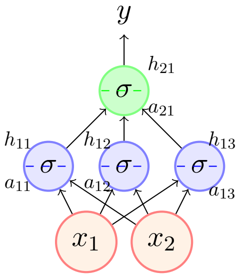
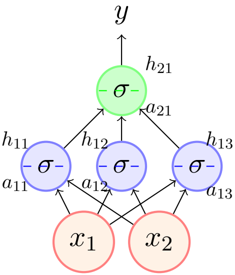
Module 9.4 : Better initialization strategies

Deep Learning has evolved

- Better optimization algorithms
- Better regularization methods
- Better activation functions
- Better weight initialization strategies



- What happens if we initialize all weights to 0?



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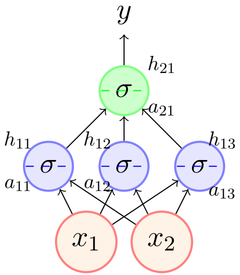

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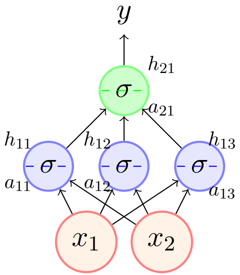

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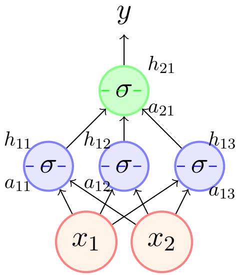

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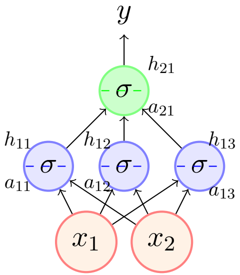
- What happens if we initialize all weights to 0?
- All neurons in layer 1 will get the same activation



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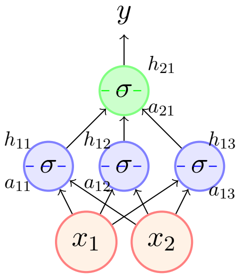

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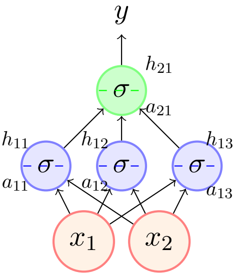

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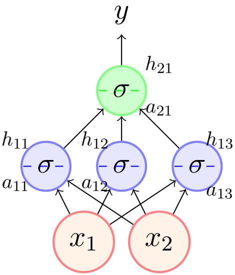

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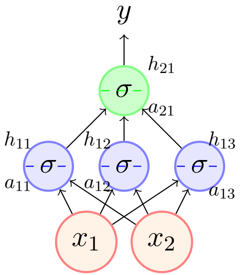

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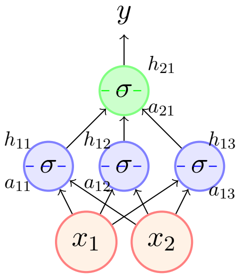

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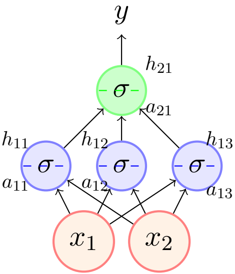

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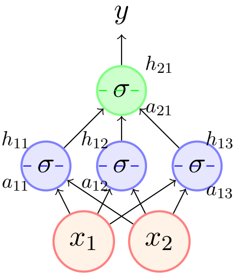

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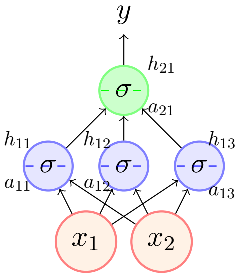
- Infact this symmetry will never break during training



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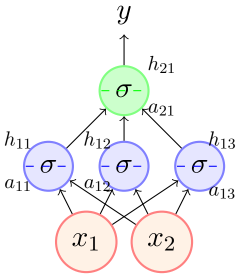

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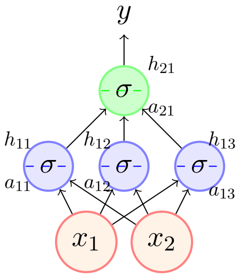
- And for all weights in layer 2 (infact, work out the math and convince yourself that all the weights in this layer will remain equal)



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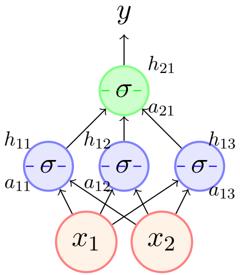

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- And for all weights in layer 2 (infact, work out the math and convince yourself that all the weights in this layer will remain equal)
- This is known as the **symmetry breaking problem**
- This will happen if all the weights in a network are initialized to the **same value**

```
D = np.random.randn(1000,500)
hidden_layer_sizes = [500]*10
nonlinearities = ['tanh']*len(hidden_layer_sizes)

act = {'relu':lambda x: np.maximum(0, x), 'tanh': lambda x: np.tanh(x),
       'sigmoid':lambda x: 1/ (1 + np.exp(-x))}
Hs = {}

for i in xrange(len(hidden_layer_sizes)):
    X = D if i == 0 else Hs[i-1]
    fan_in = X.shape[1]
    fan_out = hidden_layer_sizes[i]
    W = np.random.randn(fan_in, fan_out) *  0.01

    H = np.dot(X, W)
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We will now consider a feedforward network with:

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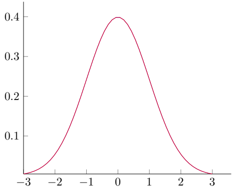
- input: 1000 points, each $\in R^{500}$
- input data is drawn from unit Gaussian



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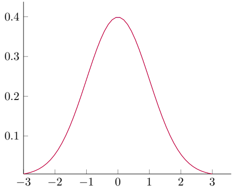

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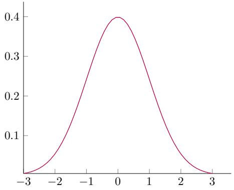

- the network has 5 layers
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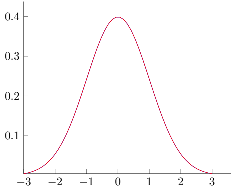

- the network has 5 layers
- each layer has 500 neurons
- we will run forward propagation on this network with different weight initializations

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- Let's try to initialize the weights to small random numbers

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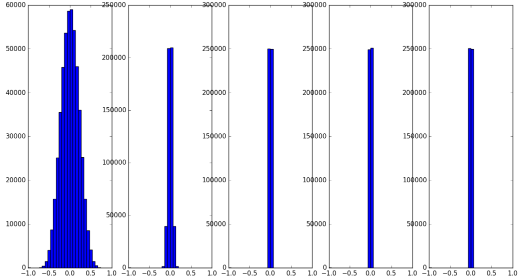
- We will see what happens to the activation across different layers



tanh activation functions

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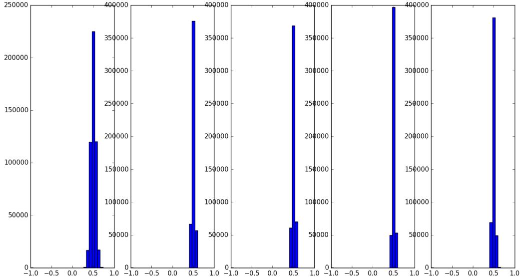

sigmoid activation functions

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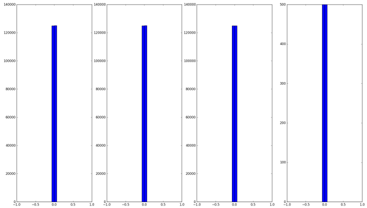

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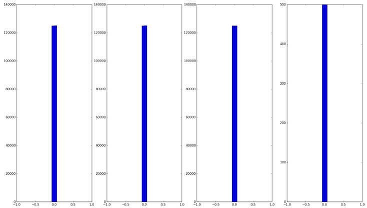

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- They will all be close to 0 (vanishing gradient problem)

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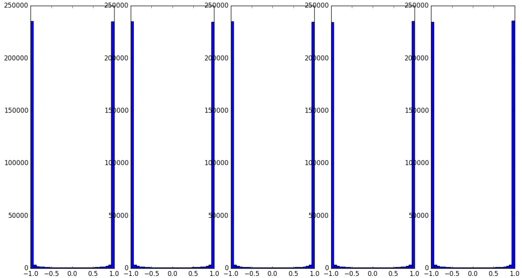

tanh activation with large weights

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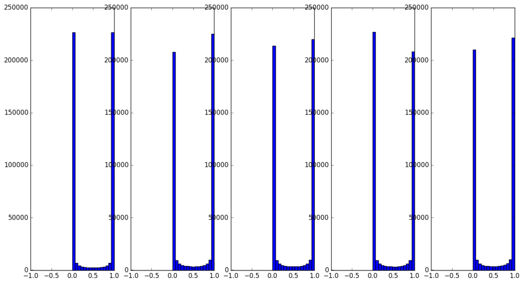

sigmoid activations with large weights

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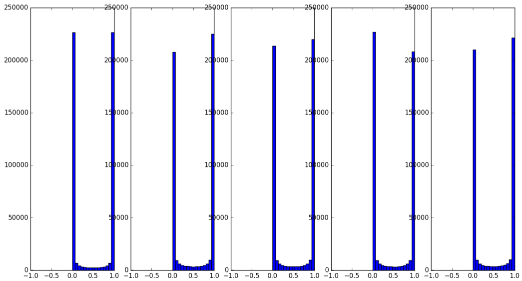
- Let us try to initialize the weights to large random numbers
- Most activations have saturated



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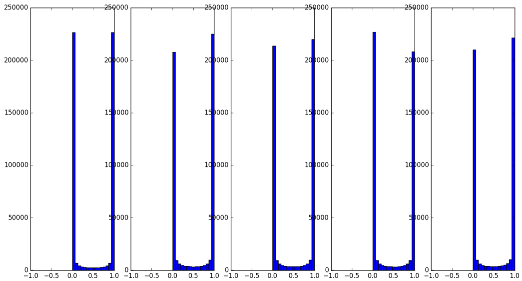

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- What happens to the gradients at saturation?

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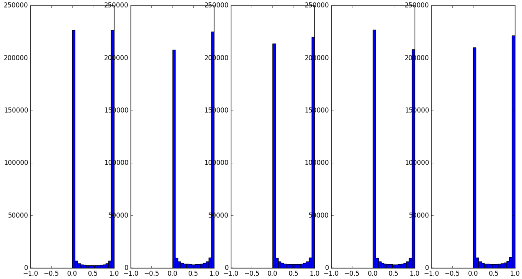

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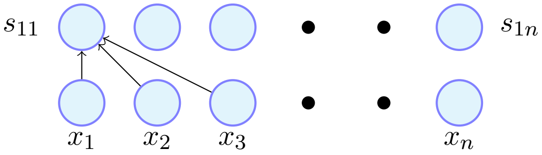
- Let us try to arrive at a more principled way of initializing weights



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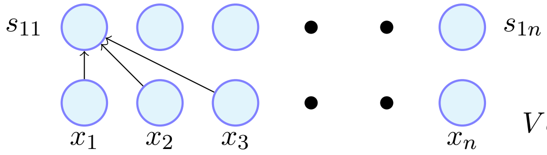

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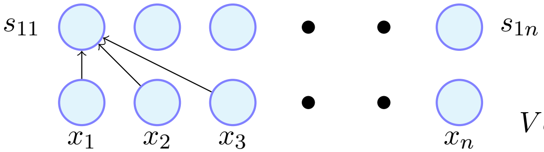

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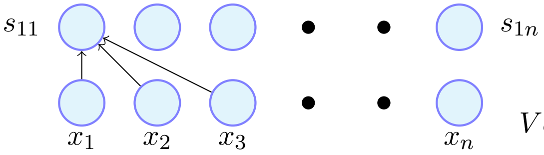

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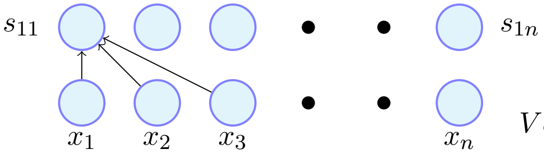

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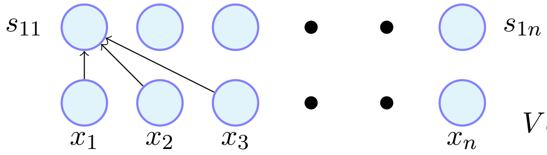

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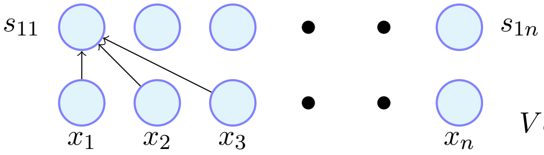

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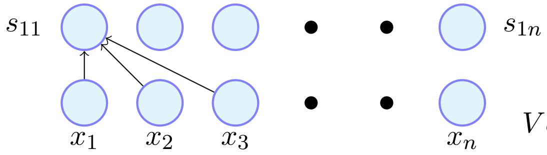

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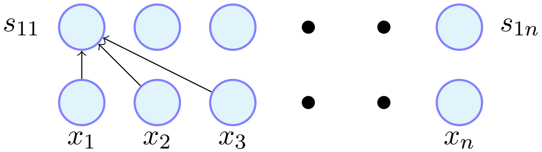

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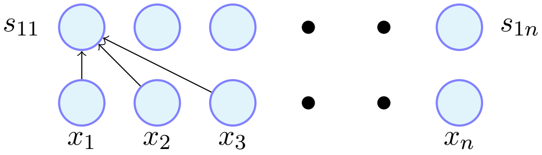

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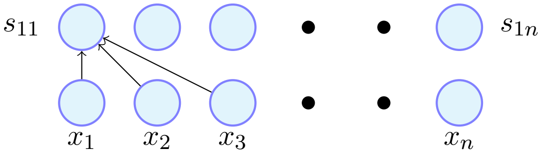

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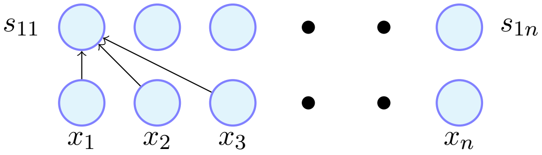

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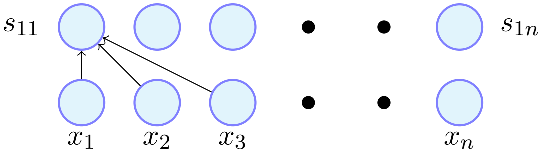

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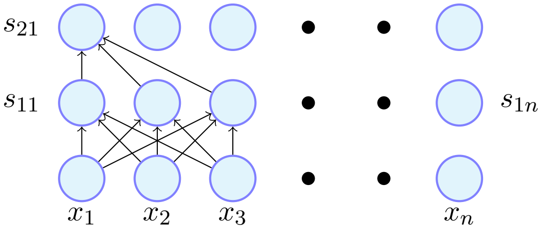

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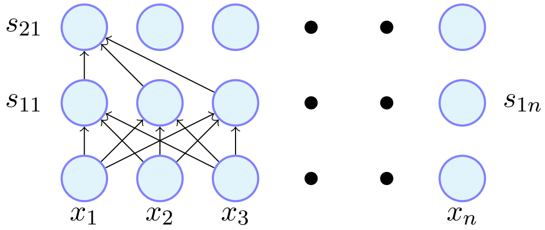

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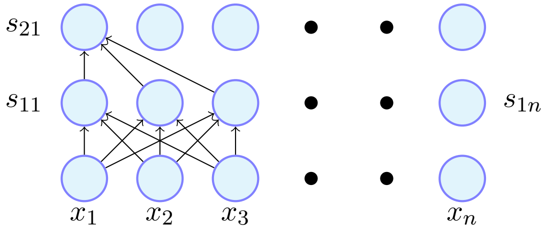

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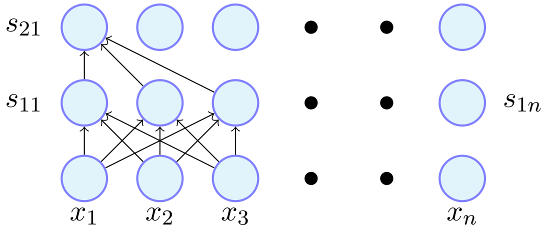

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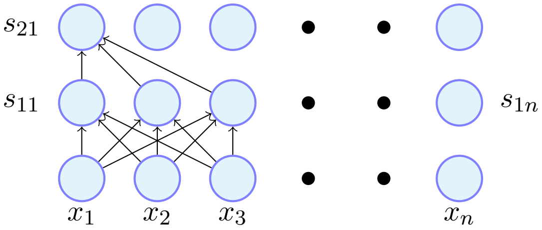

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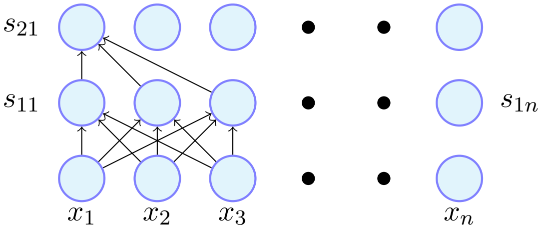

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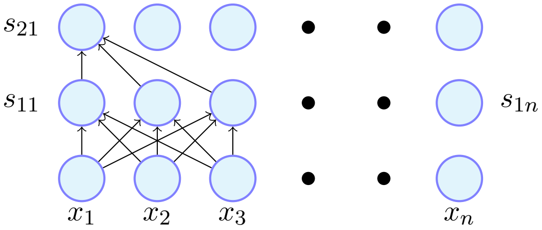

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Assuming weights across all layers have the same variance

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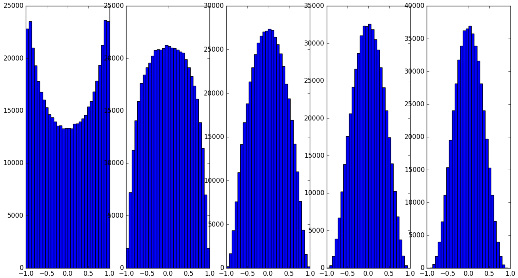

tanh activation

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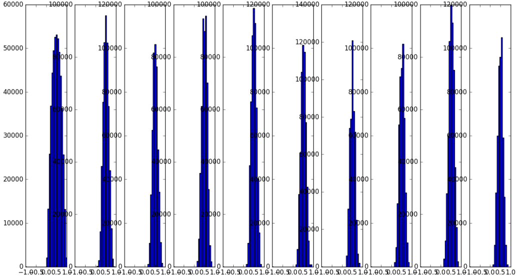

sigmoid activations

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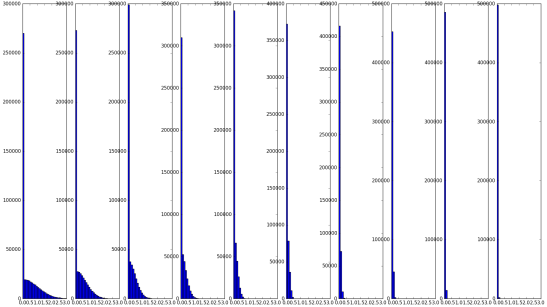
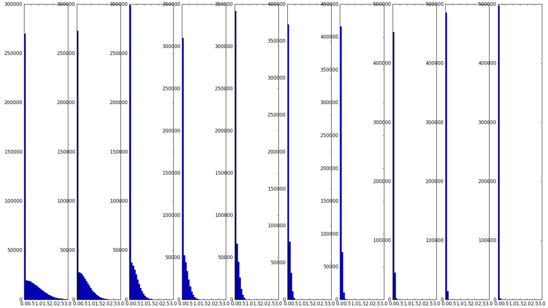


- However this does not work for ReLU neurons



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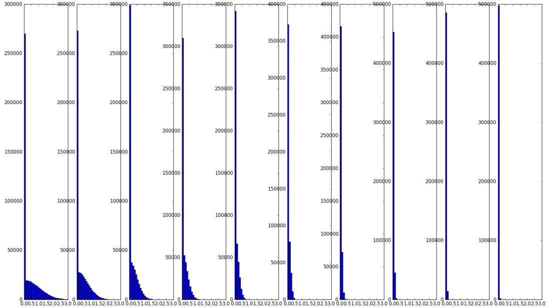

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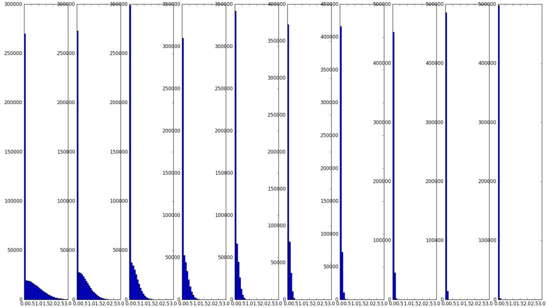

- However this does not work for ReLU neurons
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- Intuition: *He et.al.* argue that a factor of 2 is needed when dealing with ReLU Neurons
- Intuitively this happens because the range of ReLU neurons is restricted only to the positive half of the space

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- Indeed when we account for this factor of 2 we see better performance

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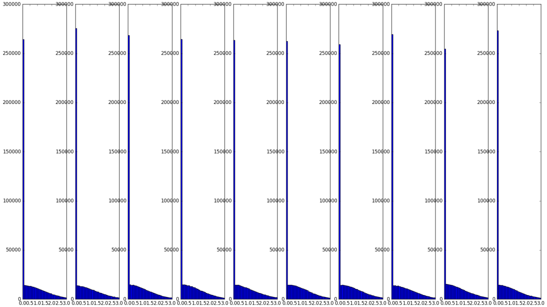

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