Chapter 7

Advanced Encryption Standard (AES)

Chapter 7

Objectives

- To review a short history of AES
- To define the basic structure of AES
- To define the transformations used by AES
- To define the key expansion process
- To discuss different implementations

7-1 INTRODUCTION

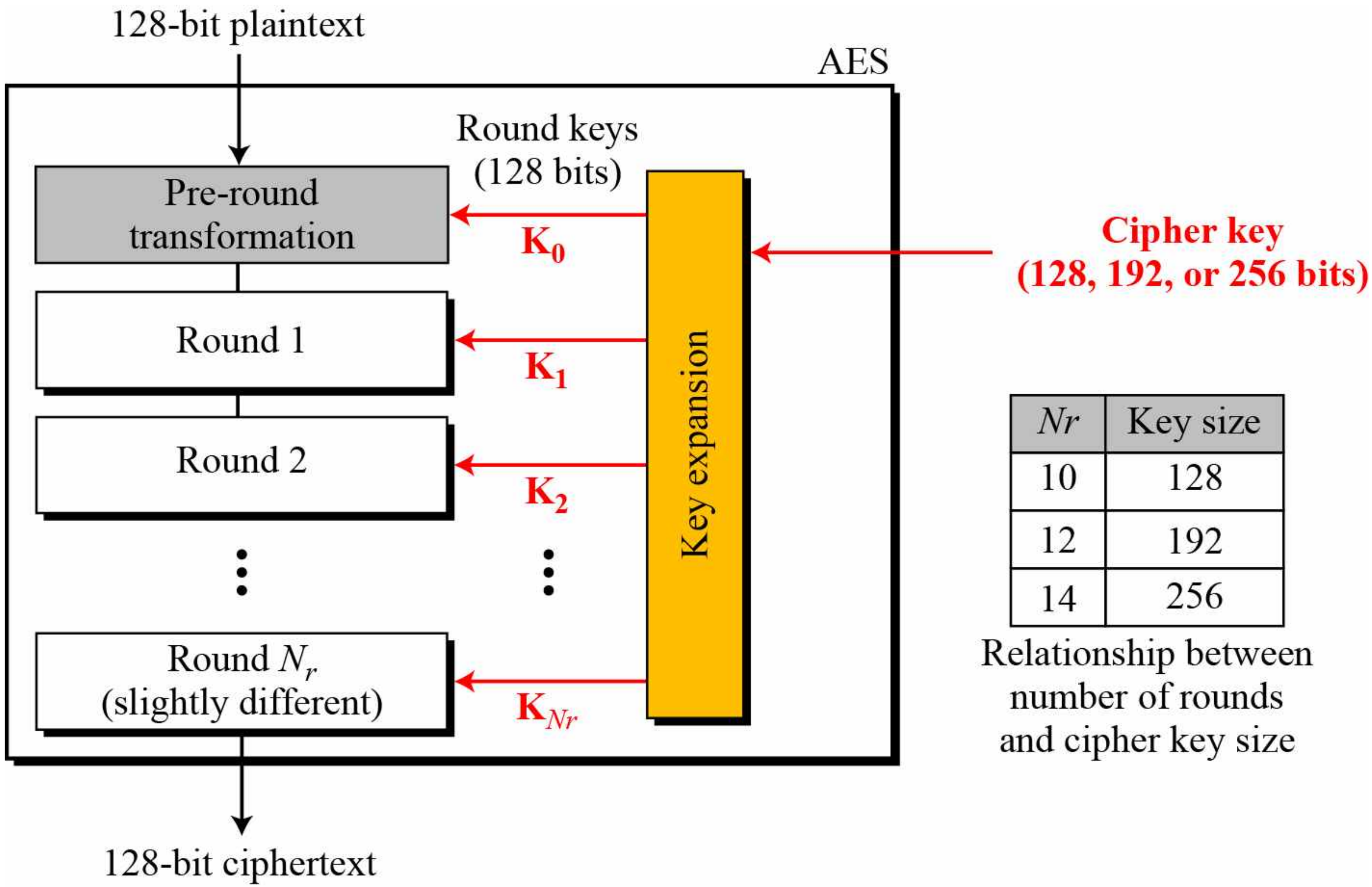
The Advanced Encryption Standard (AES) is a symmetric-key block cipher published as FIPS 197 by the National Institute of Standards and Technology in December 2001.

Candidates : 21 > 15 > 5 > 1 (Rijndael: security, cost, implementation)

AES has defined three versions, with 10, 12, and 14 rounds.

Each version uses a different cipher key size (128, 192, or 256), but the round keys are always 128 bits.

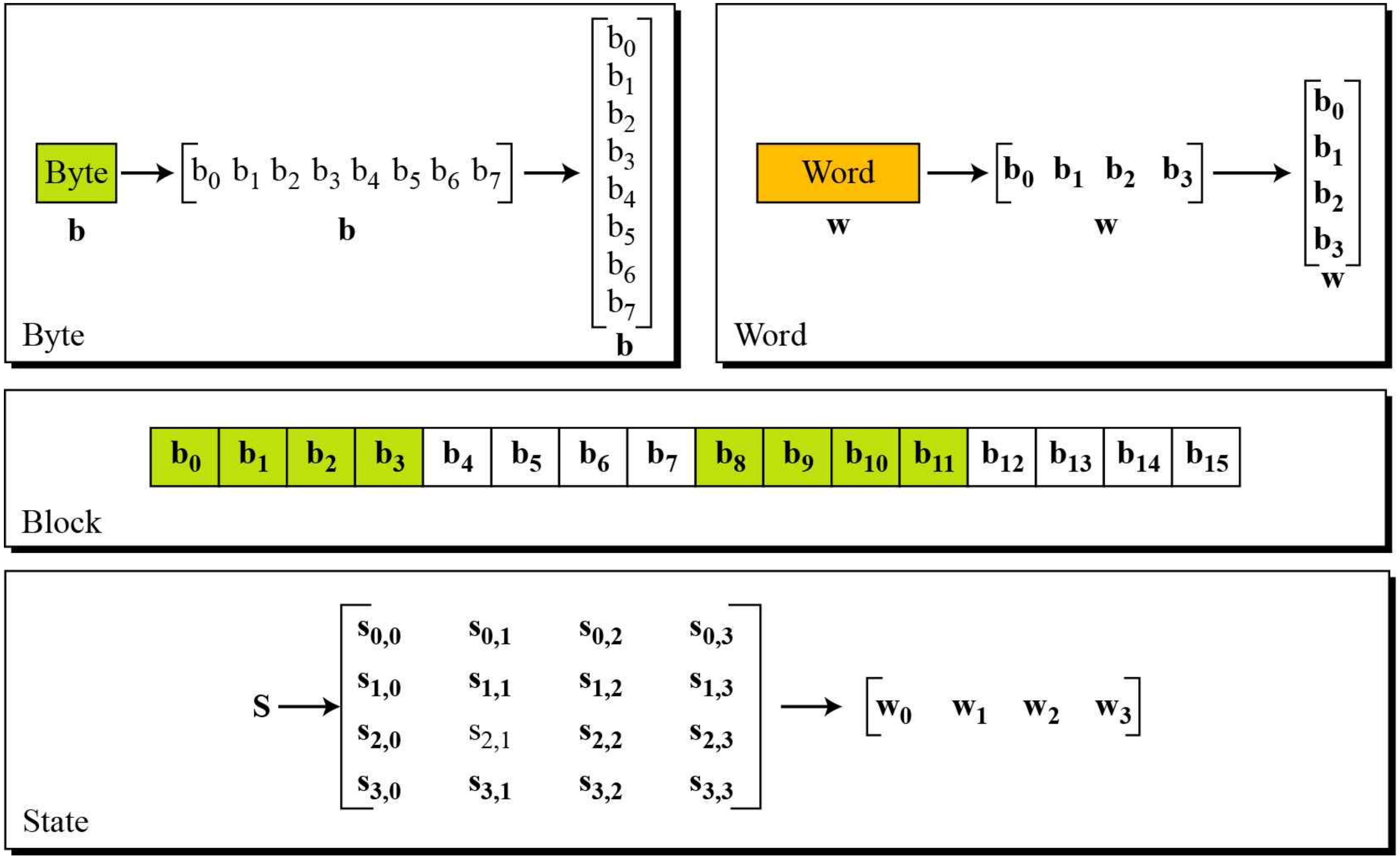
AES encryption cipher



Nr	Key size
10	128
12	192
14	256

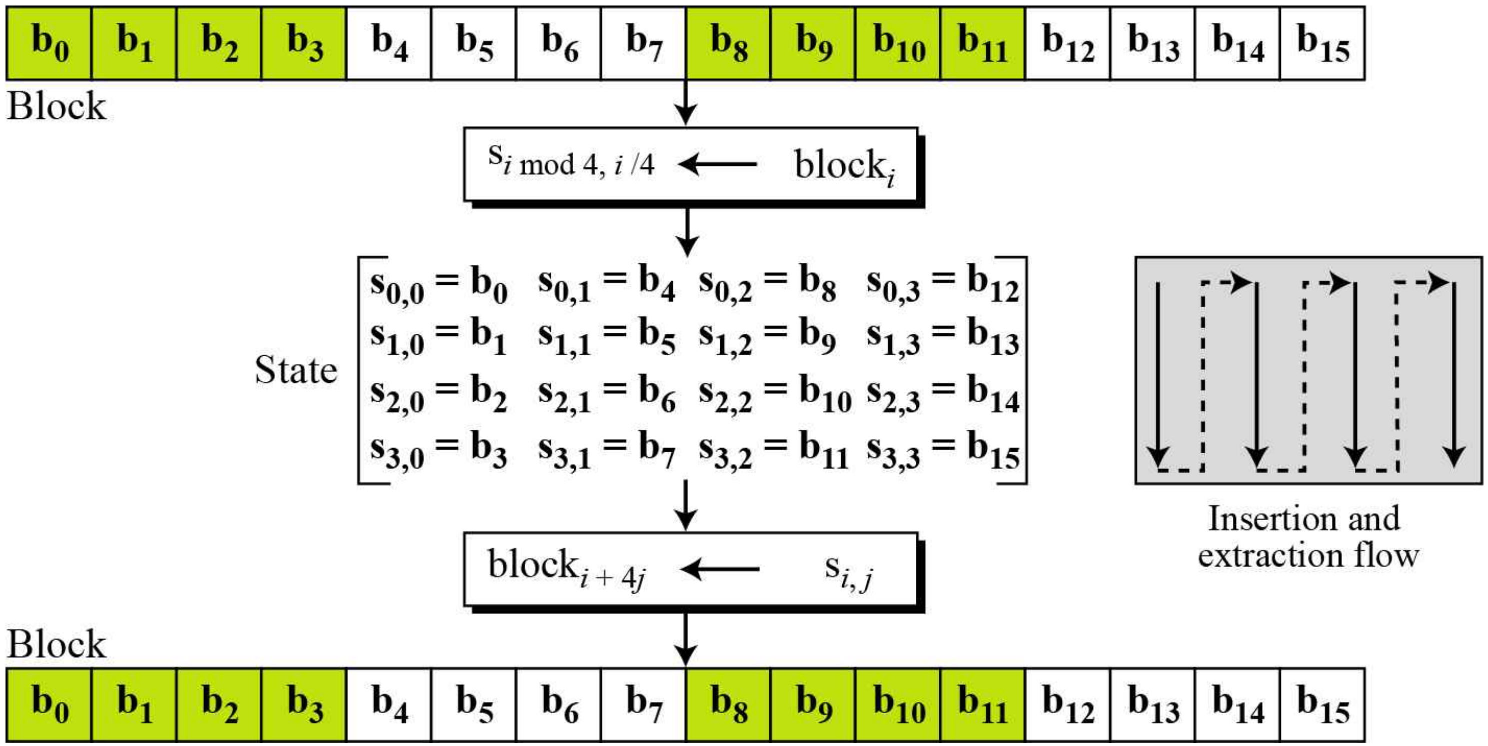
Relationship between number of rounds and cipher key size

7.1.4 Data Units.   Byte → Word → Block → State



7.1.4 Continue

Figure 6.3 Block-to-state and state-to-block transformation



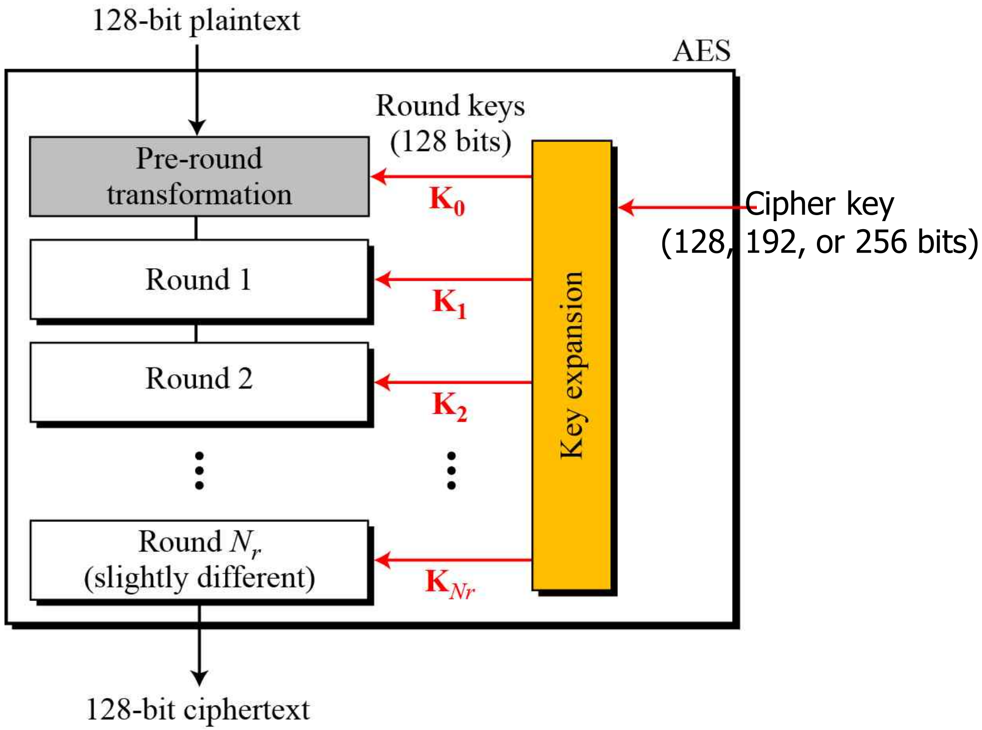
7.1.4 Continue

Figure 7.4 Changing plaintext to state

Text	A	E	S	U	S	E	S	A	M	A	T	R	I	X	Z	Z
Hexadecimal	00	04	12	14	12	04	12	00	0C	00	13	11	08	23	19	19

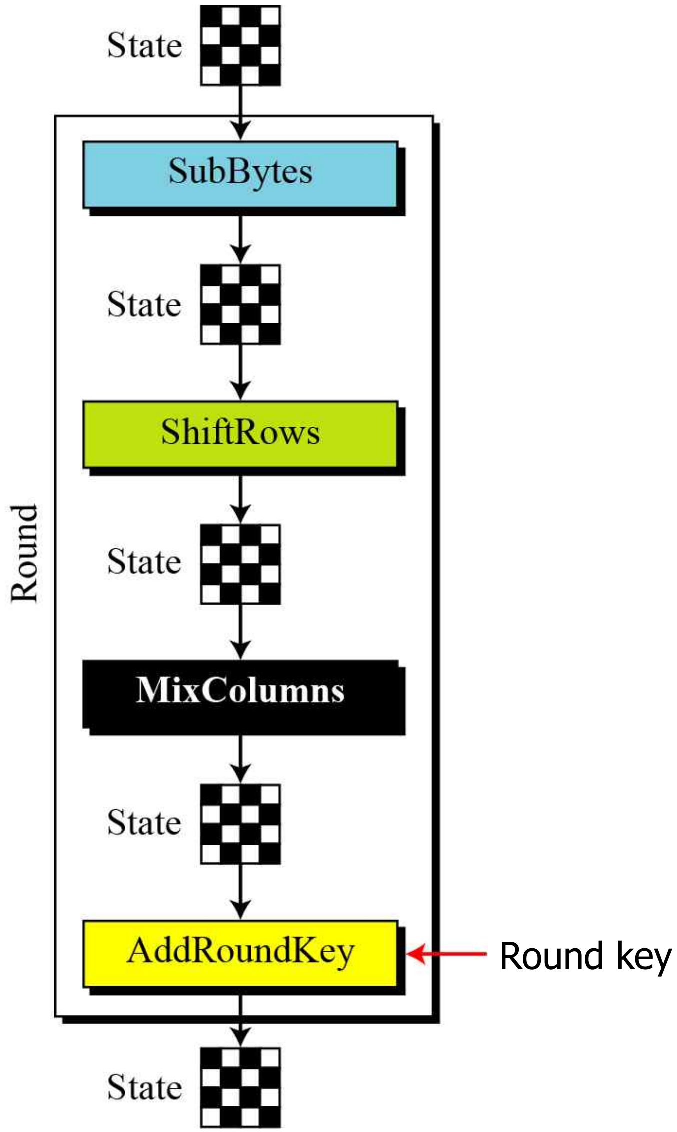
$$\begin{bmatrix} 00 & 12 & 0C & 08 \\ 04 & 04 & 00 & 23 \\ 12 & 12 & 13 & 19 \\ 14 & 00 & 11 & 19 \end{bmatrix} \text{State}$$

7.1.5 Structure of Each Round



Notes:

1. One AddRoundKey is applied before the first round.
2. The third transformation is missing in the last round.



7-2 TRANSFORMATIONS

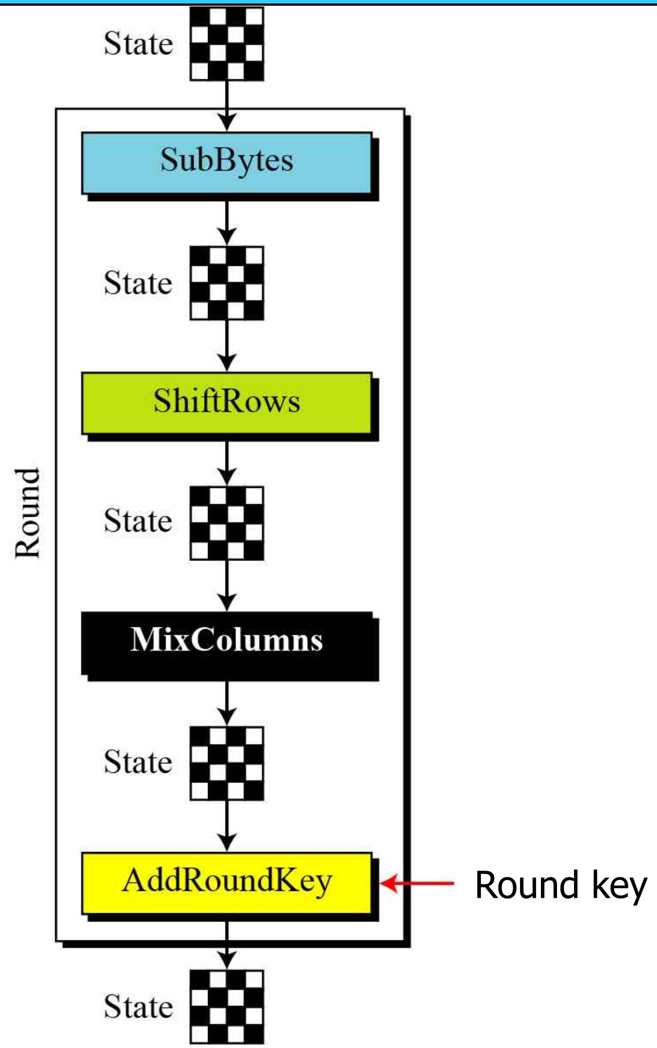
To provide security, AES uses **four types of transformations**:

Substitution

Permutation

Mixing

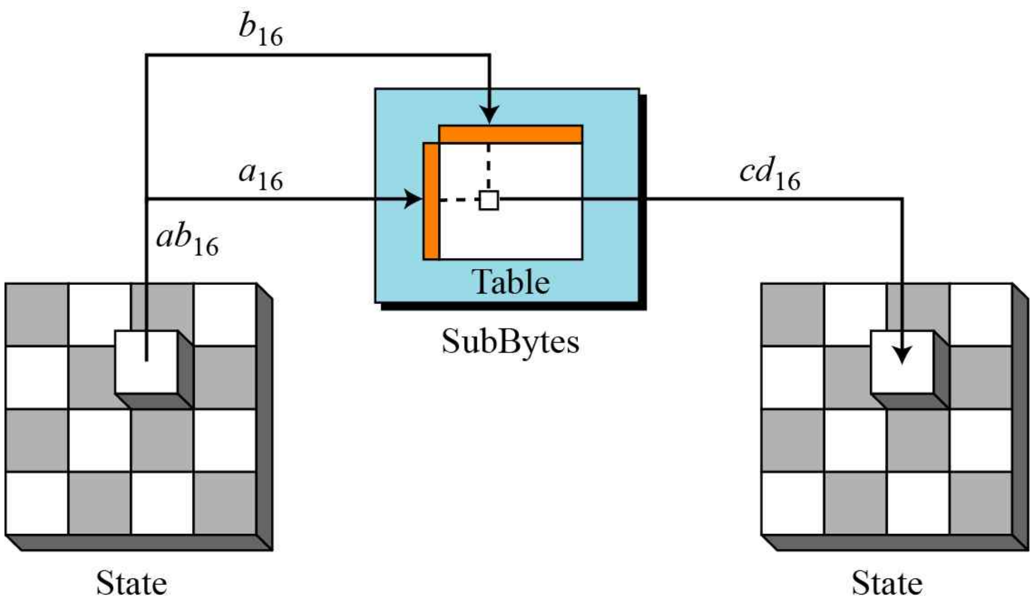
key-adding



7.2.1 Substitution

SubBytes

The first transformation, SubBytes, is used at the encryption site. To substitute a byte, we interpret the byte as two hexadecimal digits.



The SubBytes operation involves 16 independent byte-to-byte transformations.

7.2.1 Continue

Table 7.1 *SubBytes transformation table*

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*A*	*B*	*C*	*D*	*E*	*F*
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	CB	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

7.2.1 Continue

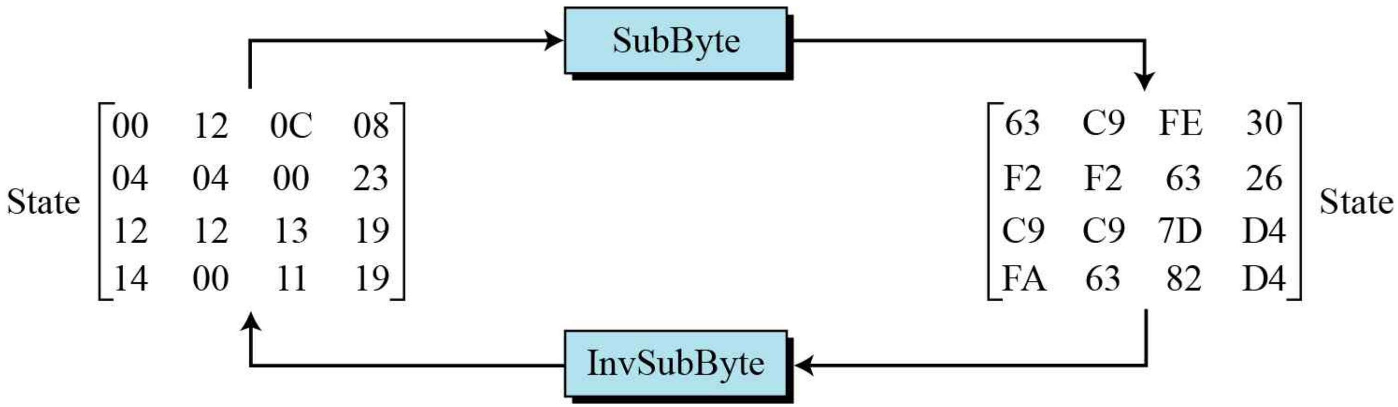
Table 7.2 *InvSubBytes transformation table*

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

7.2.1 Continue

Example 7.2

Figure 7.7 shows how a state is transformed using the SubBytes transformation. The figure also shows that the InvSubBytes transformation creates the original one. Note that if the two bytes have the same values, their transformation is also the same.



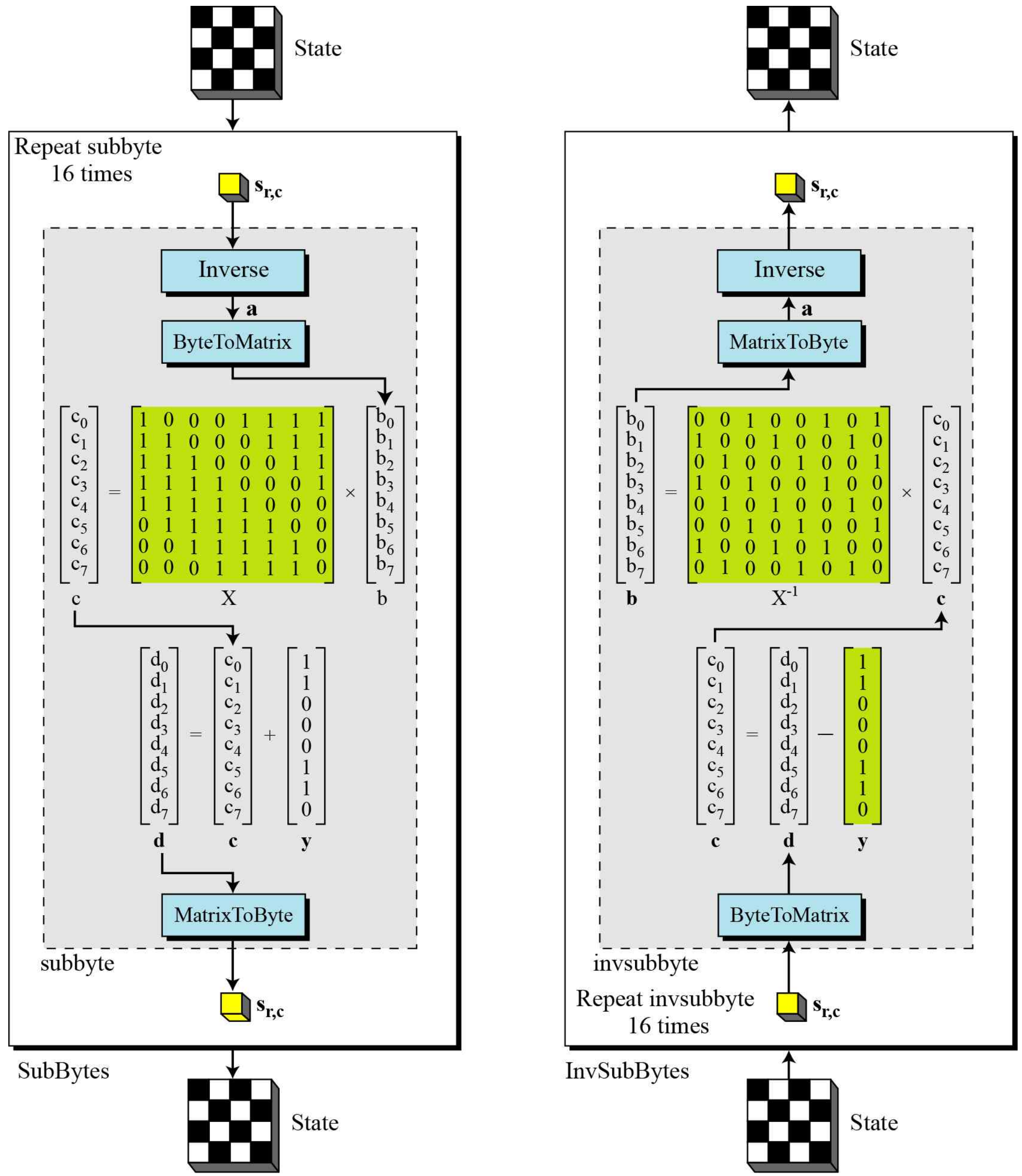
$$\text{subbyte:} \quad \rightarrow \quad \mathbf{d} = \mathbf{X}\,(s_{r,c})^{-1} \oplus \mathbf{y}$$

$$\text{invsubbyte:} \quad \rightarrow \quad [\mathbf{X}^{-1}(\mathbf{d} \oplus \mathbf{y})]^{-1} = [\mathbf{X}^{-1}(\mathbf{X}\,(s_{r,c})^{-1} \oplus \mathbf{y} \oplus \mathbf{y})]^{-1} = [(s_{r,c})^{-1}]^{-1} = s_{r,c}$$

Transformation Using the GF(2^8) Field

AES also defines the transformation algebraically using the GF(2^8) field with the irreducible polynomials

$$(x^8 + x^4 + x^3 + x + 1)$$



7.2.1 Continue

Example 7.3

Let us show how the byte 0C is transformed to FE by subbyte routine and transformed back to 0C by the invsubbyte routine.

1. *subbyte:*
   a. The multiplicative inverse of 0C in GF(2^8) field is B0, which means **b** is (10110000).
   b. Multiplying matrix **X** by this matrix results in **c** = (10011101)
   c. The result of XOR operation is **d** = (11111110), which is FE in hexadecimal.

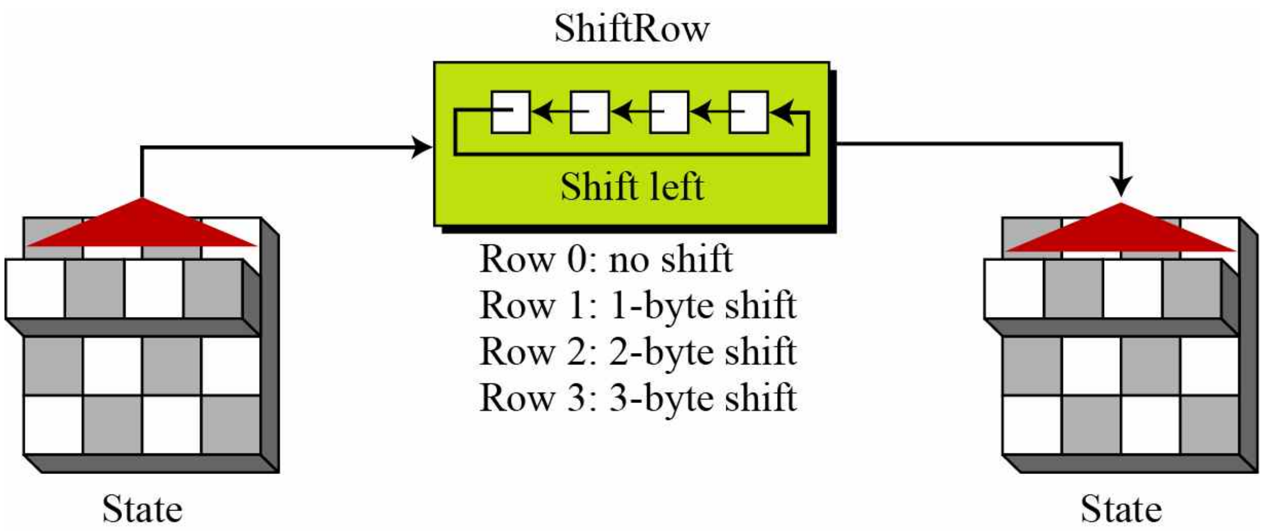
2. *invsubbyte:*
   a. The result of XOR operation is **c** = (10011101)
   b. The result of multiplying by matrix $\mathbf{X}^{-1}$ is (11010000) or B0
   c. The multiplicative inverse of B0 is 0C.

7.2.2 Permutation

Another transformation found in a round is shifting, which permutes the bytes.

ShiftRows

In the encryption, the transformation is called ShiftRows.



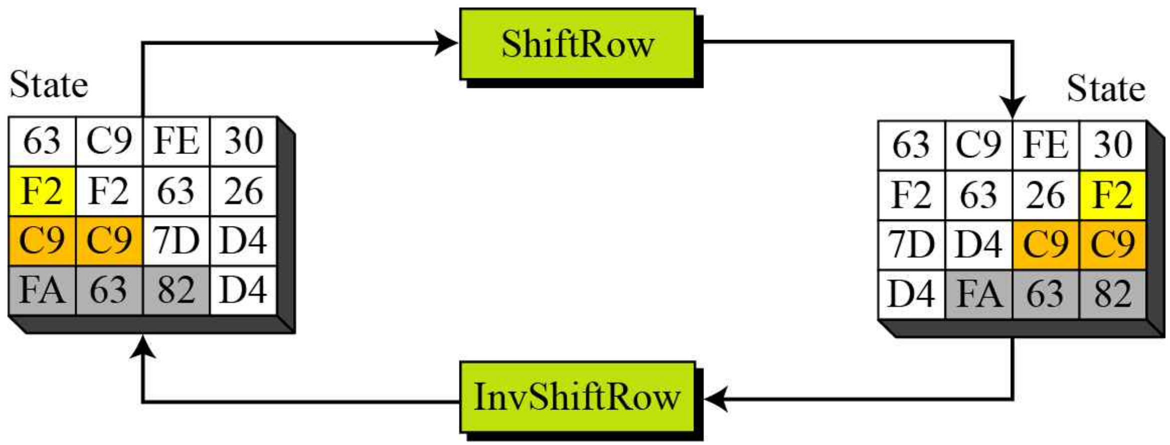
InvShiftRows

In the decryption, the transformation is called InvShiftRows and the shifting is to the right.

7.2.2 Continue

Example 7.4

Figure 7.10 shows how a state is transformed using ShiftRows transformation. The figure also shows that InvShiftRows transformation creates the original state.



7.2.3 Mixing

We need to mix bytes to provide diffusion at the bit level.

Figure 7.11 Mixing bytes using matrix multiplication

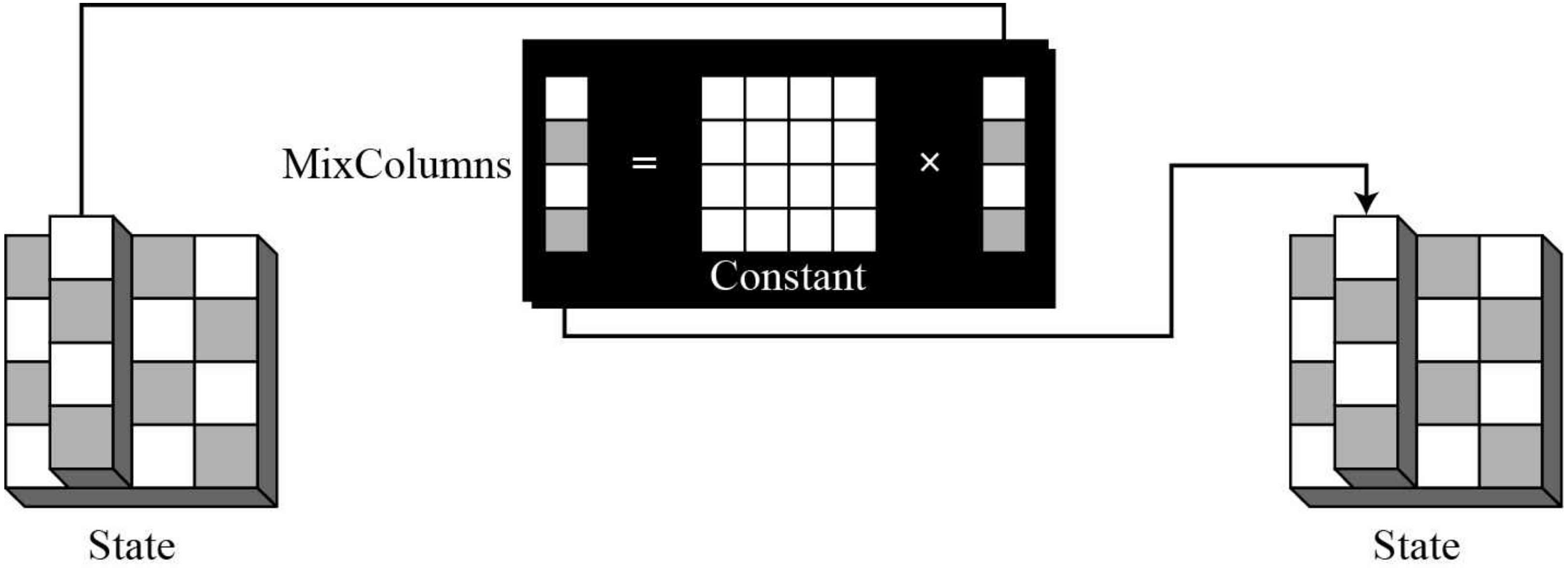
$$\begin{matrix} a\mathbf{x} + b\mathbf{y} + c\mathbf{z} + d\mathbf{t} \\ e\mathbf{x} + f\mathbf{y} + g\mathbf{z} + h\mathbf{t} \\ i\mathbf{x} + j\mathbf{y} + k\mathbf{z} + l\mathbf{t} \\ m\mathbf{x} + n\mathbf{y} + o\mathbf{z} + p\mathbf{t} \end{matrix} \rightarrow \underbrace{\begin{bmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \\ \blacksquare \end{bmatrix}}_{\text{New matrix}} = \underbrace{\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}}_{\text{Constant matrix}} \times \underbrace{\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \\ \mathbf{t} \end{bmatrix}}_{\text{Old matrix}}$$

$$\underbrace{\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix}}_{\text{C}} \xleftrightarrow{\text{Inverse}} \underbrace{\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix}}_{\text{C}^{-1}}$$

7.2.3 Continue

MixColumns

The MixColumns transformation operates at the column level; it transforms each column of the state to a new column.



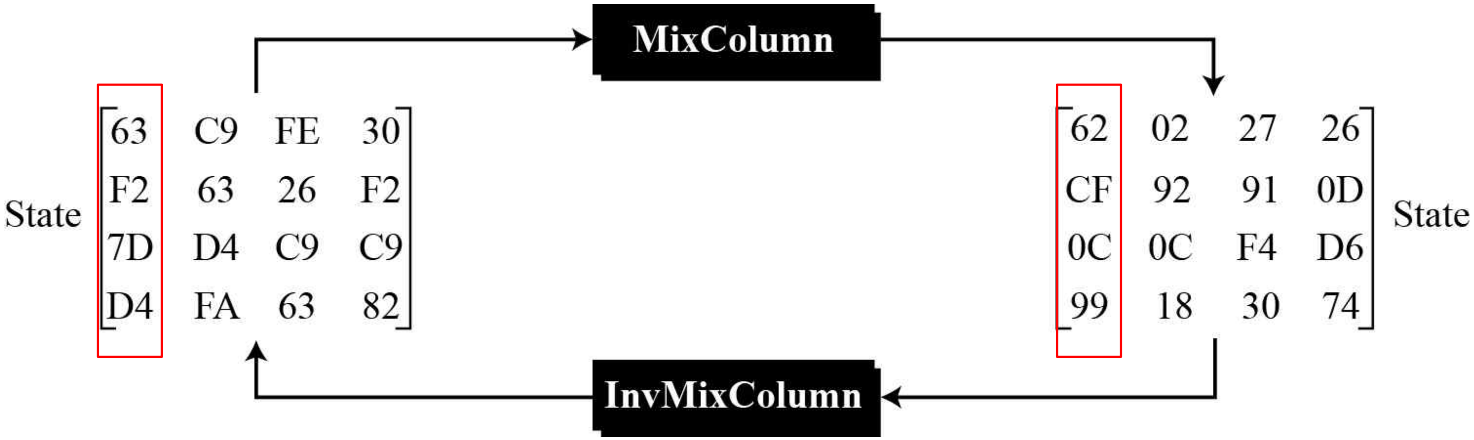
InvMixColumns

The InvMixColumns transformation is basically the same as the MixColumns transformation.

7.2.3 Continue

Example 7.5

Figure 7.14 shows how a state is transformed using the MixColumns transformation. The figure also shows that the InvMixColumns transformation creates the original one.

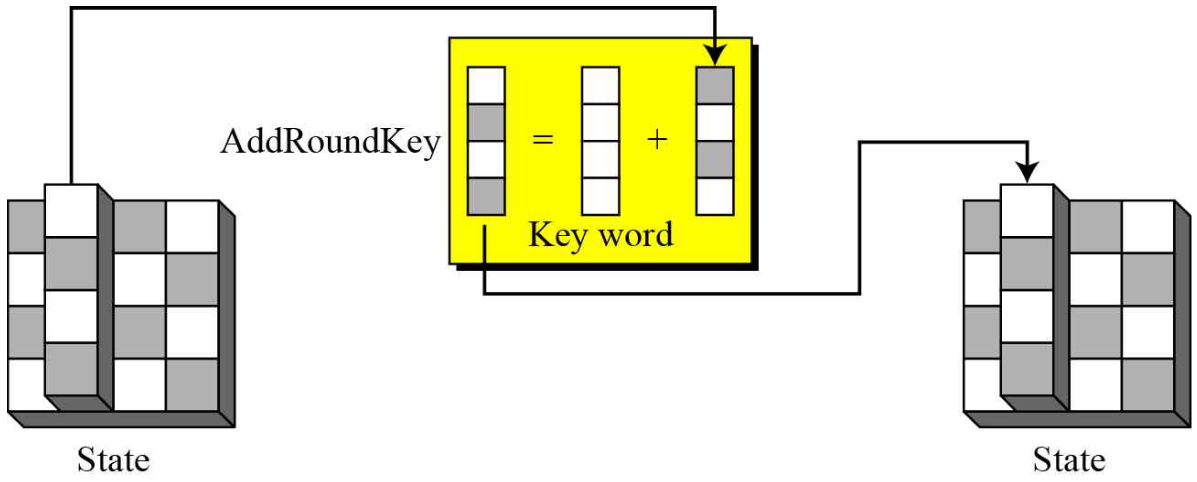


$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \cdot \begin{bmatrix} 63 \\ F2 \\ 7D \\ D4 \end{bmatrix} = \begin{bmatrix} 62 \\ CF \\ 0C \\ 99 \end{bmatrix}$$

7.2.4 Key Adding

AddRoundKey

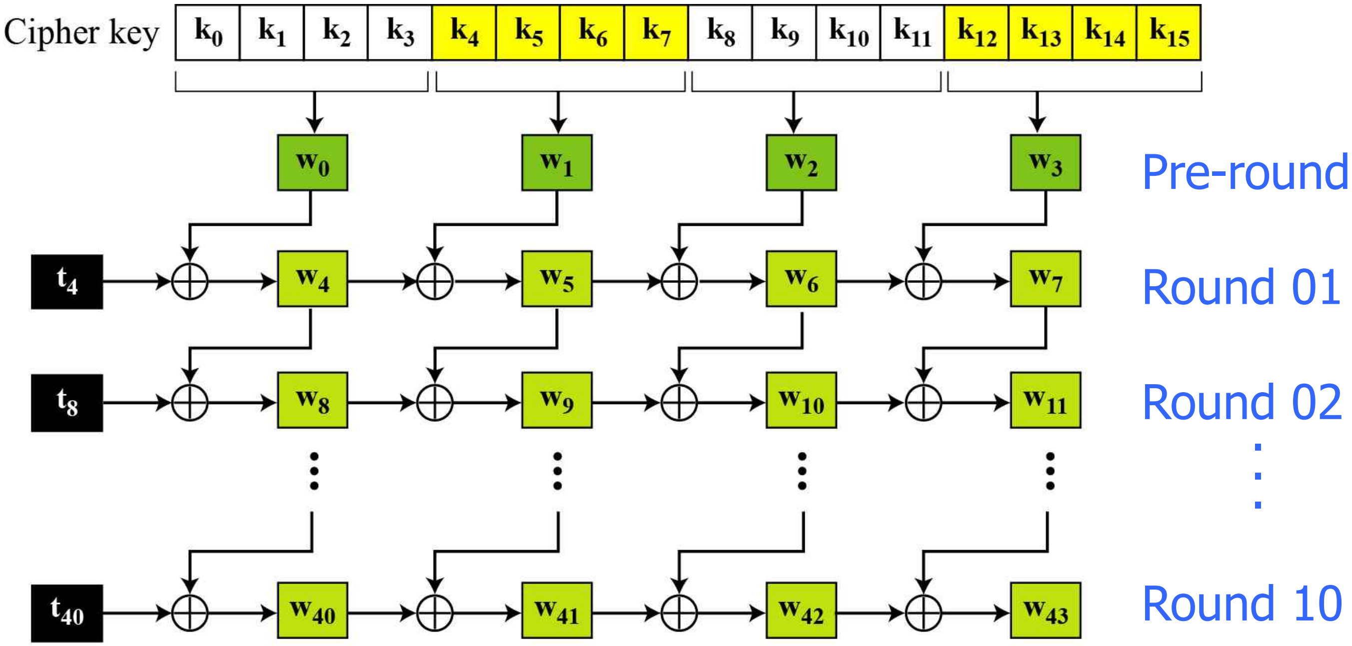
AddRoundKey proceeds one column at a time. AddRoundKey adds a round key word with each state column matrix.

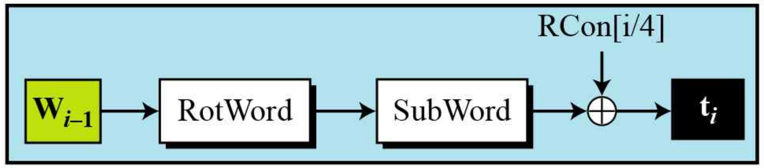


7-3 KEY EXPANSION

To create round keys for each round, AES uses a key-expansion process. If the number of rounds is N_r, the key-expansion routine creates $N_r + 1$ 128-bit round keys from one single 128-bit cipher key.

7.3.1 Key Expansion in AES-128





Making of t_i (temporary) words $i = 4\ N_r$

Table 7.4 *RCon constants*

Round	*Constant (RCon)*	*Round*	*Constant (RCon)*
1	(**01** 00 00 00)$_{16}$	6	(**20** 00 00 00)$_{16}$
2	(**02** 00 00 00)$_{16}$	7	(**40** 00 00 00)$_{16}$
3	(**04** 00 00 00)$_{16}$	8	(**80** 00 00 00)$_{16}$
4	(**08** 00 00 00)$_{16}$	9	(**1B** 00 00 00)$_{16}$
5	(**10** 00 00 00)$_{16}$	10	(**36** 00 00 00)$_{16}$

7.3.1 Continue

Example 7.6

Table 7.5 shows how the keys for each round are calculated assuming that the 128-bit cipher key agreed upon by Alice and Bob is (24 75 A2 B3 34 75 56 88 31 E2 12 00 13 AA 54 87)16.

Table 7.5 *Key expansion example*

Round	*Values of t's*	*First word in the round*	*Second word in the round*	*Third word in the round*	*Fourth word in the round*
—		w_{00} = 2475A2B3	w_{01} = 34755688	w_{02} = 31E21200	w_{03} = 13AA5487
1	AD20177D	w_{04} = 8955B5CE	w_{05} = BD20E346	w_{06} = 8CC2F146	w_{07} = 9F68A5C1
2	470678DB	w_{08} = CE53CD15	w_{09} = 73732E53	w_{10} = FFB1DF15	w_{11} = 60D97AD4
3	31DA48D0	w_{12} = FF8985C5	w_{13} = 8CFAAB96	w_{14} = 734B7483	w_{15} = 2475A2B3
4	47AB5B7D	w_{16} = B822deb8	w_{17} = 34D8752E	w_{18} = 479301AD	w_{19} = 54010FFA
5	6C762D20	w_{20} = D454F398	w_{21} = E08C86B6	w_{22} = A71F871B	w_{23} = F31E88E1
6	52C4F80D	w_{24} = 86900B95	w_{25} = 661C8D23	w_{26} = C1030A38	w_{27} = 321D82D9
7	E4133523	w_{28} = 62833EB6	w_{29} = 049FB395	w_{30} = C59CB9AD	w_{31} = F7813B74
8	8CE29268	w_{32} = EE61ACDE	w_{33} = EAFE1F4B	w_{34} = 2F62A6E6	w_{35} = D8E39D92
9	0A5E4F61	w_{36} = E43FE3BF	w_{37} = 0EC1FCF4	w_{38} = 21A35A12	w_{39} = F940C780
10	3FC6CD99	w_{40} = DBF92E26	w_{41} = D538D2D2	w_{42} = F49B88C0	w_{43} = 0DDB4F40

7.3.1 Continue

Example 7.7

Each round key in AES depends on the previous round key. The dependency, however, is nonlinear because of SubWord transformation. The addition of the round constants also guarantees that each round key will be different from the previous one.

7.3.1 Continue

Example 7.8

The two sets of round keys can be created from two cipher keys that are different only in one bit.

Cipher Key 1: 12 45 A2 A1 23 31 A4 A3 B2 CC A**A** 34 C2 BB 77 23
Cipher Key 2: 12 45 A2 A1 23 31 A4 A3 B2 CC A**B** 34 C2 BB 77 23

Table 7.6 *Comparing two sets of round keys*

R.	*Round keys for set 1*	*Round keys for set 2*	*B. D.*
—	1245A2A1 2331A4A3 B2CCAA34 C2BB7723	1245A2A1 2331A4A3 B2CCAB34 C2BB7723	01
1	F9B08484 DA812027 684D8A13 AAF6FD30	F9B08484 DA812027 684D8B13 AAF6FC30	02
2	B9E48028 6365A00F 0B282A1C A1DED72C	B9008028 6381A00F 0BCC2B1C A13AD72C	17
3	A0EAF11A C38F5115 C8A77B09 6979AC25	3D0EF11A 5E8F5115 55437A09 F479AD25	30
4	1E7BCEE3 DDF49FF6 1553E4FF 7C2A48DA	839BCEA5 DD149FB0 8857E5B9 7C2E489C	31
5	EB2999F3 36DD0605 238EE2FA 5FA4AA20	A2C910B5 7FDD8F05 F78A6ABC 8BA42220	34
6	82852E3C B4582839 97D6CAC3 C87260E3	CB5AA788 B487288D 430D4231 C8A96011	56
7	82553FD4 360D17ED A1DBDD2E 69A9BDCD	588A2560 EC0D0DED AF004FDC 67A92FCD	50
8	D12F822D E72295C0 46F948EE 2F50F523	0B9F98E5 E7929508 4892DAD4 2F3BF519	44
9	99C9A438 7EEB31F8 38127916 17428C35	F2794CF0 15EBD9F8 5D79032C 7242F635	51
10	83AD32C8 FD460330 C5547A26 D216F613	E83BDAB0 FDD00348 A0A90064 D2EBF651	52

7.3.1 Continue

Example 7.9

The concept of **weak keys**, as we discussed for DES in Chapter 6, does not apply to AES. Assume that all bits in the cipher key are 0s. The following shows the words for some rounds:

Pre-round:	00000000	00000000	00000000	00000000
Round 01:	62636363	62636363	62636363	62636363
Round 02:	9B9898C9	F9FBFBAA	9B9898C9	F9FBFBAA
Round 03:	90973450	696CCFFA	F2F45733	0B0FAC99
...	...	...	...	...
Round 10:	B4EF5BCB	3E92E211	23E951CF	6F8F188E

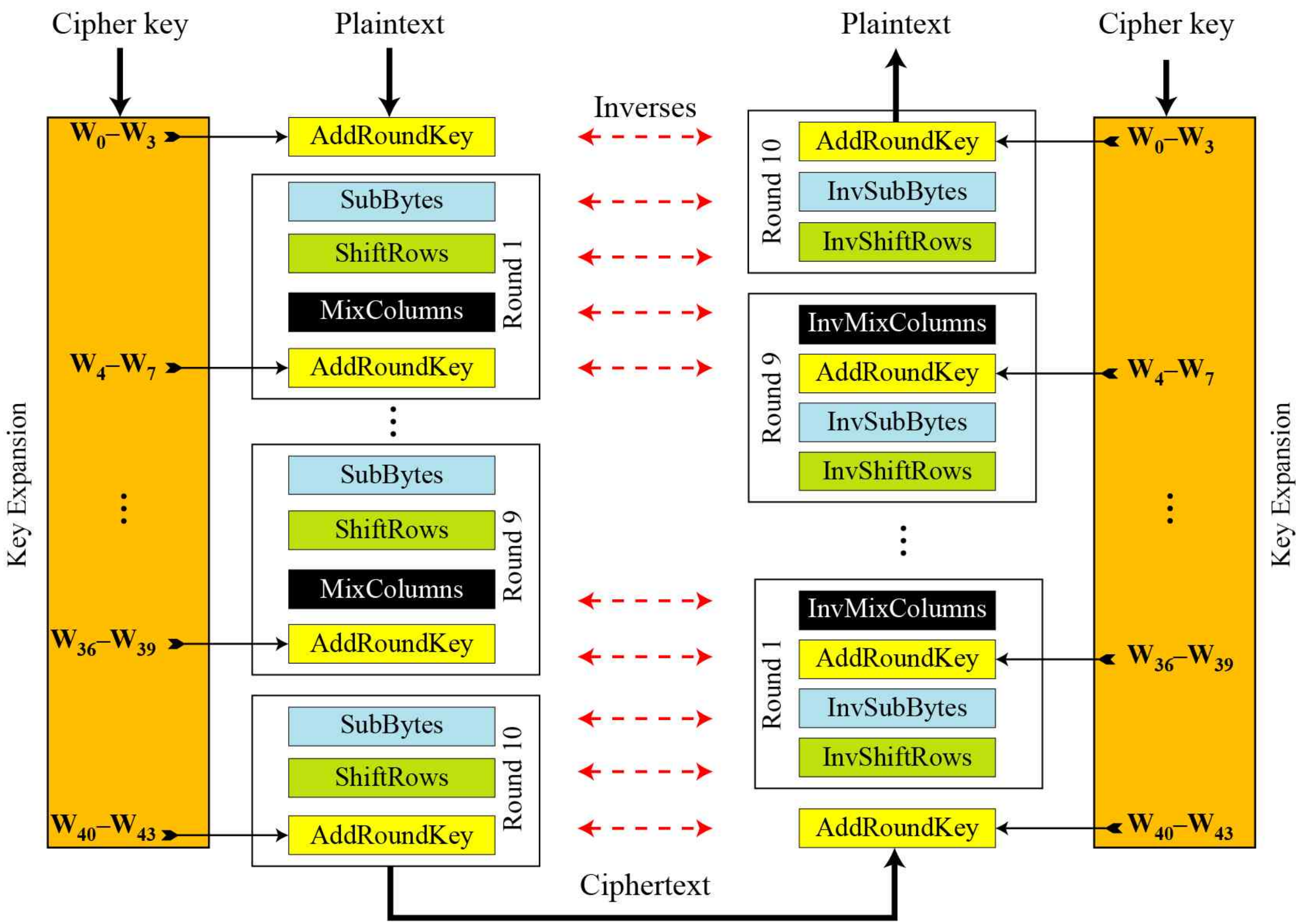
The words in the pre-round and the first round are all the same. In the second round, the first word matches with the third; the second word matches with the fourth. However, after the second round the pattern disappears; **every word is different**.

7-4 CIPHERS

AES uses four types of transformations for encryption and decryption. In the standard, the encryption algorithm is referred to as the cipher and the decryption algorithm as the inverse cipher.

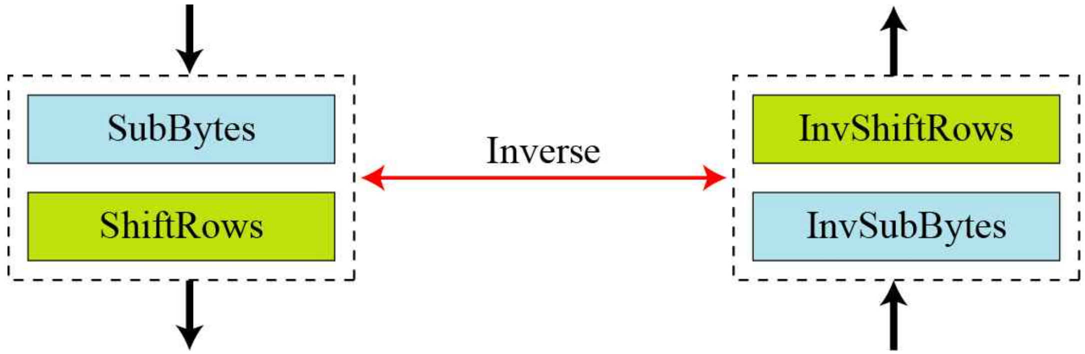
7.4.1 Original Design

Figure 7.17 Ciphers and inverse ciphers of the original design



7.4.2 Alternative Design

Invertibility of SubBytes and ShiftRows combinations



Invertibility of MixColumns and AddRoundKey combination

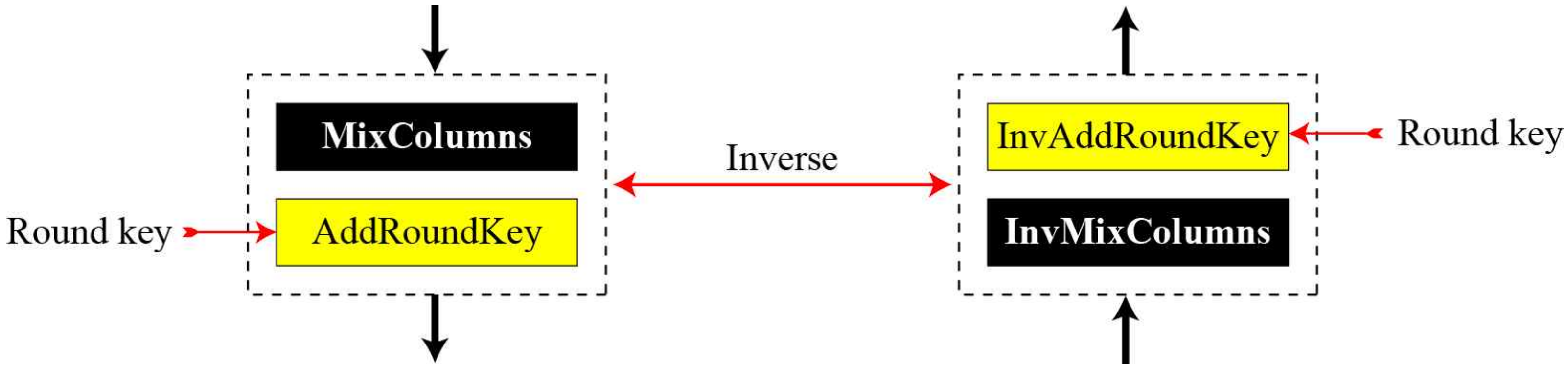
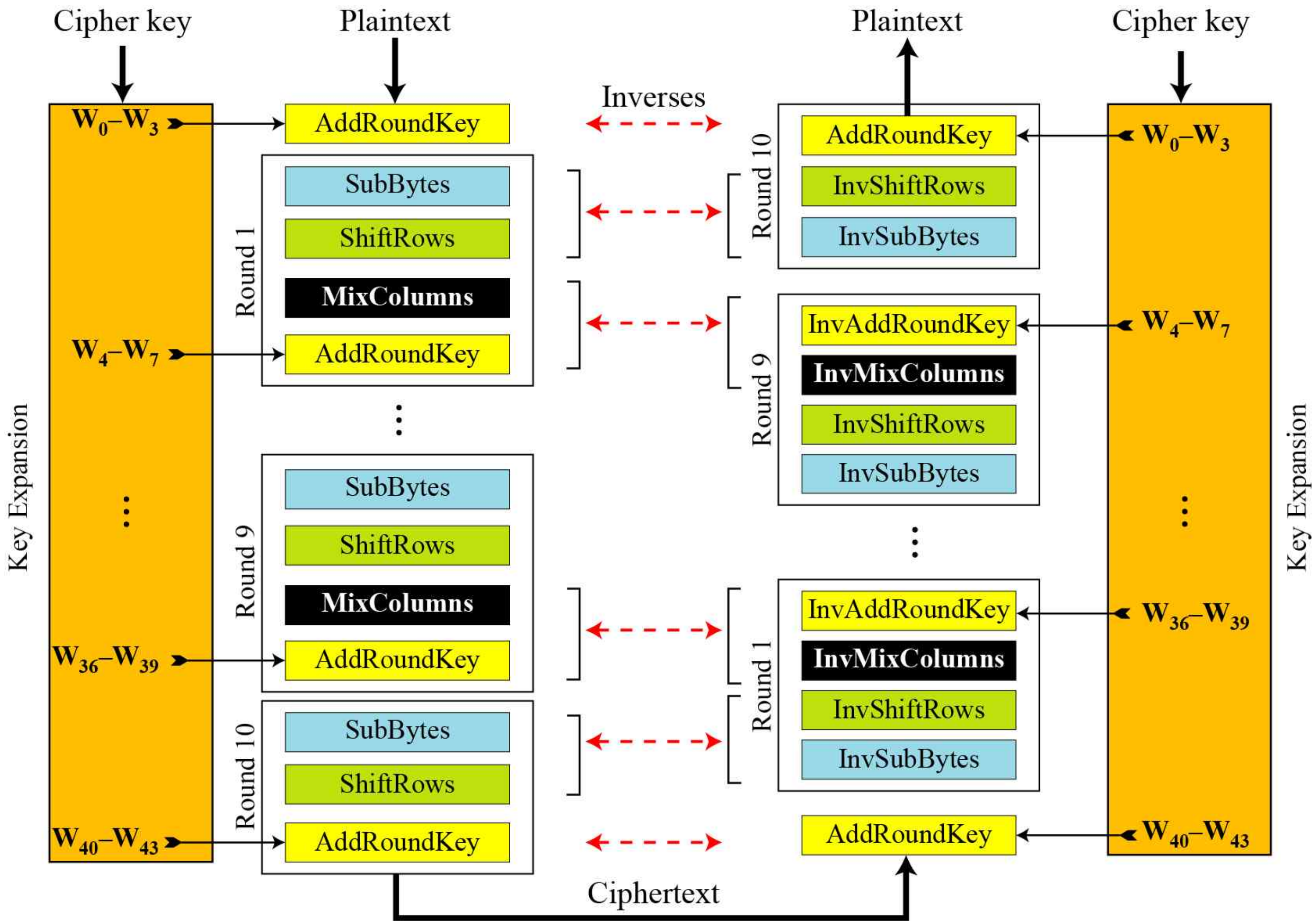


Figure 7.20 Cipher and reverse cipher in **alternate design**



7-5 Examples

Example 7.10 Randomly selected cipher key

Plaintext:	00	04	12	14	12	04	12	00	0C	00	13	11	08	23	19	19
Cipher Key:	24	75	A2	B3	34	75	56	88	31	E2	12	00	13	AA	54	87
Ciphertext:	BC	02	8B	D3	E0	E3	B1	95	55	0D	6D	FB	E6	F1	82	41

Example 7.12 When the plaintext is made of all 0s

Plaintext:	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
Cipher Key:	24	75	A2	B3	34	75	56	88	31	E2	12	00	13	AA	54	87
Ciphertext:	63	2C	D4	5E	5D	56	ED	B5	62	04	01	A0	AA	9C	2D	8D

Example 7.13 Check the avalanche effect

Plaintext 1:	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
Plaintext 2:	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	01
Ciphertext 1:	63	2C	D4	5E	5D	56	ED	B5	62	04	01	A0	AA	9C	2D	8D
Ciphertext 2:	26	F3	9B	BC	A1	9C	0F	B7	C7	2E	7E	30	63	92	73	13

Example 7.14 Using a cipher key in which all bits are 0s

Plaintext:	00	04	12	14	12	04	12	00	0c	00	13	11	08	23	19	19
Cipher Key:	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
Ciphertext:	5A	6F	4B	67	57	B7	A5	D2	C4	30	91	ED	64	9A	42	72

7-6 ANALYSIS OF AES

7.6.1 Security

AES was designed after DES. Most of the known attacks on DES were already tested on AES.

Brute-Force Attack

AES is definitely more secure than DES due to the larger-size key.

Statistical Attacks

Numerous tests have failed to do statistical analysis of the ciphertext.

Differential and Linear Attacks

There are no differential and linear attacks on AES as yet.

7.6.2 Implementation

AES can be implemented in software, hardware, and firmware. The implementation can use table lookup process or routines that use a well-defined algebraic structure.

7.6.3 Simplicity and Cost

The algorithms used in AES are so simple that they can be easily implemented using cheap processors and a minimum amount of memory.

Assignment 6, Canceled.

- Problems 17, 18, 23, 24, 26, 33 in chapter 7
- Problems 3~7 in chapter 8