Lecture 3

Sigmoid Neurons, Gradient Descent, Feedforward Neural Networks, Representation Power of Feedforward Neural Networks

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- For Module 3.4, I have borrowed ideas from the videos by Ryan Harris on "visualize backpropagation" (available on youtube)
- For Module 3.5, I have borrowed ideas from this excellent book * which is available online
- I am sure I would have been influenced and borrowed ideas from other sources and I apologize if I have failed to acknowledge them

*http://neuralnetworksanddeeplearning.com/chap4.html

Module 3.1: Sigmoid Neuron

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- Can we have a network which can (approximately) represent such functions ?
- Before answering the above question we will have to first graduate from ***perceptrons*** to ***sigmoidal neurons*** ...

Recall

- A perceptron will fire if the weighted sum of its inputs is greater than the threshold (-w_0)

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- For example, let us return to our problem of deciding whether we will like or dislike a movie
- Consider that we base our decision only on one input ($x_1 = criticsRating$ which lies between 0 and 1)
- If the threshold is 0.5 ($w_0 = -0.5$) and $w_1 = 1$ then what would be the decision for a movie with $criticsRating = 0.51$?

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- What about a movie with $criticsRating = 0.49$? (dislike)
- It seems harsh that we would like a movie with rating 0.51 but not one with a rating of 0.49

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- It is a characteristic of the perceptron function itself which behaves like a step function
- There will always be this sudden change in the decision (from 0 to 1) when $\sum_{i=1}^{n} w_i x_i$ crosses the threshold (-w_0)
- For most real world applications we would expect a smoother decision function which gradually changes from 0 to 1

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- Also the output y is no longer binary but a real value between 0 and 1 which can be interpreted as a probability
- Instead of a like/dislike decision we get the probability of liking the movie

Perceptron

$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$

$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

Sigmoid (logistic) Neuron

$$y = \frac{1}{1 + e^{-(\sum_{i=0}^{n} w_i x_i)}}$$

Perceptron

$$z = \sum_{i=1}^{n} w_i x_i$$

Sigmoid Neuron

$$z = \sum_{i=1}^{n} w_i x_i$$

Not smooth, not continuous (at $w0$), **not differentiable**

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Sigmoid Neuron

$$z = \sum_{i=1}^{n} w_i x_i$$

Smooth, continuous, **differentiable**

Module 3.2: A typical Supervised Machine Learning Setup

- What next ?

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- Before we see such an algorithm we will revisit the concept of **error**

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- This line doesn't seem to be too bad
- Sure, it misclassifies 3 blue points and 3 red points but we could live with this error in **most** real world applications
- From now on, we will accept that it is hard to drive the error to 0 in most cases and will instead aim to reach the minimum possible error

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The learning algorithm should aim to find a $\mathbf{W}$ which minimizes the above function (squared error between y and $\hat{y}$)

Module 3.3: Learning Parameters: (Infeasible) guess work

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- Further to be consistent with the literature, from now on, we will refer to w_0 as b (bias)
- Lastly, instead of considering the problem of predicting like/dislike, we will assume that we want to predict $criticsRating(y)$ given $imdbRating(x)$ (for no particular reason)

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Find w and b such that:

$$\underset{w,b}{\text{minimize}}\, \mathscr{L}(w, b) = \sum_{i=1}^{N} (y_i - f(x_i))^2$$

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What does it mean to train the network?

- Suppose we train the network with $(x, y) = (0.5, 0.2)$ and $(2.5, 0.9)$

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- $f(0.5) \rightarrow 0.2$ and $f(2.5) \rightarrow 0.9$

$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

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In other words...

- We hope to find a sigmoid function such that $(0.5, 0.2)$ and $(2.5, 0.9)$ lie on this sigmoid

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Let us see this in more detail....

(2.5, 0.9)
(0.5, 0.2)
1.0
0.8
0.6
0.4
0.2
0.0
y
−6
−4
−2
0
2
4
6
x

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- Let us try a random guess.. (say, $w = 0.5, b = 0$)
- Clearly not good, but how bad is it ?
- Let us revisit $\mathscr{L}(w, b)$ to see how bad it is ...

$$\mathscr{L}(w,b) = \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2$$

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$$= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2$$

$$
\begin{aligned}
\mathscr{L}(w,b) &= \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2 \\
&= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 \\
&= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2
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&= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 \\
&= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2 \\
&= 0.073
\end{aligned}
$$

We want $\mathscr{L}(w,b)$ to be as close to 0 as possible

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481

Oops!! this made things even worse...

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214

Perhaps it would help to push w and b in the other direction...

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028

Let us keep going in this direction, *i.e.*, increase w and decrease b

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w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
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1.42	-1.73	0.0028
1.65	-2.08	0.0003

Let us keep going in this direction, *i.e.*, increase w and decrease b

Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028
1.65	-2.08	0.0003
1.78	-2.27	0.0000

With some guess work and intuition we were able to find the right values for w and b

Let us look at something better than our "guess work" algorithm....

- Since we have only 2 points and 2 parameters $(w,\ b)$ we can easily plot $\mathscr{L}(w, b)$ for different values of $(w,\ b)$ and pick the one where $\mathscr{L}(w, b)$ is minimum

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- Since we have only 2 points and 2 parameters $(w,\ b)$ we can easily plot $\mathscr{L}(w, b)$ for different values of $(w,\ b)$ and pick the one where $\mathscr{L}(w, b)$ is minimum
- But of course this becomes intractable once you have many more data points and many more parameters !!
- Further, even here we have plotted the error surface only for a small range of $(w,\ b)$ [from $(-6, 6)$ and not from $(-\inf, \inf)$]

Let us look at the geometric interpretation of our "guess work" algorithm in terms of this error surface

(2.5, 0.9)
(0.5, 0.2)
y
x
1.0
0.8
0.6
0.4
0.2
0.0
−6
−4
−2
0
2
4
6

Random search on error surface
error
w
b
1.0
0.5
0.0
−0.5
−1.0
0.08
0.16
0.24
0.32
0.40
0.48
0.56
0.64

(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
y
x

Random search on error surface
error
w
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0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64

(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
w=-0.10, b=0.00
y
x

Random search on error surface
error
w
b

(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
w=-0.10, b=0.00
w=0.94, b=-0.95
y
x

Random search on error surface
error
w
b
0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64

(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
w=-0.10, b=0.00
w=0.94, b=-0.95
w=1.43, b=-1.74
y
x

Random search on error surface
error
w
b
0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64

(2.5, 0.9)
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w=0.50, b=0.00
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w=1.65, b=-2.08
y
x

Random search on error surface
error
w
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0.08
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(2.5, 0.9)
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w=0.50, b=0.00
w=-0.10, b=0.00
w=0.94, b=-0.95
w=1.43, b=-1.74
w=1.65, b=-2.08
w=1.79, b=-2.28
y
x

Random search on error surface
error
w
b
0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64

Module 3.4: Learning Parameters : Gradient Descent

Now let us see if there is a more efficient and principled way of doing this

Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!

vector of parameters, say, randomly initialized

$$\theta = [w, b]$$

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$$\Delta\theta = [\Delta w, \Delta b]$$

change in the values of w, b

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We moved in the direction of $\Delta\theta$

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Let us be a bit conservative: move only by a small amount η

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vector of parameters, say, randomly initialized

$$\theta = [w, b]$$

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change in the values of w, b

$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$

We moved in the direction of $\Delta\theta$

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vector of parameters, say, randomly initialized

$\theta = [w, b]$

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change in the values of w, b

We moved in the direction of $\Delta\theta$

Let us be a bit conservative: move only by a small amount η

$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$

Question: What is the right $\Delta\theta$ to use ?

vector of parameters, say, randomly initialized

$$\theta = [w, b]$$

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change in the values of w, b

We moved in the direction of $\Delta\theta$

Let us be a bit conservative: move only by a small amount η

$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$

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The answer comes from Taylor series

For ease of notation, let $\Delta\theta = u$, then from Taylor series, we have,

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$$\mathscr{L}(\theta + \eta u) = \mathscr{L}(\theta) + \eta * u^T \nabla \mathscr{L}(\theta) + \frac{\eta^2}{2!} * u^T \nabla^2 \mathscr{L}(\theta) u + \frac{\eta^3}{3!} * ... + \frac{\eta^4}{4!} * ...$$

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Note that the move (ηu) would be favorable only if,

$$\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) < 0 \ [\textit{i.e., if the new loss is less than the previous loss}]$$

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Note that the move (ηu) would be favorable only if,

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This implies,

$$u^T \nabla \mathscr{L}(\theta) < 0$$

Okay, so we have,

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But, what is the range of $u^T \nabla \mathscr{L}(\theta)$?

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Let β be the angle between u^T and $\nabla \mathscr{L}(\theta)$, then we know that,

$$-1 \leq cos(\beta) = \frac{u^T \nabla \mathscr{L}(\theta)}{||u|| * ||\nabla \mathscr{L}(\theta)||} \leq 1$$

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multiply throughout by $k = ||u|| * ||\nabla \mathscr{L}(\theta)||$

$$-k \leq k * cos(\beta) = u^T \nabla \mathscr{L}(\theta) \leq k$$

Okay, so we have,

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Let β be the angle between u^T and $\nabla \mathscr{L}(\theta)$, then we know that,

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multiply throughout by $k = ||u|| * ||\nabla \mathscr{L}(\theta)||$

$$-k \leq k * cos(\beta) = u^T \nabla \mathscr{L}(\theta) \leq k$$

Thus, $\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) = u^T \nabla \mathscr{L}(\theta) = k * cos(\beta)$ will be most negative when $cos(\beta) = -1$ *i.e.*, when β is $180°$

Gradient Descent Rule

- The direction u that we intend to move in should be at 180° w.r.t. the gradient

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Parameter Update Equations

$$w_{t+1} = w_t - \eta \nabla w_t$$
$$b_{t+1} = b_t - \eta \nabla b_t$$
$$where, \nabla w_t = \frac{\partial \mathscr{L}(w,b)}{\partial w}_{at\ w = w_t, b = b_t}, \nabla b = \frac{\partial \mathscr{L}(w,b)}{\partial b}_{at\ w = w_t, b = b_t}$$

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So we now have a more principled way of moving in the w-b plane than our "guess work" algorithm

- Let us create an algorithm from this rule ...

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Algorithm: gradient_descent()

$t \leftarrow 0;$
$max_iterations \leftarrow 1000;$
while $t < max_iterations$ **do**
  $w_{t+1} \leftarrow w_t - \eta \nabla w_t;$
  $b_{t+1} \leftarrow b_t - \eta \nabla b_t;$
end

- Let us create an algorithm from this rule ...

Algorithm: gradient_descent()

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- To see this algorithm in practice lest first derive ∇w and ∇b for our toy neural network

$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

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Let's assume there is only 1 point to fit (x, y)

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$$\mathscr{L}(w, b) = \frac{1}{2} * (f(x) - y)^2$$

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Let's assume there is only 1 point to fit (x, y)

$$\mathscr{L}(w, b) = \frac{1}{2} * (f(x) - y)^2$$

$$\nabla w = \frac{\partial \mathscr{L}(w, b)}{\partial w} = \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2]$$

$$\nabla w = \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2]$$

$$
\begin{aligned}
\nabla w &= \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2] \\
&= \frac{1}{2} * [2 * (f(x) - y) * \frac{\partial}{\partial w}(f(x) - y)]
\end{aligned}
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&= (f(x) - y) * \frac{\partial}{\partial w}(f(x)) \\
&= (f(x) - y) * \frac{\partial}{\partial w}\left(\frac{1}{1 + e^{-(wx+b)}}\right)
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$$
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&\frac{\partial}{\partial w}\left(\frac{1}{1 + e^{-(wx+b)}}\right) \\
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&= \frac{-1}{(1 + e^{-(wx+b)})^2} * (e^{-(wx+b)})\frac{\partial}{\partial w}(-(wx + b)))
\end{aligned}
$$

$$
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\nabla w &= \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2] \\
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&= \frac{-1}{(1 + e^{-(wx+b)})^2} * (e^{-(wx+b)}) \frac{\partial}{\partial w}(-(wx + b))) \\
&= \frac{-1}{(1 + e^{-(wx+b)})} * \frac{e^{-(wx+b)}}{(1 + e^{-(wx+b)})} * (-x) \\
&= \frac{1}{(1 + e^{-(wx+b)})} * \frac{e^{-(wx+b)}}{(1 + e^{-(wx+b)})} * (x)
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&= \frac{-1}{(1 + e^{-(wx+b)})} * \frac{e^{-(wx+b)}}{(1 + e^{-(wx+b)})} * (-x) \\
&= \frac{1}{(1 + e^{-(wx+b)})} * \frac{e^{-(wx+b)}}{(1 + e^{-(wx+b)})} * (x) \\
&= f(x) * (1 - f(x)) * x
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\end{aligned}
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$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

So if there is only 1 point (x, y), we have,

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So if there is only 1 point (x, y), we have,

$$\nabla w = (f(x) - y) * f(x) * (1 - f(x)) * x$$

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So if there is only 1 point (x, y), we have,

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For two points,

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So if there is only 1 point (x, y), we have,

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So if there is only 1 point (x, y), we have,

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$$\nabla b = \sum_{i=1}^{2} (f(x_i) - y_i) * f(x_i) * (1 - f(x_i))$$

```
X = [0.5, 2.5]
Y = [0.2, 0.9]
```

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X = [0.5, 2.5]
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def f(w,b,x) : #sigmoid with parameters w,b
    return 1.0 / (1.0 + np.exp(-(w*x + b)))
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def error (w, b) :
    err = 0.0
    for x,y in zip(X,Y) :
        fx = f(w,b,x)
        err += 0.5 * (fx - y) ** 2
    return err
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def grad_b(w,b,x,y) :
    fx = f(w,b,x)
    return (fx - y) * fx * (1 - fx)
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    err = 0.0
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    return err

def grad_b(w,b,x,y) :
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X = [0.5, 2.5]
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def f(w,b,x) : #sigmoid with parameters w,b
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- For the time being it suffices to know that we have an algorithm for learning the parameters of a sigmoid neuron
- So where do we head from here ?

Module 3.5: Representation Power of a Multilayer Network of Sigmoid Neurons

Representation power of a multilayer network of perceptrons

Representation power of a multilayer network of sigmoid neurons

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A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

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Representation power of a multilayer network of sigmoid neurons A multilayer network of neurons with a single hidden layer can be used to approximate any continuous function to any desired precision

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In other words, there is a guarantee that for any function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$, we can always find a neural network (with 1 hidden layer containing enough neurons) whose output g(x) satisfies $|g(x) - f(x)| < \epsilon$!!

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Proof: We will see an illustrative proof of this... [Cybenko, 1989], [Hornik, 1991]

- See this link* for an excellent illustration of this proof
- The discussion in the next few slides is based on the ideas presented at the above link

*http://neuralnetworksanddeeplearning.com/chap4.html

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- We observe that such an arbitrary function can be approximated by several "tower" functions
- More the number of such "tower" functions, better the approximation
- To be more precise, we can approximate any arbitrary function by a sum of such "tower" functions

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- We can then have a simple network which can just add them up to approximate the function
- Our job now is to figure out what is inside this blackbox

We will figure this out over the next few slides ...

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$$w = 50, b = 1, \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 2,\ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 3, \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 4,\ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 5,\ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 6,\ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 7, \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

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$$w = 50, b = 8, \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

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$$w = 50, b = 9, \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
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- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 10, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 11, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 12, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 13, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 14, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
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$$w = 50, b = 15, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

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$$w = 50, b = 16, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

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$$w = 50, b = 18, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

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$$w = 50, b = 19, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

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$$w = 50, b = 26, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
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- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 27, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
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- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 28, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 29, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
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- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 30, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

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$$w = 50, b = 32,\ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
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- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$w = 50, b = 34, \ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
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$$w = 50, b = 35,\ \sigma(x) = \frac{1}{1-\exp^{(wx+b)}}$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

- Now let us see what we get by taking two such sigmoid functions (with different b's) and subtracting one from the other

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- Now let us see what we get by taking two such sigmoid functions (with different b's) and subtracting one from the other
- Voila! We have our tower function !!

- Can we come up with a neural network to represent this operation of subtracting one sigmoid function from another ?

h21
+1
-1
h11
h12
w1,b1
w2,b2
x1

- What if we have more than one input?

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- Suppose we are trying to take a decision about whether we will find oil at a particular location on the ocean bed(Yes/No)
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- We are given some data and it seems that $y(oil|no-oil)$ is a complex function of x_1 and x_2

- What if we have more than one input?
- Suppose we are trying to take a decision about whether we will find oil at a particular location on the ocean bed(Yes/No)
- Further, suppose we base our decision on two factors: Salinity (x_1) and Pressure (x_2)
- We are given some data and it seems that $y(oil|no-oil)$ is a complex function of x_1 and x_2
- We want a neural network to approximate this function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- This is what a 2-dimensional sigmoid looks like

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- We need to figure out how to get a tower in this case

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 2, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 3, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 4, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 5, w_2 = 0, b = 0$

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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 6, w_2 = 0, b = 0$

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- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 7, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 8, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 9, w_2 = 0, b = 0$

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- We need to figure out how to get a tower in this case
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 10, w_2 = 0, b = 0$

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- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 11, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 12, w_2 = 0, b = 0$

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- We need to figure out how to get a tower in this case
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 13, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 14, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 15, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 16, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 17, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 18, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 19, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 20, w_2 = 0, b = 0$

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$w_1 = 21, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 22, w_2 = 0, b = 0$

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- We need to figure out how to get a tower in this case
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 23, w_2 = 0, b = 0$

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- We need to figure out how to get a tower in this case
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 24, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 25, w_2 = 0, b = 5$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 25, w_2 = 0, b = 10$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 25, w_2 = 0, b = 15$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 25, w_2 = 0, b = 25$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 25, w_2 = 0, b = 30$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 25, w_2 = 0, b = 35$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 25, w_2 = 0, b = 40$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 25, w_2 = 0, b = 45$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

- What if we take two such step functions (with different b values) and subtract one from the other

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- We still don't get a tower (or we get a tower which is open from two sides)

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

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- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 2, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 3, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 4, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 5, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 6, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 7, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 8, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 0, w_2 = 9, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 10, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 0, w_2 = 11, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 12, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 0, w_2 = 13, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 14, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 0, w_2 = 15, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 16, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 17, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 18, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 19, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 20, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 21, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 22, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 23, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$w_1 = 0, w_2 = 24, b = 0$

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 0, w_2 = 25, b = 5$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 0, w_2 = 25, b = 10$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 0, w_2 = 25, b = 15$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
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- And now we change b

$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

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- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 0, w_2 = 25, b = 30$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
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- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 0, w_2 = 25, b = 40$

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$$y = \frac{1}{1 + \exp^{-(w_1 x_1 + w_2 x_2 + b)}}$$

$w_1 = 0, w_2 = 25, b = 45$

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- Again, what if we take two such step functions (with different b values) and subtract one from the other
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- Notice that this open tower has a different orientation from the previous one

- Now what will we get by adding two such open towers ?

+

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h_{31} is passed through a sigmoid function with the following characteristics

- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base
- We can now pass this output through another sigmoid neuron to get the desired tower !
- We can now approximate any function by summing up many such towers

- For example, we could approximate the following function using a sum of several towers

- Can we come up with a neural network to represent this entire procedure of constructing a 3 dimensional tower ?

x1
x2
−
−
+

x1
x2
−
−
+

Think

- For 1 dimensional input we needed 2 neurons to construct a tower
- For 2 dimensional input we needed 4 neurons to construct a tower
- How many neurons will you need to construct a tower in n dimensions ?

Time to retrospect

- Why do we care about approximating any arbitrary function ?
- Can we tie all this back to the classification problem that we have been dealing with ?

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- This is what we actually want
- The illustrative proof that we just saw tells us that we can have a neural network with two hidden layers which can approximate the above function by a sum of towers
- Which means we can have a neural network which can exactly separate the blue points from the red points !!