Module 3.4: Learning Parameters : Gradient Descent

Now let us see if there is a more efficient and principled way of doing this

Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!

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$$\theta = [w, b]$$

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Question: What is the right $\Delta\theta$ to use ?

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The answer comes from Taylor series

For ease of notation, let $\Delta\theta = u$, then from Taylor series, we have,

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Note that the move (ηu) would be favorable only if,

$$\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) < 0 \ [\textit{i.e., if the new loss is less than the previous loss}]$$

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This implies,

$$u^T \nabla_\theta \mathscr{L}(\theta) < 0$$

Okay, so we have,

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But, what is the range of $u^T \nabla_\theta \mathscr{L}(\theta)$?

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Let β be the angle between u and $\nabla_\theta \mathscr{L}(\theta)$, then we know that,

$$-1 \leq cos(\beta) = \frac{u^T \nabla_\theta \mathscr{L}(\theta)}{||u|| * ||\nabla_\theta \mathscr{L}(\theta)||} \leq 1$$

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multiply throughout by $k = ||u|| * ||\nabla_\theta \mathscr{L}(\theta)||$

$$-k \leq k * cos(\beta) = u^T \nabla_\theta \mathscr{L}(\theta) \leq k$$

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$$-k \leq k * cos(\beta) = u^T \nabla_\theta \mathscr{L}(\theta) \leq k$$

Thus, $\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) = u^T \nabla_\theta \mathscr{L}(\theta) = k * cos(\beta)$ will be most negative when $cos(\beta) = -1$ *i.e.*, when β is $180°$

Gradient Descent Rule

- The direction u that we intend to move in should be at 180° w.r.t. the gradient

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Parameter Update Equations

$$w_{t+1} = w_t - \eta \nabla w_t$$
$$b_{t+1} = b_t - \eta \nabla b_t$$
$$where, \nabla w_t = \frac{\partial \mathscr{L}(w,b)}{\partial w}_{at\ w = w_t, b = b_t}, \nabla b = \frac{\partial \mathscr{L}(w,b)}{\partial b}_{at\ w = w_t, b = b_t}$$

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So we now have a more principled way of moving in the w-b plane than our "guess work" algorithm

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Algorithm: gradient_descent()

$t \leftarrow 0;$
$max_iterations \leftarrow 1000;$
while $t < max_iterations$ **do**
  $w_{t+1} \leftarrow w_t - \eta \nabla w_t;$
  $b_{t+1} \leftarrow b_t - \eta \nabla b_t;$
  $t \leftarrow t + 1;$
end

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- To see this algorithm in practice let us first derive ∇w and ∇b for our toy neural network

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$$\mathscr{L}(w, b) = \frac{1}{2} * (f(x) - y)^2$$

$$\nabla w = \frac{\partial \mathscr{L}(w, b)}{\partial w} = \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2]$$

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\nabla w &= \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2] \\
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&= (f(x) - y) * f(x) * (1 - f(x)) * x
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&= f(x) * (1 - f(x)) * x
\end{aligned}
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$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

So if there is only 1 point (x, y), we have,

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So if there is only 1 point (x, y), we have,

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$$\nabla b = \sum_{i=1}^{2} (f(x_i) - y_i) * f(x_i) * (1 - f(x_i))$$

```
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Y = [0.2, 0.9]
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X = [0.5, 2.5]
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def f(w,b,x) : #sigmoid with parameters w,b
    return 1.0 / (1.0 + np.exp(-(w*x + b)))
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def error (w, b) :
    err = 0.0
    for x,y in zip(X,Y) :
        fx = f(w,b,x)
        err += 0.5 * (fx - y) ** 2
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def grad_b(w,b,x,y) :
    fx = f(w,b,x)
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def do_gradient_descent() :
    w, b, eta, max_epochs = -2, -2, 1.0, 1000
    for i in range(max_epochs) :
        dw, db = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)
        w = w - eta * dw
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Gradient descent on the error surface

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X = [0.5, 2.5]
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def f(w,b,x) : #sigmoid with parameters w,b
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- So where do we head from here ?