

Module 5.2: Learning Parameters : Gradient Descent

Now let's see if there is a more efficient and principled way of doing this

Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!

vector of parameters,
say, randomly initialized

$$\theta = [w, b]$$

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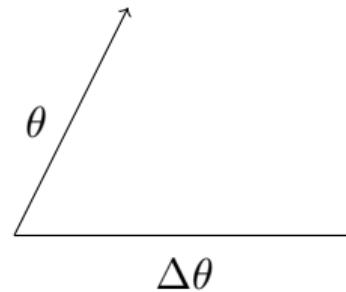
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change in the
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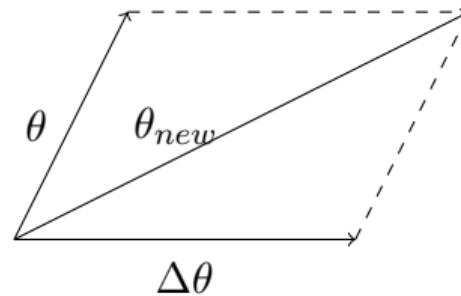


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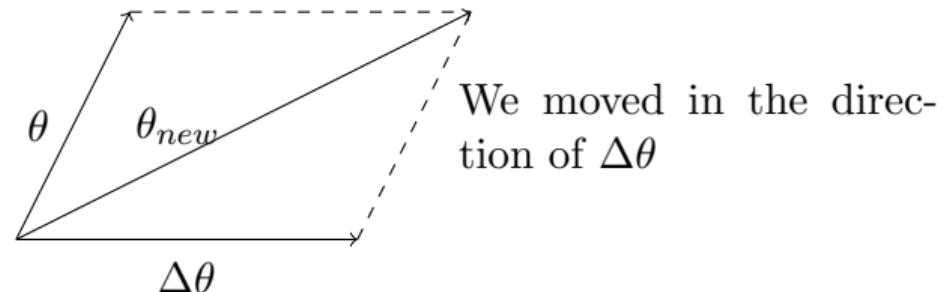
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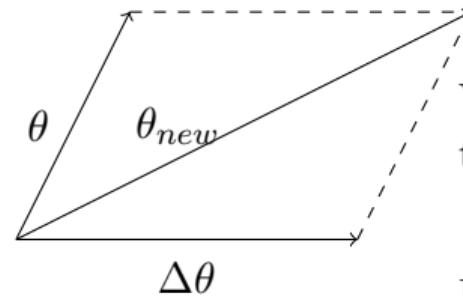
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We moved in the direction of $\Delta\theta$

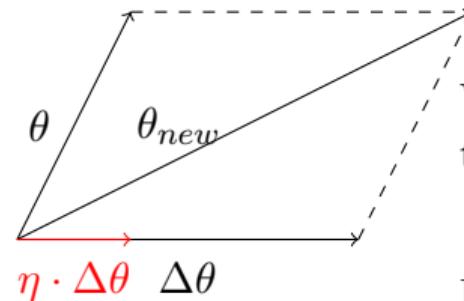
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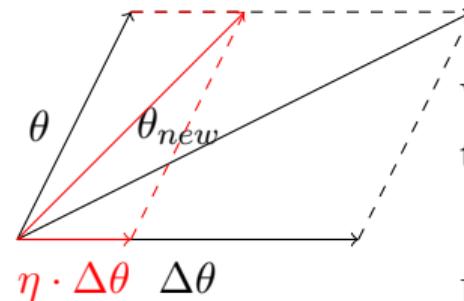
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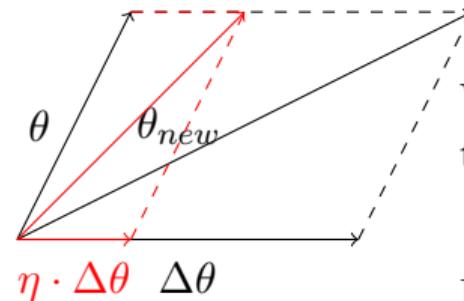
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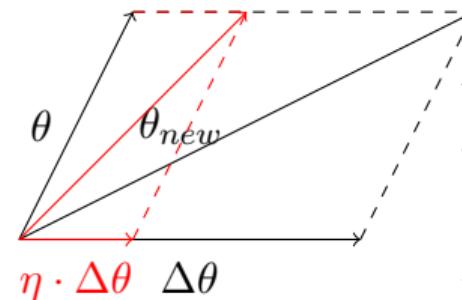
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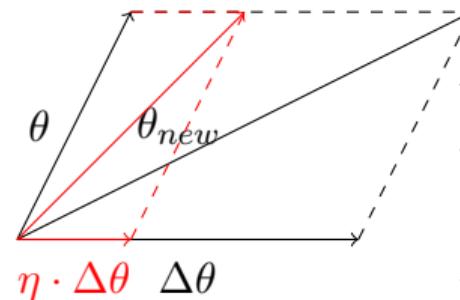
Question: What is the right $\Delta\theta$ to use?

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The answer comes from Taylor series

For ease of notation, let $\Delta\theta = u$, then from Taylor series, we have,

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$$\mathcal{L}(\theta + \eta u) - \mathcal{L}(\theta) < 0 \text{ [i.e., if the new loss is less than the previous loss]}$$

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This implies,

$$u^T \nabla \mathcal{L}(\theta) < 0$$

Okay, so we have,

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But, what is the range of $u^T \nabla \mathcal{L}(\theta)$?

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Multiply throughout by $k = \|u\| * \|\nabla \mathcal{L}(\theta)\|$

$$-k \leq k * \cos(\beta) = u^T \nabla \mathcal{L}(\theta) \leq k$$

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Thus, $\mathcal{L}(\theta + \eta u) - \mathcal{L}(\theta) = u^T \nabla \mathcal{L}(\theta) = k * \cos(\beta)$ will be most negative when $\cos(\beta) = -1$ i.e., when β is 180°

Gradient Descent Rule

- The direction u that we intend to move in should be at 180° w.r.t. the gradient

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Parameter Update Equations

$$w_{t+1} = w_t - \eta \nabla w_t$$

$$b_{t+1} = b_t - \eta \nabla b_t$$

where, $\nabla w_t = \frac{\partial \mathcal{L}(w, b)}{\partial w}$ at $w = w_t, b = b_t$, $\nabla b_t = \frac{\partial \mathcal{L}(w, b)}{\partial b}$ at $w = w_t, b = b_t$

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So we now have a more principled way of moving in the w - b plane than our “guess work” algorithm

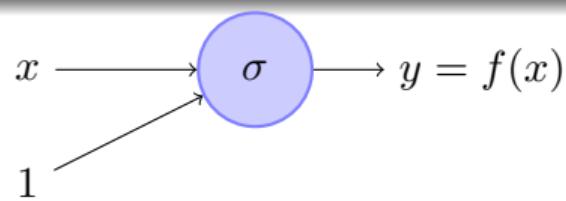
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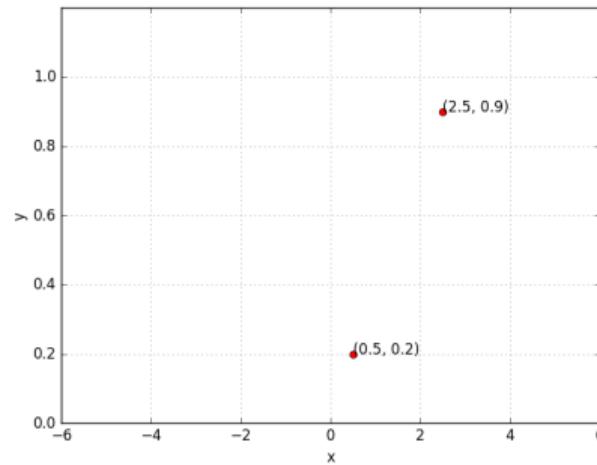
Algorithm 1: gradient_descent()

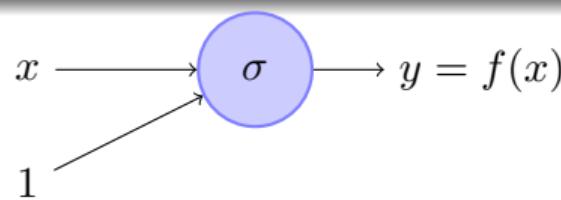
```
t ← 0;  
max_iterations ← 1000;  
while  $t < max\_iterations$  do  
    |  $w_{t+1} \leftarrow w_t - \eta \nabla w_t;$   
    |  $b_{t+1} \leftarrow b_t - \eta \nabla b_t;$   
end
```

- To see this algorithm in practice let us first derive ∇w and ∇b for our toy neural network



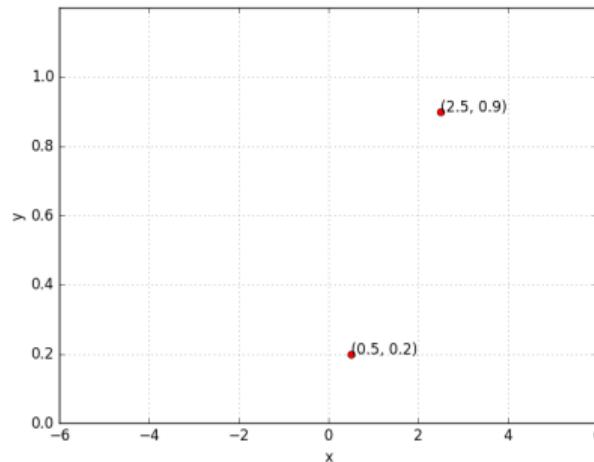
$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

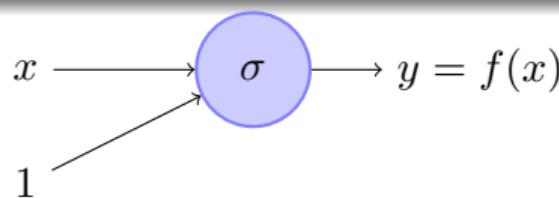




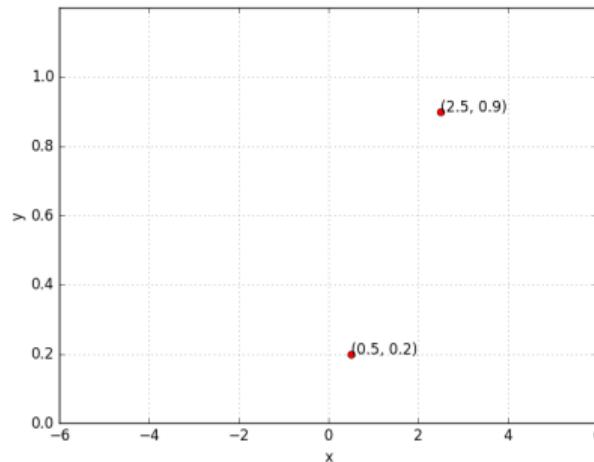
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Let's assume there is only 1 point to fit
(x, y)



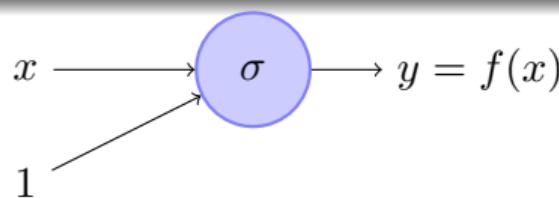


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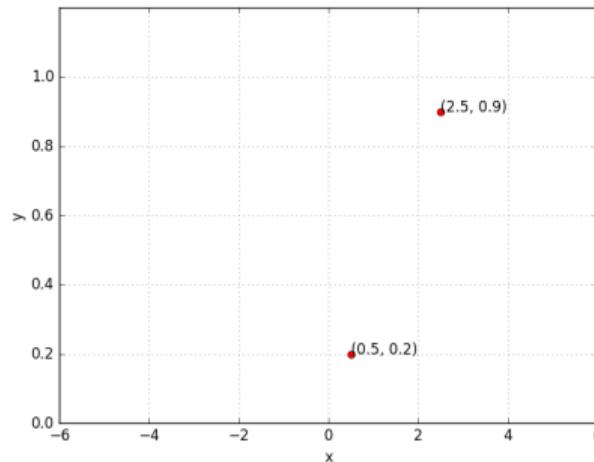


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$$\mathcal{L}(w, b) = \frac{1}{2} * (f(x) - y)^2$$



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$$\begin{aligned}\mathcal{L}(w, b) &= \frac{1}{2} * (f(x) - y)^2 \\ \nabla w &= \frac{\partial \mathcal{L}(w, b)}{\partial w} = \frac{\partial}{\partial w} \left[\frac{1}{2} * (f(x) - y)^2 \right]\end{aligned}$$

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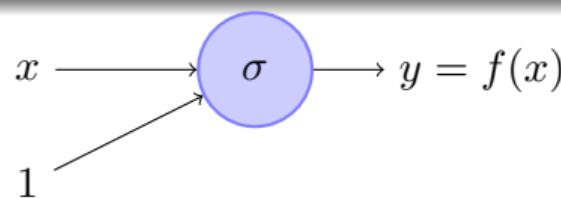
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 &= f(x) * (1 - f(x)) * x
 \end{aligned}$$

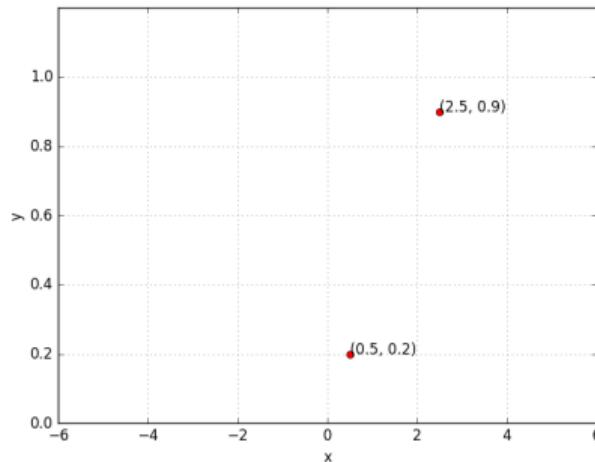
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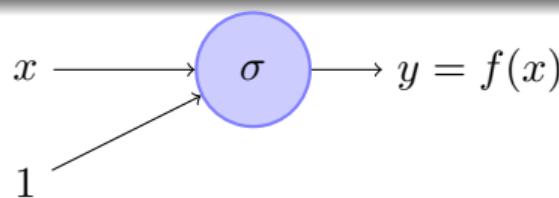
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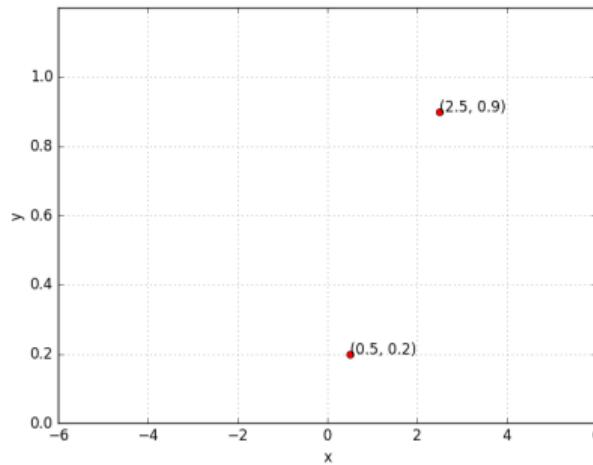
$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

So if there is only 1 point (x, y) , we have,



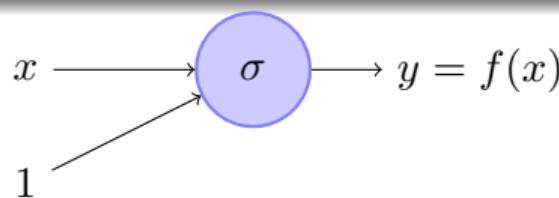


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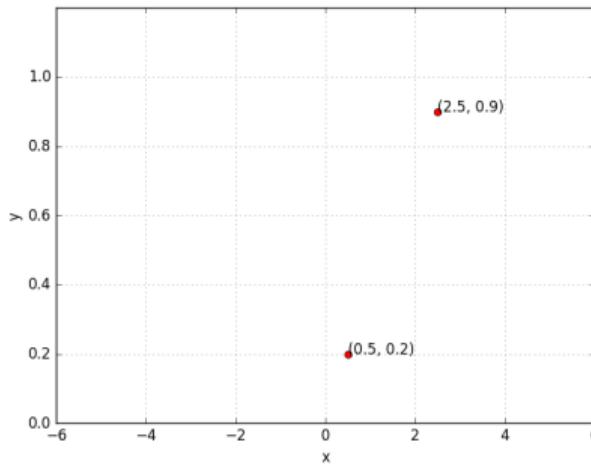


So if there is only 1 point (x, y) , we have,

$$\nabla w = (f(x) - y) * f(x) * (1 - f(x)) * x$$



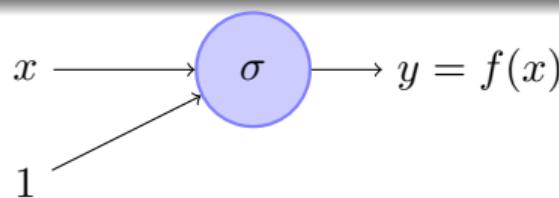
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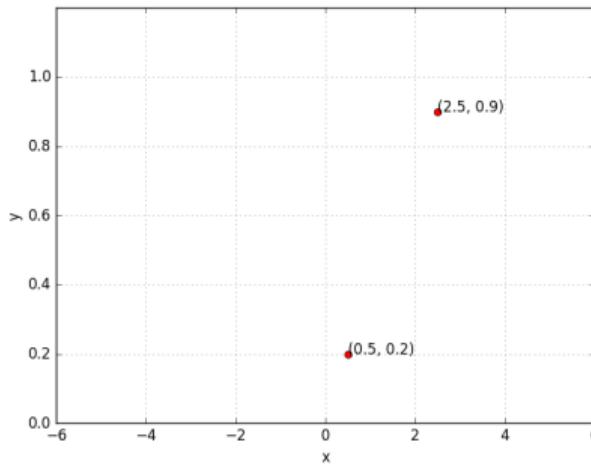
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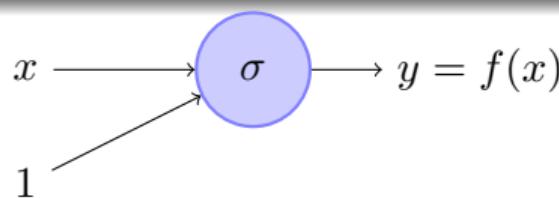


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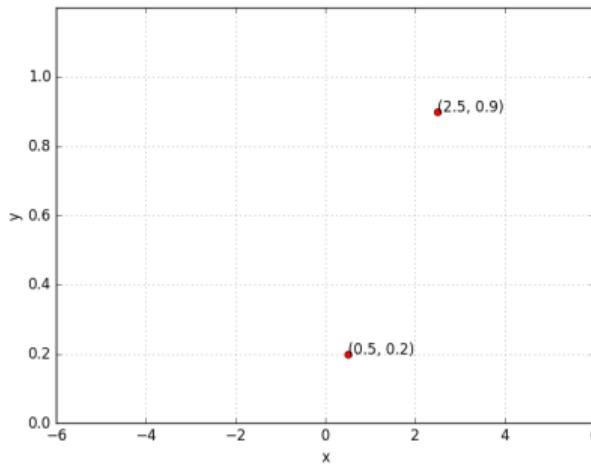
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For two points,

$$\nabla w = \sum_{i=1}^2 (f(x_i) - y_i) * f(x_i) * (1 - f(x_i)) * x_i$$



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For two points,

$$\nabla w = \sum_{i=1}^2 (f(x_i) - y_i) * f(x_i) * (1 - f(x_i)) * x_i$$

$$\nabla b = \sum_{i=1}^2 (f(x_i) - y_i) * f(x_i) * (1 - f(x_i))$$

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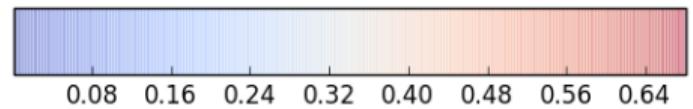
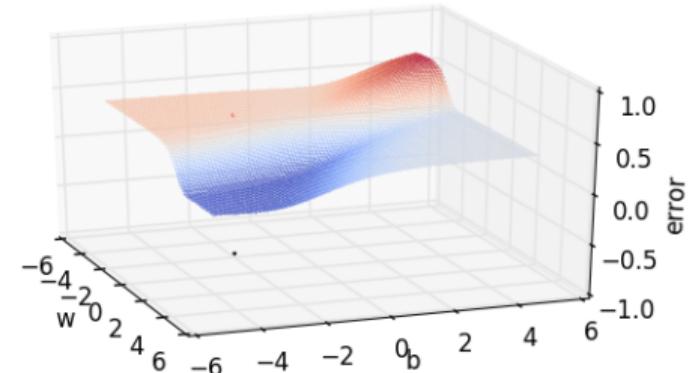
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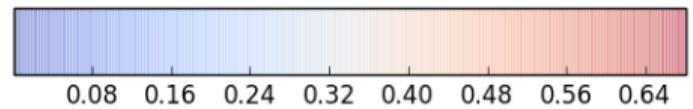
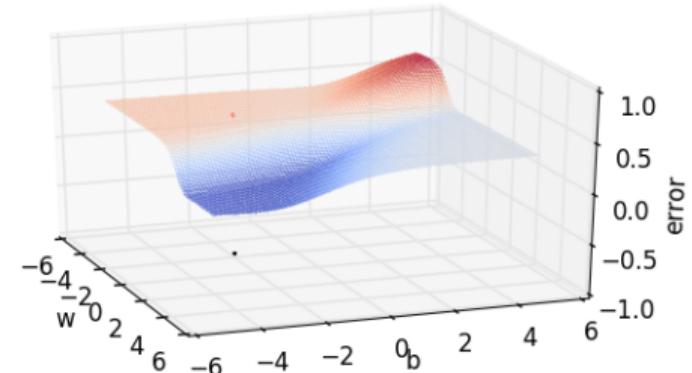
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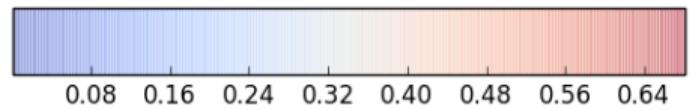
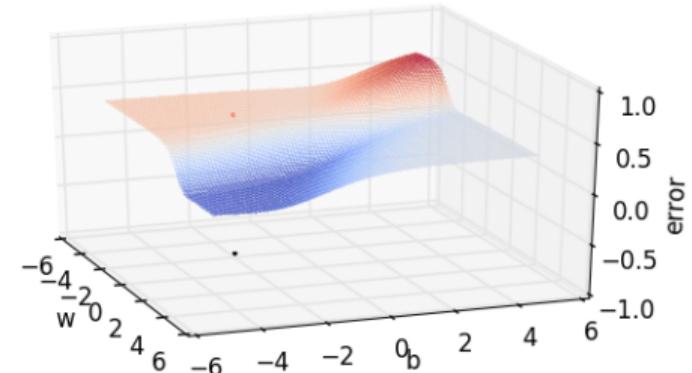
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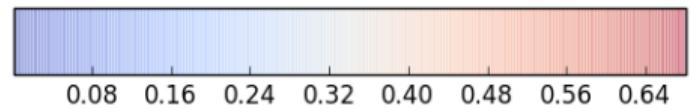
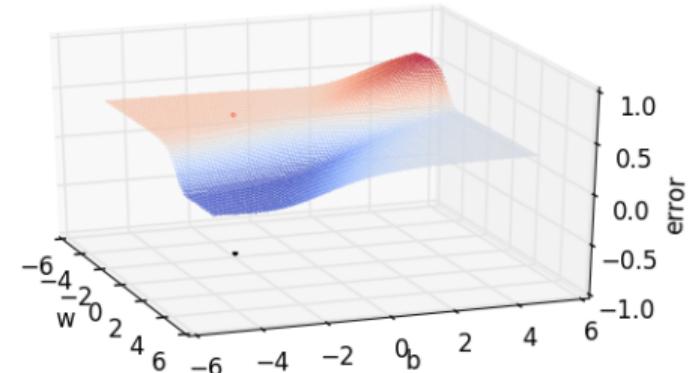
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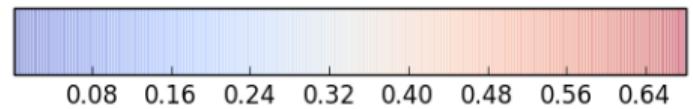
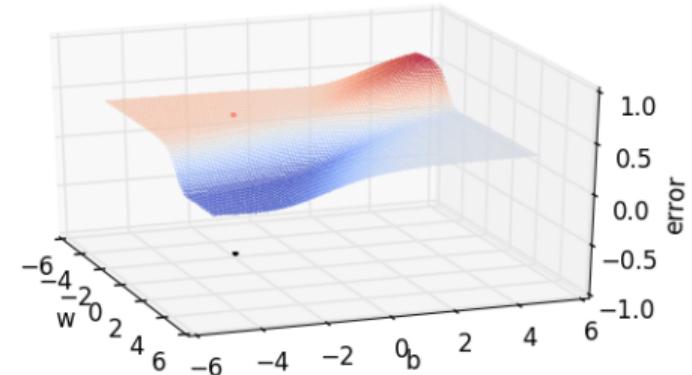
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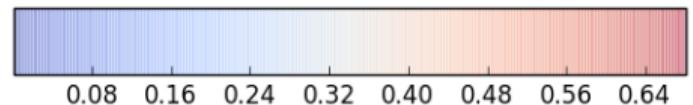
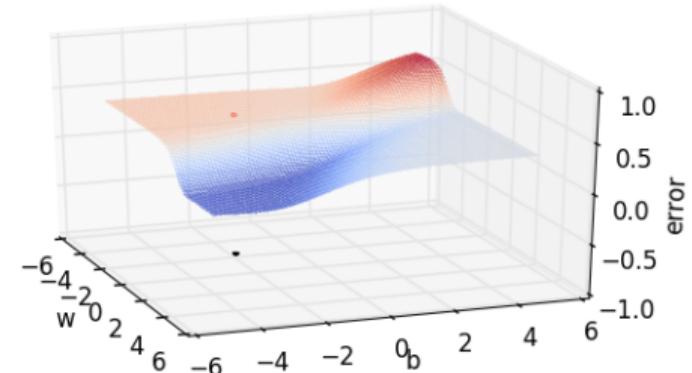
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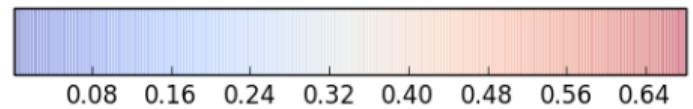
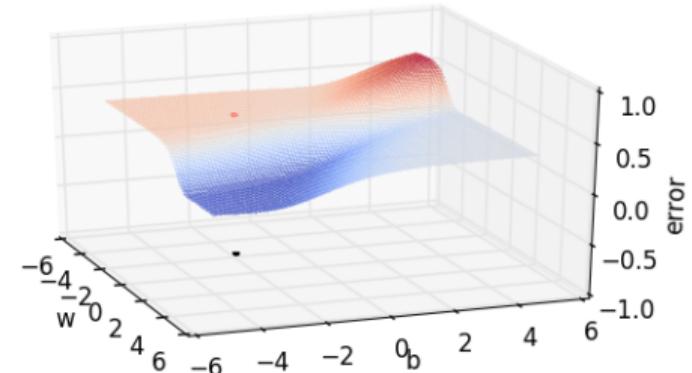
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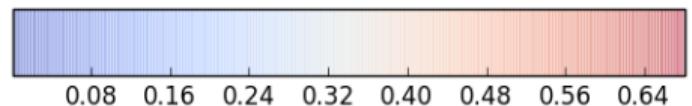
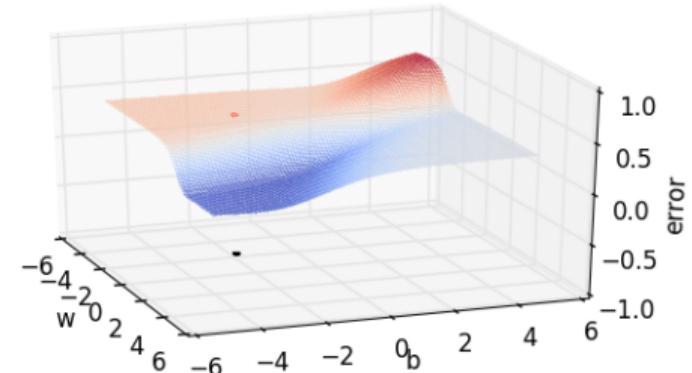
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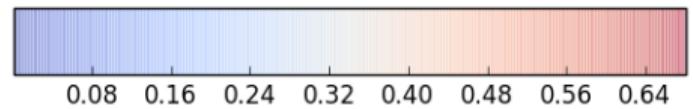
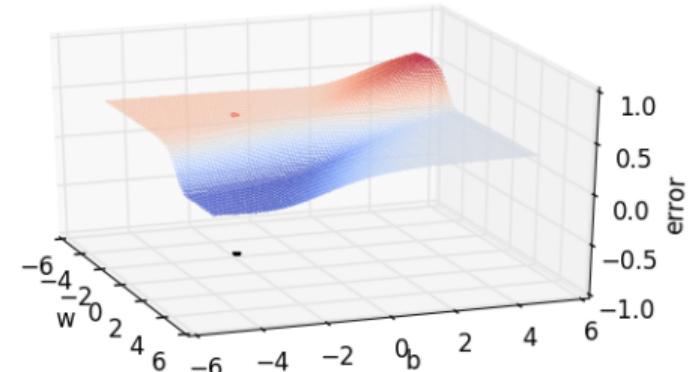
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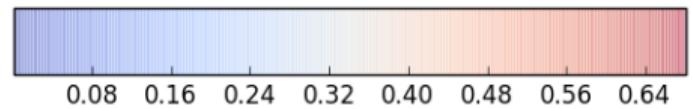
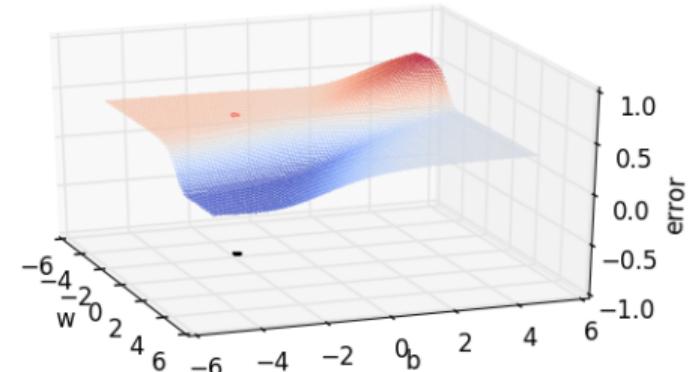
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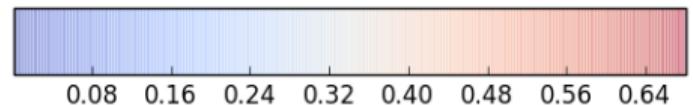
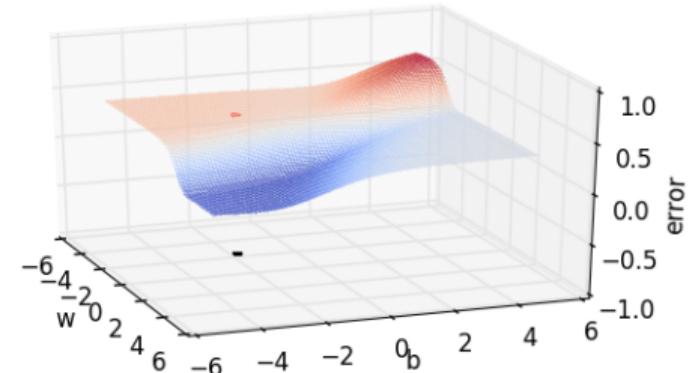
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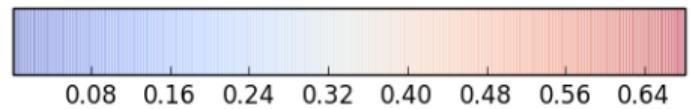
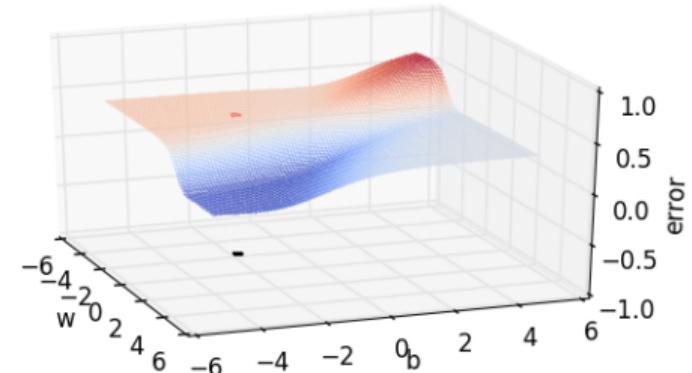
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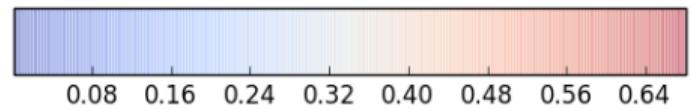
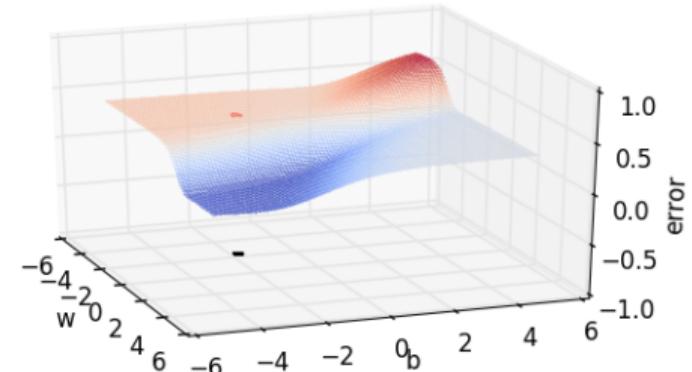
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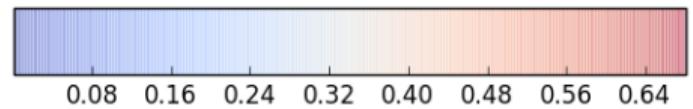
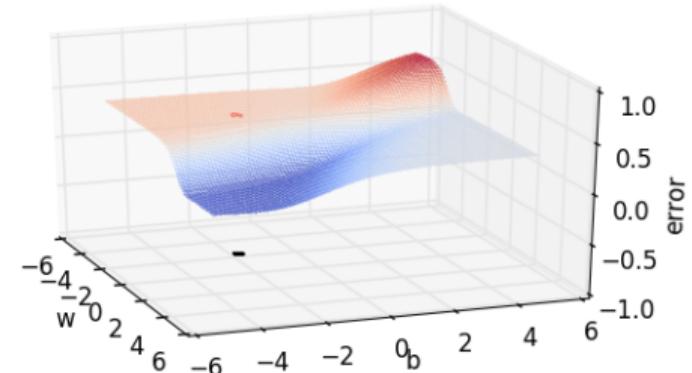
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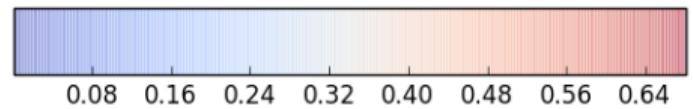
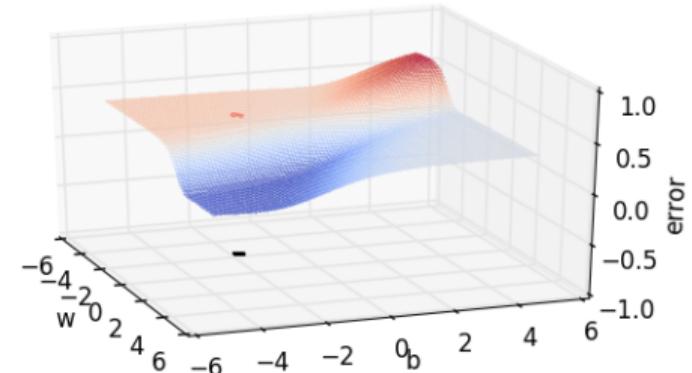
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Gradient descent on the error surface



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Y = [0.2, 0.9]

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    return 1.0 / (1.0 + np.exp(-(w*x + b)))

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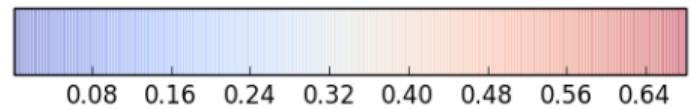
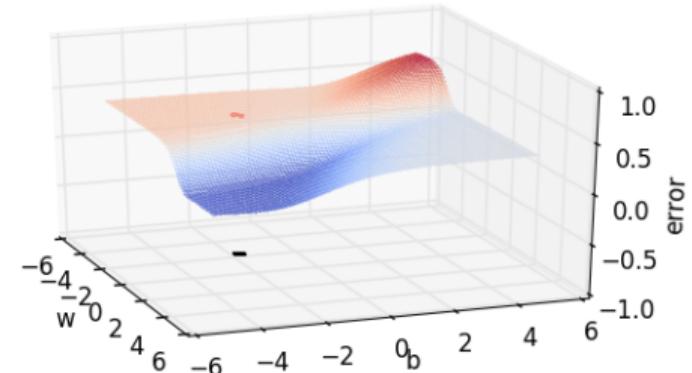
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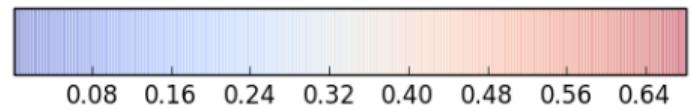
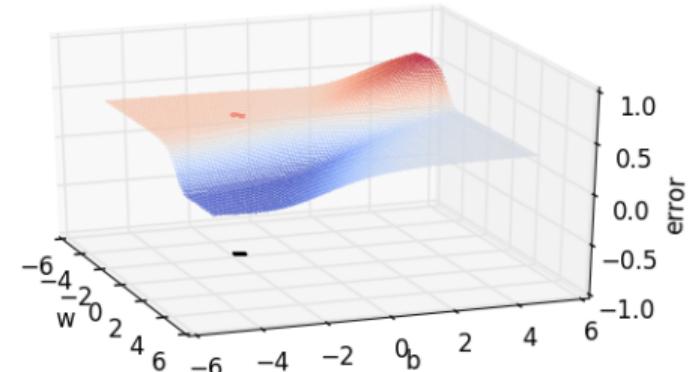
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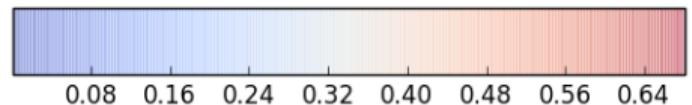
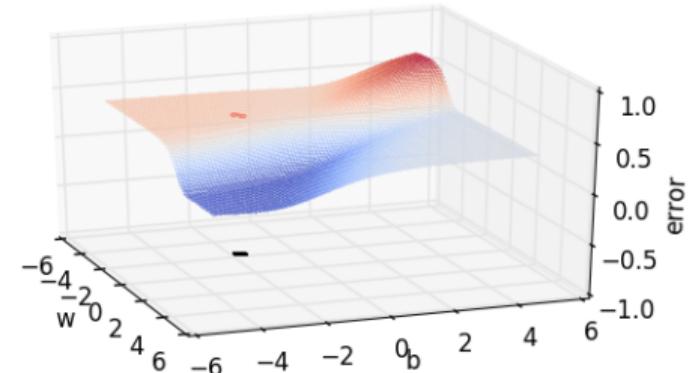
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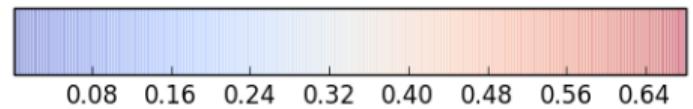
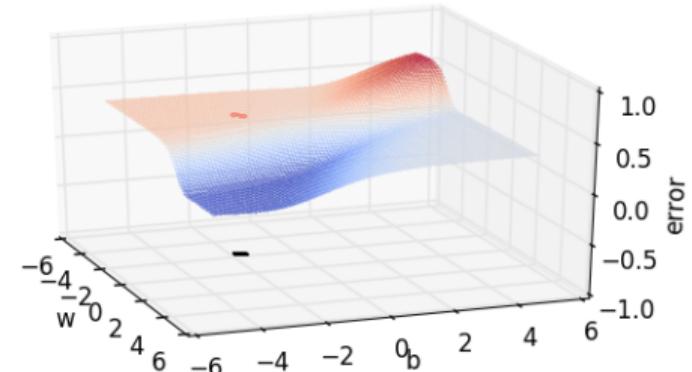
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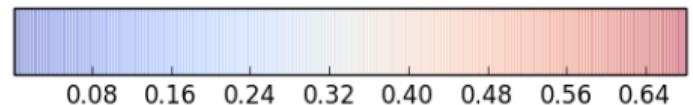
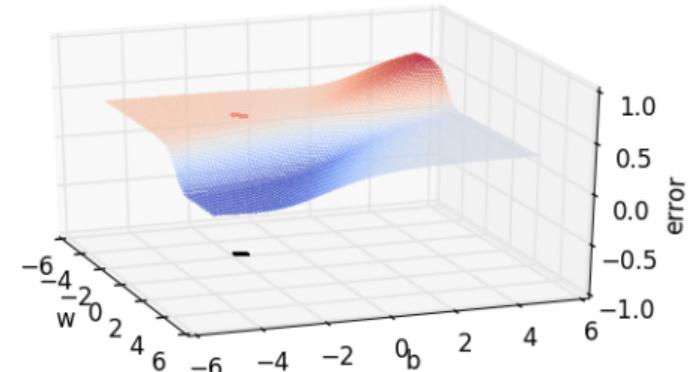
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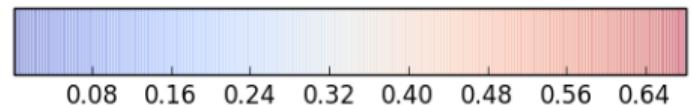
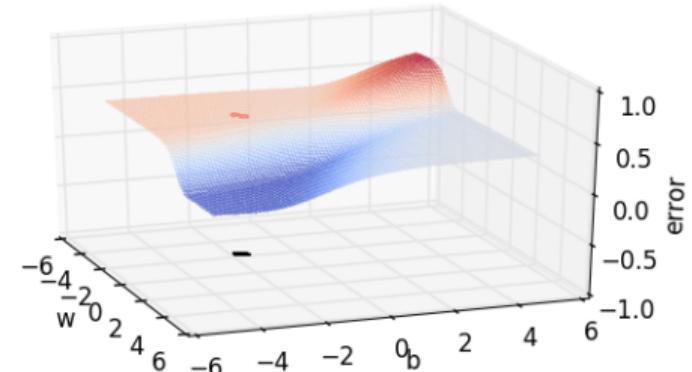
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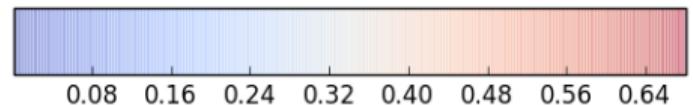
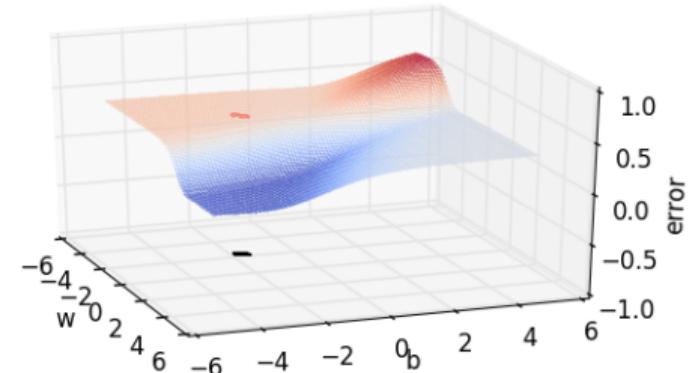
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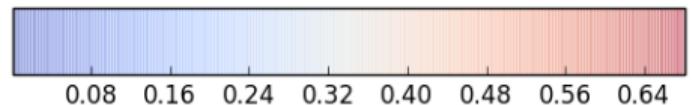
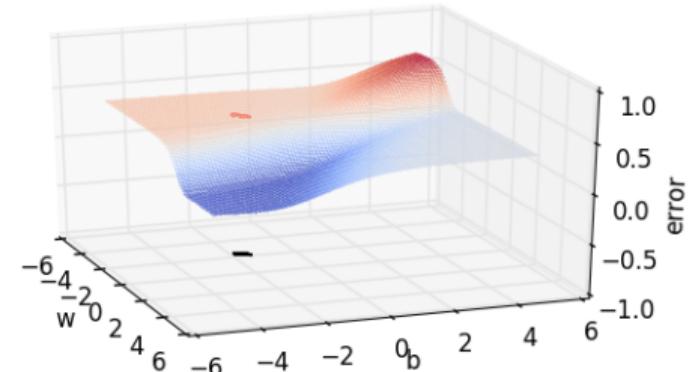
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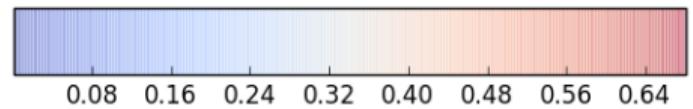
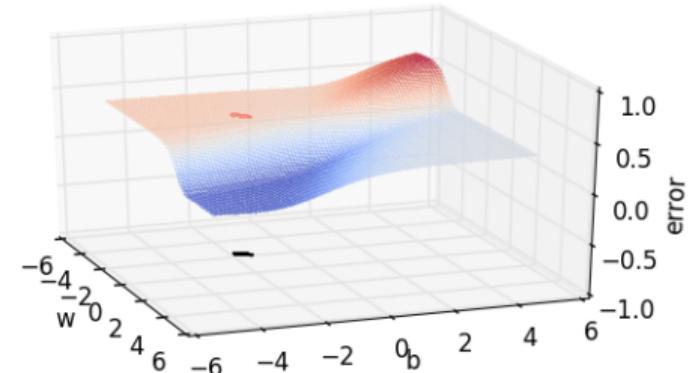
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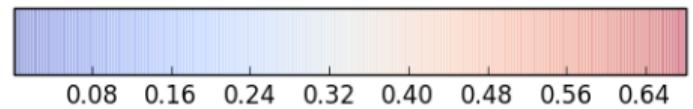
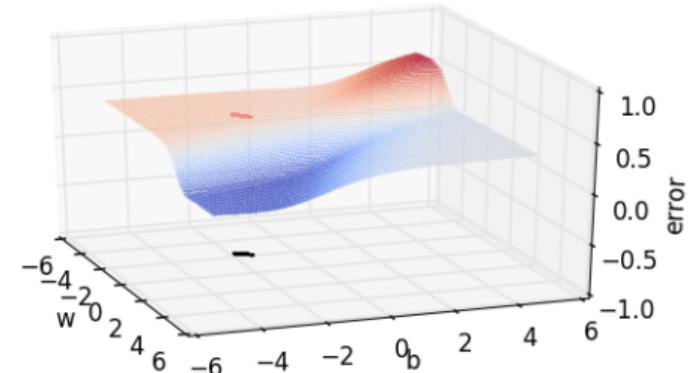
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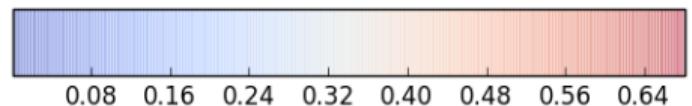
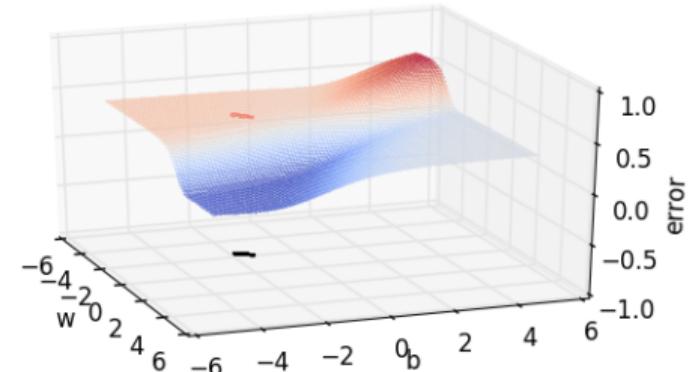
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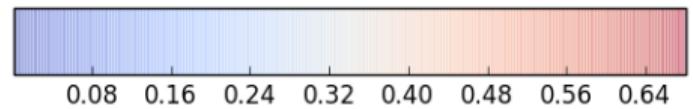
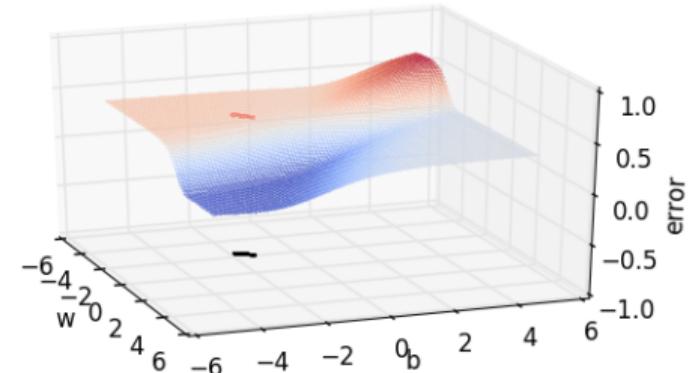
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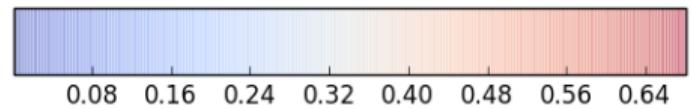
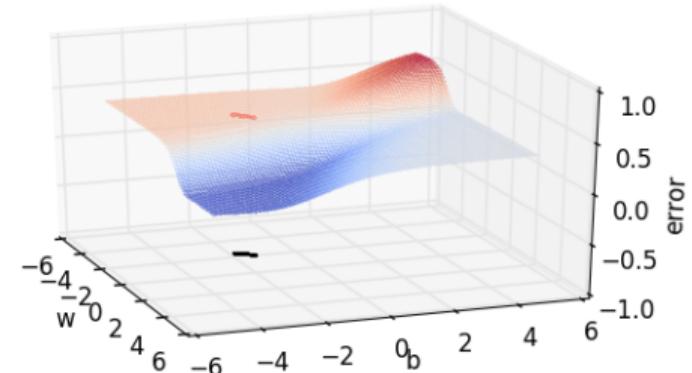
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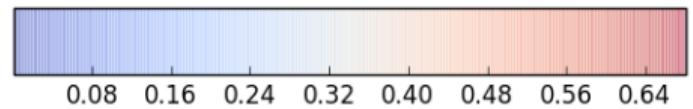
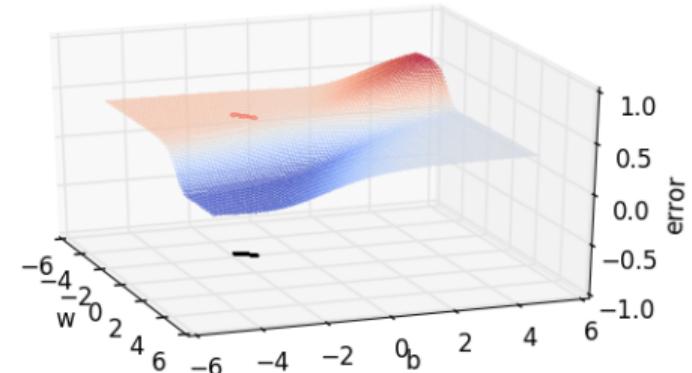
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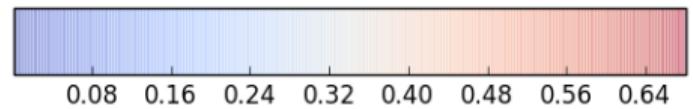
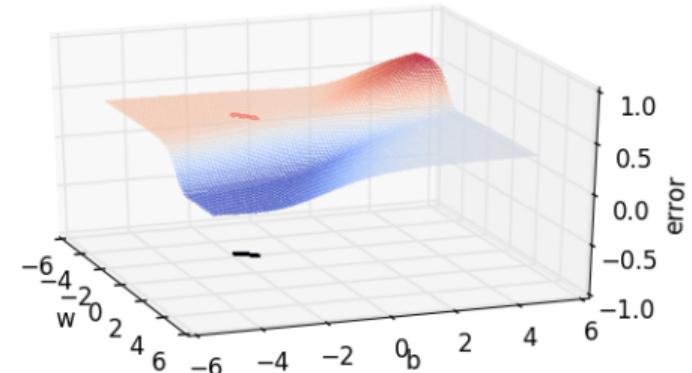
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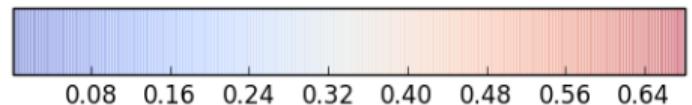
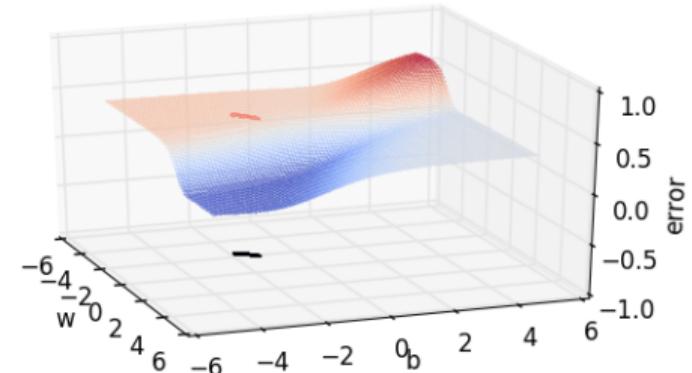
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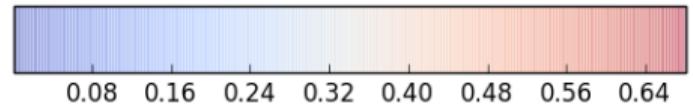
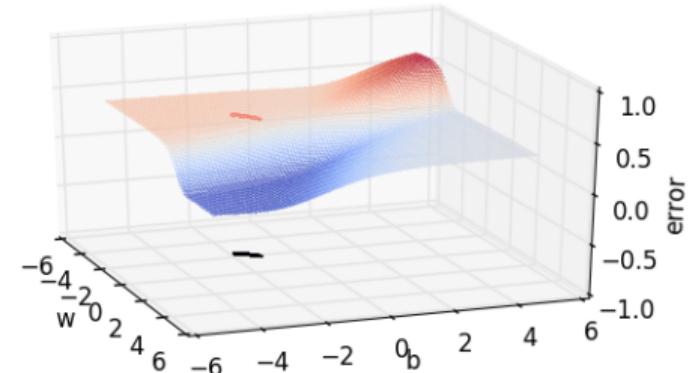
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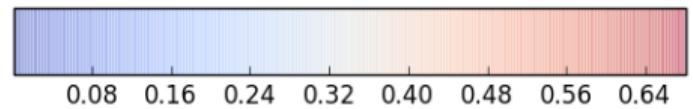
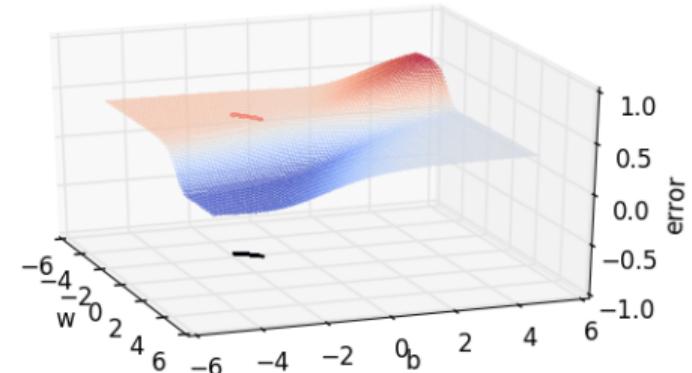
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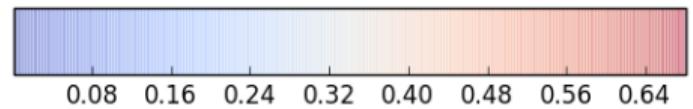
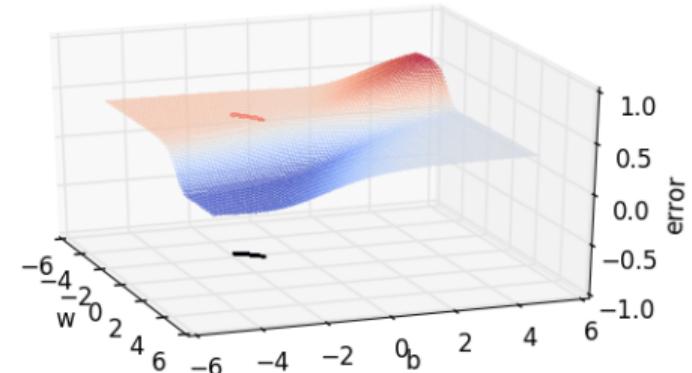
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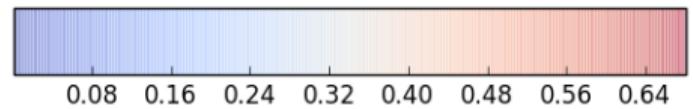
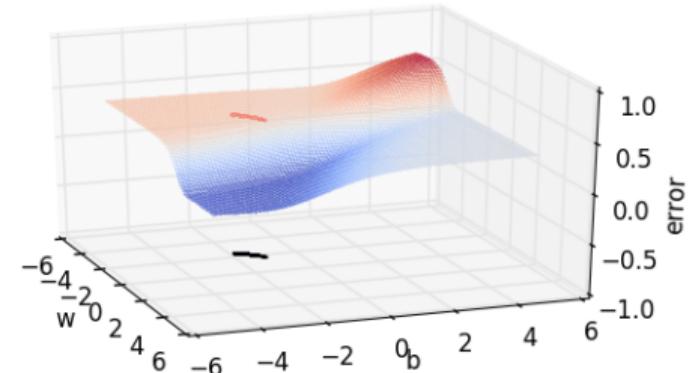
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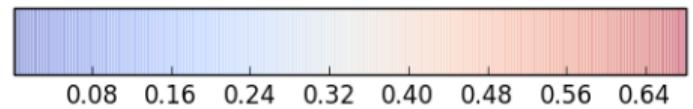
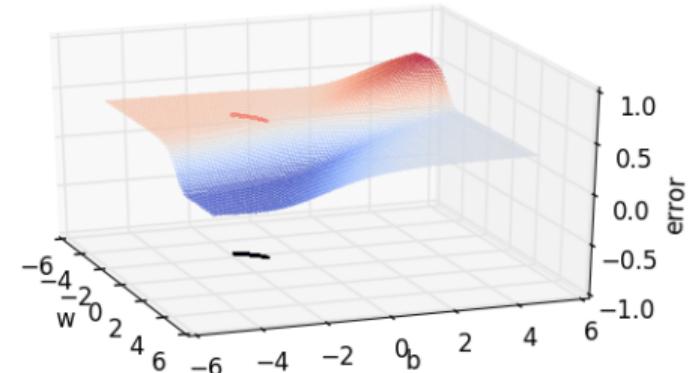
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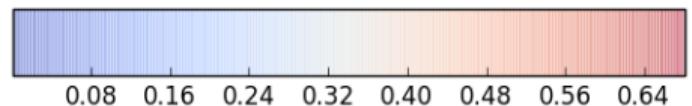
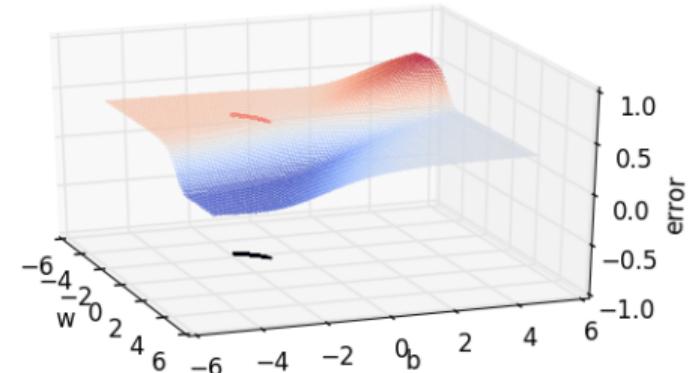
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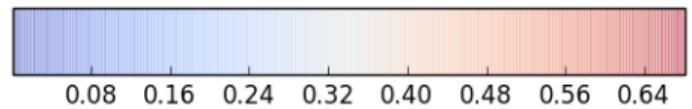
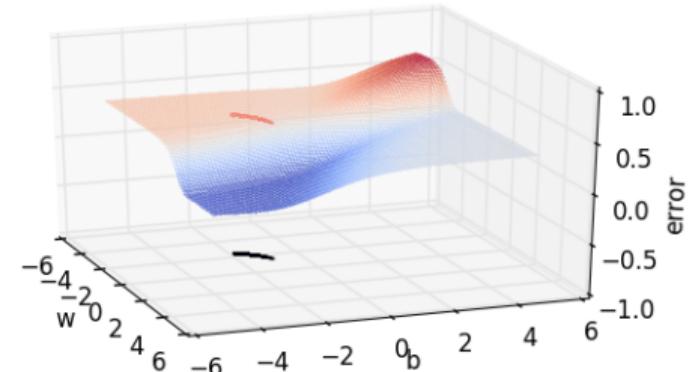
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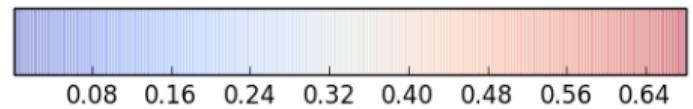
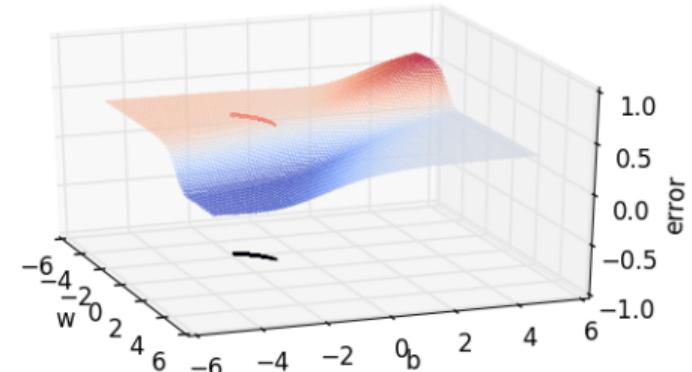
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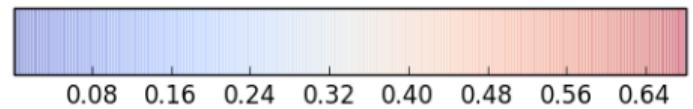
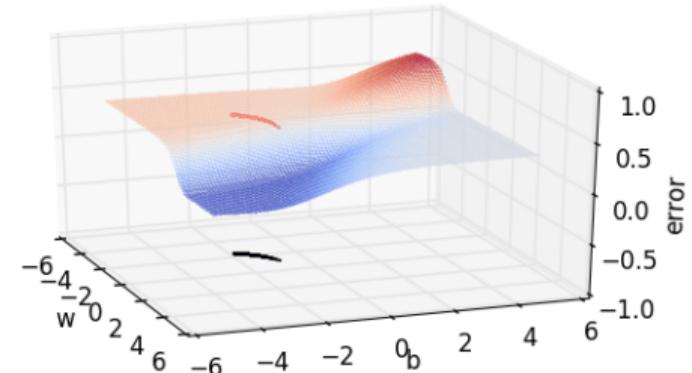
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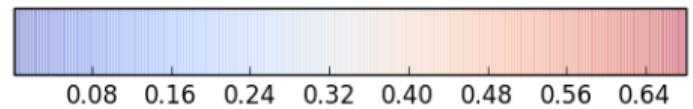
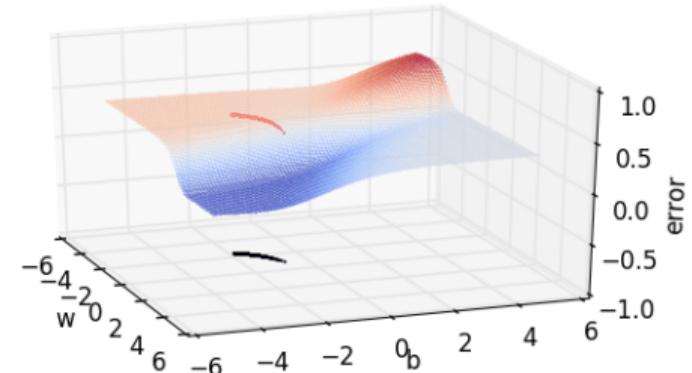
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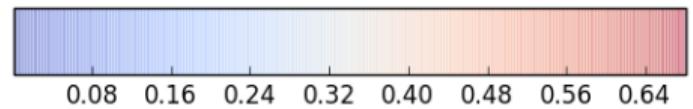
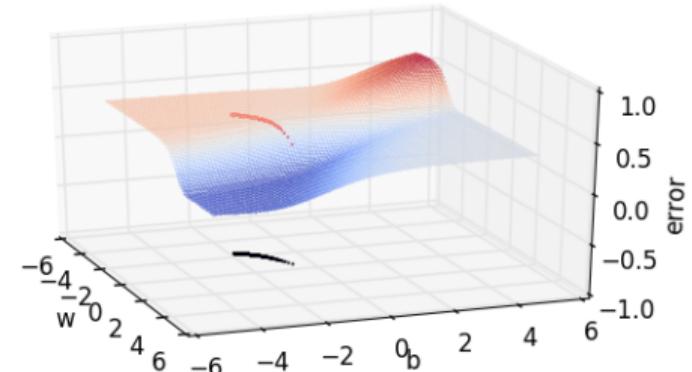
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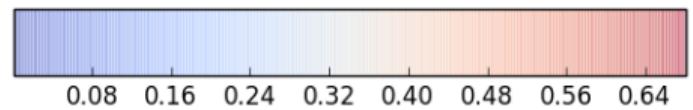
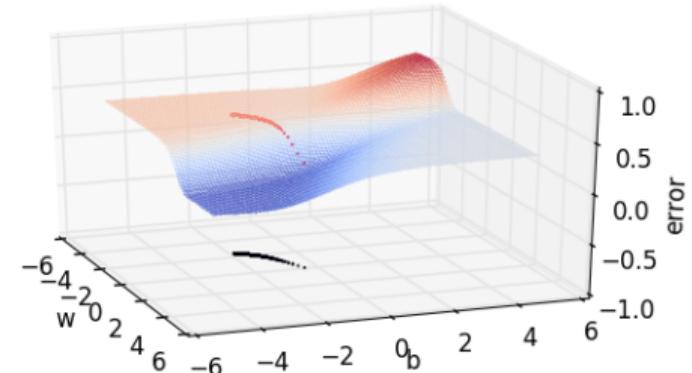
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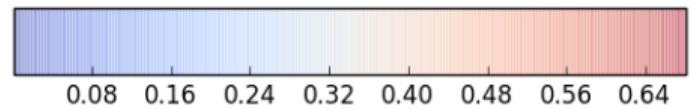
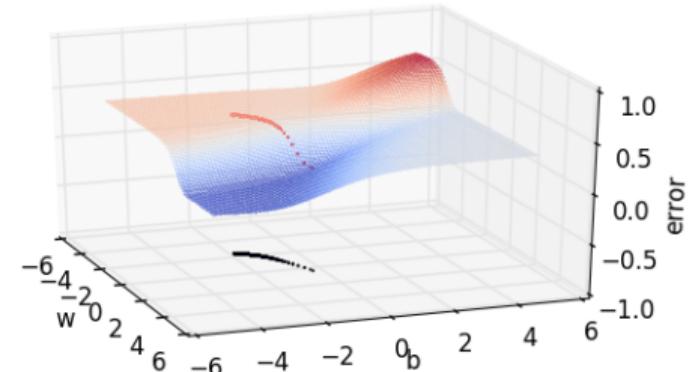
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Gradient descent on the error surface



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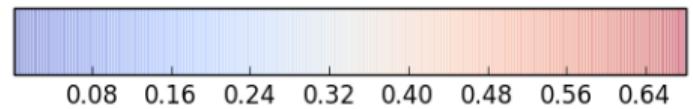
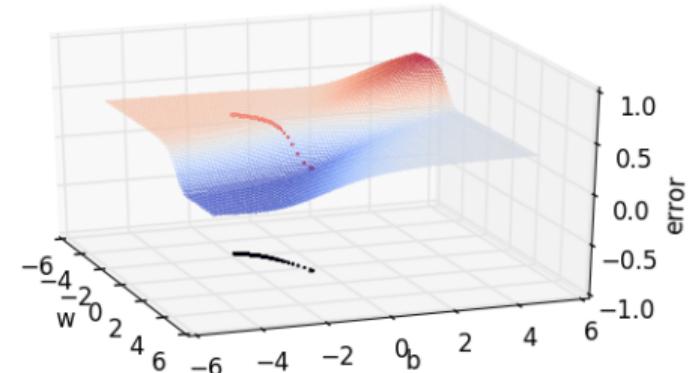
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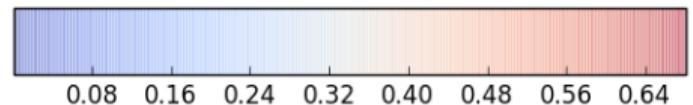
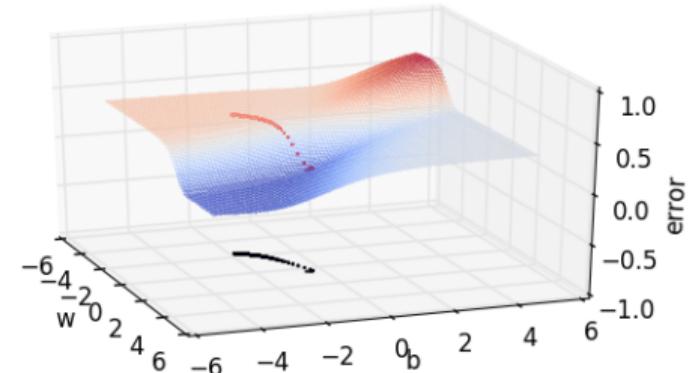
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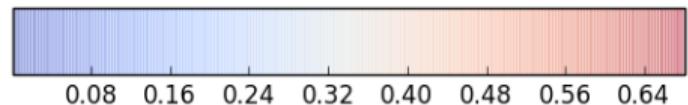
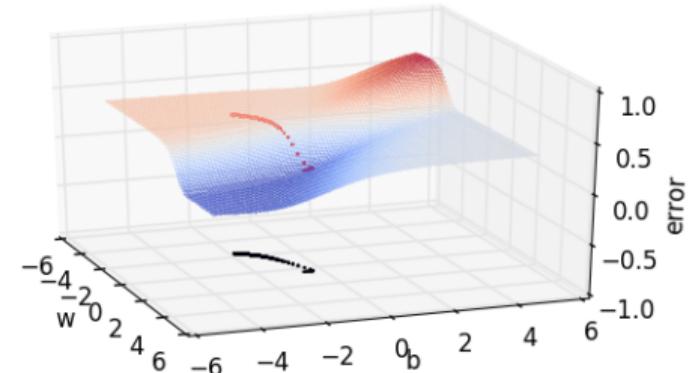
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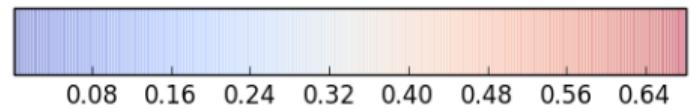
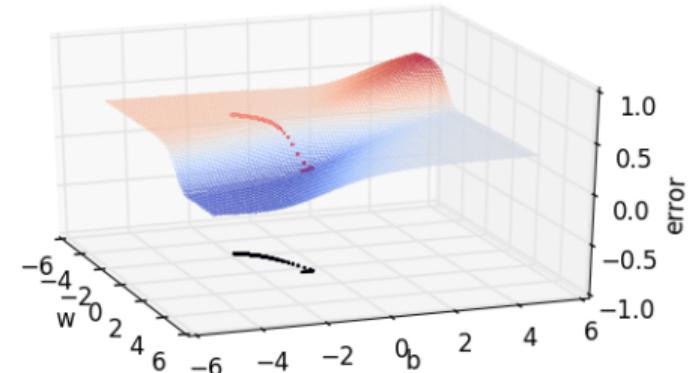
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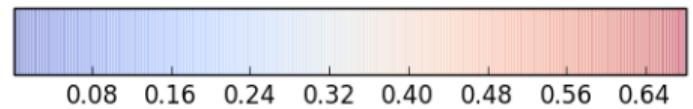
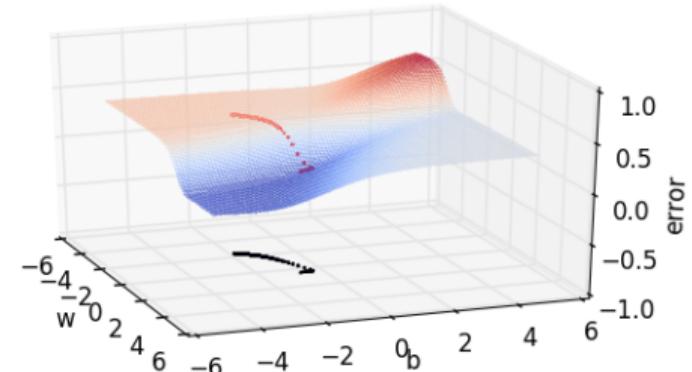
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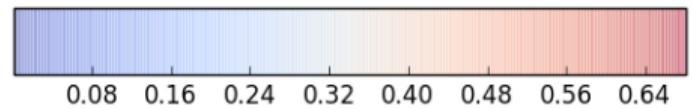
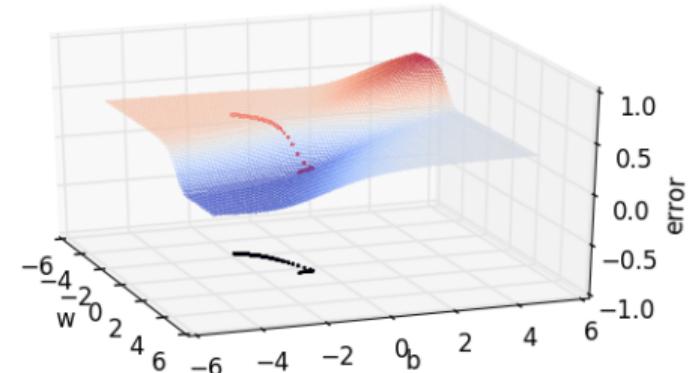
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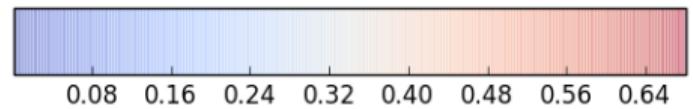
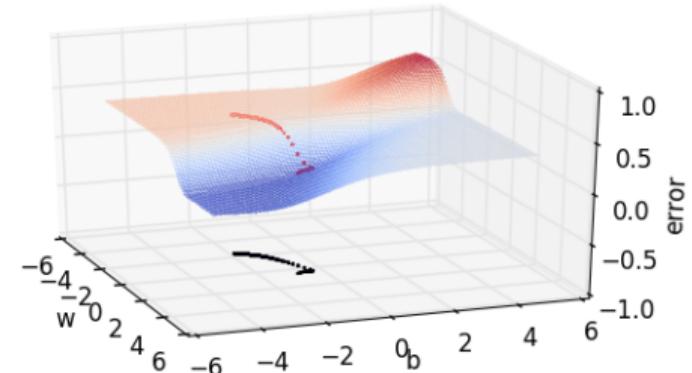
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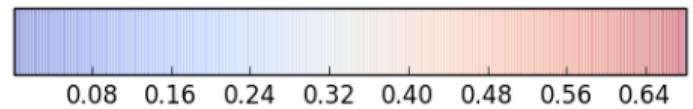
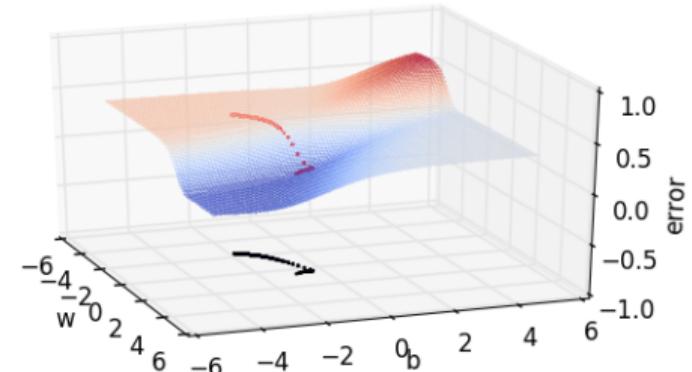
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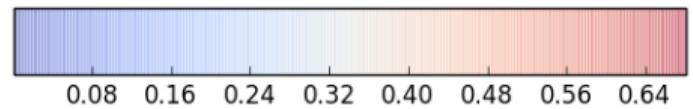
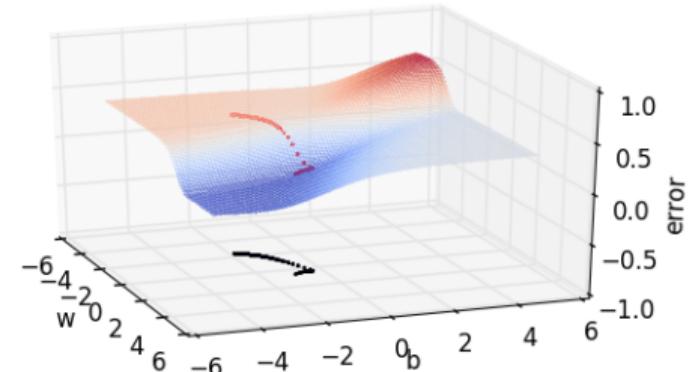
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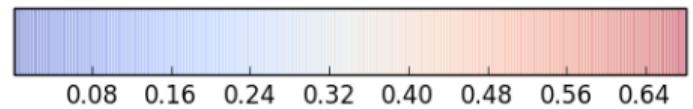
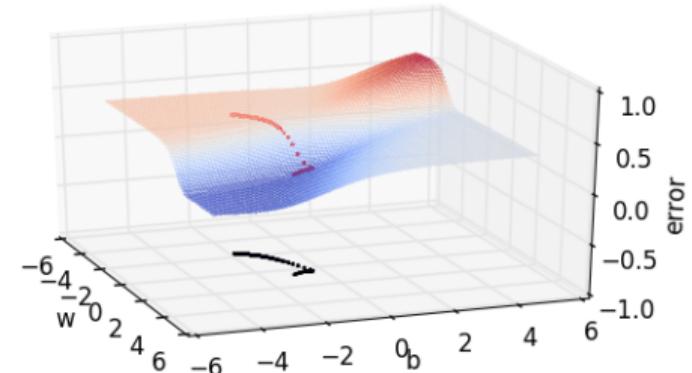
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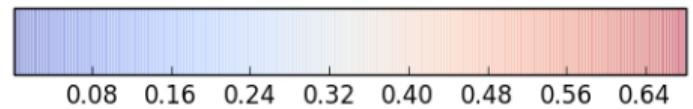
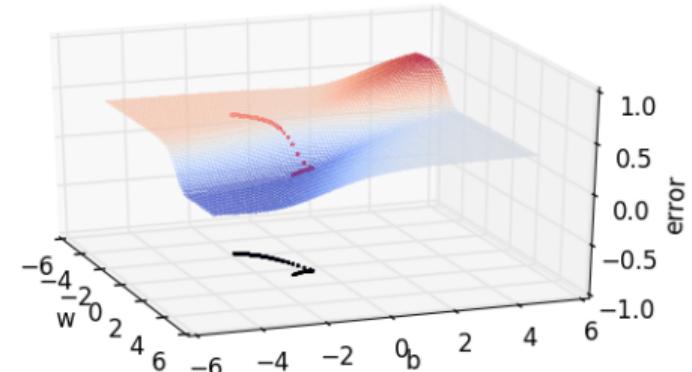
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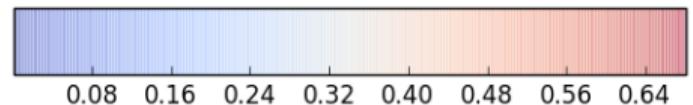
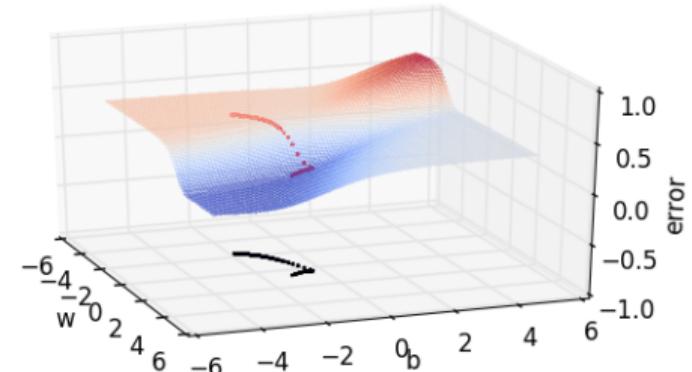
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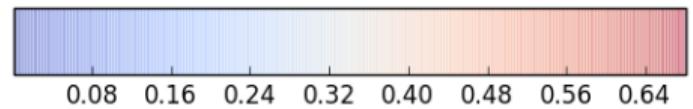
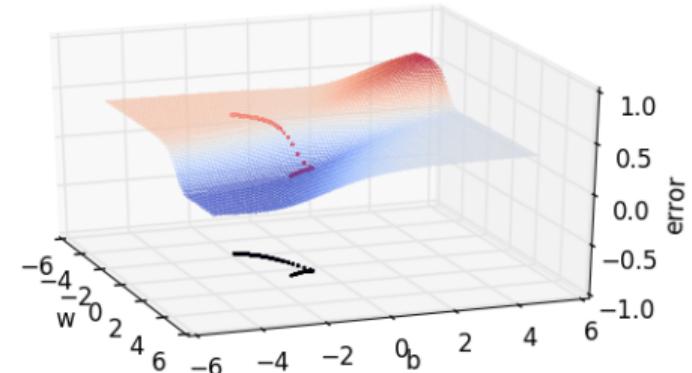
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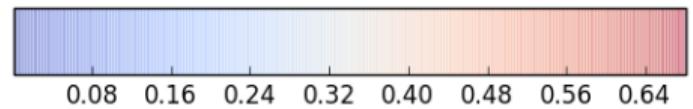
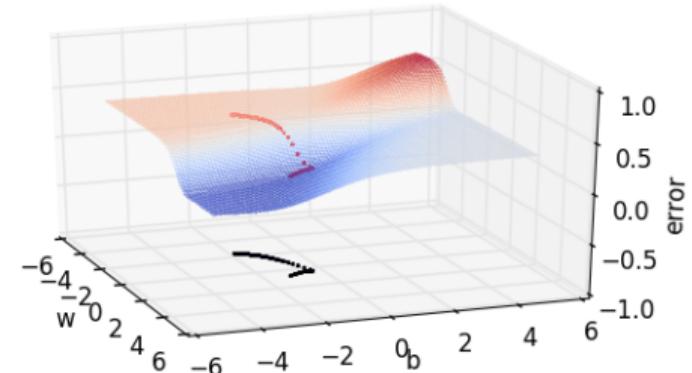
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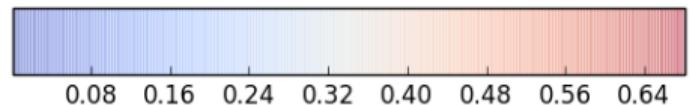
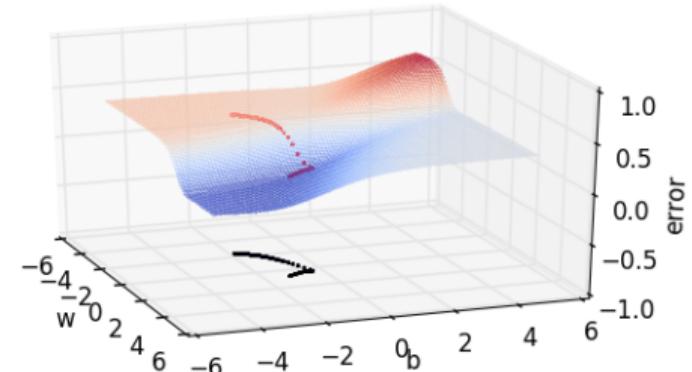
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Gradient descent on the error surface



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Y = [0.2, 0.9]

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    return 1.0 / (1.0 + np.exp(-(w*x + b)))

def error (w, b) :
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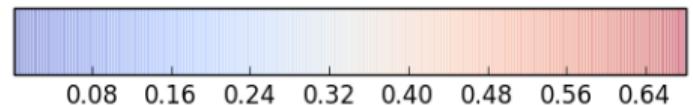
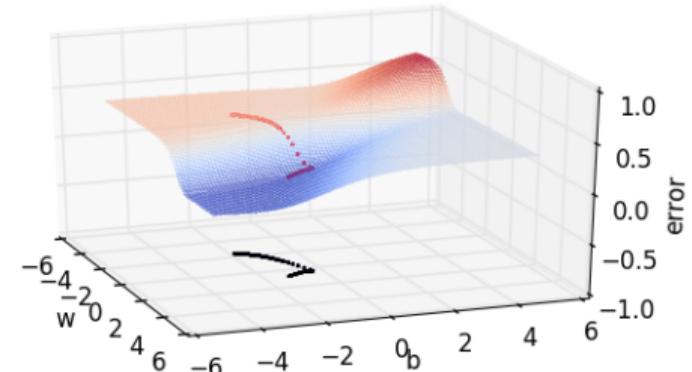
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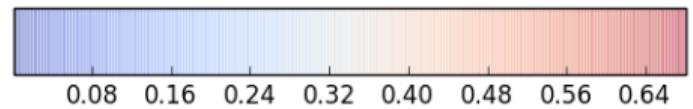
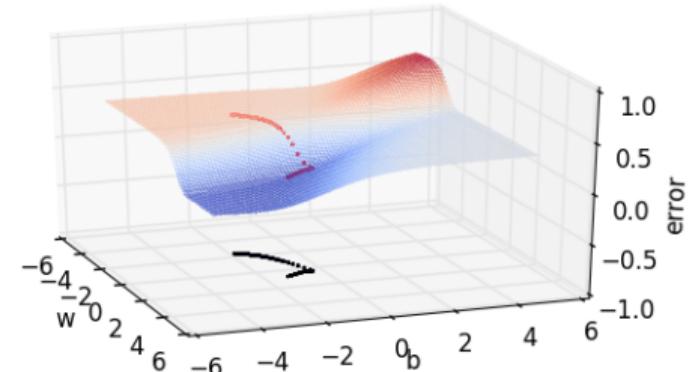
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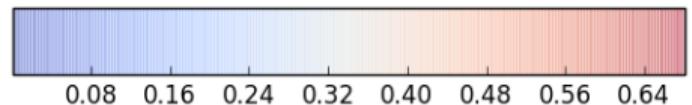
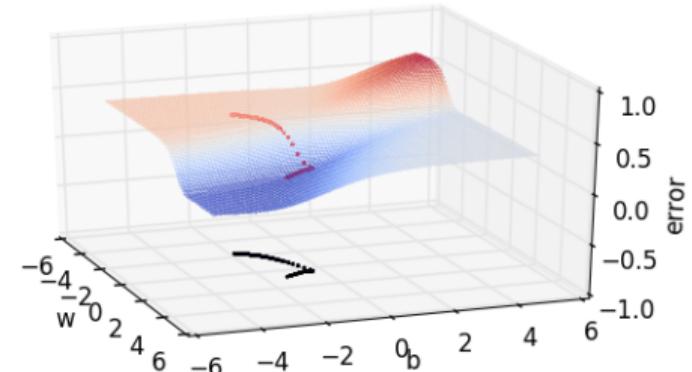
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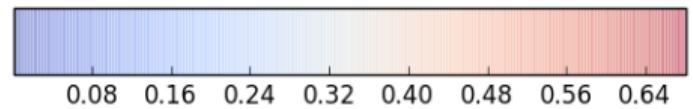
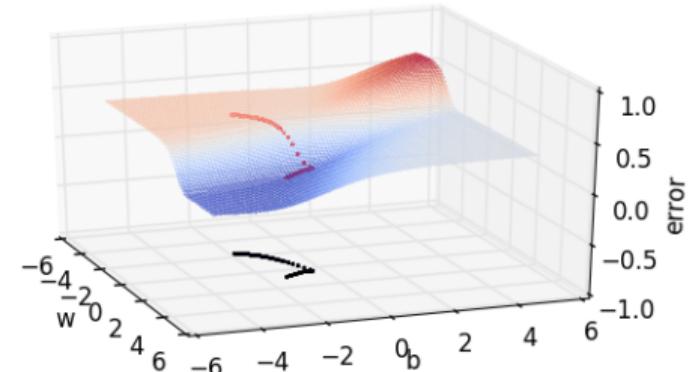
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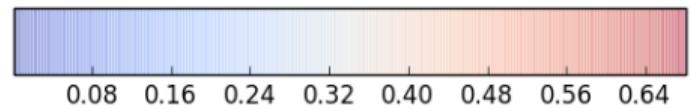
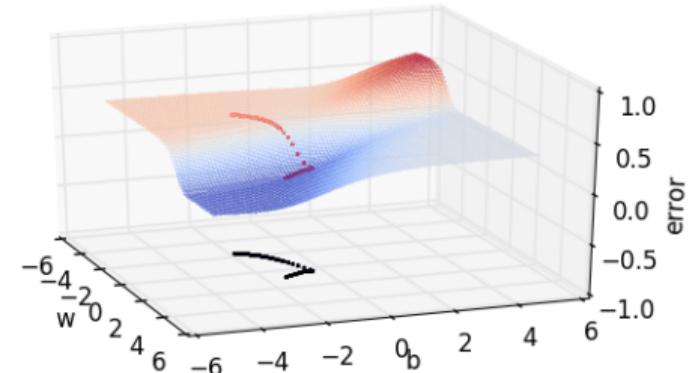
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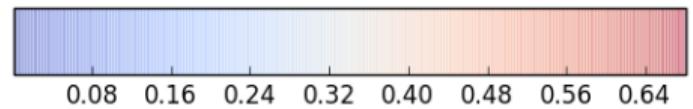
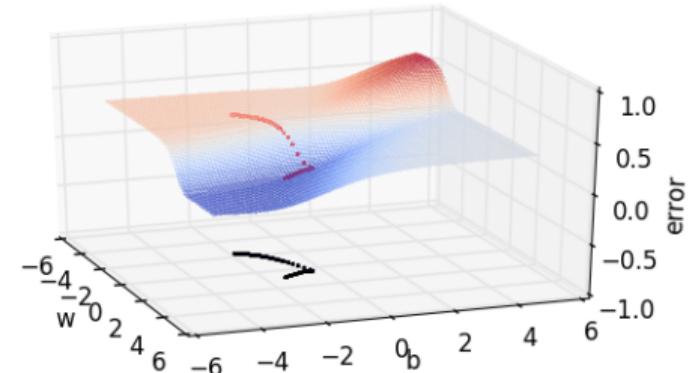
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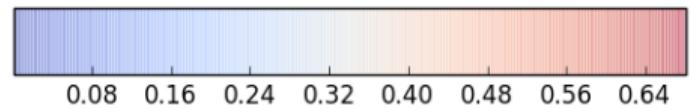
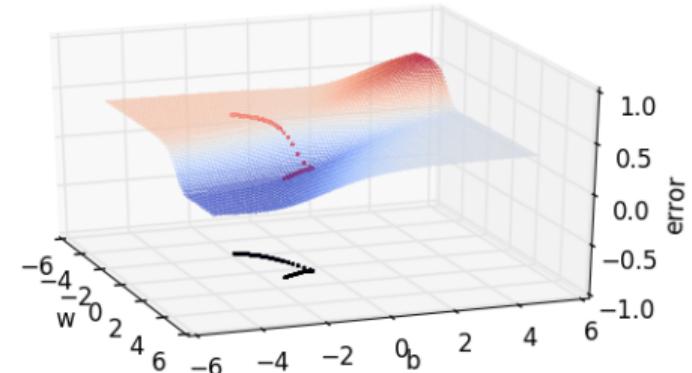
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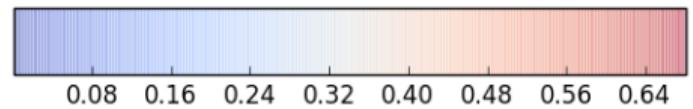
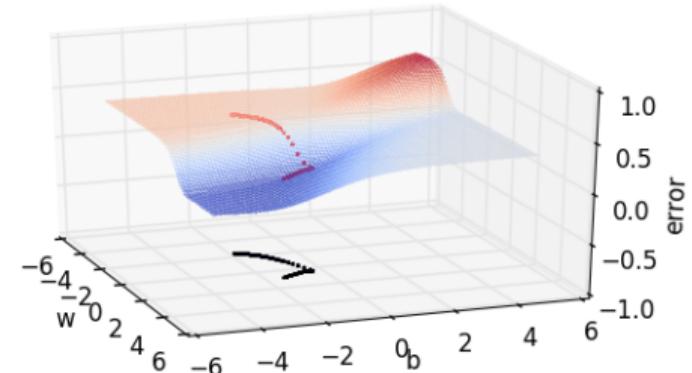
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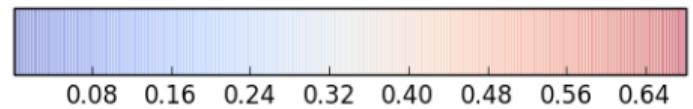
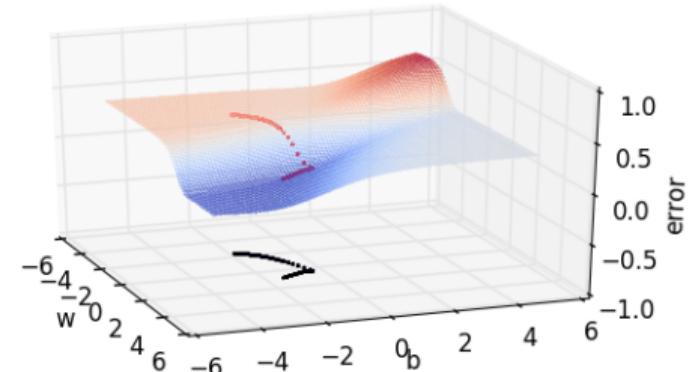
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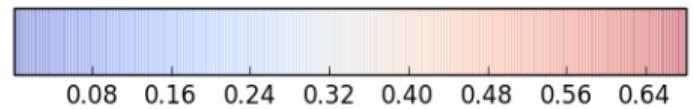
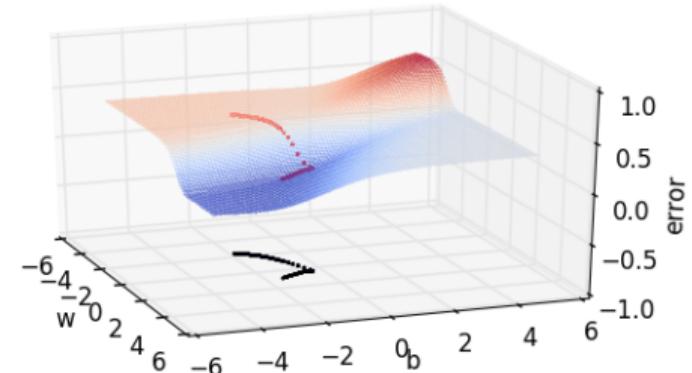
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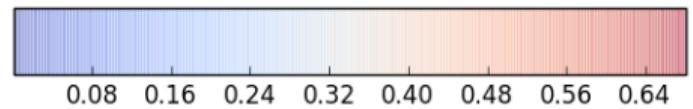
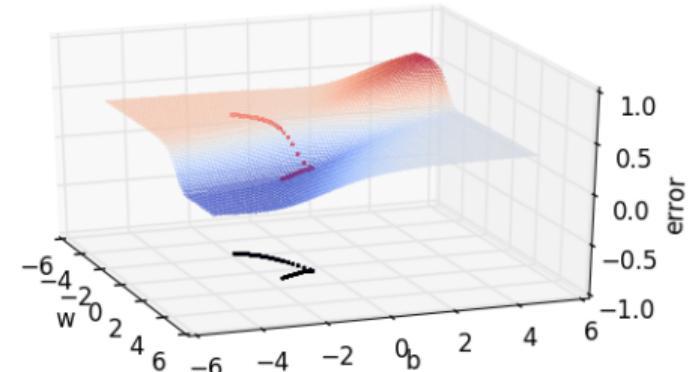
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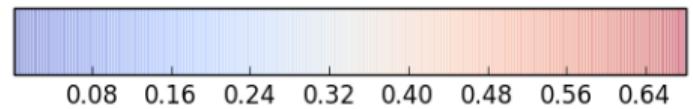
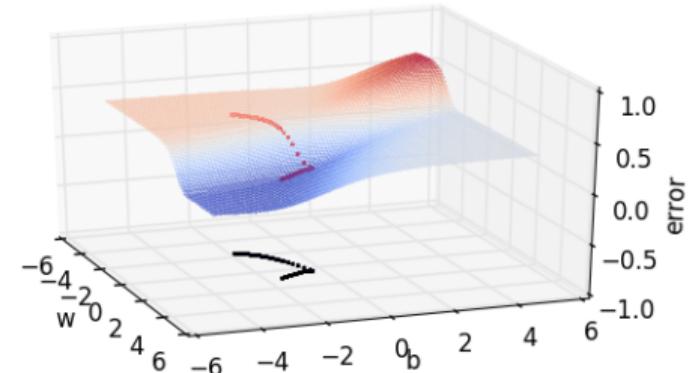
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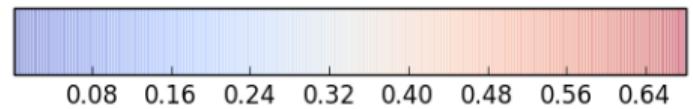
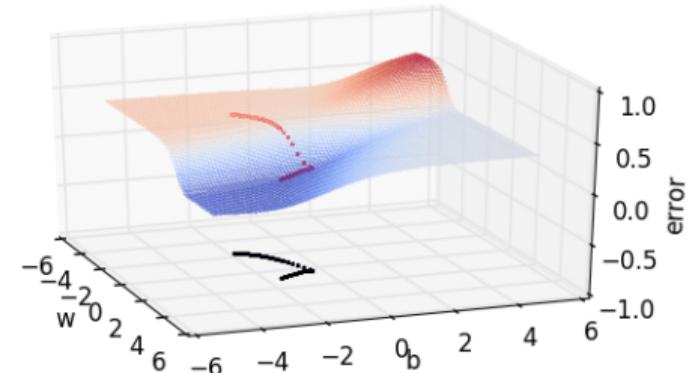
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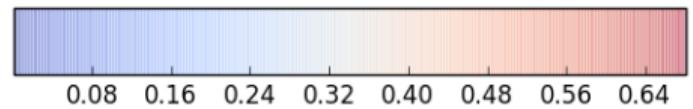
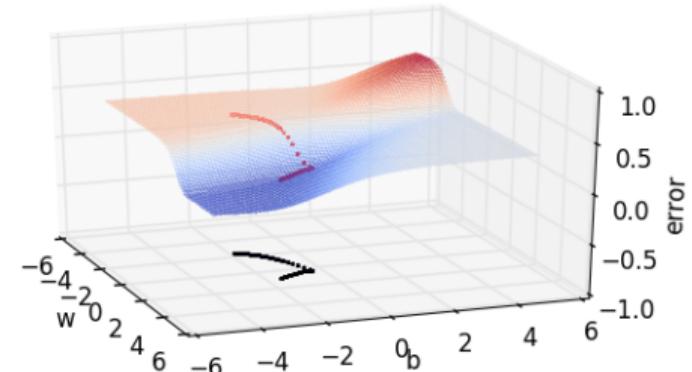
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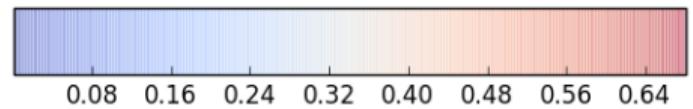
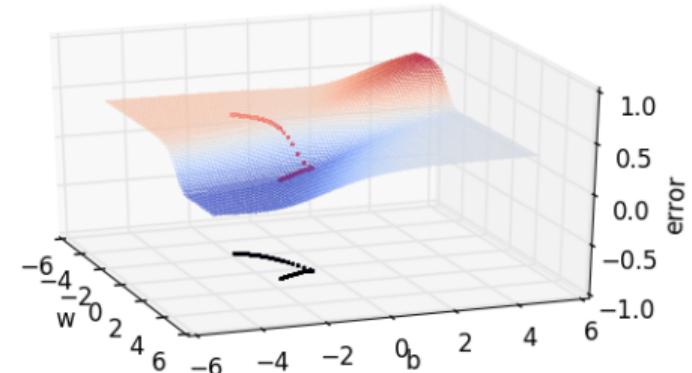
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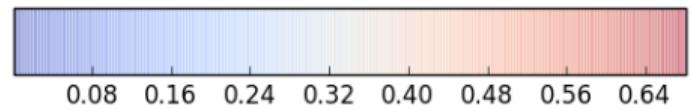
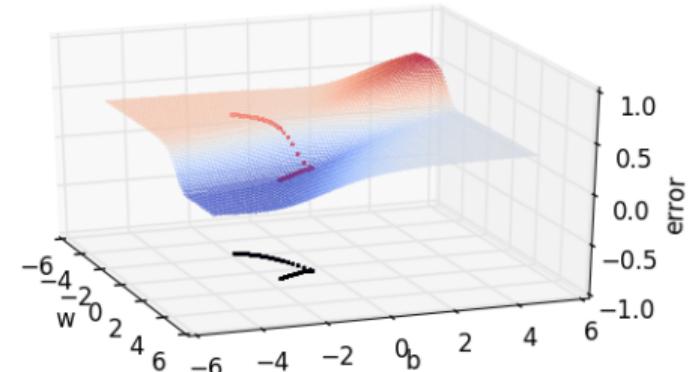
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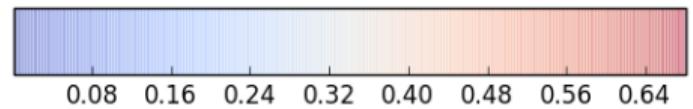
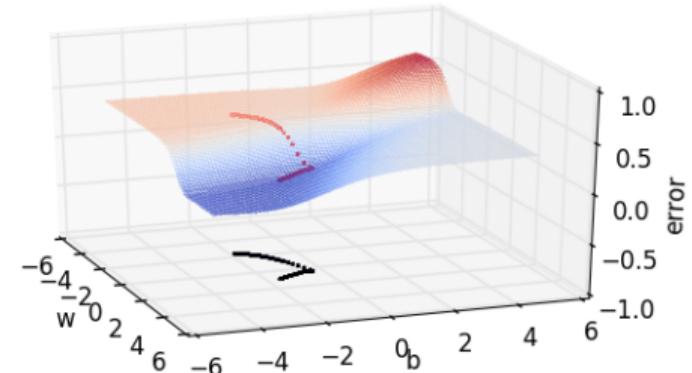
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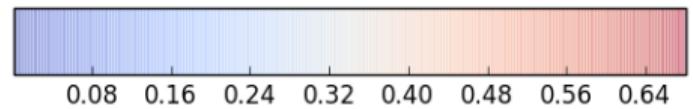
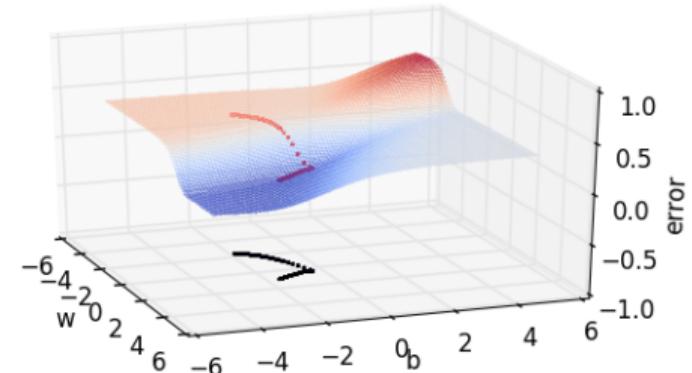
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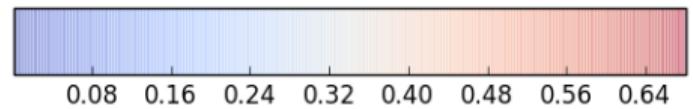
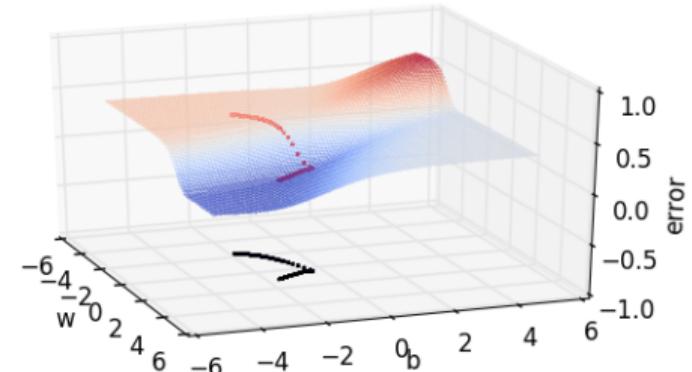
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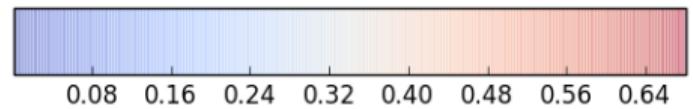
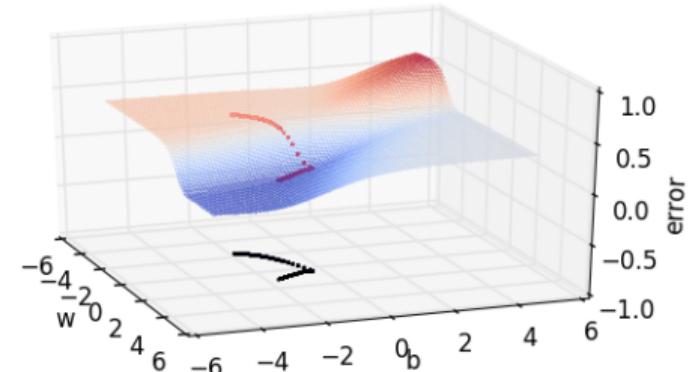
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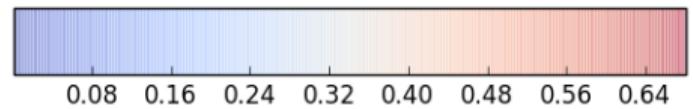
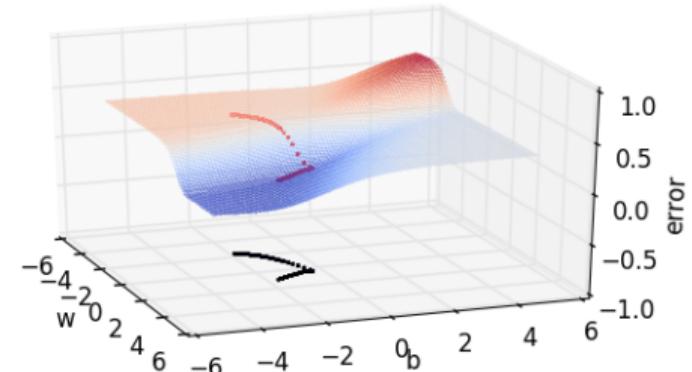
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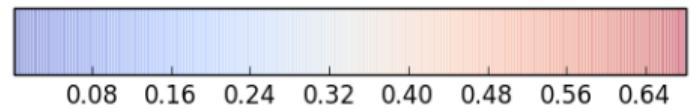
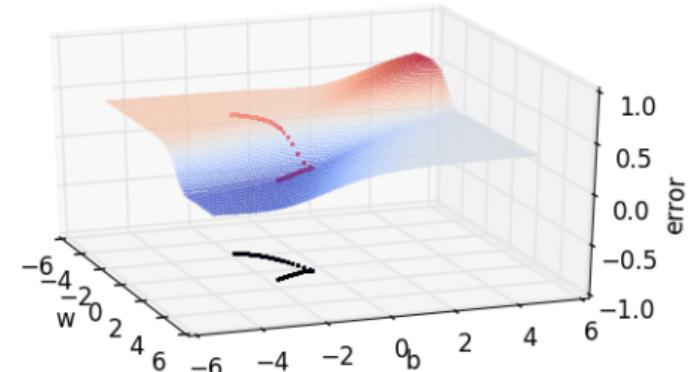
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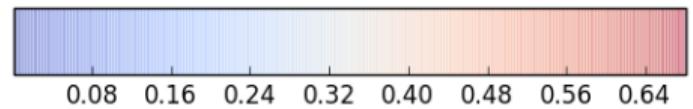
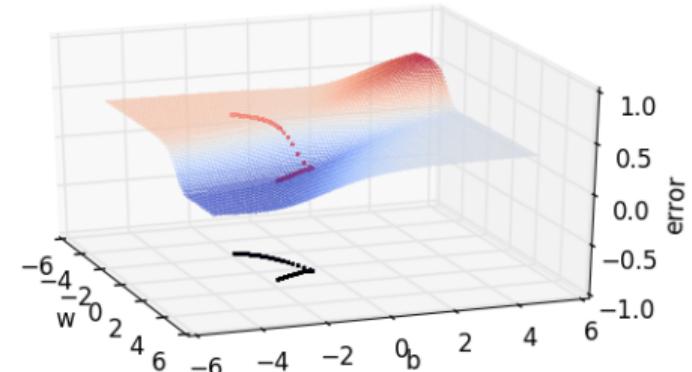
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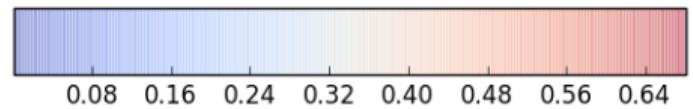
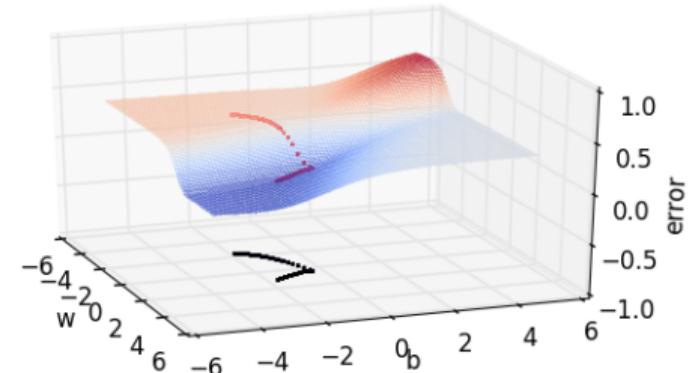
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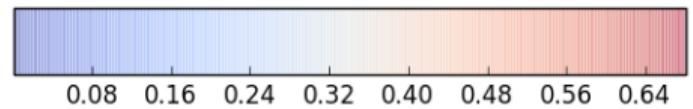
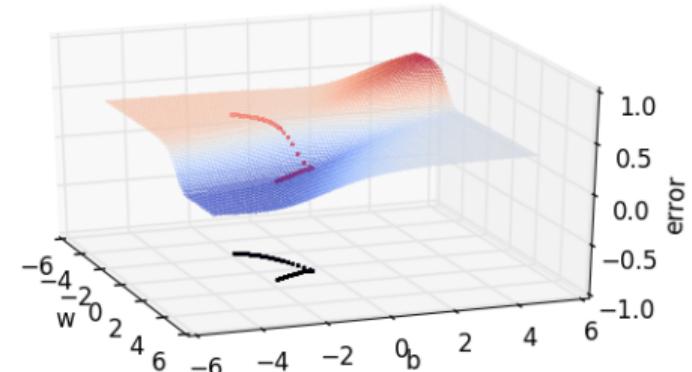
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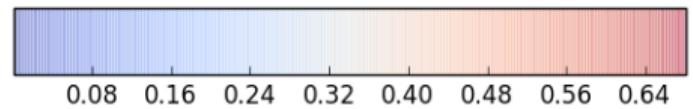
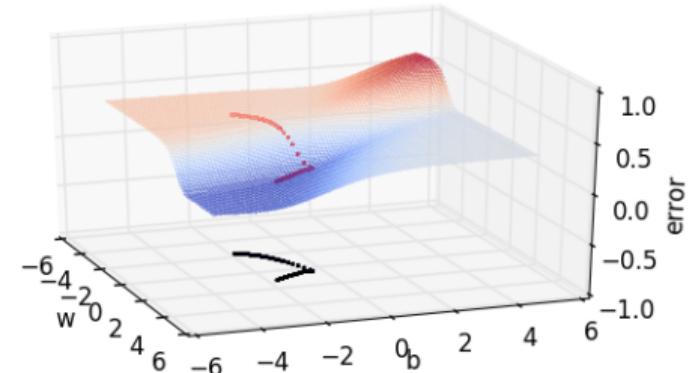
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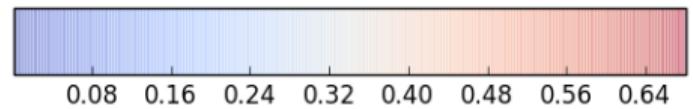
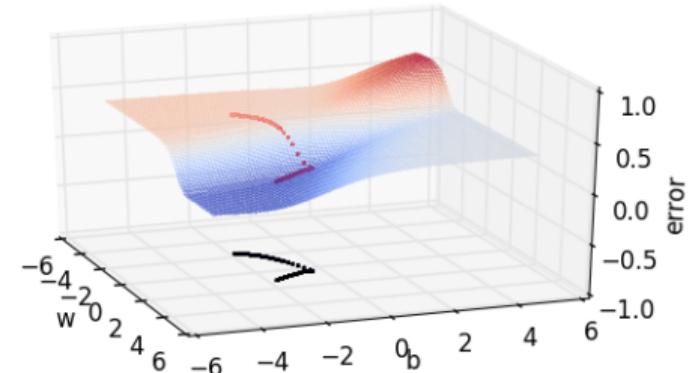
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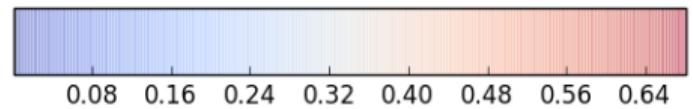
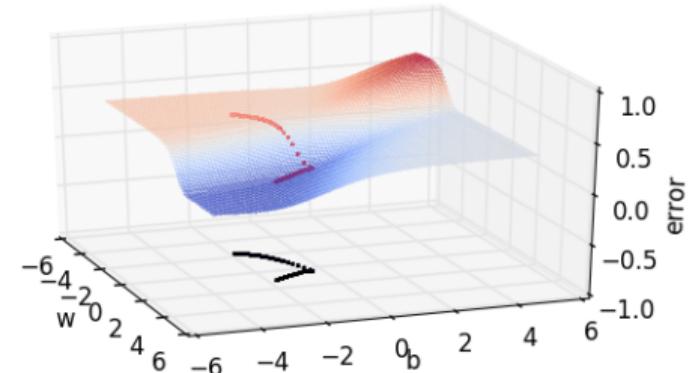
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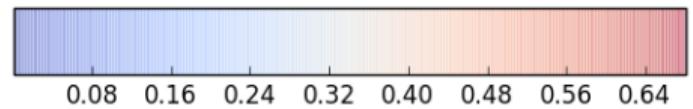
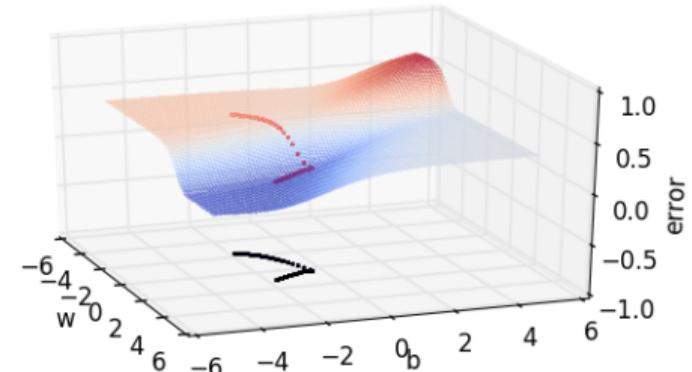
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Gradient descent on the error surface



```

X = [0.5, 2.5]
Y = [0.2, 0.9]

def f(w,b,x) : #sigmoid with parameters w,b
    return 1.0 / (1.0 + np.exp(-(w*x + b)))

def error (w, b) :
    err = 0.0
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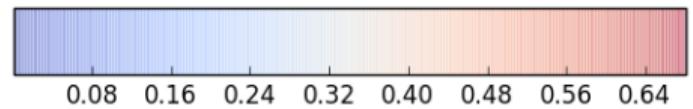
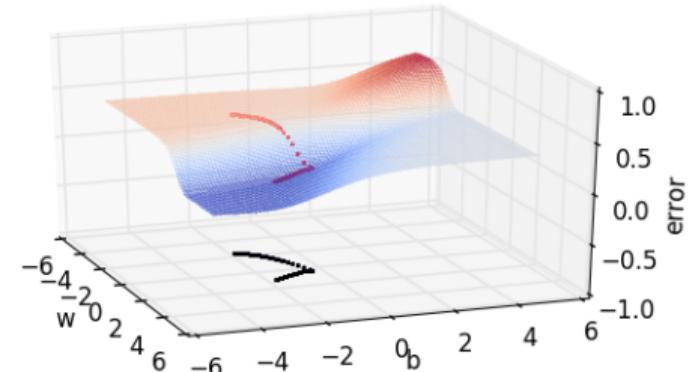
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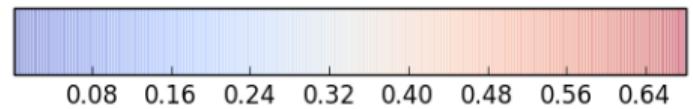
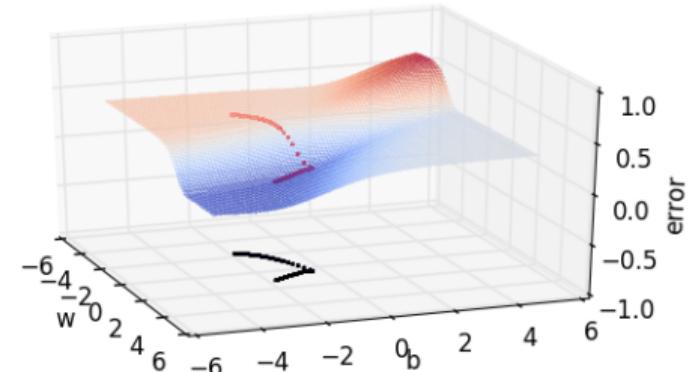
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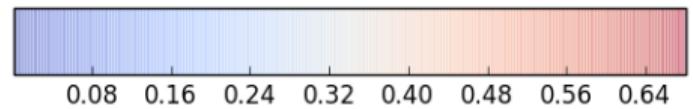
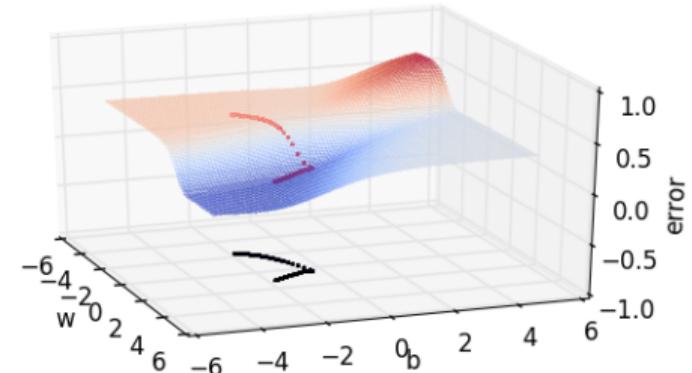
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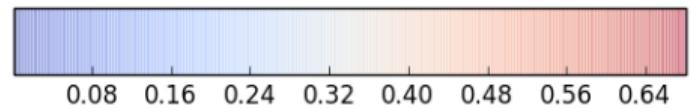
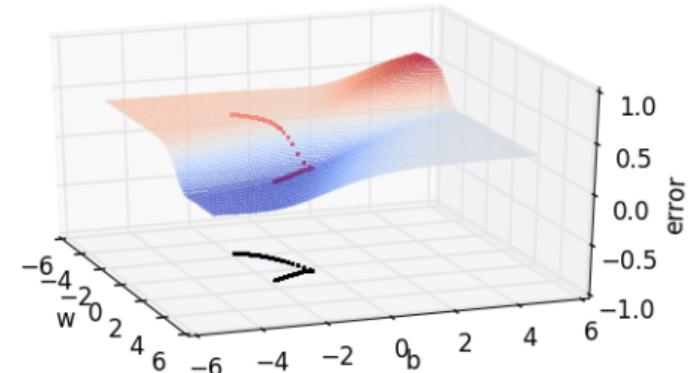
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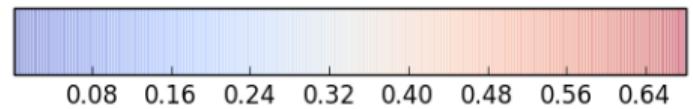
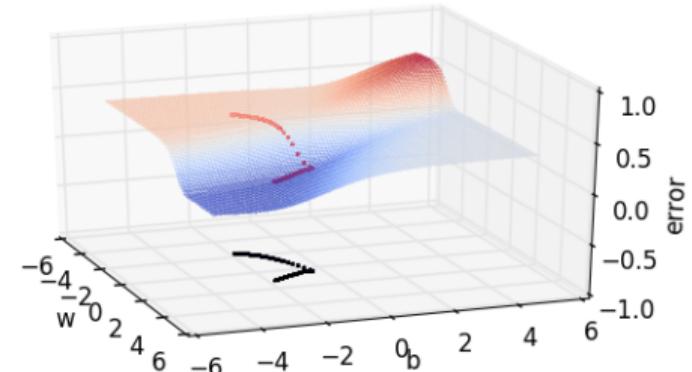
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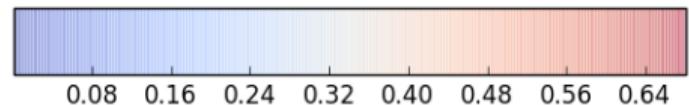
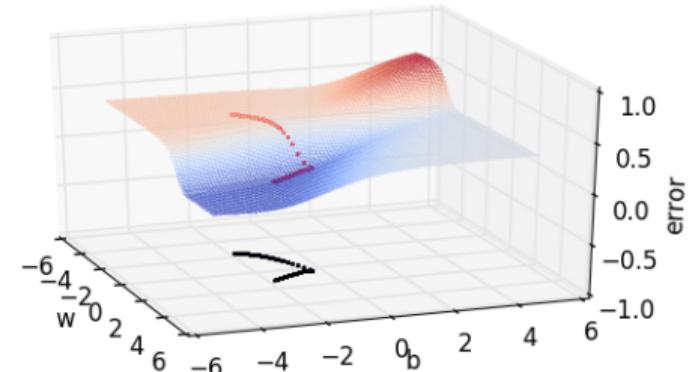
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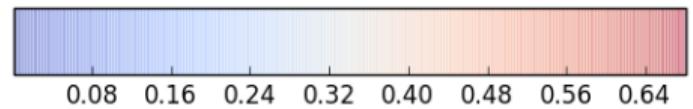
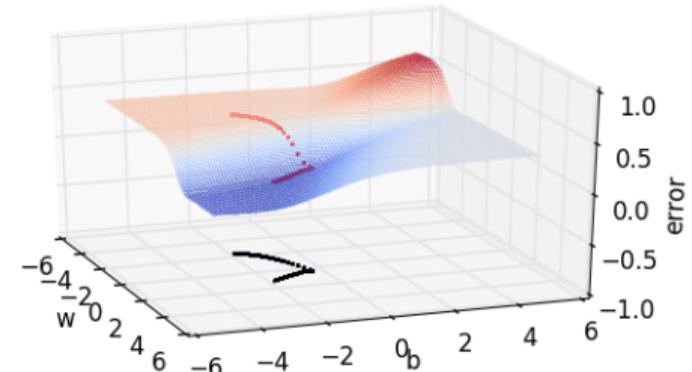
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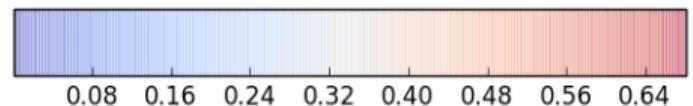
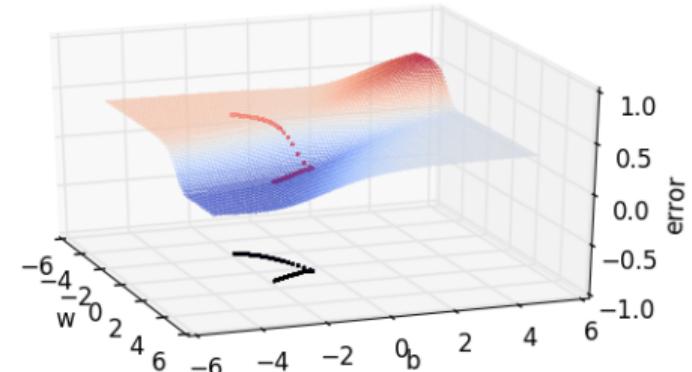
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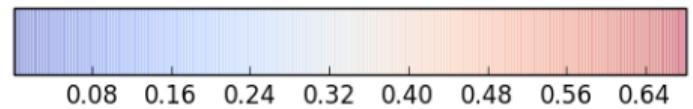
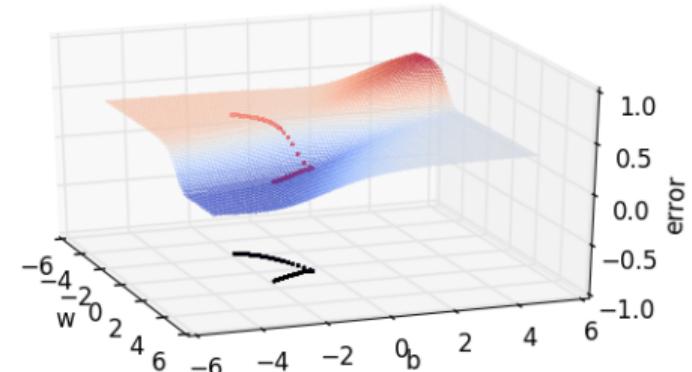
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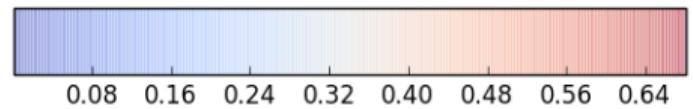
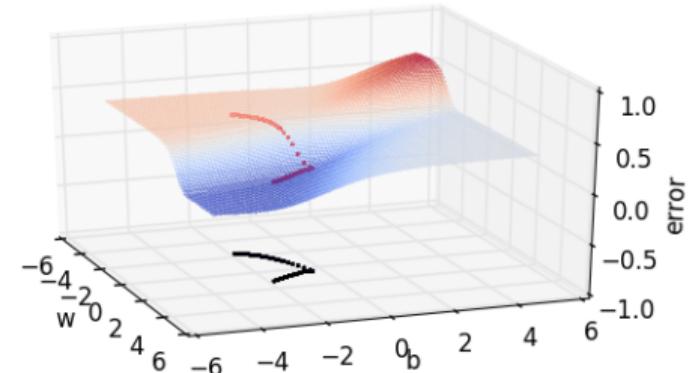
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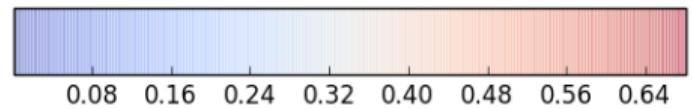
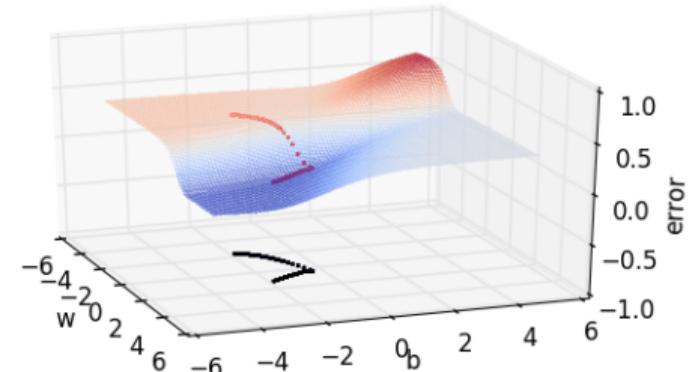
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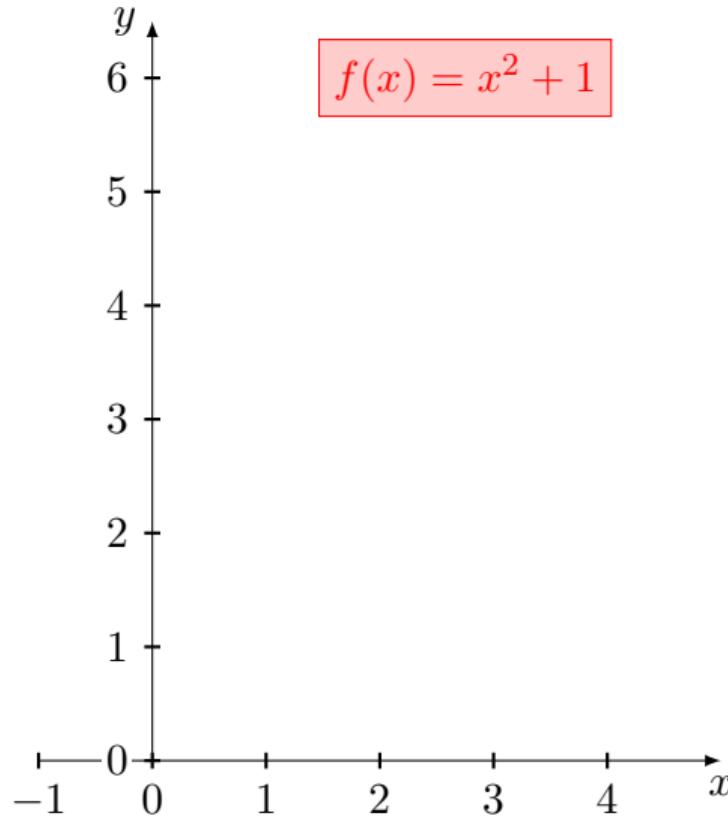
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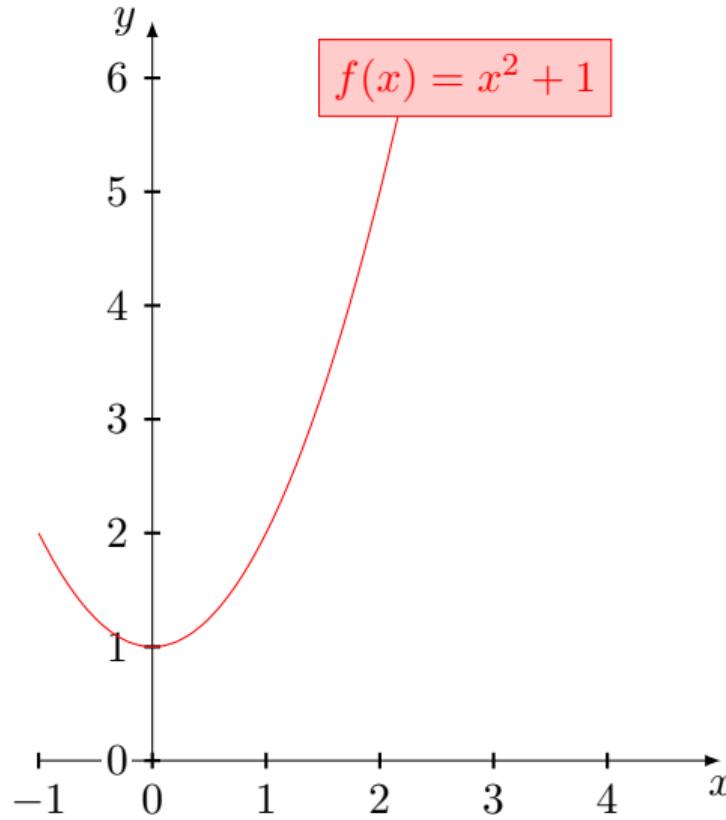
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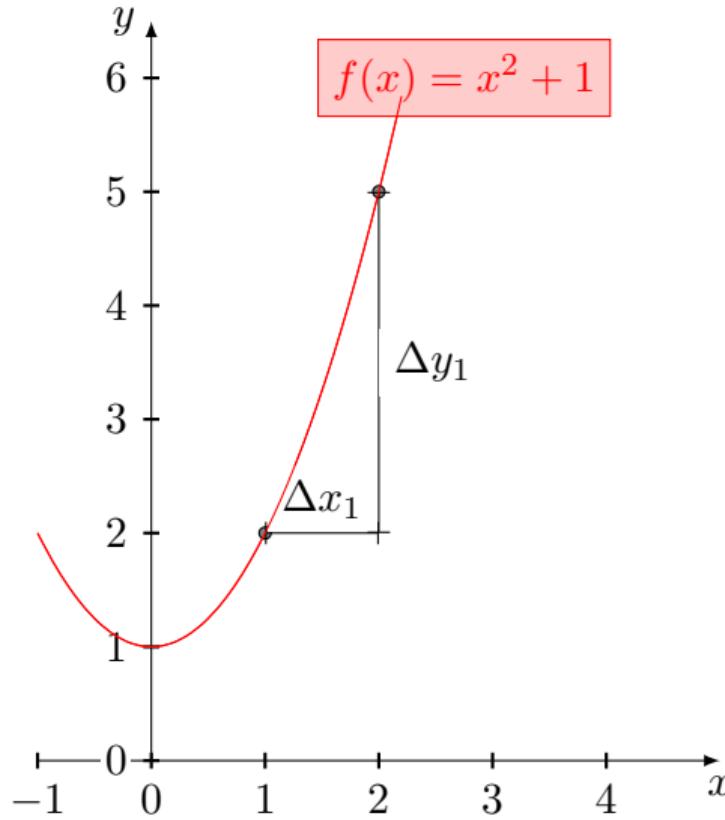
Gradient descent on the error surface



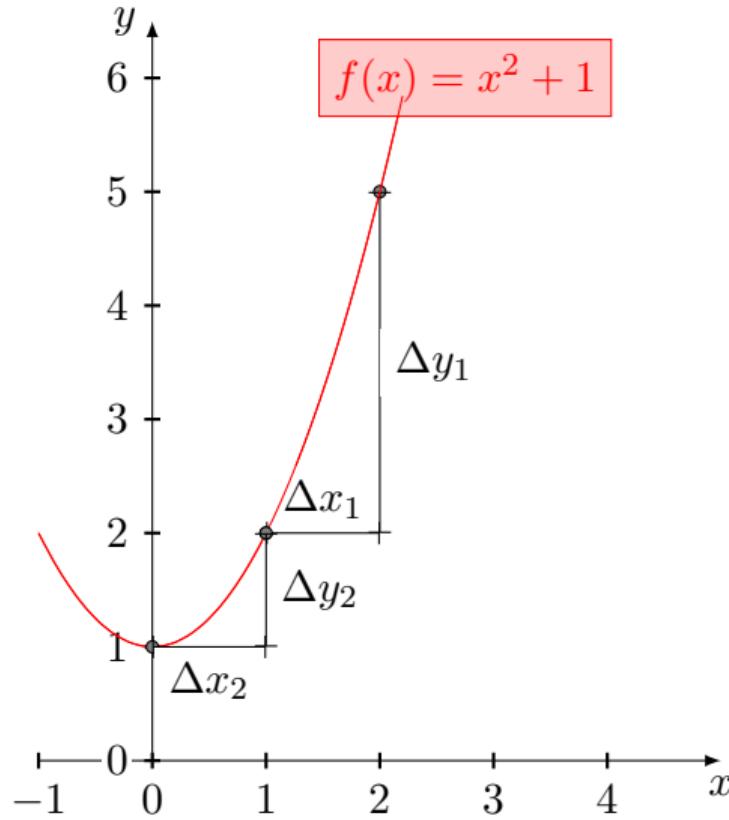




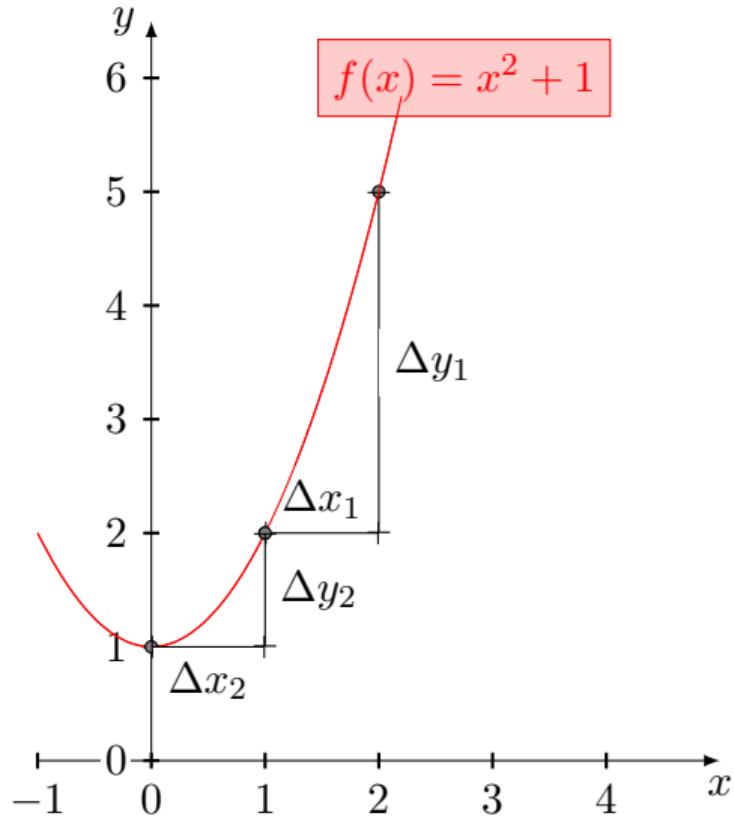
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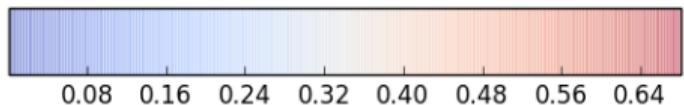
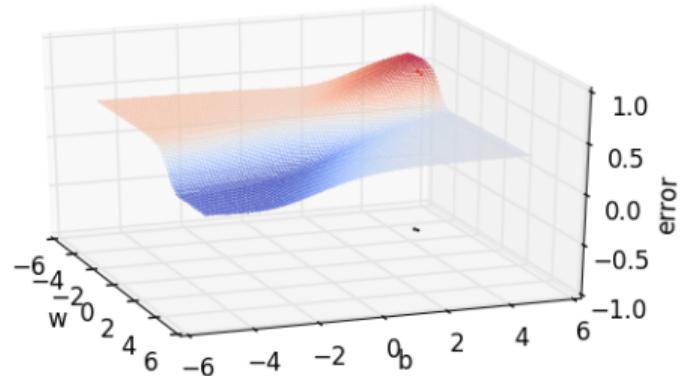


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- Recall that our weight updates are proportional to the gradient $w = w - \eta \nabla w$

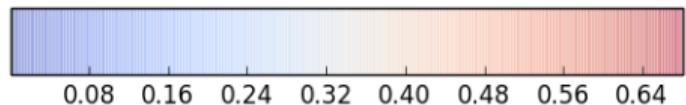
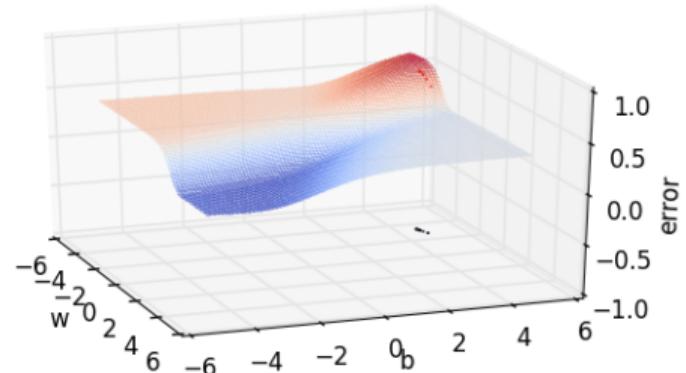


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- Recall that our weight updates are proportional to the gradient $w = w - \eta \nabla w$
- Hence in the areas where the curve is gentle the updates are small whereas in the areas where the curve is steep the updates are large

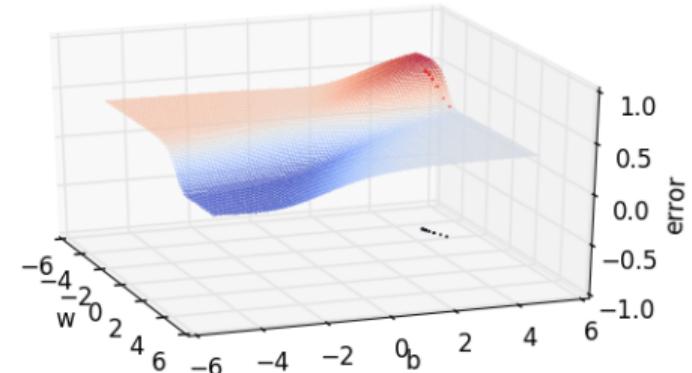
- Let's see what happens when we start from a different point



- Irrespective of where we start from once we hit a surface which has a gentle slope, the progress slows down

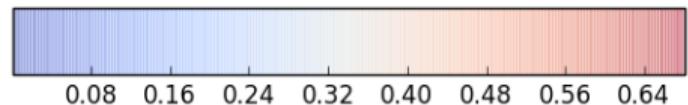
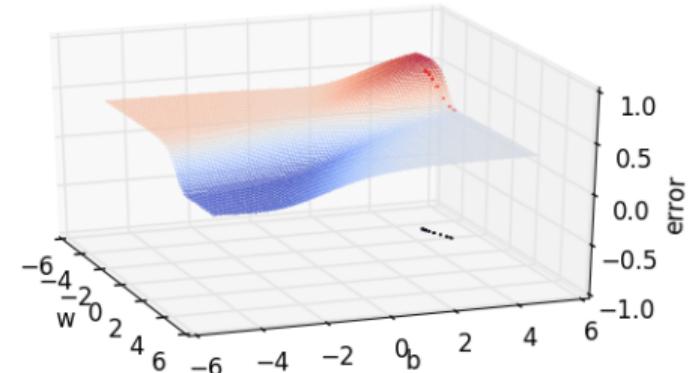


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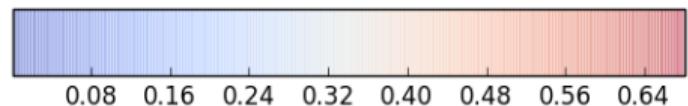
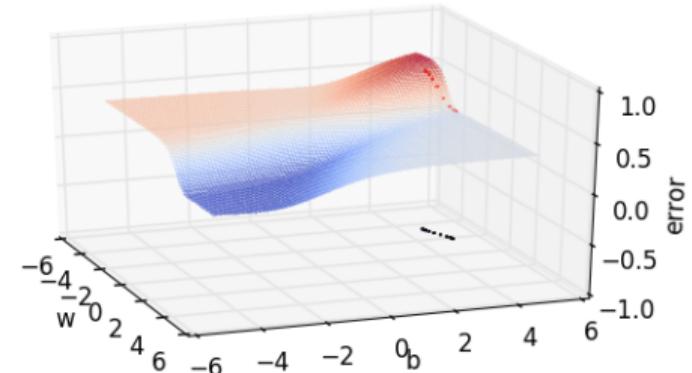


0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64

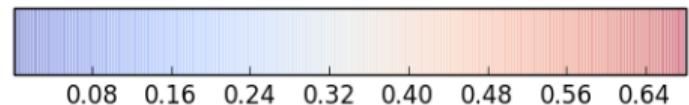
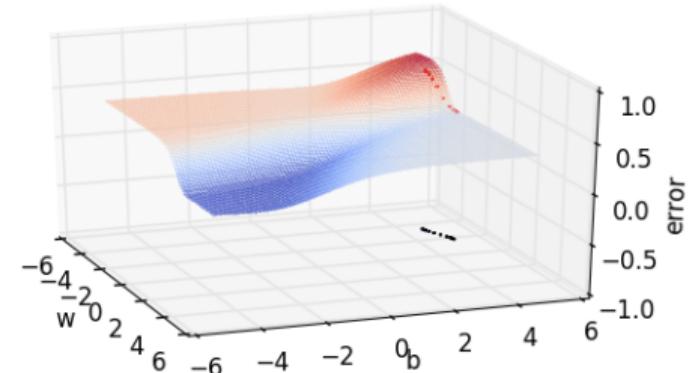
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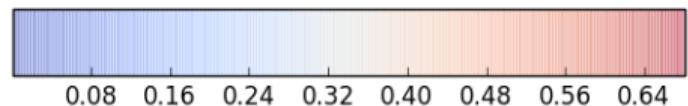
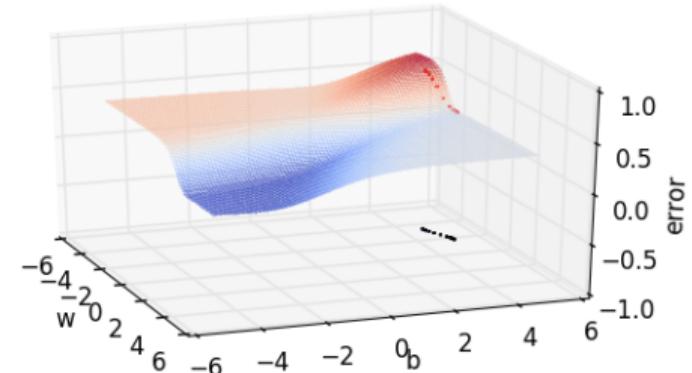
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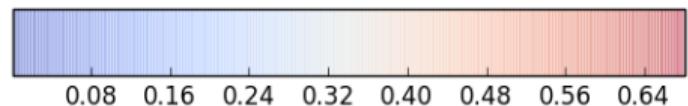
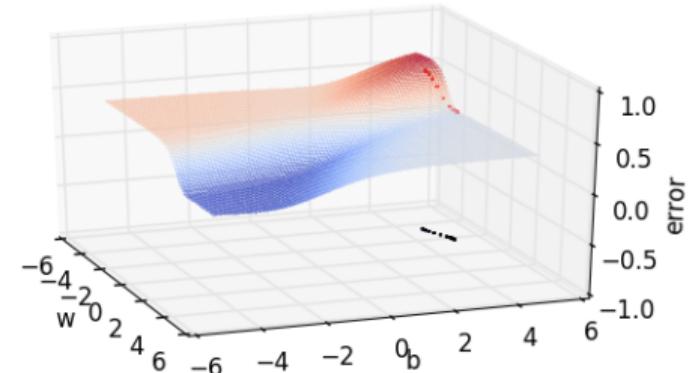
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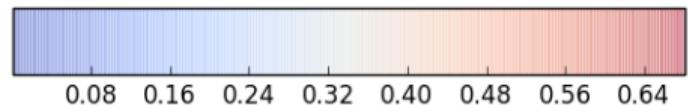
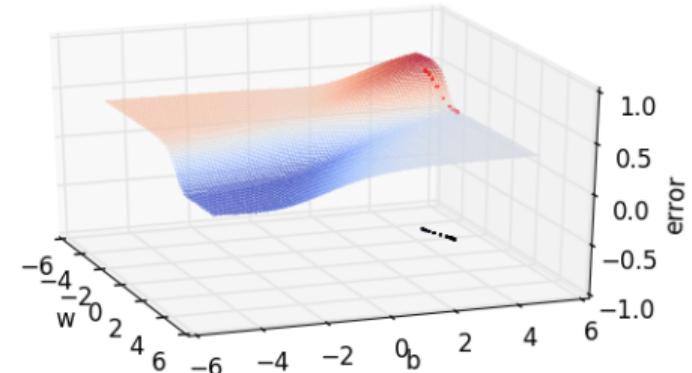
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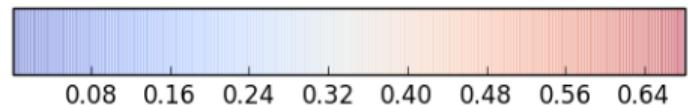
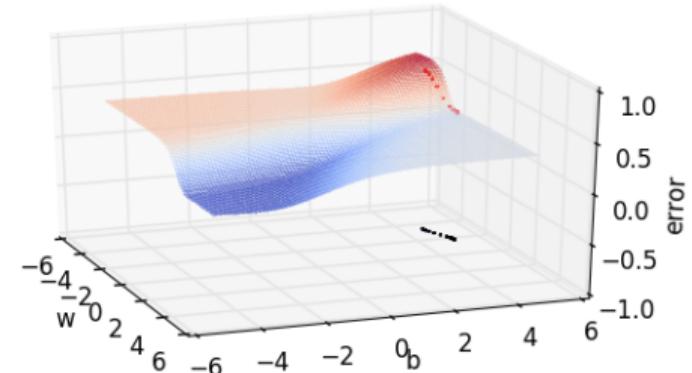
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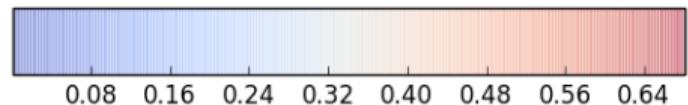
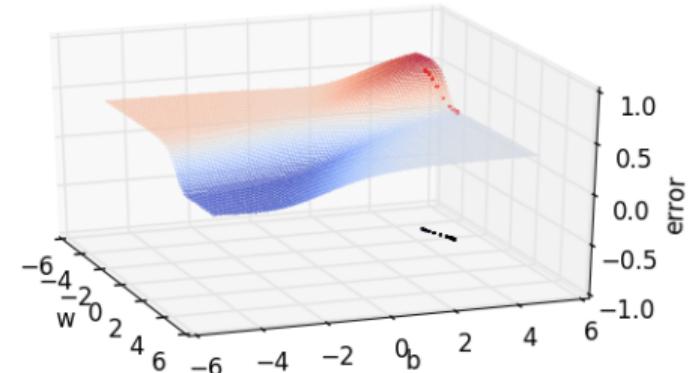
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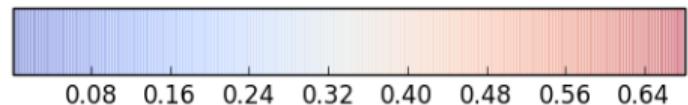
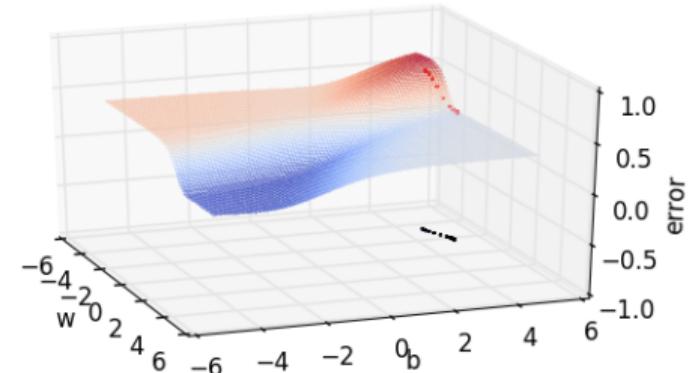
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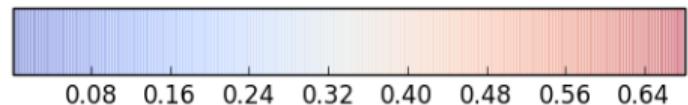
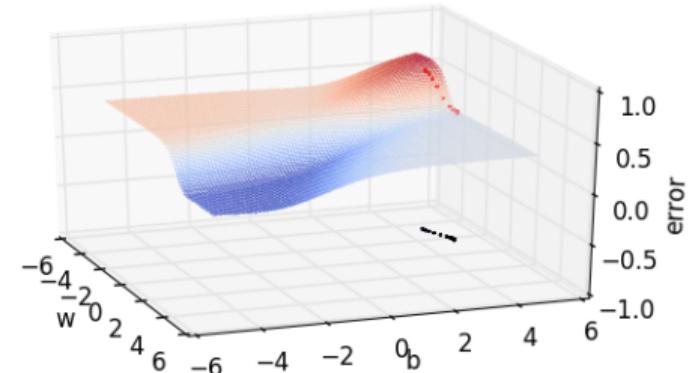
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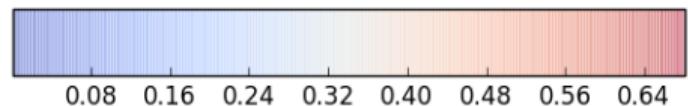
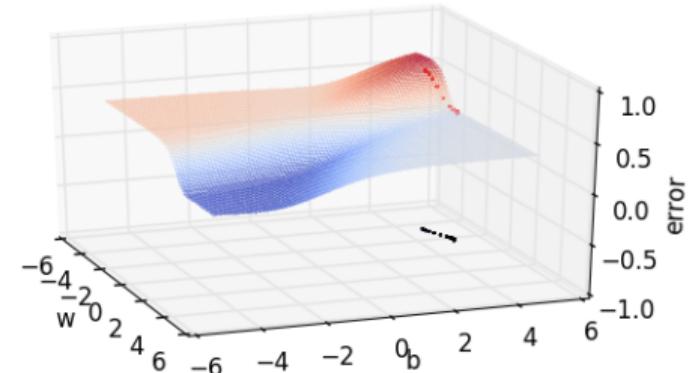
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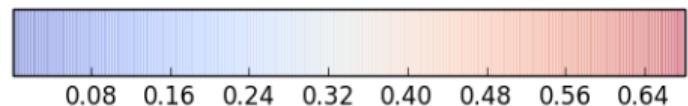
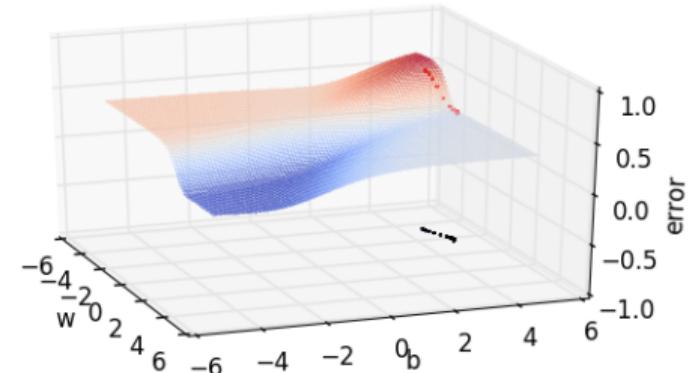
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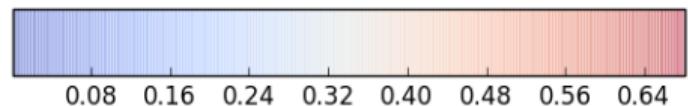
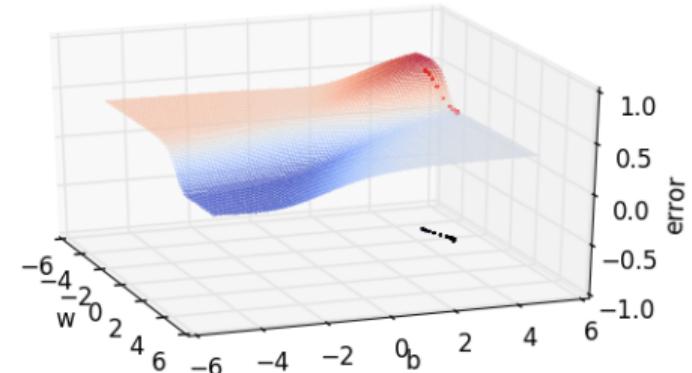
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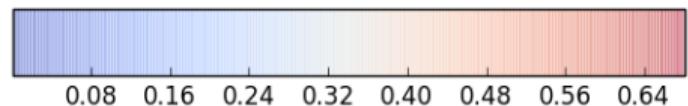
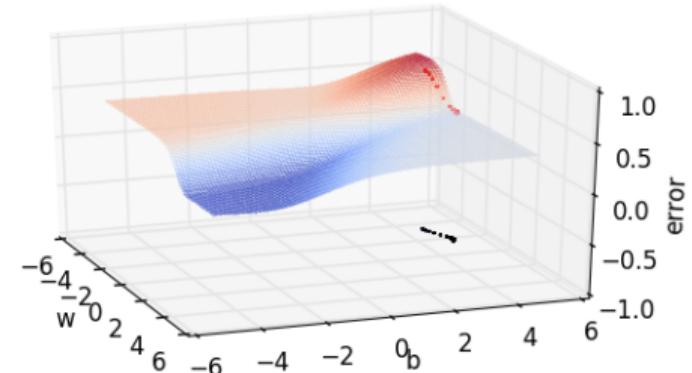
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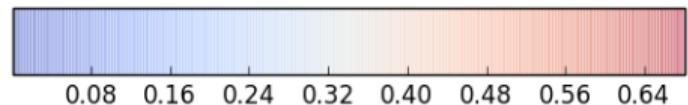
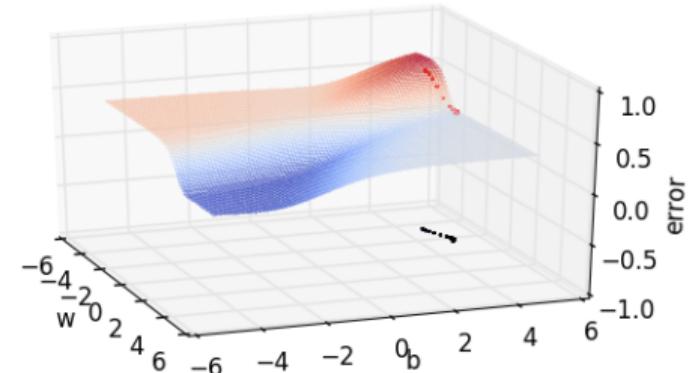
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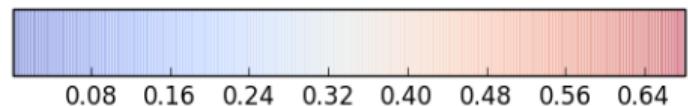
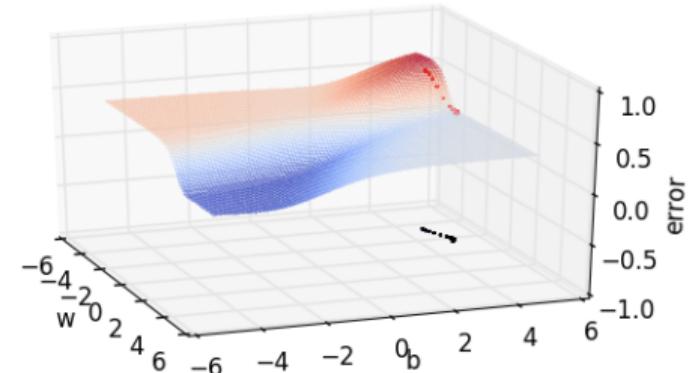
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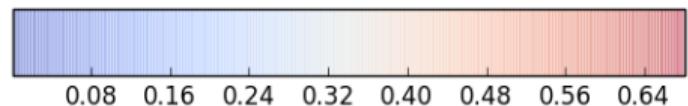
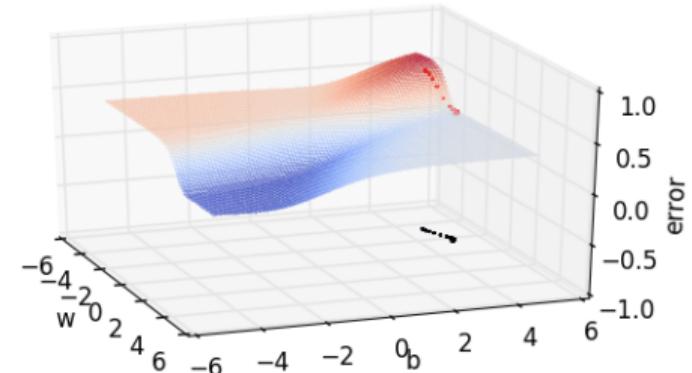
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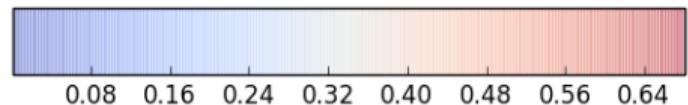
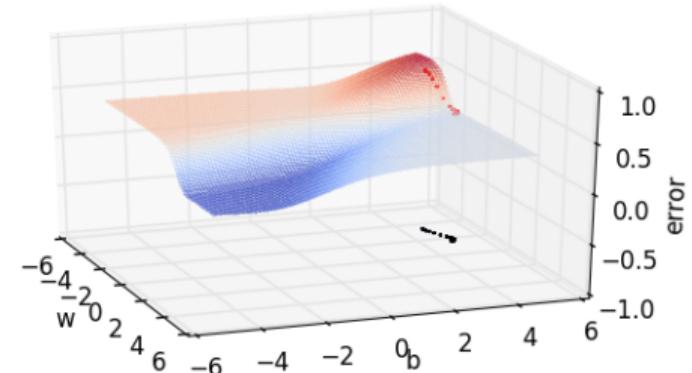
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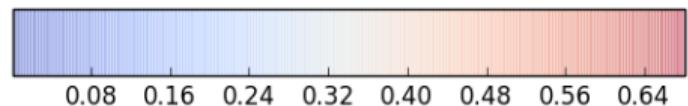
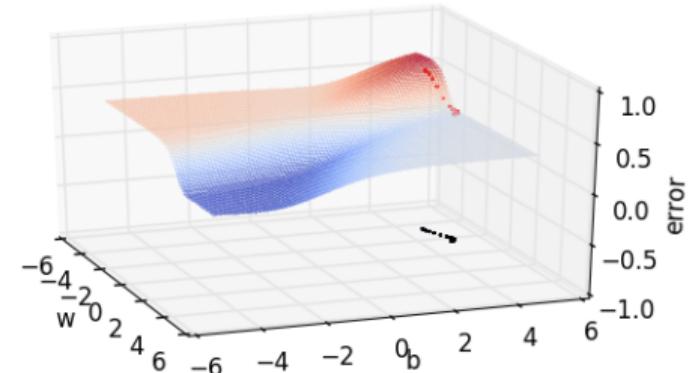
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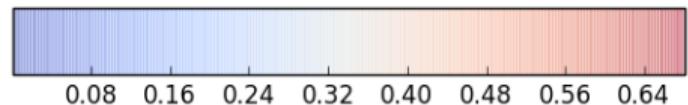
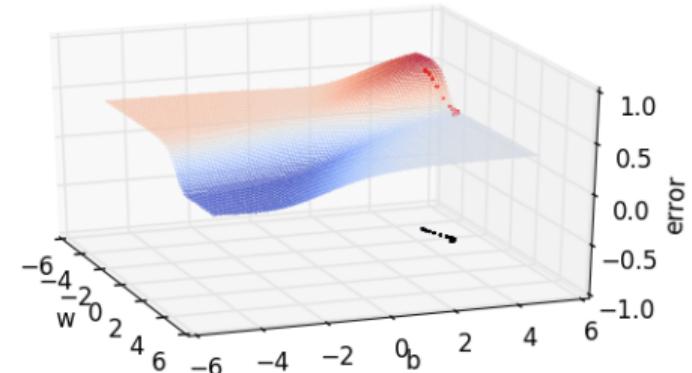
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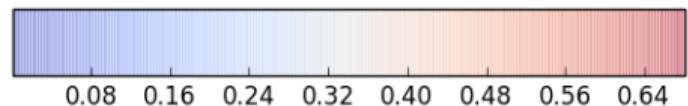
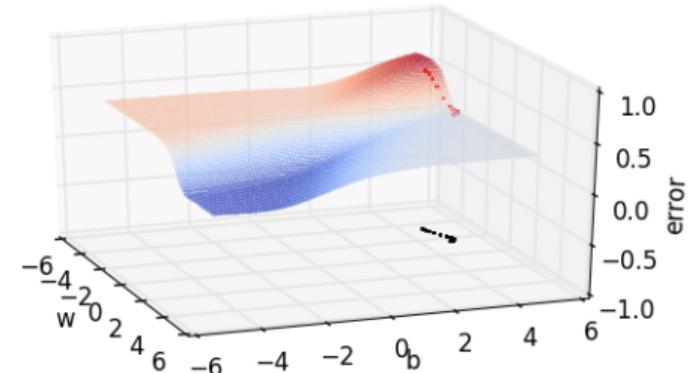
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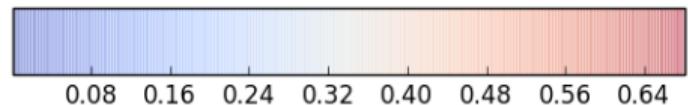
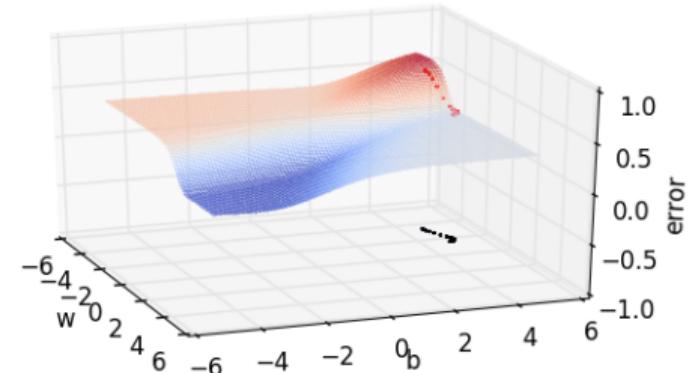
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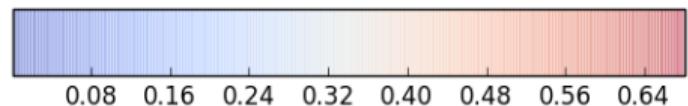
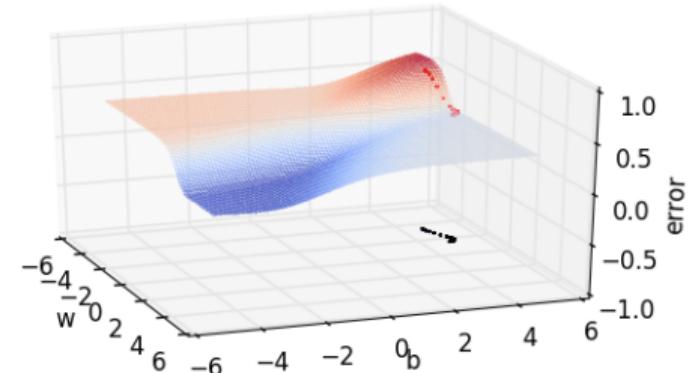
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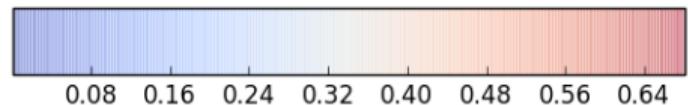
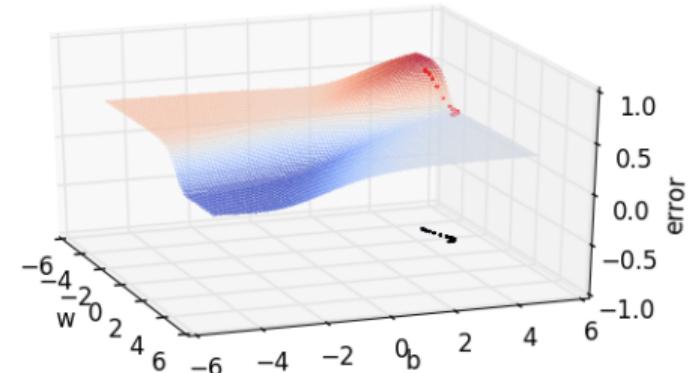
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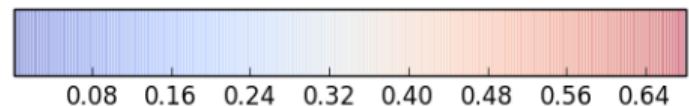
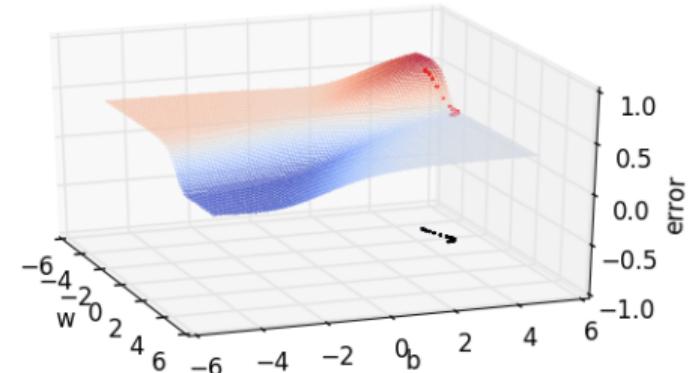
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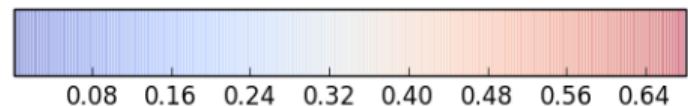
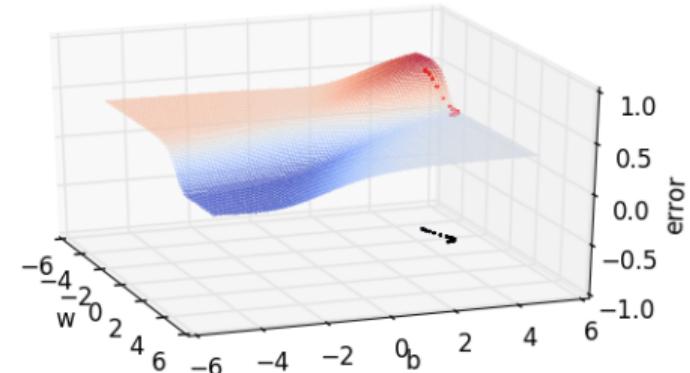
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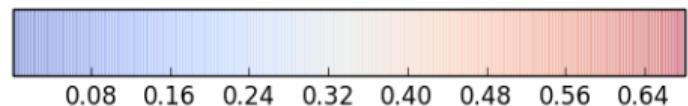
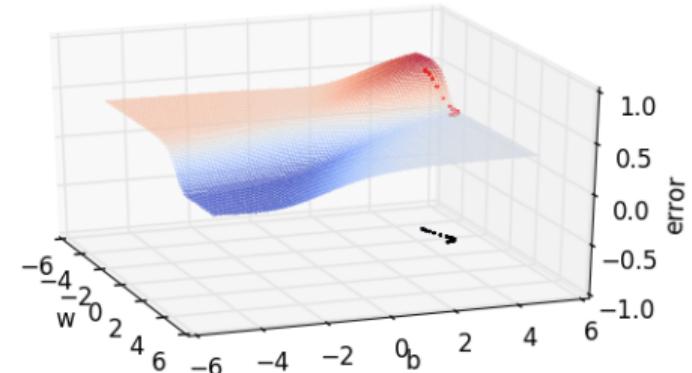
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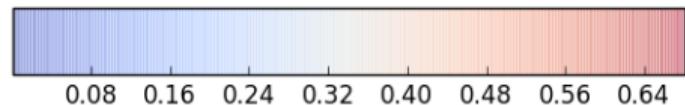
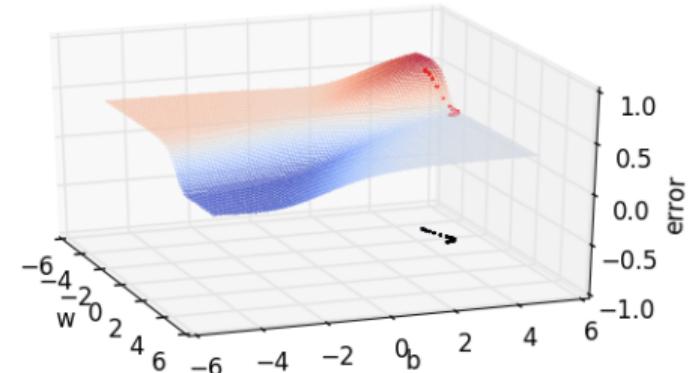
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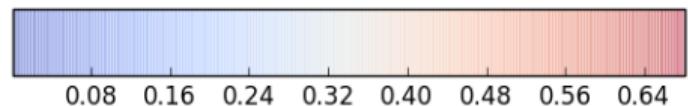
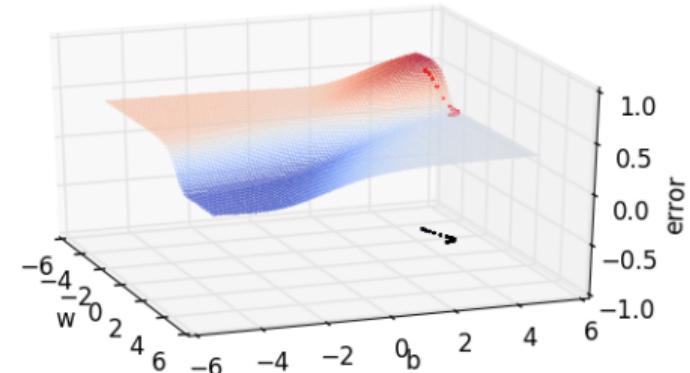
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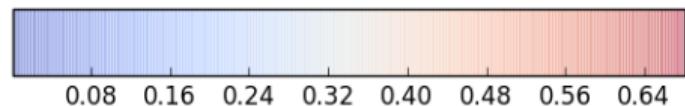
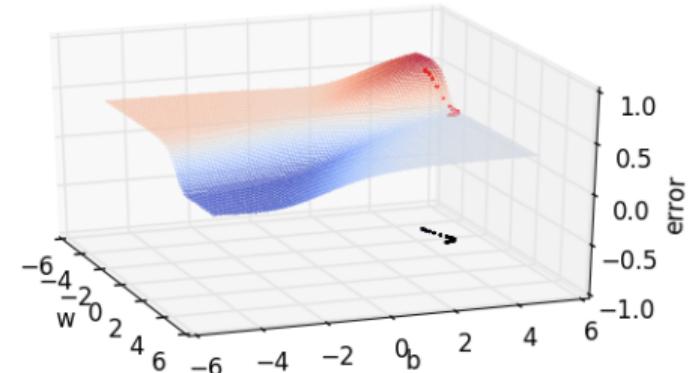
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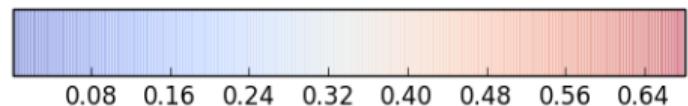
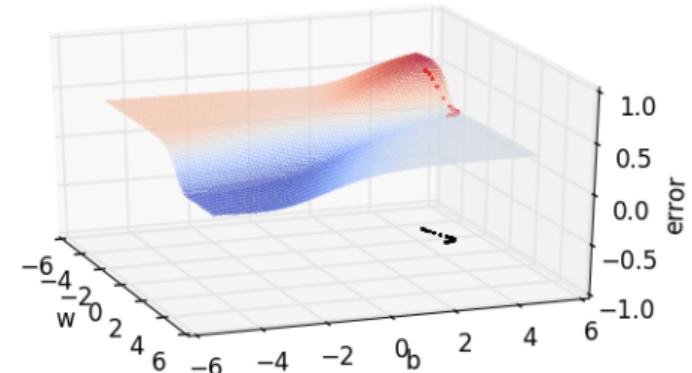
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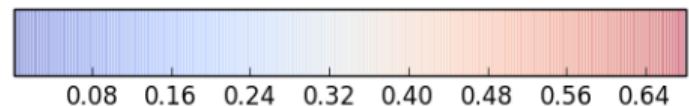
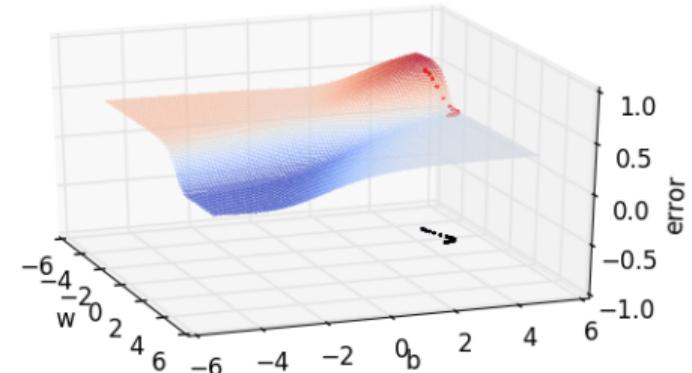
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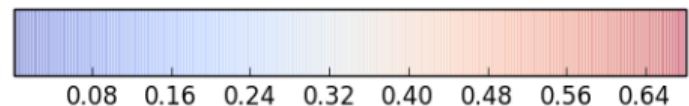
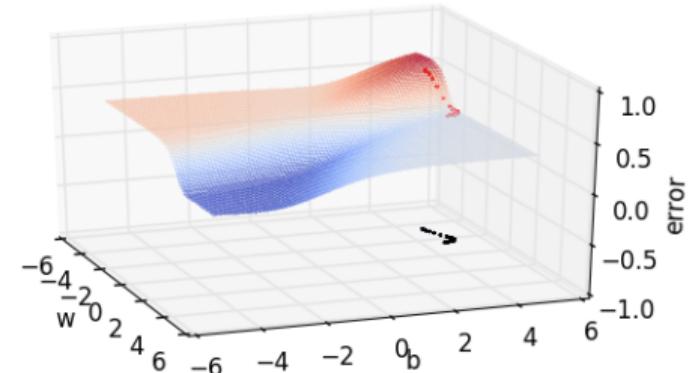
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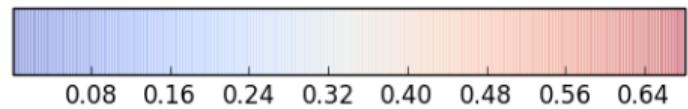
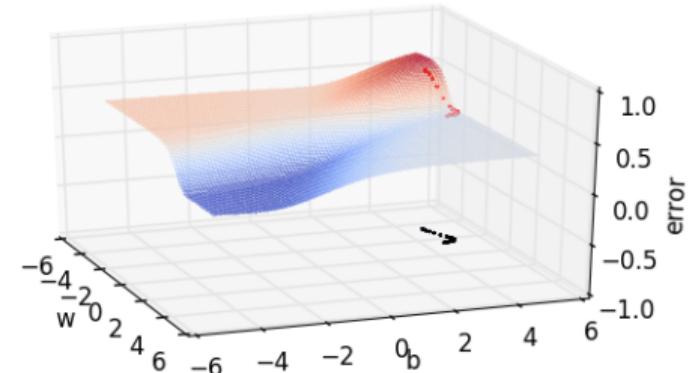
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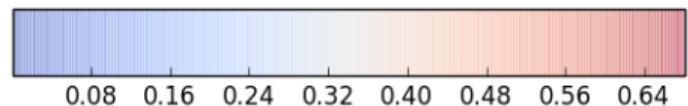
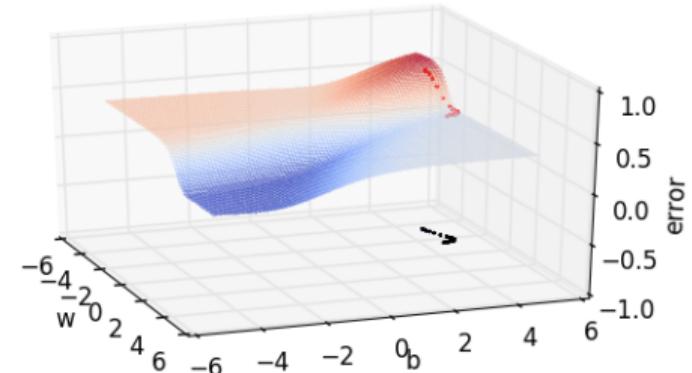
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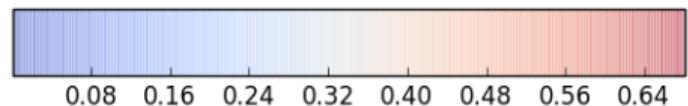
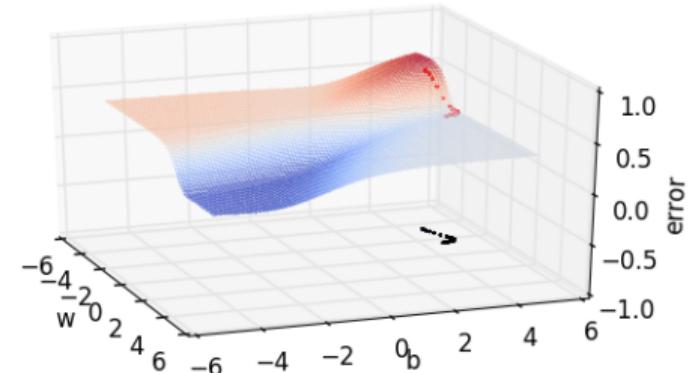
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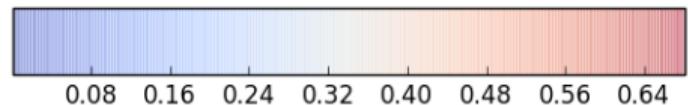
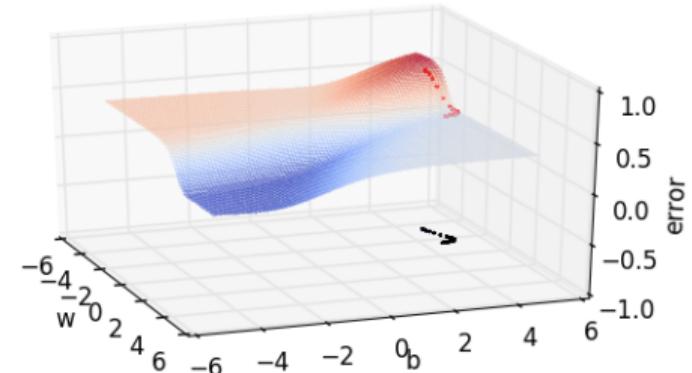
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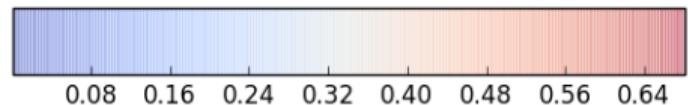
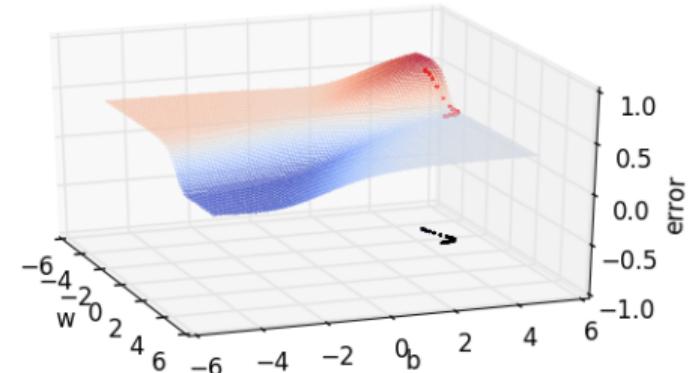
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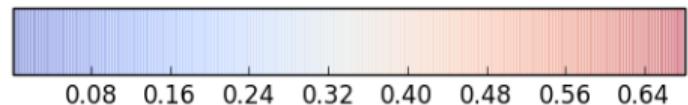
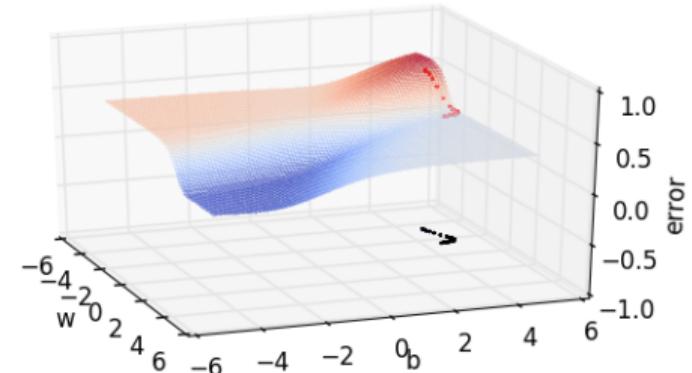
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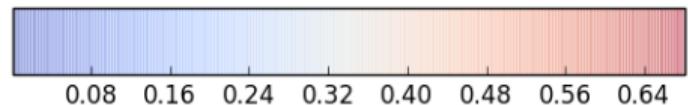
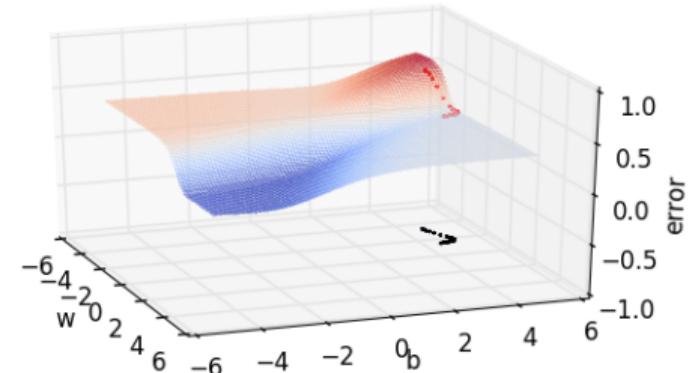
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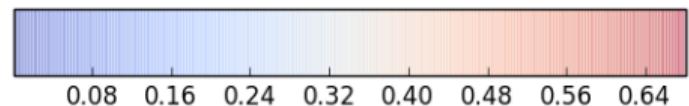
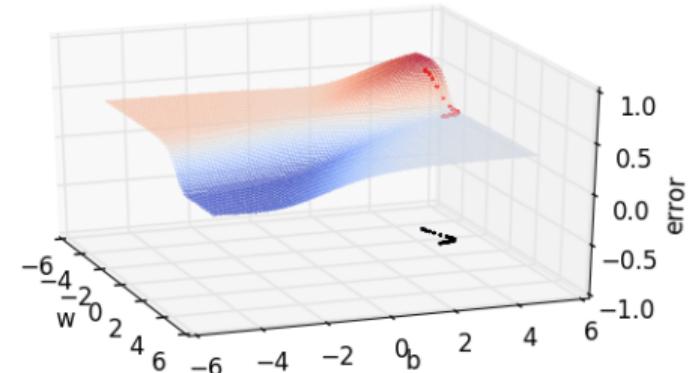
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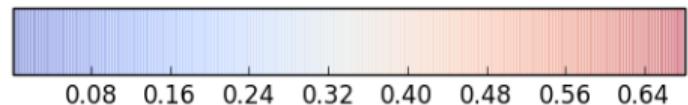
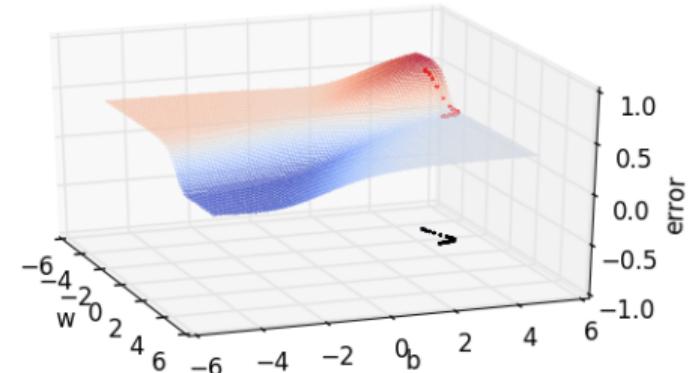
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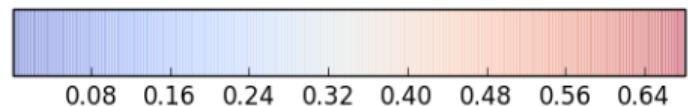
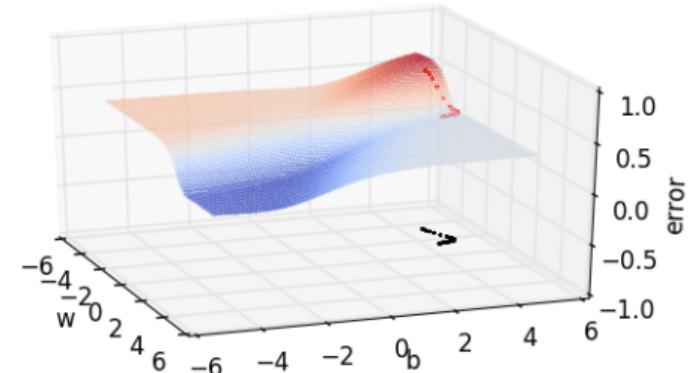
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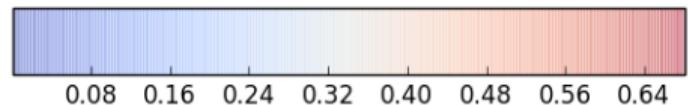
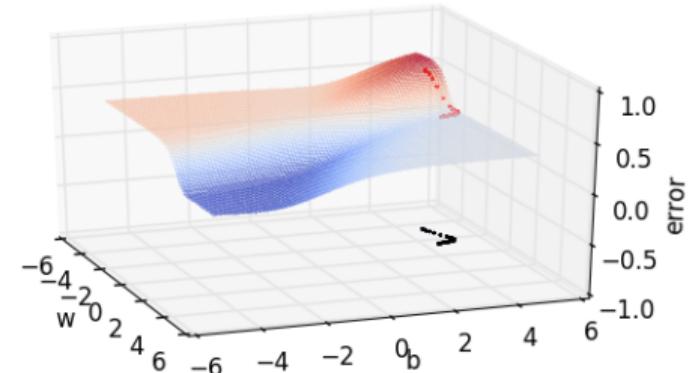
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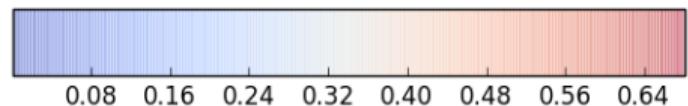
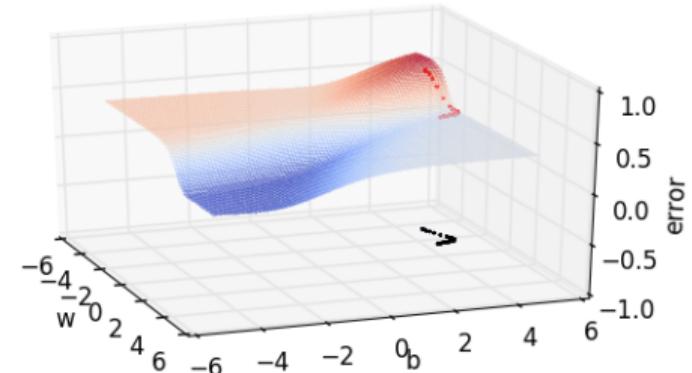
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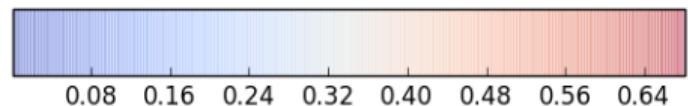
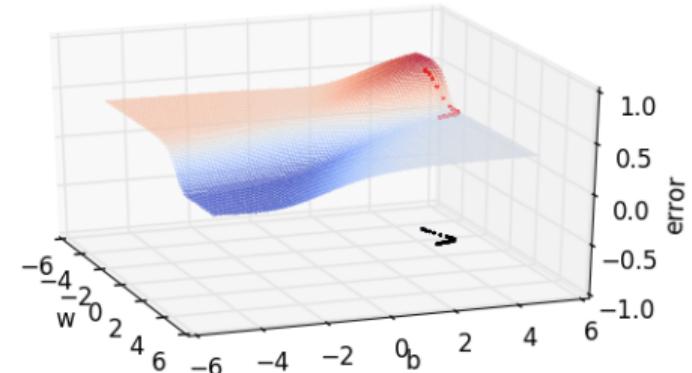
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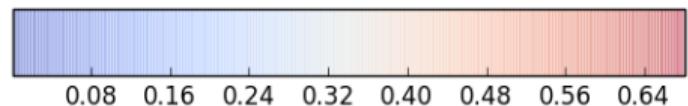
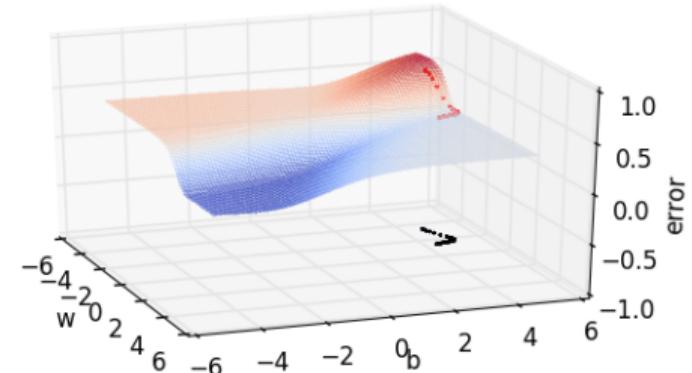
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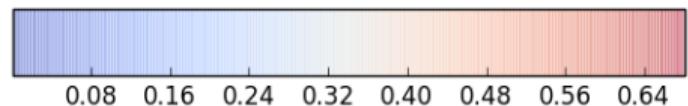
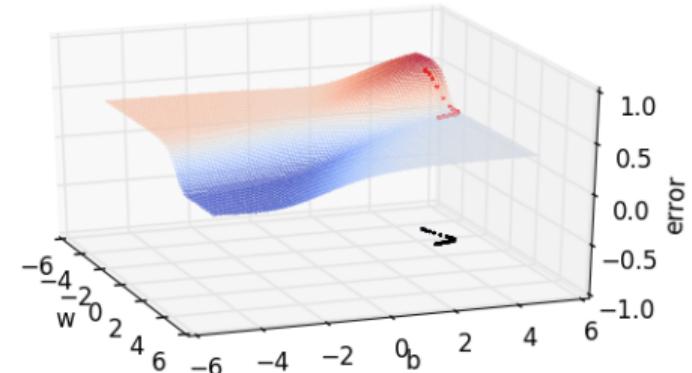
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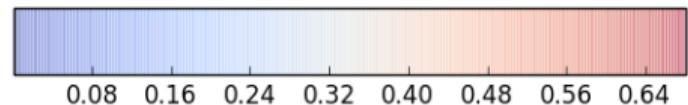
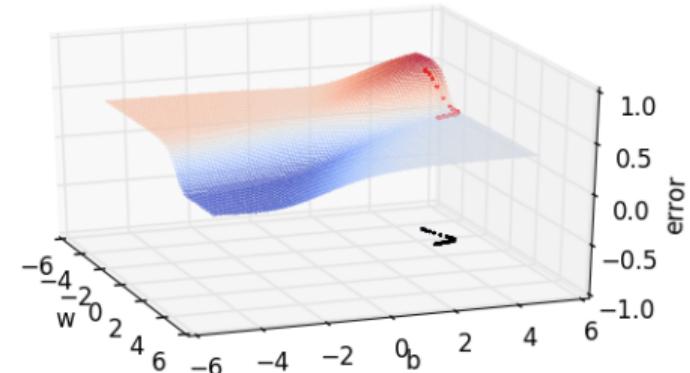
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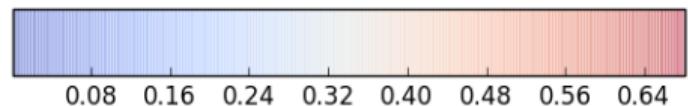
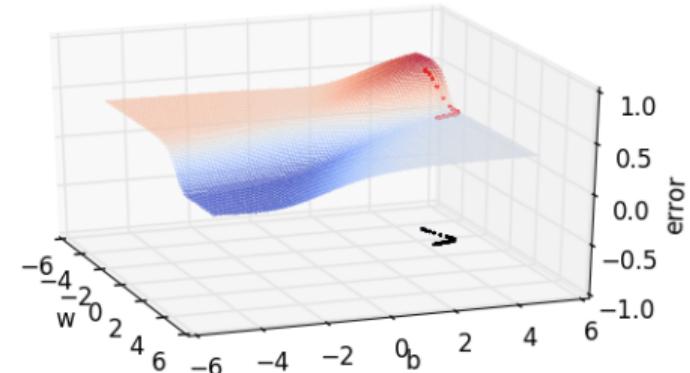
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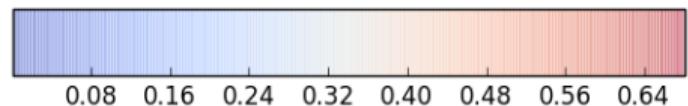
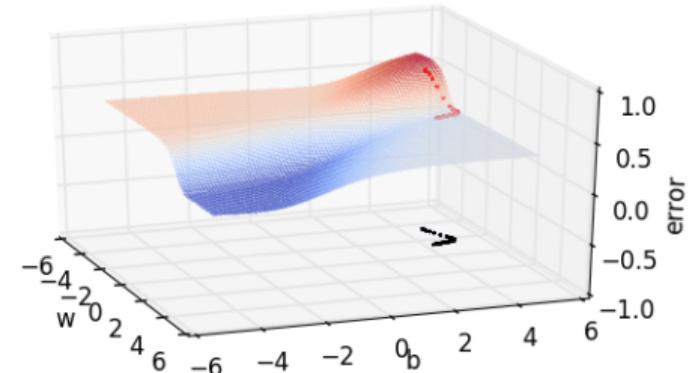
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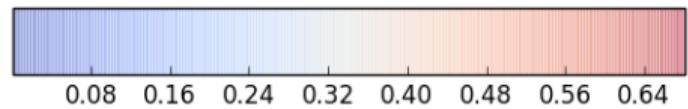
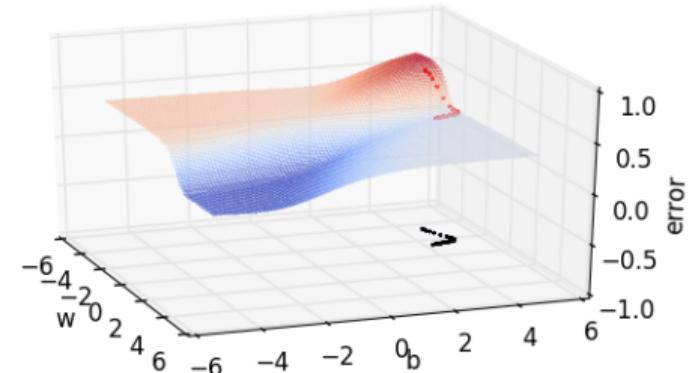
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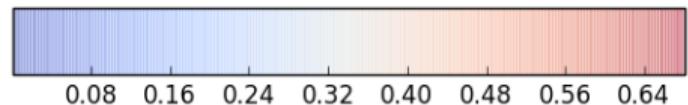
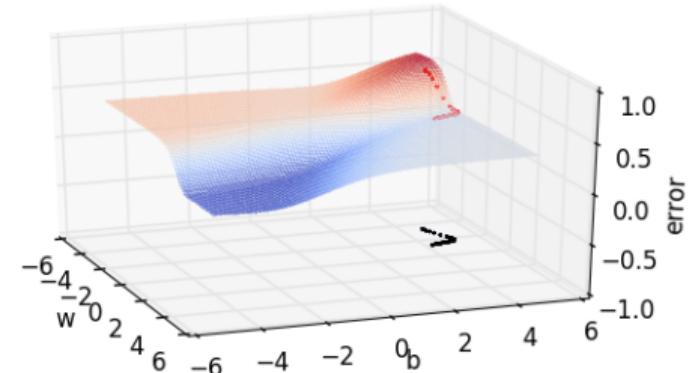
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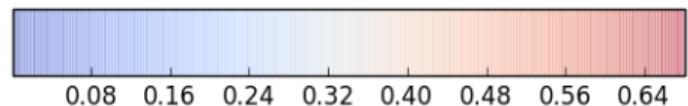
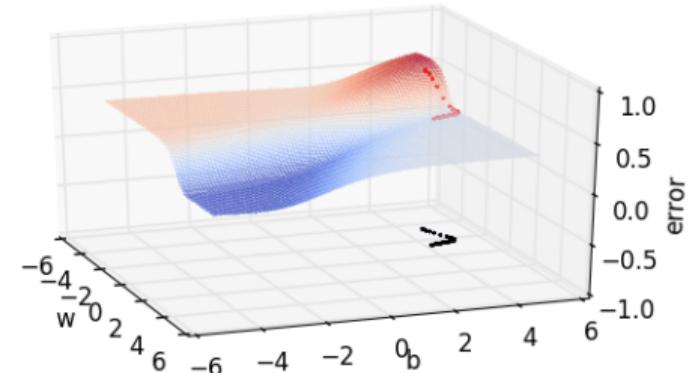
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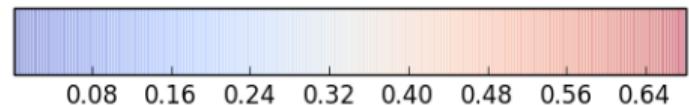
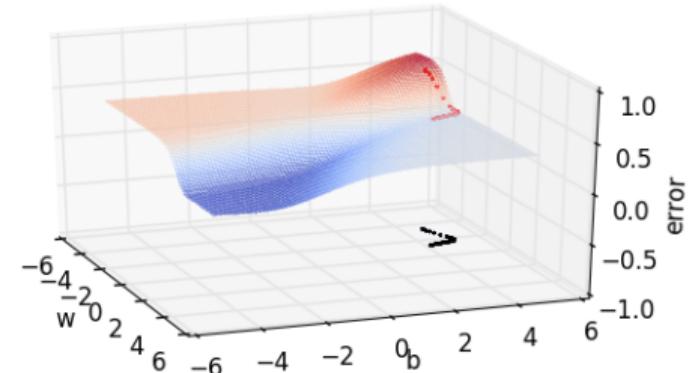
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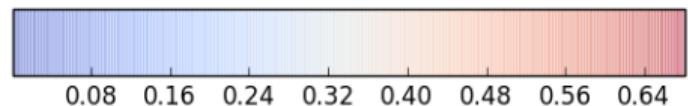
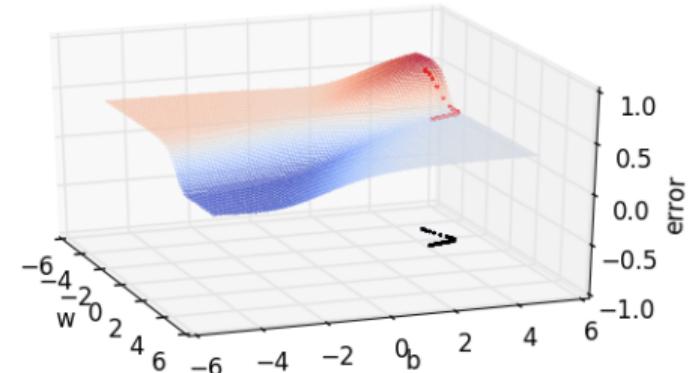
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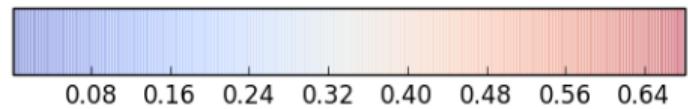
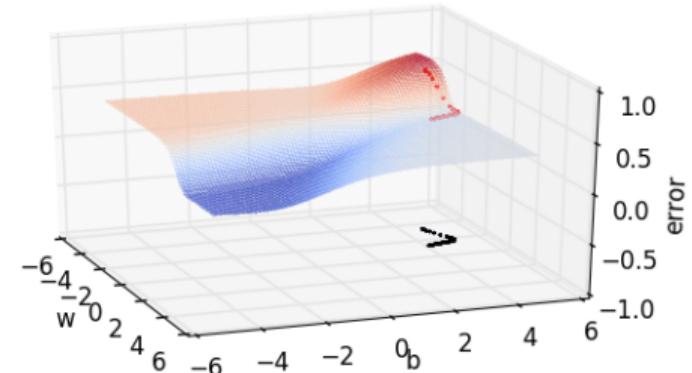
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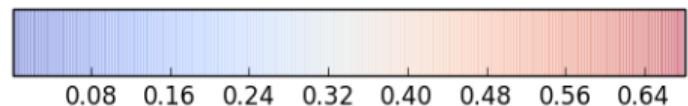
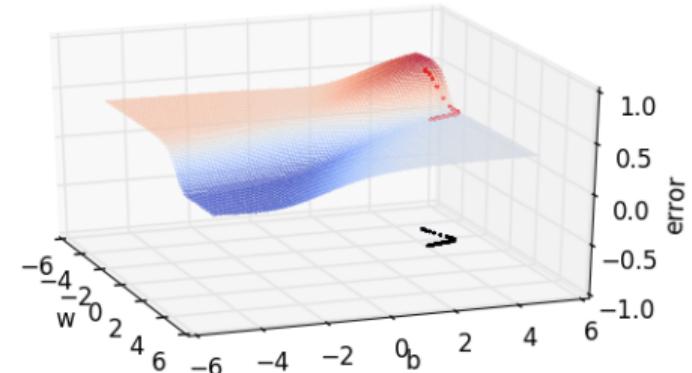
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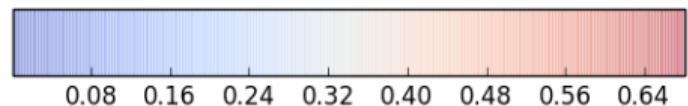
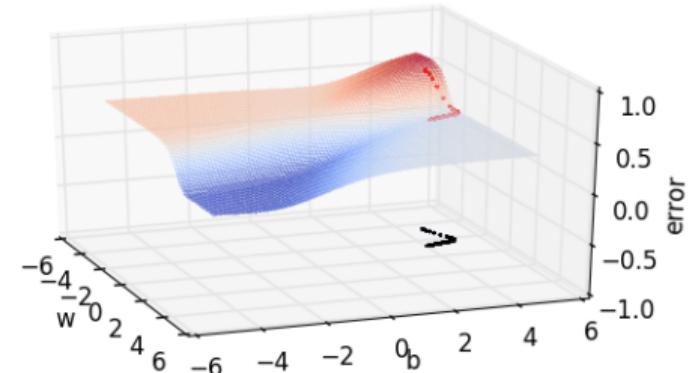
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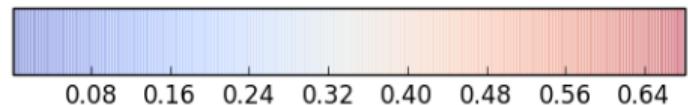
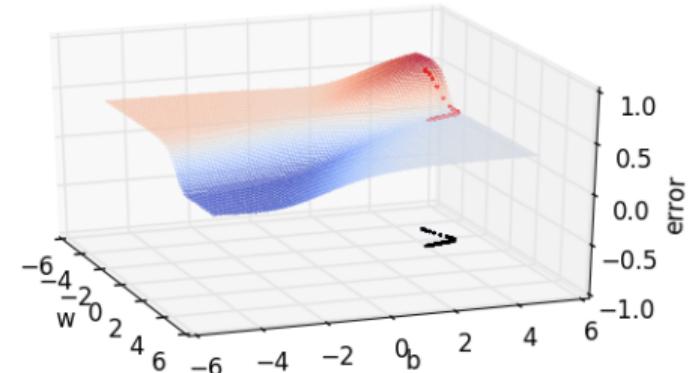
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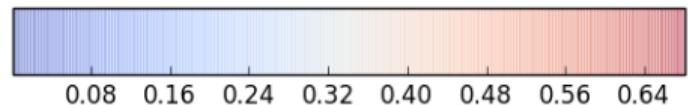
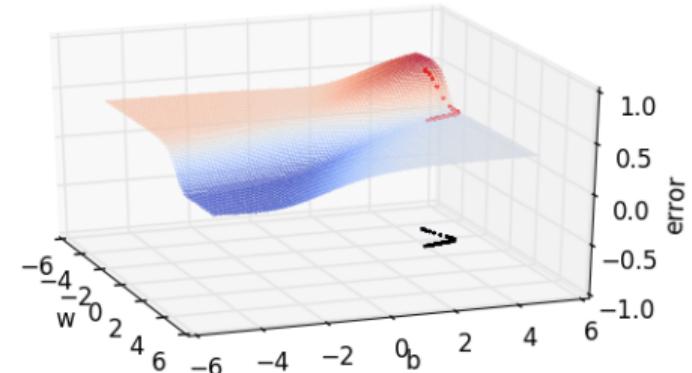
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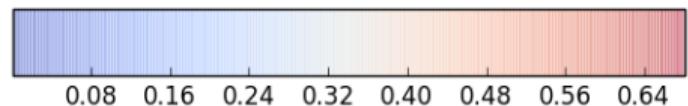
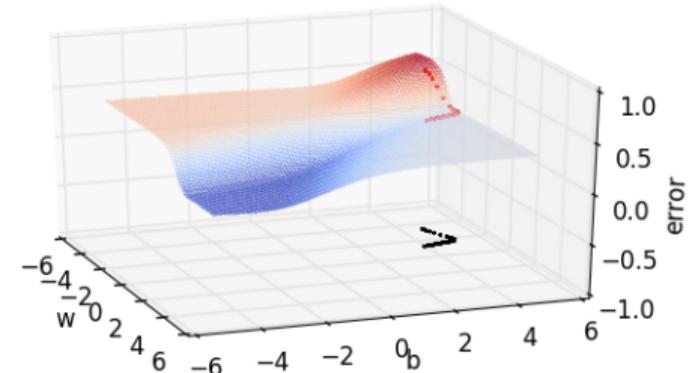
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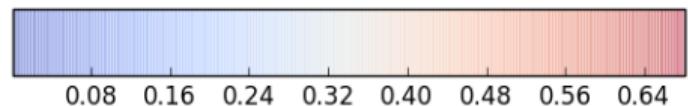
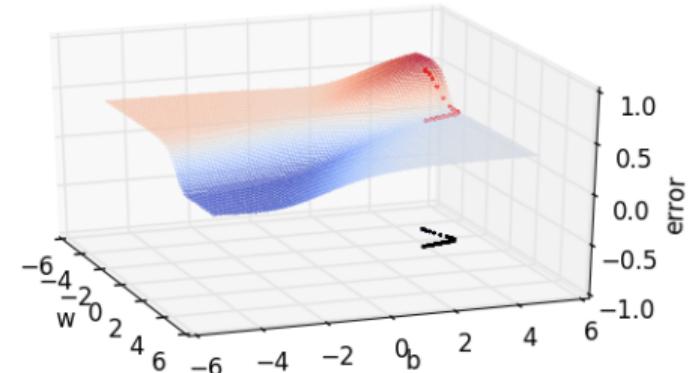
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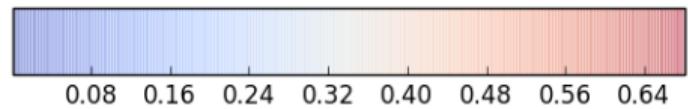
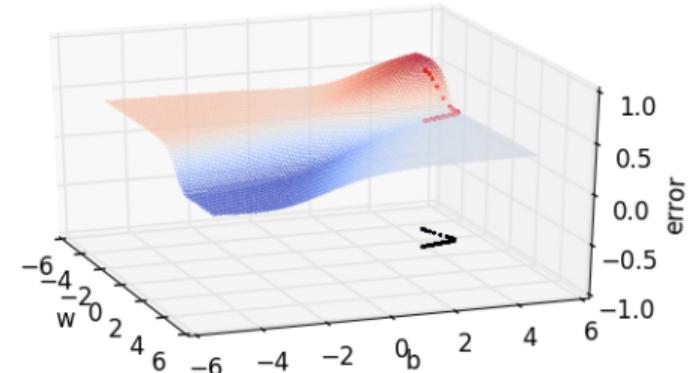
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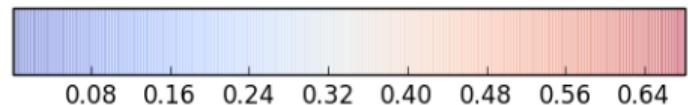
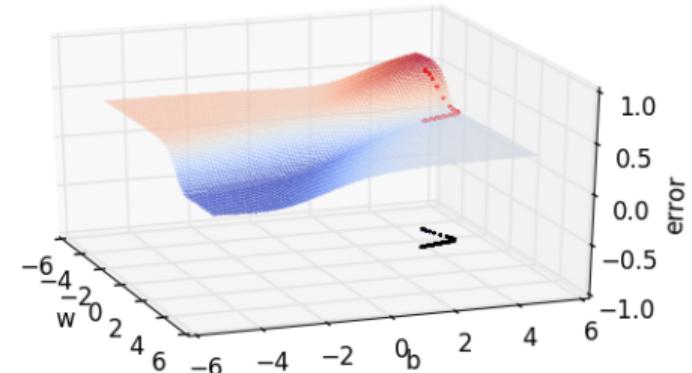
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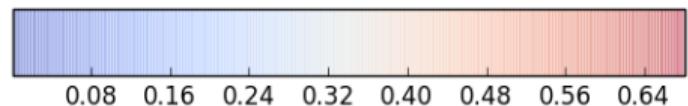
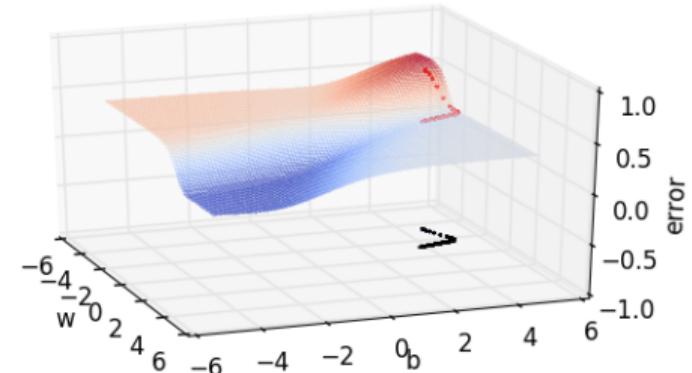
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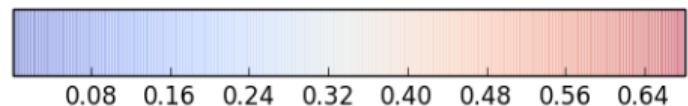
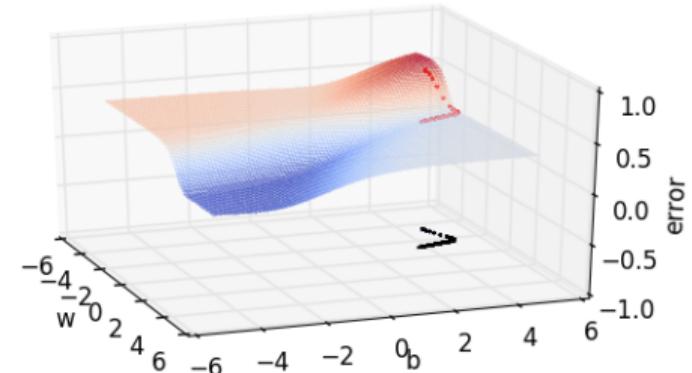
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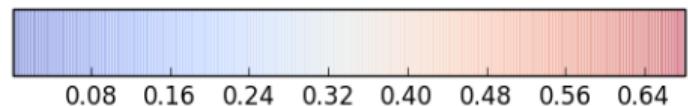
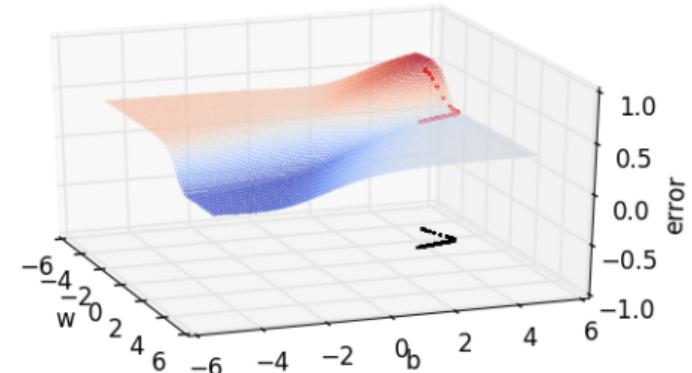
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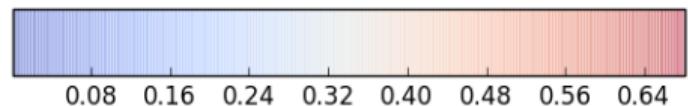
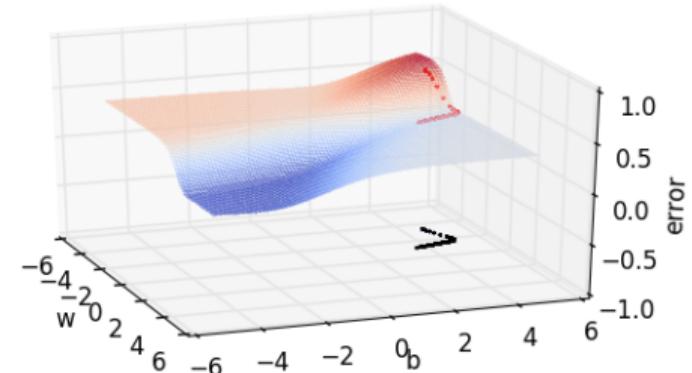
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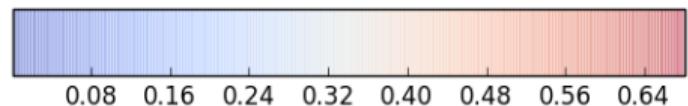
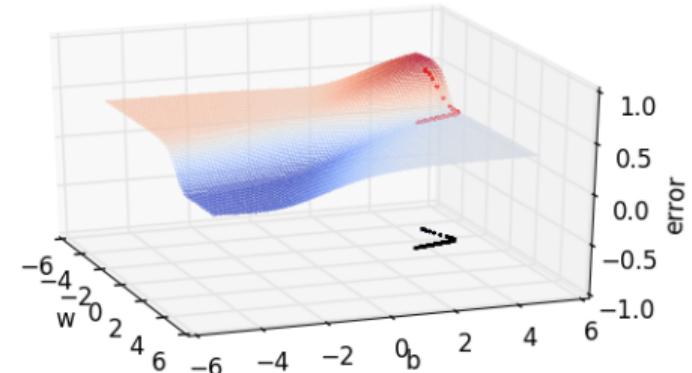
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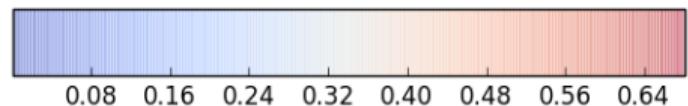
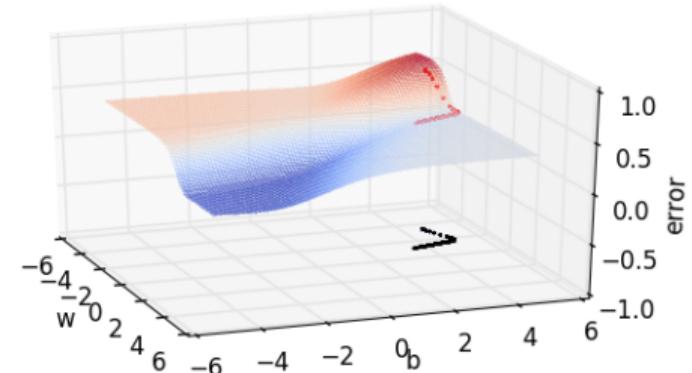
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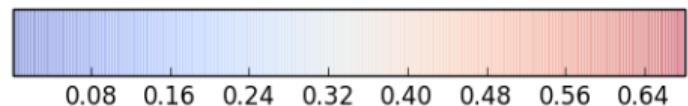
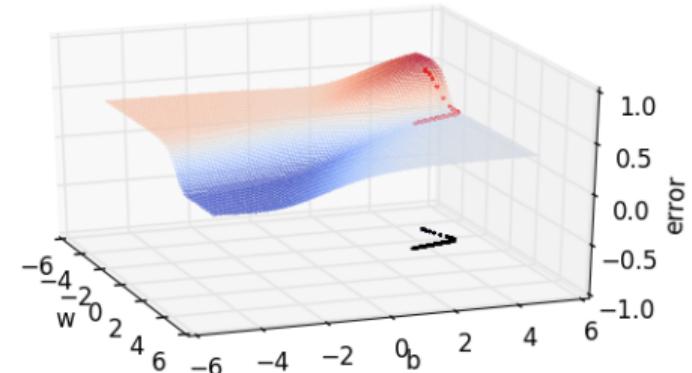
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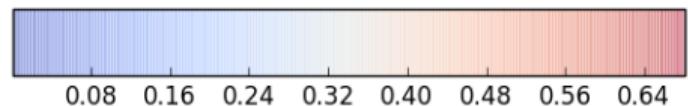
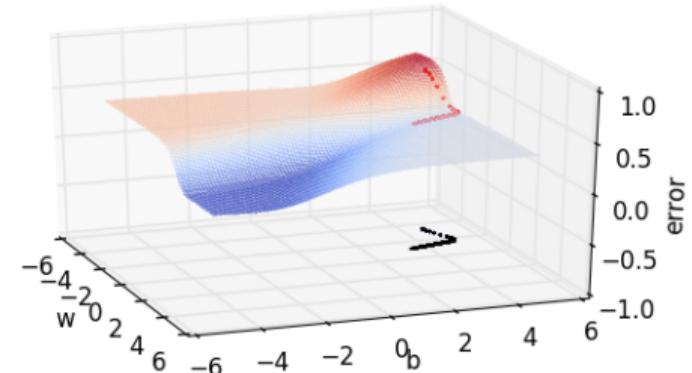
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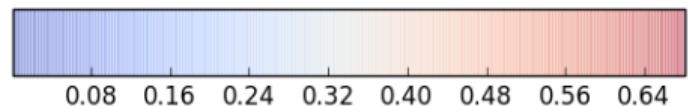
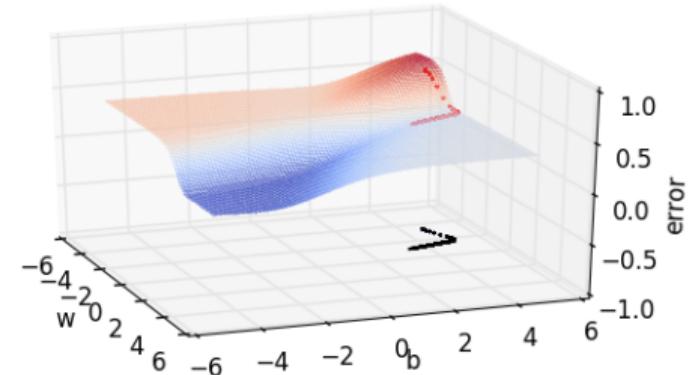
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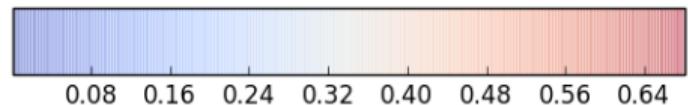
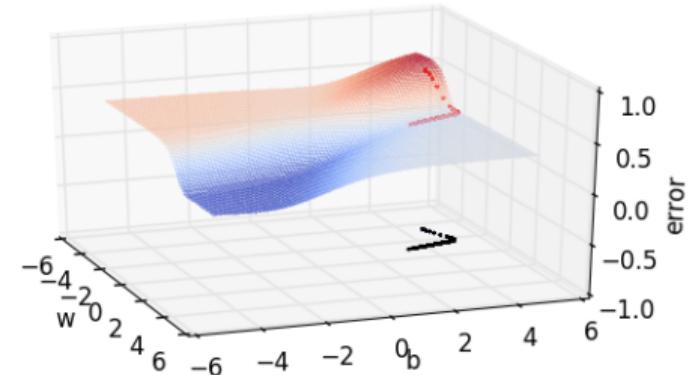
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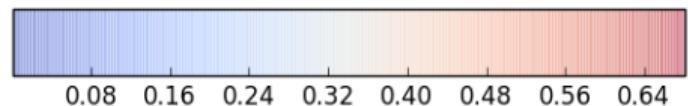
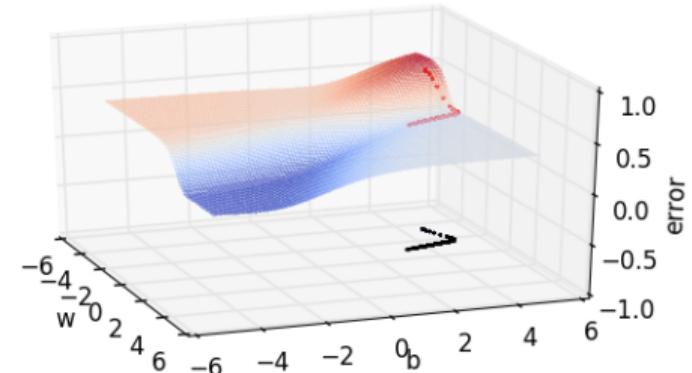
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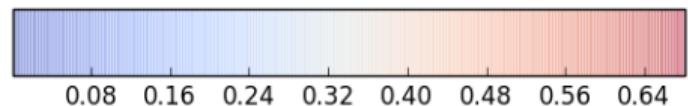
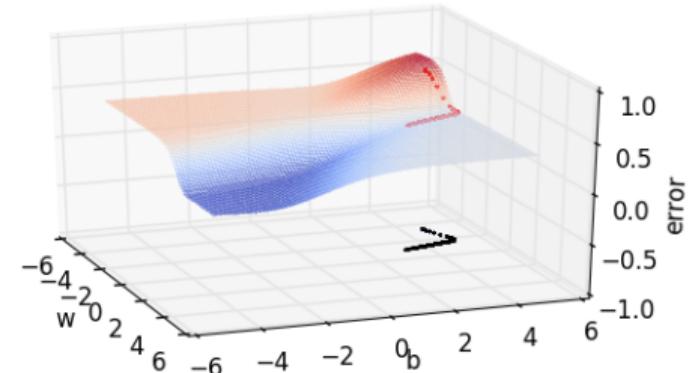
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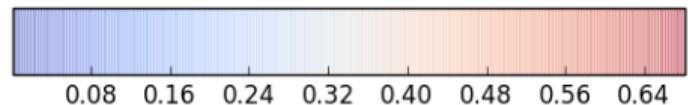
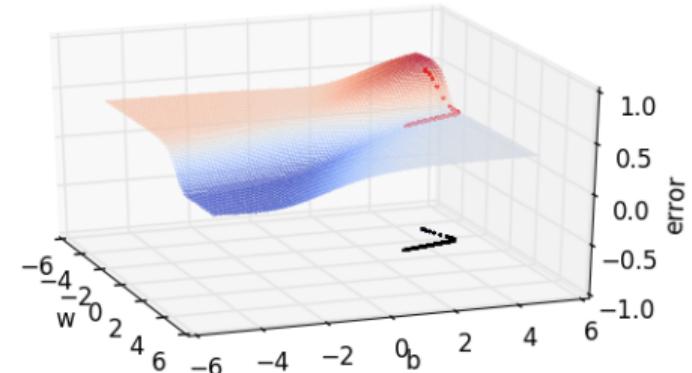
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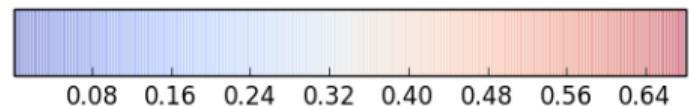
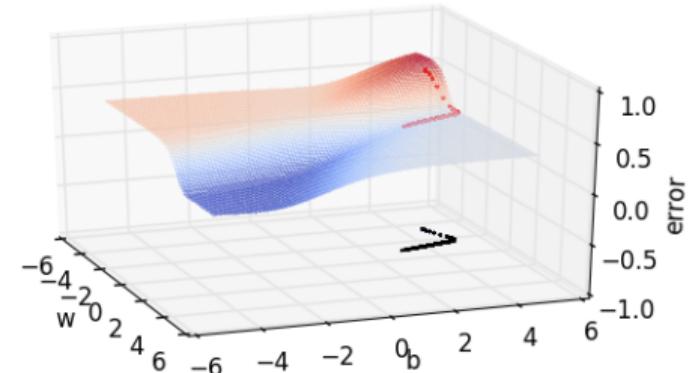
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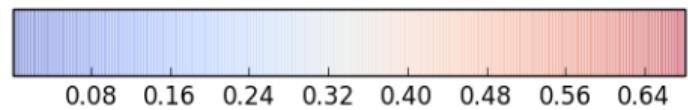
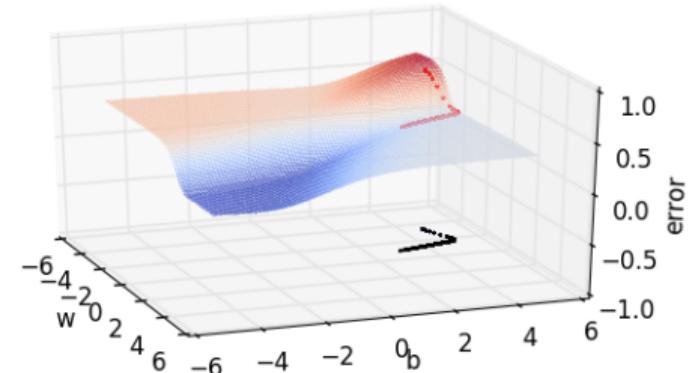
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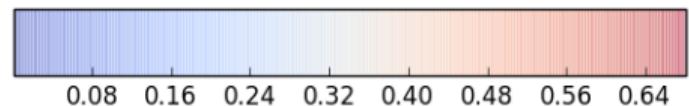
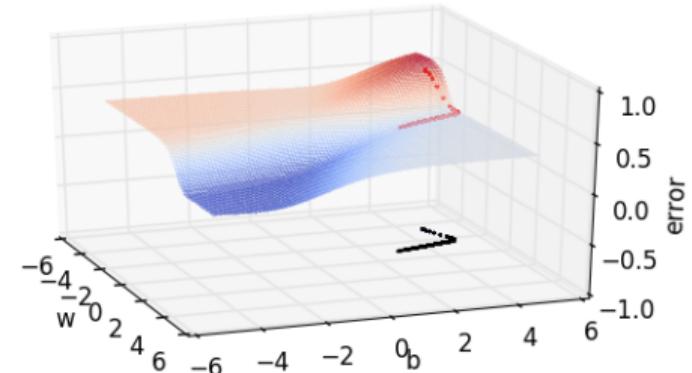
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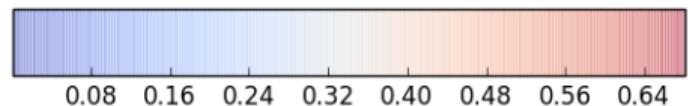
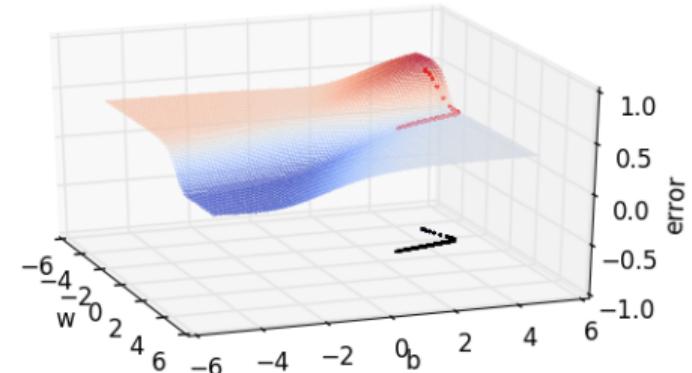
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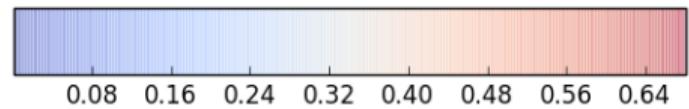
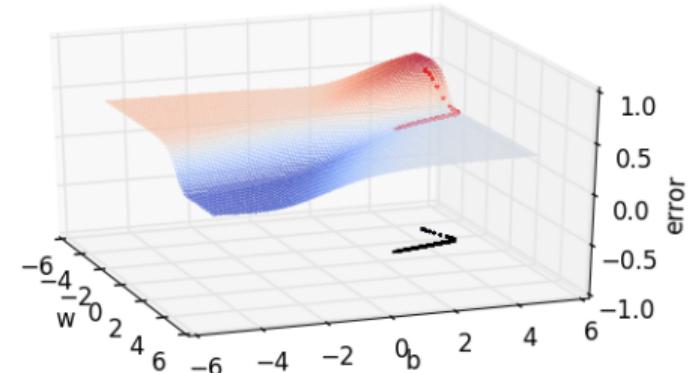
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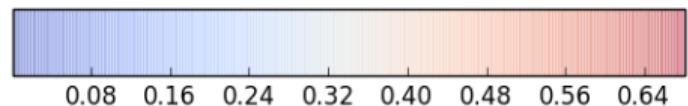
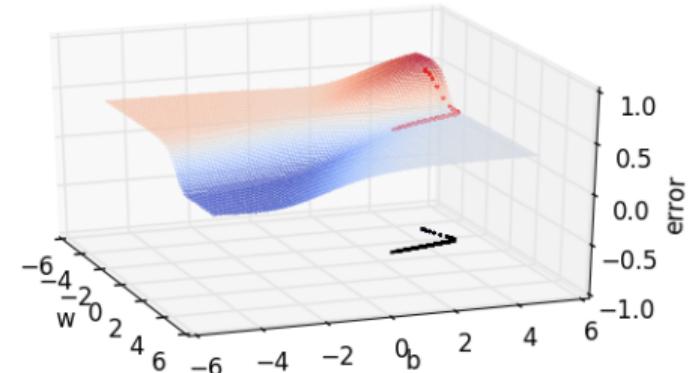
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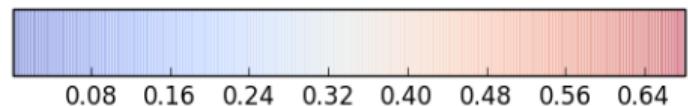
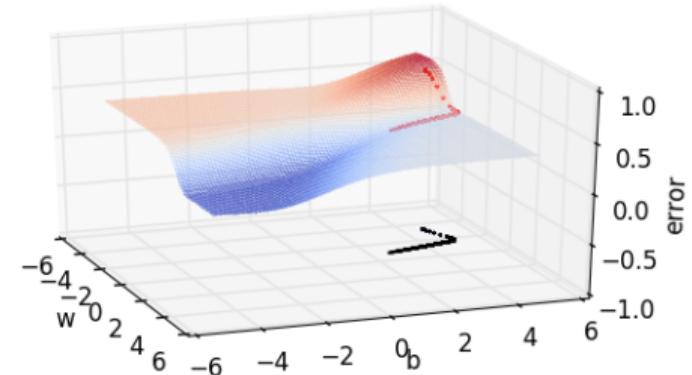
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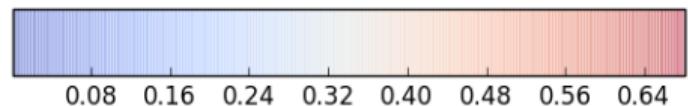
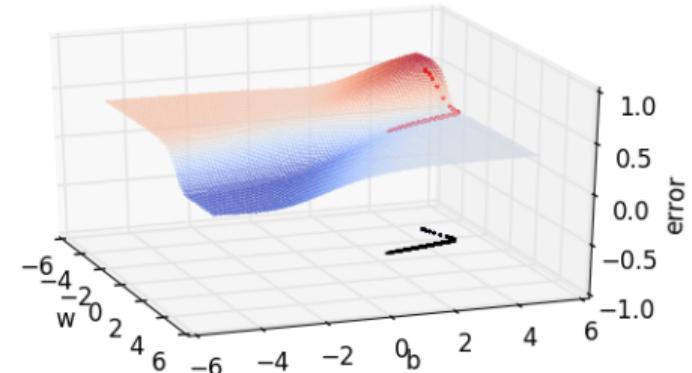
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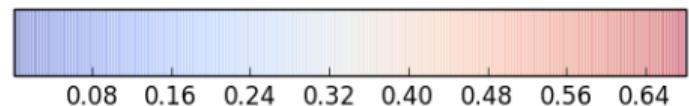
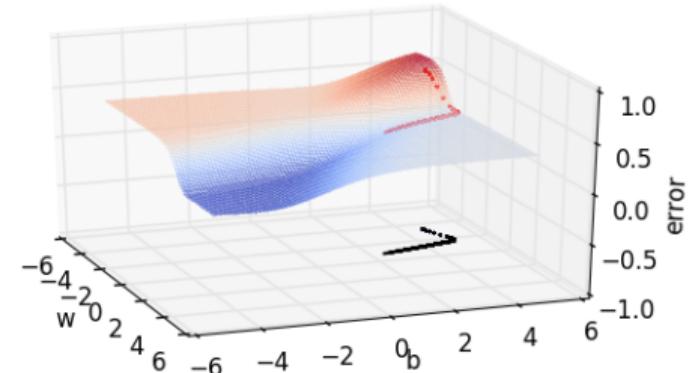
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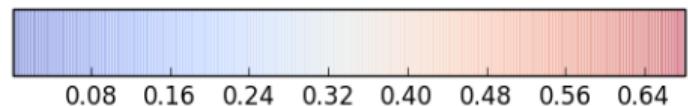
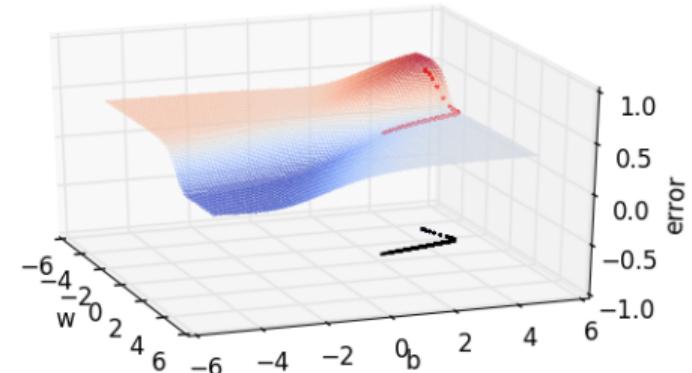
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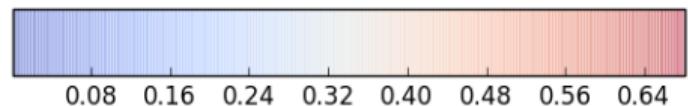
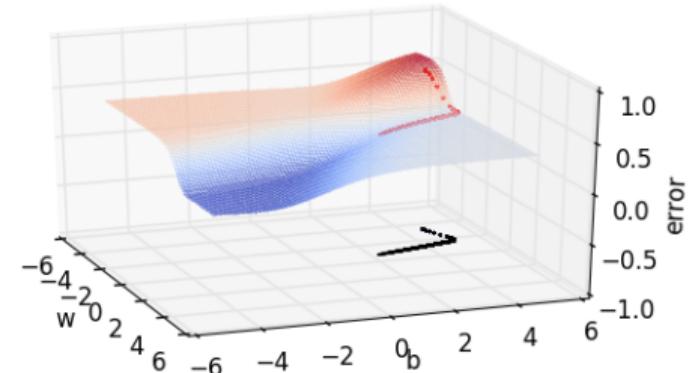
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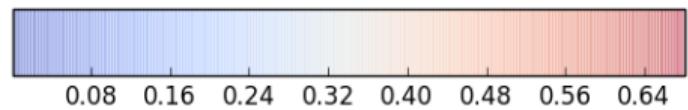
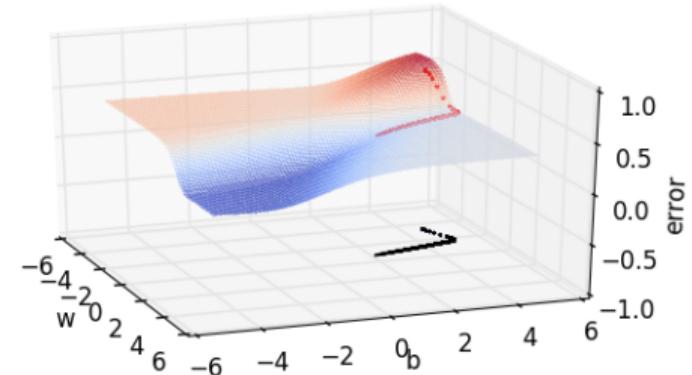
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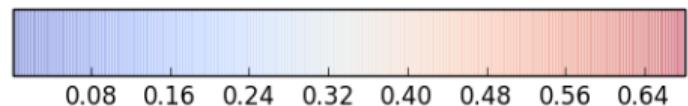
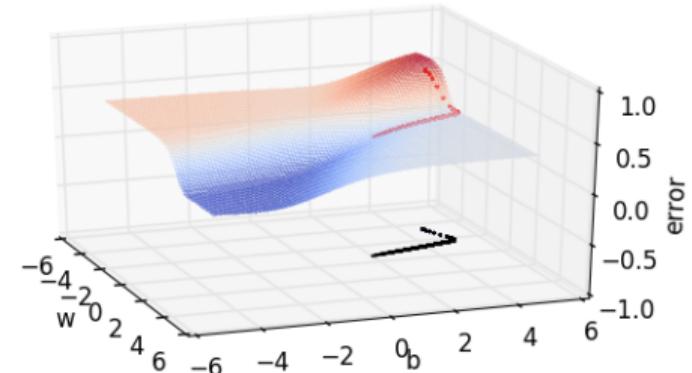
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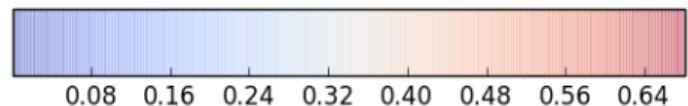
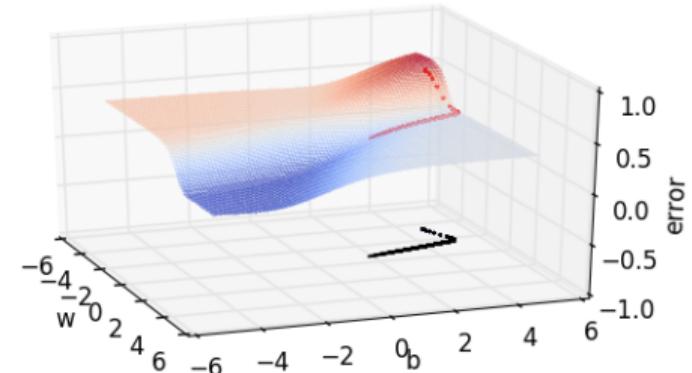
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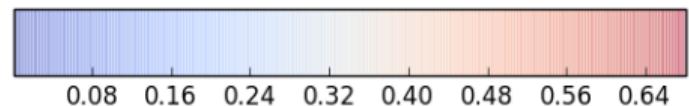
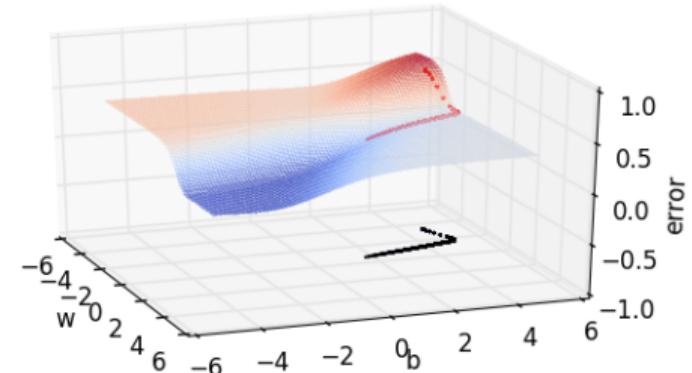
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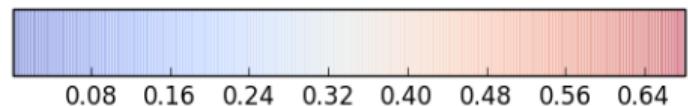
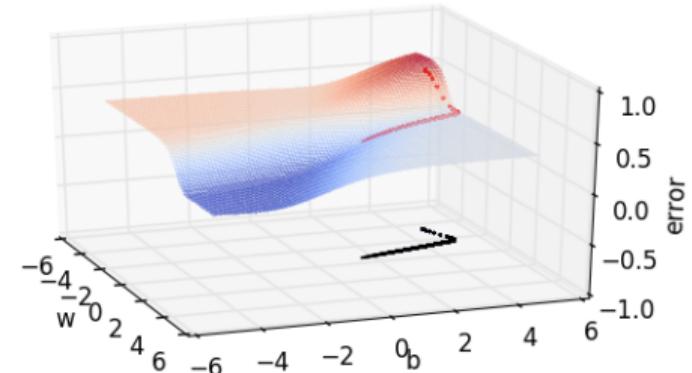
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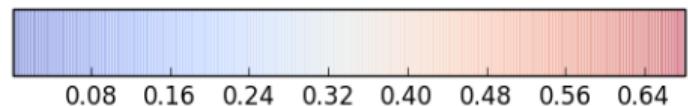
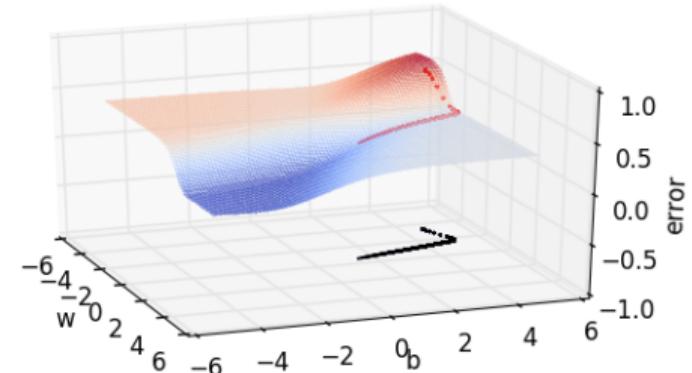
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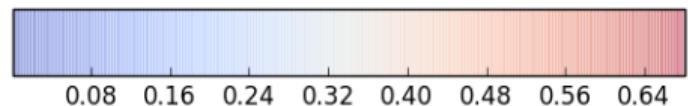
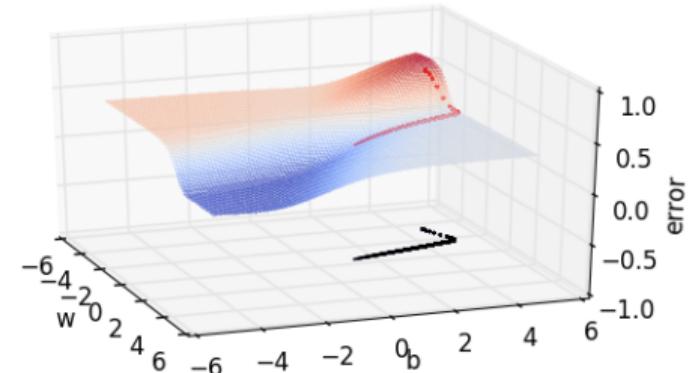
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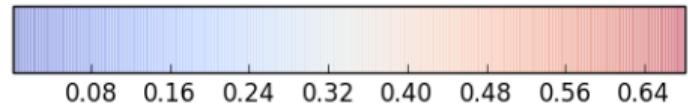
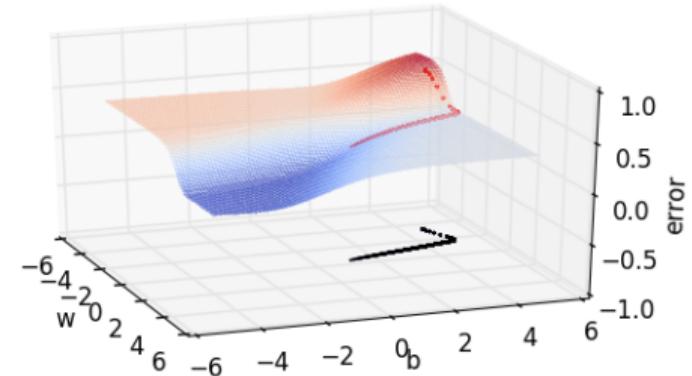
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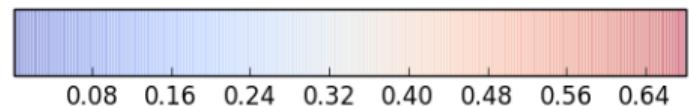
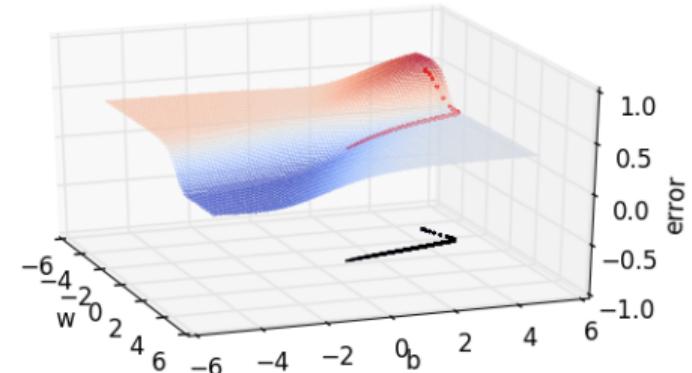
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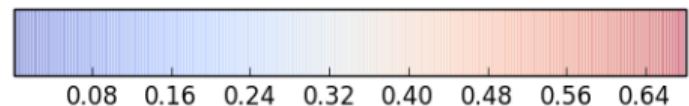
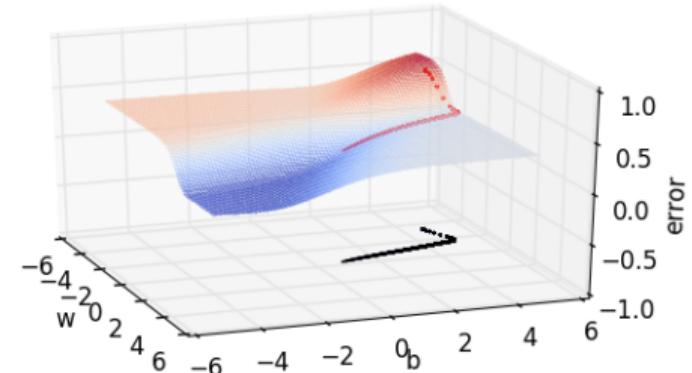
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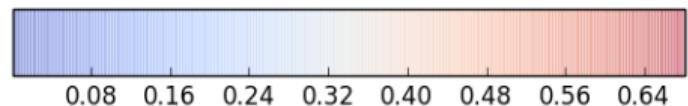
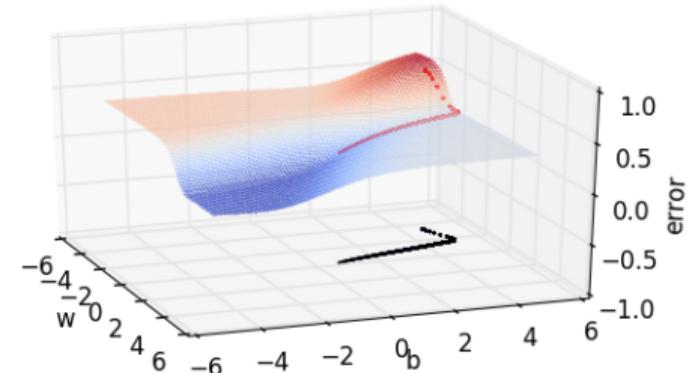
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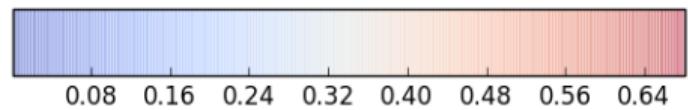
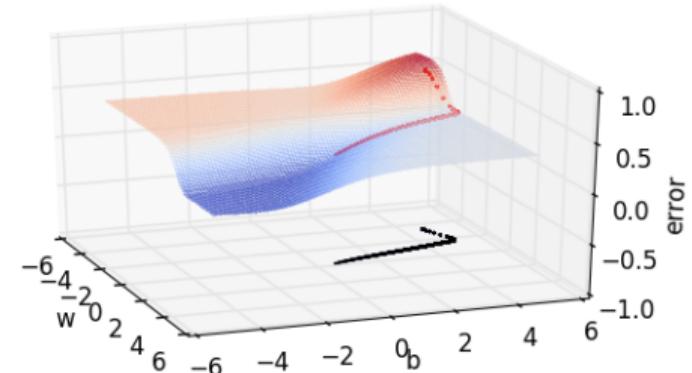
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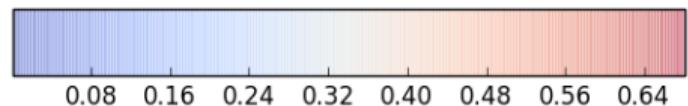
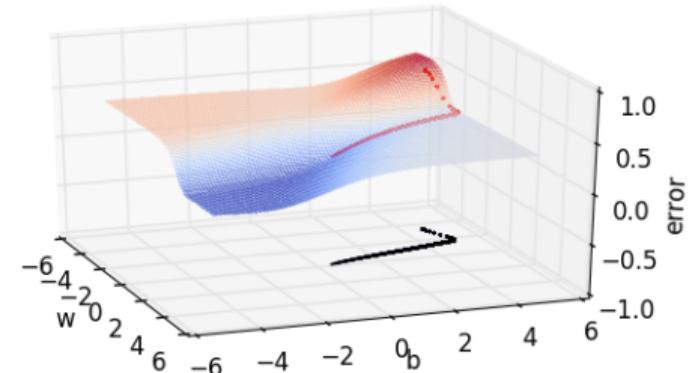
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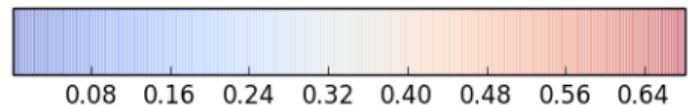
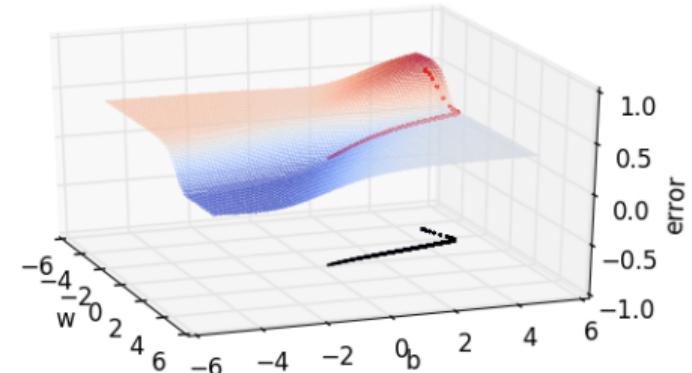
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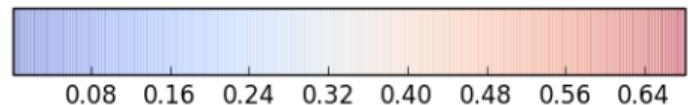
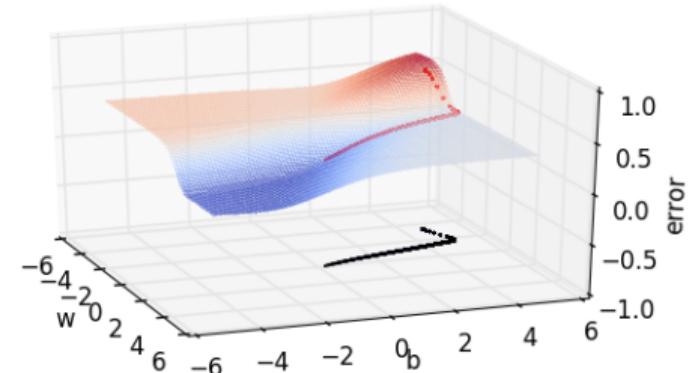
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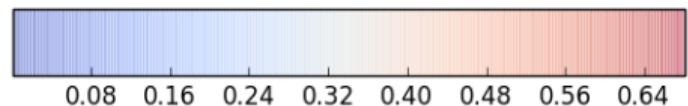
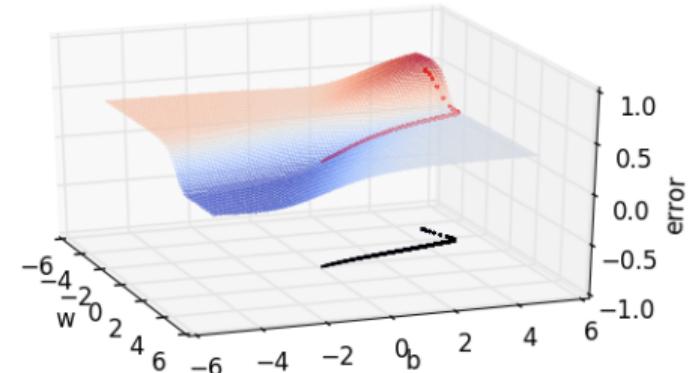
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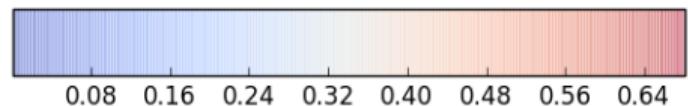
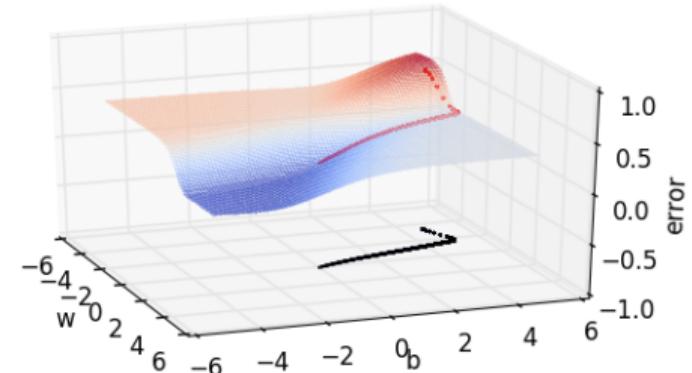
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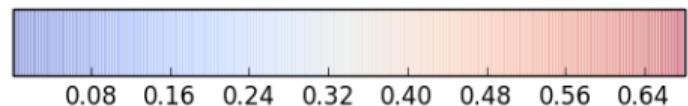
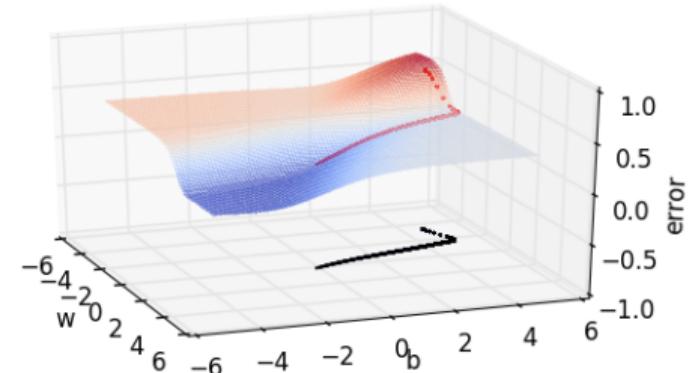
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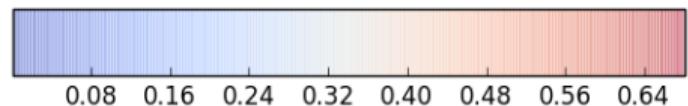
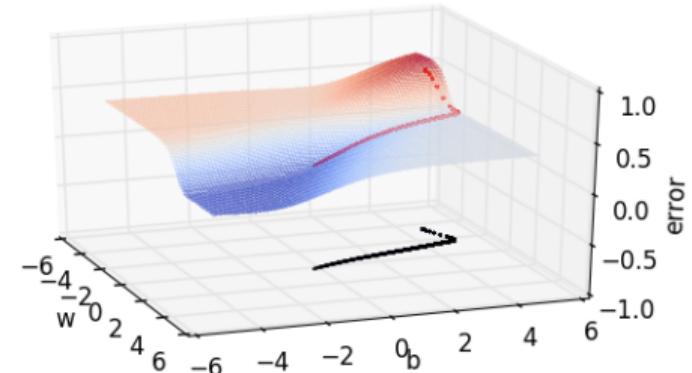
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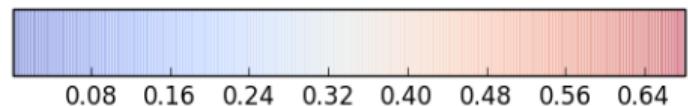
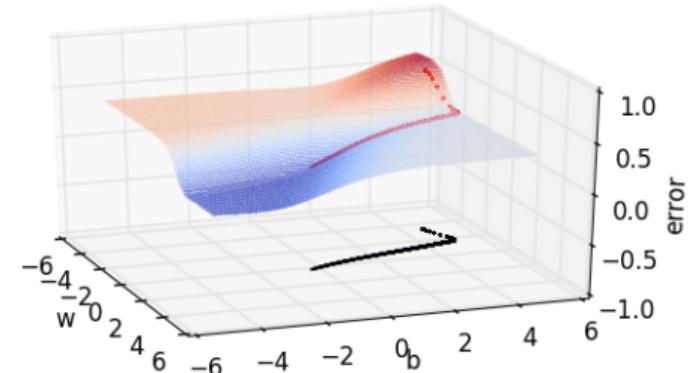
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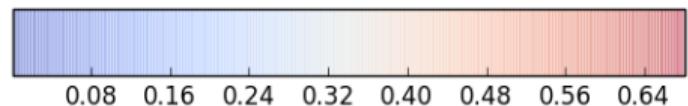
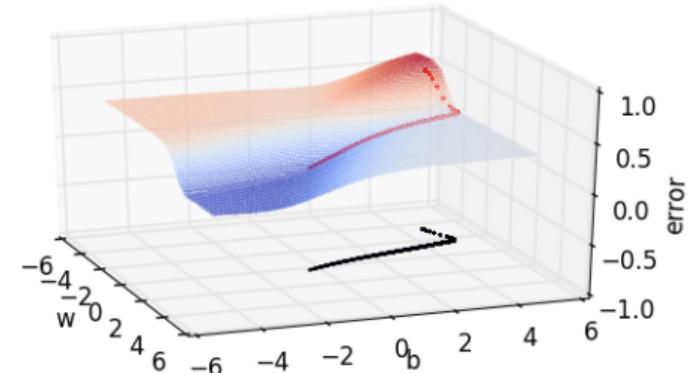
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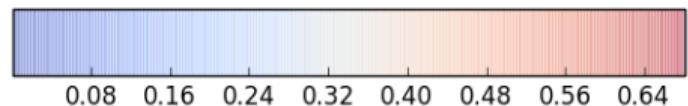
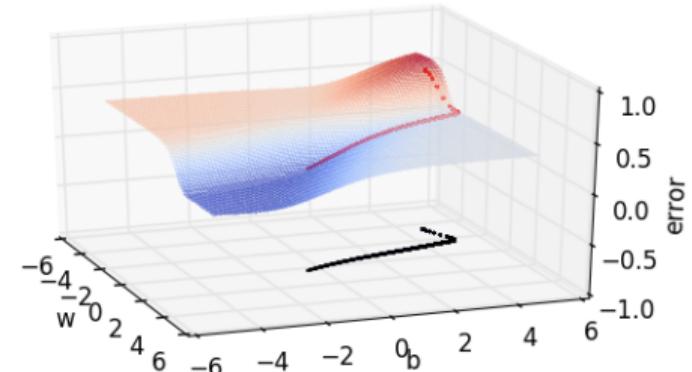
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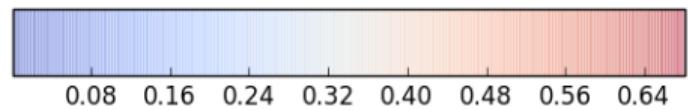
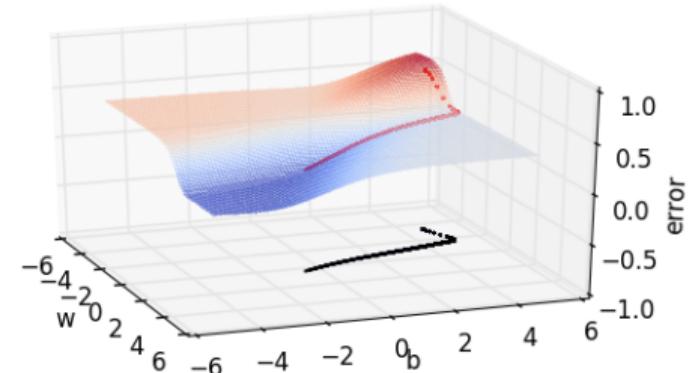
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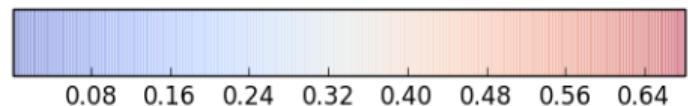
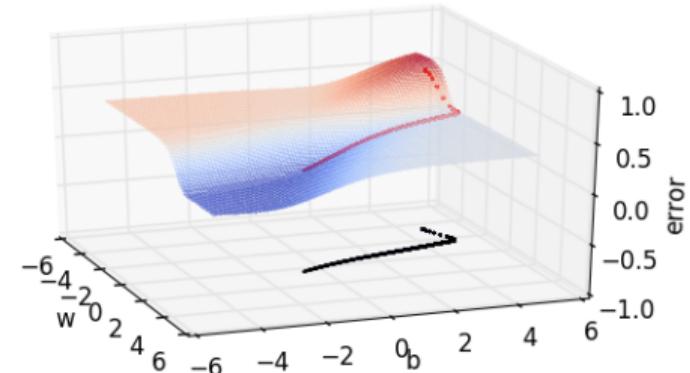
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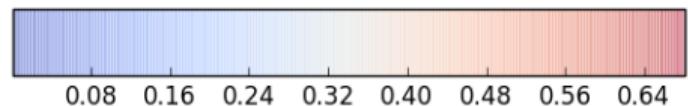
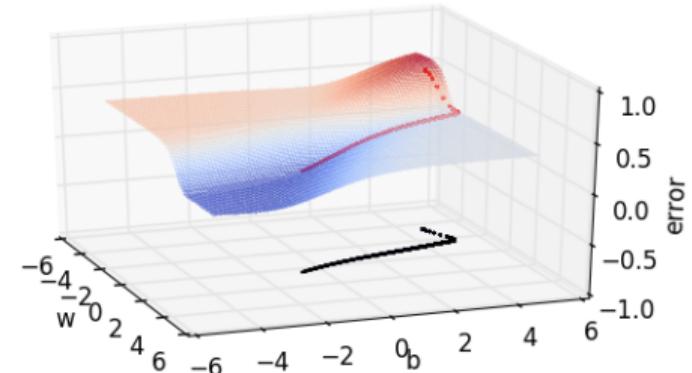
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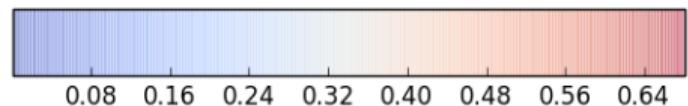
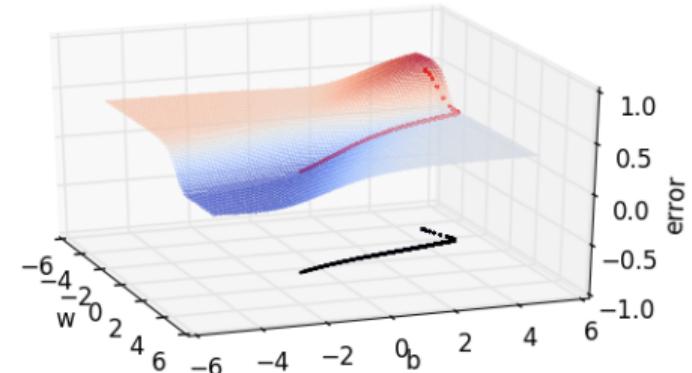
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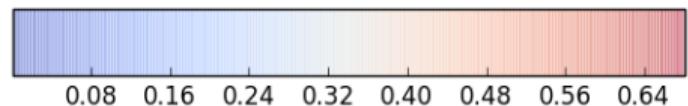
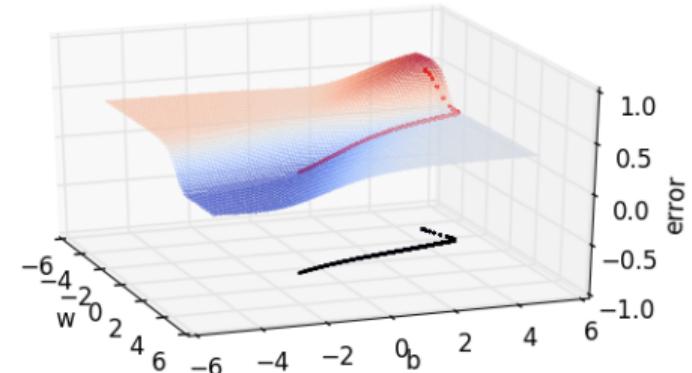
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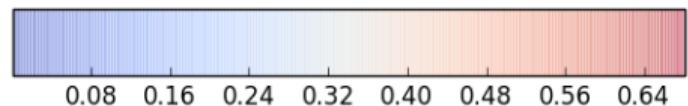
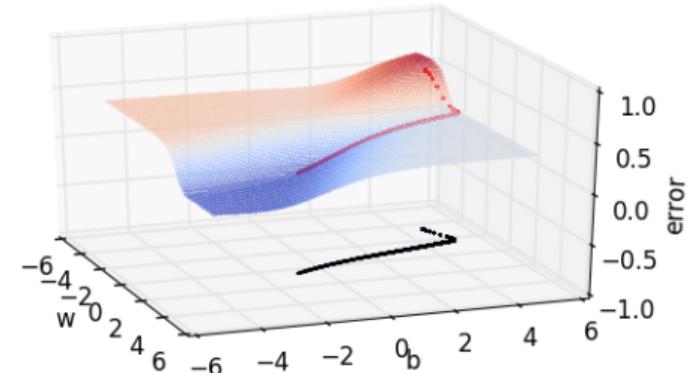
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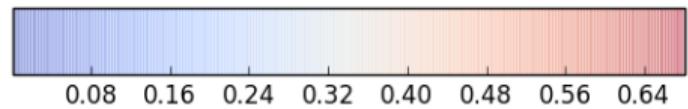
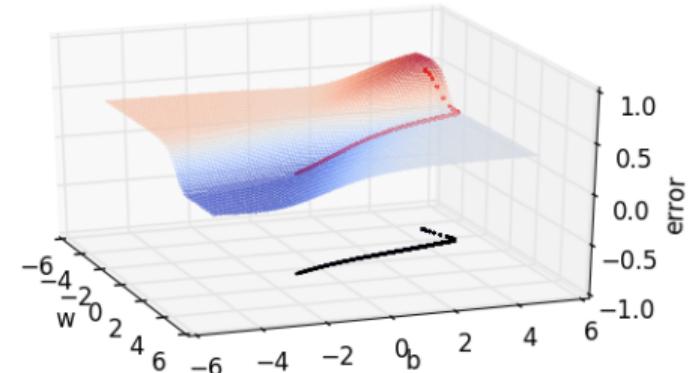
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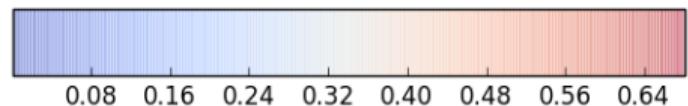
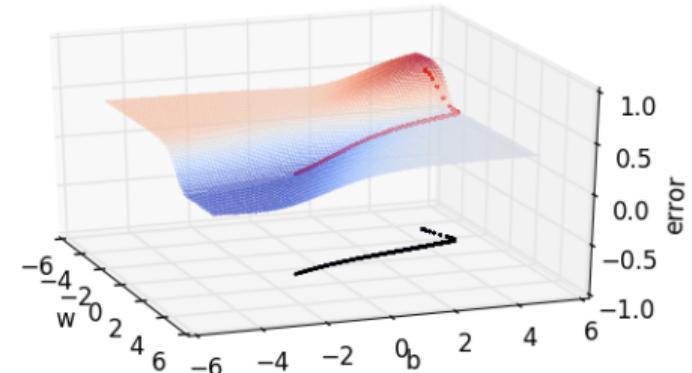
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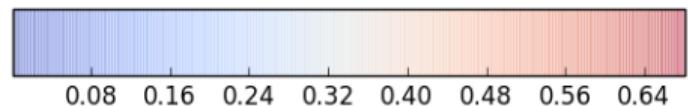
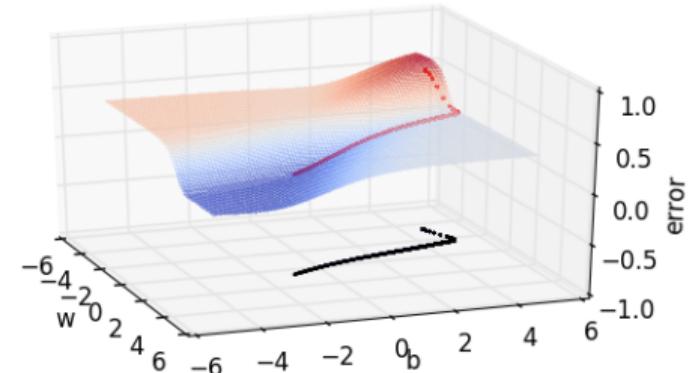
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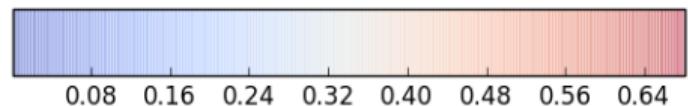
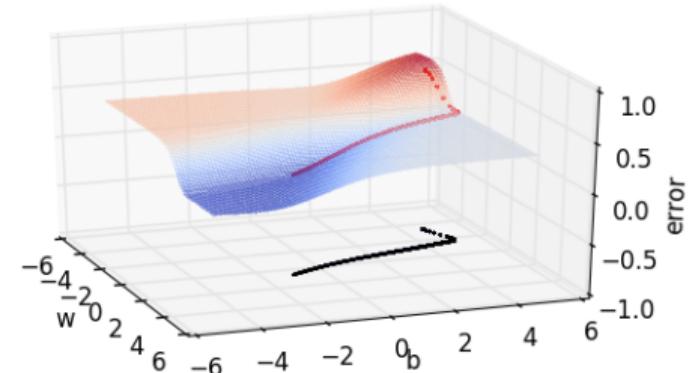
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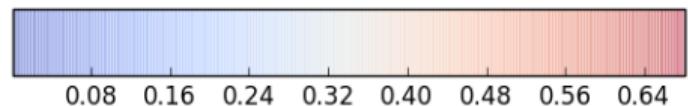
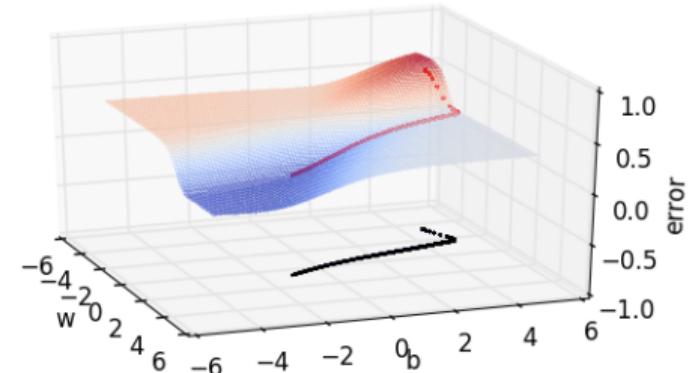
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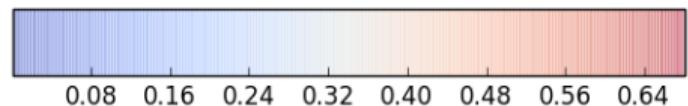
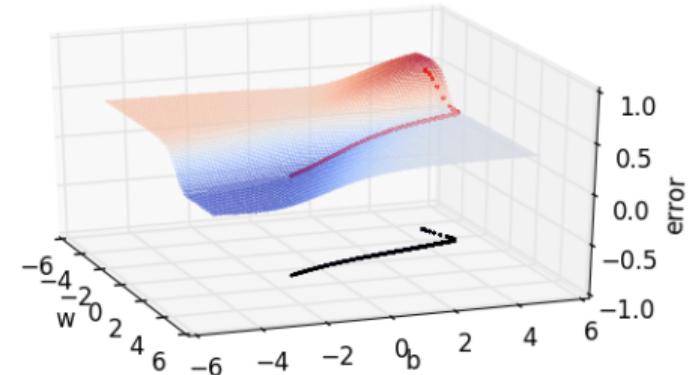
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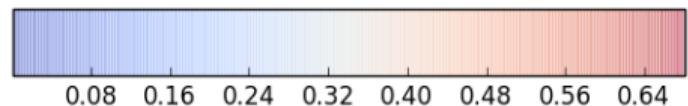
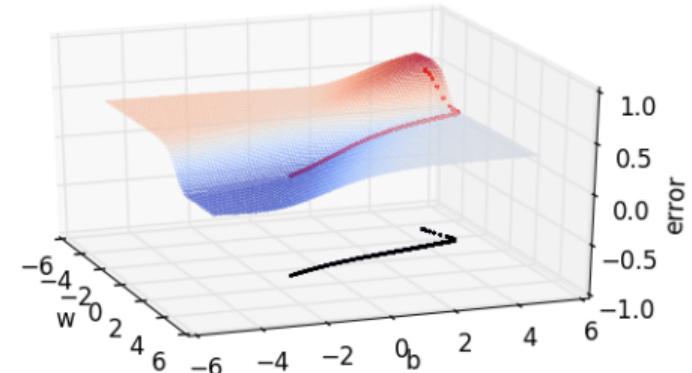
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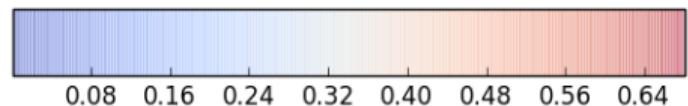
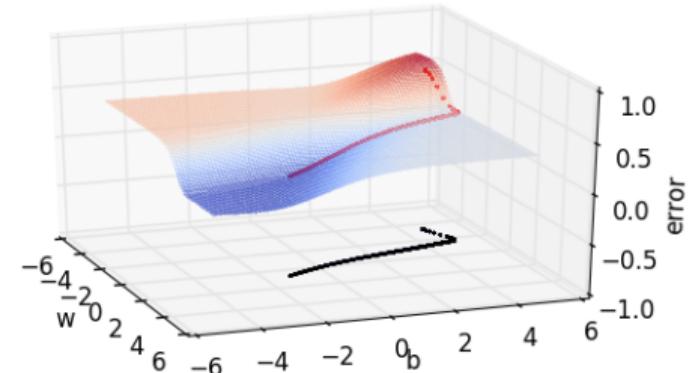
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