Module 3.5: Representation Power of a Multilayer Network of Sigmoid Neurons

Representation power of a multilayer network of perceptrons

Representation power of a multilayer network of sigmoid neurons

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A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

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Representation power of a multilayer network of sigmoid neurons

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In other words, there is a guarantee that for any function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$, we can always find a neural network (with 1 hidden layer containing enough neurons) whose output g(x) satisfies $|g(x) - f(x)| < \epsilon$!!

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Representation power of a multilayer network of sigmoid neurons

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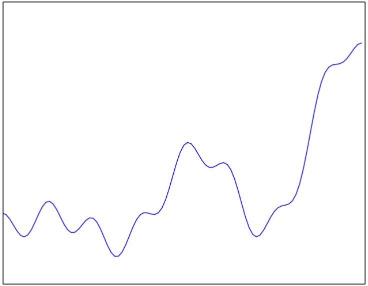
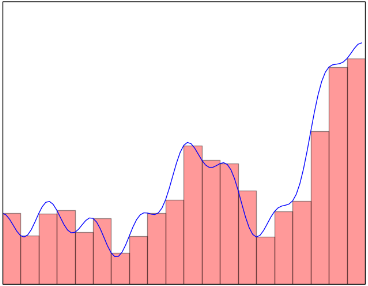
Proof: We will see an illustrative proof of this... [Cybenko, 1989], [Hornik, 1991]

- See this link* for an excellent illustration of this proof
- The discussion in the next few slides is based on the ideas presented at the above link

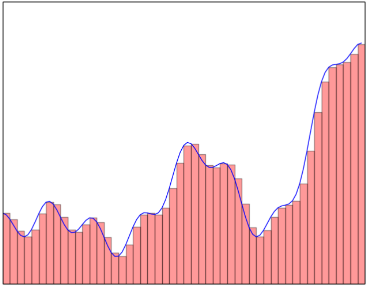
*http://neuralnetworksanddeeplearning.com/chap4.html



- We are interested in knowing whether a network of neurons can be used to represent an arbitrary function (like the one shown in the figure)



- We are interested in knowing whether a network of neurons can be used to represent an arbitrary function (like the one shown in the figure)
- We observe that such an arbitrary function can be approximated by several "tower" functions



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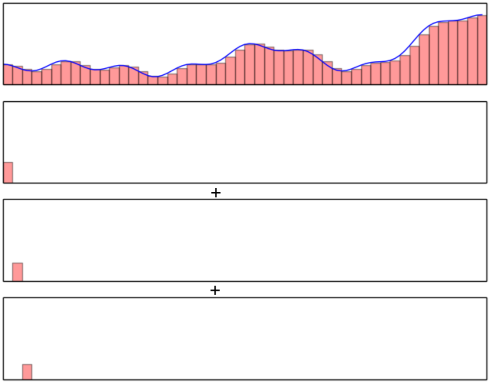
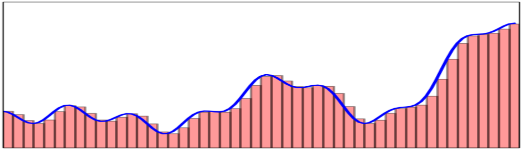
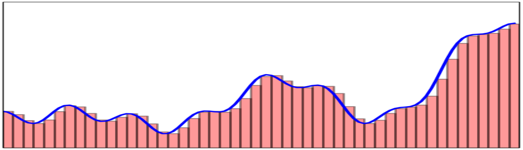


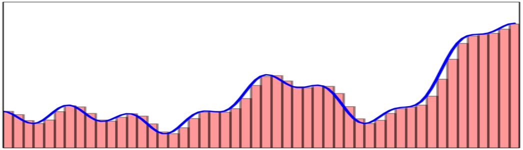
- We are interested in knowing whether a network of neurons can be used to represent an arbitrary function (like the one shown in the figure)
- We observe that such an arbitrary function can be approximated by several "tower" functions
- More the number of such "tower" functions, better the approximation
- To be more precise, we can approximate any arbitrary function by a sum of such "tower" functions

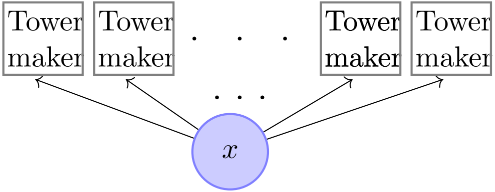


- We make a few observations

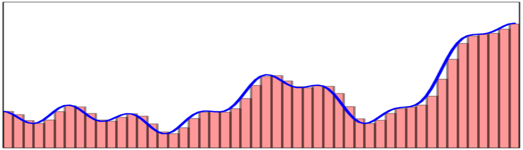


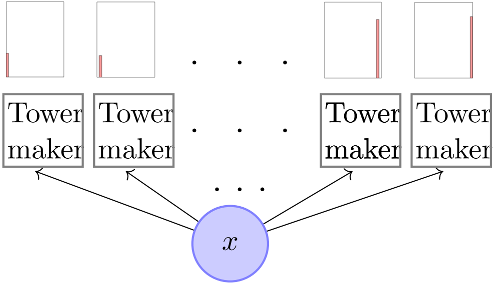
- We make a few observations
- All these "tower" functions are similar and only differ in their heights and positions on the x-axis





- We make a few observations
- All these "tower" functions are similar and only differ in their heights and positions on the x-axis
- Suppose there is a black box which takes the original input (x) and constructs these tower functions





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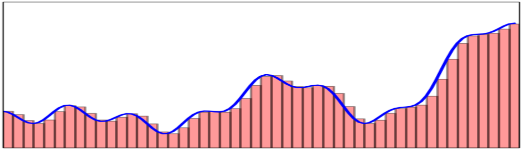
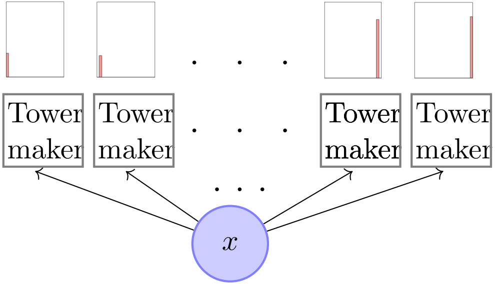
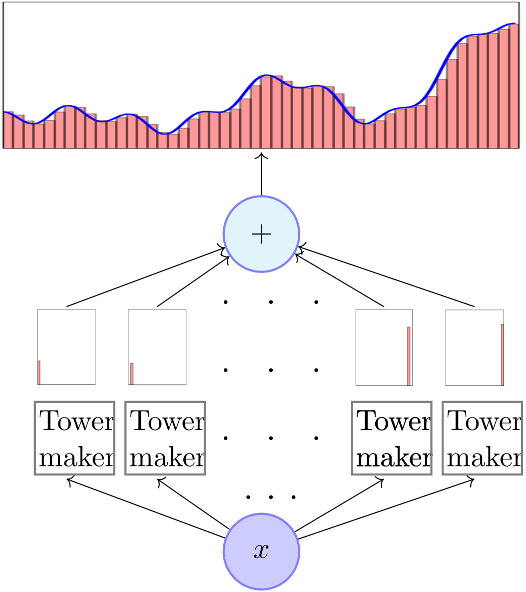




- We make a few observations
- All these "tower" functions are similar and only differ in their heights and positions on the x-axis
- Suppose there is a black box which takes the original input (x) and constructs these tower functions
- We can then have a simple network which can just add them up to approximate the function



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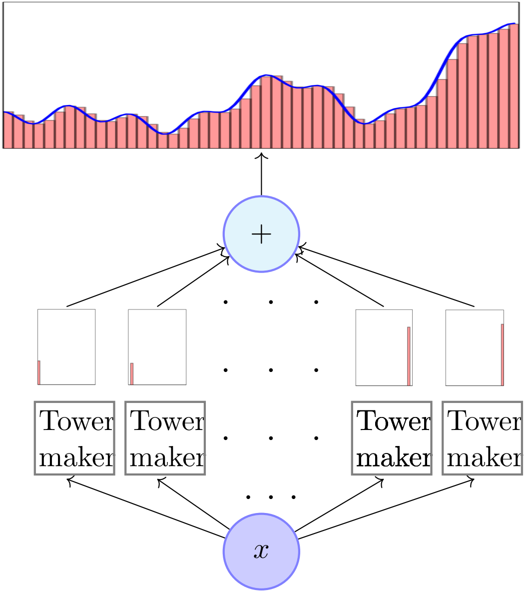
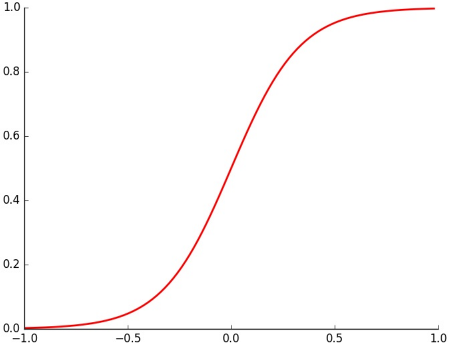
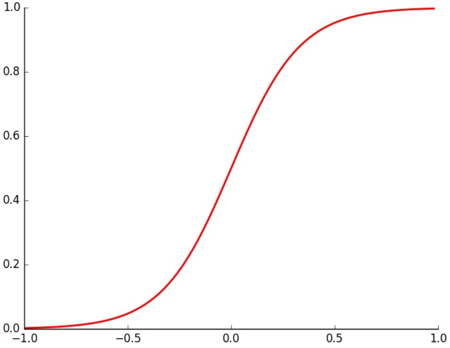


- We make a few observations
- All these "tower" functions are similar and only differ in their heights and positions on the x-axis
- Suppose there is a black box which takes the original input (x) and constructs these tower functions
- We can then have a simple network which can just add them up to approximate the function
- Our job now is to figure out what is inside this blackbox

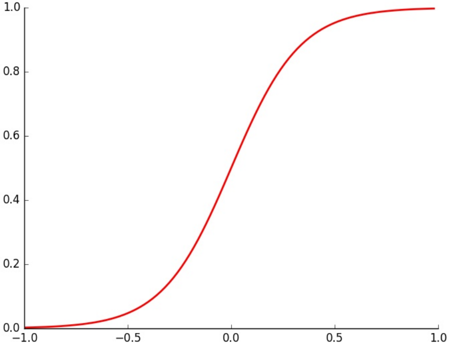
We will figure this out over the next few slides ...



- If we take the logistic function and set w to a very high value we will recover the step function

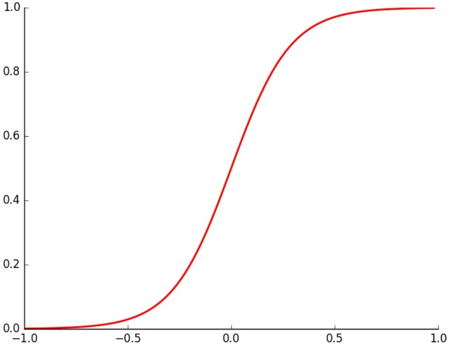


- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



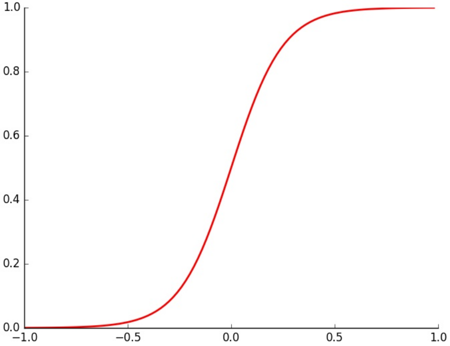
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



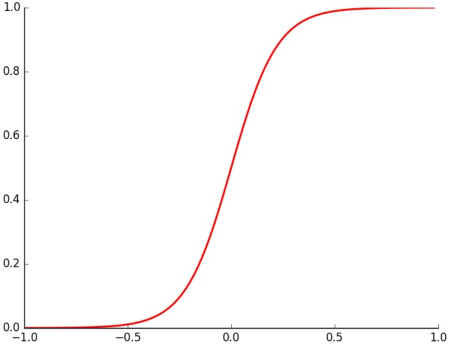
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 1$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



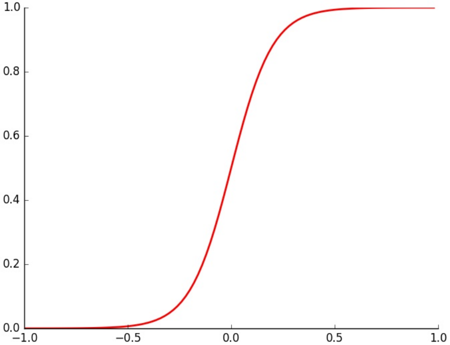
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 2$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



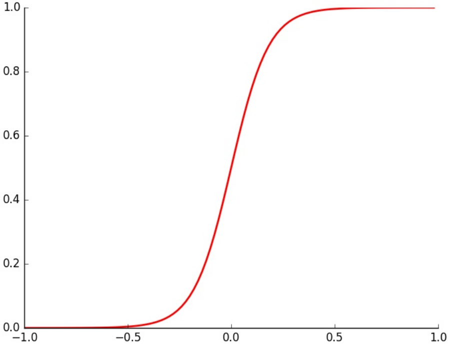
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 3$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



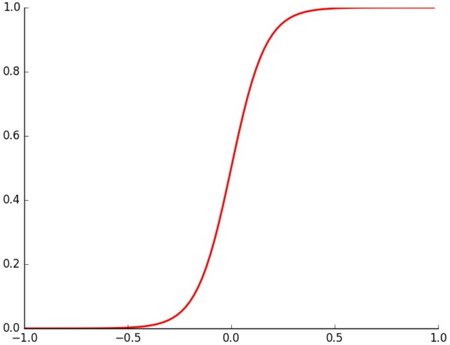
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 4$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



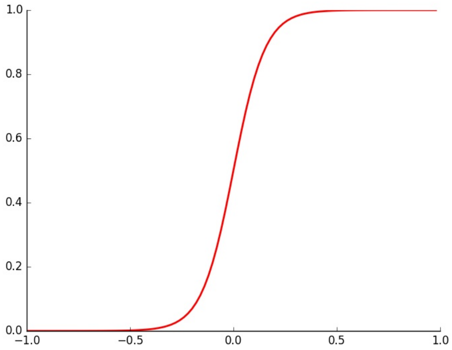
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 5$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



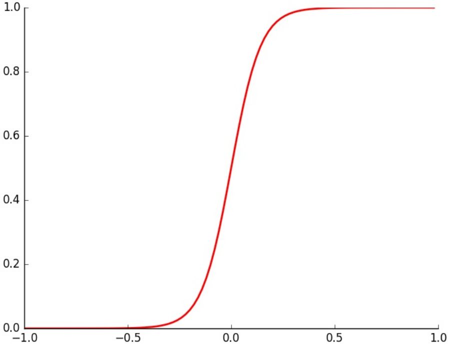
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 6$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



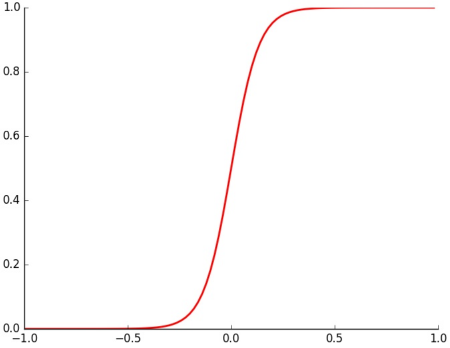
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 7$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 8$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 9$$

- If we take the logistic function and set w to a very high value we will recover the step function
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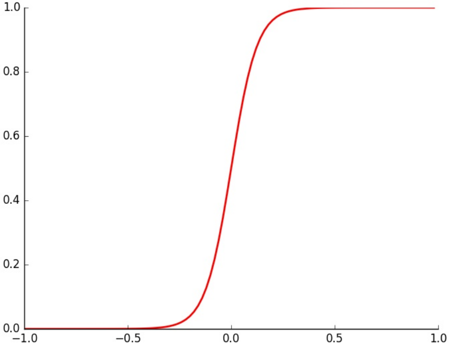
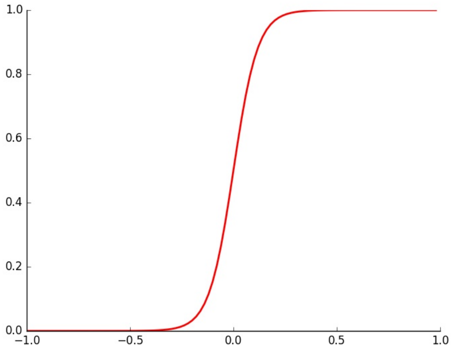


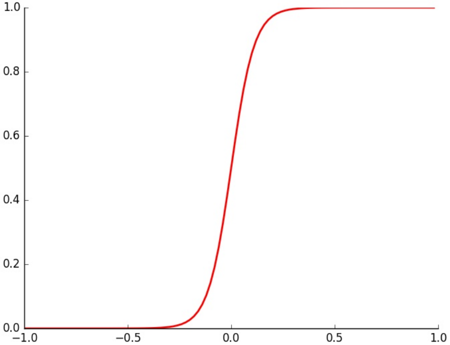
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 10$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



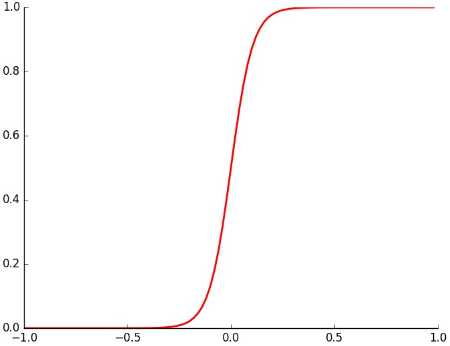
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 11$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



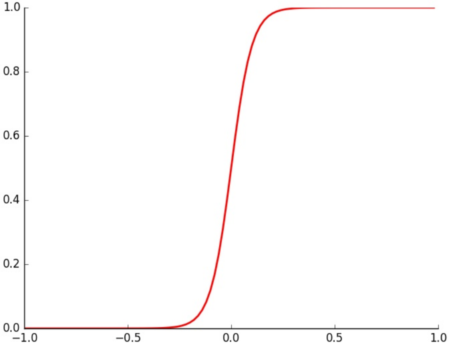
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 12$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



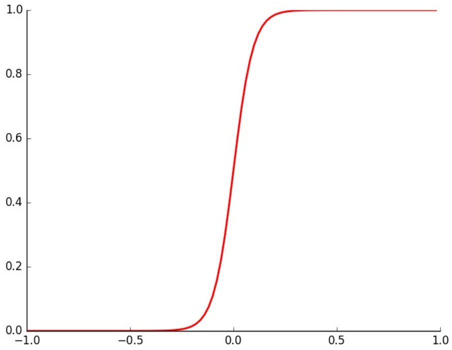
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 13$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



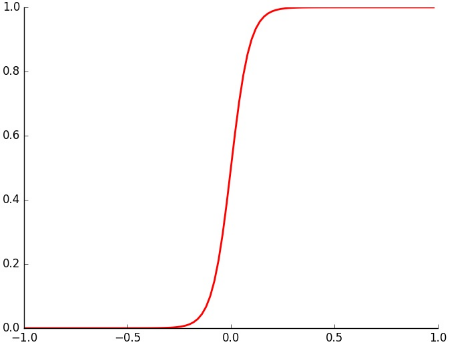
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 14$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



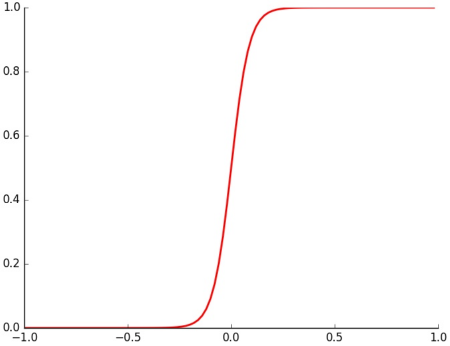
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 15$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



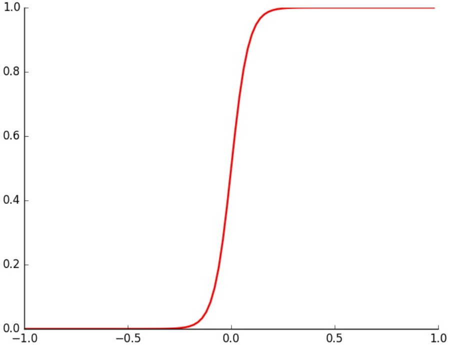
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 16$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



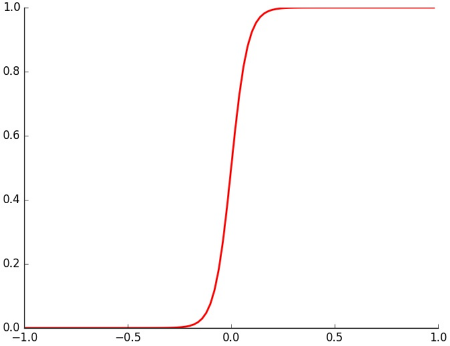
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 17$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



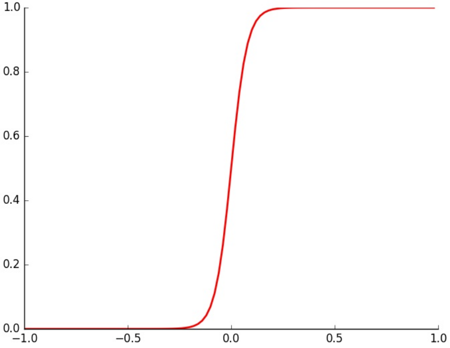
$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}}$ $w = 50, b = 18$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



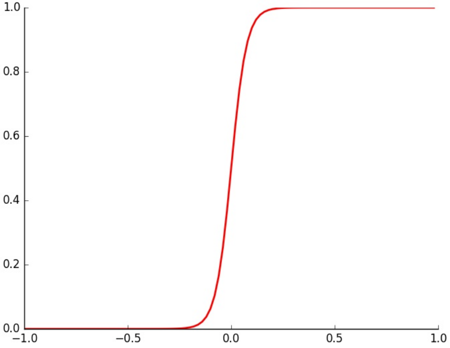
$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}}$ $w = 50, b = 19$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



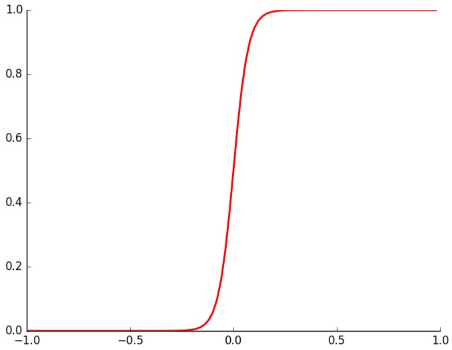
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 20$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



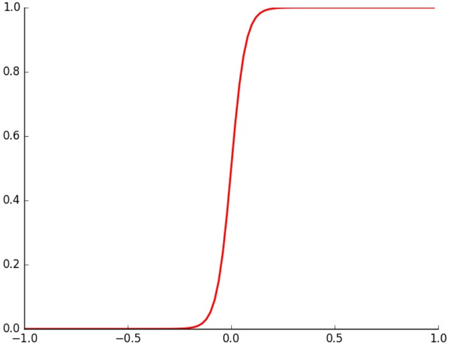
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 21$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



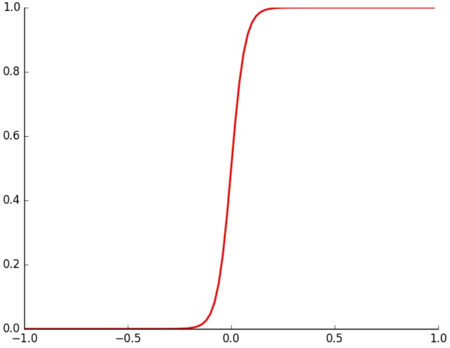
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 22$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



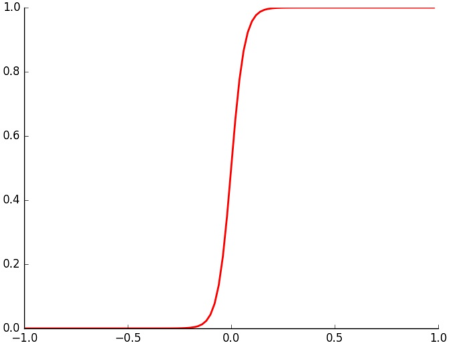
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 23$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



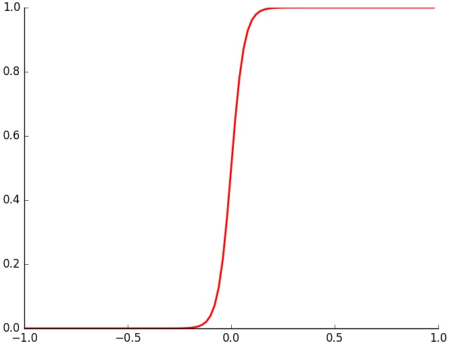
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 24$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



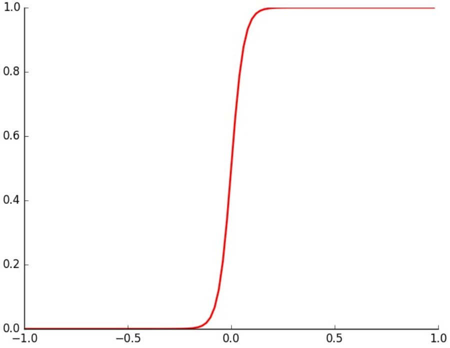
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 25$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



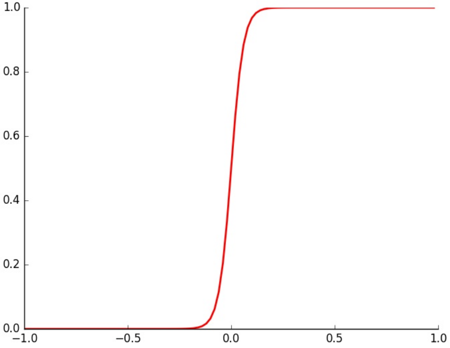
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 26$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



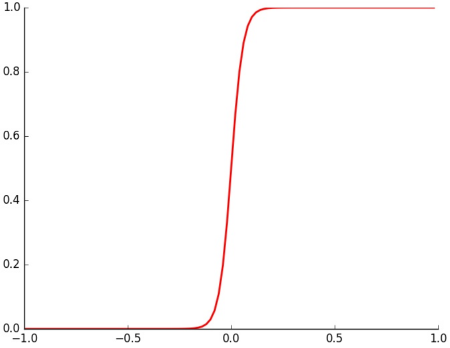
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 27$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



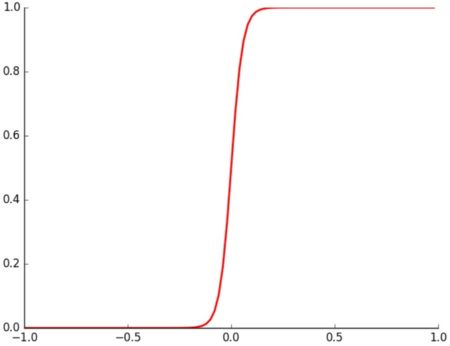
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 28$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



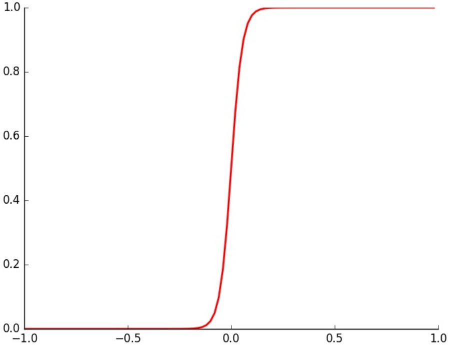
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 29$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



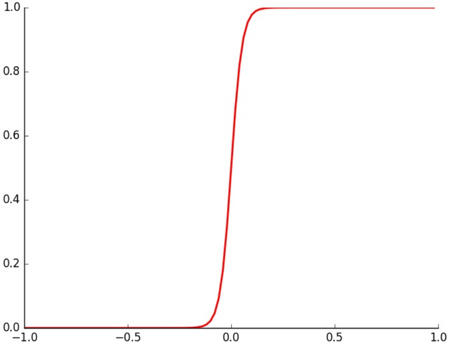
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 30$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



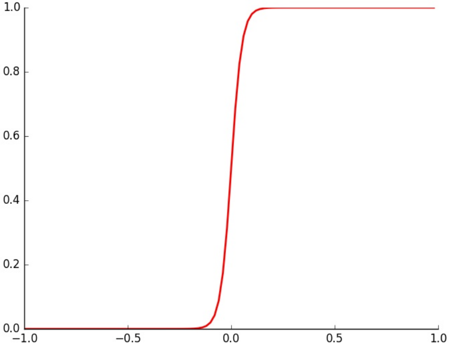
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 31$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



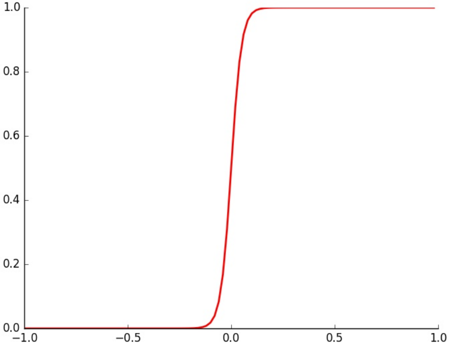
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 32$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



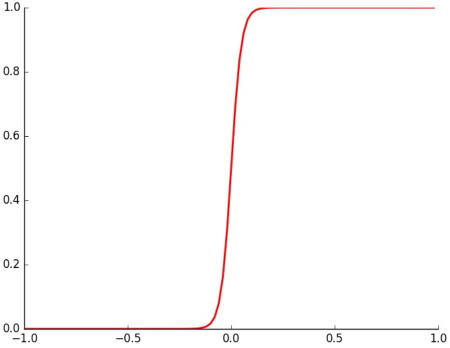
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 33$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



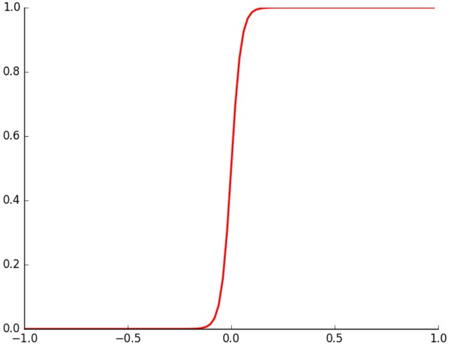
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 34$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



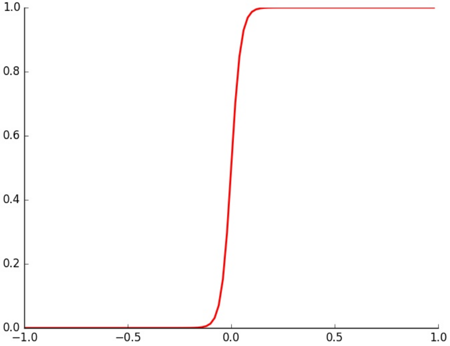
$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}}$ $w = 50, b = 35$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



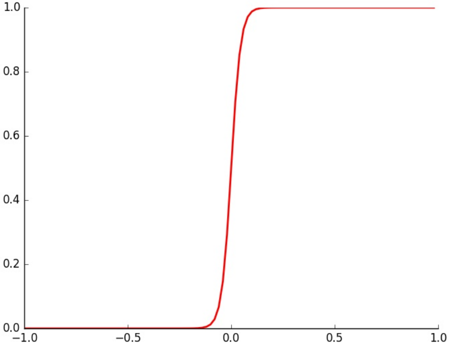
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 36$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



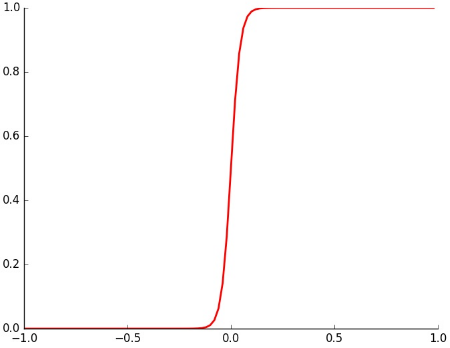
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 37$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



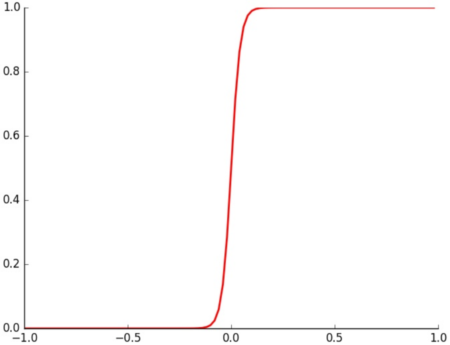
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 38$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



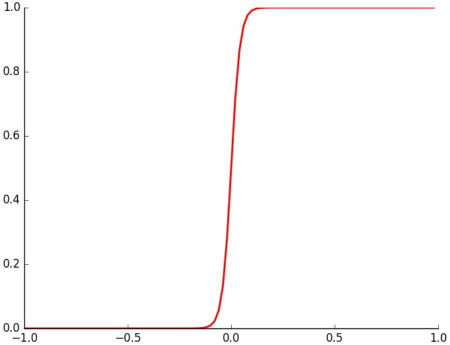
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 39$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



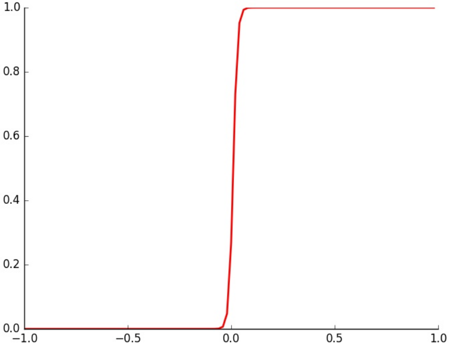
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 40$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



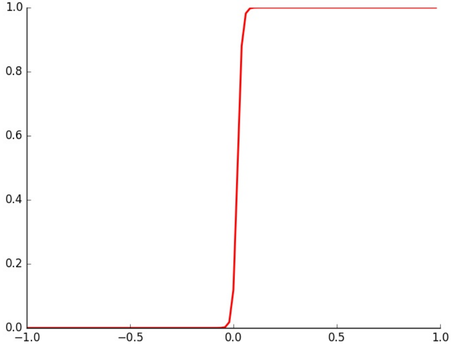
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 41$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



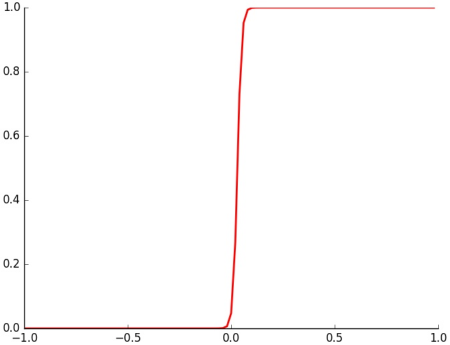
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 1$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



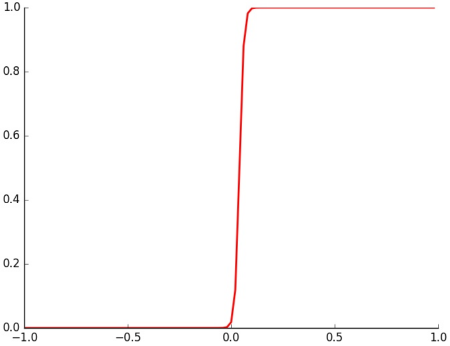
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 2$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



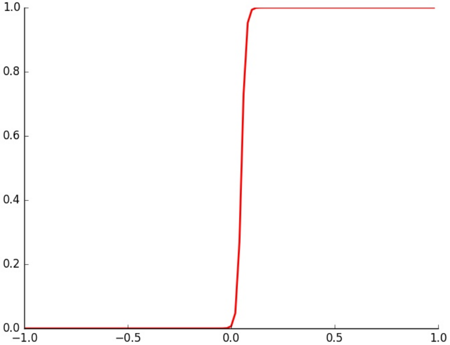
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 3$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



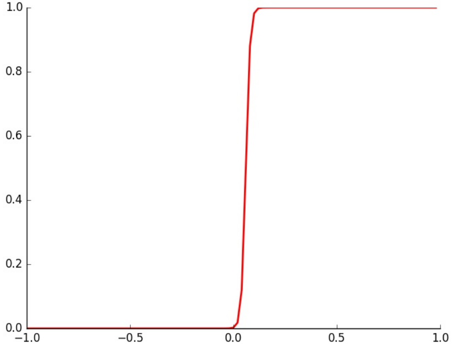
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 4$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



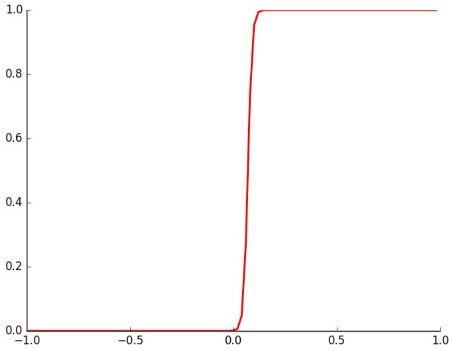
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \; w = 50, b = 5$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



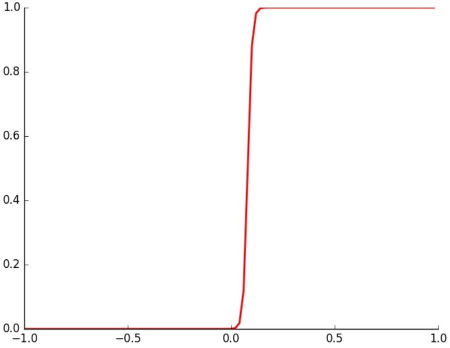
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 6$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



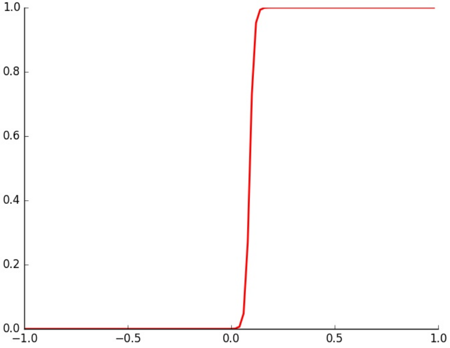
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 7$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



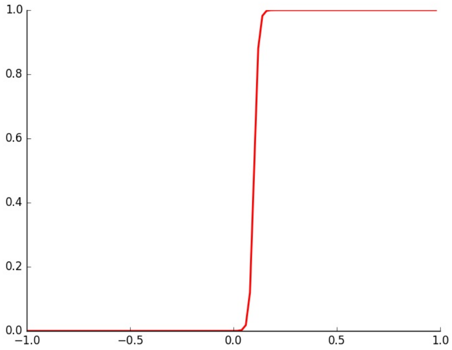
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 8$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



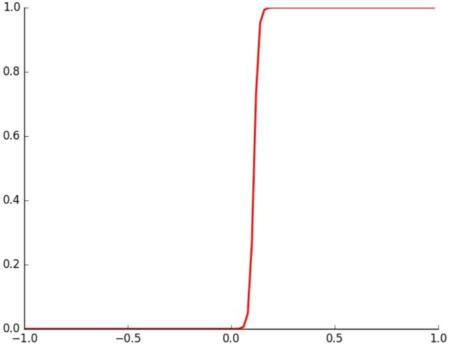
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 9$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



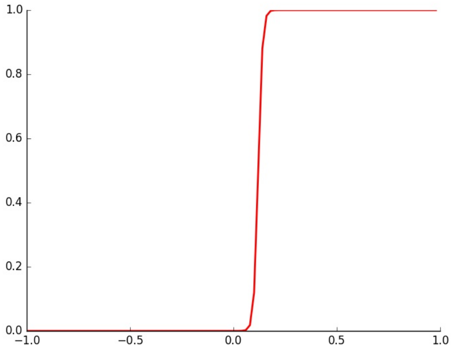
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 10$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



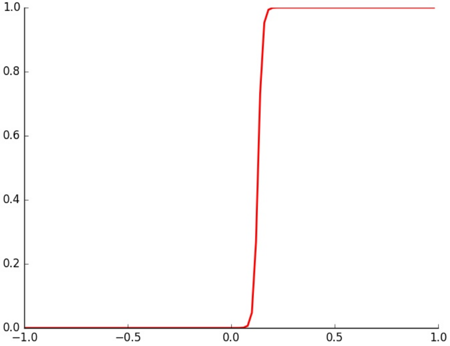
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 11$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



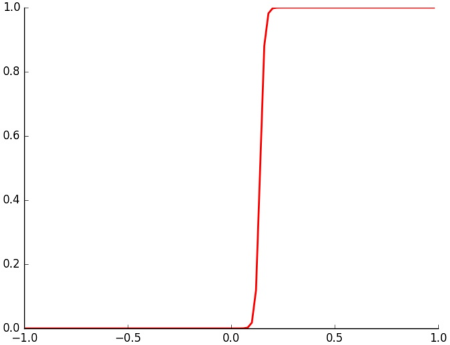
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 12$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



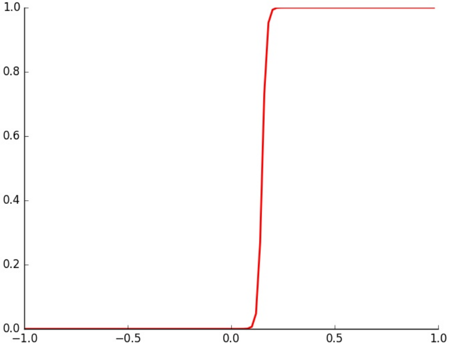
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 13$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



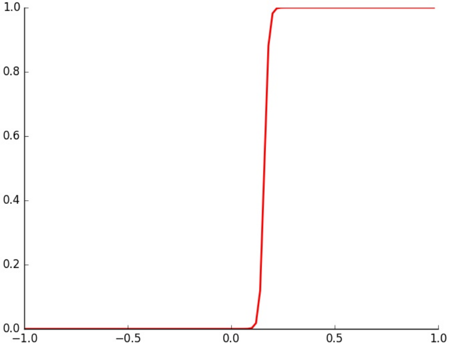
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 14$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



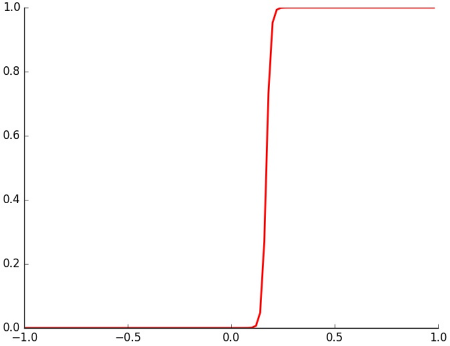
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 15$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



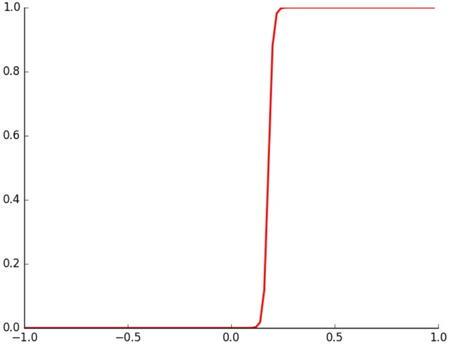
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 16$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



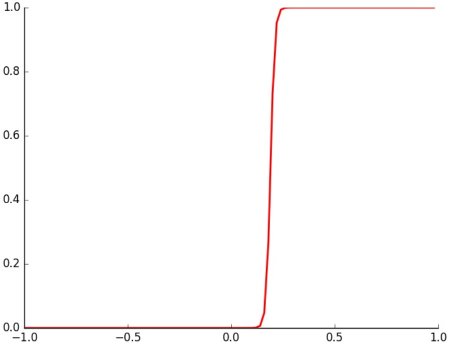
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 17$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



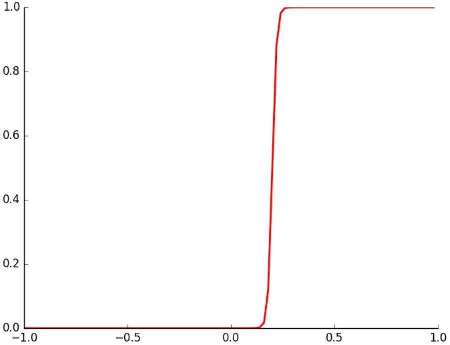
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 18$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



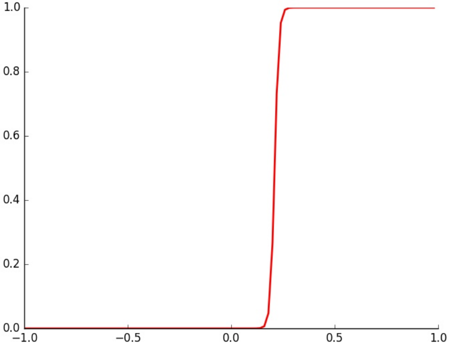
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 19$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



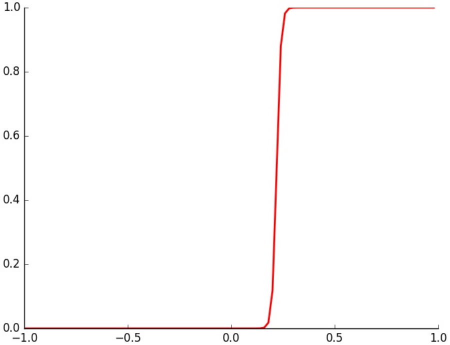
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 20$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



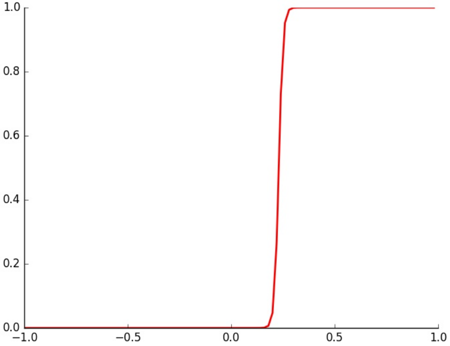
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 21$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



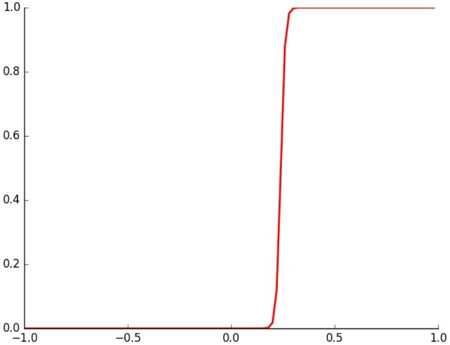
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 22$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



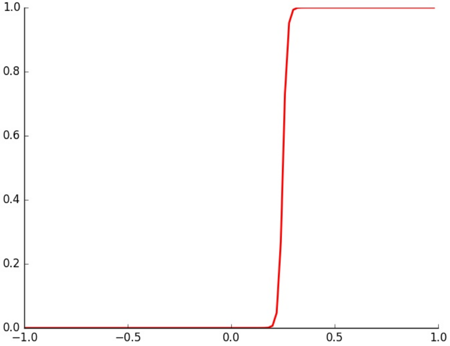
$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}}$ $w = 50, b = 23$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



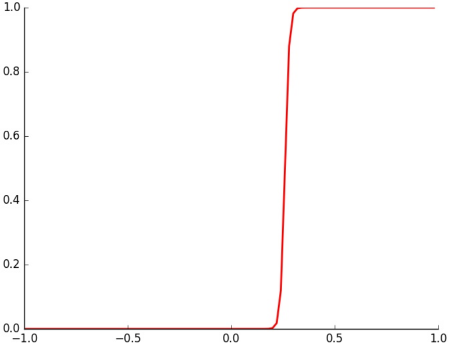
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 24$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



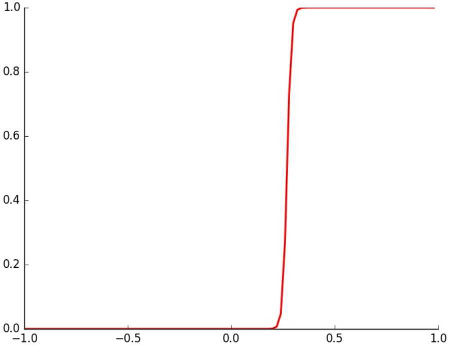
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 25$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



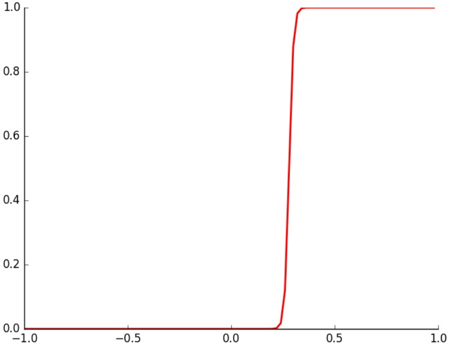
$$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}} \quad w = 50, b = 26$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



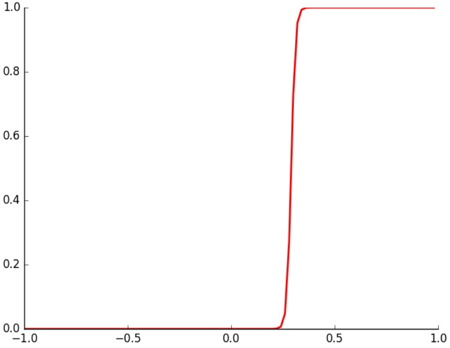
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 27$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



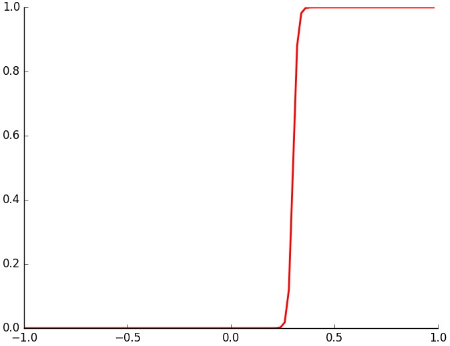
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 28$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



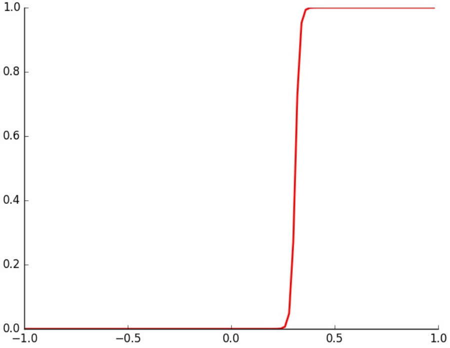
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 29$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



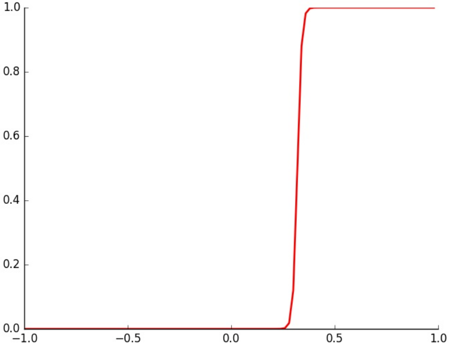
$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}}$ $w = 50, b = 30$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



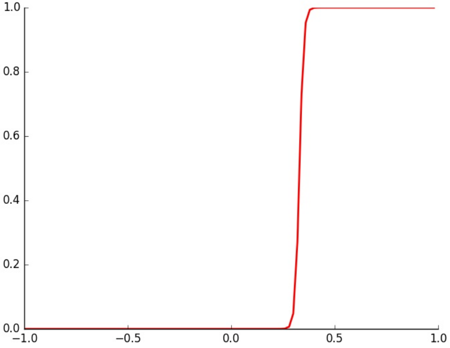
$\sigma(x) = \frac{1}{1-e^{-(wx+b)}}$ $w = 50, b = 31$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



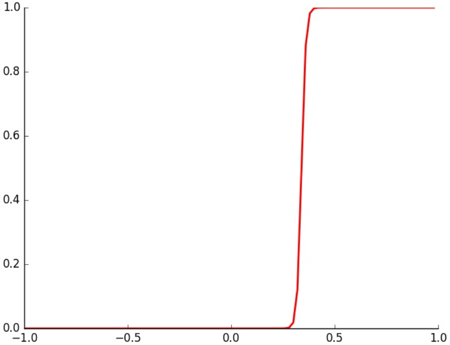
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 32$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



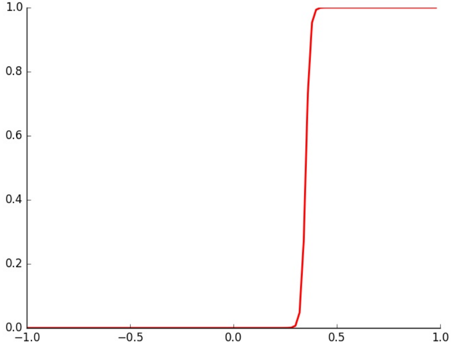
$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 33$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



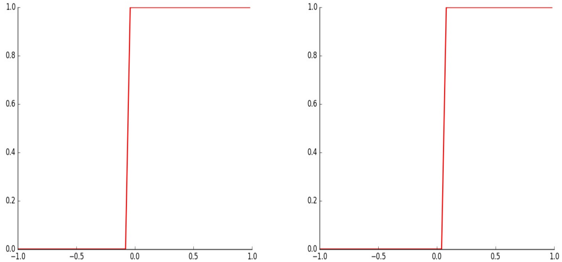
$\sigma(x) = \frac{1}{1 - e^{-(wx+b)}}$ $w = 50, b = 34$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

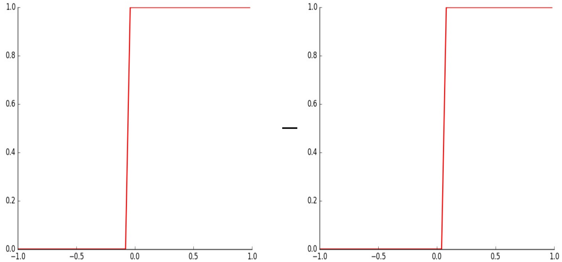


$$\sigma(x) = \frac{1}{1-e^{-(wx+b)}} \quad w = 50, b = 35$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



- Now let us see what we get by taking two such sigmoid functions (with different b's) and subtracting one from the other



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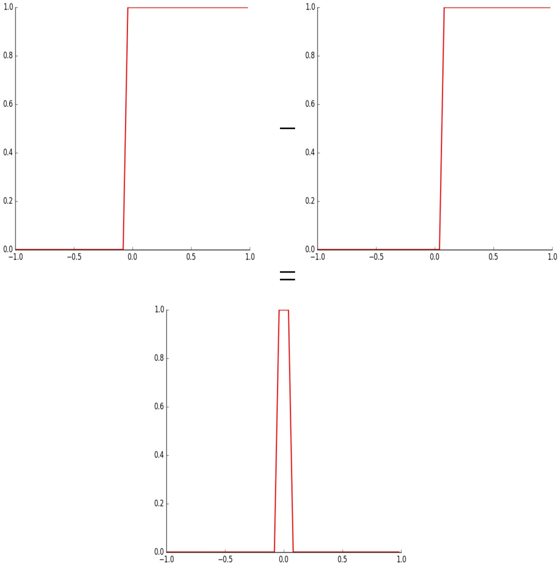
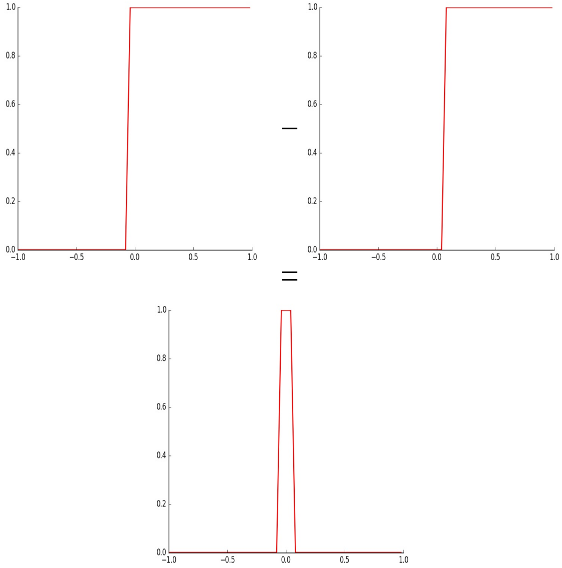
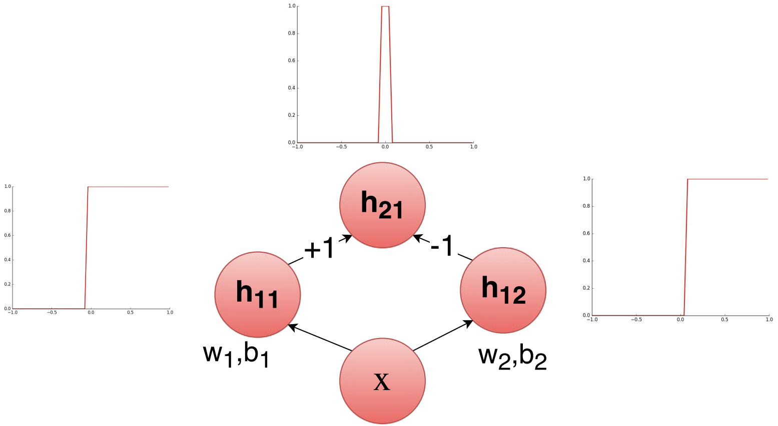


- Now let us see what we get by taking two such sigmoid functions (with different b's) and subtracting one from the other



- Now let us see what we get by taking two such sigmoid functions (with different b's) and subtracting one from the other
- Voila! We have our tower function !!

- Can we come up with a neural network to represent this operation of subtracting one sigmoid function from another ?


h21
+1
-1
h11
h12
w1,b1
x
w2,b2

- What if we have more than one input?

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- Suppose we are trying to take a decision about whether we will find oil at a particular location on the ocean bed(Yes/No)

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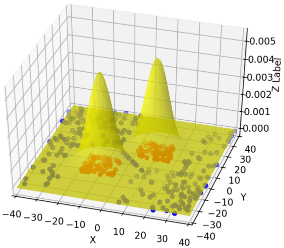
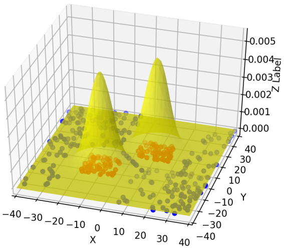
- Suppose we are trying to take a decision about whether we will find oil at a particular location on the ocean bed(Yes/No)
- Further, suppose we base our decision on two factors: Salinity (x_1) and Pressure (x_2)

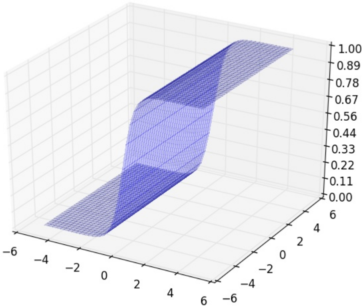


- What if we have more than one input?
- Suppose we are trying to take a decision about whether we will find oil at a particular location on the ocean bed(Yes/No)
- Further, suppose we base our decision on two factors: Salinity (x_1) and Pressure (x_2)
- We are given some data and it seems that y(oil|no-oil) is a complex function of x_1 and x_2



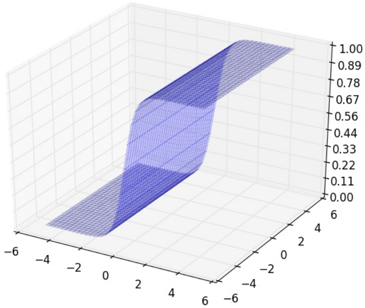
- What if we have more than one input?
- Suppose we are trying to take a decision about whether we will find oil at a particular location on the ocean bed(Yes/No)
- Further, suppose we base our decision on two factors: Salinity (x_1) and Pressure (x_2)
- We are given some data and it seems that y(oil|no-oil) is a complex function of x_1 and x_2
- We want a neural network to approximate this function

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



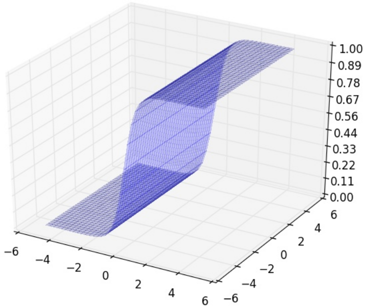
- This is what a 2-dimensional sigmoid looks like

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case

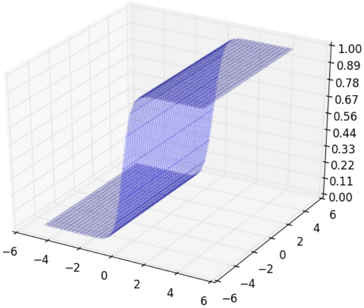
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 2, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

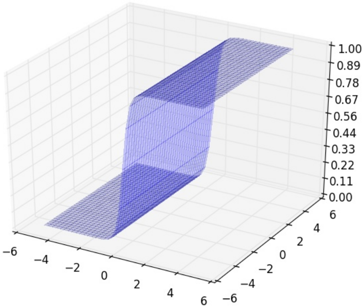
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 3, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

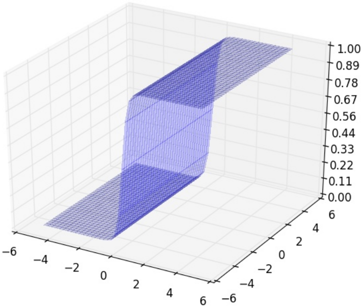
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 4, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

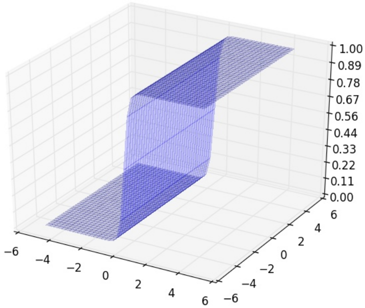
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 5, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

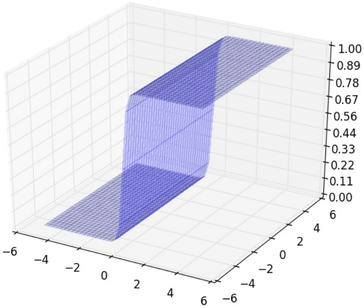
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 6, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

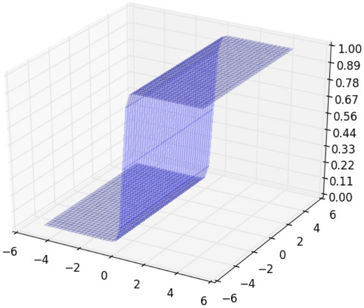
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 7, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

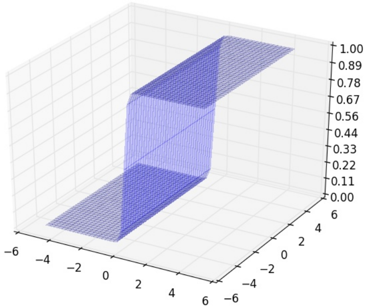
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 8, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

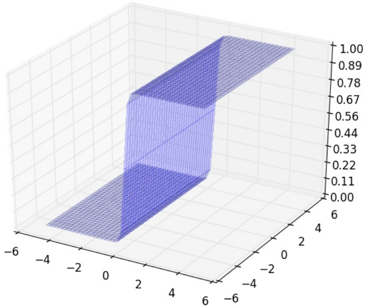
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 9, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

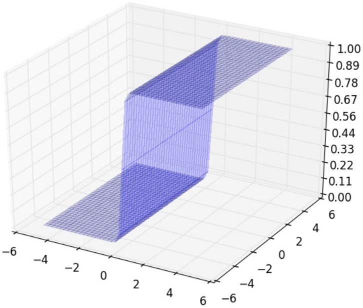
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 10, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

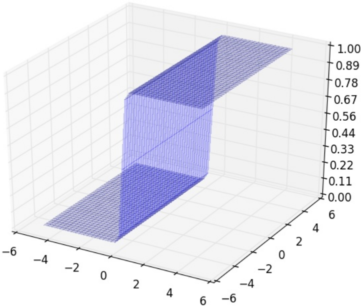
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 11, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

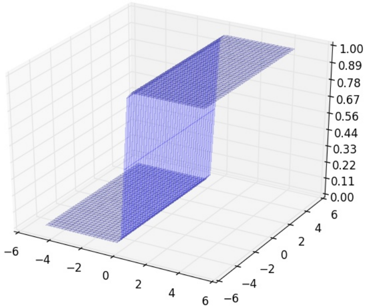
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 12, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

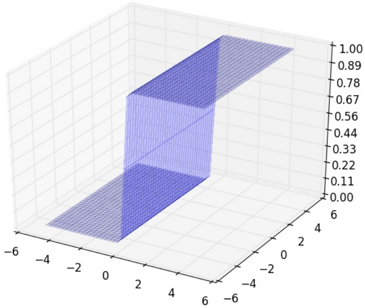
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 13, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

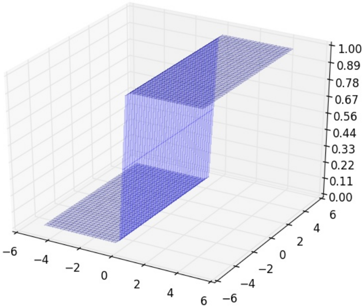
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 14, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

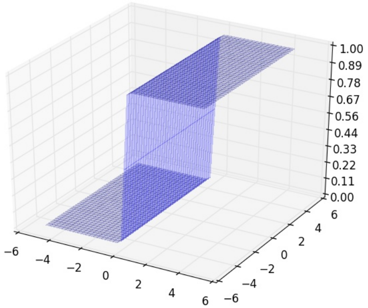
$$y = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2 + b)}}$$



$w_1 = 15, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

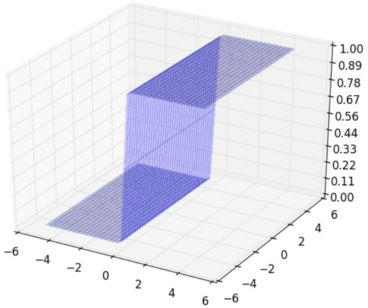
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 16, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

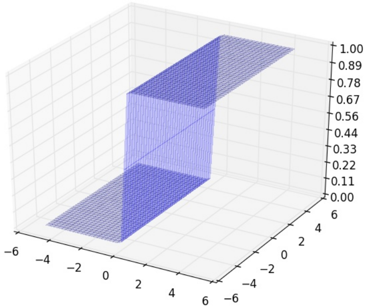
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 17, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

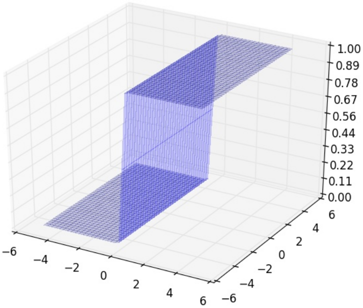
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 18, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

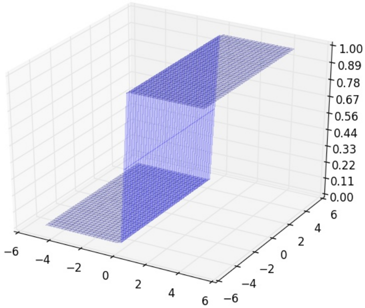
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 19, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

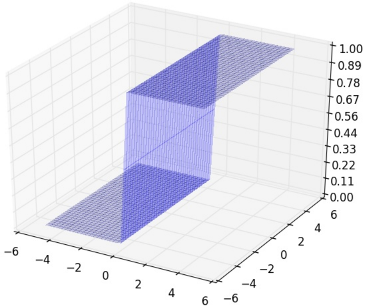
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 20, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

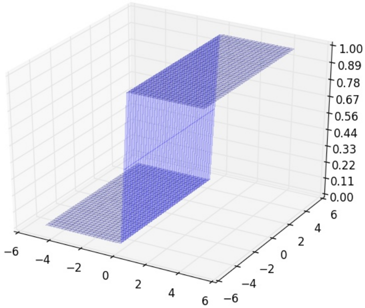
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 21, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

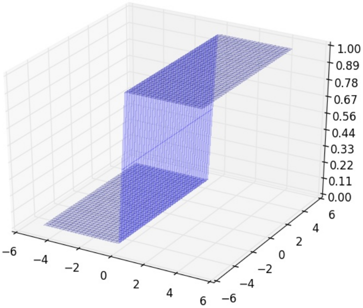
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 22, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

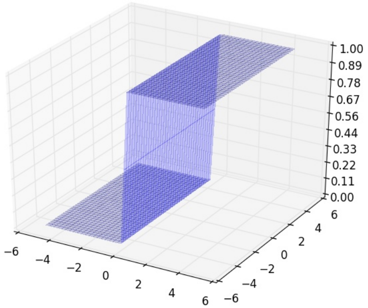
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 23, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

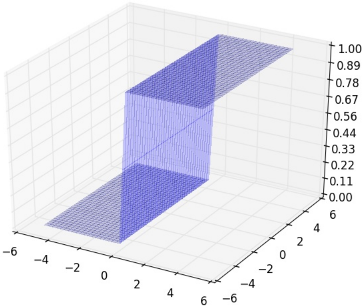
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$$w_1 = 24, w_2 = 0, b = 0$$

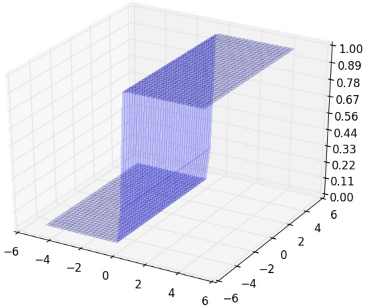
- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

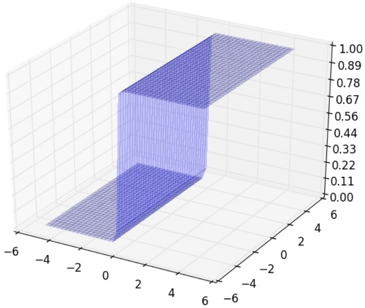
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 5$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

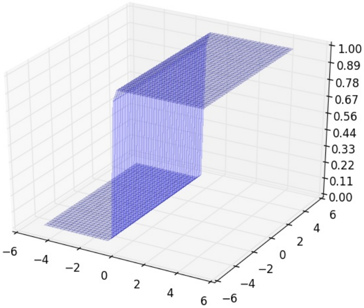
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 10$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

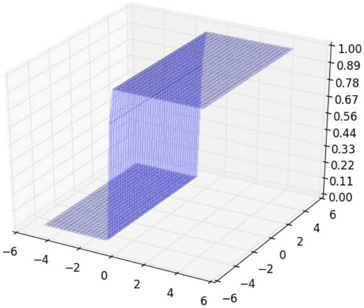
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 15$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

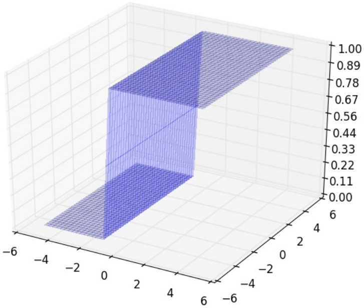
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 20$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

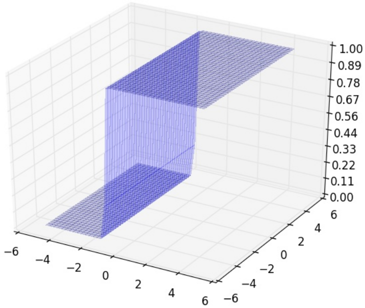
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 25$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

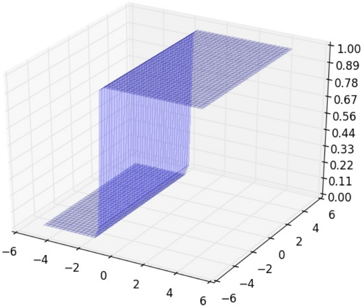
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 30$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

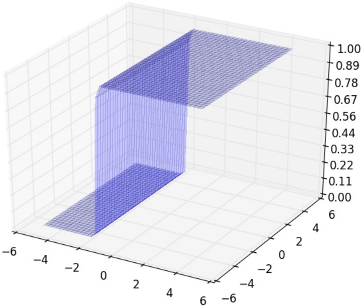
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 35$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

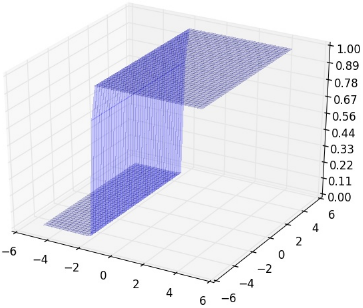
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 40$

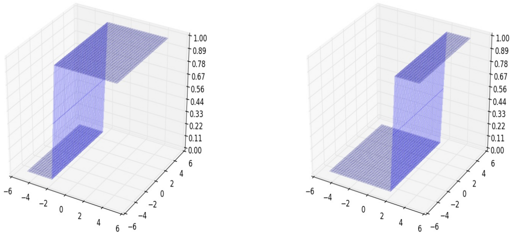
- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

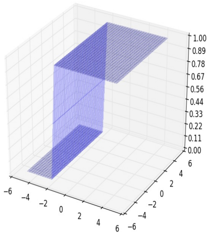


$w_1 = 25, w_2 = 0, b = 45$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



- What if we take two such step functions (with different b values) and subtract one from the other



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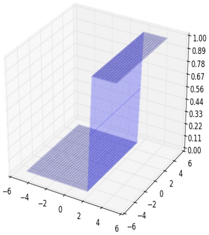
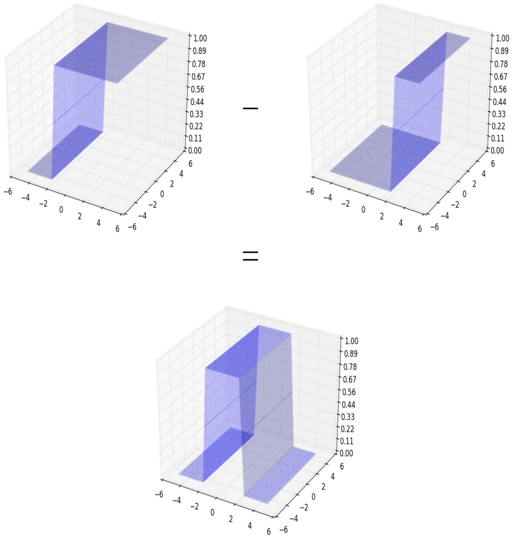
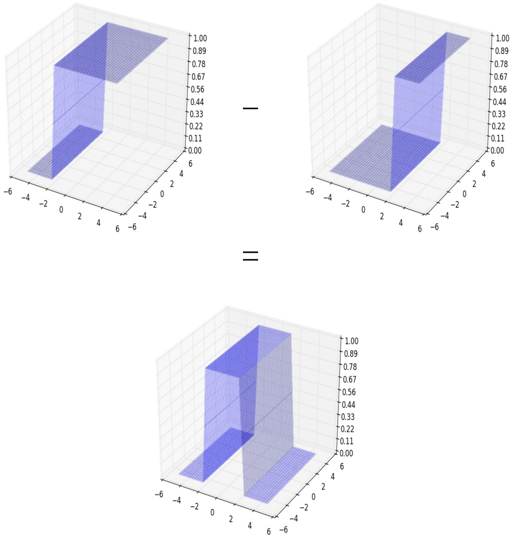


- What if we take two such step functions (with different b values) and subtract one from the other



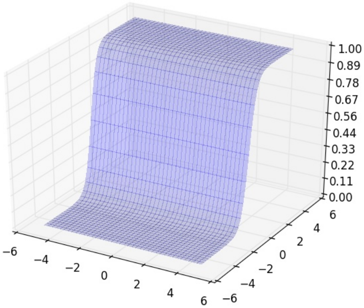
- What if we take two such step functions (with different b values) and subtract one from the other



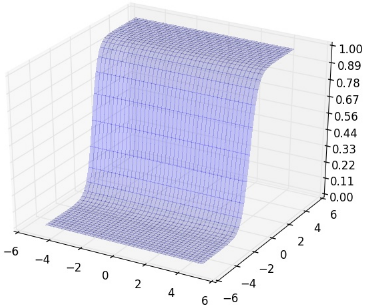
- What if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

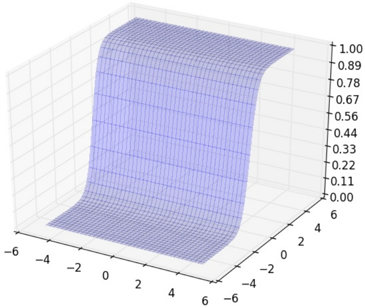


$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

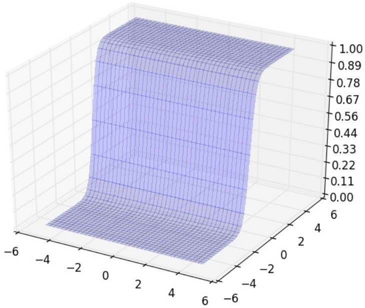
$$y = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2 + b)}}$$



$w_1 = 0, w_2 = 2, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

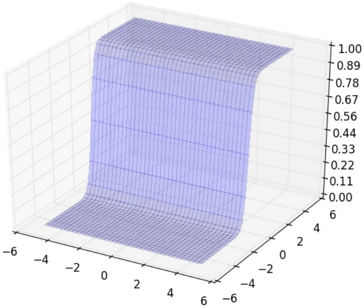
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 3, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

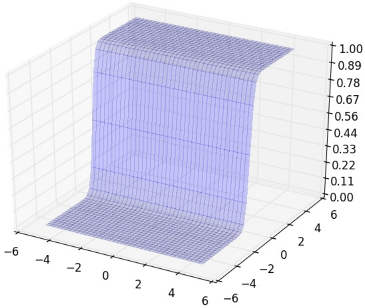
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$$w_1 = 0, w_2 = 4, b = 0$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

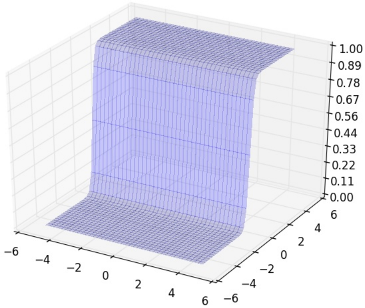
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 5, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

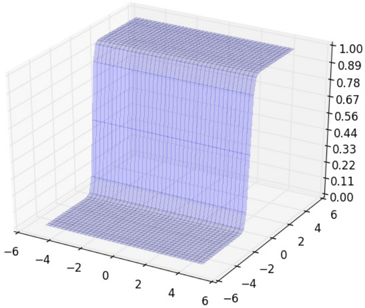
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 6, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

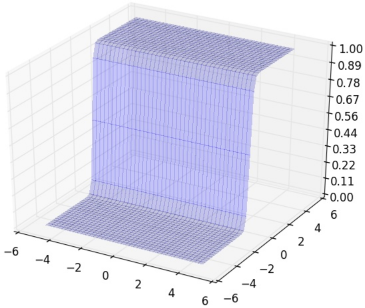


$w_1 = 0, w_2 = 7, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

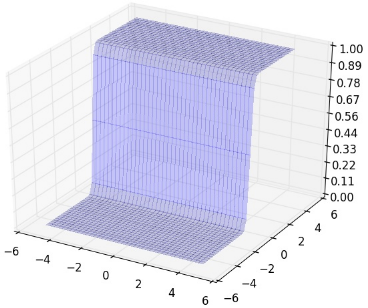
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation



$w_1 = 0, w_2 = 8, b = 0$

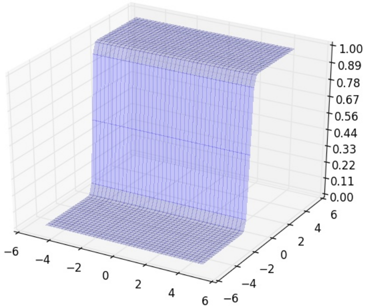
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 9, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

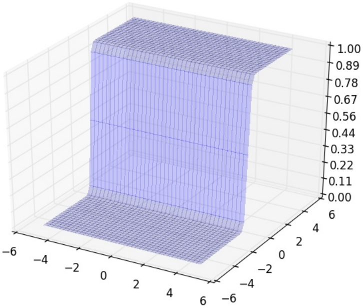
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 10, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

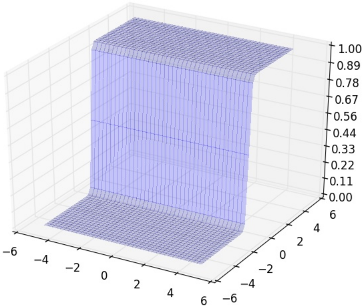
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 11, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

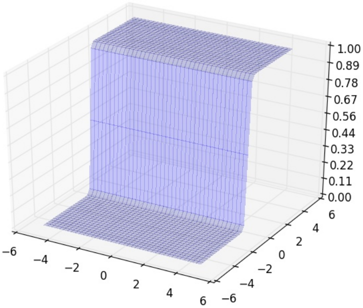
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 12, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

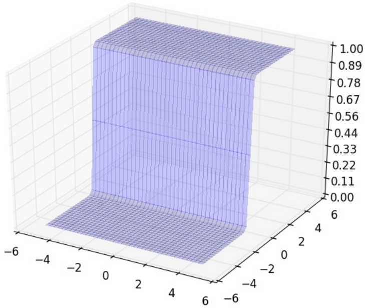
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 13, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

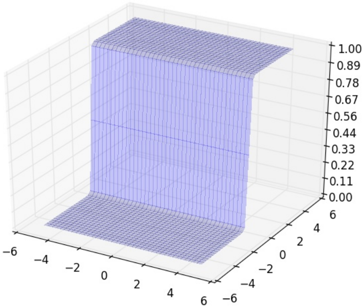


$w_1 = 0, w_2 = 14, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

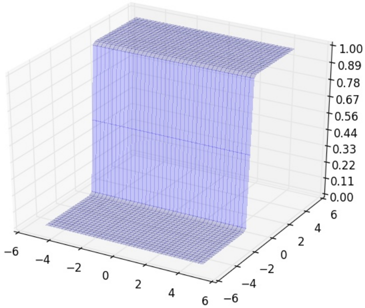
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation



$w_1 = 0, w_2 = 15, b = 0$

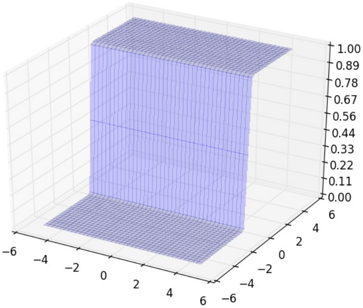
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 16, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

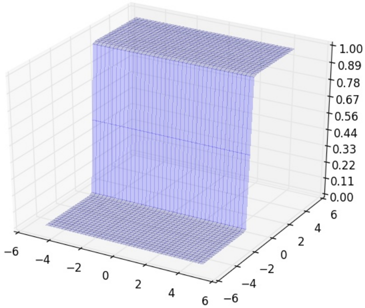
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 17, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

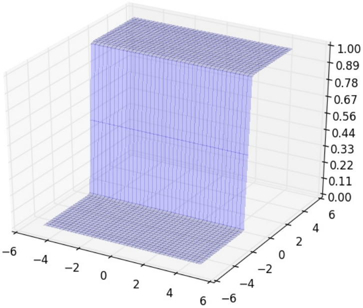
$$y = \frac{1}{1+e^{-(w_1x_1+w_2x_2+b)}}$$



$w_1 = 0, w_2 = 18, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

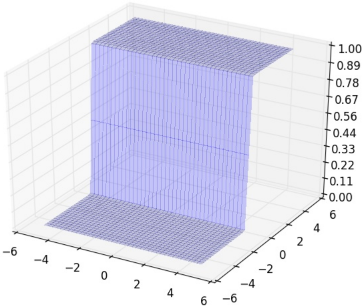
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 19, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

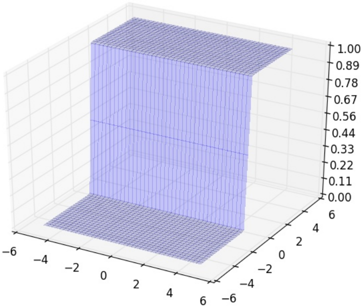
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 20, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

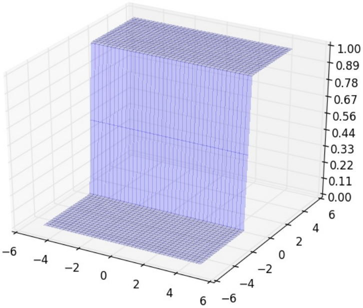


$w_1 = 0, w_2 = 21, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

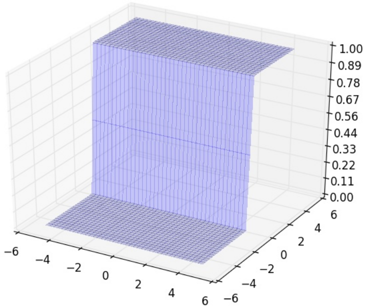
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation



$w_1 = 0, w_2 = 22, b = 0$

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

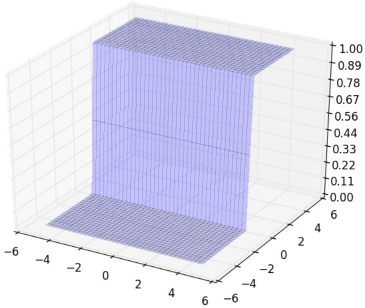


$w_1 = 0, w_2 = 23, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

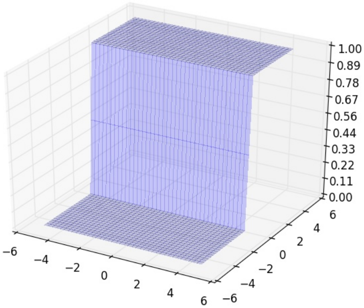
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation



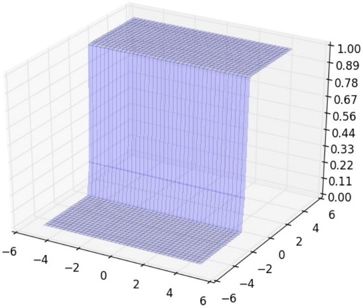
$w_1 = 0, w_2 = 24, b = 0$

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

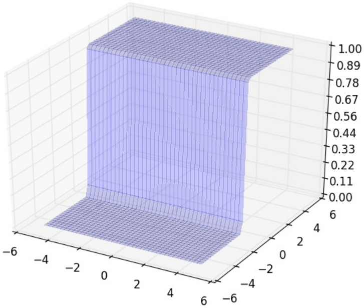
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 5$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

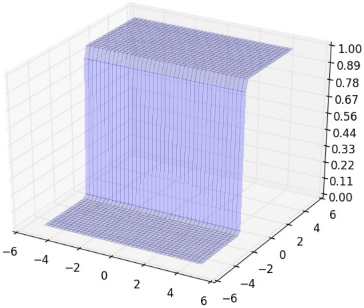
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 10$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

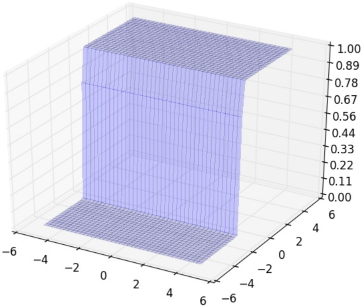
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 15$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

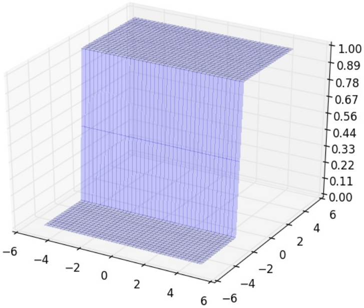
$$y = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 20$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

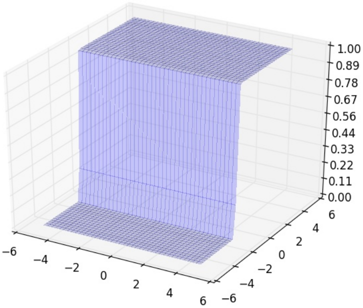
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 25$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

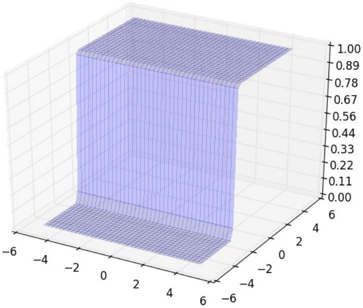
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 30$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 35$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
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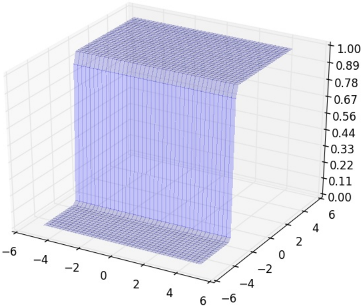
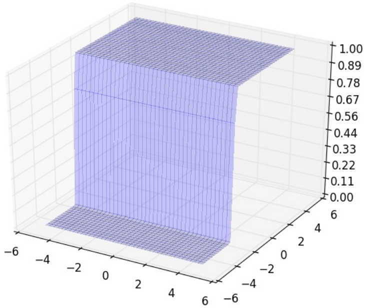
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 40$

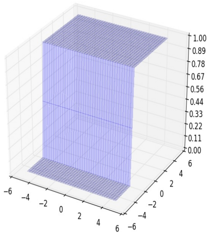
- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

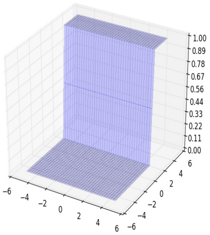
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



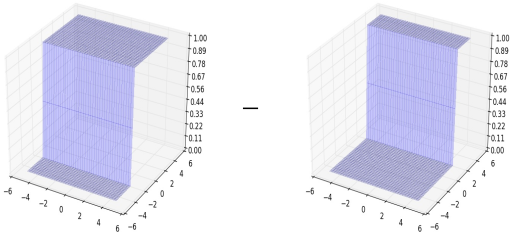
$w_1 = 0, w_2 = 25, b = 45$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b





- Again, what if we take two such step functions (with different b values) and subtract one from the other



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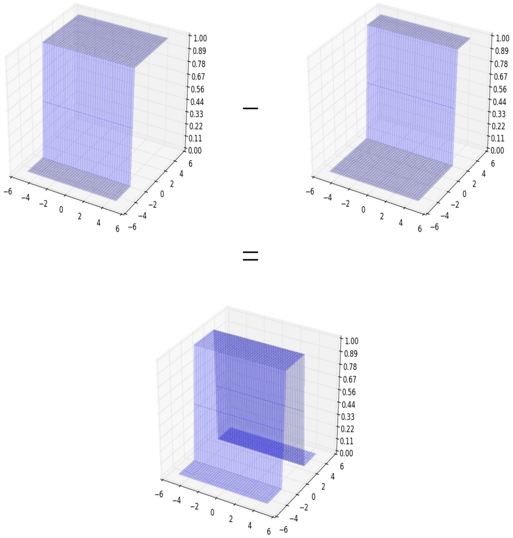

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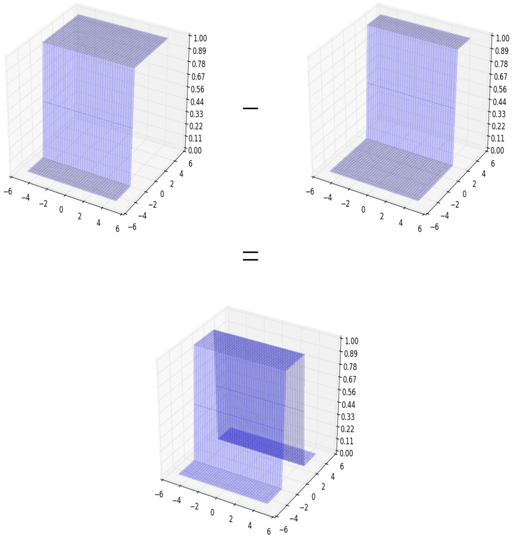
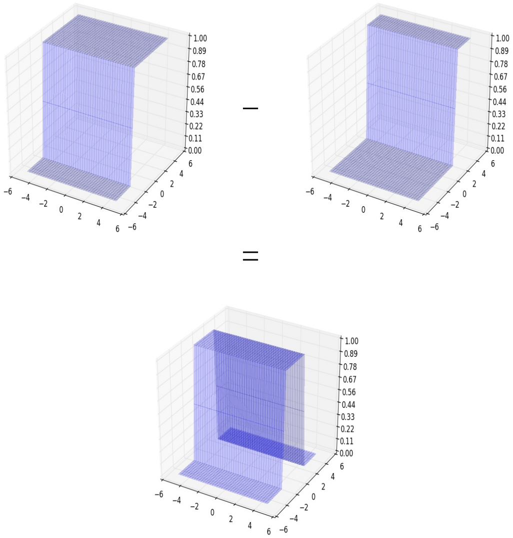
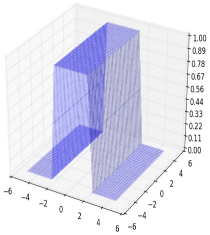
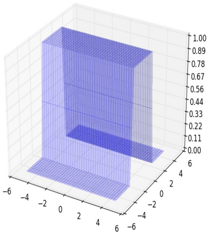


- Again, what if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)

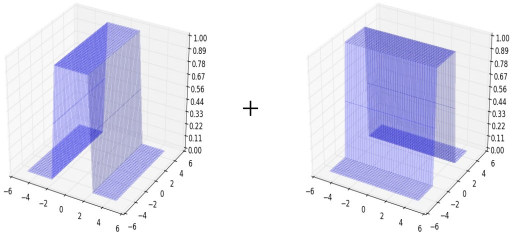


- Again, what if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)
- Notice that this open tower has a different orientation from the previous one





- Now what will we get by adding two such open towers ?



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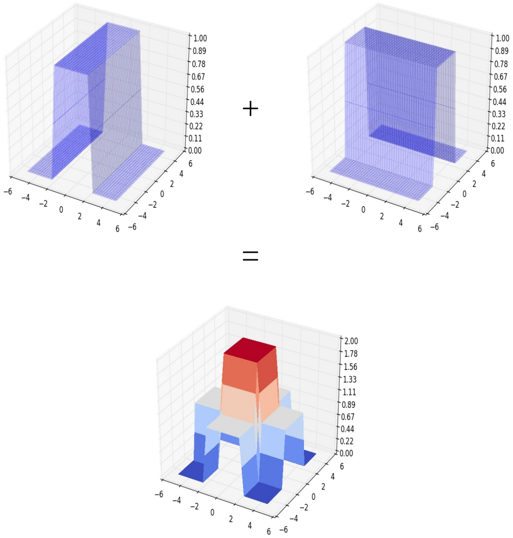
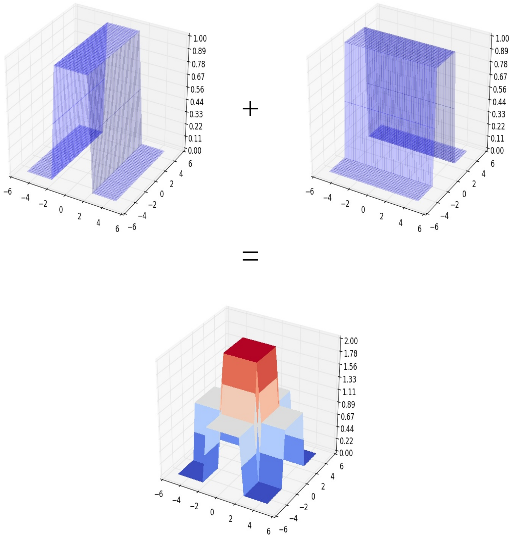
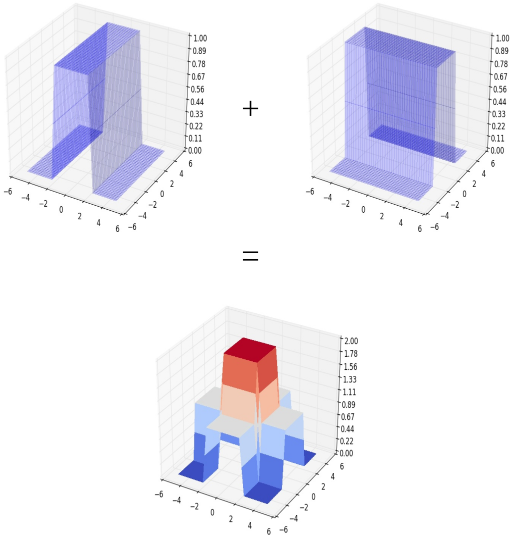
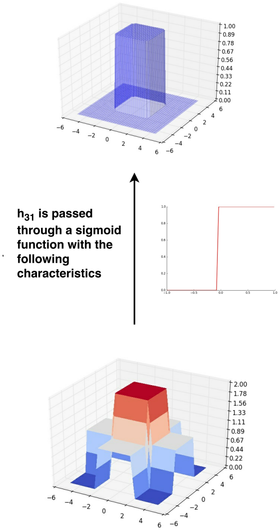




- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base



- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base
- We can now pass this output through another sigmoid neuron to get the desired tower !



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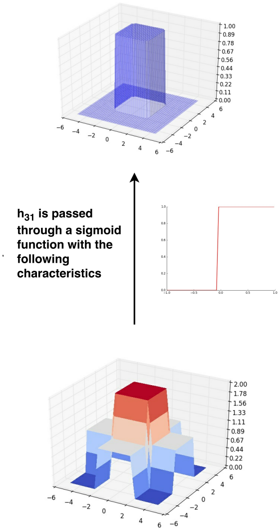
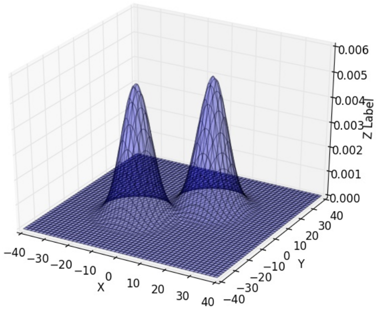
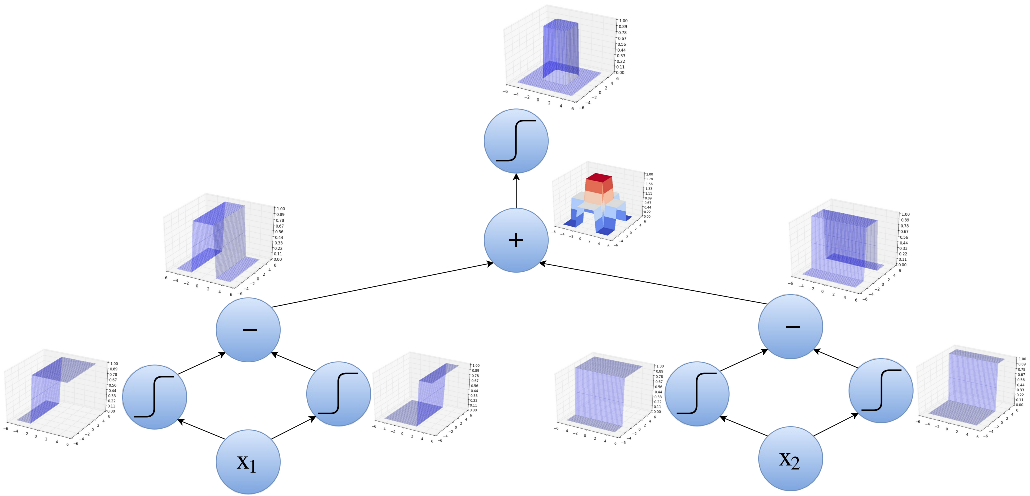


- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base
- We can now pass this output through another sigmoid neuron to get the desired tower !
- We can now approximate any function by summing up many such towers

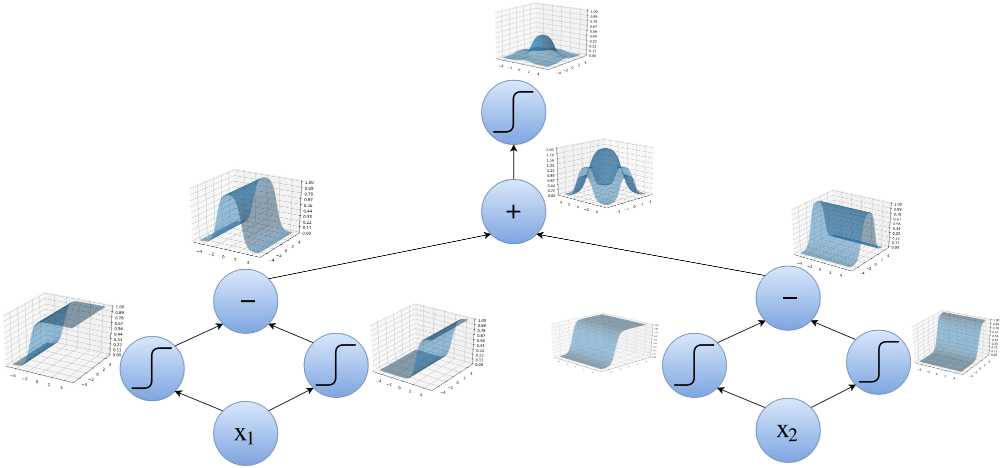


- For example, we could approximate the following function using a sum of several towers

- Can we come up with a neural network to represent this entire procedure of constructing a 3 dimensional tower ?



+
−
−
x1
x2

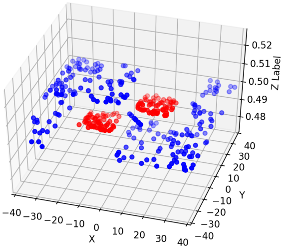

+
−
−
x_1
x_2

Think

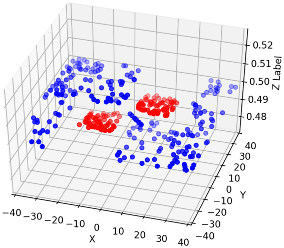
- For 1 dimensional input we needed 2 neurons to construct a tower
- For 2 dimensional input we needed 4 neurons to construct a tower
- How many neurons will you need to construct a tower in n dimensions ?

Time to retrospect

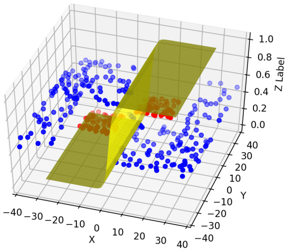
- Why do we care about approximating any arbitrary function ?
- Can we tie all this back to the classification problem that we have been dealing with ?



- We are interested in separating the blue points from the red points



- We are interested in separating the blue points from the red points
- Suppose we use a single sigmoidal neuron to approximate the relation between $x = [x_1, x_2]$ and y



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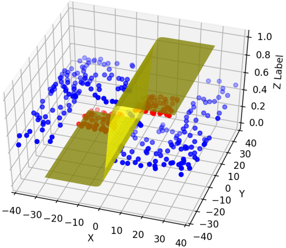

- We are interested in separating the blue points from the red points
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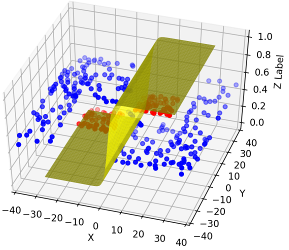
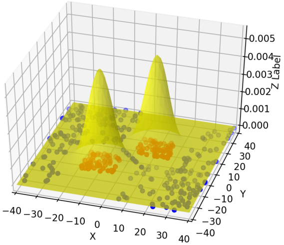
- Obviously, there will be errors (some blue points get classified as 1 and some red points get classified as 0)





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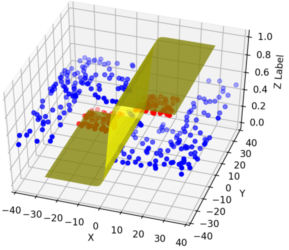
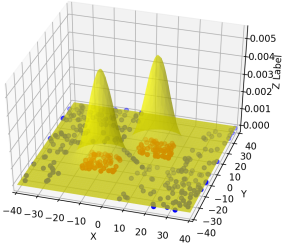
- This is what we actually want





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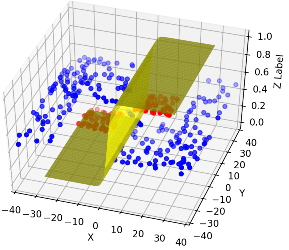
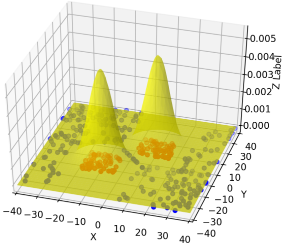
- This is what we actually want
- The illustrative proof that we just saw tells us that we can have a neural network with two hidden layers which can approximate the above function by a sum of towers





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- This is what we actually want
- The illustrative proof that we just saw tells us that we can have a neural network with two hidden layers which can approximate the above function by a sum of towers
- Which means we can have a neural network which can exactly separate the blue points from the red points !!