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Less Computational Unscented Kalman Filter for Practical State Estimation of Small Scale Unmanned Helicopters

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Abstract—**This paper presents the unscented Kalman filter (UKF) with reduced simplex sigma-point for the navigation system in a small scale unmanned helicopter. UKF is widely applied to nonlinear systems. However, the disadvantage of traditional UKF is the high computational cost caused by the unscented transformation step. The computational cost is proportional to the number of the constructed sigma-points. Therefore a reduced simplex sigma-point selection is proposed to be practically applied for the sensor fusion on the unmanned helicopter. The simulation and experimental results verify the computational load reduction.**

Index Terms: Unscented Kalman Filter, Sigma points, Sensor fusion, Unmanned helicopter.

I. INTRODUCTION

Navigation is an essential element of unmanned helicopters to ensure the safe and efficient flights. There are multiple sensors in most navigation systems of the unmanned helicopters such as GPS, compass, IMU etc. to be fused together in order to obtain more accurate estimation of the current position, the attitude and the linear velocities of the unmanned helicopters [1]. Extended Kalman filter (EKF) provides the most common approach for sensor fusions of many nonlinear systems [2-4]. Since the navigation system of the unmanned helicopter is highly nonlinear, the linearization of the target system model in EKF would result in poor performance [5]. Hence UKF is a better solution that can avoid the estimation error caused by linearization using the unscented transformation (UT) method [6-8]. However, the unscented transformation introduces much more computational load to the system while the computation ability of most small scale helicopters is very limited. Therefore this paper proposes a less computational UKF practically applied to the state estimation of a small-scale unmanned helicopter for the use of the flight control system.

The unscented transformation involves transformation of nonlinear model at a set of the deterministically chosen points called the sigma points. Some researchers have recently developed different algorithms for how to select these sigma points in the UKF. The traditional UT was first introduced in [5] that required $2n + 1$ sigma points where n is the number of the augmented states. For the systems with a very high sampling rate, the large number of sigma points can cause significant computational burden. Ref. [9] reduced the number of sigma points to n+1 by choosing points matching the first two moments and minimizing the third order moments (skew). The resulting sigma points were called the minimal skew simplex sigma points. However, the radius of the bounding sphere of the points was $2^{n/2}$. Therefore, at even relatively low dimensions, there are potential problems with numerical stability. A new sigma point selection method called spherical simplex sigma point selection was further developed in [10] where a set of sigma points were chosen to lie on a hyper sphere. For an n-dimensional space, only n+2 points are required. The radius that bounds the points is proportional to $\sqrt{n}$ and the weight associated with each point is proportional to $1/n$. This paper is based on the spherical simplex sigma point selection method.

The EKF is firstly used in the estimation of the attitude of the helicopter in[11]. However, the performance was limited due to the information lost resulted from the first-order linearization. Then the UKF with the symmetric sigma points construction was applied in [12, 13] where UKF has better performance in the respect of the estimation precision. But their computational load is higher because more sigma points were constructed for the UT transformation. The reduced computational load is an important factor in improving the overall performance of controller especially for the small unmanned helicopter platform with the computational load constraints. The spherical simplex unscented Kalman filter has not been applied in the field of the helicopter navigation system. Hence this technique is proposed in this paper to reduce the number of sigma points in the UT transformation for lowering the computational cost.

The main contribution of this paper involves development and experimental implementation of the spherical simplex unscented Kalman filter for greatly reducing the computational cost with reasonable precision and stability in the state estimation of the navigation system for the small scale unmanned helicopter. This practical method is generally applicable to solving other complex navigation problems with high dimensions.

This paper is organized as follows: In Section II, modeling of the navigation system of the helicopter is

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presented. In Section III, the spherical simplex unscented Kalman filter is presented and applied on a small-scaled unmanned helicopter. Simulation and experiments are presented and discussed in Section IV. Conclusions are made in Section V.

II. Modeling of The Navigation System for The Helicopter

The GPS, compass and IMU are used in the navigation system of the unmanned helicopter in this paper. In order to estimate the attitude, the linear velocities and the position of the helicopter accurately, an IMU, a GPS and a compass are designed to be combined together as an example of the proposed sensor fusion technique. The angular velocities and the accelerometers from the IMU serve as the inputs for the navigation integration while the estimation of the attitude, the linear velocities and the position of the helicopter are the outputs for the use of the feedback loop of the flight controller. The position and the linear velocities from the GPS are treated as the measurements to be fused for more accurate state estimates of the helicopter.

The NED frame that is fixed to the tangential plane (at any point) of the earth-surface-ellipsoid serves as the navigation frame with three orthogonal axes (*x*, *y*, *z*), Fig. 1. The axis *x* always points to the north-pole of the earth and the axis z points downward to earth center. In the Earth-centered, Earth-fixed frame (ECEF), Fig. 1, the Z_{ECEF} axis is parallel and aligned with the direction of the Earth's rotation. In the equatorial plane (X_{ECEF}-Y_{ECEF}), the X_{ECEF} axis intersects the Greenwich meridian and the equator. The origin of the body frame is coincident with the helicopter gravity center while the axes (x_b , y_b , z_b) change the directions as the helicopter rotates, Fig. 2.

The angular velocities and accelerations from the IMU are with respect to the body frame, denoted as the vectors

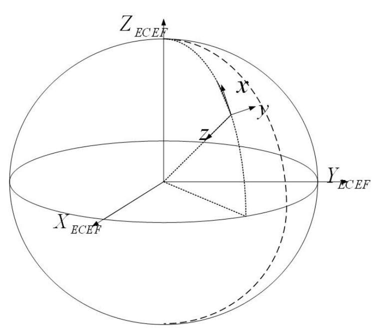


Fig.1. The navigation frame of the helicopter

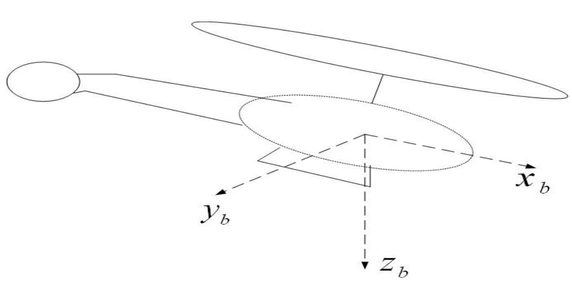


Fig. 2. The body frame of the helicopter.

$\omega_{ib} = \begin{bmatrix} p & q & r \end{bmatrix}^T$ and $a_{ib} = [a_x \quad a_x \quad a_x]^T$. The state vector of the navigation system is:

$$\begin{aligned} x &= [p \quad v \quad e]^T \\ &= [x \quad y \quad z \quad v_x \quad v_y \quad v_z \quad e_0 \quad e_1 \quad e_2 \quad e_3]^T \end{aligned} \tag{1}$$

where $p=[x \ \ y \ \ z]^T$ is the position vector in the END frame, $v=[v_x \ \ v_y \ \ v_z]^T$ is the linear velocity vector in the END frame and $e = [e_0 \quad e_1 \quad e_2 \quad e_3]^T$ is the unit quaternion of the attitude.

Hence the continues state equation of the navigation system for the helicopter is obtained as,

$$p = \dot{v} \tag{2}$$

$$\dot{v} = (T_{ib})(a_{ib} - a_{\bar{r}imu}) + [0 \quad 0 \quad 1]^T g \tag{3}$$

$$\dot{e} = -\frac{1}{2}\tilde{\Omega} e \tag{4}$$

where T_{ib} denotes the direction cosine matrix (DCM) transforming the vectors from the body frame to the NED reference frame. The DCM is given by,

$$T_{ib} = 2\begin{bmatrix} 0.5 - e_2^2 - e_3^2 & e_1e_2 - e_0e_3 & e_1e_3 + e_0e_2 \\ e_1e_2 + e_0e_3 & 0.5 - e_1^2 - e_3^2 & e_2e_3 - e_0e_1 \\ e_1e_3 - e_0e_2 & e_2e_3 + e_0e_1 & 0.5 - e_1^2 - e_2^2 \end{bmatrix}. \tag{5}$$

The g represents the gravity, and the $a_{\bar{r}imu}$ represents the component of the linear acceleration that caused by the difference between the mounting position of IMU and the center of gravity of the helicopter. The variable, $a_{\bar{r}imu}$, is given by

$$a_{\bar{r}imu} = \dot{\omega} \times \bar{r}_{imu} + \omega \times (\omega \times \bar{r}_{imu}) \tag{6}$$

where $\bar{r}_{imu}$ is the location vector of the IMU in the body frame (with the origin at the center of gravity). $\tilde{\Omega}$ is the 4×4 skew-symmetric matrix composed of the gyro measurements and the gyro bias $[p_b \quad q_b \quad r_b]$ that is obtained as,

$$\tilde{\Omega} = \begin{bmatrix} 0 & p - p_b & q - q_b & r - r_b \\ -(p - p_b) & 0 & -(r - r_b) & q - q_b \\ -(q - q_b) & r - r_b & 0 & -(p - p_b) \\ -(r - r_b) & -(q - q_b) & p - p_b & 0 \end{bmatrix} \tag{7}$$

Since the measurements from the GPS, z_{GPS}, are relative to the ECEF frame, these measurement data from the GPS should be transformed into the END frame. Then we have the new measurement vector, z_m, as

$$\begin{aligned} z_m &= h(x) \\ &= [x \quad y \quad z \quad v_x \quad v_y \quad v_z] \end{aligned} \tag{8}$$

III. Spherical Simplex Unscented Kalman Filter

The spherical simplex UKF is presented in this section based on the state equation and the measurement model of the navigation system in Section II. The recursion method is used here. The initial state estimate and the initial covariance are respectively defined as,

$$\hat{x}_0 = E\left[x_0\right] \tag{9}$$

$$P_0 = E\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right] \tag{10}$$

The UKF is based on the unscented transformation where the selected sigma points are transformed through the nonlinear process and the measurement models. The unscented transformation refers to the procedure to construct a set of the sigma points X_i and the associated weights W_i for the given mean $\hat{x}$ and the covariance P that satisfy:

$$\begin{cases} \sum_{i=1}^{n} W_i = 1 \\ \sum_{i=1}^{n} W_i X_i = \hat{x}_0 \\ \sum_{i=1}^{n} W_i \left(X_i - \hat{x}_0\right)\left(X_i - \hat{x}_0\right) = P_0 \end{cases} \tag{11}$$

where n is the number of sigma points. In the case of $\hat{x}_0$ = 0 and P= I, the spherical simplex sigma points are selected as Algorithm I according to Eq. (11). Hence the chosen $n+2$ sigma points are distributed on the hyper sphere that is located at the origin of the n-dimensional state set[10].

Algorithm I Spherical simplex sigma point selection

Choose： $0 \le W_0 \le 1$

Choose weight sequence ($i = 1 \cdots n+1$):

$$W_i = (1 - W_0)/\left(n+1\right)$$

Initialize vector sequence as:

$$X_0^1 = [0], X_1^1 = [-\frac{1}{\sqrt{2W_1}}], X_2^1 = [\frac{1}{\sqrt{2W_1}}]$$

The vector sequence for the n-dimensional state are expanded as,

$$X_i^n = \begin{cases} \begin{bmatrix} X_0^{n-1} \\ 0 \end{bmatrix} & i = 0 \\ \begin{bmatrix} X_i^{n-1} \\ -\frac{1}{\sqrt{n\left(n+1\right)W_1}} \end{bmatrix} & i = 1,..,n \\ \begin{bmatrix} 0_n \\ \frac{n}{\sqrt{n\left(n+1\right)W_1}} \end{bmatrix} & i = n+1 \end{cases}$$

These sigma points determined by Algorithm I are defined as the initial values of the constructed sigma points, X_{init}, e.g., $X_{init} = \left\{X_0^{n}, \ \cdots \ X_i^{n}, \ \cdots \ X_{n+1}^{n}\right\}$.

So when $\hat{x}_{k-1}$ (the state estimate at the time k-1) is the center of the hyper sphere, sigma points are constructed as the following,

$$X_{k-1,i} = \hat{x}_{k-1} + (\sqrt{P_{k-1}})^T X_{init}(i)\,. \tag{12}$$

Then the sigma points are passed through the nonlinear process model:

$$X^*_{k|k-1,i} = f\left[X_{k-1,i}, u_{k-1}\right]. \tag{13}$$

Hence the mean is predicted by the following equation as:

$$\hat{x}_k^- = \sum_{i=0}^{n+1} W_i X^*_{k|k-1,i}\,. \tag{14}$$

The state covariance is then approximated by

$$P_k^- = \sum_{i=0}^{n+1} W_i \left[X^*_{k|k-1,i} - \hat{x}_k^-\right]\left[X^*_{k|k-1,i} - \hat{x}_k^-\right]^T + Q \tag{15}$$

where Q is the process noise mean. This prediction is then corrected using the current measurement.

Passing the sigma points through the measurement model yields a transformed set of sigma points according to:

$$Z_{k|k-1,i} = h[X^*_{k|k-1,i}] \tag{16}$$

The predicted measurement mean is therefore represented as:

$$\hat{z}_k^- = \sum_{i=0}^{n+1} W_i Z_{k|k-1,i}\,. \tag{17}$$

Given the measurement noise covariance mean R_{meas}, the observation covariance and the state-observation correlation matrices are determined as,

$$P_{z_k z_k} = \sum_{i=0}^{n+1} W_i \left[Z_{k|k-1,i} - \hat{z}_k^-\right]\left[Z_{k|k-1,i} - \hat{z}_k^-\right]^T + R_{meas} \tag{18}$$

$$P_{x_k z_k} = \sum_{i=0}^{n+1} W_i \left[X^*_{k|k-1,i} - \hat{x}_k^-\right]\left[Z_{k|k-1,i} - \hat{z}_k^-\right]^T \tag{19}$$

Finally the Kalman gain is computed and the final estimations of the states and the corresponding covariance are derived as,

$$K_k = P_{x_k z_k} P_{z_k z_k}^{-1} \tag{20}$$

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - \hat{z}_k^-) \tag{21}$$

$$P_k = P_k^- - K_k P_{z_k z_k} K_k^T \tag{22}$$

IV. Simulation and Experiments

A. Methods and Procedures

The spherical simplex unscented Kalman filter for the navigation system of the helicopter was simulated in MATLAB. The reference attitude, position and linear velocities were given. The process white noise and the measurement white noise were added to the ideal model for simulating the uncertainties in the actual system. In order to evaluate the performance of the proposed method for the navigation system with different level complexities, simulations were divided into two cases. In Case 1, only attitude is estimated where the state equation of the navigation system is four-dimensional, i.e., Eq. (4). In the Case 2, the full state equation with ten dimensions, i.e., Eq. (2)-(4) was adopted.

The experimental platform is the helicopter model JR 260, Fig.3. This helicopter is 1.35 meter long. Its weight is 5 kg and the diameter of the main rotor is 1.5m. The processor for the experiment is LPC2368 running at 72MHz. The NAV440 from Crossbow Technology is used as the INS-GPS unit that mounted on the bottom of the fuselage. The output of NAV 400 is taken as the reference because of its high accuracy. Note that this setup is only for the experiments to verify the proposed method. Practically, this high precision combo is not properly applied because of its heavy weight. Most small scale unmanned helicopters have very low payload and most of the payload is expected to carry more batteries or gas for longer flight. Instead, the light inertial and GPS sensors are usually independently mounted on the helicopter platform and the proposed algorithm is applied to fuse them to get the state estimation for use of the flight controller. The full dimensional model of the navigation system (Case 2) is applied throughout the experiments.

The performances were judged based upon accuracy and the time spent for the whole process. Since the tests were carried out under the same circumstances every time, the assumption is made that the time spent by the filter algorithm is directly related to the computational load. The root mean squared error (RMSE) was used to determine the accuracy of the estimation. The RMSE is defined by:

$$\mathrm{RMSE}(\hat{x}) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\hat{x}_k - x_k\right)^2} \tag{23}$$

The following two algorithms will be compared with the proposed technique (SPSGUKF) in the experiments:

- UKF, with 2n + 1 sigma points (traditional UKF);
- Square-root implementation of the UKF (SRUKF).

Square root UKF (SRUKF) can increase the accuracy of the estimation in theory. Since our main concern is the computational load and the accuracy, this type of the filter is chosen to compare with the traditional UKF and the SPSGUKF.

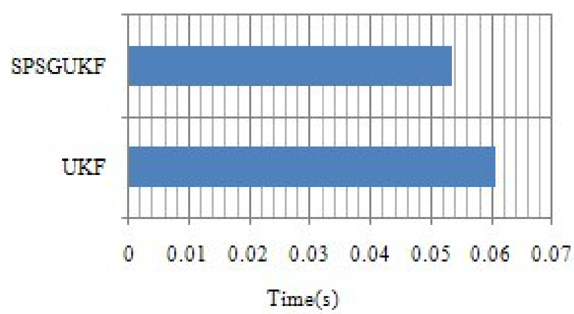
Fig.3 The unmanned helicopter used for the experiments.

B. Results and Discussion

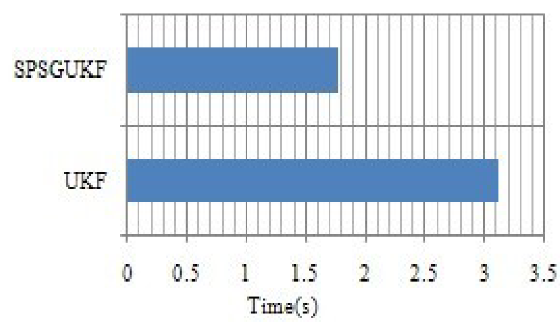
The computational load comparisons in Case 1 and Case 2 are shown in Fig.4 (a)-(b). The simulation results show that the proposed method (SPSGUKF) is slightly faster than the traditional UKF in Case 1 while the computation load is greatly reduced in Case 2 with the same sampling time 0.01 s. According to the simulation results, the traditional UKF has more computational load because of more steps in construction of the sigma points. The time reduction of the proposed method is 43% in average for all the tests in Case 2 compared with the traditional UKF. It is concluded that the proposed SPSGUKF has much more advantage in computational speed if the system is more complex and higher dimensional.

The accuracy comparison in the simulation for Case 2, Fig. 5, shows that the estimation accuracy of the proposed method is slightly lost compared with the UKF.

The computational load comparison in the experiment is



(a) Case 1: For the four dimensional system state equation



(b) Case 2: For the ten dimensional system state equation
Fig.4 The computational load comparison in the simulation

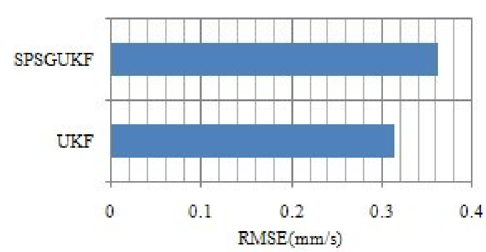


Fig.5. The north-velocity accuracy comparison in the simulation for Case 2.

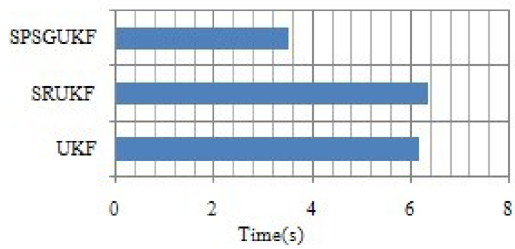


Fig.6 The computational load comparison in the experiment

shown in Fig. 6. The experiment results are pretty consistent with the simulations. The proposed method only spent 3.5 seconds for the whole estimation process while the UKF spent 6.2 seconds. The computation time is almost 50% reduced. This amount of time reduction is significant for improvement of the implementation ability of the small-scale unmanned helicopters.

The RMSE of the attitudes, the linear velocities and the positions in the experiments are shown in Table 1. According to Table 1, we can see that all the estimation errors are within the reasonable ranges for the helicopters. The velocity estimation in the North direction is plotted in the Fig. 7 as a example where the performance of the proposed algorithm is compared with the no-filter case. The velocity estimation errors in the North direction are presented in Fig. 8 using the high precision IMU as the reference. According to Fig.8, the SPSGUKF is less accurate than the traditional UKF but still applicable for the unmanned helicopters.

This significant reduction in the computational load can greatly improve the performance of all the low-power embedded computer systems. The future work, therefore, will focus on more advanced navigation applications of the proposed algorithm to fully take advantage of its fast characteristic such as Simultaneous Localization and Mapping (SLAM).

V. CONCLUSIONS

In this paper, we have presented the Unscented Kalman filter with spherical simplex sigma points (SPSGUKF) and its implementation on the high dimensional navigation system of small-scale unmanned helicopters. Through the comparison of three kinds of UKF in the simulations and experiments, the proposed algorithm has been verified in the navigation system with much less computational cost while keeping the accuracy acceptable. This method is generally applicable to other high dimensional navigation systems. Hence, applications for solving higher dimensional navigation problems such as SLAM will be the focus of the future work.

Table.1 The RSME comparisons in the three different algorithms

		UKF	SRUKF	SPSGUKF
Position (m)	North	0.00016	0.00015	0.00053
	East	0.00034	0.00033	0.00074
	Down	3.42E-06	3.42E-06	3.57E-05
Velocity (m/s)	North	0.00025	0.00026	0.00115
	East	0.0003	0.0004	0.0010
	Down	0.0010	0.0011	0.0083
Attitude (rad)	Pitch	0.0028	0.0024	0.0038
	Roll	0.0058	0.0068	0.0078
	Yaw	0.0069	0.0079	0.0089

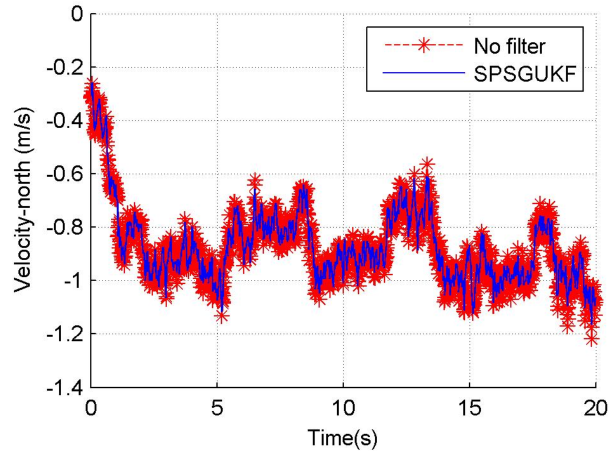


Fig.7 The velocity comparison in north direction

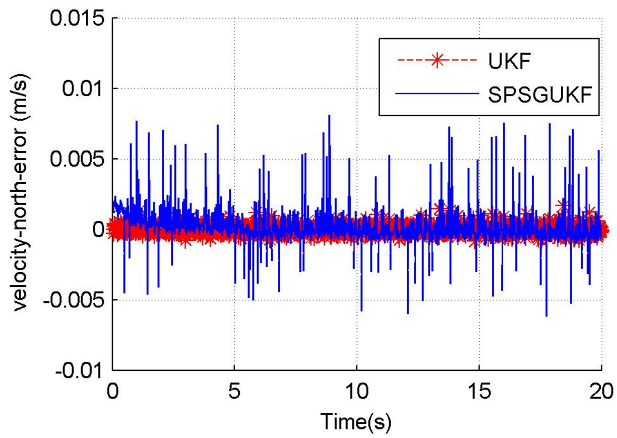


Fig.8 The velocity error comparison in north direction

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