski pospešek

$$g = l_0 \cdot \left(\frac{2\pi}{T}\right)^2 \left(1 + \frac{1}{2}\sin^2\frac{\alpha}{2} + \frac{2}{5}\left(\frac{r}{l_0}\right)^2 - \frac{1}{6}\frac{m_2}{m_k} + \left(1+k\right)\frac{\rho_{Cu}}{\rho_{Fe}}\right)$$

$$g = 219\,cm\left(1 \pm 0{,}01cm\right)\cdot\left(\frac{2\pi}{2{,}97\,s\,(\pm 0{,}3325\,\%)}\right)^2\cdot\left[1 + \frac{1}{2}\sin^2\left(\frac{2{,}185^\circ\,(\pm 0{,}001^\circ)}{2}\right) \longrightarrow \right.$$

$$\left. - \frac{1}{6\cdot 9{,}4\,kg} + 1{,}6\cdot\frac{1{,}2}{7800}\right]$$

$$g = 219\,cm\left(1 \pm 0{,}0046\right)\cdot\left(\frac{2\pi}{2{,}97\,s\,(\pm 2{,}18\,\%)}\right)^2\cdot\left[1 + \frac{1}{2}\sin^2\left(1{,}0925^\circ\,(1 \pm 0{,}05\,\%)\right) \longrightarrow \right.$$

$$\left. + \frac{2}{5}\left(\frac{0{,}0615\,m\,(1 \pm 8{,}13\,\%)}{2{,}19\,m\,(1 \pm 0{,}5\,\%)}\right)^2 - \frac{1}{564\,kg} + 1{,}6\,\frac{1{,}2}{7800}\right] =$$

$$g = 9{,}89109 \approx 9{,}89\ m\cdot s^{-2} \pm 0{,}09\ m\,s^{-2}$$

Absolutna napaka g_1

$$\left(\delta_{l_0} + 2\delta_T\right)\cdot\frac{\frac{2}{5}\frac{r^2}{l_0}\left(2\delta_{l_0} + 2\delta_r\right) + \frac{m_2}{6m_k}\left(\delta_m + \delta_2\right) + \left(\frac{1}{2\pi}\right)\left(2\delta_\Delta + \delta_{\sin\alpha}\right)}{1 + \left(\frac{1}{2}\right)\sin^2\left(\frac{\alpha}{2}\right) + \frac{2}{5}\left(\frac{r}{l_0}\right)^2 - \frac{m_2}{6m_k} + \left(1+k\right)\frac{\rho_2}{\rho_{Fe}} + \left(\frac{\Delta}{2\pi}\right)^2} =$$

$$= 2{,}3\cdot 10^{-3}$$

$$g = 9{,}88879\ m\,s^{-2}$$

$$g_1 = l_0\left(\frac{2\pi}{T}\right)^2 = 2{,}19\,m\left(\frac{2\pi}{2{,}97\,s}\right)^2 = 9{,}801464\ m\,s^{-2}$$