Proceedings of the 2009 IEEE
International Conference on Information and Automation
June 22 -25, 2009, Zhuhai/Macau, China

New Method for Kinematic Analysis of a Hybrid Manipulator

Jun Shi, Xiaorui Zhu, *Member*, Zexiang Li, *Fellow, IEEE*

***Abstract*—This paper presents a new method to analyze the kinematics of a widely-used commercial hybrid manipulator IRB260 from the ABB Company. This manipulator includes two parallel four-bar mechanisms that make the kinematics and dynamic analysis complicated. In this paper, the rigorous analytical method is totally based on the exponential coordinate description and the screw theory. This method has created the virtual open-chain manipulator that is equivalent to the hybrid manipulator. Then the kinematics can be solved using the routine steps for the virtual open-chain manipulators. The simulation results show that this new method results in the correct kinematics of this hybrid manipulator.**

Index Terms: Hybrid, Double four-bar, Kinematics, Jacobian, Screw theory.

I. Introduction

This paper introduces a new simple method for kinematic analysis of a hybrid serial-parallel manipulator based on the theories of screws. There are different techniques for kinematic analysis of serial manipulators, such as using the description of Euler angles [1], the screw theories [2, 3] and Quaternions [4], etc. Comparing with the others, the screw theories have two advantages [2]. One is to avoid the singularities introduced by analysis. The other is to provide a very geometric description to the rigid motions that could greatly simplify the analysis of mechanisms.

Kinematic analysis has been a hot topic for parallel and serial-parallel manipulators for years. The parallel mechanisms are the most challenging part to analyze for hybrid serial-parallel manipulators. In most cases, the kinematics was obtained by geometrical constraints between the parallel links [5-7]. However, this traditional technique would become more difficult as more-than-one parallel mechanisms appear in the system, and the group of equations can not be easily solved to obtain the kinematics for hybrid serial-parallel manipulators.

The main contribution of this paper is the development of a new systematic technique to simplify the analysis of the kinematics for a commercial hybrid manipulator in robotics industry, and to make the foundation for further analysis.

X. Zhu is with the Harbin Institute of Technology Shenzhen Graduate School, Shenzhen, Guangdong 518055, China (086-0755-26033788; zhuxiaorui@hotmail.com).

J. Shi is with the Harbin Institute of Technology Shenzhen Graduate School, Shenzhen, Guangdong 518055, China (hit.shi.j@gmail.com,)

Z. Li is with the Hong Kong University of Science and Technology, Hong Kong (eezxli@ee.ust.hk)

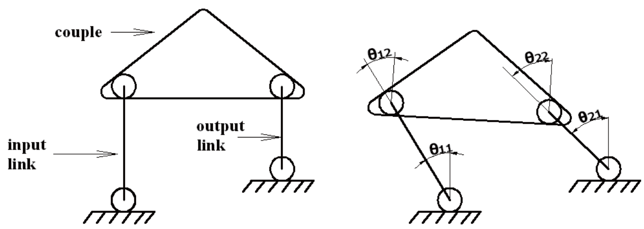
II. Background

The four-bar mechanisms are widely used in different kinds of industrial manipulators. The four-bar mechanism, as in Fig. 1 (a), has an input link, an output link, and the rigid body that connects the input and output links called the coupler. The four-bar linkage is a mechanism such that the input link and output link satisfy a given functional relationship, therefore creating a type of mechanical computer, Fig. 1 (b). The main role of the parallel four-bar mechanisms used in the manipulators is to conduct more payloads and achieve more complex motion such that the heavy parts of the system would not be applied directly on the upper arms of the robot.

Two parallel four-bar mechanisms are connected to form a new mechanism called double four-bar mechanism that creates the complexity for kinematics analysis of each link. IRB260 made by ABB Company, Fig. 2, is a typical one using this mechanism, and in this paper this type of robots will be analyzed in kinematics for illustration of the proposed simple method.

In this robot, there are two parallel four-bar mechanisms forming a double four-bar mechanism. The coupler of the lower one is the "ground frame" of the upper one. Since the four links constitutes a parallelogram and one link is fixed, the coupler can only achieve the planar translation no matter the angle that the input link rotates. Therefore the end-effector of the robot can only obtain the parallel translation. This relationship can be easily understood in Fig. 3. This mechanism makes it easy to control the motion of the end-effector especially for packing robots.

III. Modeling



(a) (b)

Fig. 1. Four-bar mechanism

Fig. 2. The photos of the ABB IRB260 [8].

As the description above, this robot is not an open-chain system because a rotation of the end-effector is confined. The traditional method is to find out each possible open-loop of the forward mapping and write out the kinematics equation for each loop (called the structure equation). Then the constraints between different loops are introduced into the group of the structure equations. However, solving these equations under the constraints is challenging for this double four-bar mechanism. In this situation, we propose a new simple method to solve the kinematics problem for this industrial manipulator.

The structure of the robot (IRB260) is first simplified as Fig. 4 where the dark lines represent the equivalent open-chain loop we are about to create. The idea is that if we can transform this double four-bar mechanism into the equivalent main open-chain one, the problem will be simpler to work out. Note that there are four motor-driven joints in this robot. Since the axis 1 and 5 are only rotating along the z axis, so they can be ignored temporarily. Due to the exiting double four-bar mechanism, the axis 2 and 3 can not affect the rotation of the end-effector along the X axis. It is noticed that the join 4 is not driven by any motor. However, it can rotate a special angle to make the end-effector keep horizontal due to the parallel mechanism.

Let us image that we delete the double four-bar system but add a virtual motor for actuating the join 4, as shown in Fig. 5. It is assumed that this virtual motor always rotates the same amount of the angle as the summation of rotation angles of the joint 2 and joint 3 but in an opposite direction, i.e., $\theta_4 = -(\theta_2 + \theta_3)$. This assumption is reasonable because in this case the virtual motor can counteract the rotation that

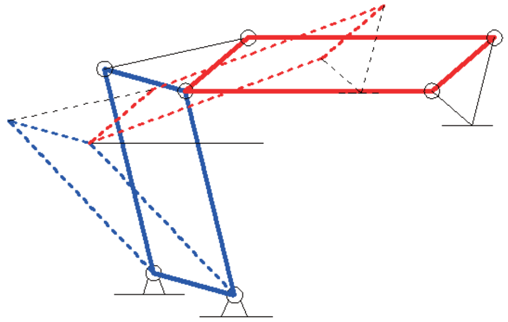


Fig. 3. The diagram of the hybrid manipulator in two positions.

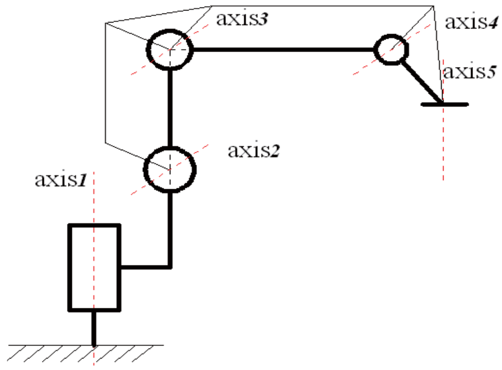


Fig. 4. Simplified structure of IRB260

the motor 2 and motor 3 give along the axis X such that the motion of the end-effector can also keep horizontal. Instead the robot mechanism is transformed to an equivalent open-chain loop with five axes as shown in Fig. 4.

IV. Kinematics

A. Forward Kinematics

Since this hybrid mechanism has been transformed to a pure open-chain mechanism, we can start to use the screw theories to solve the kinematics problem.

According to the screw theories [2], the motion of a revolute joint can correspond to a zero-pitch screw about an axis in the unit axis w direction passing through the point q. Therefore the motion can be parameterized with two vectors, w and q, seen in Fig. 5 where w represents the unit rotation axis and q represents the coordinate of any point on the axis w. Both of them are represented in the inertial coordinate frame. The point q is usually chosen for convenience of computation. Notice that the newly created open-chain loop has five revolute joints. Hence the corresponding vectors w_i and q_i for the i-th joint are obtained as, i=1,2,…,5, seen in Fig. 6,

$$q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{1}$$

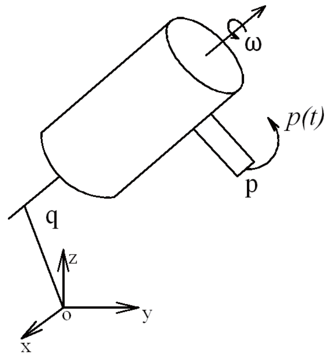


Fig. 5. A revolute joint [2].

$$q_2 = \begin{bmatrix} 0 \\ l_1 \\ l_2 \end{bmatrix} \quad \omega_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

$$q_3 = \begin{bmatrix} 0 \\ l_1 \\ l_2 + l_3 \end{bmatrix} \quad \omega_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

$$q_4 = \begin{bmatrix} 0 \\ l_1 + l_4 \\ l_2 + l_3 \end{bmatrix} \quad \omega_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

$$q_5 = \begin{bmatrix} 0 \\ l_1 + l_4 + l_6 \\ l_2 + l_3 - l_5 \end{bmatrix} \quad \omega_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

The general rigid body motions consist of rotation and translation. The Euclidean group $SE(3)$ is defined as the configuration space of the rigid body motion, i.e.

$$SE(3) = \{(p, R) : p \in R^3, R \in SO(3)\} = R^3 \times SO(3) \tag{6}$$

where p is the point vector of the origin of the body frame from the inertial frame and R is the orientation matrix of the body frame relative to the inertial frame after the movement.

In order to find an exponential coordinate for the configuration space $SE(3)$, the group $se(3)$ is defined as,

$$se(3) = \{(v, \hat{\omega}) : v \in R^3, \hat{\omega} \in so(3)\} \tag{7}$$

where $v = -\omega \times q$ and so(3) is the space of 3×3 skew-symmetric matrices. Therefore the corresponding twist $\hat{\xi} \in se(3)$ is represented in homogeneous coordinates as,

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \tag{8}$$

Then we define a twist coordinate $\xi : (v, \omega)$ (6-dimensional vector) for one link.

$$\xi = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} \tag{9}$$

According to the Proposition 2.8 in [2], the exponential mapping $e^{\hat{\xi}\theta}$ can be derived as,

$$e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})(\omega \times v) + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix} \tag{10}$$

Hence the connection between the twist coordinate and the transformation matrix of the rigid motion has been built already as,

$$g(\theta) = e^{\hat{\xi}\theta} g(0) \tag{11}$$

So the forward kinematics map g_{st} is the product of the exponentials of the twist for each link as [2, 9],

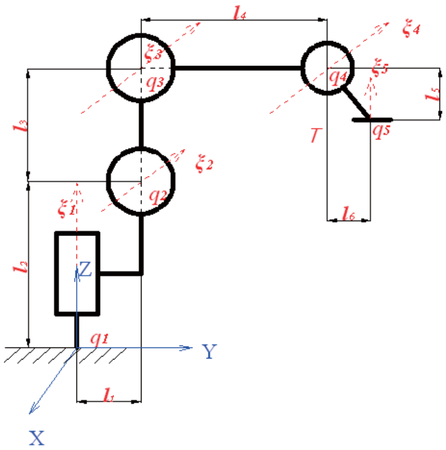


Fig. 6. Final simplified structure as an open-chain mechanism

$$g_{st}(\theta) = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} g_{st}(0) \tag{12}$$

where i=1,2,…5, $g_{st}(0)$ is the initial configuration of the manipulator and $e^{\hat{\xi}_i\theta_i}$ can be computed according to the Equation (10). Substitute the Equations (1)-(5), (9) and the virtual constraint $\theta_4 = -(\theta_2 + \theta_3)$ into the Equation (10). Then according to the Equation (12), we can get the final expression of the forward kinematics map $g_{st}(\theta)$, as following:

$$\begin{bmatrix} c(\theta_1+\theta_5) & -s(\theta_1+\theta_5) & 0 & -(l_1 + l_3\,s\theta_2 + l_4\,c(\theta_2+\theta_3) + l_6)\,s\theta_1 \\ s(\theta_1+\theta_5) & c(\theta_1+\theta_5) & 0 & (l_1 + l_3\,s\theta_2 + l_4\,c(\theta_2+\theta_3) + l_6)\,c\theta_1 \\ 0 & 0 & 1 & l_2 + l_3\,c\theta_2 - l_4\,s(\theta_2+\theta_3) - l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

where the notation $c(\theta_i) = \cos\theta_i$, $s(\theta_i) = \sin\theta_i$.

Similarly, the inverse kinematics problem can be solved. We will not discuss it in this paper.

B. Spatial Jacobian

Spatial Jacobian matrix represents the relationship between the velocity of the joint angles and the end-effector. The word spatial here means relative to the inertial coordinate frame. Although there are five joints in this model, only four of them, the joints 1, 2, 3, and 5, are actuated. Therefore, we must make some mathematic transformation on spatial Jacobian calculation of open-chain manipulators.

First, the instantaneous velocity of the end-effector is given by the twist as,

$$\hat{V}^s_{st} = \dot{g}_{st}(\theta) g^{-1}_{st}(\theta) \tag{14}$$

Then applying the chain rule to the Equation (14), we can get,

$$\begin{aligned}\hat{V}_{st}^{s}&=\sum_{i=1}^{5}\left(\frac{\partial g_{st}}{\partial\theta_i}\dot{\theta}_i\right)g_{st}^{-1}(\theta)=\sum_{i=1}^{5}\left(\frac{\partial g_{st}}{\partial\theta_i}g_{st}^{-1}(\theta)\right)\dot{\theta}_i\\&=\left(\frac{\partial g_{st}}{\partial\theta_1}g_{st}^{-1}(\theta)\right)\dot{\theta}_1+\left(\frac{\partial g_{st}}{\partial\theta_2}g_{st}^{-1}(\theta)\right)\dot{\theta}_2+\left(\frac{\partial g_{st}}{\partial\theta_3}g_{st}^{-1}(\theta)\right)\dot{\theta}_3\\&+\left(\frac{\partial g_{st}}{\partial\theta_4}g_{st}^{-1}(\theta)\right)\dot{\theta}_4+\left(\frac{\partial g_{st}}{\partial\theta_5}g_{st}^{-1}(\theta)\right)\dot{\theta}_5\end{aligned}\tag{15}$$

Let

$$\left(\frac{\partial g_{st}}{\partial\theta_i}g_{st}^{-1}\right)^{\vee}=\xi_i' \tag{16}$$

where the v operator is defined to extract the 6-dimensional vector which parameterizes a twist. Then applying the condition $\theta_4=-(\theta_2+\theta_3)$, the Equation (15) becomes,

$$\begin{aligned}&\left(\frac{\partial g_{st}}{\partial\theta_1}g_{st}^{-1}(\theta)\right)\dot{\theta}_1+\left(\frac{\partial g_{st}}{\partial\theta_2}g_{st}^{-1}(\theta)\right)\dot{\theta}_2+\left(\frac{\partial g_{st}}{\partial\theta_3}g_{st}^{-1}(\theta)\right)\dot{\theta}_3\\&+\left(\frac{\partial g_{st}}{\partial\theta_4}g_{st}^{-1}(\theta)\right)\dot{\theta}_4+\left(\frac{\partial g_{st}}{\partial\theta_5}g_{st}^{-1}(\theta)\right)\dot{\theta}_5\\&=\hat{\xi}_1\dot{\theta}_1+\hat{\xi}'_2\dot{\theta}_2+\hat{\xi}'_3\dot{\theta}_3+\hat{\xi}'_4(-\dot{\theta}_2-\dot{\theta}_3)+\hat{\xi}'_5\dot{\theta}_5\\&=\hat{\xi}_1\dot{\theta}_1+(\hat{\xi}'_2-\hat{\xi}'_4)\dot{\theta}_2+(\hat{\xi}'_3-\hat{\xi}'_4)\dot{\theta}_3+\hat{\xi}'_5\dot{\theta}_5\\&=\begin{bmatrix}\hat{\xi}_1 & \hat{\xi}'_2-\hat{\xi}'_4 & \hat{\xi}'_3-\hat{\xi}'_4 & \hat{\xi}'_5\end{bmatrix}\dot{\theta}\end{aligned}\tag{17}$$

Since $V_{st}^{s}=J_{st}^{s}(\theta)\dot{\theta}$, the spatial manipulator Jocobian can be extracted as,

$$J_{st}^{s}(\theta)=\begin{bmatrix}\xi_1 & \xi_2'-\xi_4' & \xi_3'-\xi_4' & \xi_5'\end{bmatrix} \tag{18}$$

For the i-th joint, we have

$$\begin{aligned}\left(\frac{\partial g_{st}}{\partial\theta_i}g_{st}^{-1}\right)\dot{\theta}_i&=\left(e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_{i-1}\theta_{i-1}}\hat{\xi}_i e^{\hat{\xi}_i\theta_i}\cdots e^{\hat{\xi}_n\theta_n}g_{st}(0)g_{st}^{-1}\right)\dot{\theta}_i\\&=\left(e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_{i-1}\theta_{i-1}}\hat{\xi}_i e^{-\hat{\xi}_{i-1}\theta_{i-1}}\cdots e^{-\hat{\xi}_1\theta_1}\right)\dot{\theta}_i\end{aligned}\tag{19}$$

Then,

$$\xi_i'=\left(\frac{\partial g_{st}}{\partial\theta_i}g_{st}^{-1}\right)^{\vee}=Ad_{(e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_{i-1}\theta_{i-1}})}\xi_i \tag{20}$$

where Ad represents the adjoint transformation and the lower index represents the associated transformation matrix after the rotation of the joint i-1. For example, for the rigid transformation g [2],

$$Ad_g=\begin{bmatrix}R & \hat{p}R\\ 0 & R\end{bmatrix} \tag{21}$$

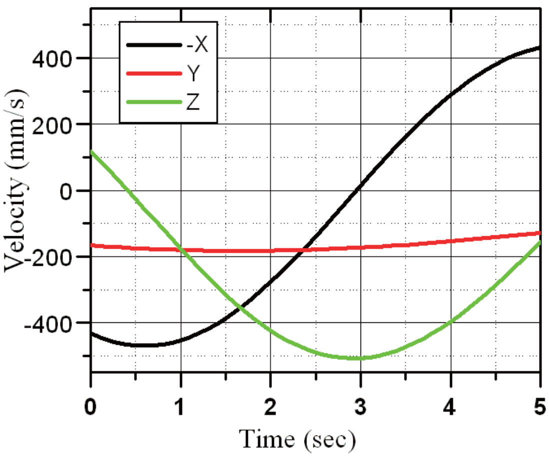
$$g=\begin{bmatrix}R & p\\ 0 & 1\end{bmatrix} \tag{22}$$

Therefore the spatial manipulator Jacobian for this manipulator can be solved. The velocity of the origin of the end-effector q_5 relative to the spatial (inertial) frame is then given by

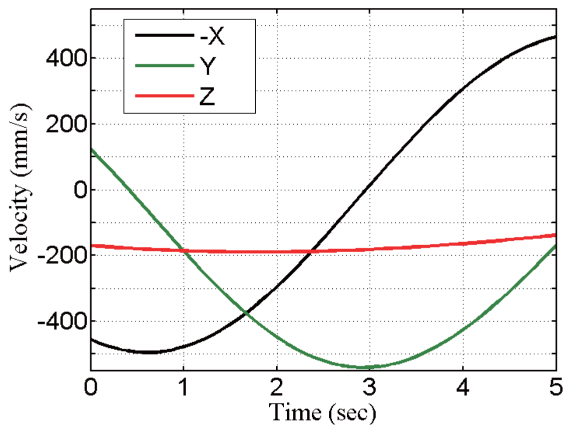
$$v_{q_5}^{s}=\hat{V}_{st}^{s}q_5^{s} \tag{23}$$

V. Verification

If we can check that the Jacobian matrix is correct, all the kinematics problems talked above can be verified. It is easy to check the velocity of the end-effector given the velocities of each joint of the manipulator. In order to obtain the velocity data, a mechanical model is built for the commercial manipulator IRB260 in ADAMS® according to the file downloaded from the official website of the ABB. Given the arbitrary angular speed to axis 1, 2, 3, and 5, we can obtain the linear velocity vector $v_{q_5}^{s}$ of the end-effector in spatial coordinate. According to the derivation in the previous Section in this paper we can also get the vector $v_{q_5}^{s}$ using the proposed method. Let $\omega_1=-30^\circ/s$, $\omega_2=10^\circ/s$, $\omega_3=5^\circ/s$, $\omega_5=0$ (because it has no effect on the $v_{q_5}^{s}$). Then we plot the



(a) $v_{q_5}^{s}$ curves simulated from ADAMS



(b) $v_{q_5}^{s}$ curves calculated from MATLAB

Fig. 7. The Comparison of $v_{q_5}^{s}$ from ADAMS and MATLAB.

vector $v_{q_5}^s$ in three orthogonal directions from two resources, Fig. 7. One group is from the real model in ADAMS®, Fig. 7(a) and the other is from the calculation of MATLAB® using the method proposed in this paper, Fig. 7(b).

From this comparison it is obvious to see that two groups of curves are almost the same, Fig. 7. Fig. 8 shows the subtle difference in the corresponding directions. These little errors are identified from the modeling or calculation approximation in ADAMS® and MATLAB®. Hence it is concluded that the new proposed technique is valid for the hybrid manipulator.

I. CONCLUSIONS

The simple method is proposed in this paper to make analysis of the hybrid manipulator. The virtual axis is added and the restriction between this virtual axis and the other neighboring axes is assumed. The system can be changed into an equivalent open-chain mechanism with these assumptions, and then everything becomes simpler to calculate. The method is verified using MATLAB® and ADAMS®. It can also be widely extended to other hybrid mechanisms, especially more-than-one four-bar linkage.

REFERENCES

[1] J. J. Craig, *Introduction to Robotics: Mechanics and Control*, third ed.: Addison-Wesley, 2005.

[2] R. M. Murray, Z. Li, and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation*: CRC Press, 1994.

[3] J. M. Selig, *Geometrical methods in robotics*: Springer, 1996.

[4] H. Goldstein, *Classical Mechanics*, second ed.: Addison-Wesley, 1980.

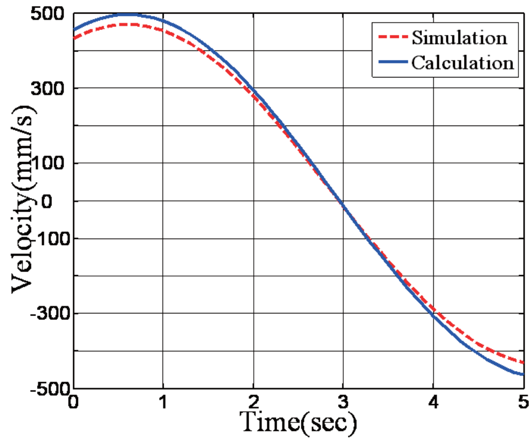
[5] M. Z. Huang, S.-H. Ling, and Y. Sheng, "A study of velocity kinematics for hybrid manipulators with parallel-series configurations," in *International conference on Robotics and Automation*, Atlanta, GA, USA, 1993, pp. 456-461.

[6] T. K. Tanev, "Kinematics of a hybrid (parallel-serial) robot manipulator," *Mechansim and Machine Theory,* vol. 35, pp. 1183-1196, 2000.

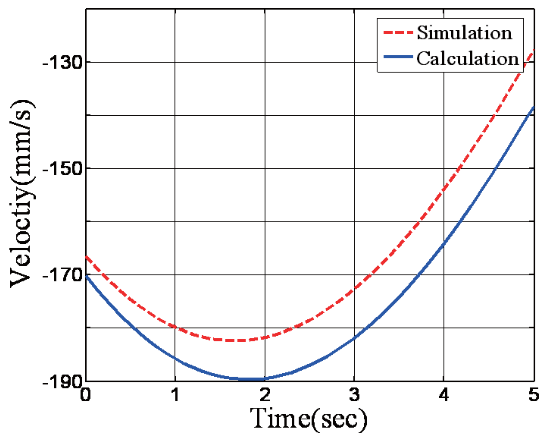
[7] X. Z. Zheng, H. Z. Bin, and Y. G. Luo, "Kinematic analysis of a hybrid serial-parallel manipulator," *International Journal of Advanced Manufacturing Technology,* vol. 23, pp. 925-930, 2004.

[8] "www.abb.com."

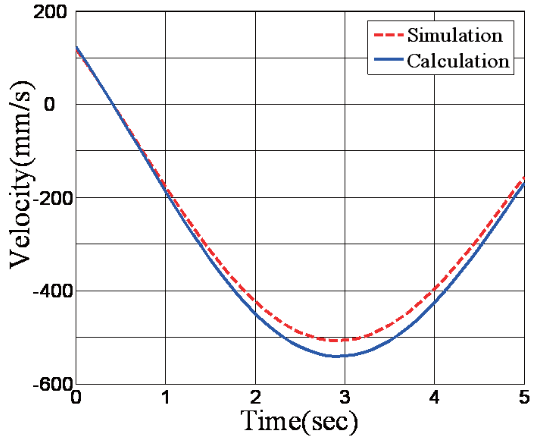
[9] R. W. Brockett, "Robotic manipulators and the product of exponentials formula," in *Mathematical Theory of Networks and Systems*, P. A. Fuhrman, Ed.: Springer-Verlag, 1984, pp. 120-129.



(a) Comparison of curves of $v_{q_5}^s$ vector in X direction



(b) Comparison of curves of $v_{q_5}^s$ vector in Y direction



(c) Comparison of curves of $v_{q_5}^s$ vector in Z direction

Fig. 8. The comparison errors of $v_{q_5}^s$ in three directions.