CS7015 (Deep Learning) : Lecture 21

Variational Autoencoders

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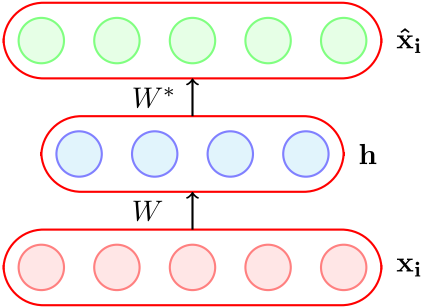
Department of Computer Science and Engineering
Indian Institute of Technology Madras

Module 21.1: Revisiting Autoencoders



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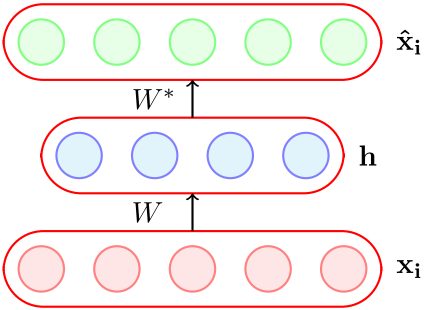

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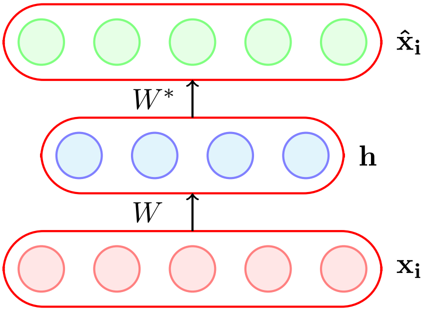

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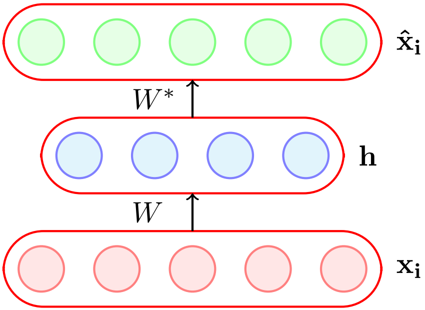

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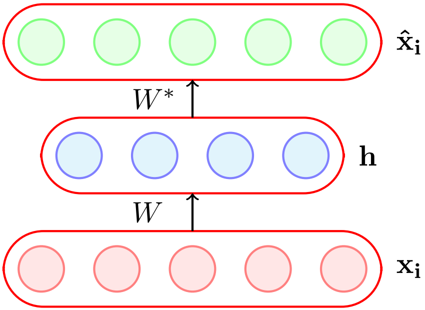
- The training happens using the following objective function

$$\min_{W,W^*,\mathbf{c},\mathbf{b}} \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} (\hat{x}_{ij} - x_{ij})^2$$



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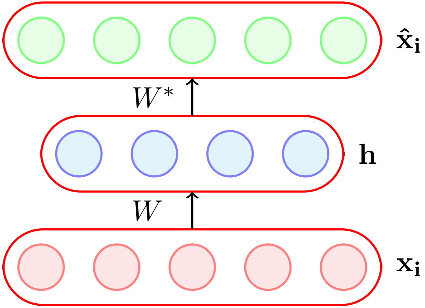

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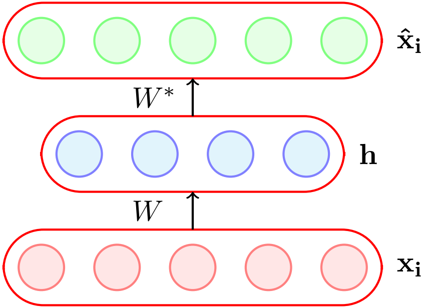

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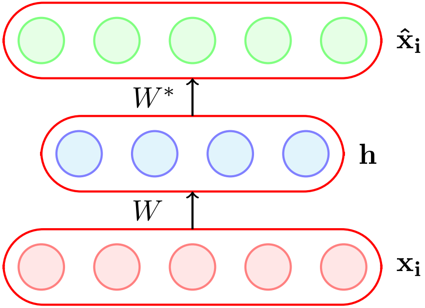

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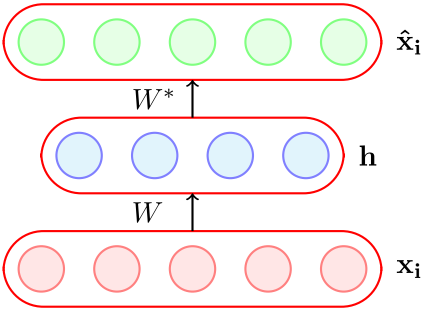

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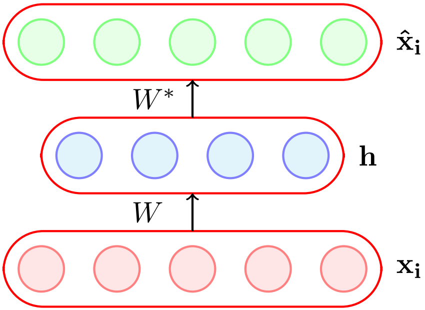

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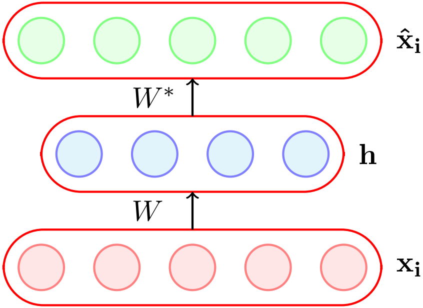
- Let us revisit *generation* in the context of autoencoders?



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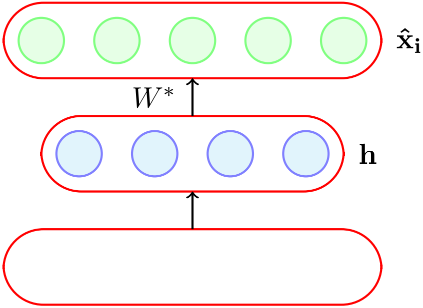

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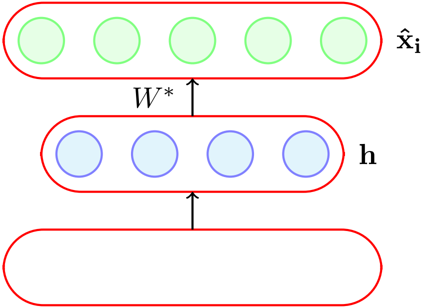

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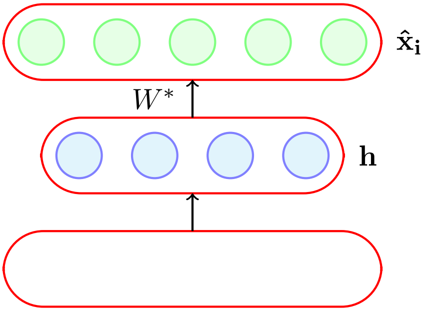

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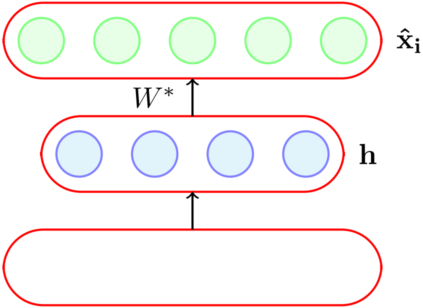

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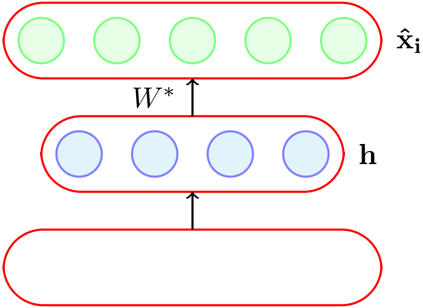
- h is a very high dimensional vector and only a few vectors in this space would actually correspond to meaningful latent representations of our input
- So of all the possible value of h which values should I feed to the decoder (we had asked a similar question before: slide 67, bullet 5 of lecture 19)



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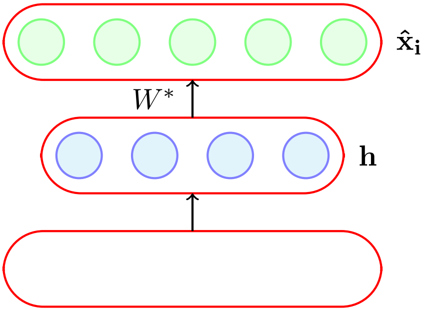

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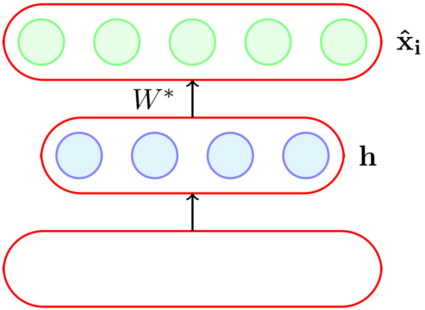

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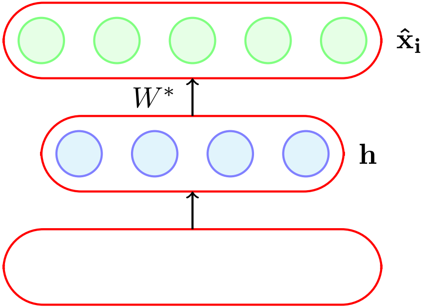

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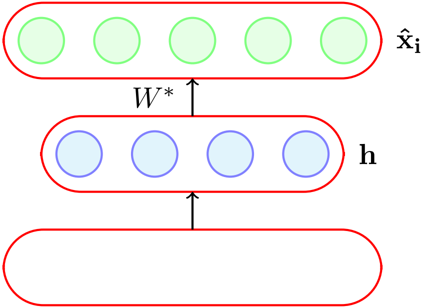

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- We will now look at variational autoencoders which have the same structure as autoencoders but they learn a distribution over the hidden variables

Module 21.2: Intuition behind Variational Autoencoders

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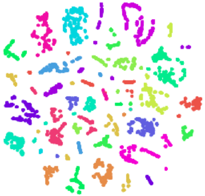

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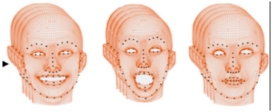

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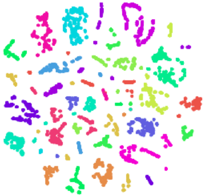
- We are also interested in generation (i.e., given a hidden representation generate an X)



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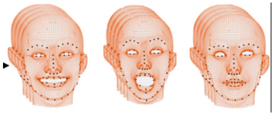

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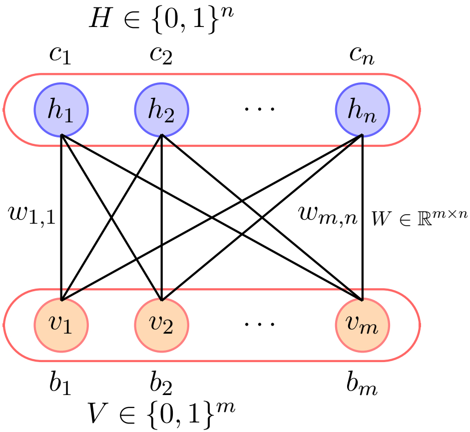
- In probabilistic terms we are interested in $P(z|X)$ and $P(X|z)$



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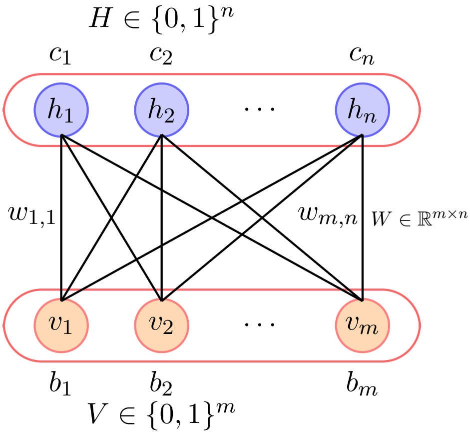

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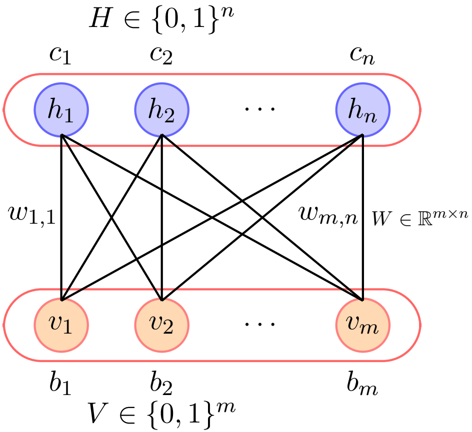

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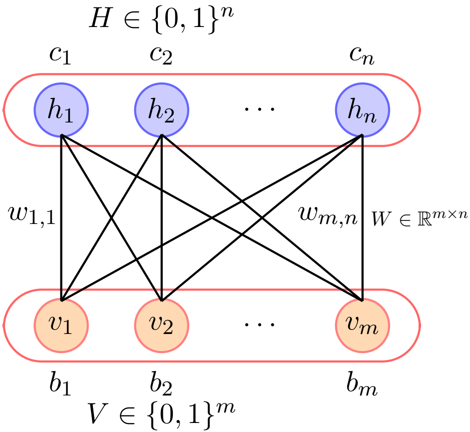

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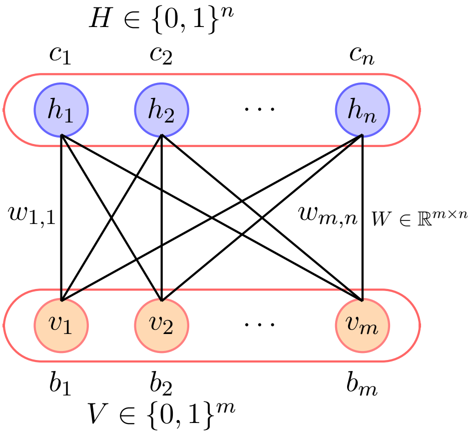

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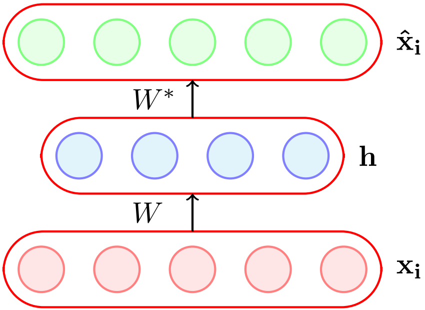
- **Structural assumptions:** We assumed certain independencies in the Markov Network
- **Computational:** When training with Gibbs Sampling we have to run the Markov Chain for many time steps which is expensive
- **Approximation:** When using Contrastive Divergence, we approximate the expectation by a point estimate



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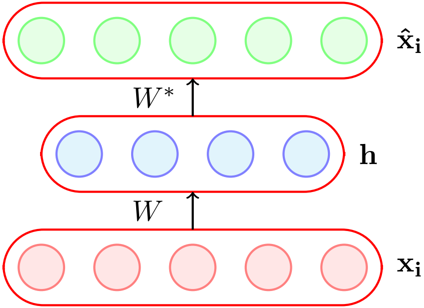

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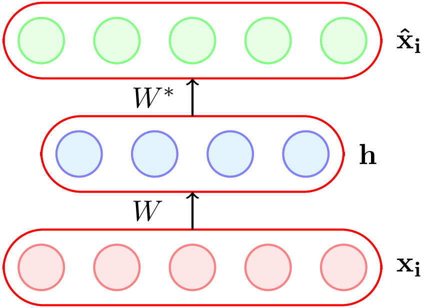
- Recall that in RBMS we learned the parameters by solving the following optimization problem

$$maximize \prod_{i=1}^{N} P(X = x_i)$$



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- Recall that in RBMS we learned the parameters by solving the following optimization problem

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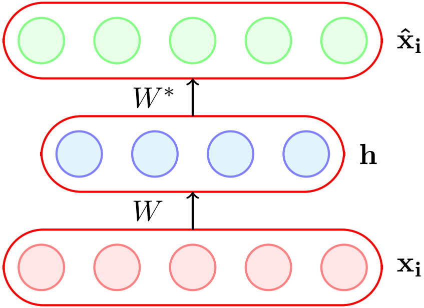
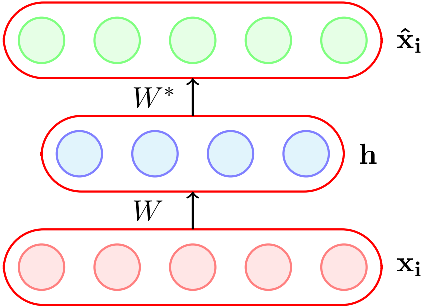
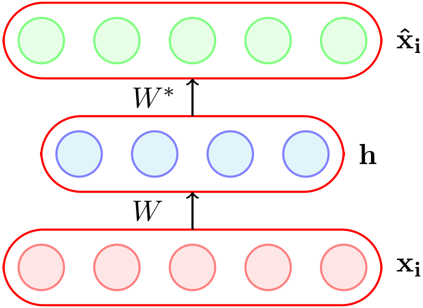
- In VAEs also we will start with the same objective function but use a different trick to solve the optimization problem (we will get there soon)



- Goal 1: Learn a distribution over the latent variables ($P(z|X)$)



- Goal 1: Learn a distribution over the latent variables ($P(z|X)$)
- Goal 2: Learn a distribution over the visible variables ($P(X|z)$)



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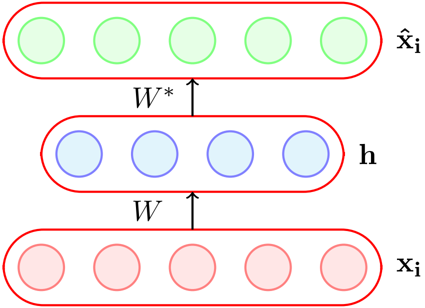

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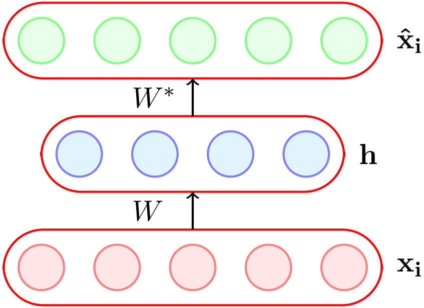

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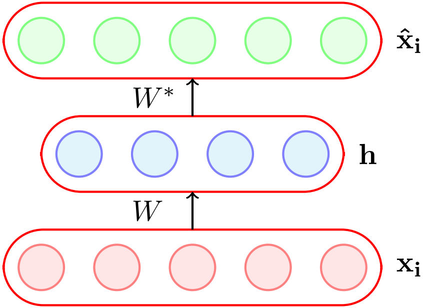
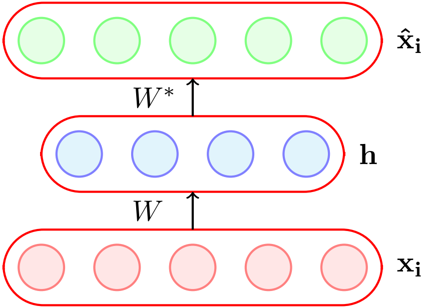
- Suppose we assume a certain form for the distribution $P(z|X)$
- For example, let us assume that $P(z|X)$ is a Normal distribution
- In RBMs, we had assumed a certain structure (independencies) for the joint distribution whereas here we are assuming a certain family form for the distribution $P(z|X)$



- Now given the assumption that $P(z|X)$ is a Normal distribution can you think of adapting the autoencoder architecture to learn the distribution $P(z|X)$ instead of learning z



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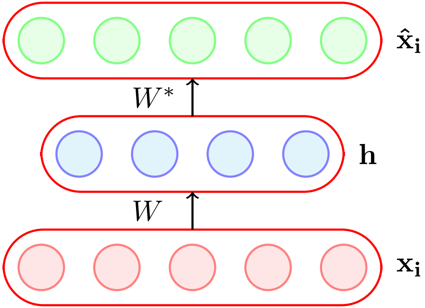
- What do we mean when we say we want to learn a distribution? We mean that we want to learn the parameters of the distribution



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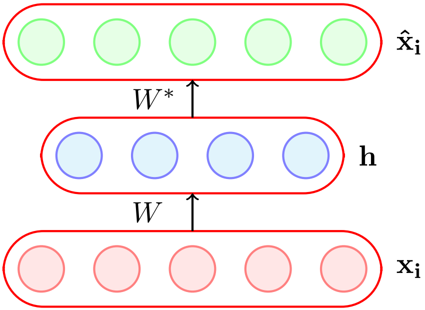

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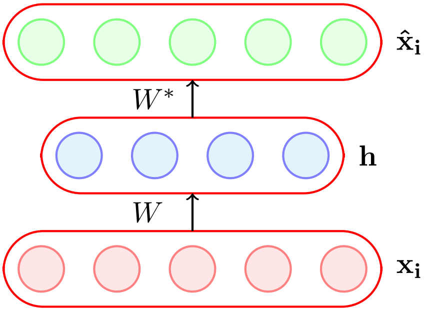

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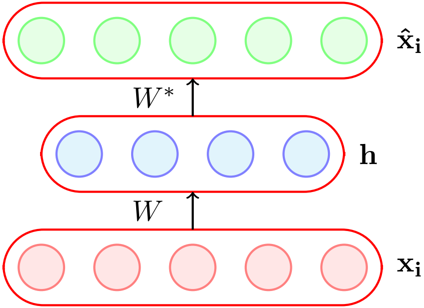
- Well, we could design the encoder to predict the mean and variance of $P(z|X)$



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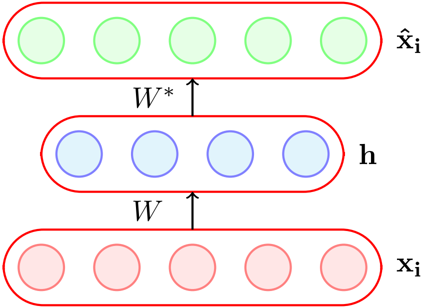

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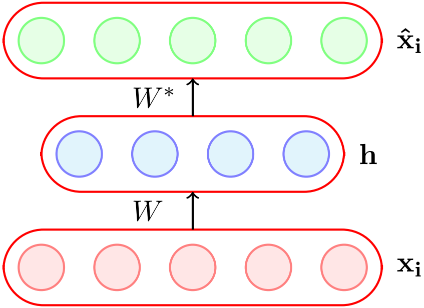

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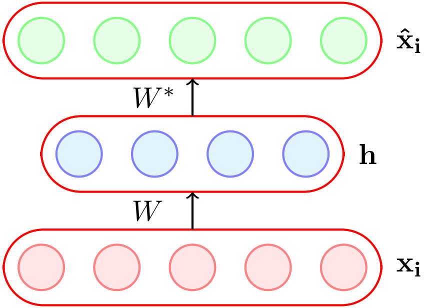

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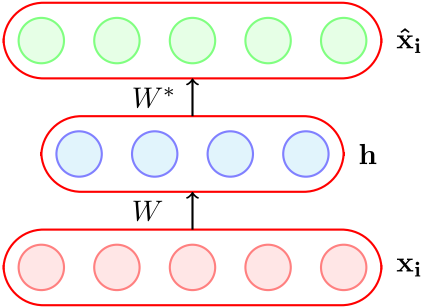

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- **Spoiler Alert:** We will be making some assumptions but to see why these assumptions make sense we will first revisit the concept of latent variables

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- For example, once the digit, size, angle thickness, position, *etc.* are fixed we know exactly what the image should look like
- Of course, we are assuming that we have enough latent variables to capture all characteristics of handwritten digits

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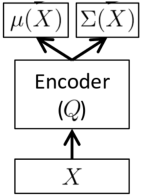
- Well, we could use a deep neural network which is good at approximating arbitrary complex functions (recall Universal Approximation Theorem)
- With this intuition, we will now look at the full picture and discuss the objective function

- First, the task of the encoder is to predict the parameters (μ, σ) of $P(z|X)$



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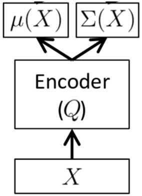

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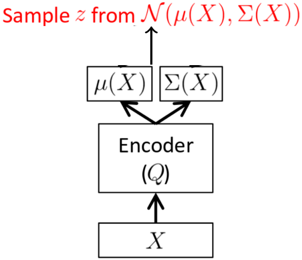

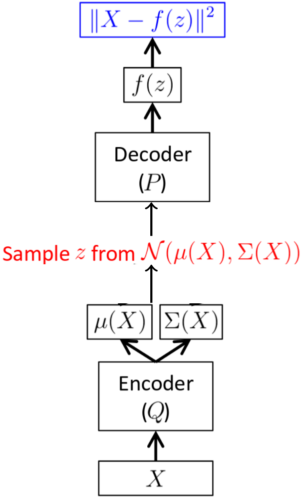

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- So given a training sample X, the encoder will first produce $\mu(X), \sigma(X)$
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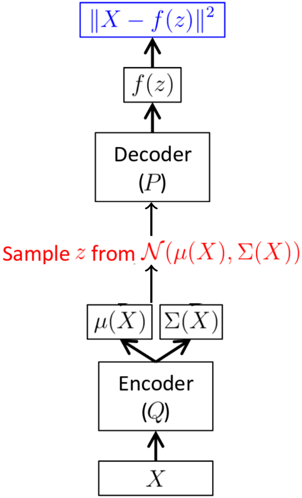
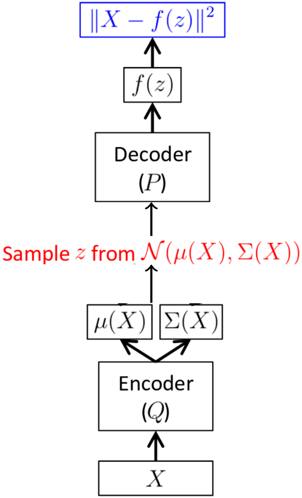
- The job of the decoder is to then reconstruct X from this sampled z



- Given this setup what should be the loss function for the decoder

$$||X - f(z;\theta)||^2$$



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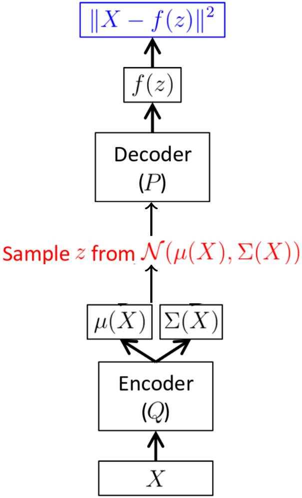

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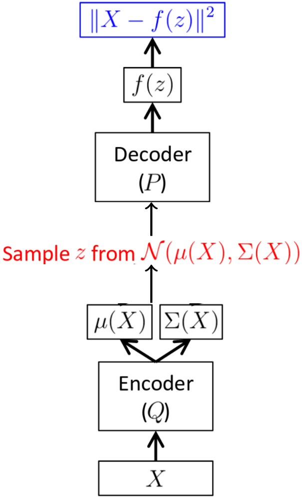

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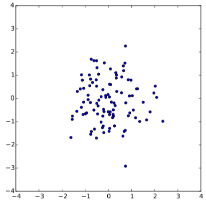
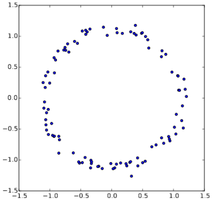
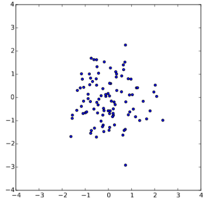
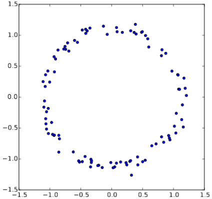
- Well if we knew the true $P(z)$ then we could have minimized the KL divergence between $Q(z|X)$ and $P(z)$
- But we don't know what $P(z)$ is so we will make an assumption that $P(z)$ is $N(0, I)$





- Isn't this is very strong assumption ?





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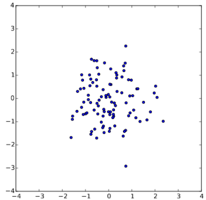

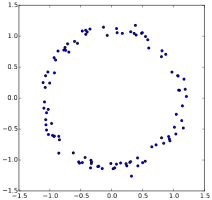

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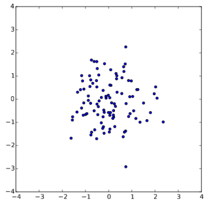

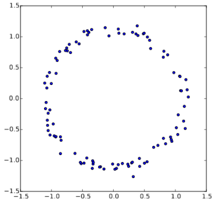

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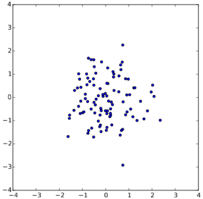

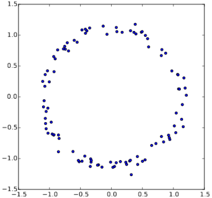

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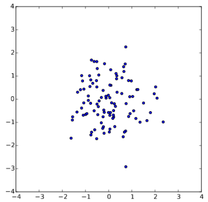
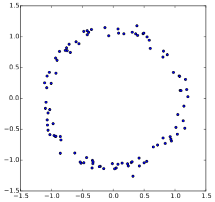
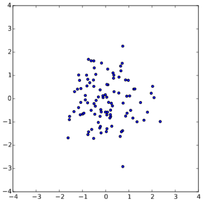
- Step 1: Start with a set of d variables that are normally distributed (that's exactly what we are assuming for $P(z)$)
- Step 2: Mapping these variables through a sufficiently complicated function (that's exactly what the first few layers of the decoder can do)

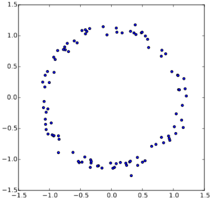




- In particular, note that in the adjoining example if z is 2-D and normally distributed then $g(z)$ is roughly ring shaped (giving us the distribution in the bottom figure)

$$g(z) = \frac{z}{10} + \frac{z}{||z||}$$





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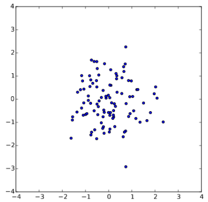

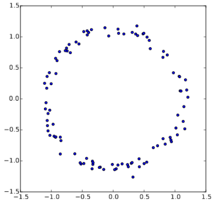

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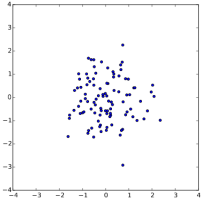

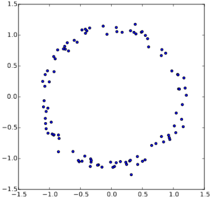

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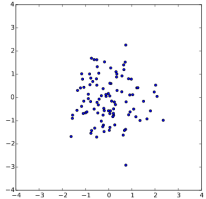
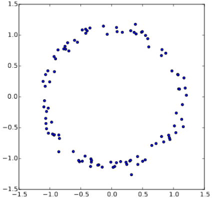
- The above argument suggests that even if we start with normally distributed variables the initial layers of the encoder could learn a complex transformation of these variables say $f(z, \theta_1)$ if required





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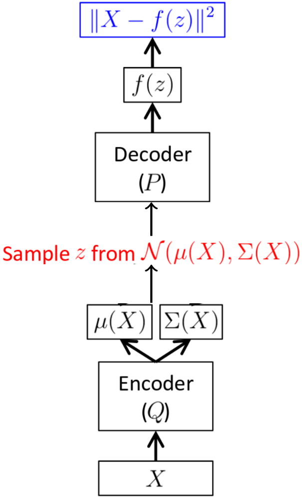
- The objective function of the decoder will ensure that an appropriate transformation of z is learnt to reconstruct X



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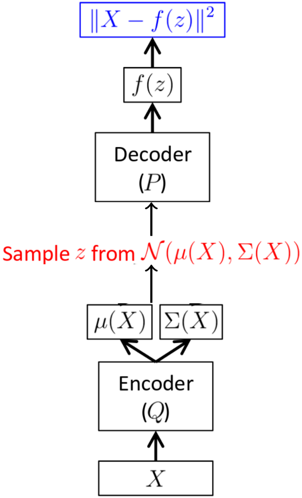

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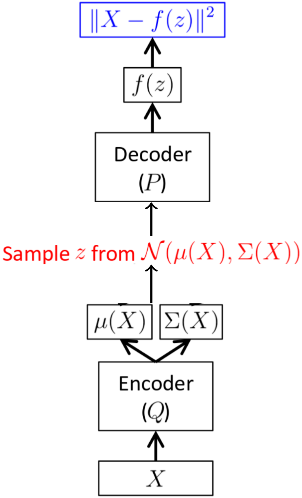

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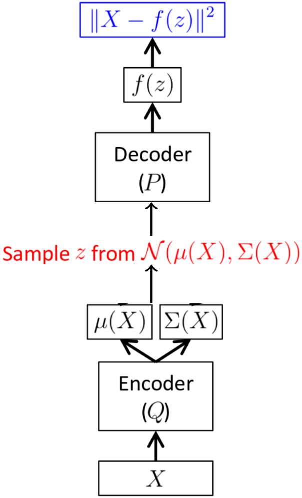
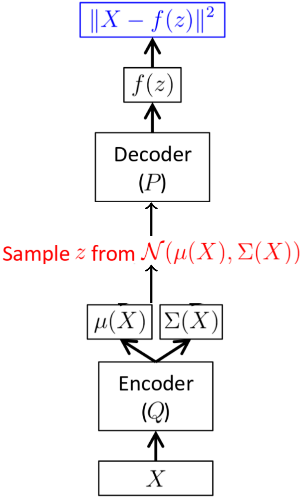
- That completes the full picture and we will summarize the discussion on the next slide

- Encoder: Generates the μ and σ of $Q(z|X)$





- Encoder: Generates the μ and σ of $Q(z|X)$
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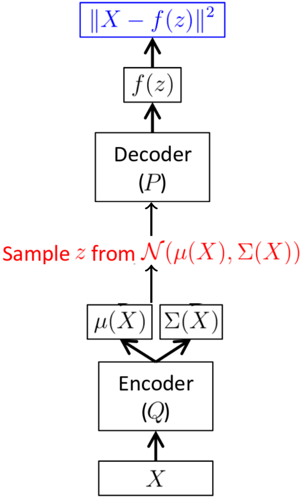

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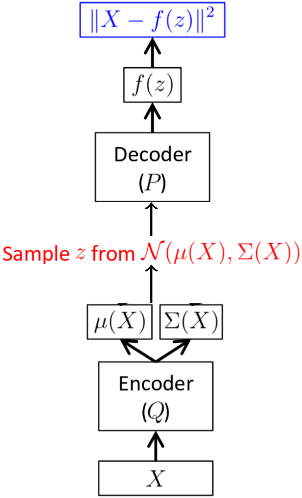

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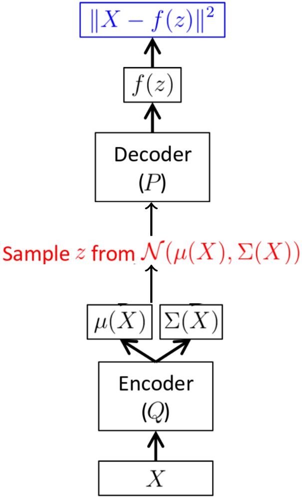

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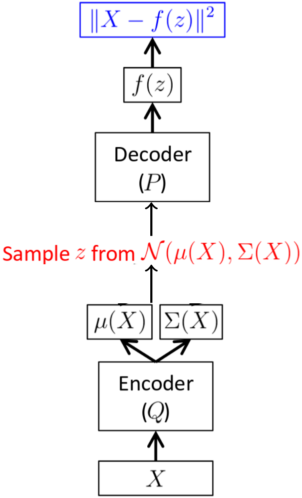
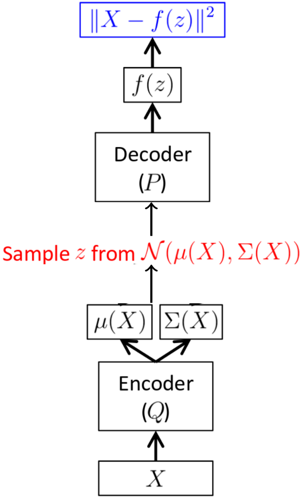
$$min \quad ||X - f(z; \theta)||^2$$

- There are still a few pieces missing and we will get back to them later



- This was a very simplistic (non-rigorous) introduction to VAEs



- This was a very simplistic (non-rigorous) introduction to VAEs
- We will now do a more rigorous discussion on the Math behind VAEs