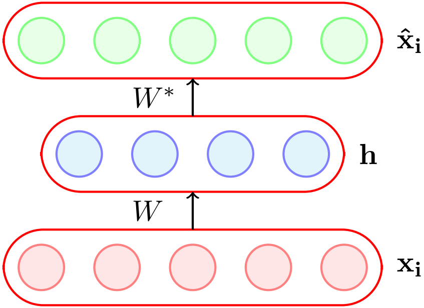
Module 21.1: Revisiting Autoencoders



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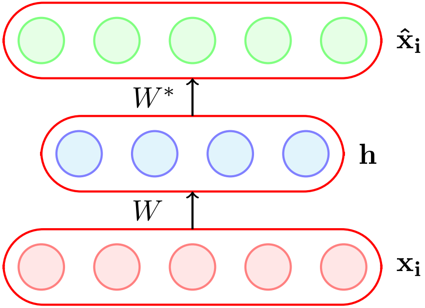

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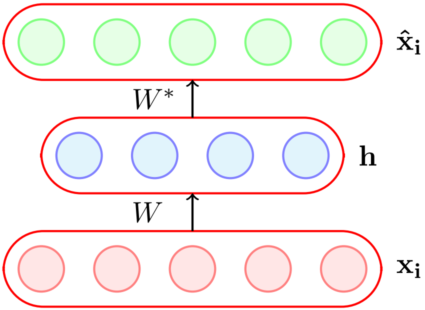

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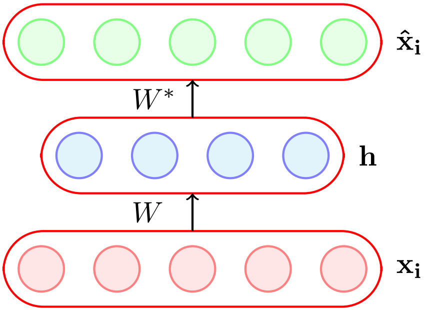

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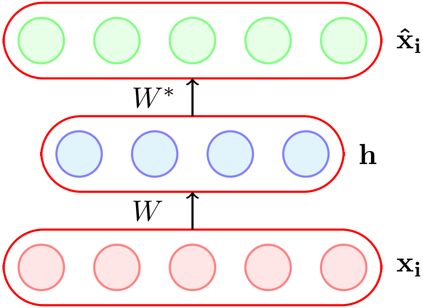
- An autoencoder contains an encoder which takes the input X and maps it to a hidden representation
- The decoder then takes this hidden representation and tries to reconstruct the input from it as $\tilde{(X)}$
- The training happens using the following objective function

$$\min_{W,W^*,\mathbf{c},\mathbf{b}} \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} (\hat{x}_{ij} - x_{ij})^2$$



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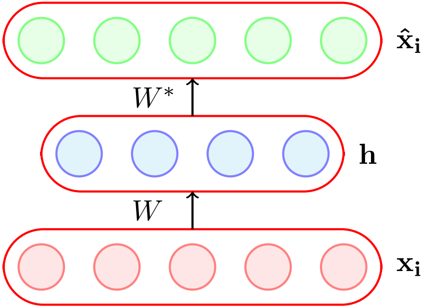

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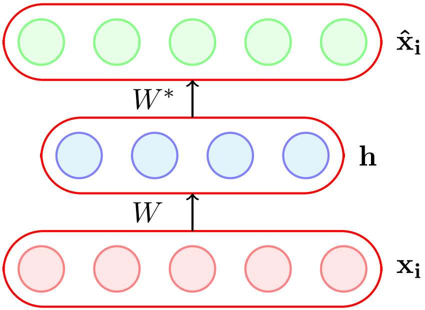

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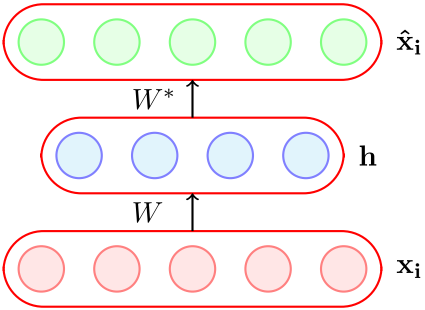

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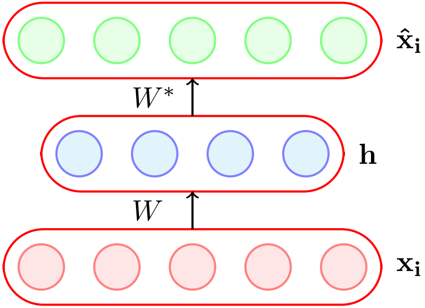

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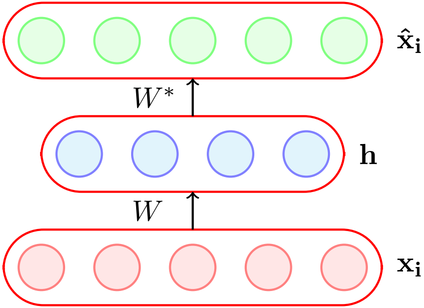

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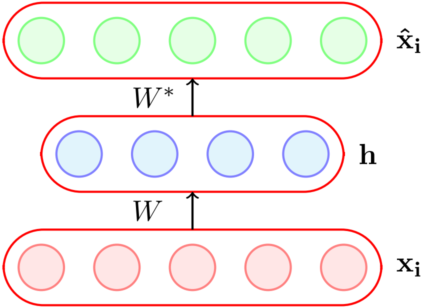
- Let us revisit *generation* in the context of autoencoders?



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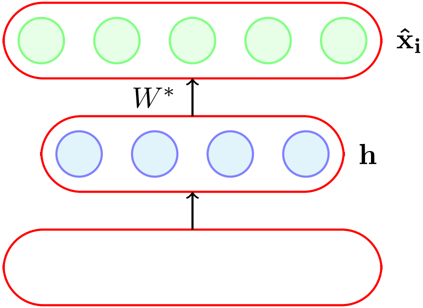

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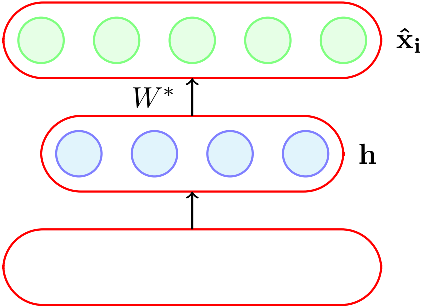
- In other words, once the autoencoder is trained can I remove the encoder, feed a hidden representation h to the decoder and decode a $\tilde{X}$ from it ?



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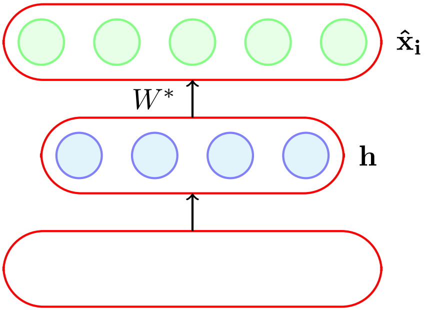
- In principle, yes, but in practice there is a problem with this approach



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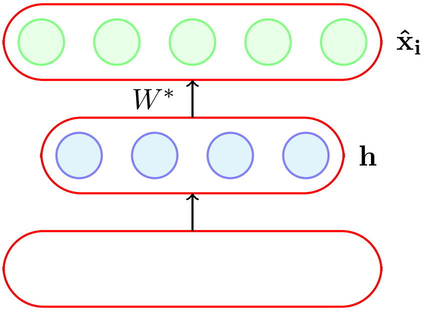

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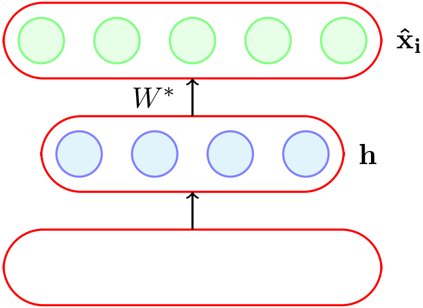
- h is a very high dimensional vector and only a few vectors in this space would actually correspond to meaningful latent representations of our input
- So of all the possible value of h which values should I feed to the decoder (we had asked a similar question before: slide 67, bullet 5 of lecture 19)



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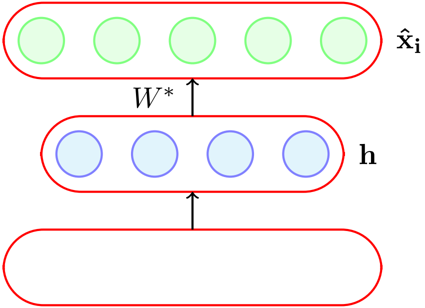

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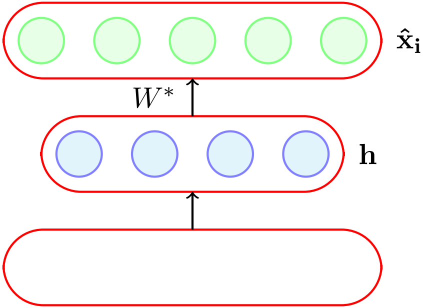

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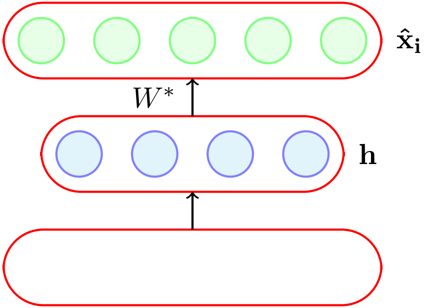

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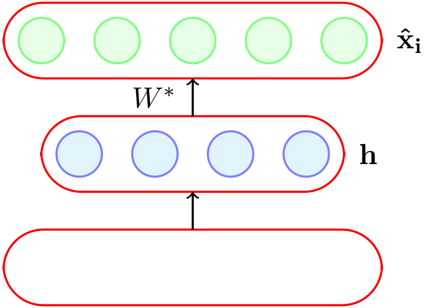

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- They learn a hidden representation h but not a distribution $P(h|X)$
- We will now look at variational autoencoders which have the same structure as autoencoders but they learn a distribution over the hidden variables