Lecture 3

Sigmoid Neurons, Gradient Descent, Feedforward Neural Networks, Representation Power of Feedforward Neural Networks

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Module 3.1: Sigmoid Neuron

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- Can we have a network which can (approximately) represent such functions ?
- Before answering the above question we will have to first graduate from ***perceptrons*** to ***sigmoidal neurons*** ...

Recall

- A perceptron will fire if the weighted sum of its inputs is greater than the threshold (-w_0)

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- If the threshold is 0.5 ($w_0 = -0.5$) and $w_1 = 1$ then what would be the decision for a movie with $criticsRating = 0.51$?

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- What about a movie with $criticsRating = 0.49$? (dislike)
- It seems harsh that we would like a movie with rating 0.51 but not one with a rating of 0.49

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- It is a characteristic of the perceptron function itself which behaves like a step function
- There will always be this sudden change in the decision (from 0 to 1) when $\sum_{i=1}^{n} w_i x_i$ crosses the threshold (-w_0)
- For most real world applications we would expect a smoother decision function which gradually changes from 0 to 1

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- Instead of a like/dislike decision we get the probability of liking the movie

Perceptron

$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$

$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

Sigmoid (logistic) Neuron

$$y = \frac{1}{1 + e^{-(\sum_{i=0}^{n} w_i x_i)}}$$

Perceptron

Not smooth, not continuous (at $w0$), **not differentiable**

Sigmoid Neuron

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Module 3.2: A typical Supervised Machine Learning Setup

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- Before we see such an algorithm we will revisit the concept of **error**

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- This line doesn't seem to be too bad
- Sure, it misclassifies 3 blue points and 3 red points but we could live with this error in **most** real world applications
- From now on, we will accept that it is hard to drive the error to 0 in most cases and will instead aim to reach the minimum possible error

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The learning algorithm should aim to find a w which minimizes the above function (squared error between y and $\hat{y}$)

Module 3.3: Learning Parameters: (Infeasible) guess work

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- Lastly, instead of considering the problem of predicting like/dislike we will assume that we want to predict $criticsRating(y)$ given $imdbRating(x)$ (for no particular reason)

x
w
σ
$\hat{y} = f(x)$
1
b

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Training objective

Find w and b such that:

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What does it mean to train the network?

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In other words...

- We hope to find a sigmoid function such that $(0.5, 0.2)$ and $(2.5, 0.9)$ lie on this sigmoid

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Let us see this in more detail....

(2.5, 0.9)
(0.5, 0.2)
1.0
0.8
0.6
0.4
0.2
0.0
y
−6
−4
−2
0
2
4
6
x

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- Let us try a random guess.. (say, $w = 0.5, b = 0$)
- Clearly not good, but how bad is it ?
- Let us revisit $\mathscr{L}(w, b)$ to see how bad it is ...

$$\mathscr{L}(w,b) = \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2$$

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$$= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2$$

$$
\begin{aligned}
\mathscr{L}(w,b) &= \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2 \\
&= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 \\
&= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2
\end{aligned}
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&= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2 \\
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\end{aligned}
$$

We want $\mathscr{L}(w,b)$ to be as close to 0 as possible

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730

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w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481

Oops!! this made things even worse...

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214

Perhaps it would help to push w and b in the other direction...

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028

Let us keep going in this direction, *i.e.*, increase w and decrease b

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1.65	-2.08	0.0003

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0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028
1.65	-2.08	0.0003
1.78	-2.27	0.0000

With some guess work and intuition we were able to find the right values for w and b

Let us look at something better than our "guess work" algorithm....

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- But of course this becomes intractable once you have many more data points and many more parameters !!
- Further, even here we have plotted the error surface only for a small range of $(w,\ b)$ [from $(-6, 6)$ and not from $(-\inf, \inf)$]

Let us look at the geometric interpretation of our "guess work" algorithm in terms of this error surface

(2.5, 0.9)
(0.5, 0.2)
y
x
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0.8
0.6
0.4
0.2
0.0
−6
−4
−2
0
2
4
6

Random search on error surface
error
w
b
1.0
0.5
0.0
−0.5
−1.0
0.08
0.16
0.24
0.32
0.40
0.48
0.56
0.64

(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
y
x

Random search on error surface
error
w
b

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w=0.50, b=0.00
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w=1.65, b=-2.08
w=1.79, b=-2.28
y
x

Random search on error surface
error
w
b
0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64

Module 3.4: Learning Parameters : Gradient Descent

- With this setup in mind, we will now focus on this **model** and discuss an **algorithm** for learning the **parameters** of this model from some given **data**

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- For ease of explanation, we will consider a very simplified version of the model having just 1 input
- Further to be consistent with the literature, from now on we will refer to w_0 as b (bias)
- Lastly, instead of considering the problem of predicting like/dislike we will assume that we want to predict $criticsRating(y)$ given $imdbRating(x)$ (for no particular reason)

x
σ
$\hat{y} = f(x)$
1

Input for training

$\{x_i, y_i\}_{i=1}^{N} \rightarrow N$ pairs of (x, y)

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$\{x_i, y_i\}_{i=1}^{N} \rightarrow N$ pairs of (x, y)

Training objective

Find w and b such that:

$$\underset{w,b}{\text{minimize}}\, \mathscr{L}(w, b) = \sum_{i=1}^{N} (y_i - f(x_i))^2$$

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What does it mean to train the network?

- Suppose we train the network with $(x, y) = (0.5, 0.2)$ and $(2.5, 0.9)$

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In other words...

- We hope to find a sigmoid function such that $(0.5, 0.2)$ and $(2.5, 0.9)$ lie on this sigmoid

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Let us see this in more detail....

(2.5, 0.9)
(0.5, 0.2)
1.0
0.8
0.6
0.4
0.2
0.0
y
−6
−4
−2
0
2
4
6
x

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- Let us try a random guess.. (say, $w = 0.5, b = 0$)
- Clearly not good, but how bad is it ?
- Let us revisit $\mathscr{L}(w, b)$ to see how bad it is ...

$$\mathscr{L}(w,b) = \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2$$

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$$= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2$$

$$
\begin{aligned}
\mathscr{L}(w,b) &= \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2 \\
&= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 \\
&= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{L}(w,b) &= \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2 \\
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\end{aligned}
$$

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&= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2 \\
&= 0.073
\end{aligned}
$$

We want $\mathscr{L}(w,b)$ to be as close to 0 as possible

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481

Oops!! this made things even worse...

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214

Perhaps it would help to push w and b in the other direction...

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028

Let us keep going in this direction, *i.e.*, increase w and decrease b

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1.65	-2.08	0.0003
1.78	-2.27	0.0000

With some guess work and intuition we were able to find the right values for w and b

Let us look at something better than our "guess work" algorithm....

- Since we have only 2 points and 2 parameters $(w,\ b)$ we can easily plot $\mathscr{L}(w, b)$ for different values of $(w,\ b)$ and pick the one where $\mathscr{L}(w, b)$ is minimum

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- But of course this becomes intractable once you have many more data points and many more parameters !!

- Since we have only 2 points and 2 parameters $(w,\ b)$ we can easily plot $\mathscr{L}(w, b)$ for different values of $(w,\ b)$ and pick the one where $\mathscr{L}(w, b)$ is minimum
- But of course this becomes intractable once you have many more data points and many more parameters !!
- Further, even here we have plotted the error surface only for a small range of $(w,\ b)$ [from $(-6, 6)$ and not from $(-\inf, \inf)$]

Let us look at the geometric interpretation of our "guess work" algorithm in terms of this error surface

(2.5, 0.9)
(0.5, 0.2)
y
x
1.0
0.8
0.6
0.4
0.2
0.0
−6
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Random search on error surface
error
w
b
1.0
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Random search on error surface
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0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64

Now let us see if there is a more efficient and principled way of doing this

Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!

vector of parameters, say, randomly initialized

$$\theta = [w, b]$$

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$$\theta = [w, b]$$

$$\Delta\theta = [\Delta w, \Delta b]$$

change in the values of w, b

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Let us be a bit conservative: move only by a small amount η

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change in the values of w, b

$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$

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Question: What is the right $\Delta\theta$ to use ?

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The answer comes from Taylor series

For ease of notation, let $\Delta\theta = u$, then from Taylor series, we have,

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$$\mathscr{L}(\theta + \eta u) = \mathscr{L}(\theta) + \eta * u^T \nabla \mathscr{L}(\theta) + \frac{\eta^2}{2!} * u^T \nabla^2 \mathscr{L}(\theta) u + \frac{\eta^3}{3!} * ... + \frac{\eta^4}{4!} * ...$$

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Note that the move (ηu) would be favorable only if,

$$\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) < 0 [\textit{i.e., if the new loss is less than the previous loss}]$$

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This implies,

$$u^T \nabla \mathscr{L}(\theta) < 0$$

Okay, so we have,

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But, what is the range of $u^T \nabla \mathscr{L}(\theta)$?

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$$-1 \leq cos(\beta) = \frac{u^T \nabla \mathscr{L}(\theta)}{||u|| * ||\nabla \mathscr{L}(\theta)||} \leq 1$$

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multiply throughout by $k = ||u|| * ||\nabla \mathscr{L}(\theta)||$

$$-k \leq k * cos(\beta) = u^T \nabla \mathscr{L}(\theta) \leq k$$

Okay, so we have,

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multiply throughout by $k = ||u|| * ||\nabla \mathscr{L}(\theta)||$

$$-k \leq k * cos(\beta) = u^T \nabla \mathscr{L}(\theta) \leq k$$

Thus, $\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) = u^T \nabla \mathscr{L}(\theta) = k * cos(\beta)$ will be most negative when $cos(\beta) = -1$ *i.e.*, when β is $180°$

Gradient Descent Rule

- The direction u that we intend to move in should be at 180° w.r.t. the gradient

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Parameter Update Equations

$$w_{t+1} = w_t - \eta \nabla w_t$$
$$b_{t+1} = b_t - \eta \nabla b_t$$
$$where, \nabla w_t = \frac{\partial \mathscr{L}(w,b)}{\partial w}_{at\ w = w_t, b = b_t}, \nabla b = \frac{\partial \mathscr{L}(w,b)}{\partial b}_{at\ w = w_t, b = b_t}$$

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So we now have a more principled way of moving in the w-b plane than our "guess work" algorithm

- Let us create an algorithm from this rule ...

- Let us create an algorithm from this rule ...

Algorithm: gradient_descent()

$t \leftarrow 0$;
$max_iterations \leftarrow 1000$;
while $t < max_iterations$ **do**
  $w_{t+1} \leftarrow w_t - \eta \nabla w_t$;
  $b_{t+1} \leftarrow b_t - \eta \nabla b_t$;
end

- To see this algorithm in practice lest first derive ∇w and ∇b for our toy neural network

$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

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Let's assume there is only 1 point to fit (x, y)

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Let's assume there is only 1 point to fit (x, y)

$$\mathscr{L}(w, b) = \frac{1}{2} * (f(x) - y)^2$$

$$\nabla w = \frac{\partial \mathscr{L}(w, b)}{\partial w} = \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2]$$

$$\nabla w = \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2]$$

$$
\begin{aligned}
\nabla w &= \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2] \\
&= \frac{1}{2} * [2 * (f(x) - y) * \frac{\partial}{\partial w}(f(x) - y)]
\end{aligned}
$$

$$
\begin{aligned}
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X = [0.5, 2.5]
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def f(w,b,x) : #sigmoid with parameters w,b
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def error (w, b) :
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X = [0.5, 2.5]
Y = [0.2, 0.9]

def f(w,b,x) : #sigmoid with parameters w,b
    return 1.0 / (1.0 + np.exp(-(w*x + b)))

def error (w, b) :
    err = 0.0
    for x,y in zip(X,Y) :
        fx = f(w,b,x)
        err += 0.5 * (fx - y) ** 2
    return err

def grad_b(w,b,x,y) :
    fx = f(w,b,x)
    return (fx - y) * fx * (1 - fx)

def grad_w(w,b,x,y) :
    fx = f(w,b,x)
    return (fx - y) * fx * (1 - fx) * x

def do_gradient_descent() :
    w, b, eta, max_epochs = -2, -2, 1.0, 1000
    for i in range(max_epochs) :
        dw, db = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)
        w = w - eta * dw
        b = b - eta * db
```

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Gradient descent on the error surface

- Later on in the course we will look at gradient descent in much more detail and discuss its variants

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- Later on in the course we will look at gradient descent in much more detail and discuss its variants
- For the time being it suffices to know that we have an algorithm for learning the parameters of a sigmoid neuron
- So where do we head from here ?

Representation Power of Multilayer Network of Sigmoid Neurons

Representation power of a multilayer network of perceptrons

Representation power of a multilayer network of sigmoid neurons

Representation power of a multilayer network of perceptrons A

multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

Representation power of a multilayer network of sigmoid neurons

Representation power of a multilayer network of perceptrons

A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

Representation power of a multilayer network of sigmoid neurons

A multilayer network of neurons with a single hidden layer can be used to approximate any continuous function to any desired precision

Representation power of a multilayer network of perceptrons

A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

Representation power of a multilayer network of sigmoid neurons

A multilayer network of neurons with a single hidden layer can be used to approximate any continuous function to any desired precision In other words, there is a guarantee that for any function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$, we can always find a neural network (with 1 hidden layer containing enough neurons) whose output g(x) satisfies $|g(x) - f(x)| < \epsilon$!!

Representation power of a multilayer network of perceptrons

A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

Representation power of a multilayer network of sigmoid neurons

A multilayer network of neurons with a single hidden layer can be used to approximate any continuous function to any desired precision In other words, there is a guarantee that for any function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$, we can always find a neural network (with 1 hidden layer containing enough neurons) whose output g(x) satisfies $|g(x) - f(x)| < \epsilon$!!

Proof: We will see an illustrative proof of this... [Cybenko, 1989], [Hornik, 1991]

- See this link* for an excellent illustration of this proof
- The discussion in the next few slides is based on the ideas presented at the above link

*http://neuralnetworksanddeeplearning.com/chap4.html

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- We are interested in knowing whether a network of neurons can be used to represent an arbitrary function (like the one shown in the figure)
- We observe that such an arbitrary function can be approximated by several "tower" functions
- More the number of such "tower" functions, better the approximation
- To be more precise, we can approximate any arbitrary function by a sum of such "tower" functions

- We make a few observations

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- We make a few observations
- All these "tower" functions are similar and only differ in their heights and positions on the x-axis
- Suppose there is a black box which takes the original input (x) and constructs these tower functions
- We can then have a simple network which can just add them up to approximate the function
- Our job now is to figure out what is inside this blackbox

We will figure this out over the next few slides ...

- If we take the logistic function and set w to a very high value we will recover the step function

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w

$w = 6, b = 0, 3$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w

$w = 7, b = 0, 4$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w

$w = 8, b = 0, 5$

- If we take the logistic function and set w to a very high value we will recover the step function
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$w = 9, b = 0,6$

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$w = 10, b = 0, 7$

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$w = 11, b = 0,8$

$w = 12, b = 0, 9$

- If we take the logistic function and set w to a very high value we will recover the step function
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$w = 13, b = 0, 10$

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$w = 14, b = 0, 11$

$w = 15, b = 0, 12$

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$w = 16, b = 0, 13$

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$w = 17, b = 0,14$

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$w = 18, b = 0,15$

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$w = 19, b = 0,16$

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$w = 20, b = 0, 17$

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- Let us see what happens as we change the value of w

$w = 21, b = 0, 18$

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- Let us see what happens as we change the value of w

$w = 22, b = 0, 19$

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- Let us see what happens as we change the value of w

$w = 23, b = 0, 20$

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$w = 24, b = 0,21$

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$w = 25, b = 0, 22$

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$w = 26, b = 0,23$

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$w = 27, b = 0,24$

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$w = 28, b = 0,25$

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$w = 29, b = 0,26$

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$w = 30, b = 0,27$

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$w = 31, b = 0,28$

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$w = 32, b = 0,29$

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$w = 33, b = 0,30$

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$w = 34, b = 0,31$

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$w = 35, b = 0,32$

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$w = 36, b = 0,33$

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$w = 37, b = 0,34$

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$w = 38, b = 0,35$

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$w = 39, b = 0,36$

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$w = 40, b = 0,37$

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$w = 41, b = 0,38$

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$w = 42, b = 0,39$

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$w = 43, b = 0,40$

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- Let us see what happens as we change the value of w

$w = 44, b = 0,41$

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- Let us see what happens as we change the value of w

$w = 45, b = 0, 42$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w

$w = 46, b = 0,43$

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- Let us see what happens as we change the value of w

$w = 47, b = 0,44$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w

$w = 50, b = 1$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$w = 50, b = 2$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$w = 50, b = 3$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
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$w = 50, b = 4$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
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$w = 50, b = 5$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$w = 50, b = 6$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$w = 50, b = 7$

- If we take the logistic function and set w to a very high value we will recover the step function
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$w = 50, b = 8$

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$w = 50, b = 9$

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$w = 50, b = 12$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
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$w = 50, b = 14$

- If we take the logistic function and set w to a very high value we will recover the step function
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$w = 50, b = 24$

- If we take the logistic function and set w to a very high value we will recover the step function
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$w = 50, b = 25$

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$w = 50, b = 26$

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$w = 50, b = 28$

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$w = 50, b = 29$

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- Let us see what happens as we change the value of w
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$w = 50, b = 30$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$w = 50, b = 31$

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- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$w = 50, b = 32$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$w = 50, b = 33$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$w = 50, b = 34$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$w = 50, b = 35$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

- Now let us see what we get by taking two such sigmoid functions (with different bs) and subtracting one from the other

−

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- Now let us see what we get by taking two such sigmoid functions (with different bs) and subtracting one from the other

- Now let us see what we get by taking two such sigmoid functions (with different bs) and subtracting one from the other
- Voila! We have our tower function !!

- Can we come up with a neural network to represent this operation of subtracting one sigmoid function from another ?

h21
h11
h12
x

What if we have more than 1 input ?

- This is what a 3-dimensional sigmoid looks like

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 2, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 3, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 4, w_2 = 0, b = 0$

$w_1 = 5, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 6, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 7, w_2 = 0, b = 0$

$w_1 = 8, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 9, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 10, w_2 = 0, b = 0$

$w_1 = 11, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
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$w_1 = 12, w_2 = 0, b = 0$

$w_1 = 13, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 14, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 15, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

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- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 16, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 17, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 18, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 19, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 20, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 21, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 22, w_2 = 0, b = 0$

$w_1 = 23, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$w_1 = 24, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$w_1 = 25, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$w_1 = 25, w_2 = 0, b = 5$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$w_1 = 25, w_2 = 0, b = 10$

$w_1 = 25, w_2 = 0, b = 15$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$w_1 = 25, w_2 = 0, b = 20$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$w_1 = 25, w_2 = 0, b = 25$

$w_1 = 25, w_2 = 0, b = 30$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$w_1 = 25, w_2 = 0, b = 35$

$w_1 = 25, w_2 = 0, b = 40$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$w_1 = 25, w_2 = 0, b = 45$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

- What if we take two such step functions (with different b values) and subtract one from the other

- What if we take two such step functions (with different b values) and subtract one from the other

- What if we take two such step functions (with different b values) and subtract one from the other

- What if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$$w_1 = 0, w_2 = 2, b = 0$$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 3, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 4, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 5, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 6, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 7, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 8, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 9, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 10, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 11, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 12, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 13, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 14, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 15, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 16, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$$w_1 = 0, w_2 = 17, b = 0$$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$$w_1 = 0, w_2 = 18, b = 0$$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 19, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 20, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 21, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$$w_1 = 0, w_2 = 22, b = 0$$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$$w_1 = 0, w_2 = 23, b = 0$$

$w_1 = 0, w_2 = 24, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

$w_1 = 0, w_2 = 25, b = 0$

$w_1 = 0, w_2 = 25, b = 5$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

$w_1 = 0, w_2 = 25, b = 10$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

$w_1 = 0, w_2 = 25, b = 15$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

$w_1 = 0, w_2 = 25, b = 20$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

$w_1 = 0, w_2 = 25, b = 25$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

$w_1 = 0, w_2 = 25, b = 30$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

$w_1 = 0, w_2 = 25, b = 35$

$w_1 = 0, w_2 = 25, b = 40$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

$w_1 = 0, w_2 = 25, b = 45$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b

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- Again, what if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)
- Notice that this open tower has a different orientation from the previous one

- Now what will we get by adding two such open towers ?

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- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base

- Now what will we get by adding two such open towers ?
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- We can now pass this output through another sigmoid neuron to get the desired tower !

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- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base
- We can now pass this output through another sigmoid neuron to get the desired tower !
- We can now approximate any function by summing up many such towers

- For example, we could approximate the following function using a sum of several towers

- Can we come up with a neural network to represent this entire procedure of constructing a 3 dimensional tower ?

x1
x2

x_1
x_2

Think

- For 1 dimensional input we needed 2 neurons to construct a tower
- For 2 dimensional input we needed 4 neurons to construct a tower
- How many neurons will you need to construct a tower in n dimensions ?

Time to retrospect

- Why do we care about approximating any arbitrary function ?
- Can we tie all this back to the classification problem that we have been dealing with ?

- We are interested in separating the blue points from the red points

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- The illustrative proof that we just saw tells us that we can have a neural network with two hidden layers which can approximate the above function by a sum of towers

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- This is what we actually want
- The illustrative proof that we just saw tells us that we can have a neural network with two hidden layers which can approximate the above function by a sum of towers
- Which means we can have a neural network which can exactly separate the blue points from the red points !!