Lecture 3

Sigmoid Neurons, Gradient Descent, Feedforward Neural Networks, Representation Power of Feedforward Neural Networks

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Mitesh M. Khapra

Department of Computer Science and Engineering
Indian Institute of Technology Madras

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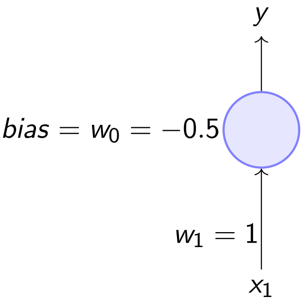
- Can we have a network which can (approximately) represent such functions ?
- Before answering the above question we will have to first graduate from **perceptrons** to **sigmoidal neurons** ...

Recall

- A perceptron will fire if the weighted sum of its inputs is greater than the threshold (-w_0)



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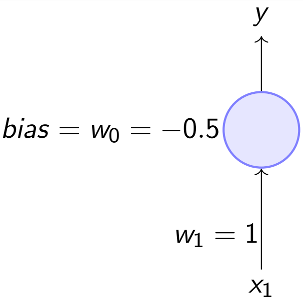

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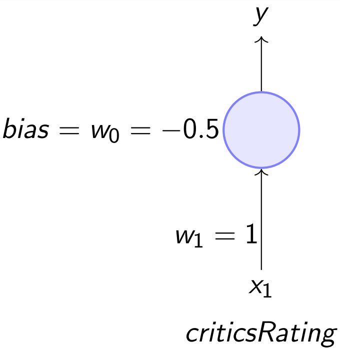

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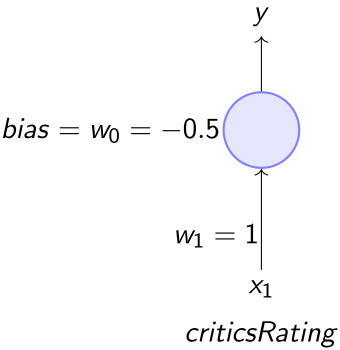

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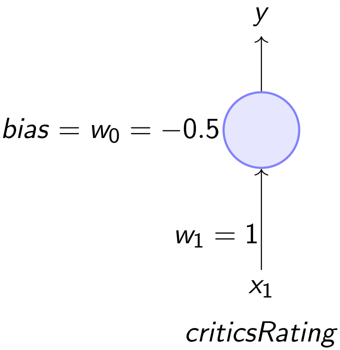

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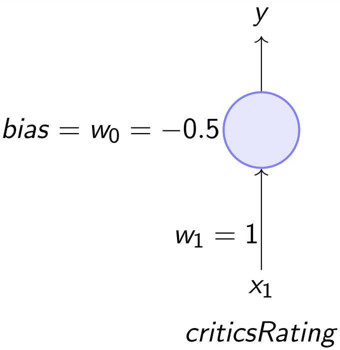

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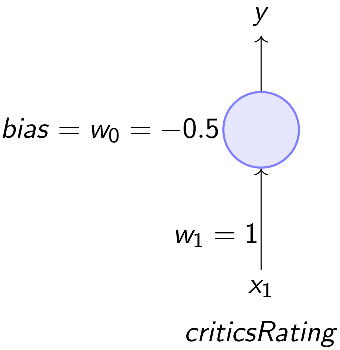

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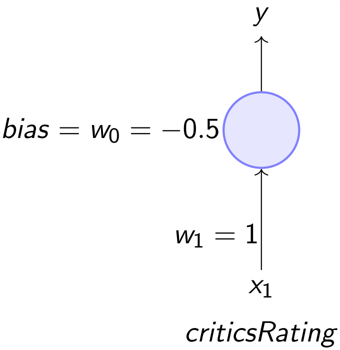

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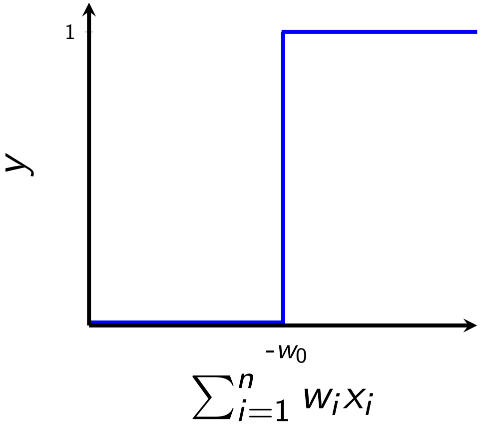
- What about a movie with $criticsRating = 0.49$? (dislike)
- It seems harsh that we would like a movie with rating 0.51 but not one with a rating of 0.49

- This behavior is not a characteristic of the specific problem we chose or the specific weight and threshold that we chose



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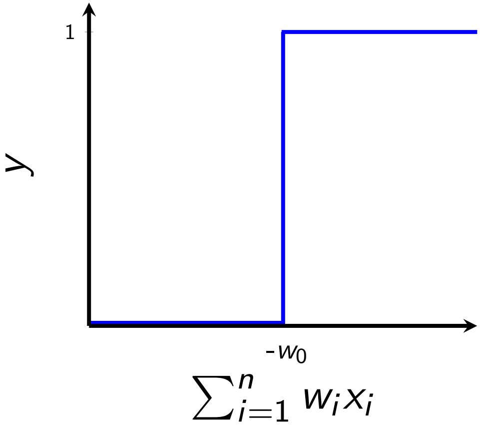

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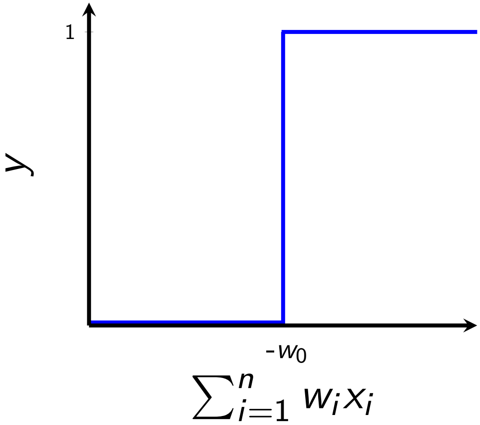

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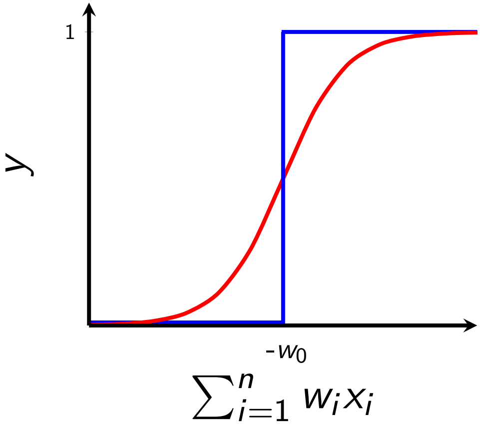
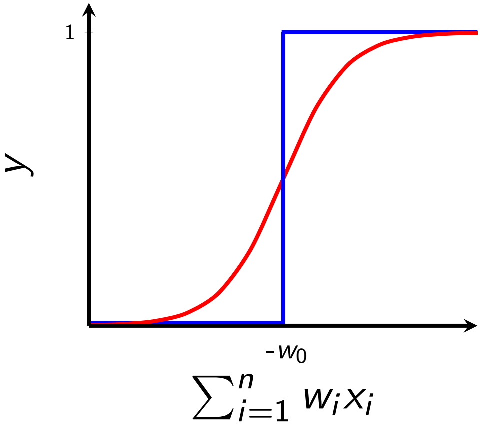
- For most real world applications we would expect a smoother decision function which gradually changes from 0 to 1



- Introducing sigmoid neurons where the output function is much smoother than the step function



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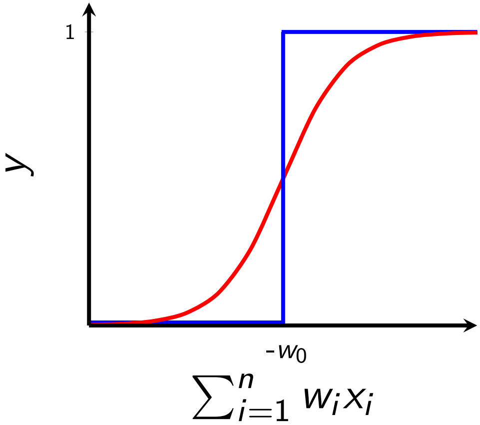

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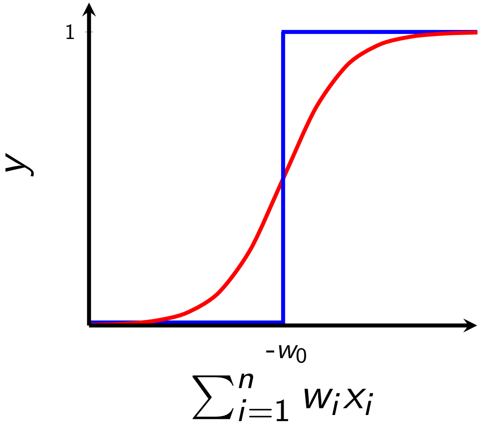

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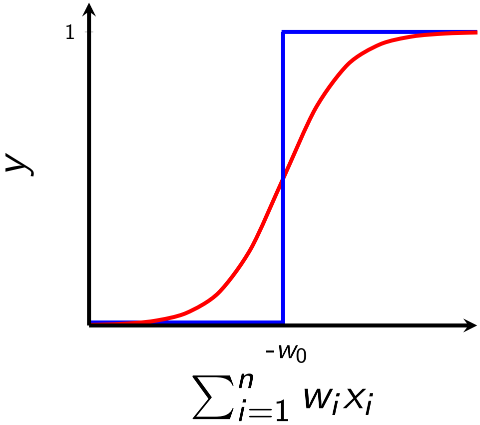

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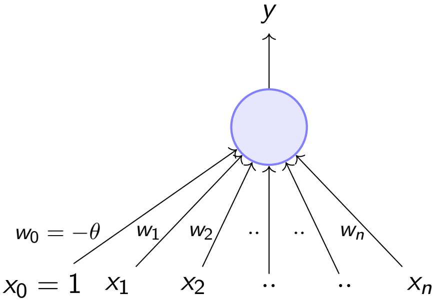
- Instead of a like/dislike decision we get the probability of liking the movie

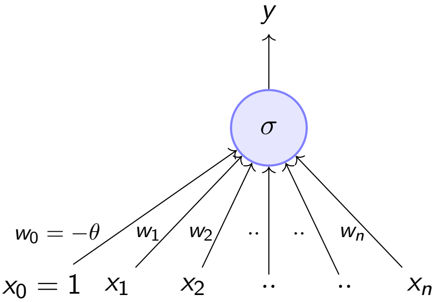
Perceptron



$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$

$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

Sigmoid (logistic) Neuron



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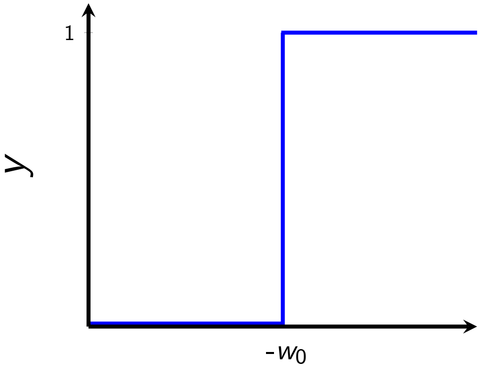
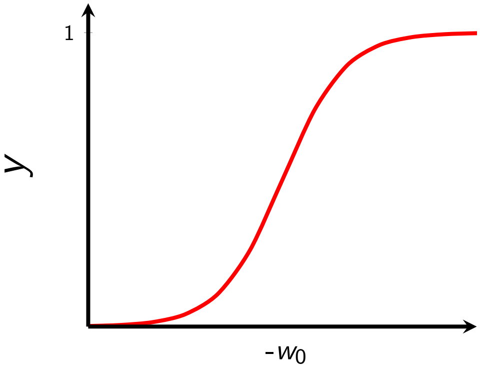


$\sum_{i=1}^{n} w_i x_i$

Not smooth, not continuous (at $w0$), **not differentiable**

Sigmoid Neuron



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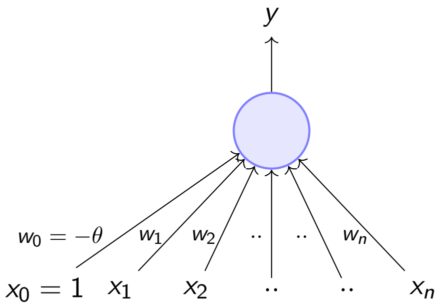

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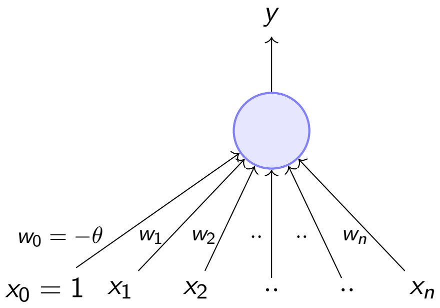

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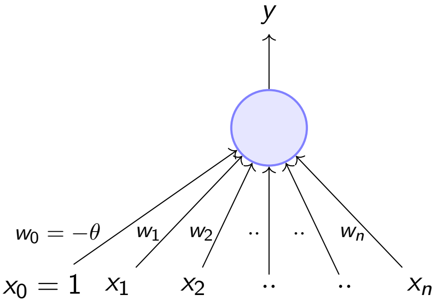

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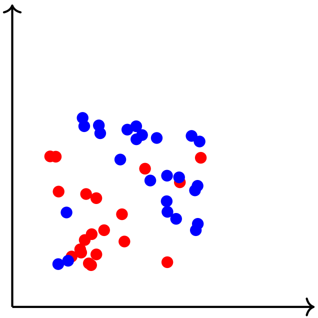
- Before we see such an algorithm we will revisit the concept of **error**



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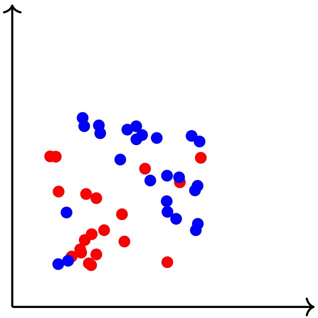

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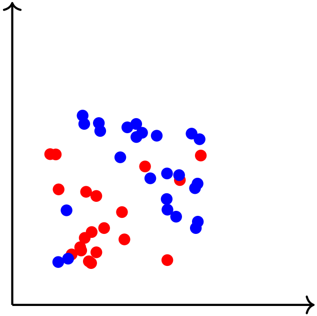

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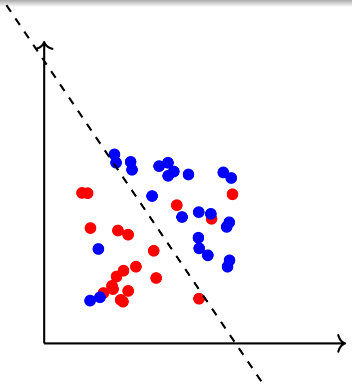

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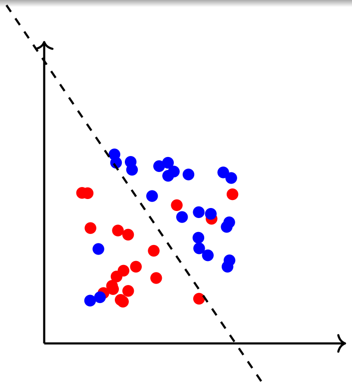

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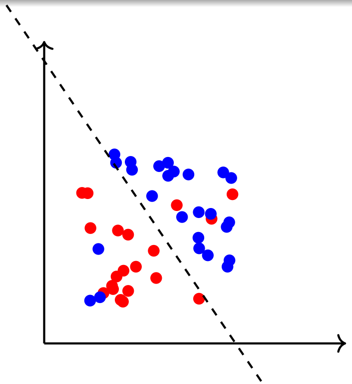

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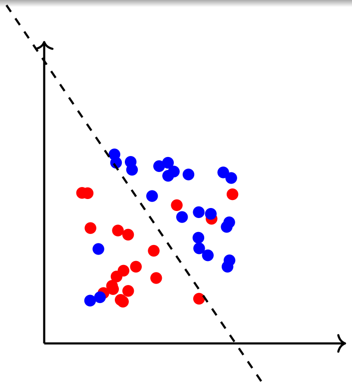

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- From now on, we will accept that it is hard to drive the error to 0 in most cases and will instead aim to reach the minimum possible error

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The learning algorithm should aim to find a w which minimizes the above function (squared error between y and $\hat{y}$)

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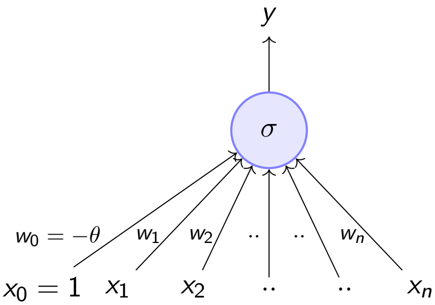
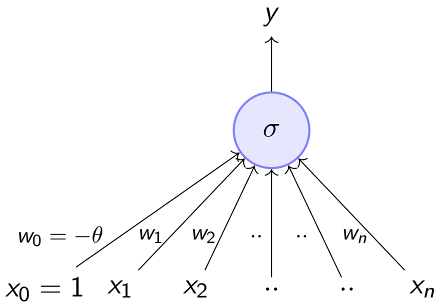
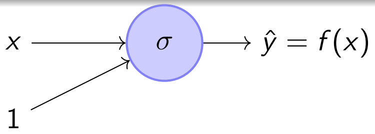


- With this setup in mind, we will now focus on this **model** and discuss an **algorithm** for learning the **parameters** of this model from some given **data**

Sigmoid (logistic) Neuron



- With this setup in mind, we will now focus on this **model** and discuss an **algorithm** for learning the **parameters** of this model from some given **data**
- σ stands for the sigmoid function (logistic function in this case)



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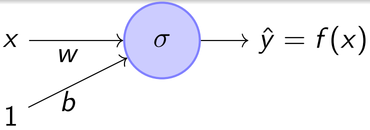
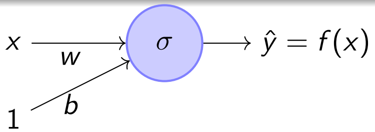
- σ stands for the sigmoid function (logistic function in this case)
- For ease of explanation, we will consider a very simplified version of the model having just 1 input



$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

- With this setup in mind, we will now focus on this **model** and discuss an **algorithm** for learning the **parameters** of this model from some given **data**
- σ stands for the sigmoid function (logistic function in this case)
- For ease of explanation, we will consider a very simplified version of the model having just 1 input
- Further to be consistent with the literature, from now on we will refer to w_0 as b (bias)



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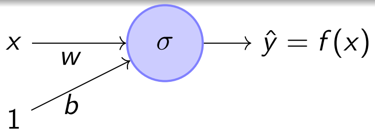
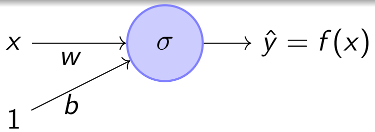
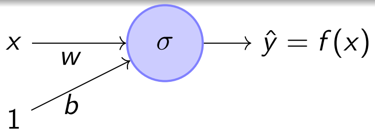
- With this setup in mind, we will now focus on this **model** and discuss an **algorithm** for learning the **parameters** of this model from some given **data**
- σ stands for the sigmoid function (logistic function in this case)
- For ease of explanation, we will consider a very simplified version of the model having just 1 input
- Further to be consistent with the literature, from now on we will refer to w_0 as b (bias)
- Lastly, instead of considering the problem of predicting like/dislike we will assume that we want to predict $criticsRating(y)$ given $imdbRating(x)$ (for no particular reason)


x
w
σ
$\hat{y} = f(x)$
1
b



Input for training

$\{x_i, y_i\}_{i=1}^{N} \rightarrow N$ pairs of (x, y)



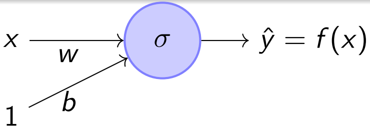
Input for training

$\{x_i, y_i\}_{i=1}^{N} \rightarrow N$ pairs of (x, y)

Training objective

Find w and b such that:

$$\underset{w,b}{\text{minimize}}\, \mathscr{L}(w, b) = \sum_{i=1}^{N} (y_i - f(x_i))^2$$



$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

Input for training

$\{x_i, y_i\}_{i=1}^{N} \rightarrow N$ pairs of (x, y)

Training objective

Find w and b such that:

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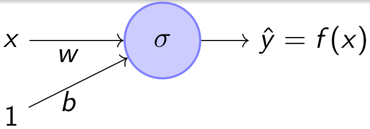
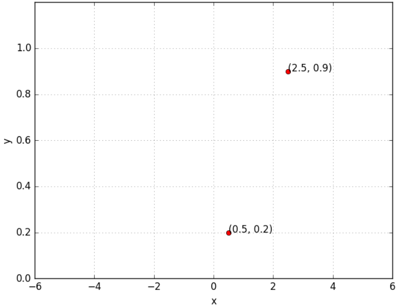


$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

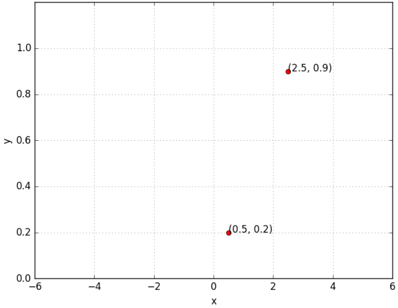


What does it mean to train the network?

- Suppose we train the network with $(x, y) = (0.5, 0.2)$ and $(2.5, 0.9)$

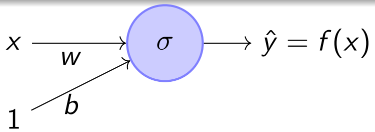
$$x \xrightarrow{w} \sigma \longrightarrow \hat{y} = f(x)$$
$$1 \xrightarrow{b} \sigma$$

$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

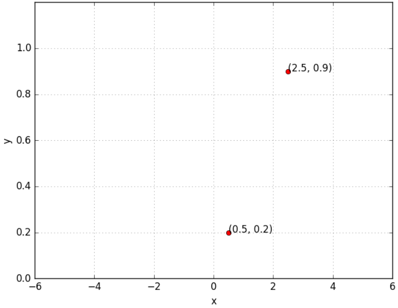


What does it mean to train the network?

- Suppose we train the network with $(x, y) = (0.5, 0.2)$ and $(2.5, 0.9)$
- At the end of training we expect to find w*, b* such that:



$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$



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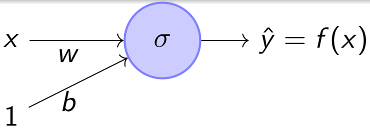
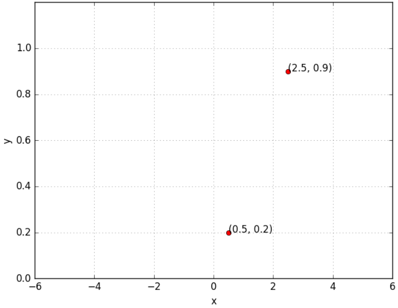
- Suppose we train the network with $(x, y) = (0.5, 0.2)$ and $(2.5, 0.9)$
- At the end of training we expect to find w*, b* such that:
- $f(0.5) \rightarrow 0.2$ and $f(2.5) \rightarrow 0.9$



$$f(x) = \frac{1}{1+e^{-(w \cdot x+b)}}$$



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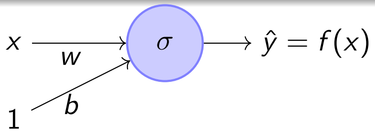
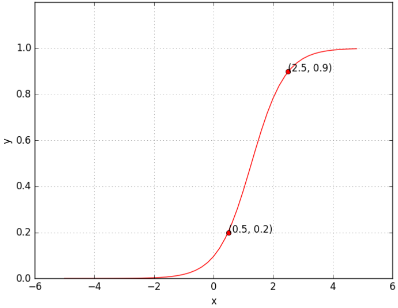
In other words...

- We hope to find a sigmoid function such that $(0.5, 0.2)$ and $(2.5, 0.9)$ lie on this sigmoid



$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$



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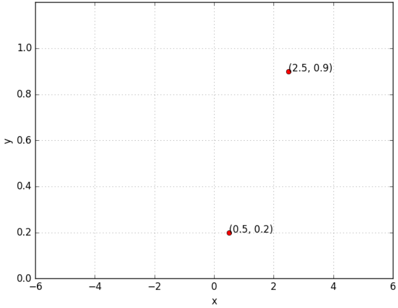
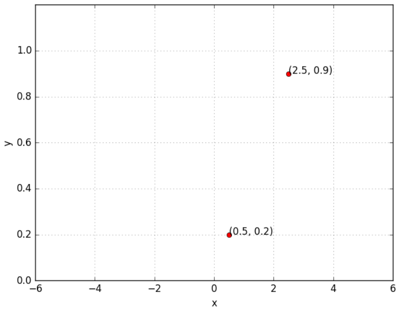
In other words...

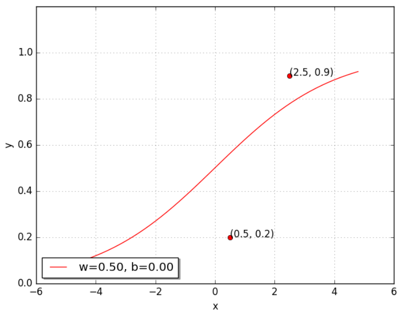
- We hope to find a sigmoid function such that $(0.5, 0.2)$ and $(2.5, 0.9)$ lie on this sigmoid

Lets see this in more detail....

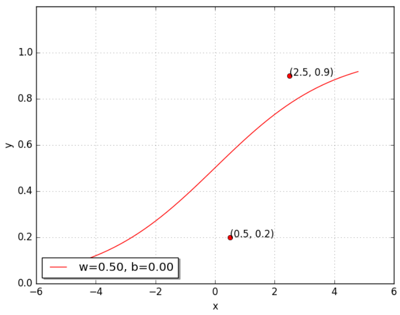

1.0
0.8
0.6
0.4
0.2
0.0
y
(2.5, 0.9)
(0.5, 0.2)
−6
−4
−2
0
2
4
6
x



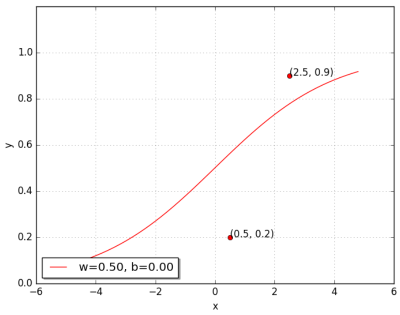
- Can we try to find such a $w*, b*$ manually



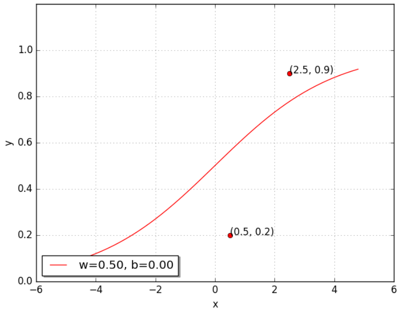
- Can we try to find such a $w*, b*$ manually
- Lets try a random guess.. (say, $w = 0.5, b = 0$)



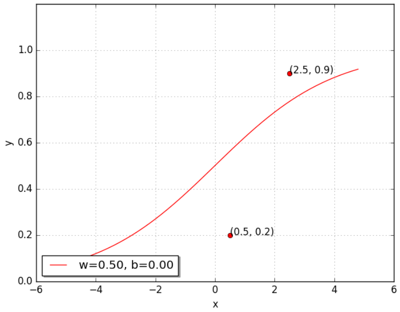
- Can we try to find such a $w*, b*$ manually
- Lets try a random guess.. (say, $w = 0.5, b = 0$)
- Clearly not good, but how bad is it ?



- Can we try to find such a $w*, b*$ manually
- Lets try a random guess.. (say, $w = 0.5, b = 0$)
- Clearly not good, but how bad is it ?
- Lets revisit $\mathscr{L}(w, b)$ to see how bad it is ...

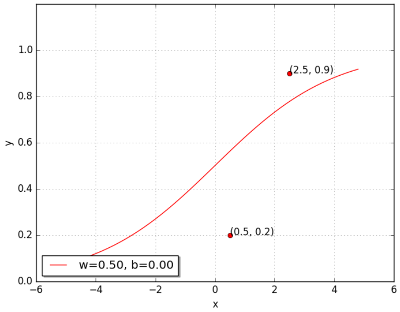


$$\mathscr{L}(w, b) = \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2$$

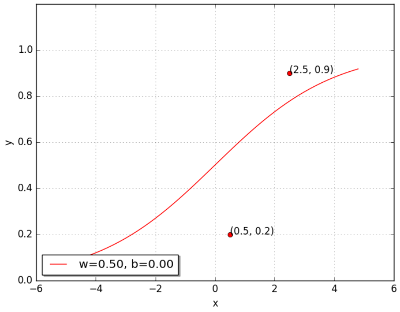


$$\mathscr{L}(w,b) = \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2$$

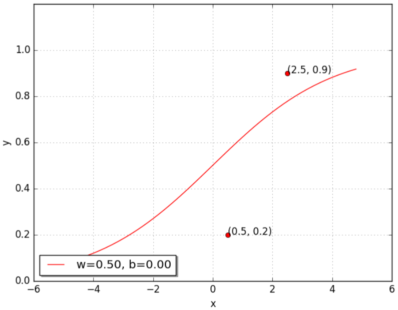
$$= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2$$



$$
\begin{aligned}
\mathscr{L}(w,b) &= \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2 \\
&= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 \\
&= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2
\end{aligned}
$$



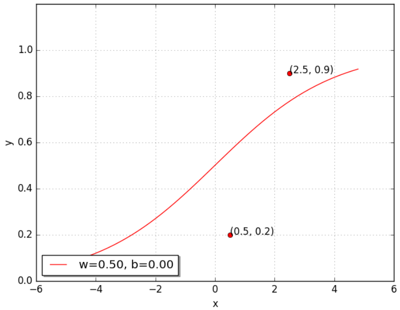
$$
\begin{aligned}
\mathscr{L}(w,b) &= \frac{1}{2} * \sum_{i=1}^{N}(y_i - f(x_i))^2 \\
&= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 \\
&= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2 \\
&= 0.073
\end{aligned}
$$



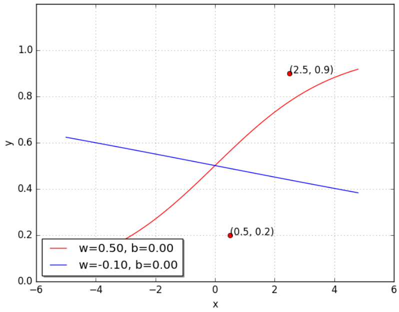
$$
\begin{aligned}
\mathscr{L}(w, b) &= \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2 \\
&= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 \\
&= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2 \\
&= 0.073
\end{aligned}
$$

We want $\mathscr{L}(w, b)$ to be as close to 0 as possible

Lets try some other values of w, b



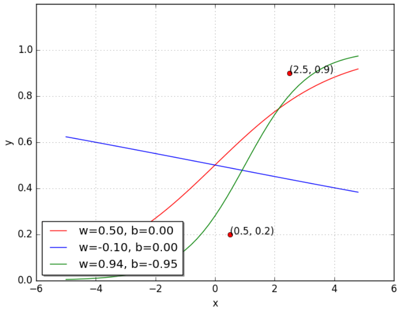
w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730



Lets try some other values of w, b

w	b	$\mathscr{L}(w, b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481

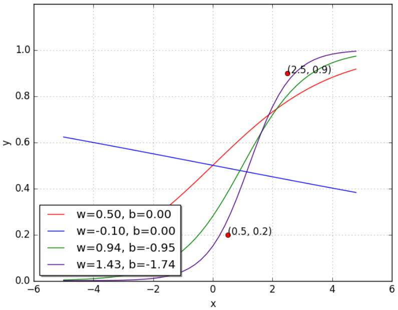
Oops!! this made things even worse...



Lets try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214

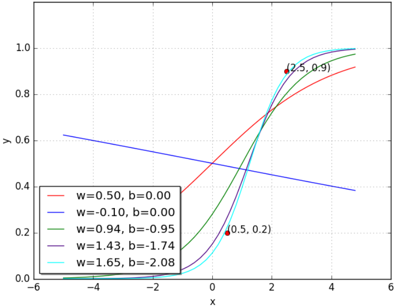
Perhaps it would help to push w and b in the other direction...



Lets try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028

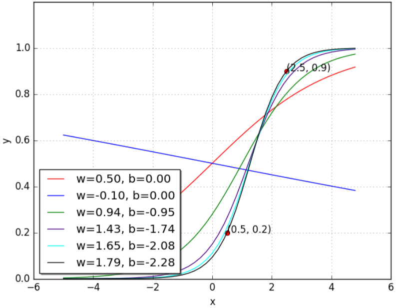
Lets keep going in this direction, *i.e.*, increase w and decrease b



Lets try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028
1.65	-2.08	0.0003

Lets keep going in this direction, *i.e.*, increase w and decrease b



Lets try some other values of w, b

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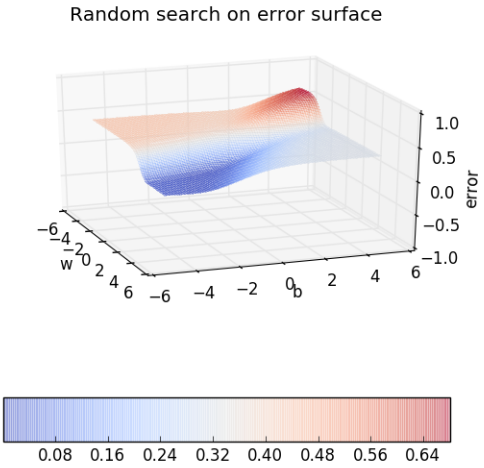
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028
1.65	-2.08	0.0003
1.78	-2.27	0.0000

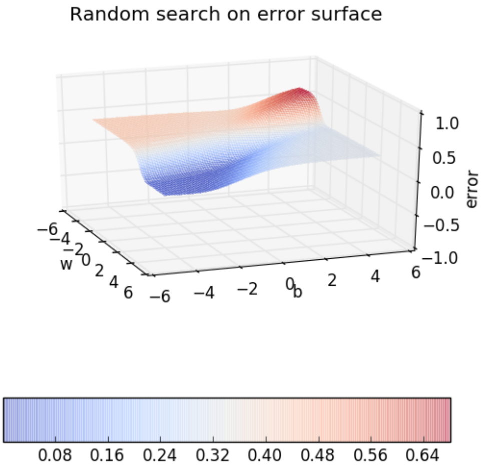
With some guess work and intuition we were able to find the right values for w and b

Lets look at something better than our "guess work" algorithm....

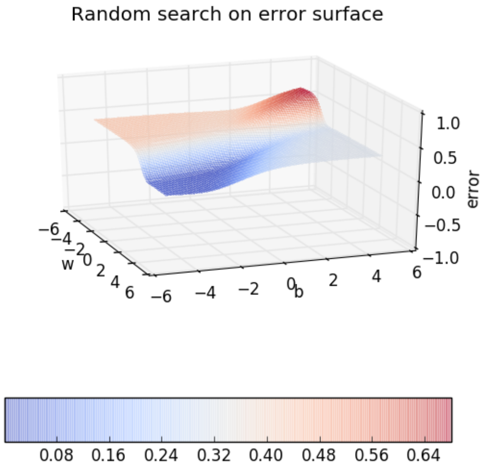
- Since we have only 2 points and 2 parameters $(w,\ b)$ we can easily plot $\mathscr{L}(w, b)$ for different values of $(w,\ b)$ and pick the one where $\mathscr{L}(w, b)$ is minimum



- Since we have only 2 points and 2 parameters (w, b) we can easily plot $\mathscr{L}(w, b)$ for different values of (w, b) and pick the one where $\mathscr{L}(w, b)$ is minimum

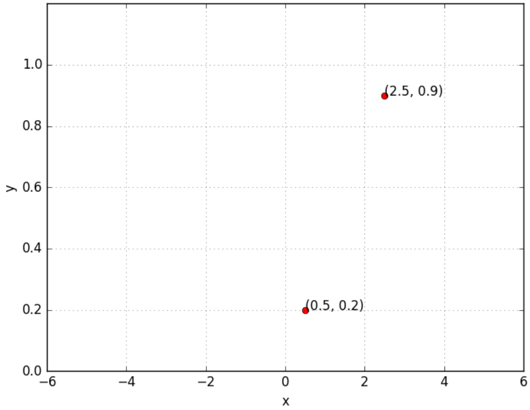


- Since we have only 2 points and 2 parameters $(w,\ b)$ we can easily plot $\mathscr{L}(w, b)$ for different values of $(w,\ b)$ and pick the one where $\mathscr{L}(w, b)$ is minimum
- But of course this becomes intractable once you have many more data points and many more parameters !!

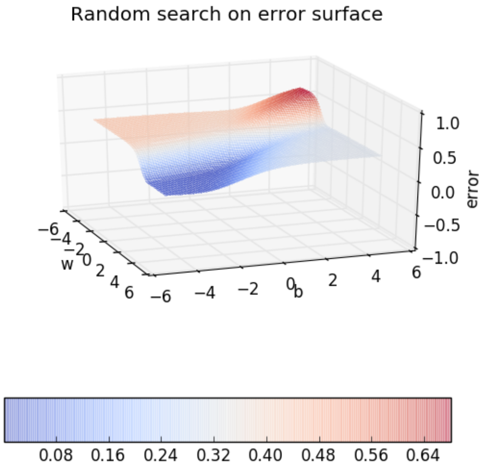


- Since we have only 2 points and 2 parameters $(w,\ b)$ we can easily plot $\mathscr{L}(w,b)$ for different values of $(w,\ b)$ and pick the one where $\mathscr{L}(w,b)$ is minimum
- But of course this becomes intractable once you have many more data points and many more parameters !!
- Further, even here we have plotted the error surface only for a small range of $(w,\ b)$ [from $(-6,6)$ and not from $(-\inf,\inf)$]

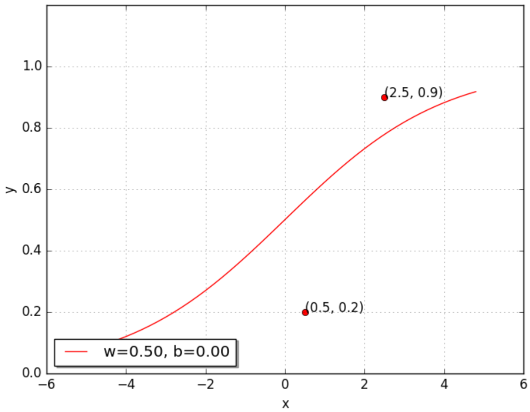
Lets look at the geometric interpretation of our "guess work" algorithm in terms of this error surface

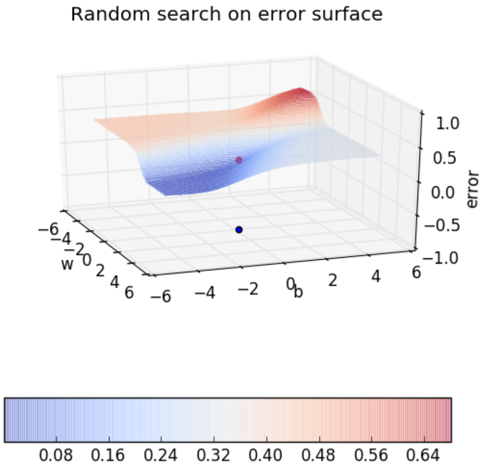


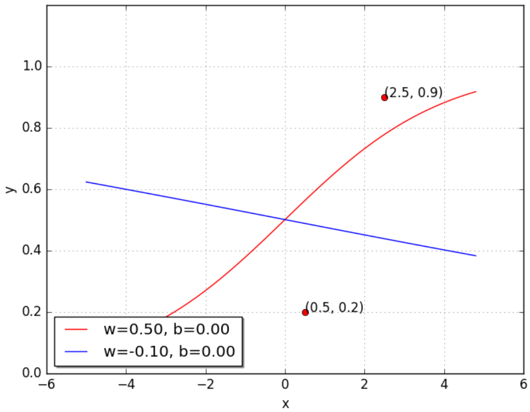
1.0
0.8
0.6
0.4
0.2
0.0
y
−6
−4
−2
0
2
4
6
x
(2.5, 0.9)
(0.5, 0.2)

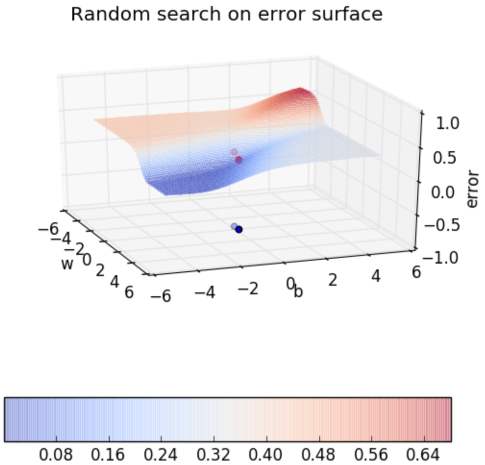


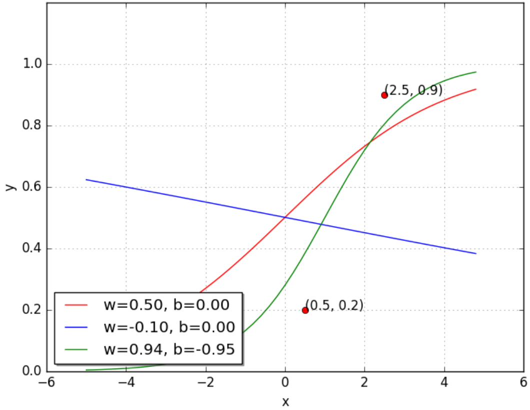
Random search on error surface
error
1.0
0.5
0.0
−0.5
−1.0
w
b
−6
−4
−2
0
2
4
6
0.08
0.16
0.24
0.32
0.40
0.48
0.56
0.64


(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
y
x
1.0
0.8
0.6
0.4
0.2
0.0
−6
−4
−2
0
2
4
6

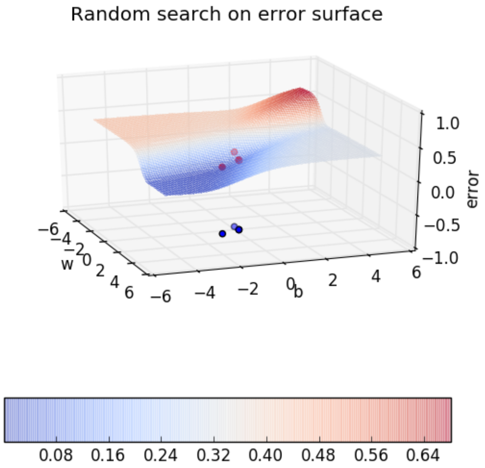

Random search on error surface
w
b
error
1.0
0.5
0.0
−0.5
−1.0
0.08
0.16
0.24
0.32
0.40
0.48
0.56
0.64


(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
w=-0.10, b=0.00
y
x
1.0
0.8
0.6
0.4
0.2
0.0
−6
−4
−2
0
2
4
6

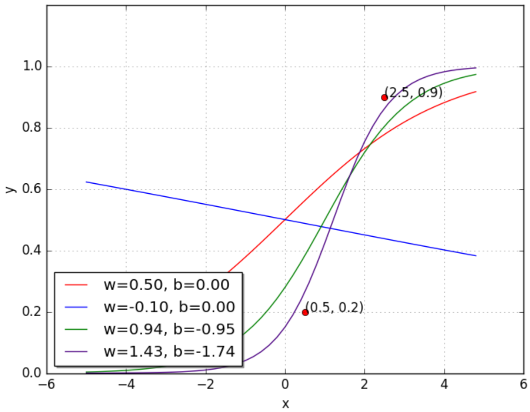

Random search on error surface
w
b
error
1.0
0.5
0.0
−0.5
−1.0
0.08
0.16
0.24
0.32
0.40
0.48
0.56
0.64



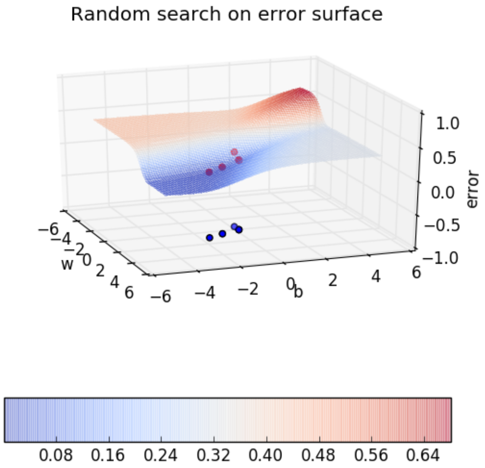
1.0
0.8
0.6
0.4
0.2
0.0
y
−6
−4
−2
0
2
4
6
x
(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
w=-0.10, b=0.00
w=0.94, b=-0.95



Random search on error surface
1.0
0.5
0.0
−0.5
−1.0
error
w
b
0.08
0.16
0.24
0.32
0.40
0.48
0.56
0.64



(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
w=-0.10, b=0.00
w=0.94, b=-0.95
w=1.43, b=-1.74
y
x



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0.08
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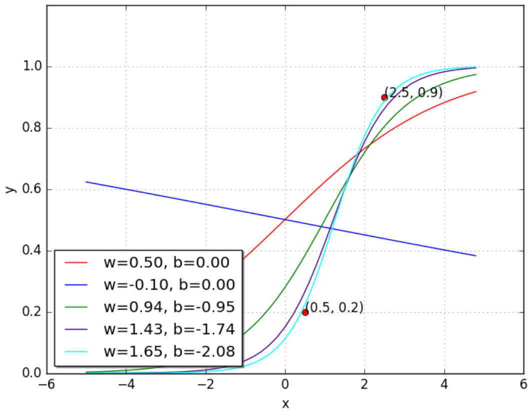

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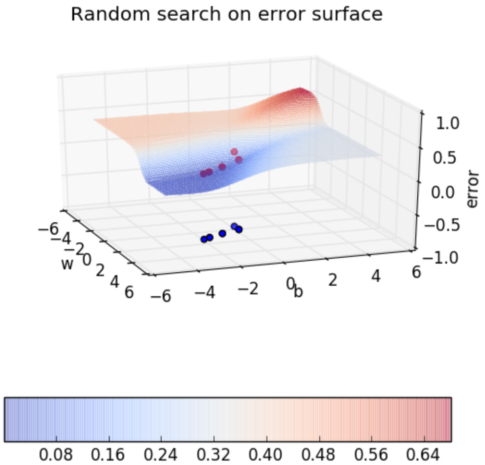

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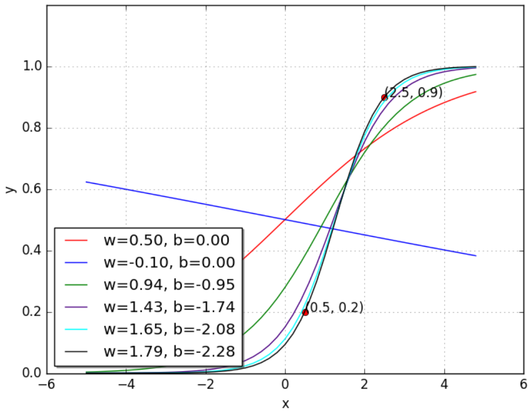

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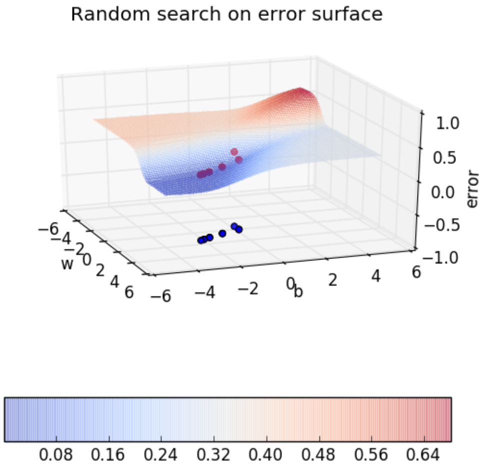
w=1.43, b=-1.74
w=1.65, b=-2.08
w=1.79, b=-2.28
y
x



Random search on error surface
w
b
error
0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64

Now lets see if there is a more efficient and principled way of doing this

Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!

vector of parameters, say, randomly initialized

$$\theta = [w, b]$$

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$$\theta = [w, b]$$

$$\Delta\theta = [\Delta w, \Delta b]$$

change in the
values of w, b

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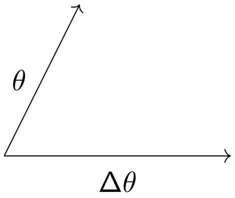

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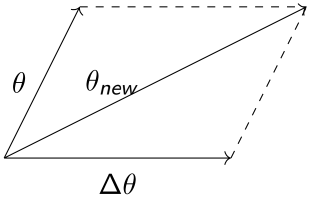

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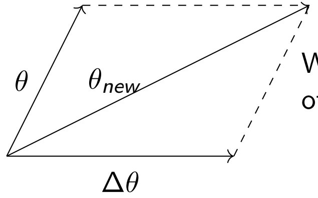

We moved in the direction of $\Delta\theta$

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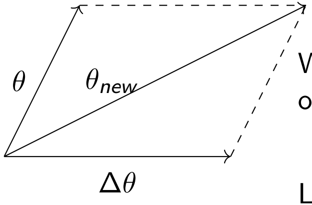

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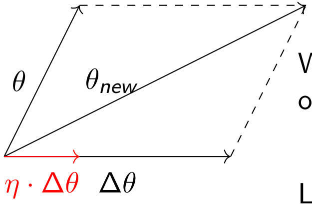

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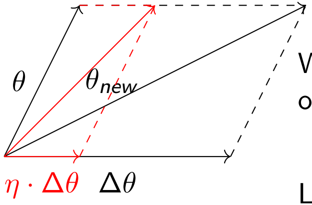

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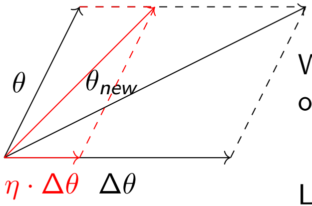

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$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$

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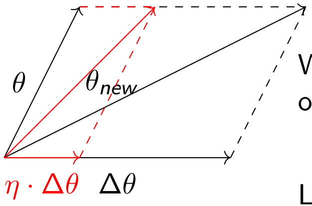

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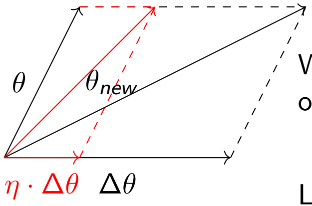

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The answer comes from Taylor series

For ease of notation, let $\Delta\theta = u$, then from Taylor series, we have,

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Note that the move (ηu) would be favorable only if,

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This implies,

$$u^T \nabla \mathscr{L}(\theta) < 0$$

Okay, so we have,

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$$-1 \leq cos(\beta) = \frac{u^T \nabla \mathscr{L}(\theta)}{||u|| * ||\nabla \mathscr{L}(\theta)||} \leq 1$$

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multiply throughout by $k = ||u|| * ||\nabla \mathscr{L}(\theta)||$

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Thus, $\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) = u^T \nabla \mathscr{L}(\theta) = k * cos(\beta)$ will be most negative when $cos(\beta) = -1$ *i.e.*, when β is $180°$

Gradient Descent Rule

- The direction u that we intend to move in should be at 180° w.r.t. the gradient

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Parameter Update Equations

$$w_{t+1} = w_t - \eta \nabla w_t$$
$$b_{t+1} = b_t - \eta \nabla b_t$$
$$where, \nabla w_t = \frac{\partial \mathscr{L}(w,b)}{\partial w}_{at\ w = w_t,\, b = b_t}, \nabla b = \frac{\partial \mathscr{L}(w,b)}{\partial b}_{at\ w = w_t,\, b = b_t}$$

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So we now have a more principled way of moving in the w-b plane than our "guess work" algorithm

- Lets create an algorithm from this rule ...

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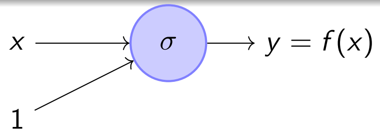
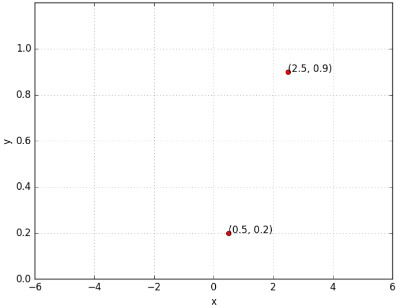
Algorithm: gradient_descent()

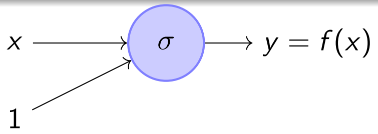
$t \leftarrow 0$;
$max\_iterations \leftarrow 1000$;
while $t < max\_iterations$ **do**
  $w_{t+1} \leftarrow w_t - \eta \nabla w_t$;
  $b_{t+1} \leftarrow b_t - \eta \nabla b_t$;
end

- To see this algorithm in practice lest first derive ∇w and ∇b for our toy neural network



$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$





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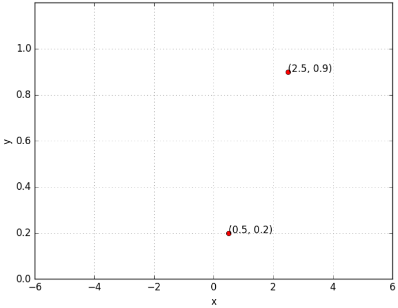
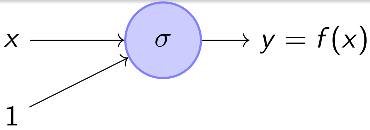


Let's assume there is only 1 point to fit (x, y)



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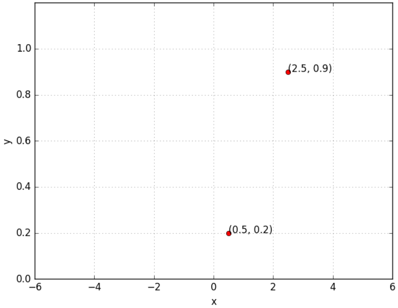

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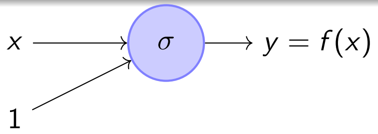
$$\mathscr{L}(w, b) = \frac{1}{2} * (f(x) - y)^2$$



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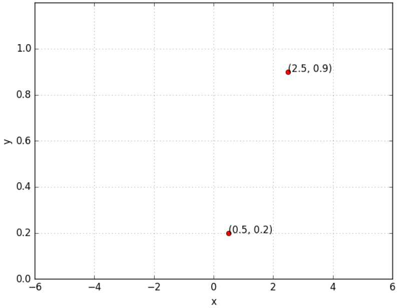

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$$\nabla w = \frac{\partial \mathscr{L}(w, b)}{\partial w} = \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2]$$

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\begin{aligned}
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&= \frac{1}{2} * [2 * (f(x) - y) * \frac{\partial}{\partial w}(f(x) - y)]
\end{aligned}
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$$\begin{aligned}&\frac{\partial}{\partial w}\left(\frac{1}{1 + e^{-(wx+b)}}\right) \\ &= \frac{-1}{(1 + e^{-(wx+b)})^2}\frac{\partial}{\partial w}(e^{-(wx+b)}))\end{aligned}$$

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$$= \frac{-1}{(1 + e^{-(wx+b)})} * \frac{e^{-(wx+b)}}{(1 + e^{-(wx+b)})} * (-x)$$
$$= \frac{1}{(1 + e^{-(wx+b)})} * \frac{e^{-(wx+b)}}{(1 + e^{-(wx+b)})} * (x)$$

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$$= \frac{-1}{(1 + e^{-(wx+b)})} * \frac{e^{-(wx+b)}}{(1 + e^{-(wx+b)})} * (-x)$$
$$= \frac{1}{(1 + e^{-(wx+b)})} * \frac{e^{-(wx+b)}}{(1 + e^{-(wx+b)})} * (x)$$
$$= f(x) * (1 - f(x)) * x$$

$$\nabla w = \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2]$$
$$= \frac{1}{2} * [2 * (f(x) - y) * \frac{\partial}{\partial w}(f(x) - y)]$$
$$= (f(x) - y) * \frac{\partial}{\partial w}(f(x))$$
$$= (f(x) - y) * \frac{\partial}{\partial w}\left(\frac{1}{1 + e^{-(wx+b)}}\right)$$
$$= (f(x) - y) * f(x) * (1 - f(x)) * x$$

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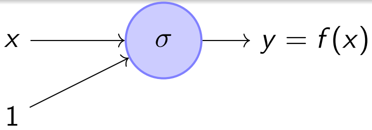
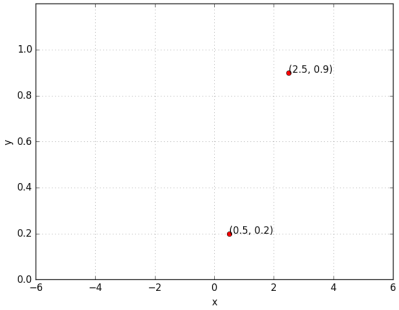


$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$



So if there is only 1 point (x, y), we have,

$$x \longrightarrow \sigma \longrightarrow y = f(x)$$

1

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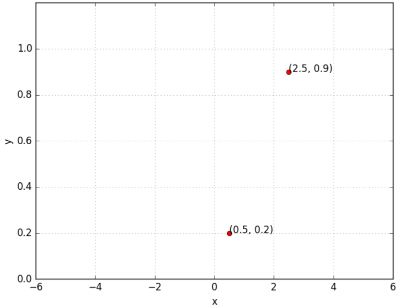

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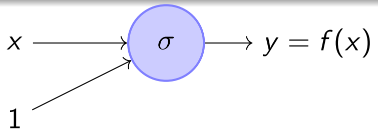

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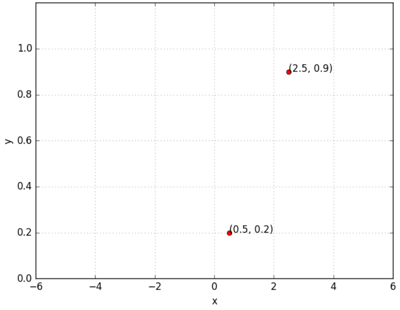

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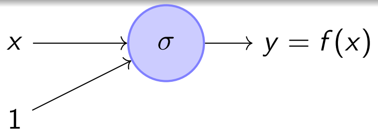
For two points,



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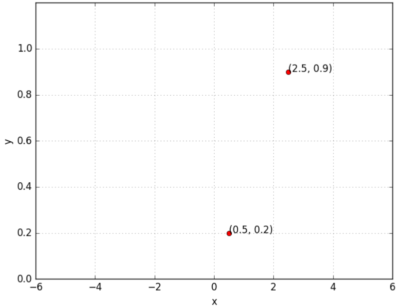

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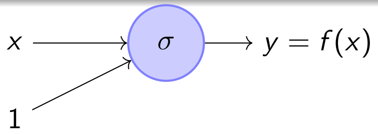

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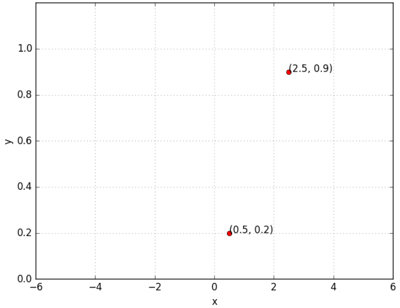

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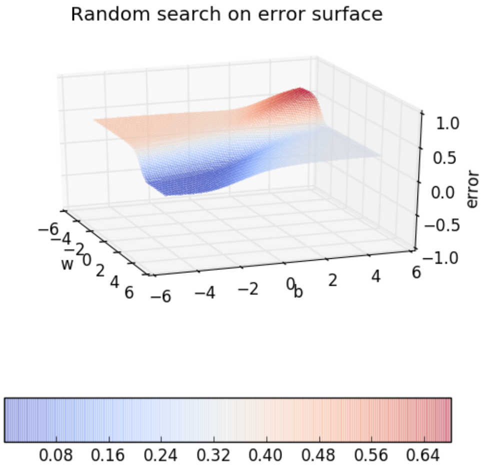

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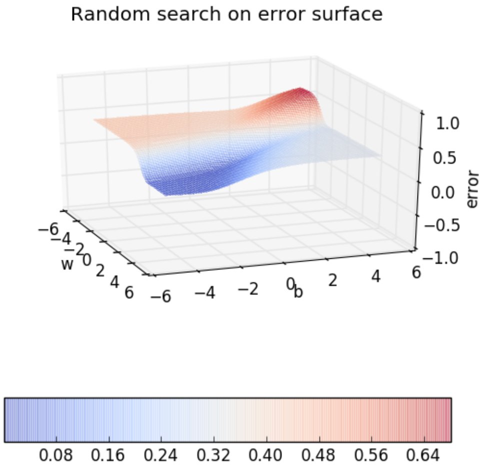

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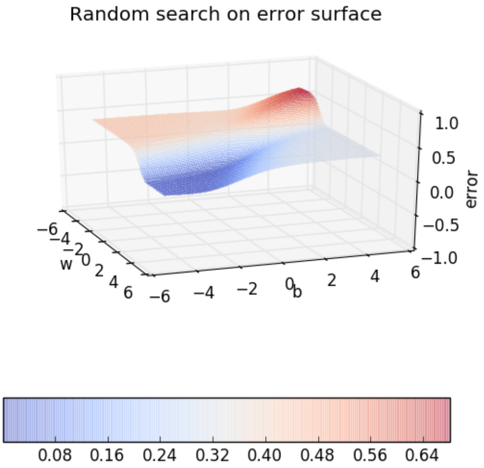

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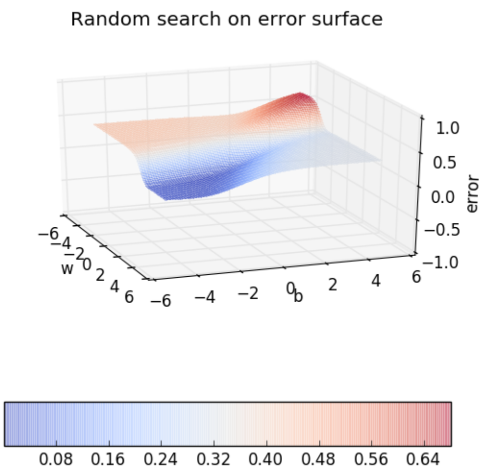

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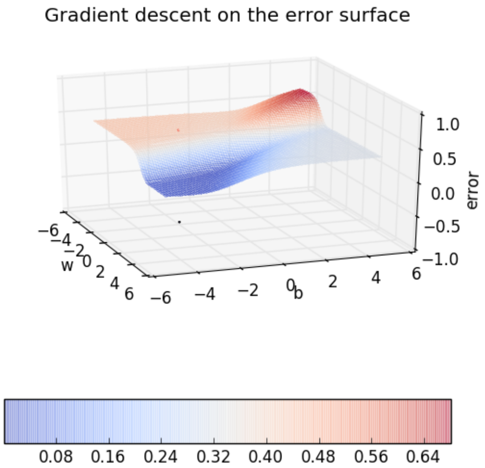

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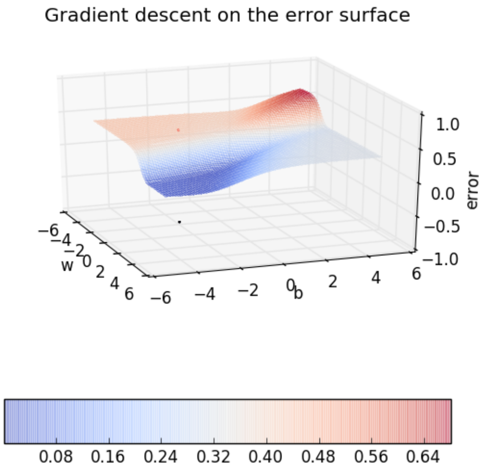

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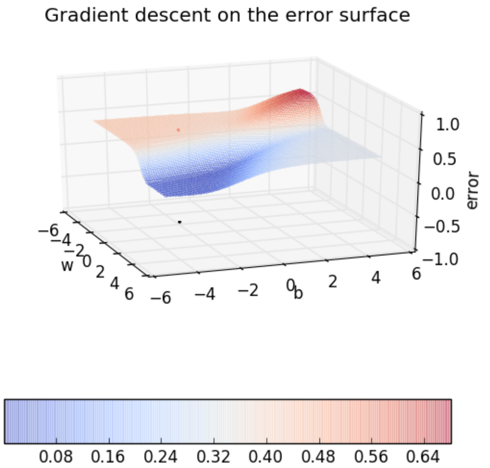

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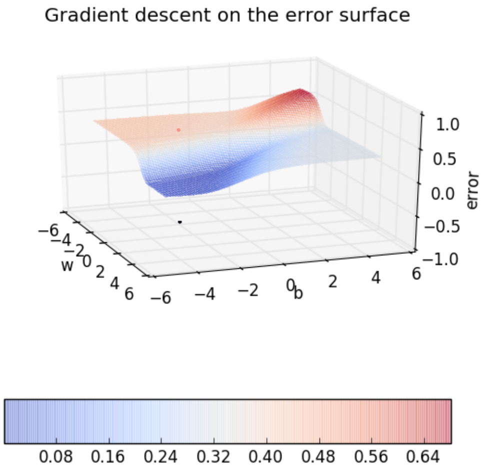

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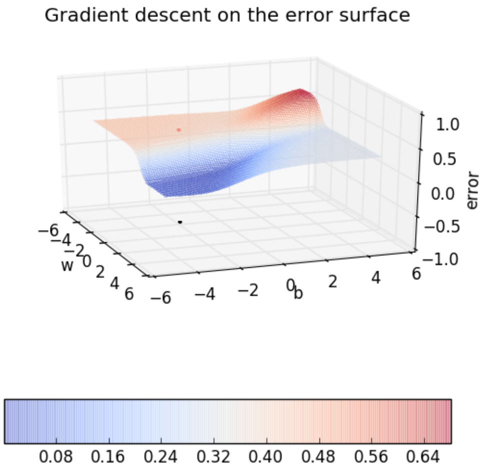

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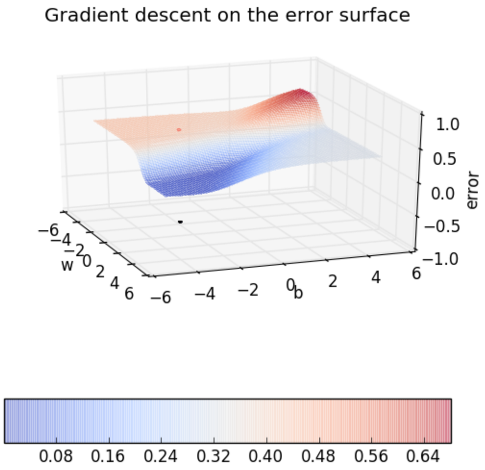

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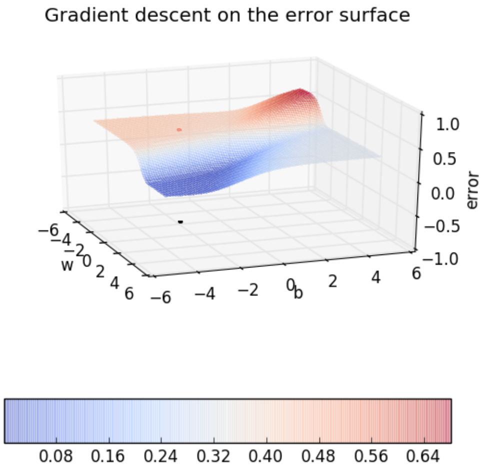

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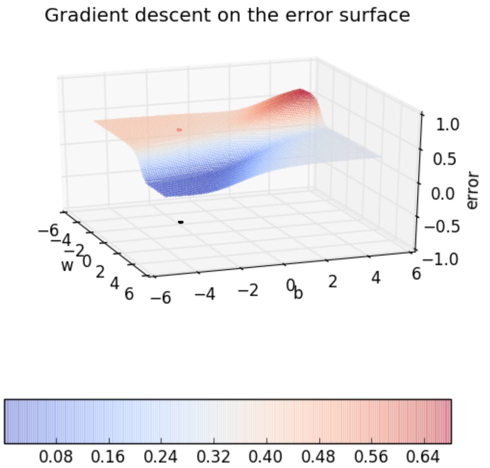

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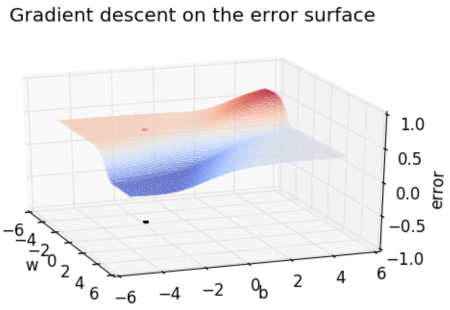

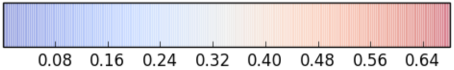

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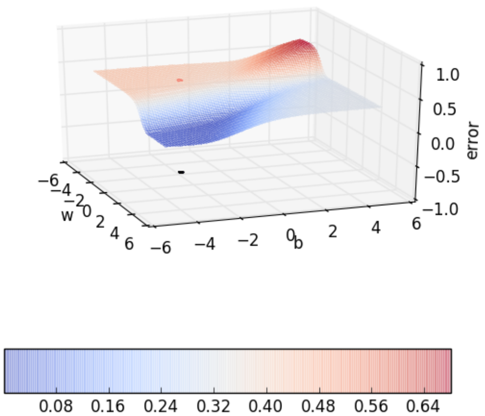
Gradient descent on the error surface



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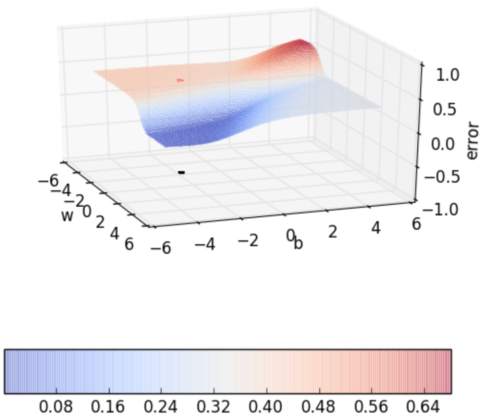

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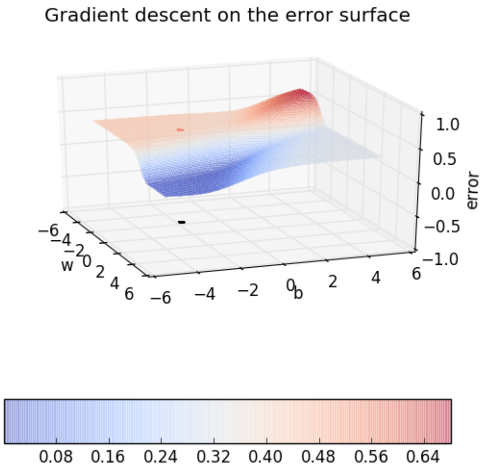

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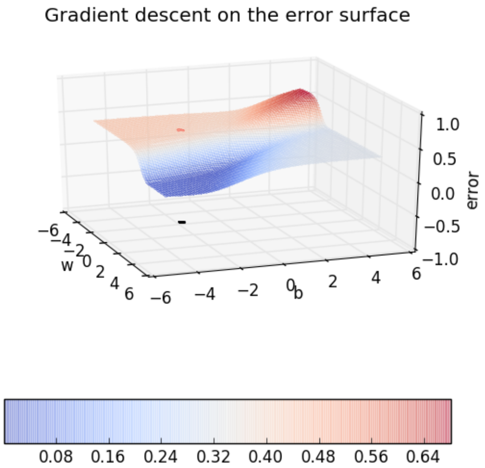

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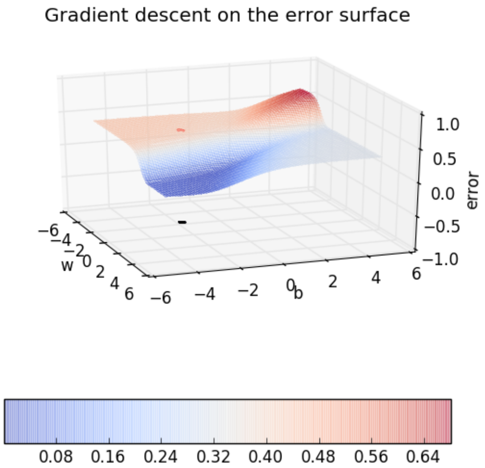

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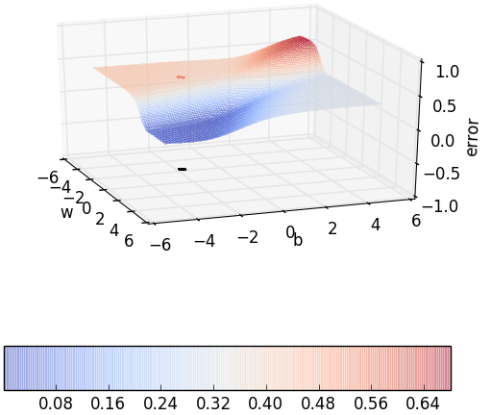
Gradient descent on the error surface



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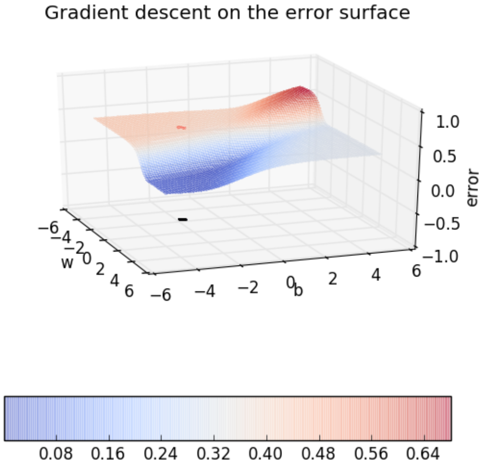

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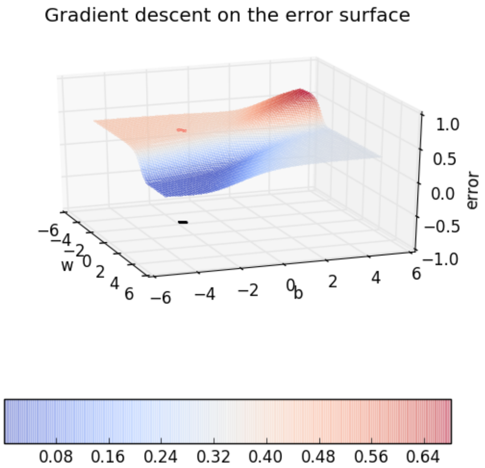

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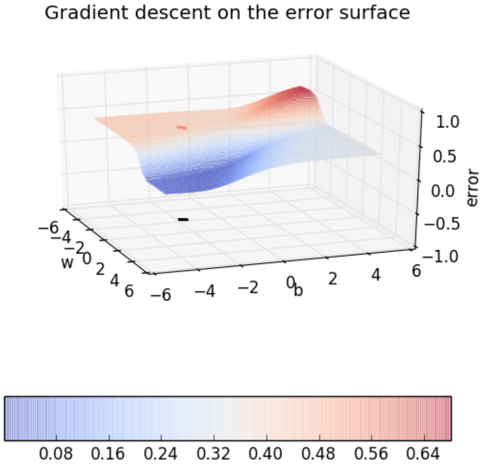

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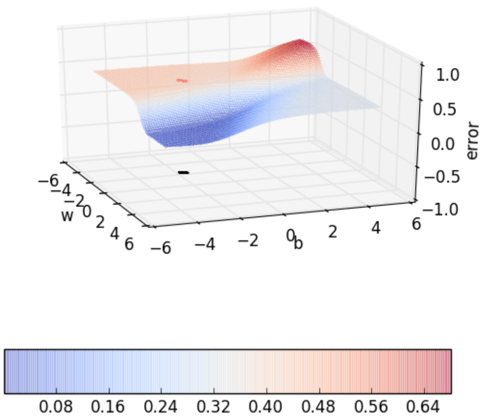
Gradient descent on the error surface



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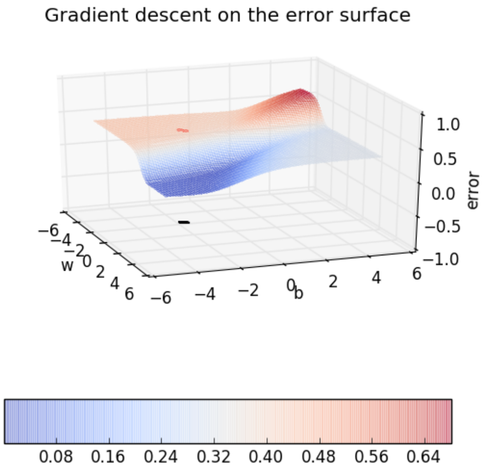

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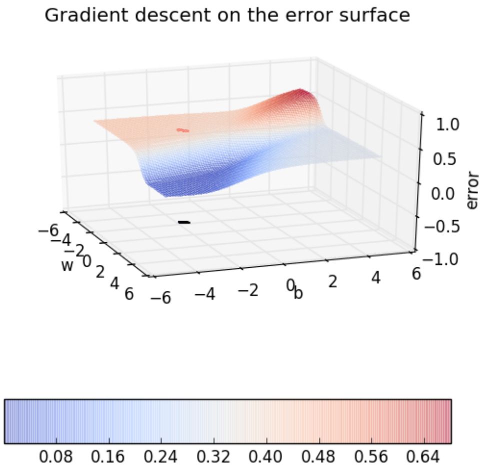

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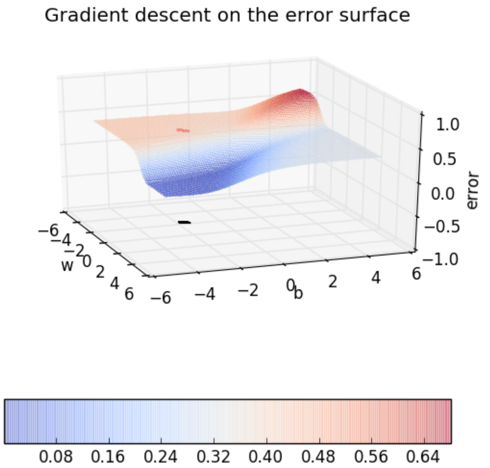

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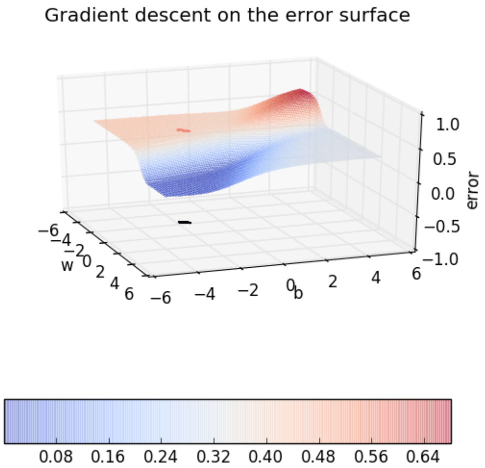

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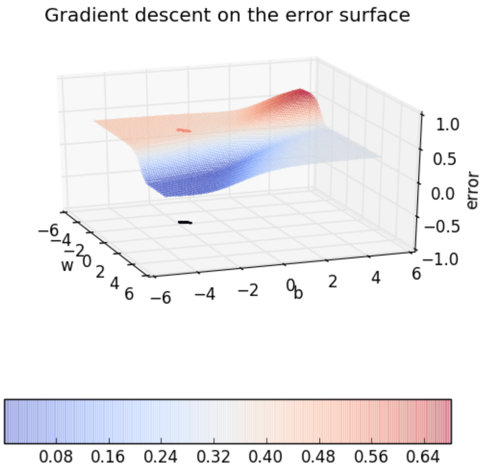

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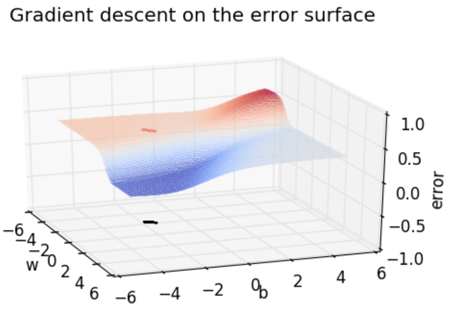

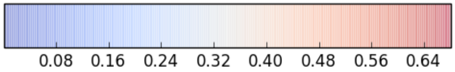

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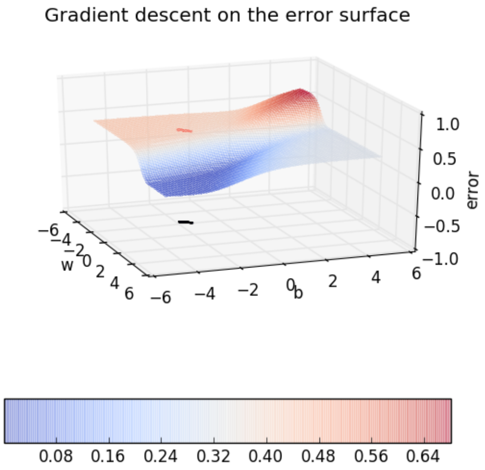

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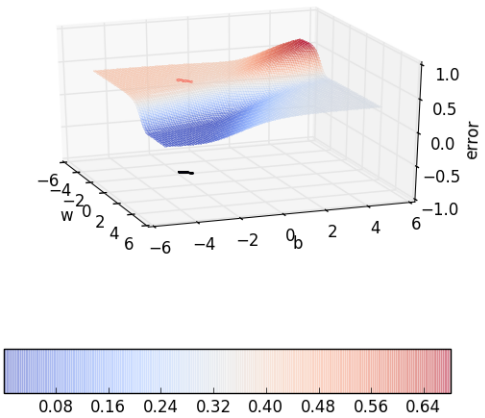
Gradient descent on the error surface



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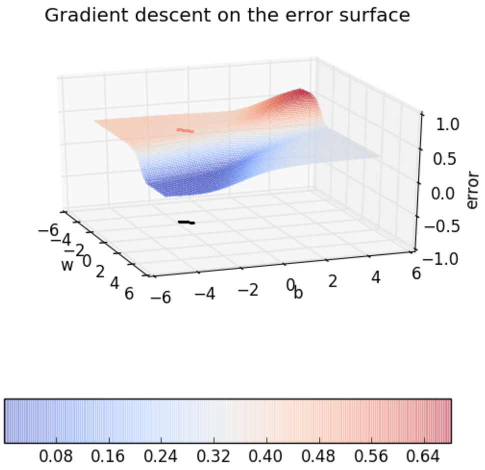

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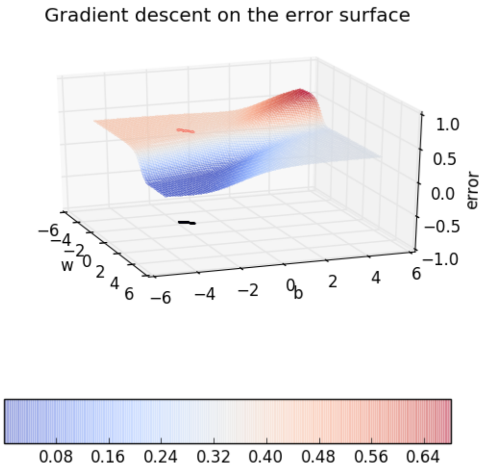

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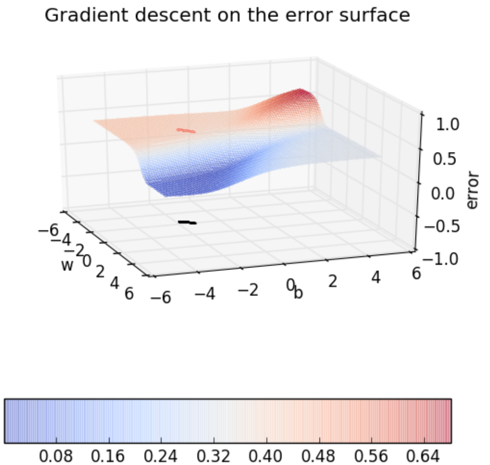

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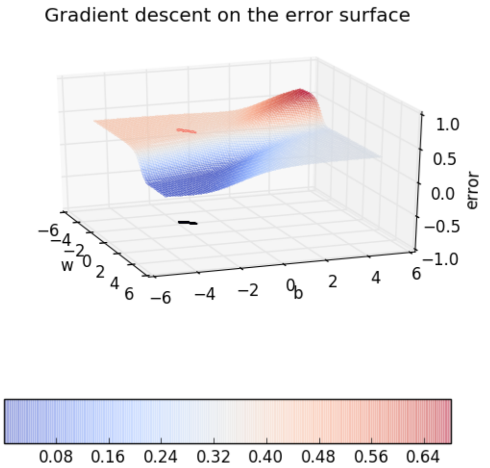

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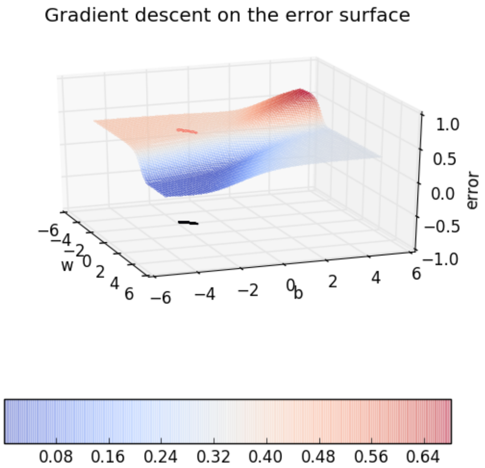

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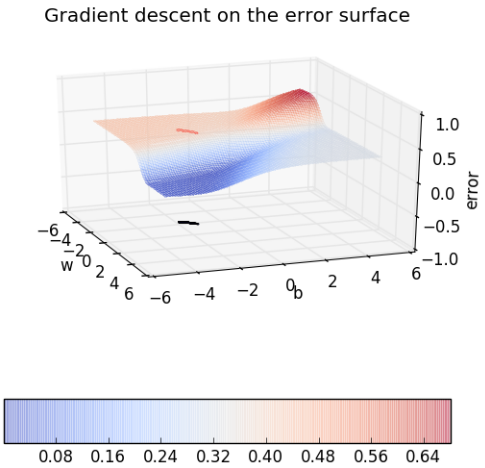

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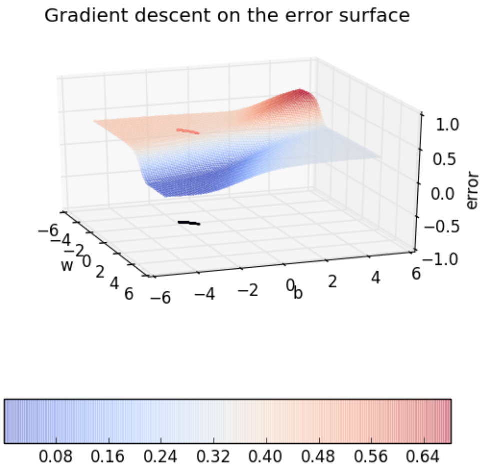

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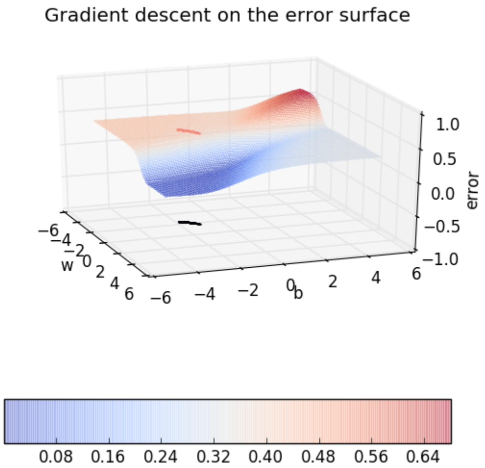

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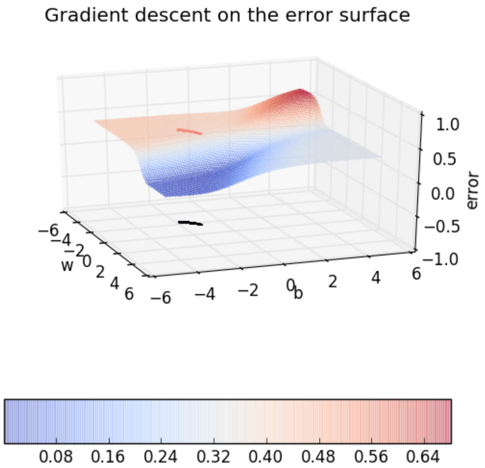

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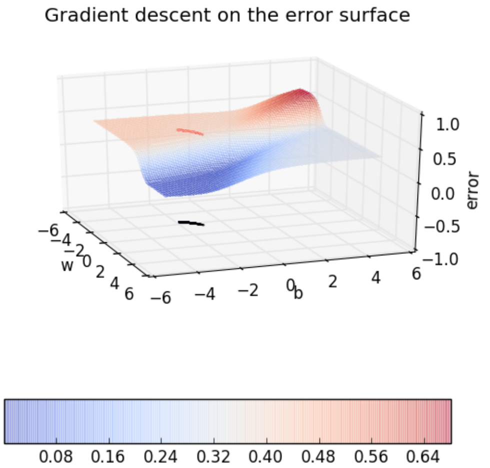

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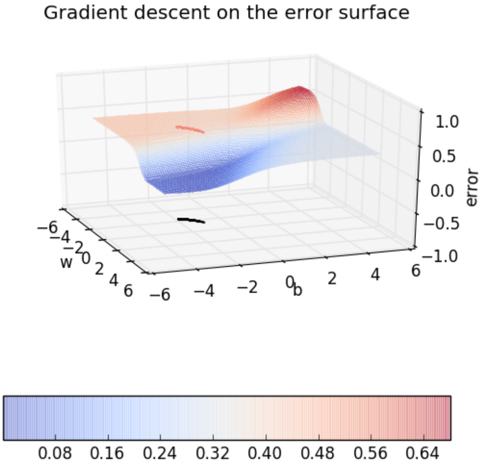

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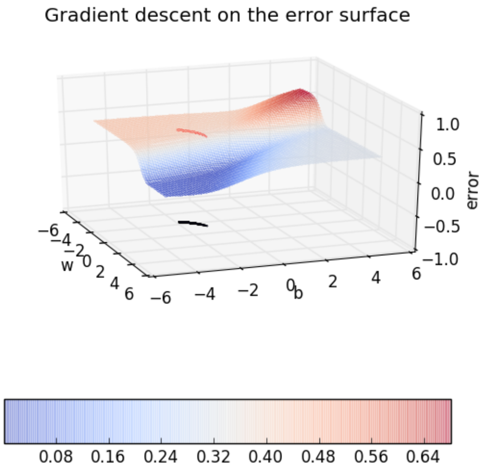

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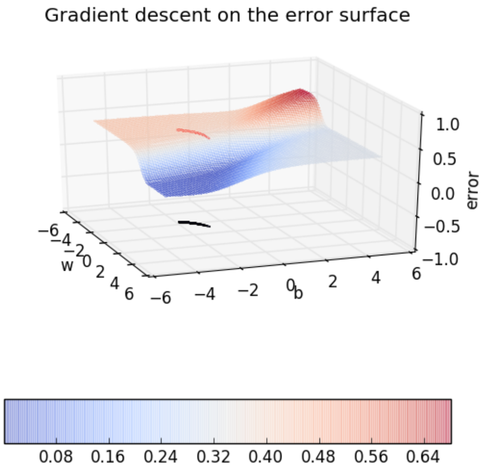

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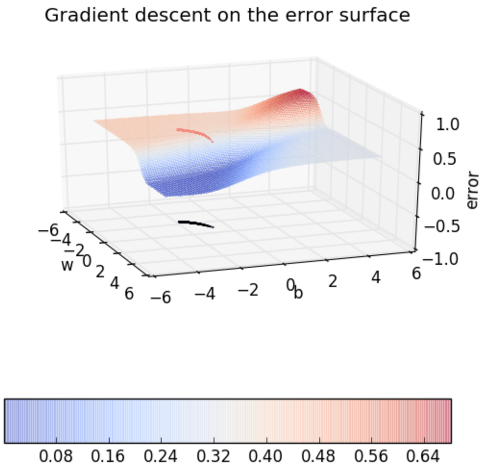

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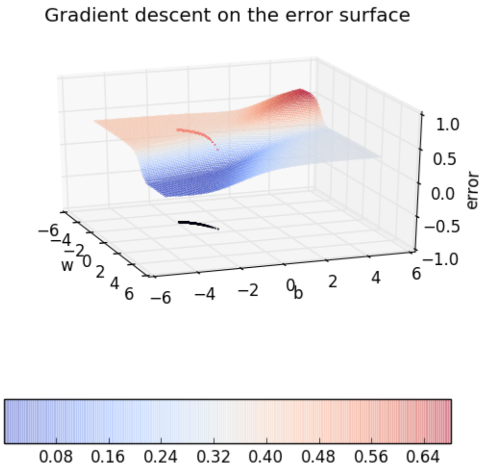

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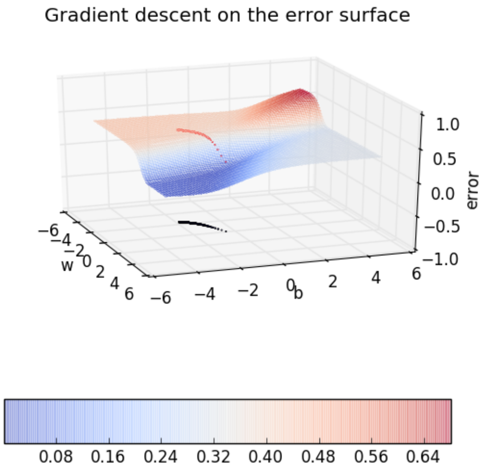

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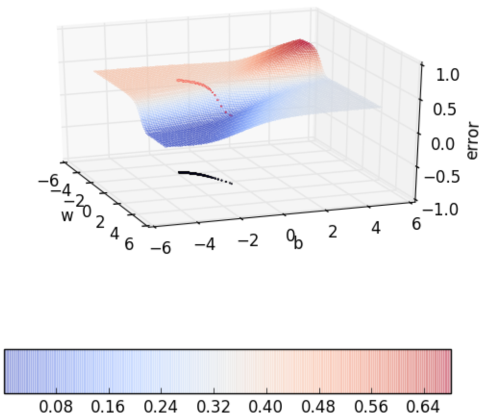
Gradient descent on the error surface



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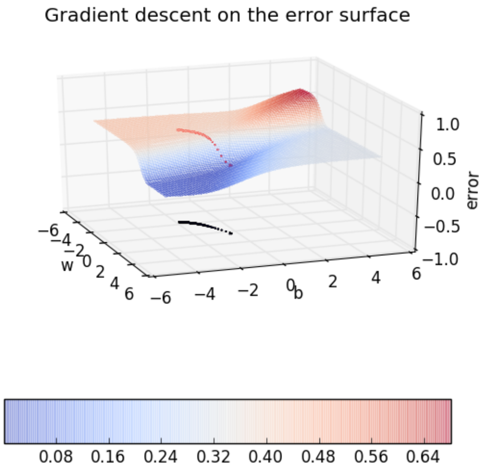

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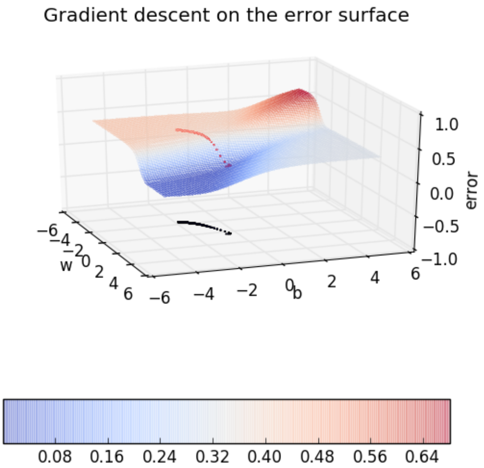

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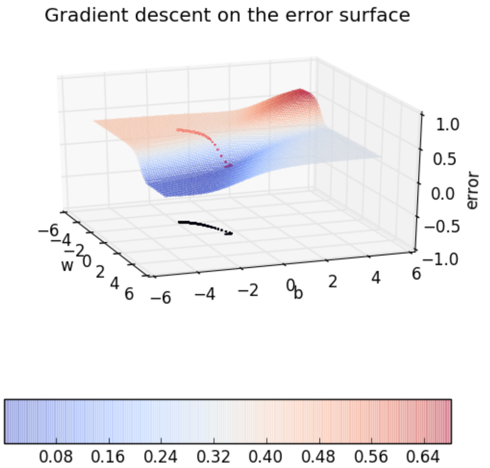

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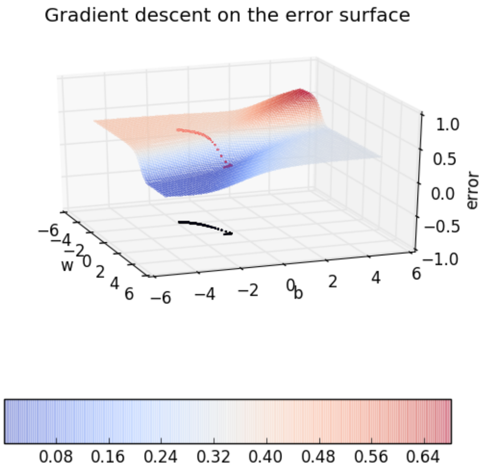

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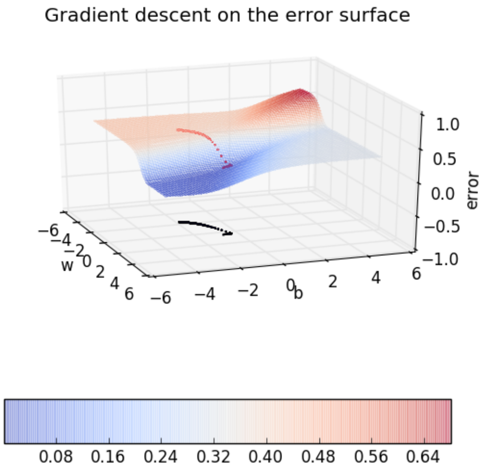

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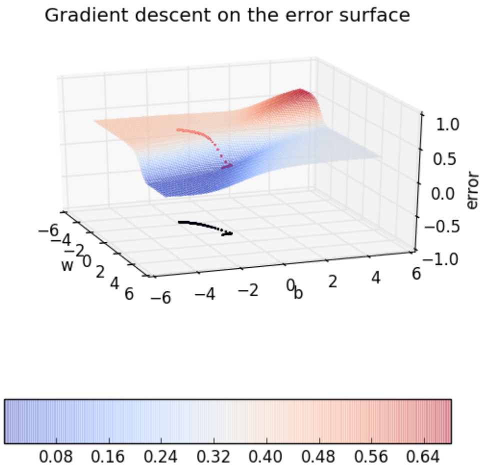

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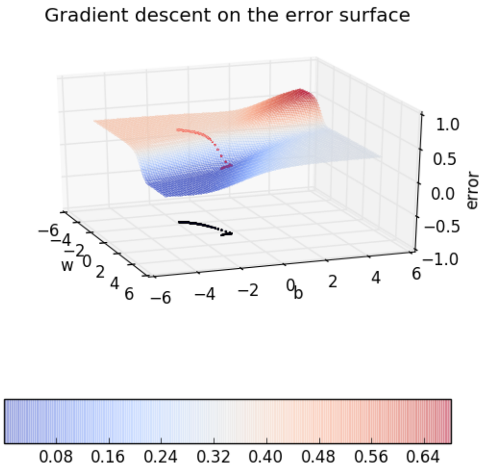

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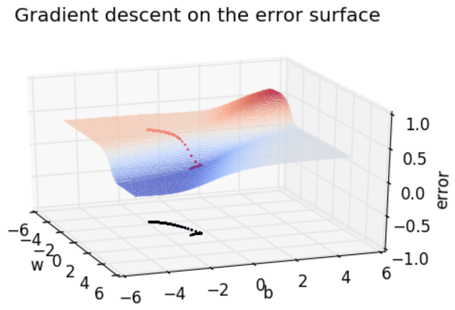

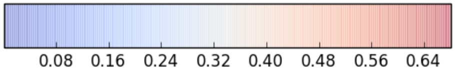

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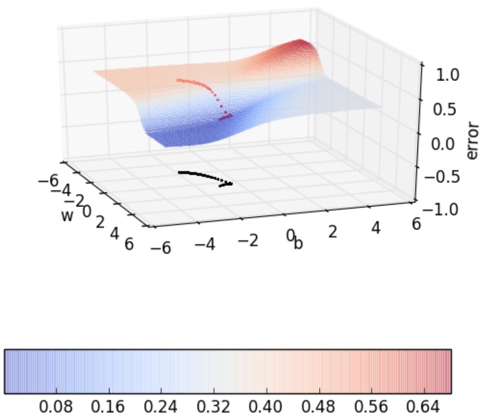
Gradient descent on the error surface



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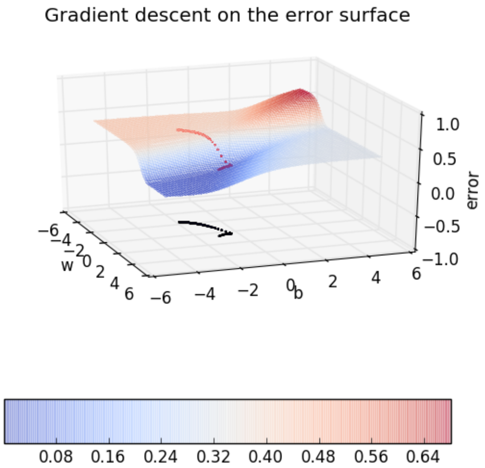

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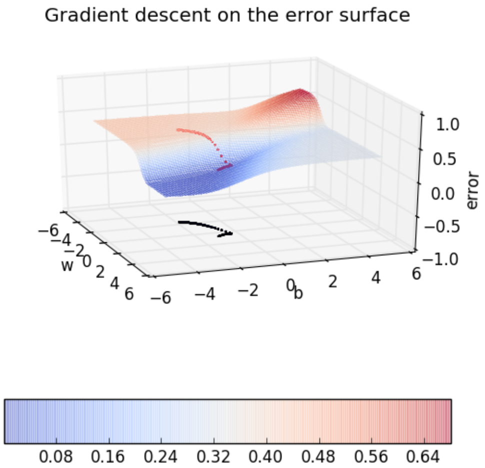

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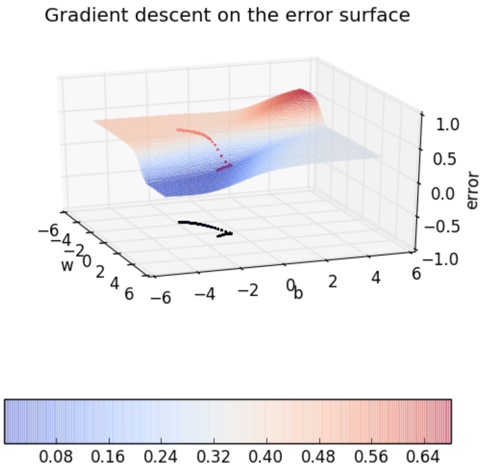

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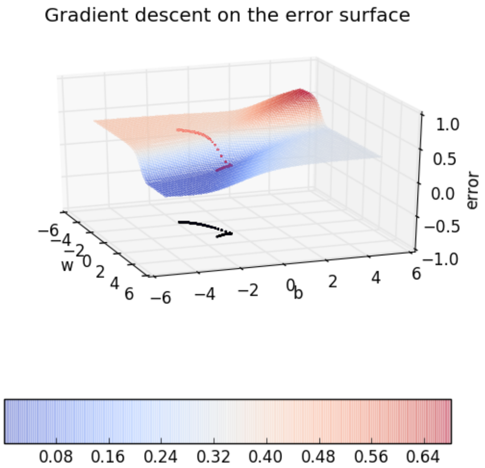

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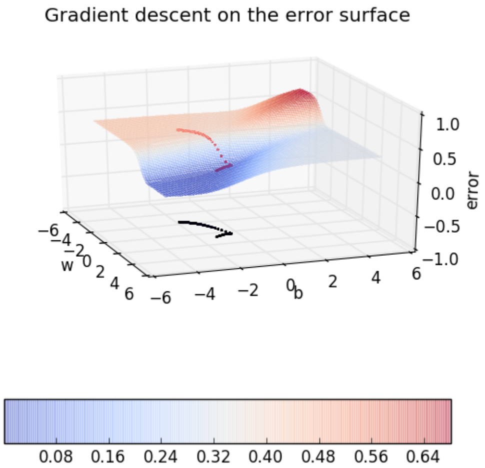

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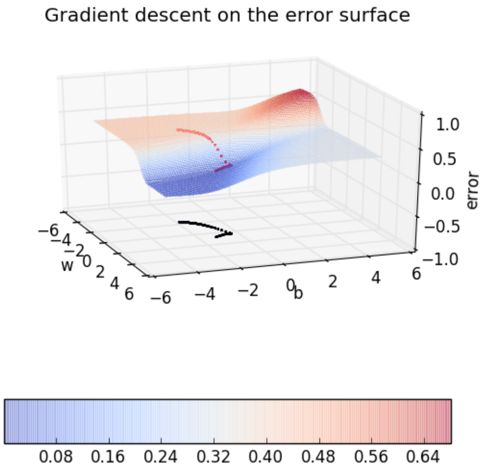

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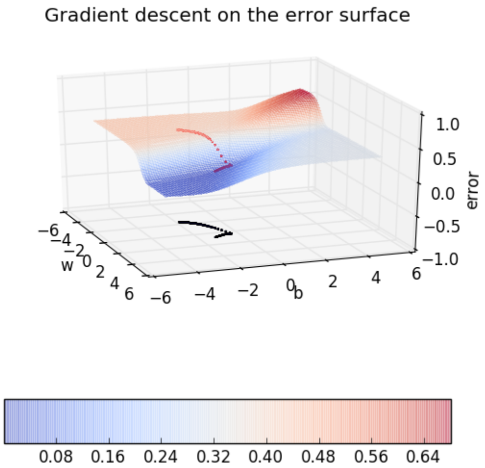

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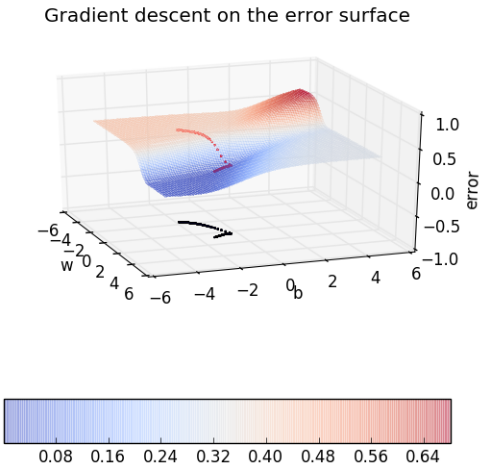

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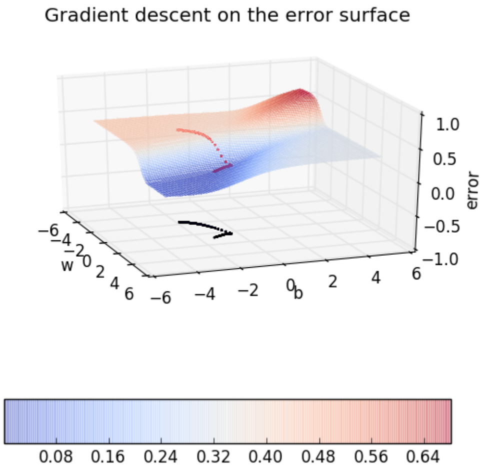

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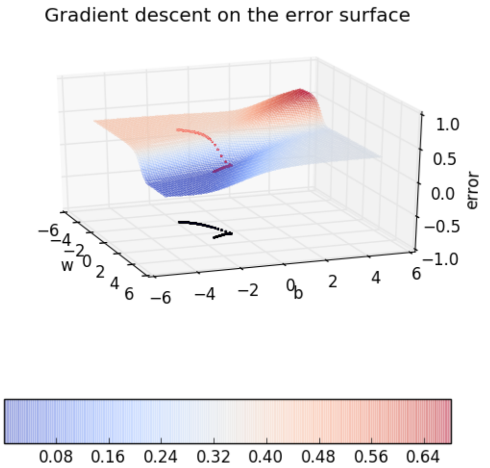

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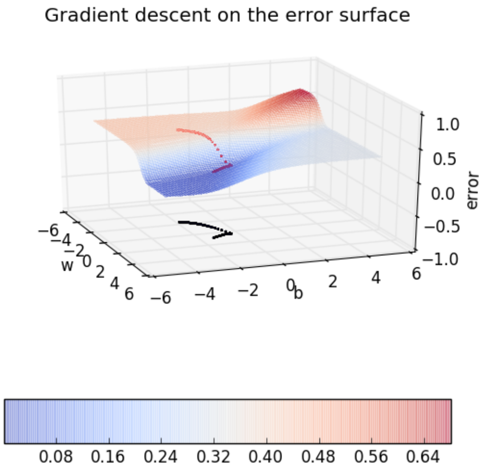

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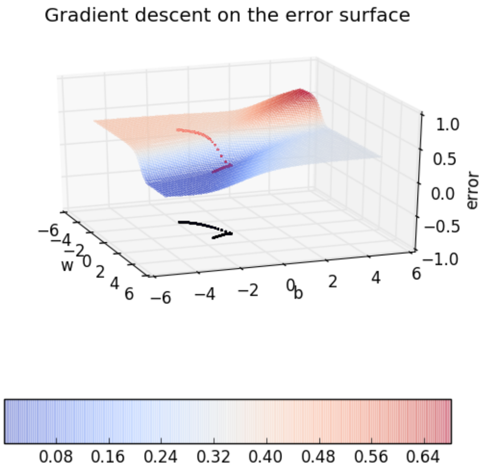

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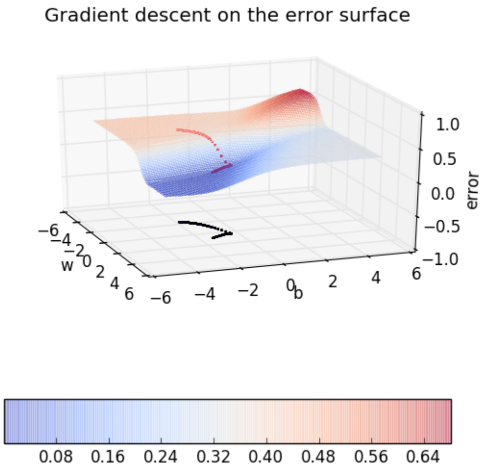

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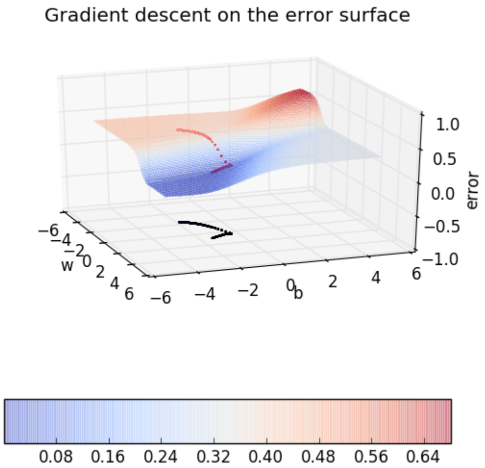

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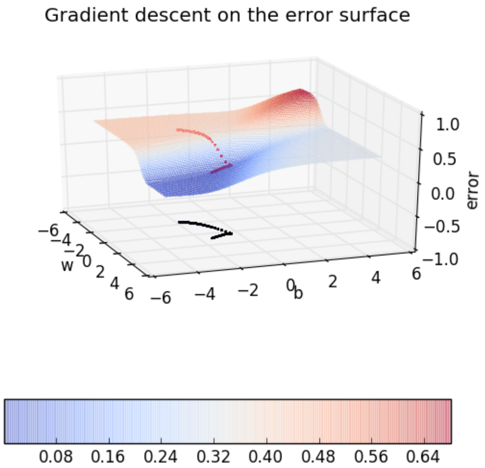

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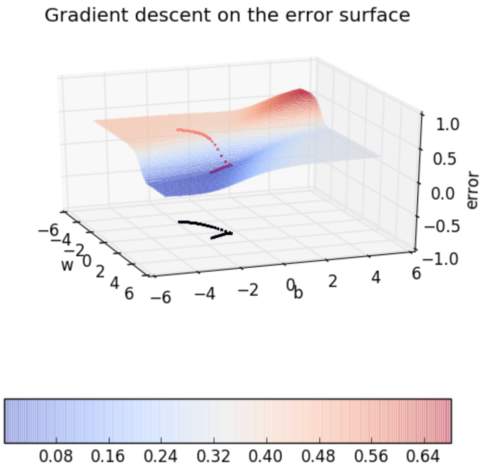

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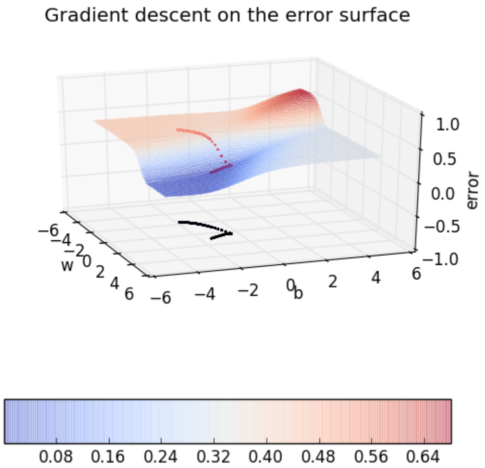

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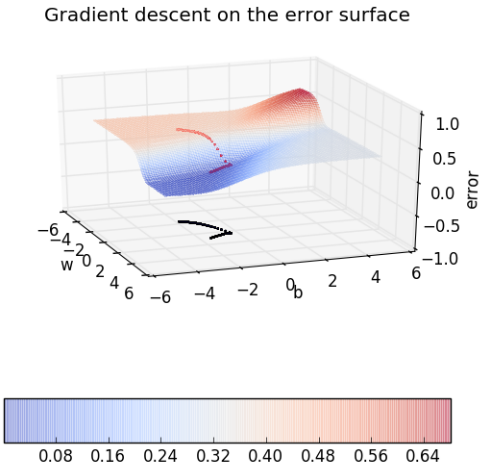

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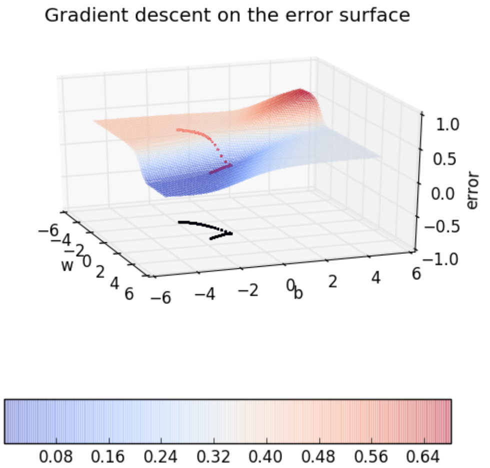

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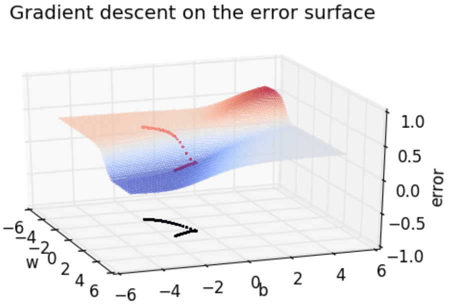

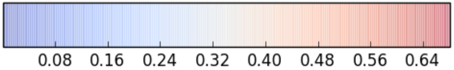

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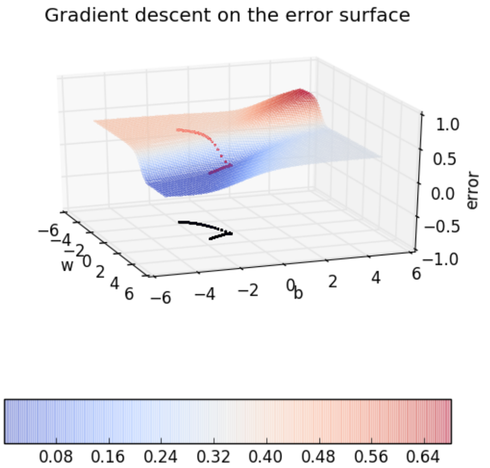

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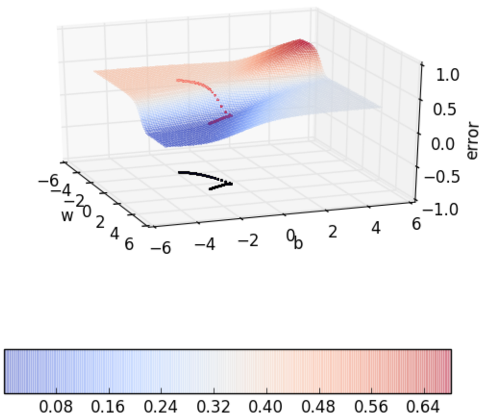
Gradient descent on the error surface



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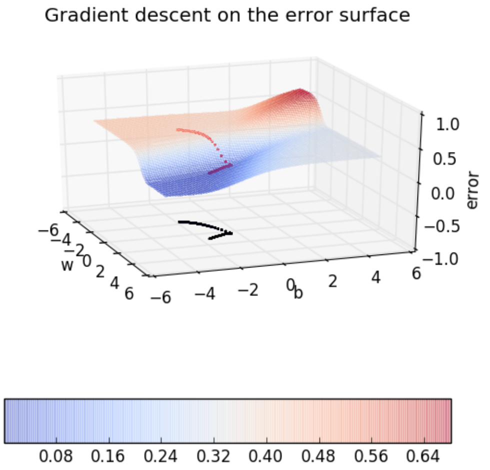

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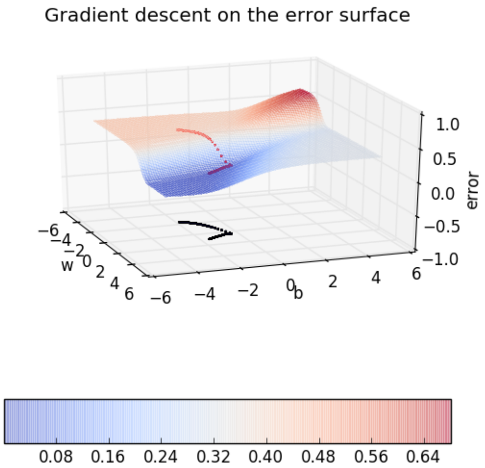

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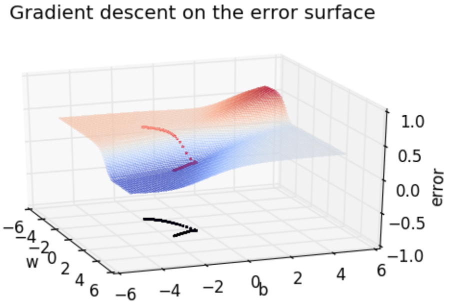

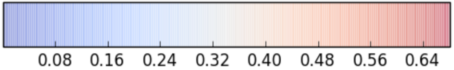

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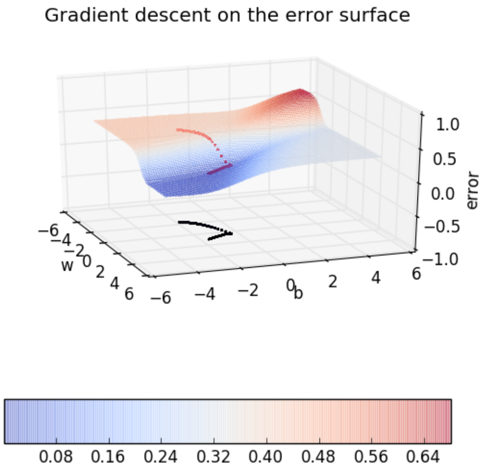

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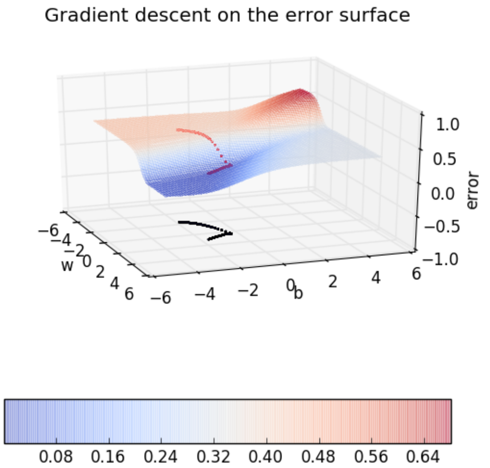

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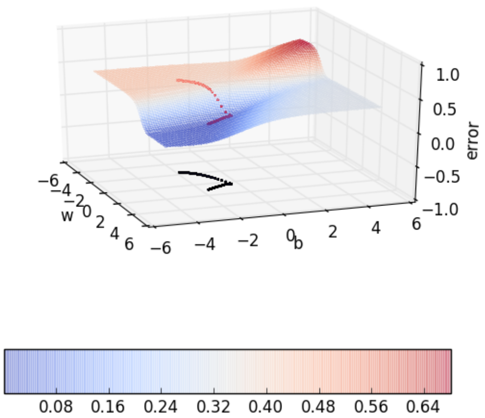
Gradient descent on the error surface



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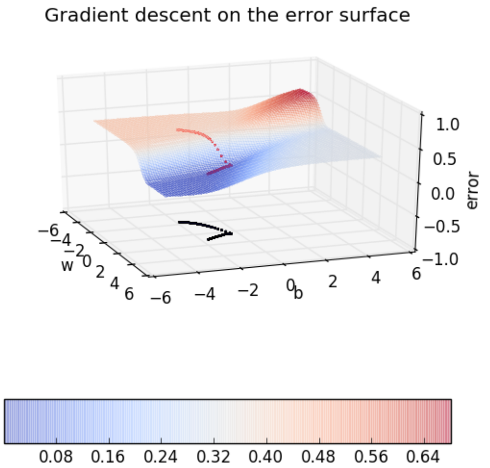

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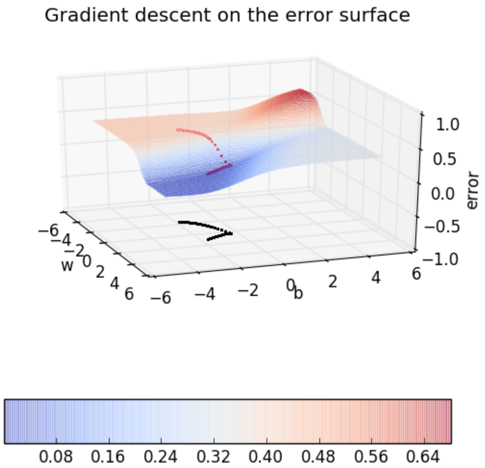

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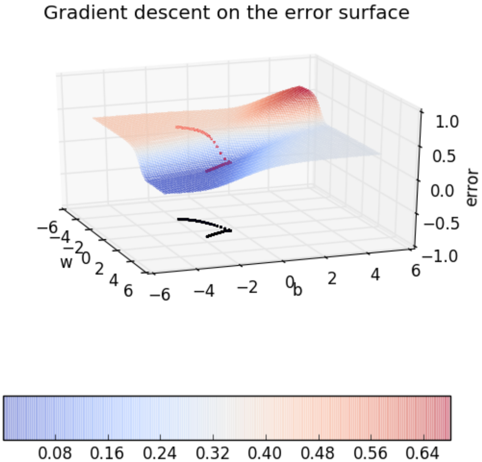

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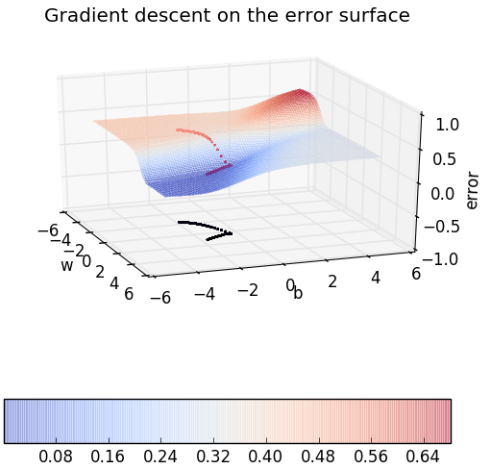

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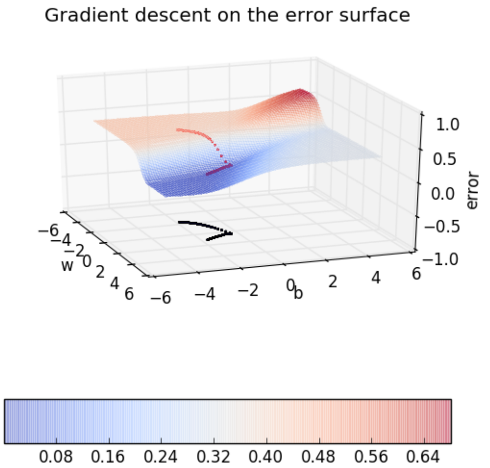

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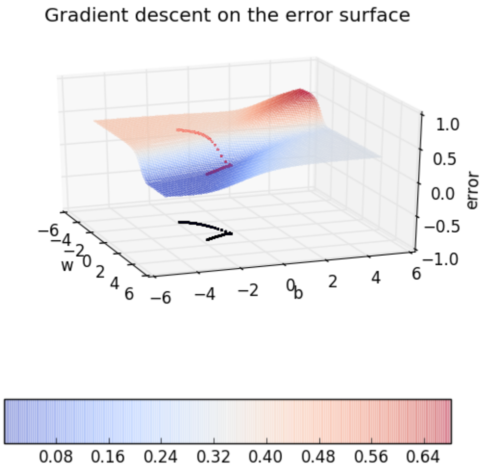

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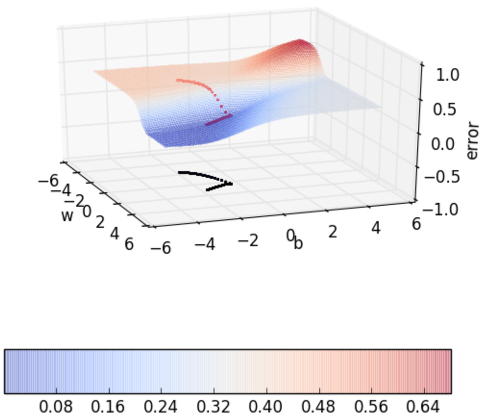
Gradient descent on the error surface



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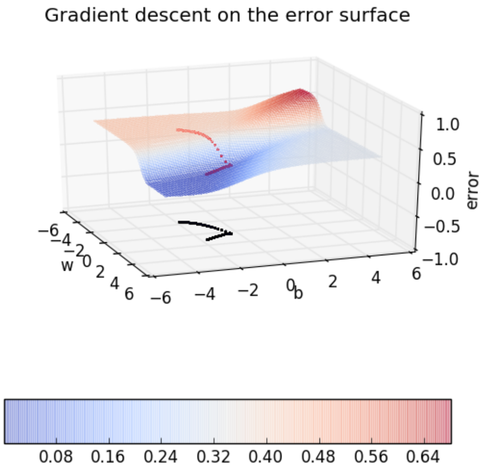

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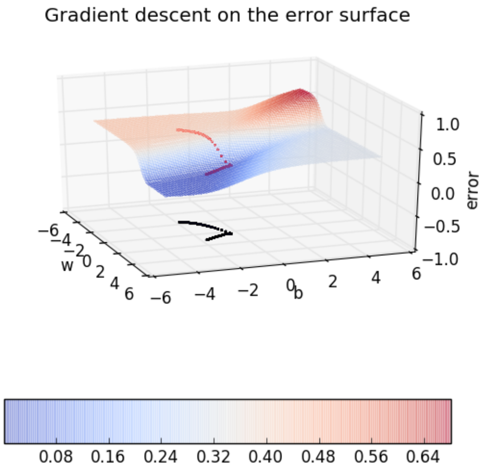

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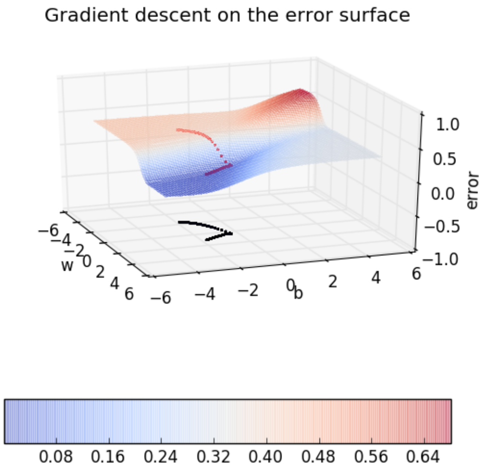

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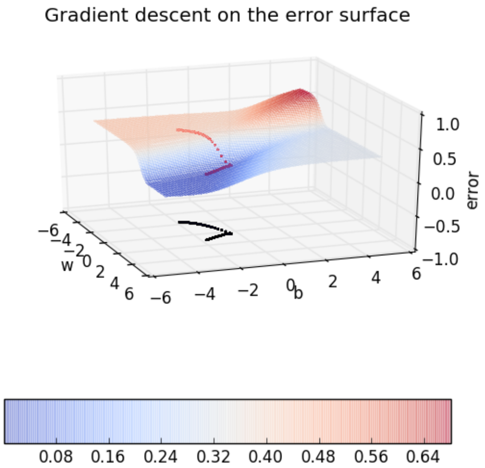

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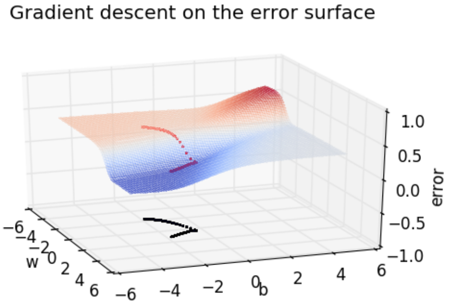

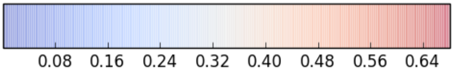

```python
X = [0.5, 2.5]
Y = [0.2, 0.9]

def f(w,b,x) : #sigmoid with parameters w,b
    return 1.0 / (1.0 + np.exp(-(w*x + b)))

def error (w, b) :
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        for x,y in zip(X, Y) :
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        w = w - eta * dw
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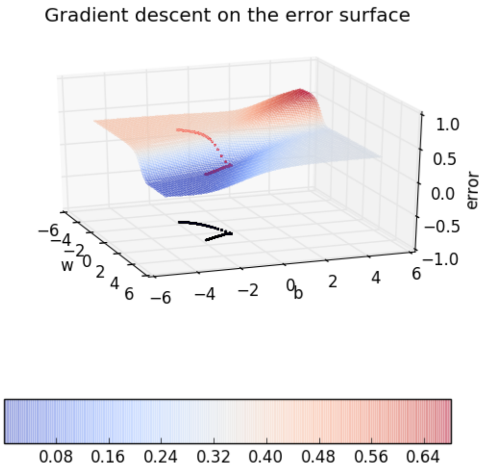

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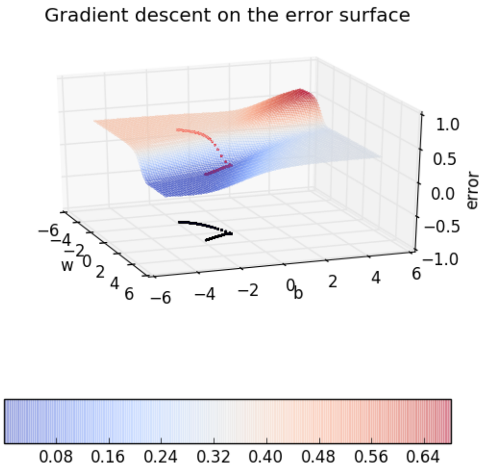

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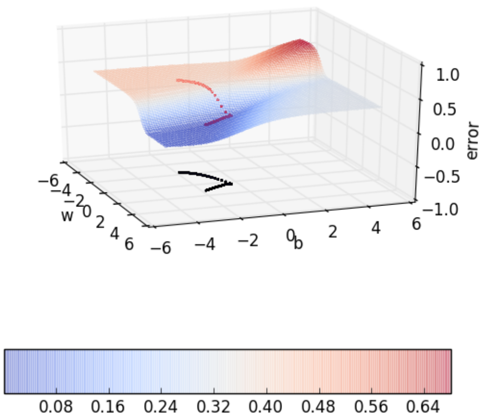
Gradient descent on the error surface



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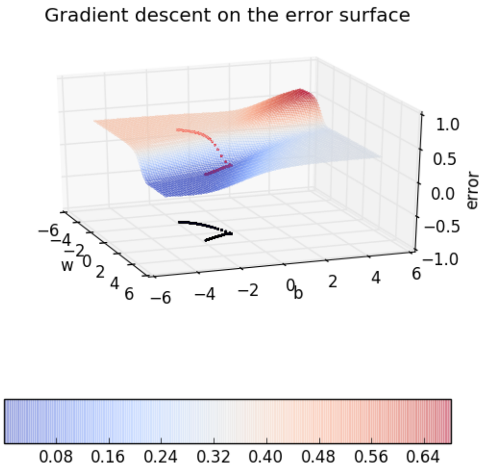

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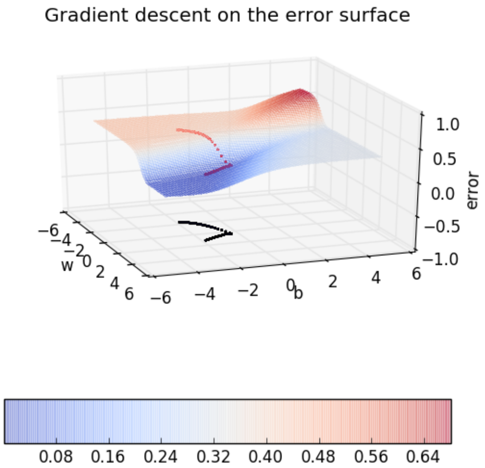

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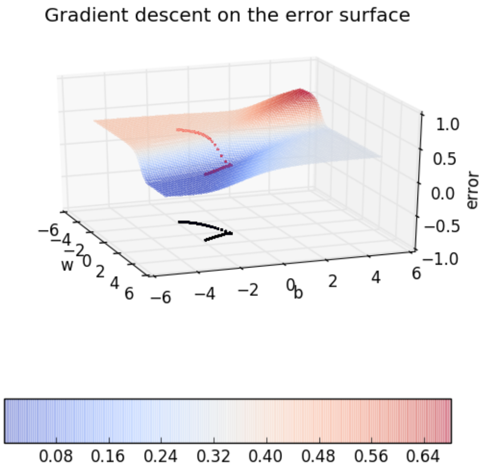

- Later on in the course we will look at gradient descent in much more detail and discuss its variants

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- For the time being it suffices to know that we have an algorithm for learning the parameters of a sigmoid neuron
- So where do we head from here ?

Representation power of a multilayer network of perceptrons

Representation power of a multilayer network of sigmoid neurons

Representation power of a multilayer network of perceptrons

A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

Representation power of a multilayer network of sigmoid neurons

Representation power of a multilayer network of perceptrons

A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

Representation power of a multilayer network of sigmoid neurons

A multilayer network of neurons with a single hidden layer can be used to approximate any continuous function to any desired precision

Representation power of a multilayer network of perceptrons

A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

Representation power of a multilayer network of sigmoid neurons

A multilayer network of neurons with a single hidden layer can be used to approximate any continuous function to any desired precision

In other words, there is a guarantee that for any function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$, we can always find a neural network (with 1 hidden layer containing enough neurons) whose output g(x) satisfies $|g(x) - f(x)| < \epsilon$!!

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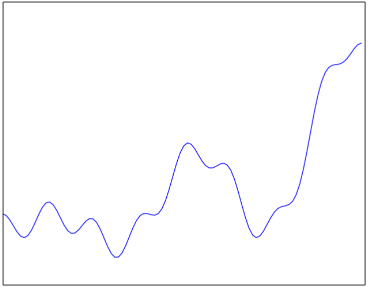
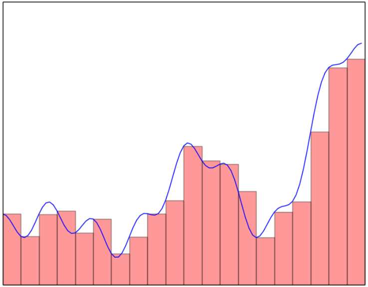
Proof: We will see an illustrative proof of this... [Cybenko, 1989], [Hornik, 1991]

- See this link* for an excellent illustration of this proof
- The discussion in the next few slides is based on the ideas presented at the above link

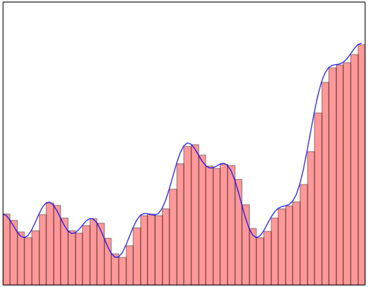
*http://neuralnetworksanddeeplearning.com/chap4.html



- We are interested in knowing whether a network of neurons can be used to represent an arbitrary function (like the one shown in the figure)



- We are interested in knowing whether a network of neurons can be used to represent an arbitrary function (like the one shown in the figure)
- We observe that such an arbitrary function can be approximated by several "tower" functions



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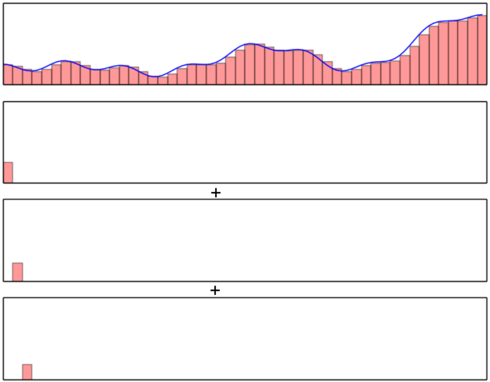

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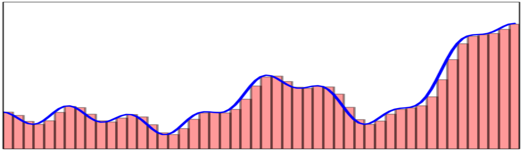
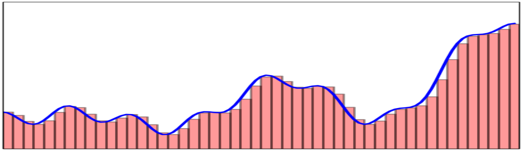
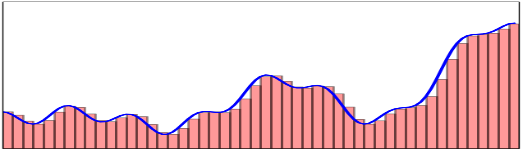
- We observe that such an arbitrary function can be approximated by several “tower” functions
- More the number of such “tower” functions, better the approximation
- To be more precise, we can approximate any arbitrary function by a sum of such “tower” functions



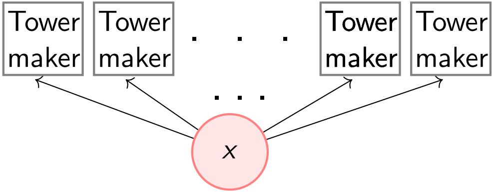
- We make a few observations

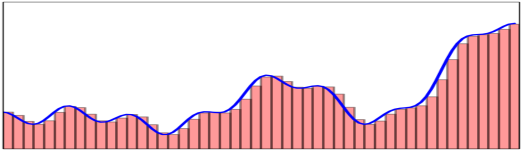


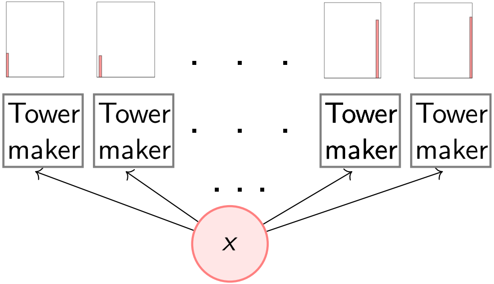
- We make a few observations
- All these "tower" functions are similar and only differ in their heights and positions on the x-axis



- We make a few observations
- All these "tower" functions are similar and only differ in their heights and positions on the x-axis
- Suppose there is a black box which takes the original input (x) and constructs these tower functions







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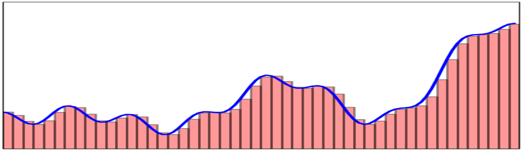

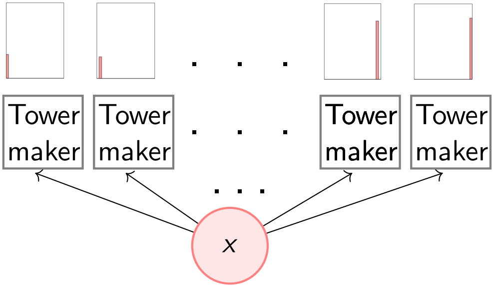

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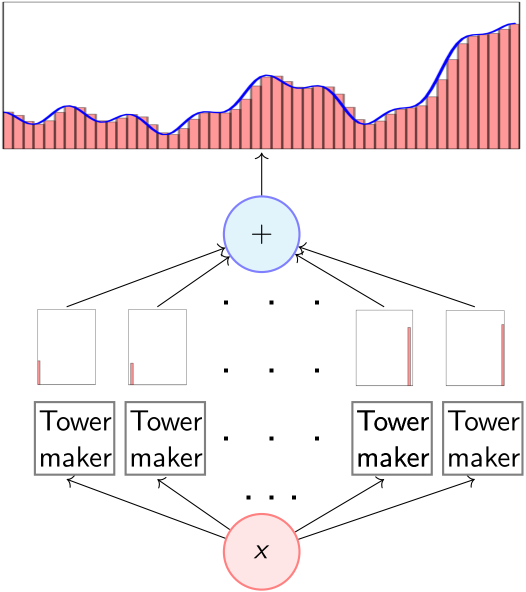
- We can then have a simple network which can just add them up to approximate the function



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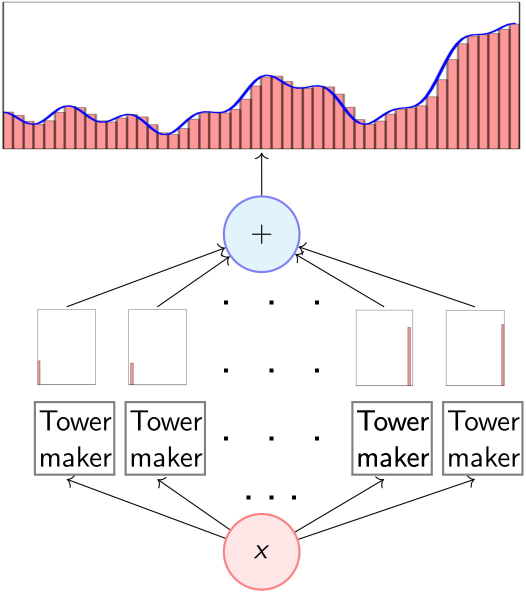

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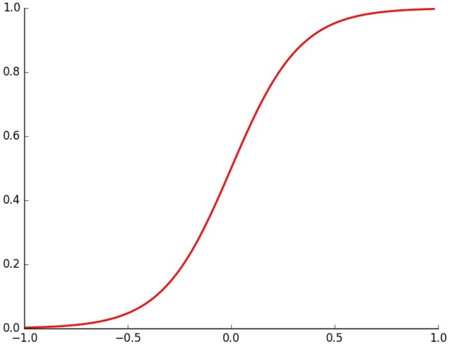
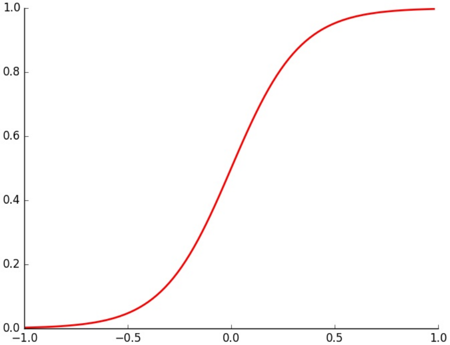
- We can then have a simple network which can just add them up to approximate the function
- Our job now is to figure out what is inside this blackbox

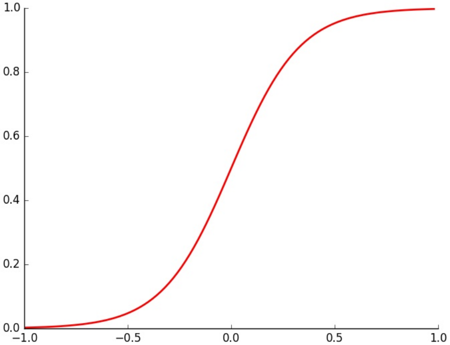
We will figure this out over the next few slides ...



- If we take the logistic function and set w to a very high value we will recover the step function



- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



$w = 6, b = 0, 3$

- If we take the logistic function and set w to a very high value we will recover the step function
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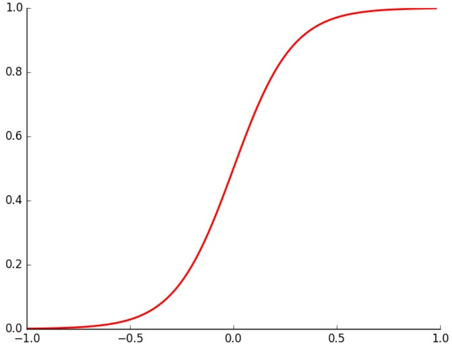

$w = 7, b = 0, 4$

- If we take the logistic function and set w to a very high value we will recover the step function
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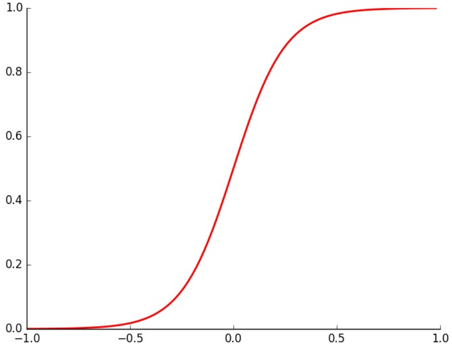

$w = 8, b = 0, 5$

- If we take the logistic function and set w to a very high value we will recover the step function
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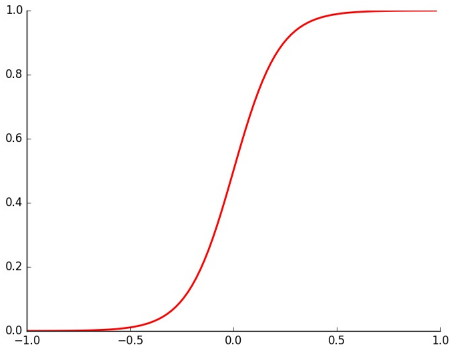

$w = 9, b = 0, 6$

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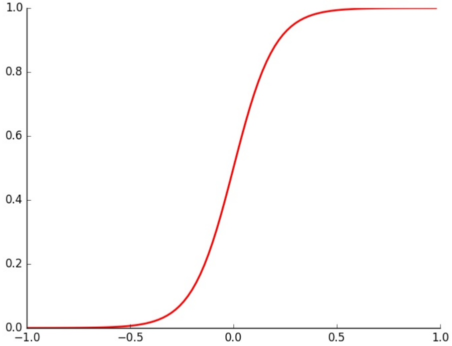
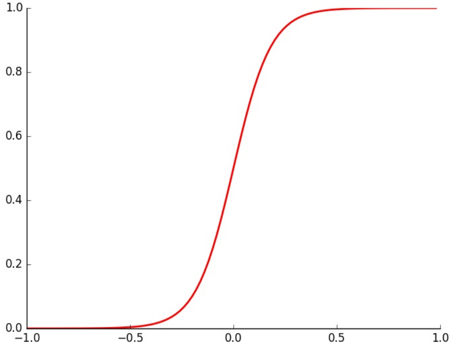


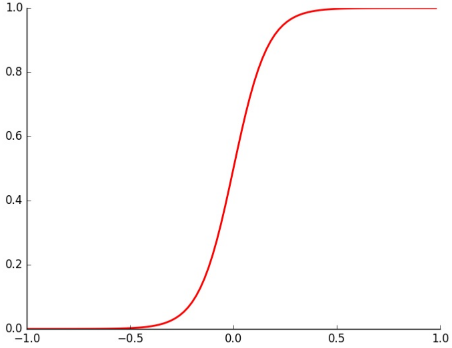
$w = 10, b = 0, 7$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



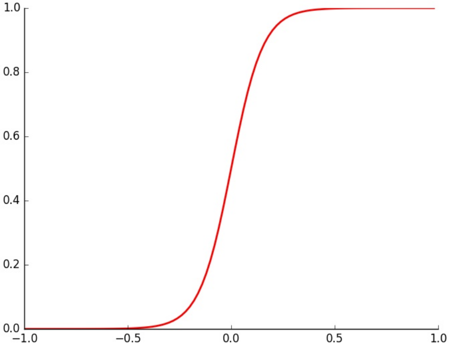
$w = 11, b = 0, 8$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



$w = 12, b = 0,9$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



$w = 13, b = 0, 10$

- If we take the logistic function and set w to a very high value we will recover the step function
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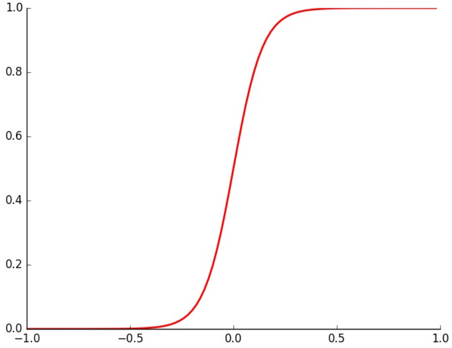

$w = 14, b = 0, 11$

- If we take the logistic function and set w to a very high value we will recover the step function
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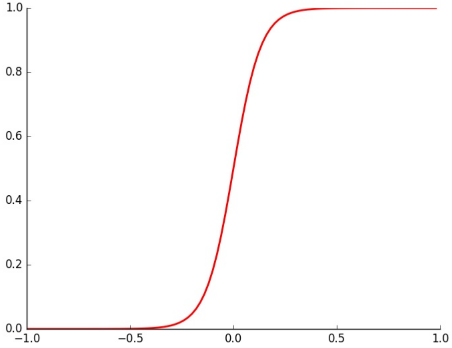

$w = 15, b = 0, 12$

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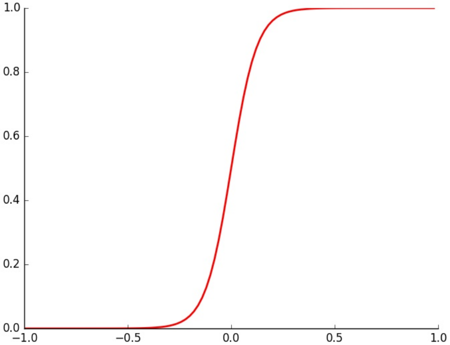

$w = 16, b = 0, 13$

- If we take the logistic function and set w to a very high value we will recover the step function
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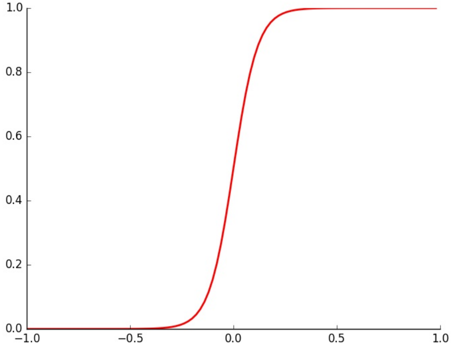
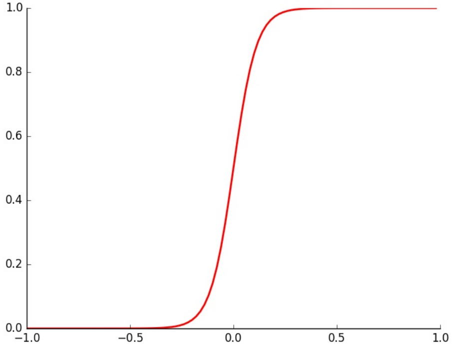


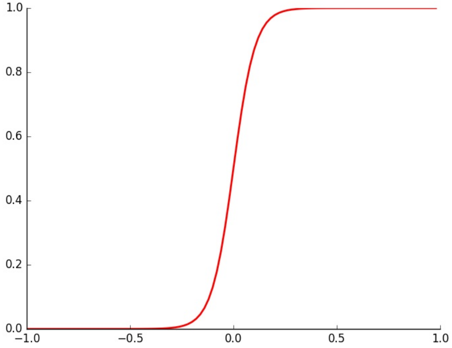
$w = 17, b = 0, 14$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



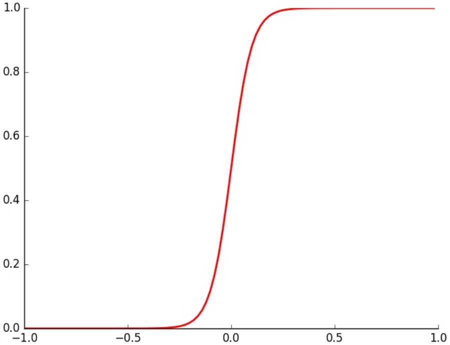
$w = 18, b = 0, 15$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



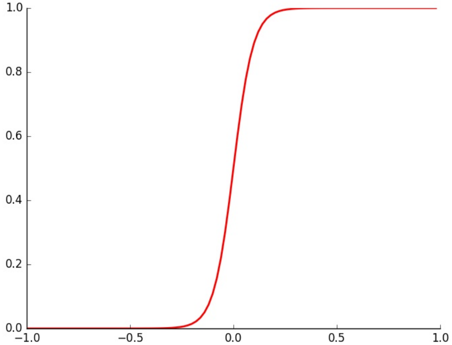
$w = 19, b = 0,16$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



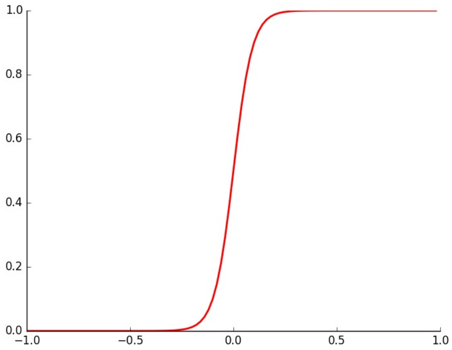
$w = 20, b = 0, 17$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



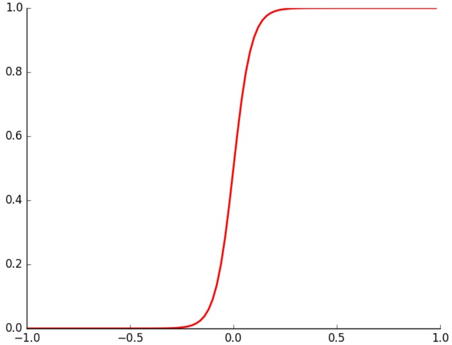
$w = 21, b = 0, 18$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



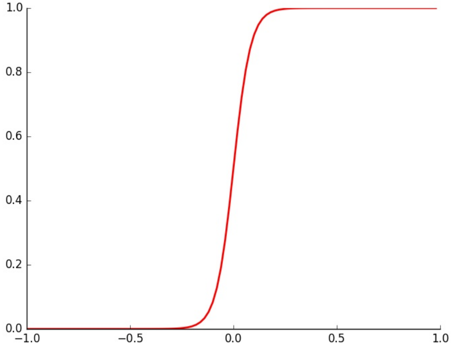
$$w = 22, b = 0, 19$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



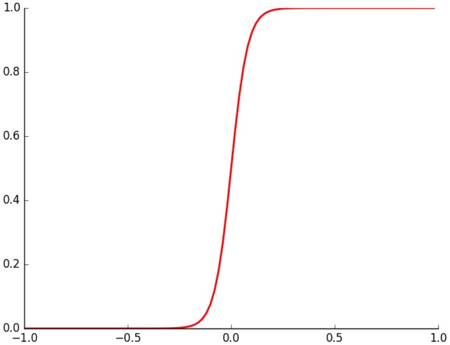
$w = 23, b = 0, 20$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



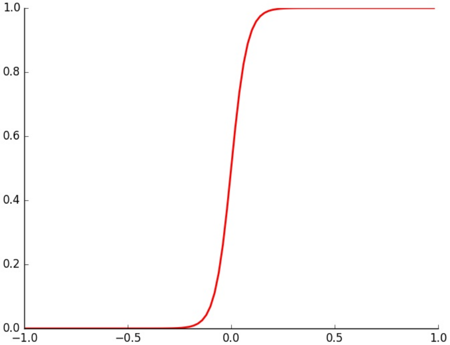
$w = 24, b = 0, 21$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



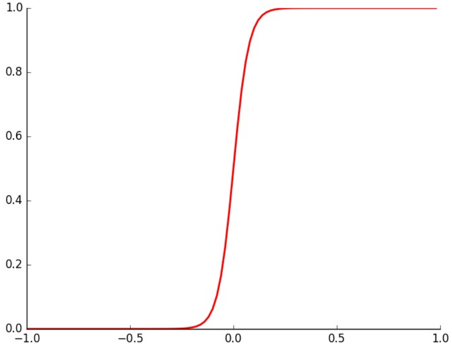
$w = 25, b = 0, 22$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



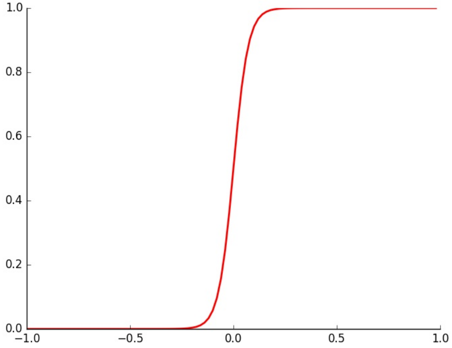
$w = 26, b = 0,23$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



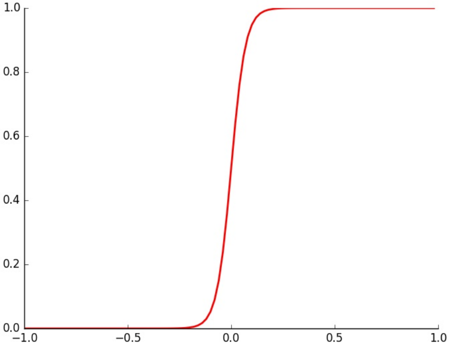
$w = 27, b = 0, 24$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



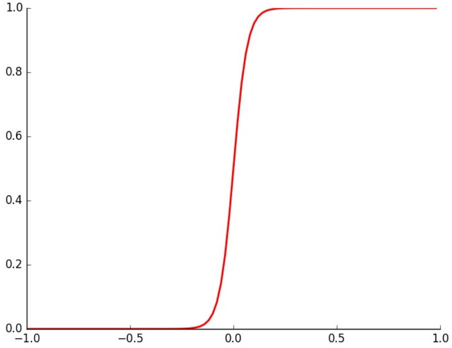
$w = 28, b = 0,25$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



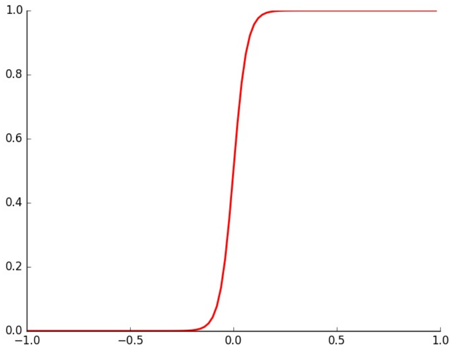
$w = 29, b = 0, 26$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



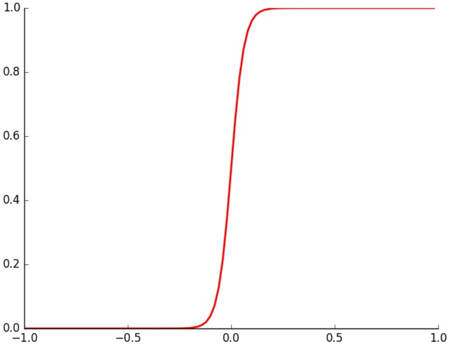
$w = 30, b = 0, 27$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



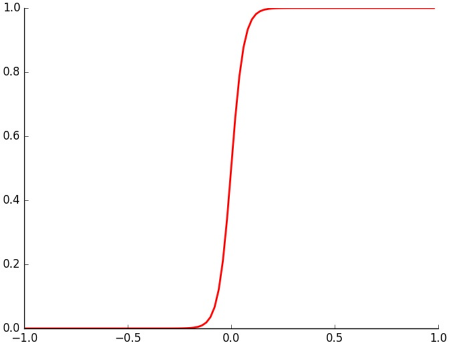
$w = 31, b = 0, 28$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



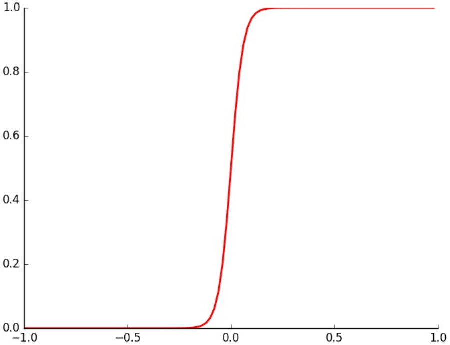
$w = 32, b = 0, 29$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



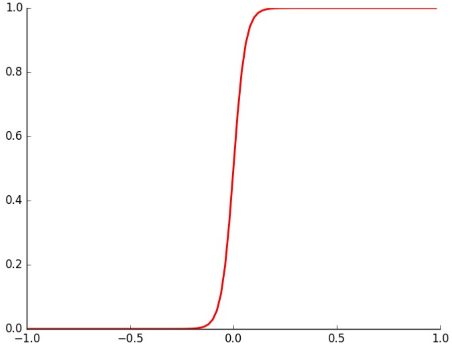
$w = 33, b = 0, 30$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



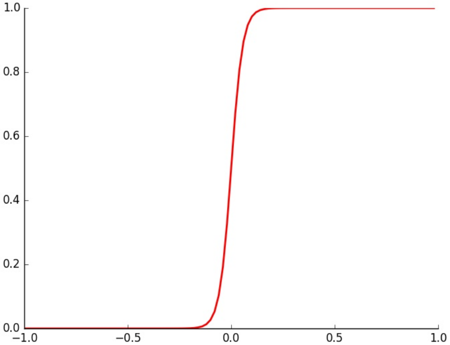
$w = 34, b = 0, 31$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



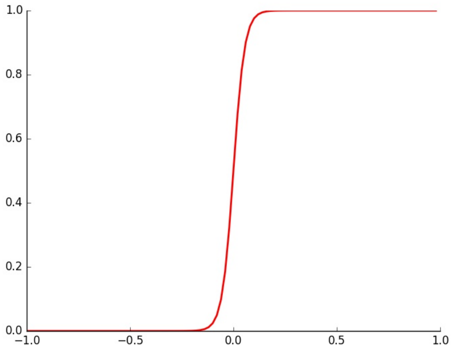
$w = 35, b = 0, 32$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



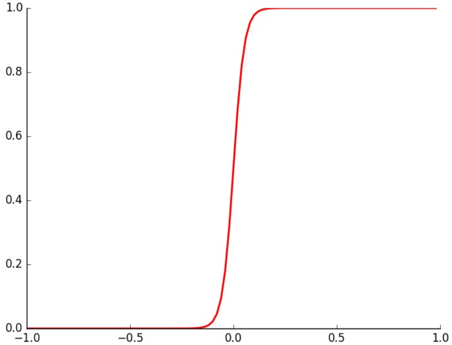
$w = 36, b = 0, 33$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



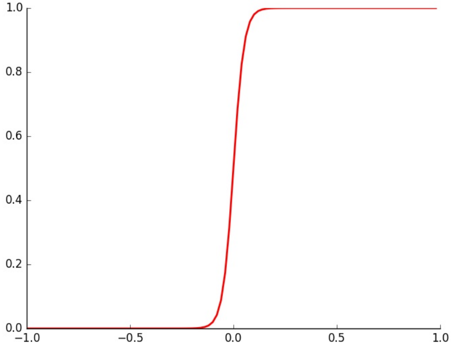
$w = 37, b = 0, 34$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



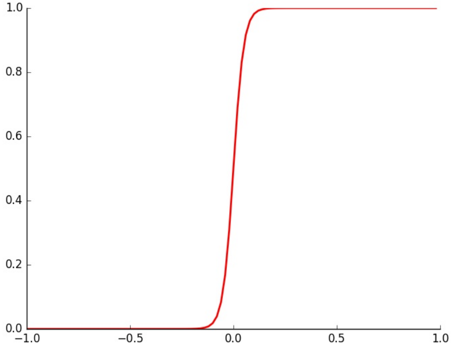
$w = 38, b = 0, 35$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



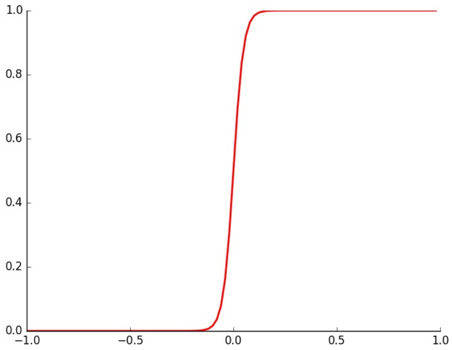
$w = 39, b = 0, 36$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



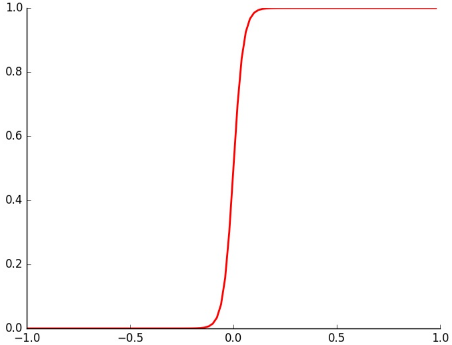
$w = 40, b = 0, 37$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



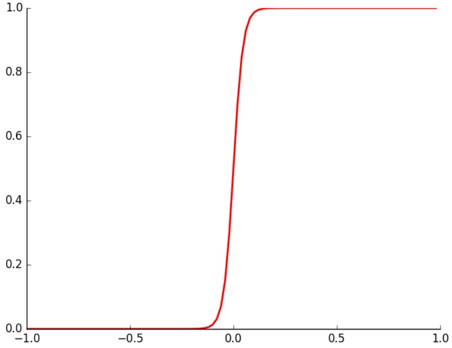
$w = 41, b = 0, 38$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



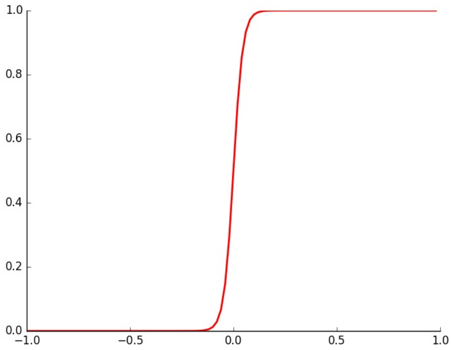
$w = 42, b = 0, 39$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



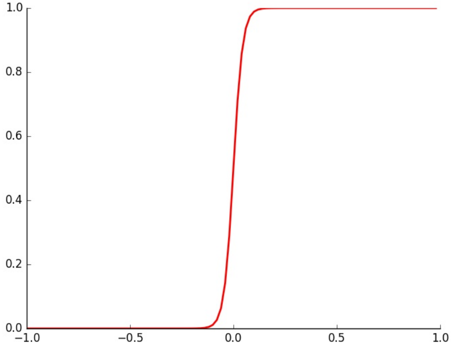
$w = 43, b = 0, 40$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



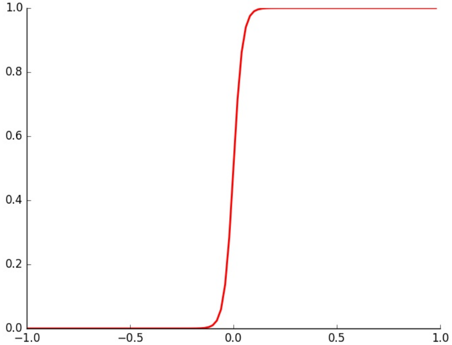
$w = 44, b = 0, 41$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



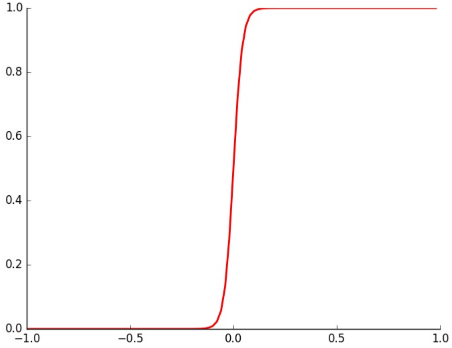
$w = 45, b = 0,42$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



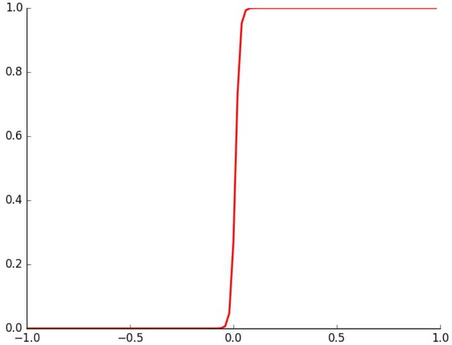
$w = 46, b = 0, 43$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



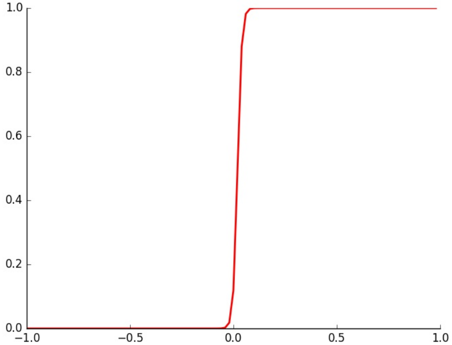
$w = 47, b = 0, 44$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



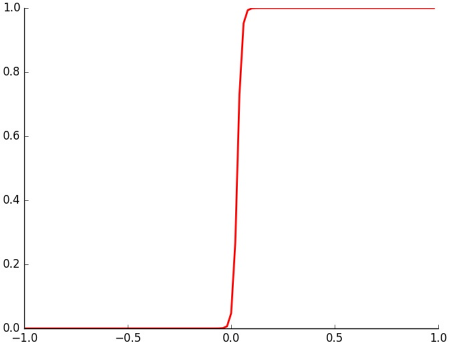
$w = 50, b = 1$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



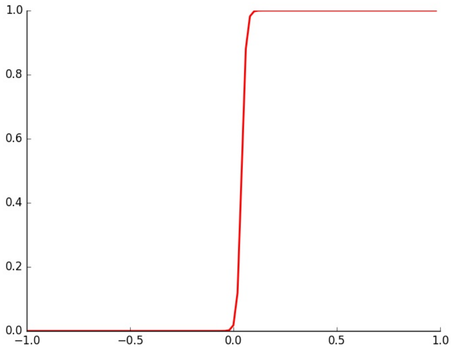
$w = 50, b = 2$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



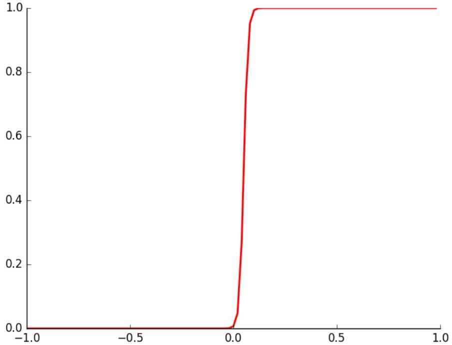
$w = 50, b = 3$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



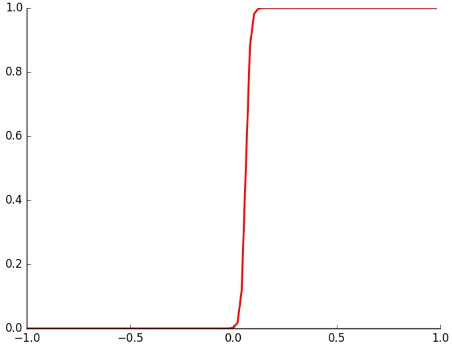
$w = 50, b = 4$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



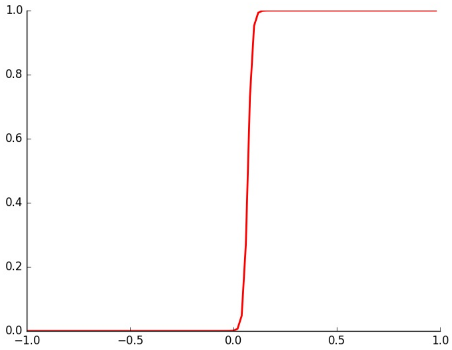
$w = 50, b = 5$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



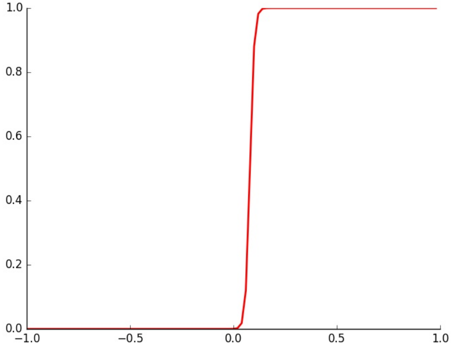
$w = 50, b = 6$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



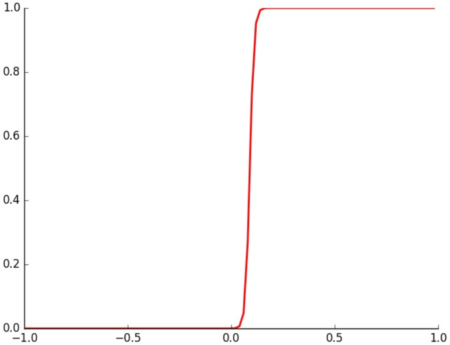
$w = 50, b = 7$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



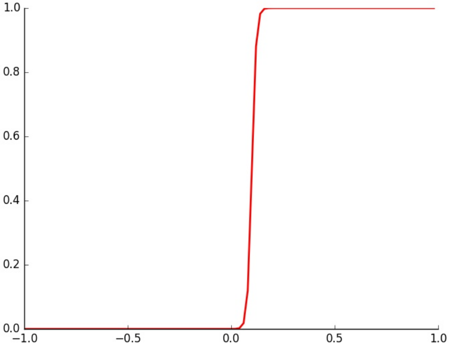
$w = 50, b = 8$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



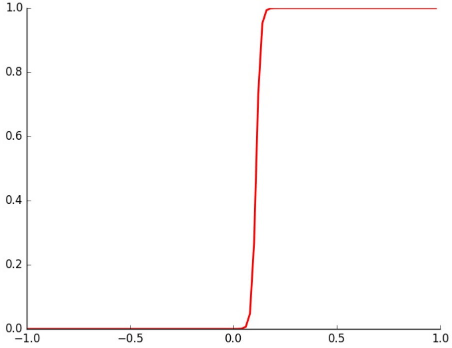
$w = 50, b = 9$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



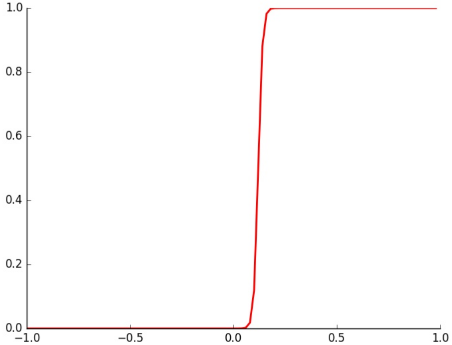
$w = 50, b = 10$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



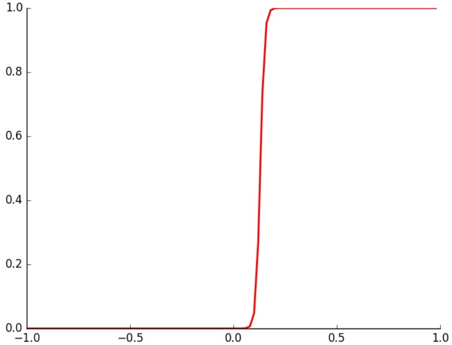
$w = 50, b = 11$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



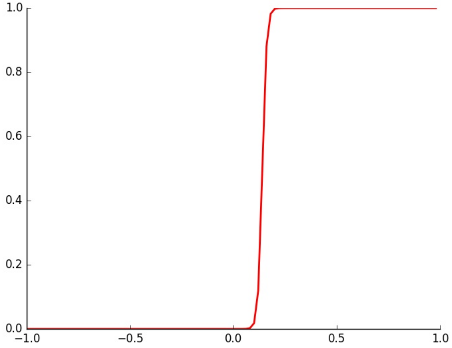
$w = 50, b = 12$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



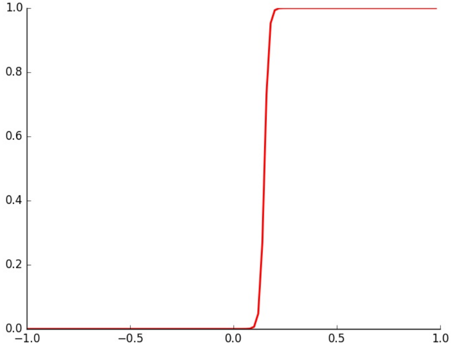
$w = 50, b = 13$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



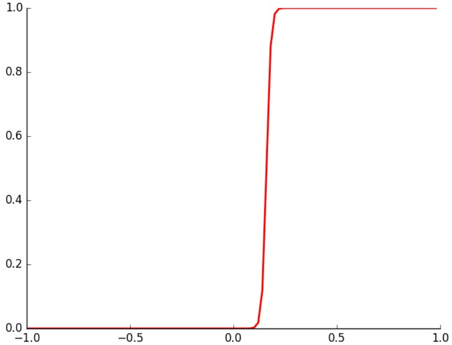
$w = 50, b = 14$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



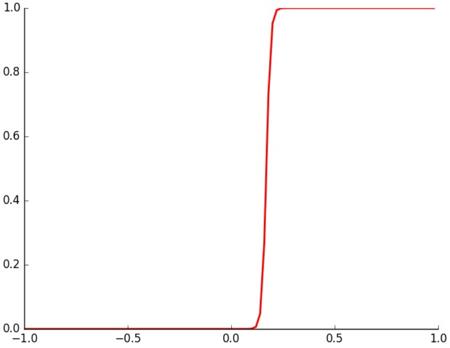
$w = 50, b = 15$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



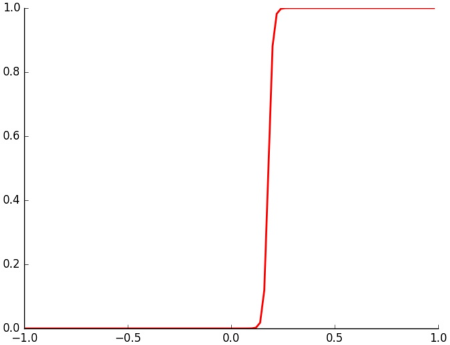
$w = 50, b = 16$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



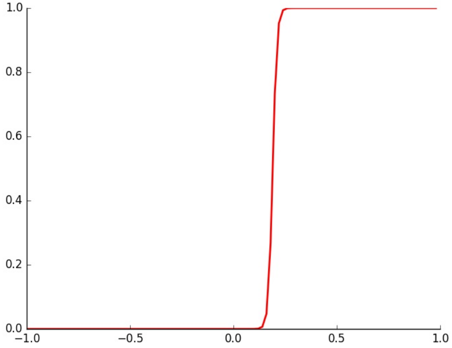
$w = 50, b = 17$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



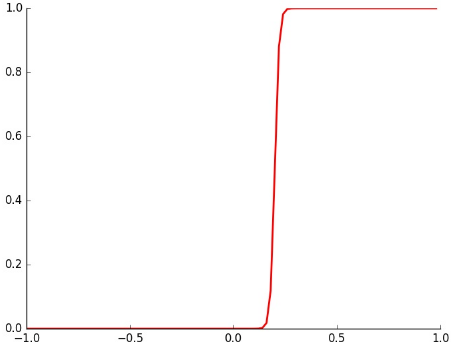
$w = 50, b = 18$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



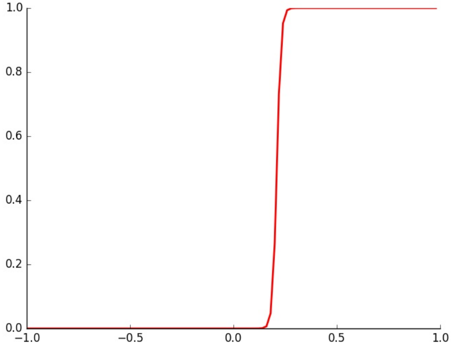
$w = 50, b = 19$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



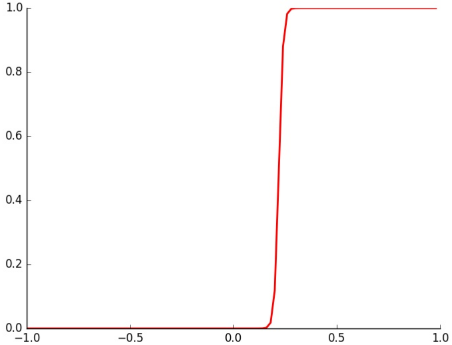
$w = 50, b = 20$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



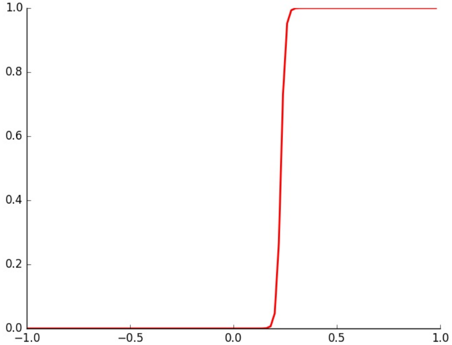
$w = 50, b = 21$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



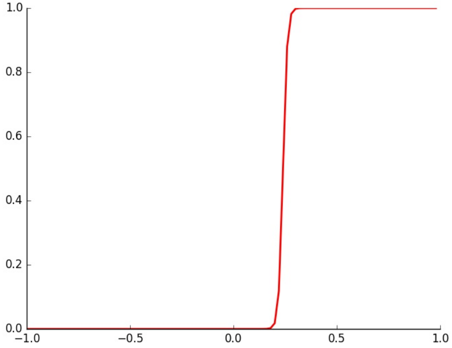
$w = 50, b = 22$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



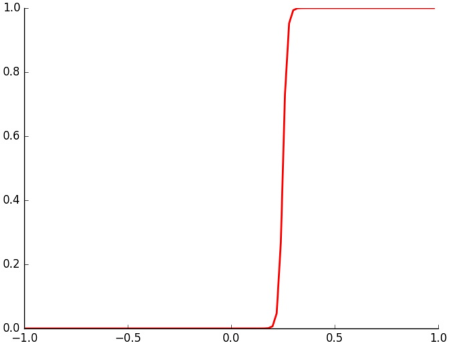
$w = 50, b = 23$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



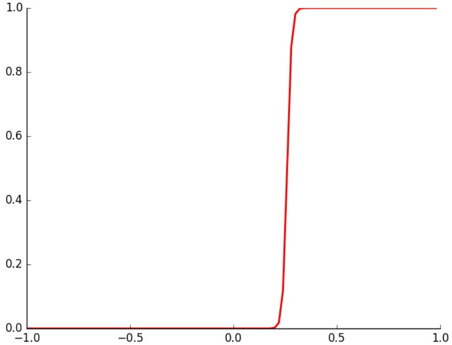
$w = 50, b = 24$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



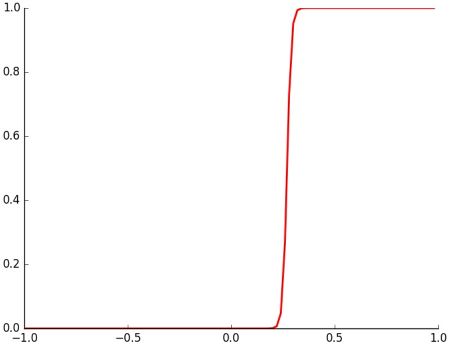
$w = 50, b = 25$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



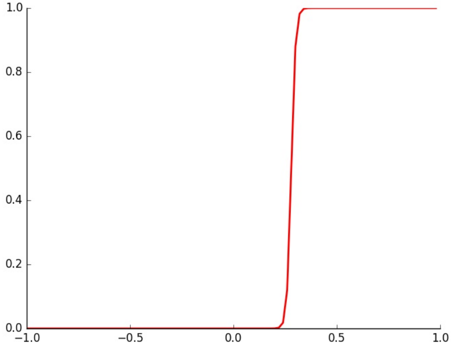
$w = 50, b = 26$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



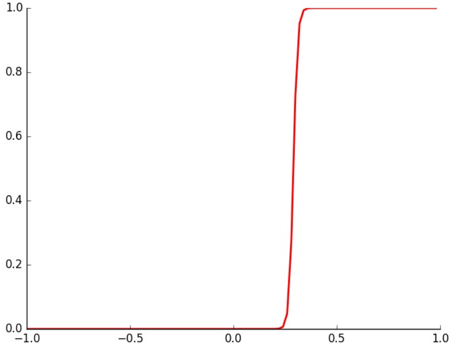
$w = 50, b = 27$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



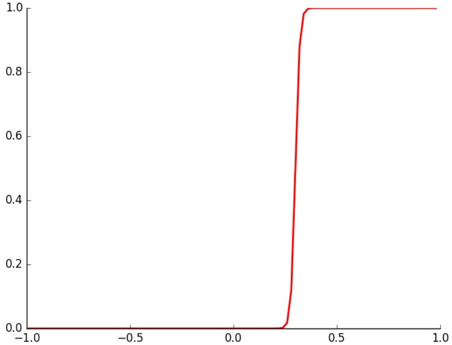
$w = 50, b = 28$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



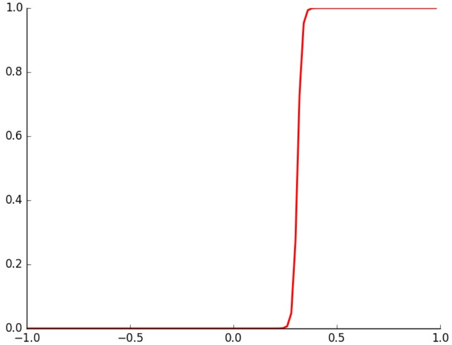
$w = 50, b = 29$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



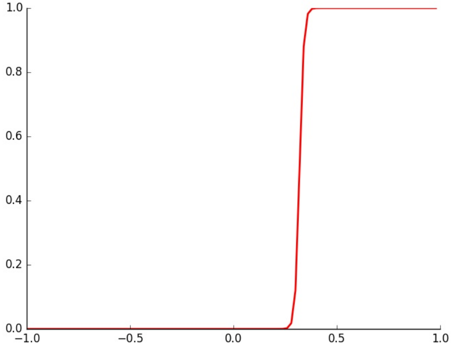
$w = 50, b = 30$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



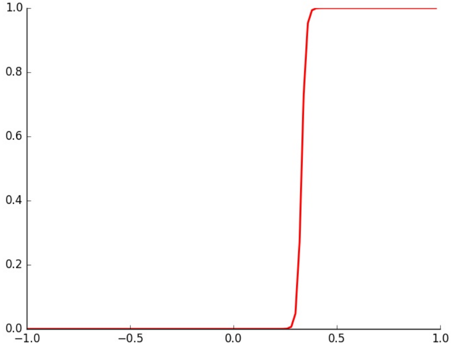
$w = 50, b = 31$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



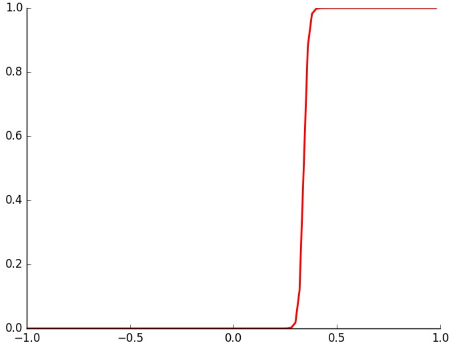
$w = 50, b = 32$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



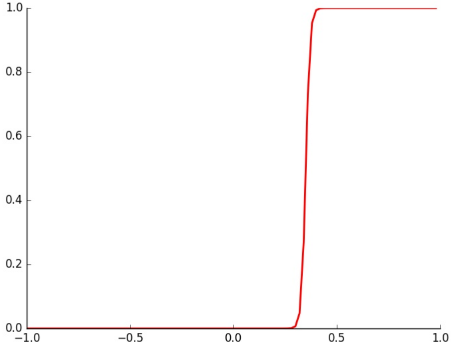
$w = 50, b = 33$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



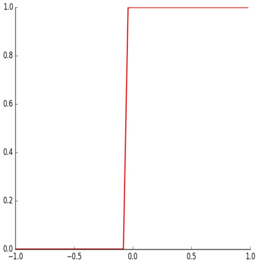
$w = 50, b = 34$

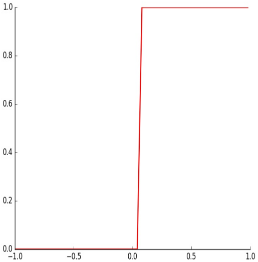
- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



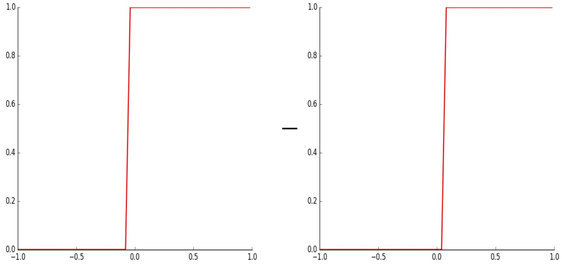
$w = 50, b = 35$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

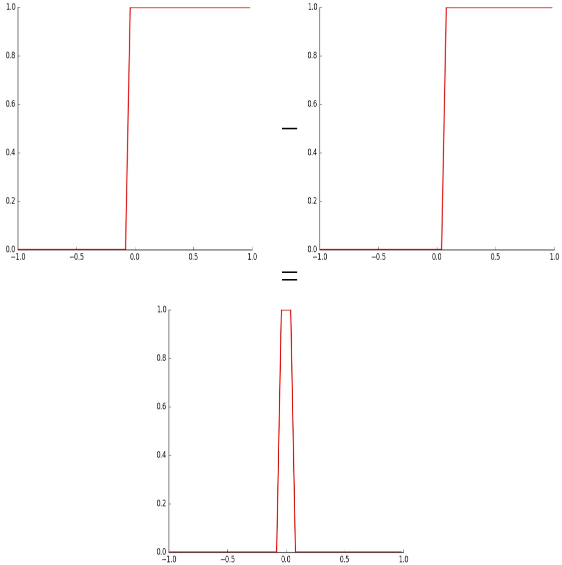




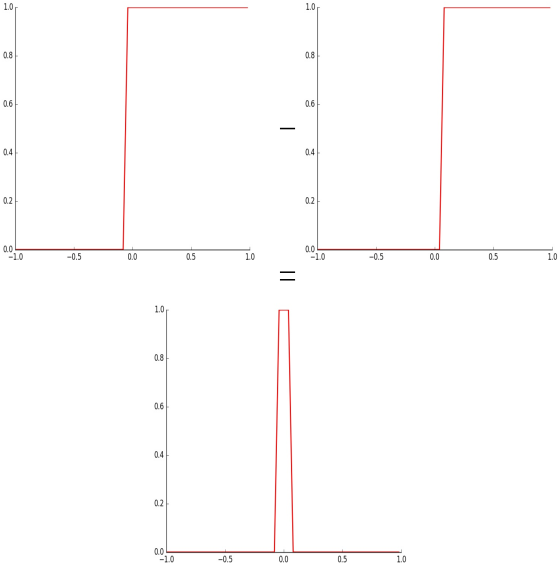
- Now lets see what we get by taking two such sigmoid functions (with different *b*s) and subtracting one from the other



- Now lets see what we get by taking two such sigmoid functions (with different *b*s) and subtracting one from the other

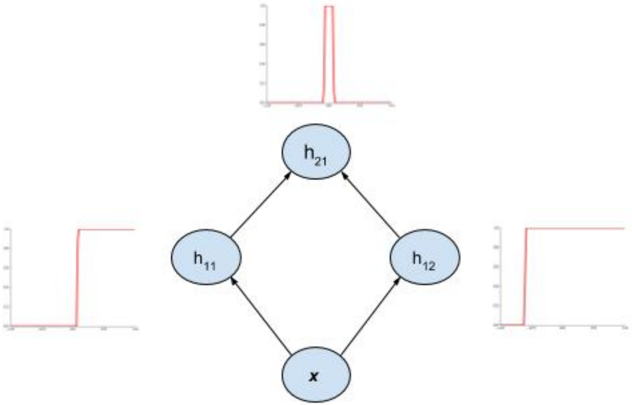


- Now lets see what we get by taking two such sigmoid functions (with different *b*s) and subtracting one from the other

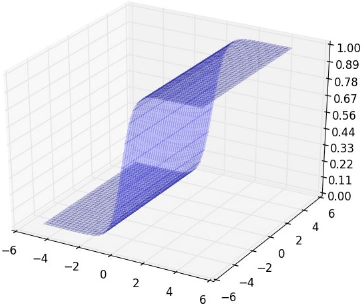


- Now lets see what we get by taking two such sigmoid functions (with different bs) and subtracting one from the other
- Voila! We have our tower function !!

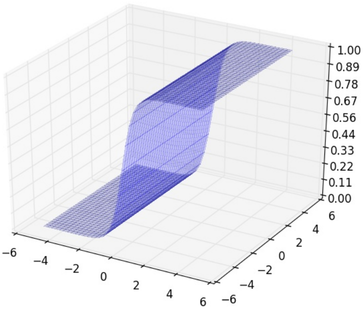
- Can we come up with a neural network to represent this operation of subtracting one sigmoid function from another ?


h21
h11
h12
x

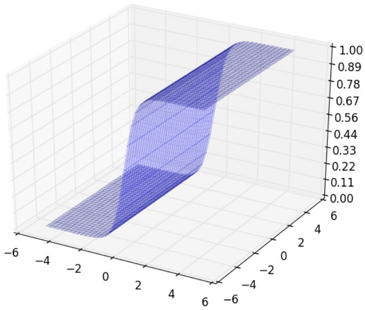
What if we have more than 1 input ?



- This is what a 3-dimensional sigmoid looks like

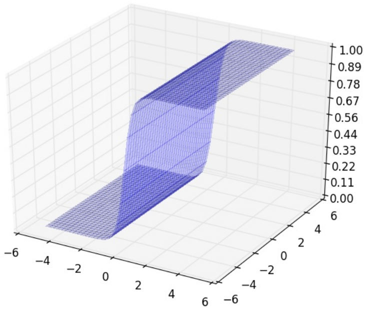


- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower



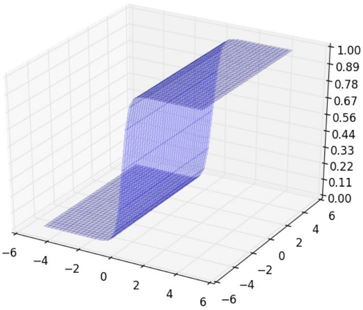
$$w_1 = 2, w_2 = 0, b = 0$$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



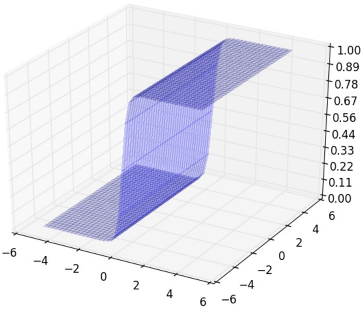
$w_1 = 3, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



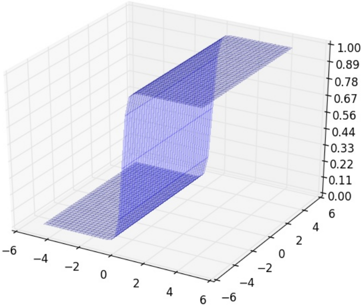
$w_1 = 4, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



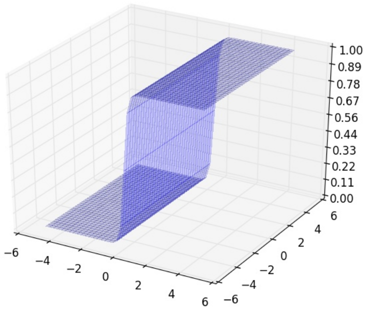
$w_1 = 5, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



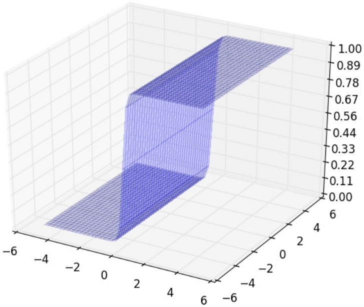
$w_1 = 6, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



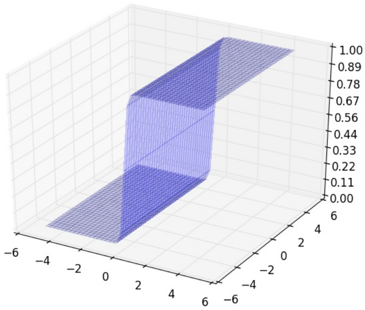
$w_1 = 7, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



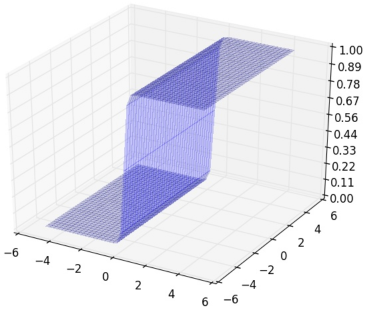
$w_1 = 8, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



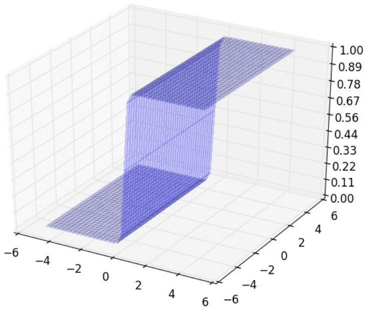
$w_1 = 9, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



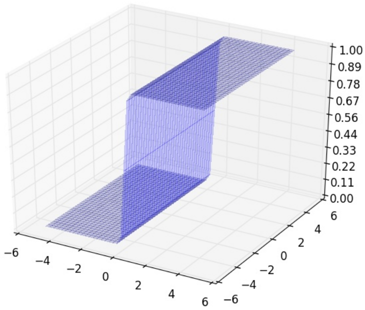
$w_1 = 10, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



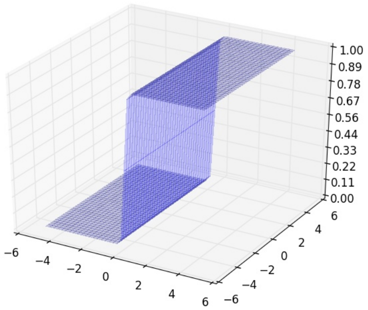
$w_1 = 11, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



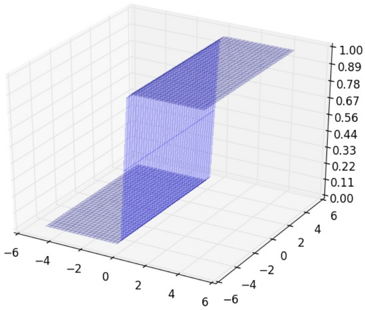
$w_1 = 12, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



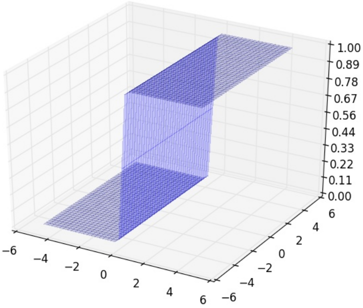
$w_1 = 13, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



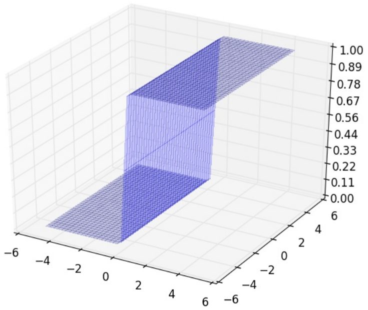
$w_1 = 14, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



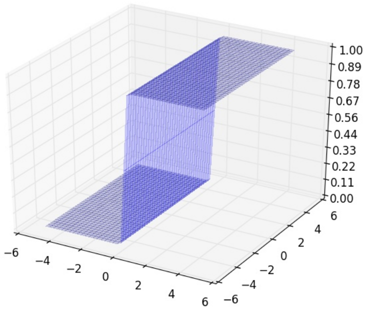
$w_1 = 15, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



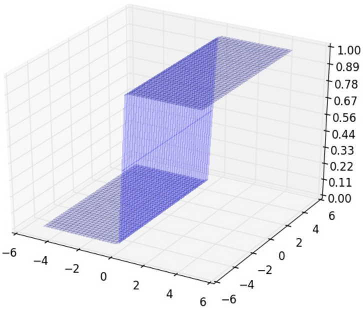
$w_1 = 16, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



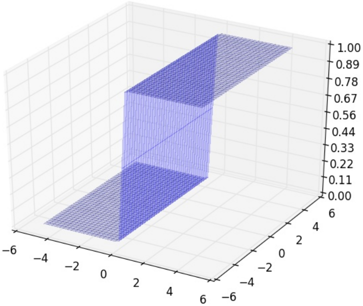
$w_1 = 17, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



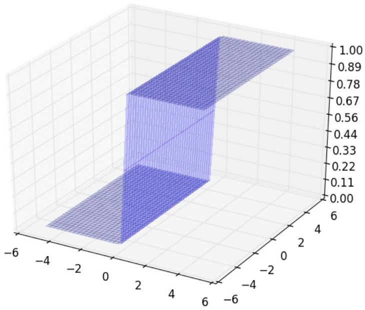
$w_1 = 18, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



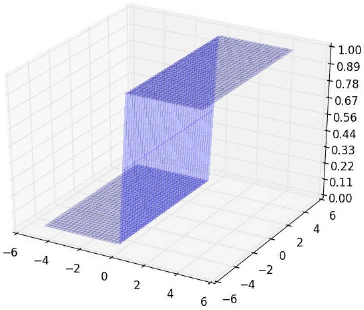
$w_1 = 19, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



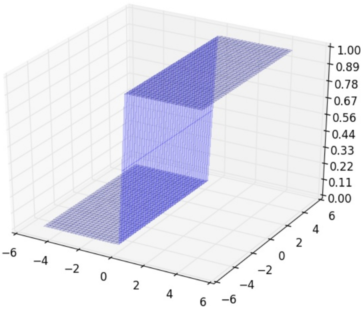
$w_1 = 20, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



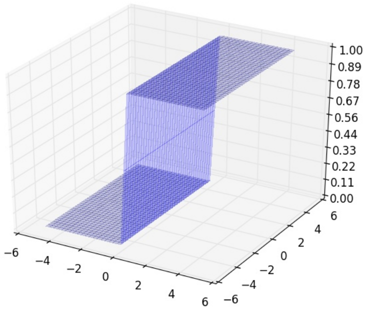
$w_1 = 21, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



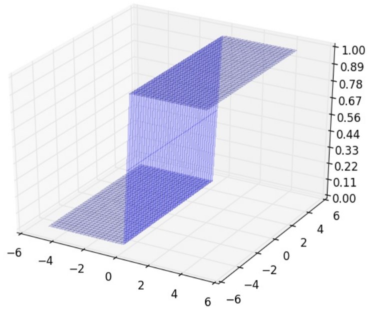
$w_1 = 22, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function



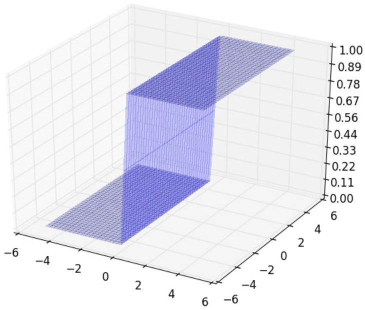
$w_1 = 23, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

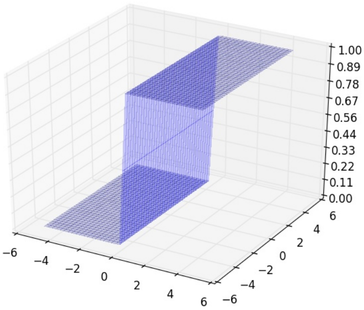


$$w_1 = 24, w_2 = 0, b = 0$$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

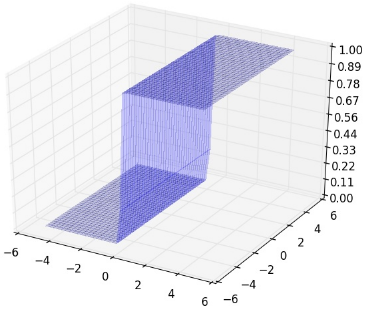


- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



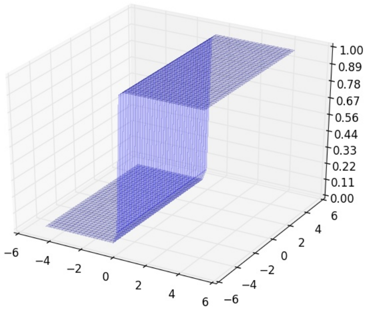
$w_1 = 25, w_2 = 0, b = 0$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



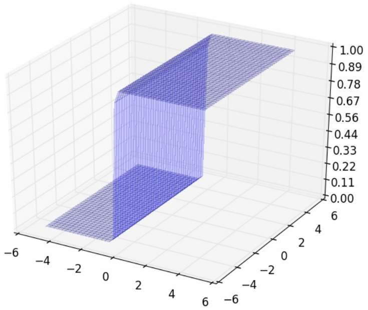
$w_1 = 25, w_2 = 0, b = 5$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



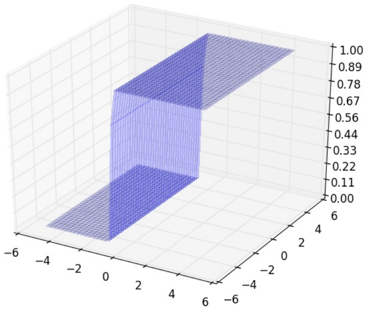
$w_1 = 25, w_2 = 0, b = 10$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



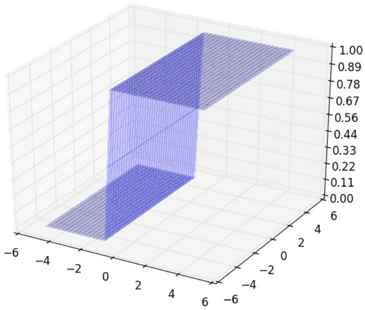
$$w_1 = 25, w_2 = 0, b = 15$$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



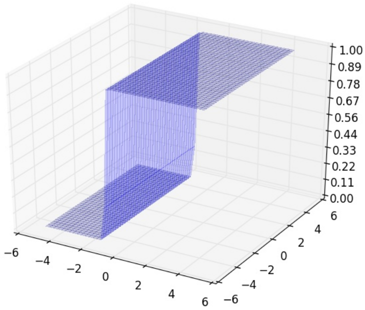
$w_1 = 25, w_2 = 0, b = 20$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



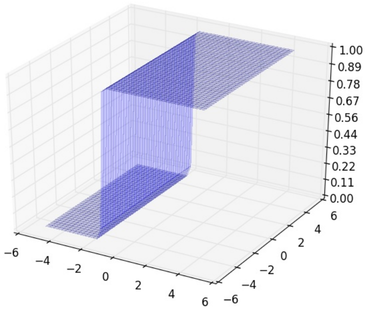
$w_1 = 25, w_2 = 0, b = 25$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



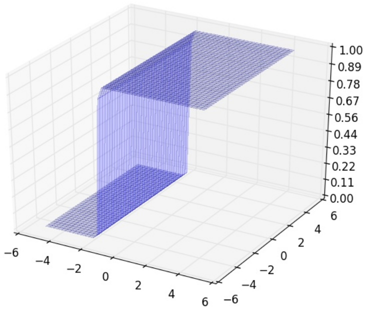
$w_1 = 25, w_2 = 0, b = 30$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



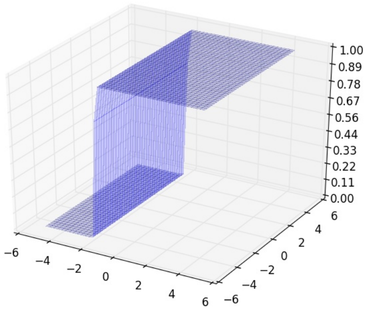
$w_1 = 25, w_2 = 0, b = 35$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



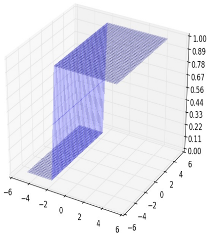
$w_1 = 25, w_2 = 0, b = 40$

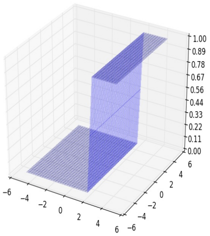
- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?



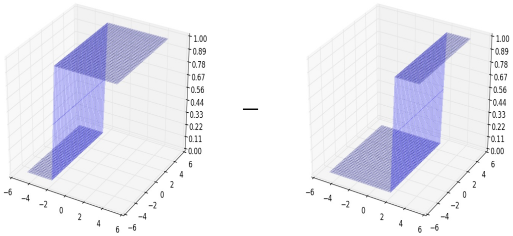
$w_1 = 25, w_2 = 0, b = 45$

- This is what a 3-dimensional sigmoid looks like
- We need to figure out how to get a 3-dimensional tower
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

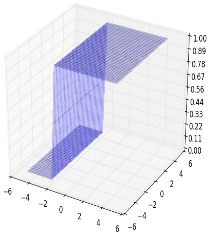




- What if we take two such step functions (with different b values) and subtract one from the other



- What if we take two such step functions (with different b values) and subtract one from the other



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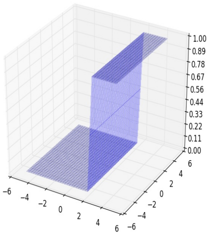

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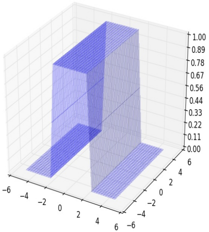
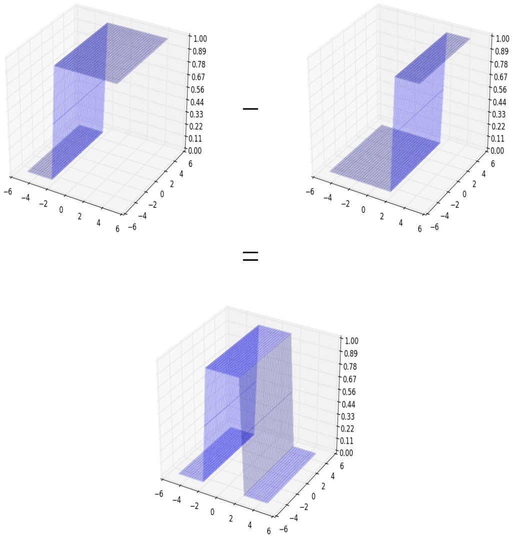
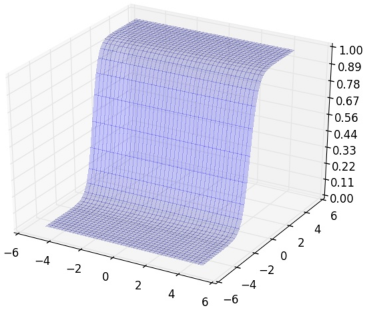


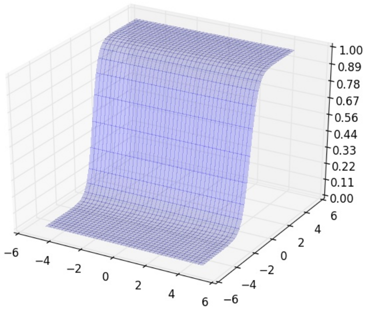
- What if we take two such step functions (with different b values) and subtract one from the other



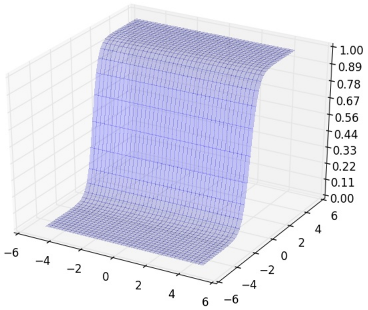
- What if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)



- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

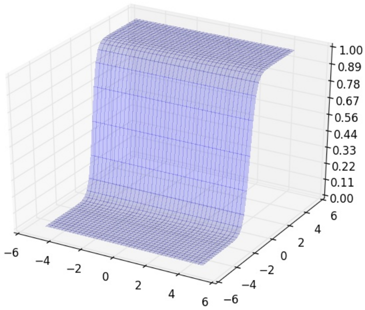


- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



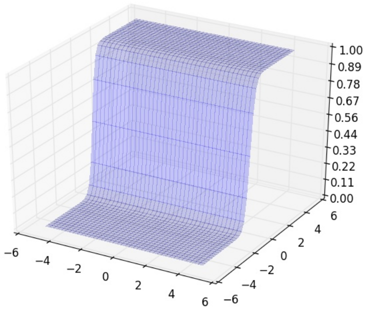
$w_1 = 0, w_2 = 2, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



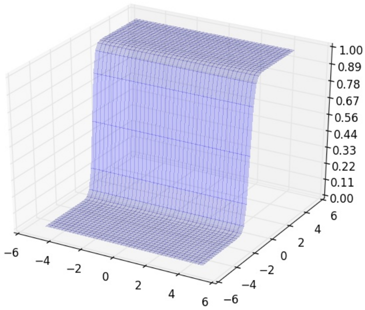
$w_1 = 0, w_2 = 3, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



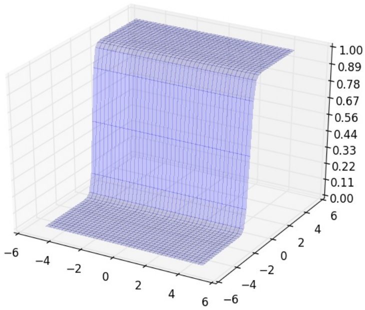
$w_1 = 0, w_2 = 4, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



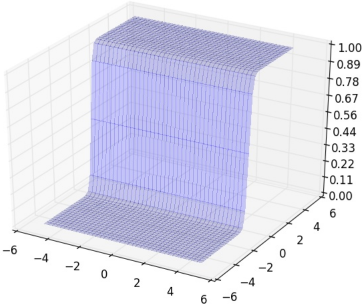
$w_1 = 0, w_2 = 5, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



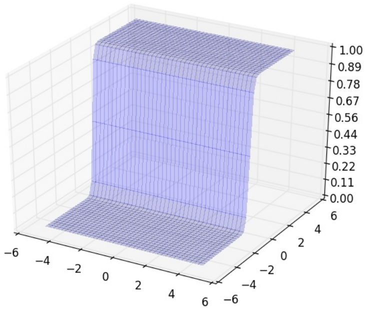
$w_1 = 0, w_2 = 6, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



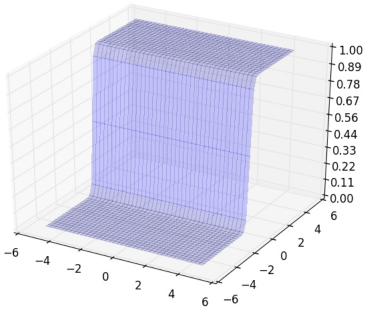
$w_1 = 0, w_2 = 7, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



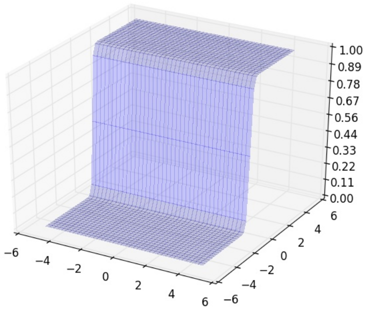
$w_1 = 0, w_2 = 8, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



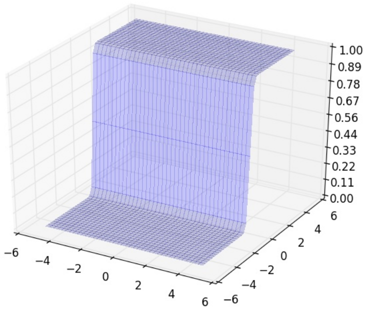
$w_1 = 0, w_2 = 9, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



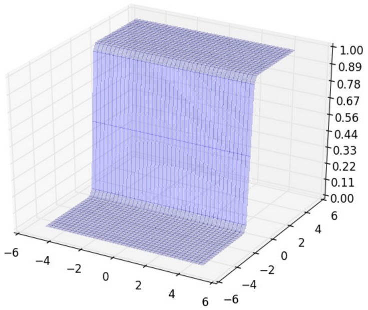
$$w_1 = 0, w_2 = 10, b = 0$$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



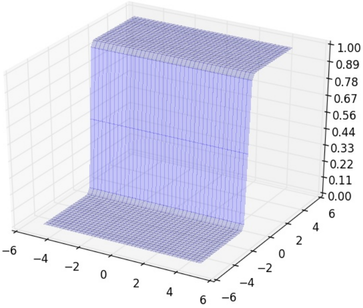
$w_1 = 0, w_2 = 11, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



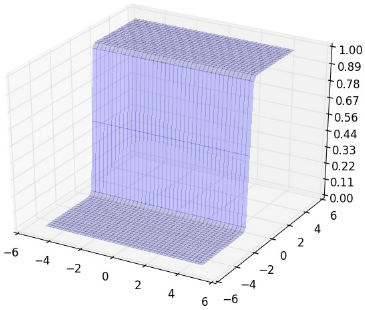
$$w_1 = 0, w_2 = 12, b = 0$$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



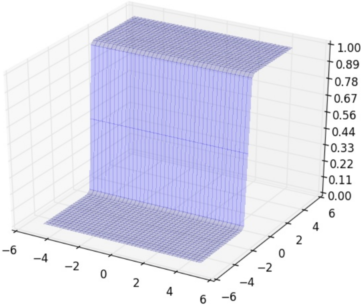
$w_1 = 0, w_2 = 13, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



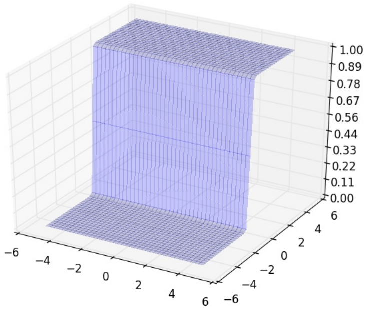
$w_1 = 0, w_2 = 14, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



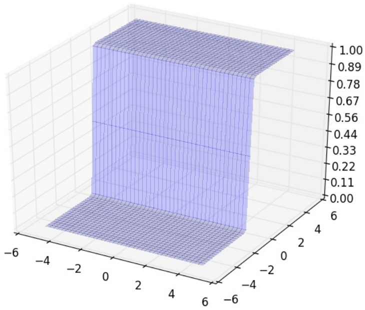
- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 15, b = 0$



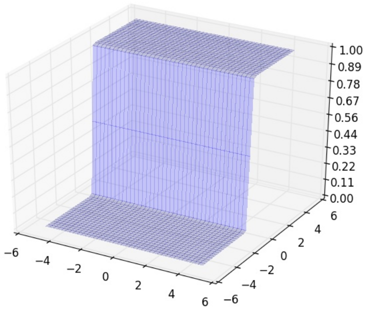
$$w_1 = 0, w_2 = 16, b = 0$$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



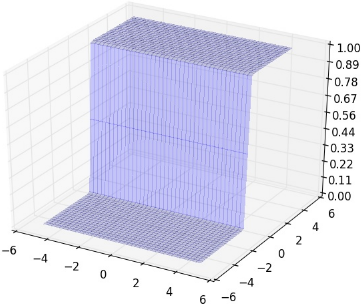
$w_1 = 0, w_2 = 17, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



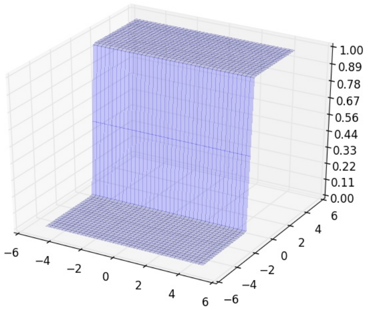
$w_1 = 0, w_2 = 18, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



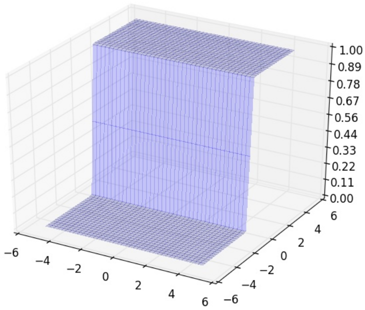
- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$$w_1 = 0, w_2 = 19, b = 0$$



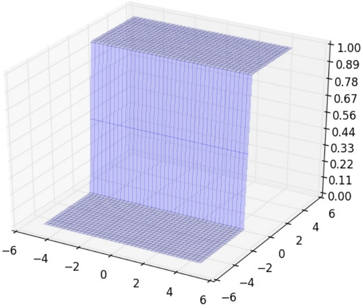
$w_1 = 0, w_2 = 20, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



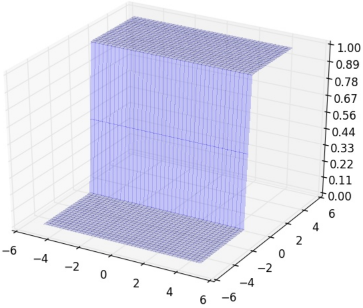
- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

$w_1 = 0, w_2 = 21, b = 0$



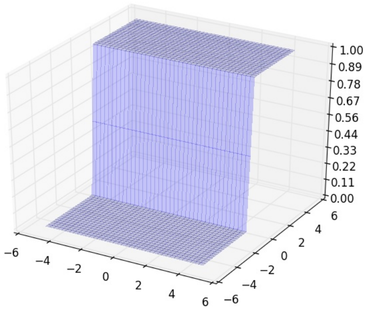
$w_1 = 0, w_2 = 22, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation



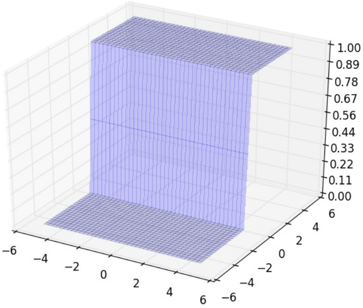
$w_1 = 0, w_2 = 23, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

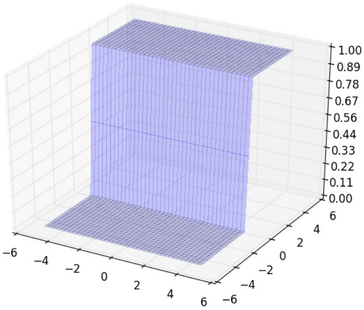


$w_1 = 0, w_2 = 24, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation

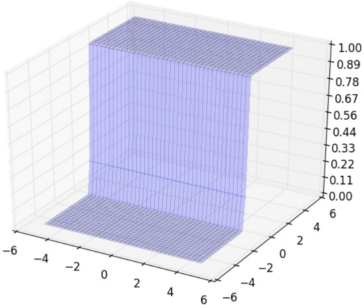


- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b



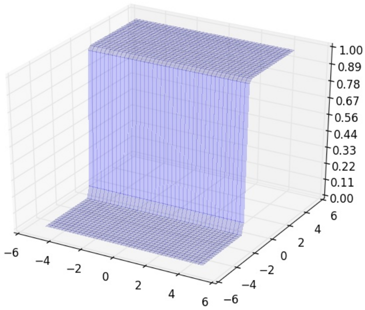
$w_1 = 0, w_2 = 25, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b



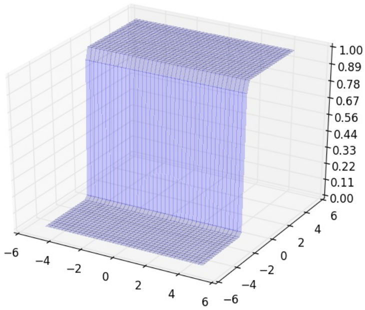
$w_1 = 0, w_2 = 25, b = 5$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b



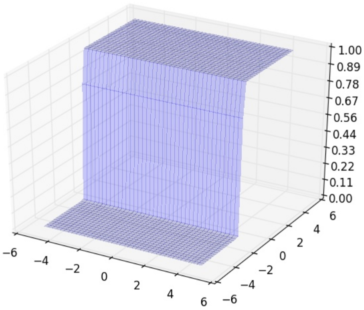
$w_1 = 0, w_2 = 25, b = 10$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b



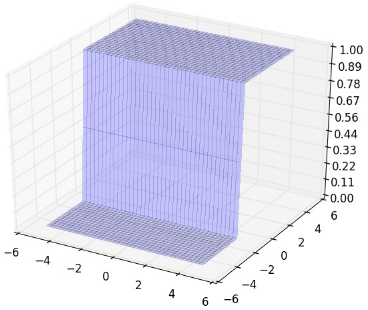
$w_1 = 0, w_2 = 25, b = 15$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b



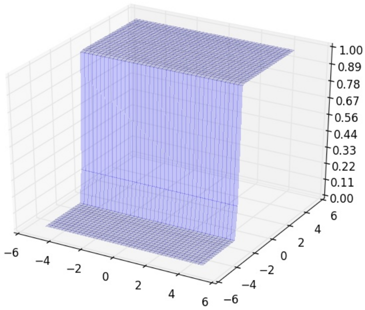
$w_1 = 0, w_2 = 25, b = 20$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b



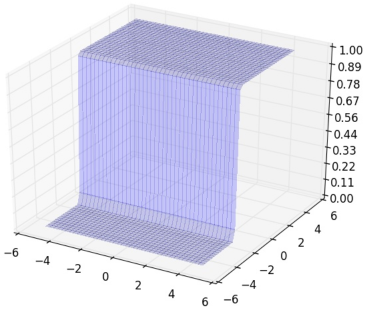
$w_1 = 0, w_2 = 25, b = 25$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b



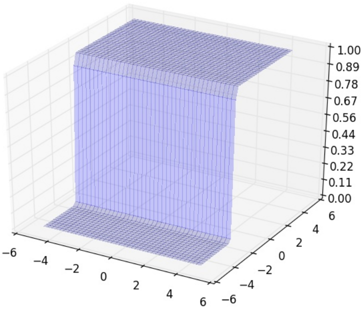
$w_1 = 0, w_2 = 25, b = 30$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b



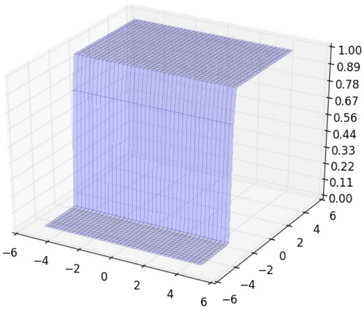
$w_1 = 0, w_2 = 25, b = 35$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b



$w_1 = 0, w_2 = 25, b = 40$

- Now let us set w_1 to 0 and adjust w_2 to get a 3-dimensional step function with a different orientation
- And now we change b



$w_1 = 0, w_2 = 25, b = 45$

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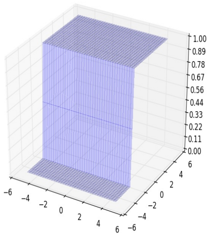
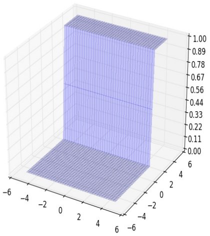
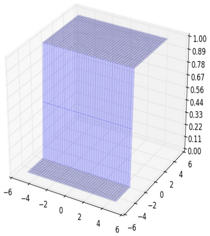




- Again, what if we take two such step functions (with different b values) and subtract one from the other



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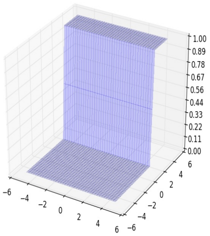

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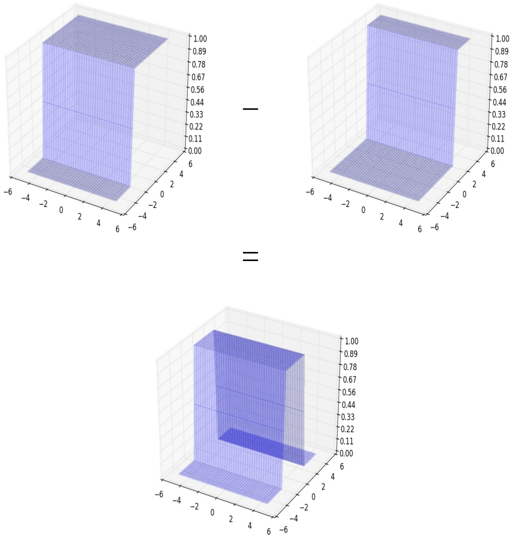

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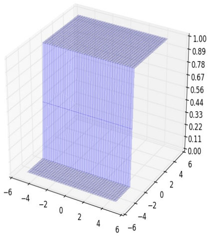

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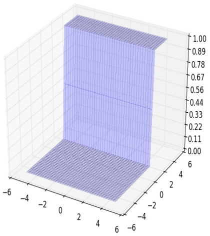

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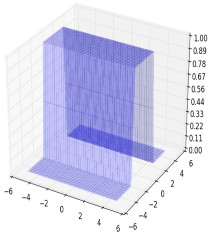
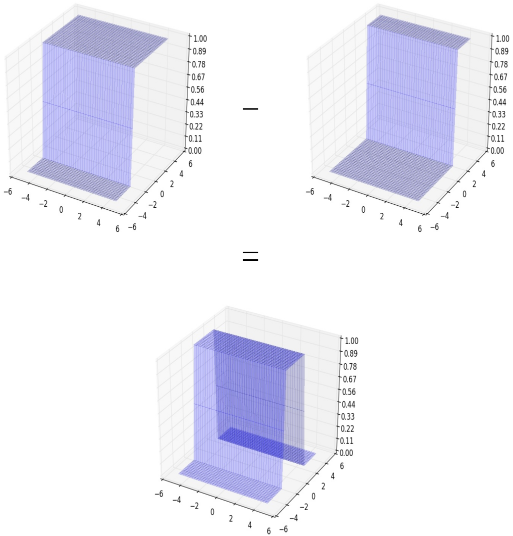
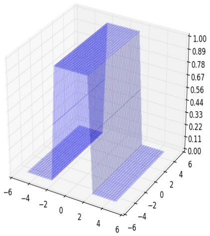
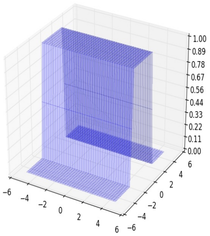


- Again, what if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)

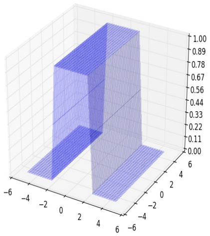


- Again, what if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)
- Notice that this open tower has a different orientation from the previous one





- Now what will we get by adding two such open towers ?



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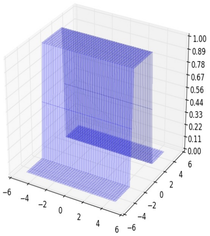

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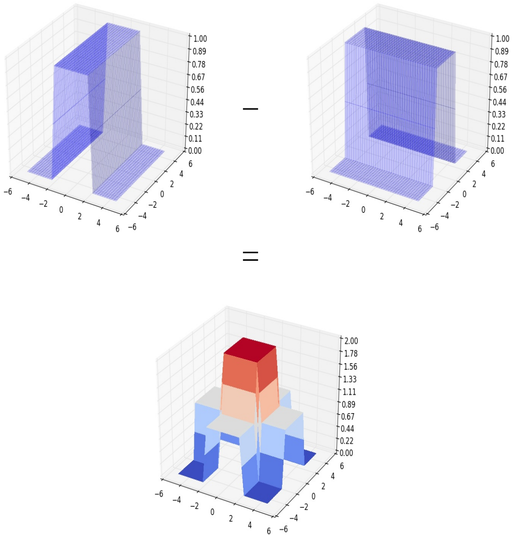

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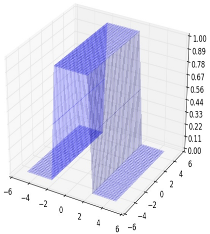

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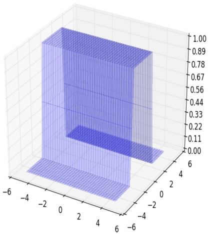

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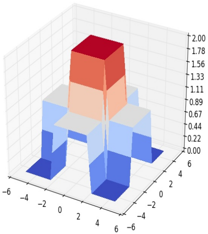
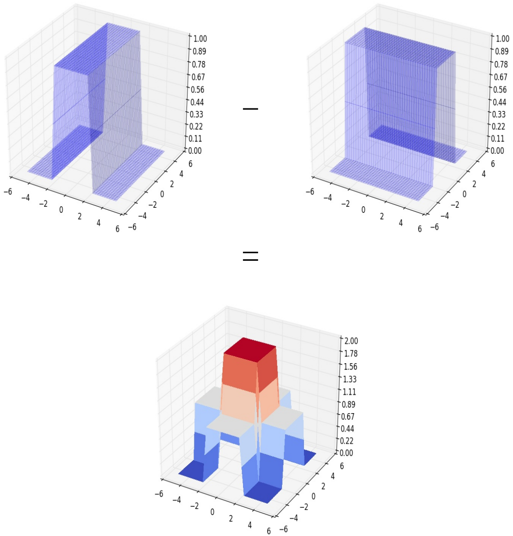
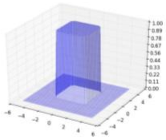
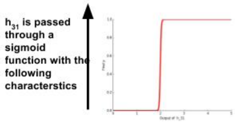


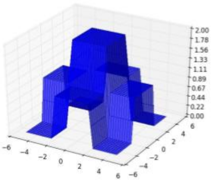
- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base



- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base
- We can now pass this output through another sigmoid neuron to get the desired tower !







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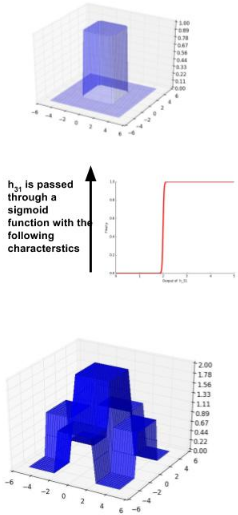
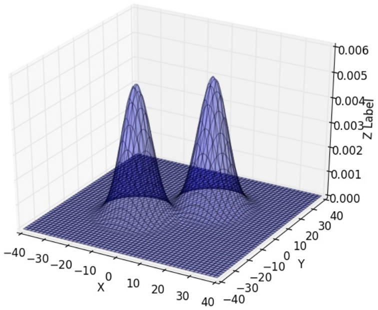
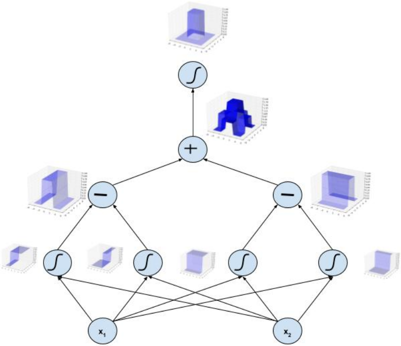


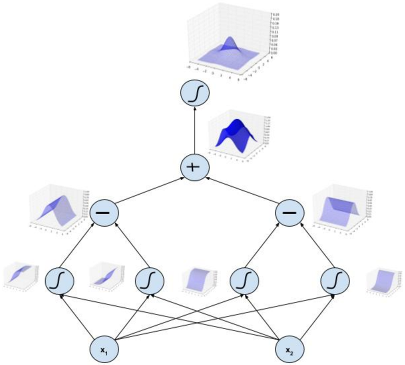
- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base
- We can now pass this output through another sigmoid neuron to get the desired tower !
- We can now approximate any function by summing up many such towers



- For example, we could approximate the following function using a sum of several towers

- Can we come up with a neural network to represent this entire procedure of constructing a 3 dimensional tower ?


+
−
−
x_1
x_2

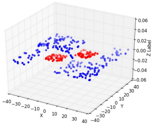

+
−
−
x_1
x_2

Think

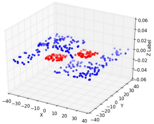
- For 1 dimensional input we needed 2 neurons to construct a tower
- For 2 dimensional input we needed 4 neurons to construct a tower
- How many neurons will you need to construct a tower in n dimensions ?

Time to retrospect

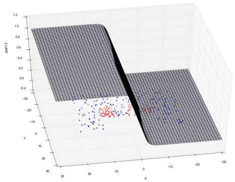
- Why do we care about approximating any arbitrary function ?
- Can we tie all this back to the classification problem that we have been dealing with ?



- We are interested in separating the blue points from the red points



- We are interested in separating the blue points from the red points
- Suppose we use a single sigmoidal neuron to approximate the relation between $x = [x_1, x_2]$ and y



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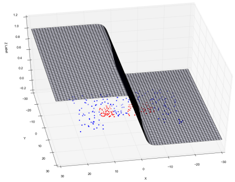

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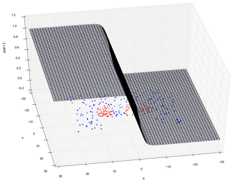
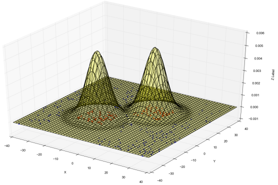
- Obviously, there will be errors (some blue points get classified as 1 and some red points get classified as 0)





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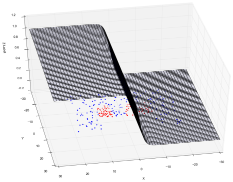
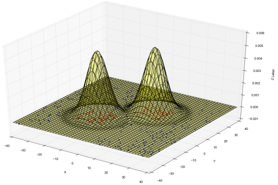
- This is what we actually want





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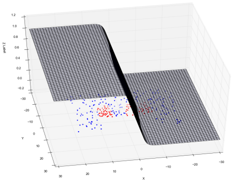
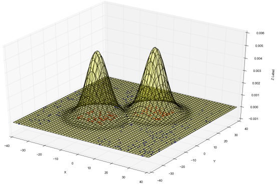
- The illustrative proof that we just saw tells us that we can have a neural network with two hidden layers which can approximate the above function by a sum of towers





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- This is what we actually want
- The illustrative proof that we just saw tells us that we can have a neural network with two hidden layers which can approximate the above function by a sum of towers
- Which means we can have a neural network which can exactly separate the blue points from the red points !!