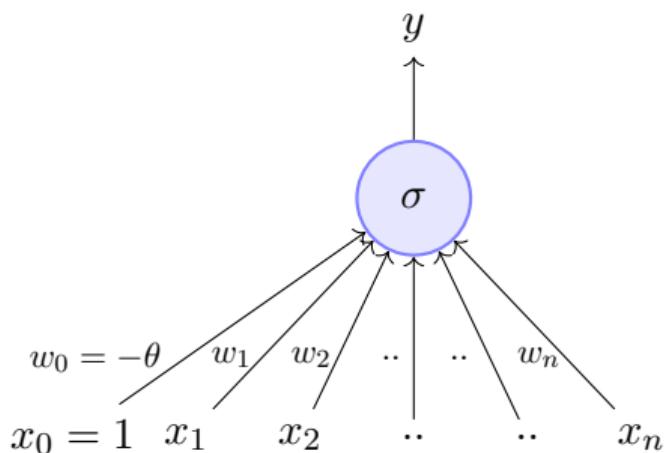
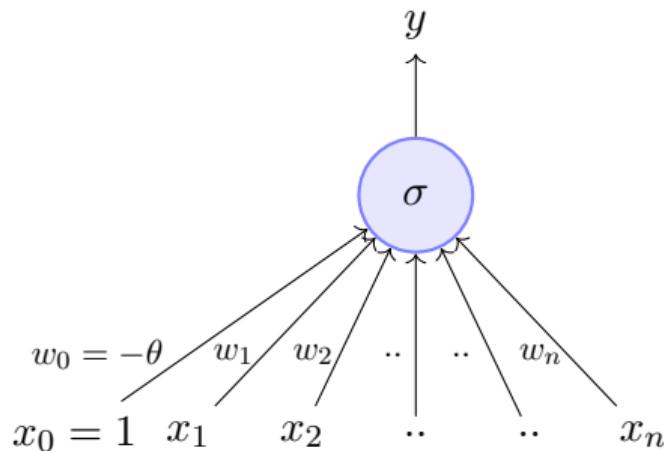


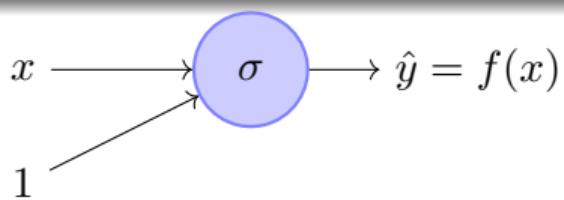
Module 3.4: Learning Parameters : Gradient Descent



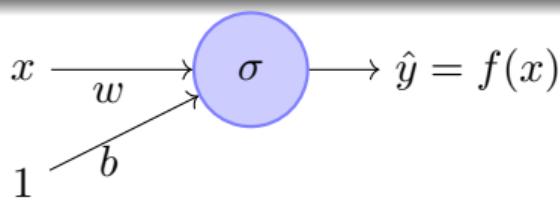
- With this setup in mind, we will now focus on this **model** and discuss an **algorithm** for learning the **parameters** of this model from some given **data**



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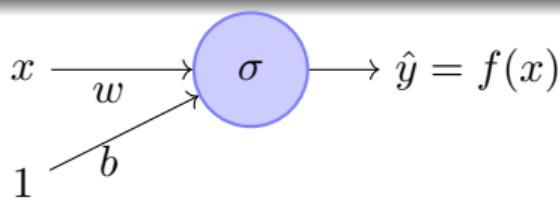


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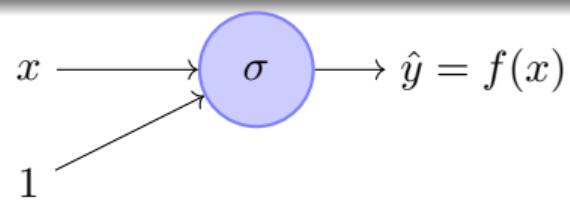
$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

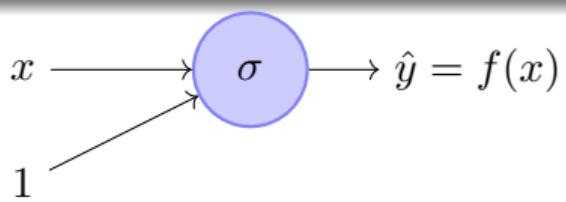
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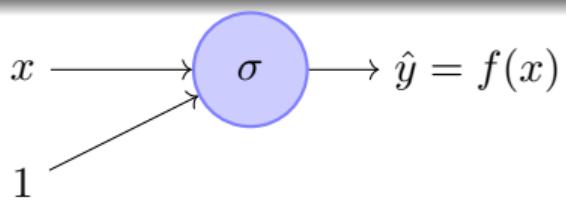
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- Further to be consistent with the literature, from now on we will refer to w_0 as b (bias)
- Lastly, instead of considering the problem of predicting like/dislike we will assume that we want to predict $\text{criticsRating}(y)$ given $\text{imdbRating}(x)$ (for no particular reason)





Input for training

$\{x_i, y_i\}_{i=1}^N \rightarrow N$ pairs of (x, y)



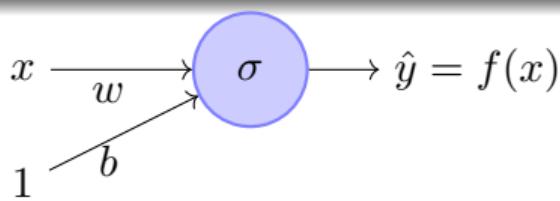
Input for training

$$\{x_i, y_i\}_{i=1}^N \rightarrow N \text{ pairs of } (x, y)$$

Training objective

Find w and b such that:

$$\underset{w,b}{\text{minimize}} \mathcal{L}(w, b) = \sum_{i=1}^N (y_i - f(x_i))^2$$



$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

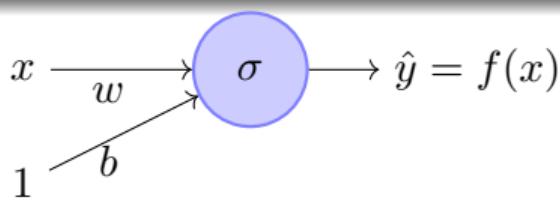
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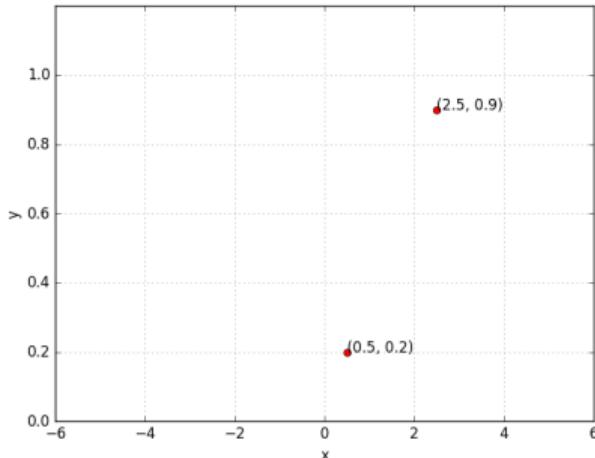
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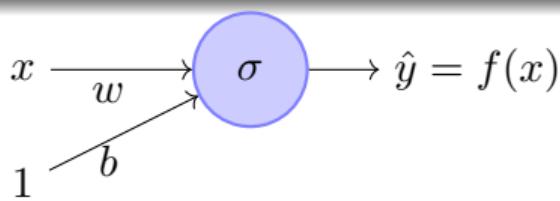


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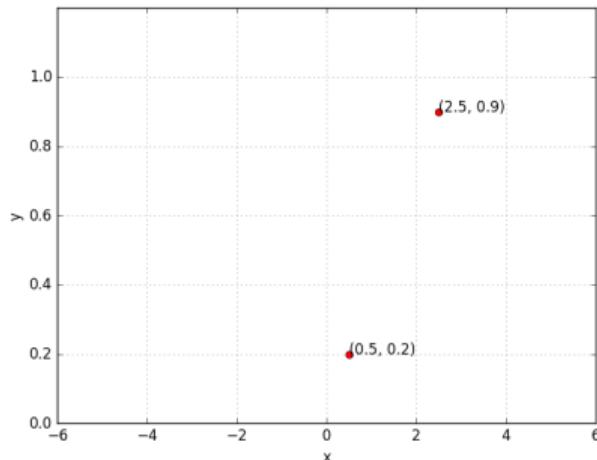


What does it mean to train the network?

- Suppose we train the network with $(x, y) = (0.5, 0.2)$ and $(2.5, 0.9)$

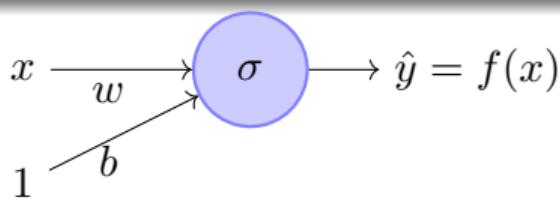


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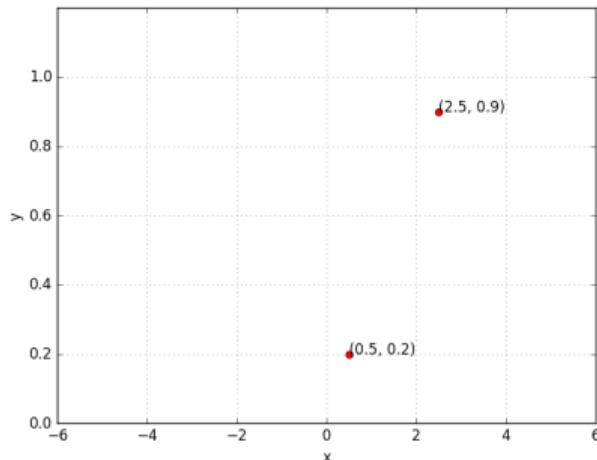


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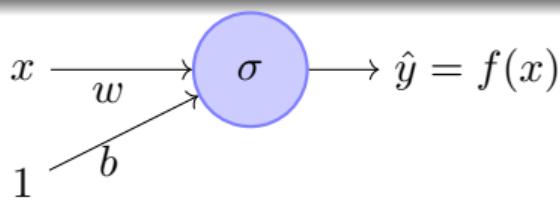


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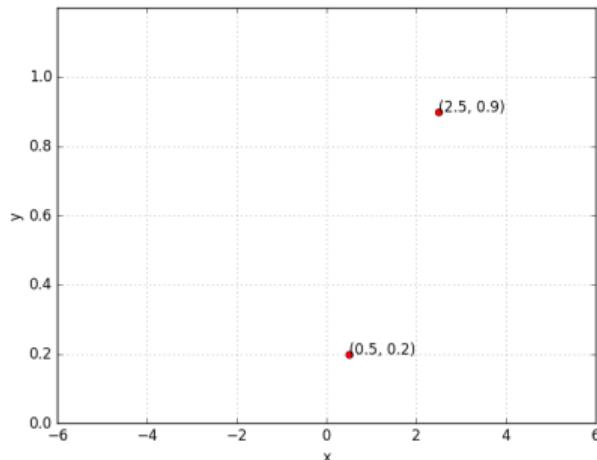


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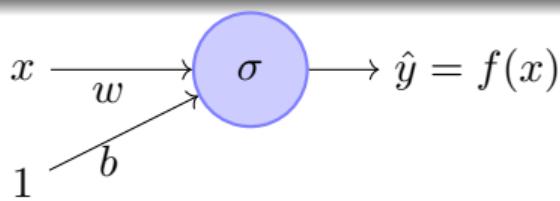


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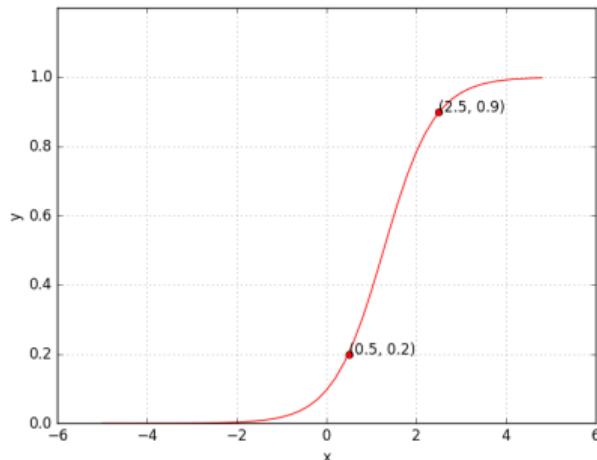
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In other words...

- We hope to find a sigmoid function such that $(0.5, 0.2)$ and $(2.5, 0.9)$ lie on this sigmoid



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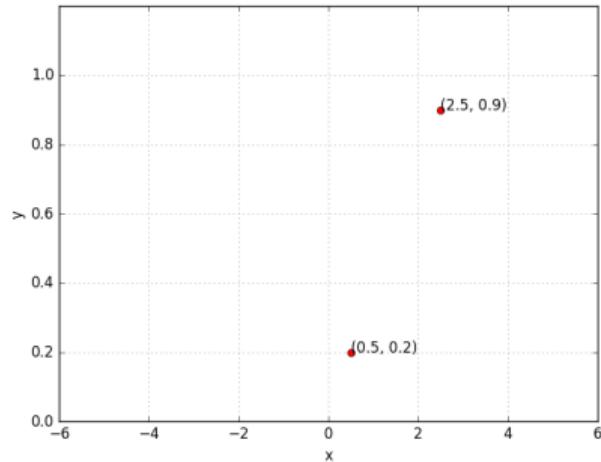
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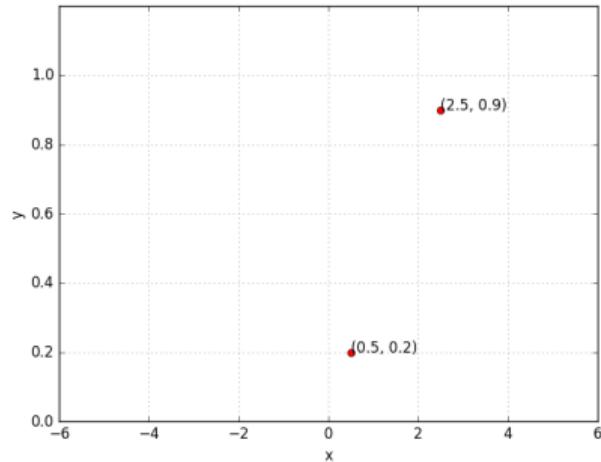
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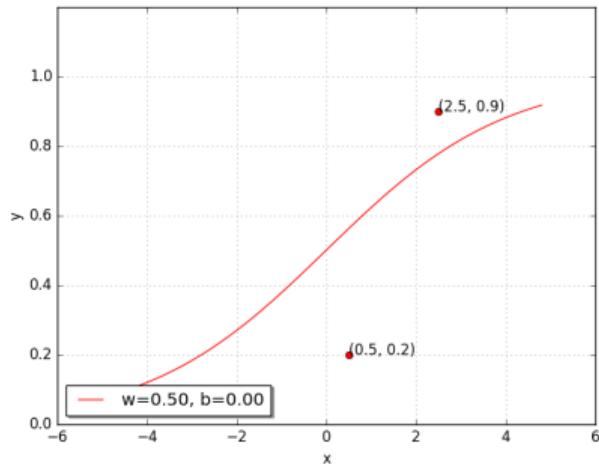
Let us see this in more detail....



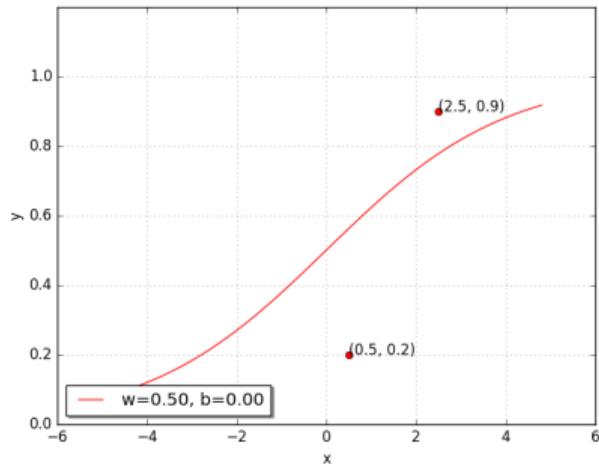
- Can we try to find such a w^*, b^* manually



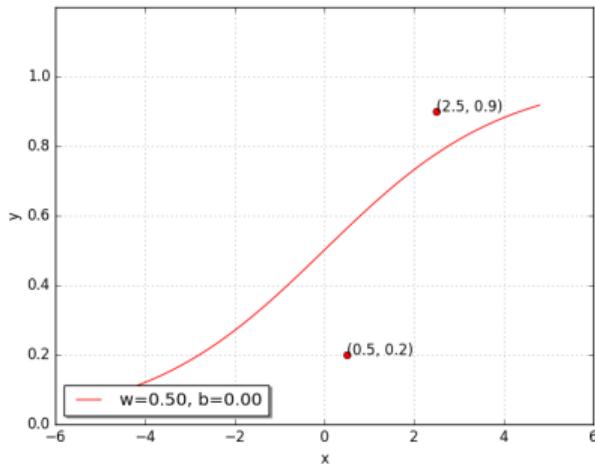
- Can we try to find such a w^*, b^* manually
- Let us try a random guess.. (say, $w = 0.5, b = 0$)

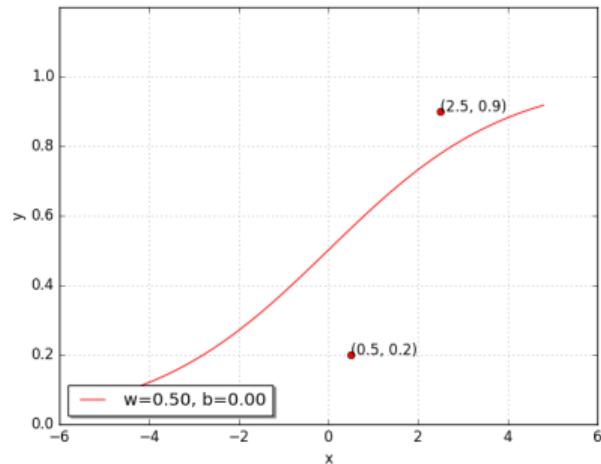


- Can we try to find such a w^*, b^* manually
- Let us try a random guess.. (say, $w = 0.5, b = 0$)
- Clearly not good, but how bad is it ?

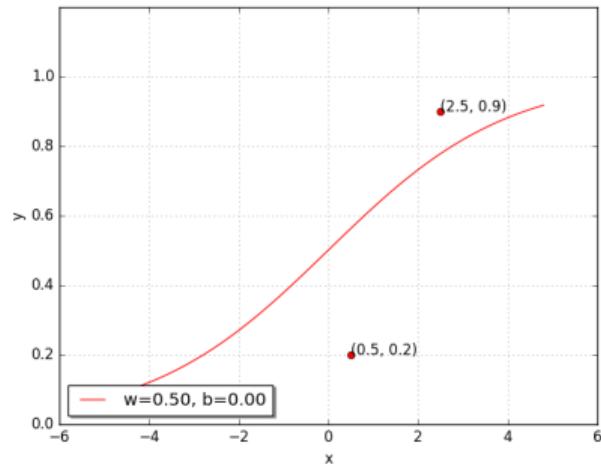


- Can we try to find such a w^*, b^* manually
- Let us try a random guess.. (say, $w = 0.5, b = 0$)
- Clearly not good, but how bad is it ?
- Let us revisit $\mathcal{L}(w, b)$ to see how bad it is ...

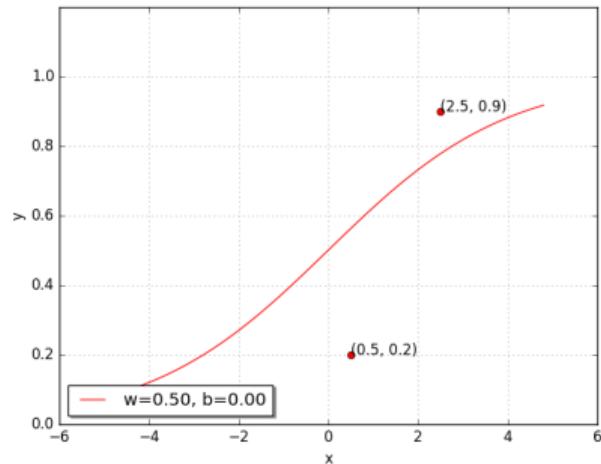




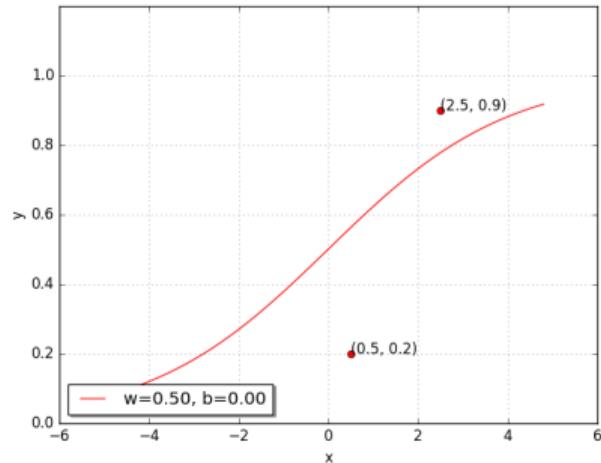
$$\mathcal{L}(w, b) = \frac{1}{2} * \sum_{i=1}^N (y_i - f(x_i))^2$$



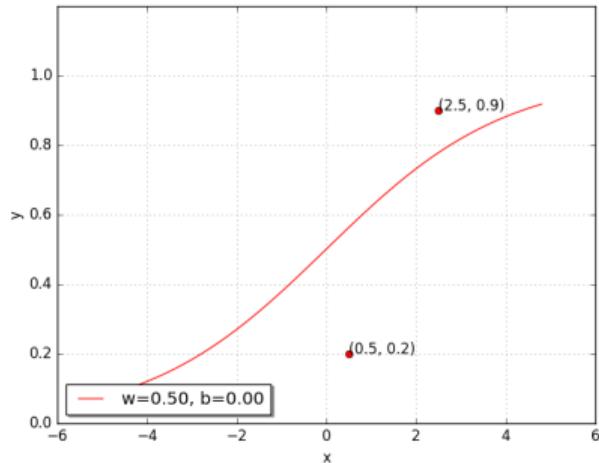
$$\begin{aligned}\mathcal{L}(w, b) &= \frac{1}{2} * \sum_{i=1}^N (y_i - f(x_i))^2 \\ &= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2\end{aligned}$$



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 \mathcal{L}(w, b) &= \frac{1}{2} * \sum_{i=1}^N (y_i - f(x_i))^2 \\
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 &= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2
 \end{aligned}$$



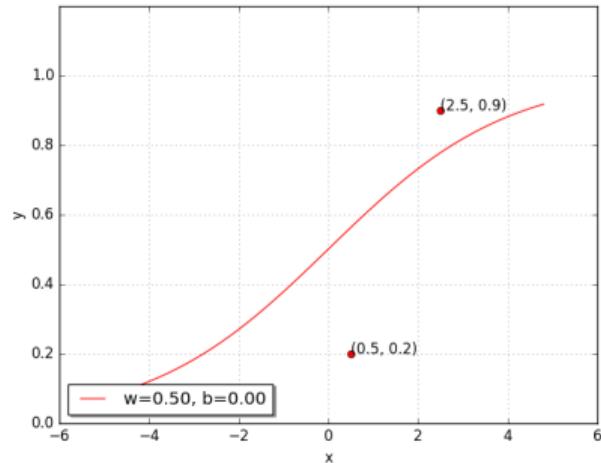
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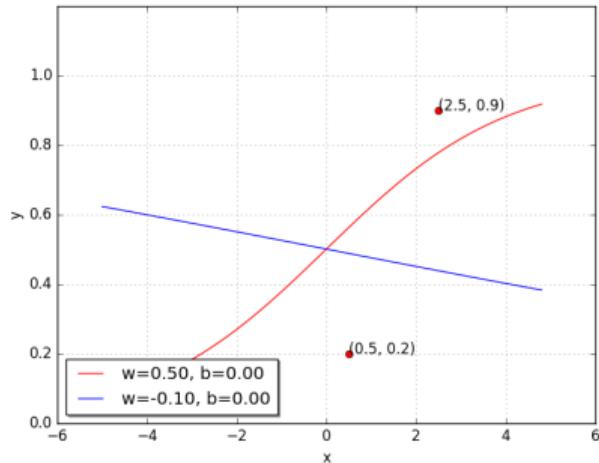
We want $\mathcal{L}(w, b)$ to be as close to 0 as possible

Let us try some other values of w, b



| w | b | $\mathcal{L}(w, b)$ |
|------|------|---------------------|
| 0.50 | 0.00 | 0.0730 |

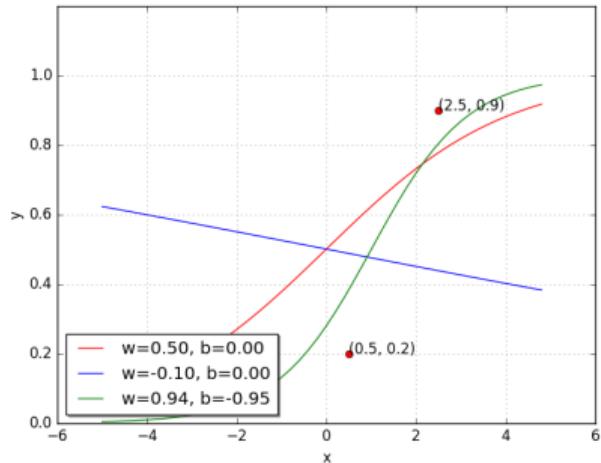
Let us try some other values of w, b



| w | b | $\mathcal{L}(w, b)$ |
|-------|------|---------------------|
| 0.50 | 0.00 | 0.0730 |
| -0.10 | 0.00 | 0.1481 |

Oops!! this made things even worse...

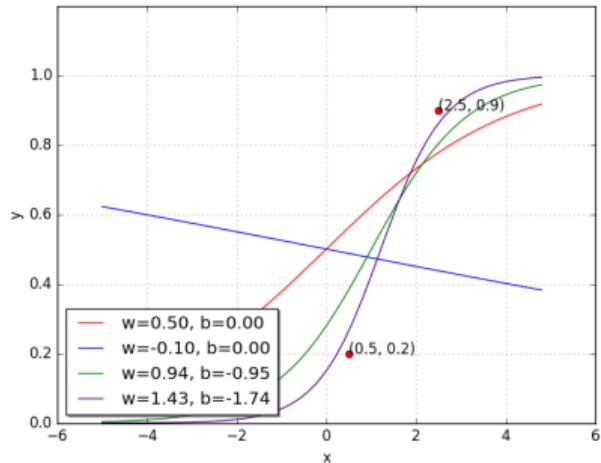
Let us try some other values of w , b



| w | b | $\mathcal{L}(w, b)$ |
|-------|-------|---------------------|
| 0.50 | 0.00 | 0.0730 |
| -0.10 | 0.00 | 0.1481 |
| 0.94 | -0.94 | 0.0214 |

Perhaps it would help to push w and b in the other direction...

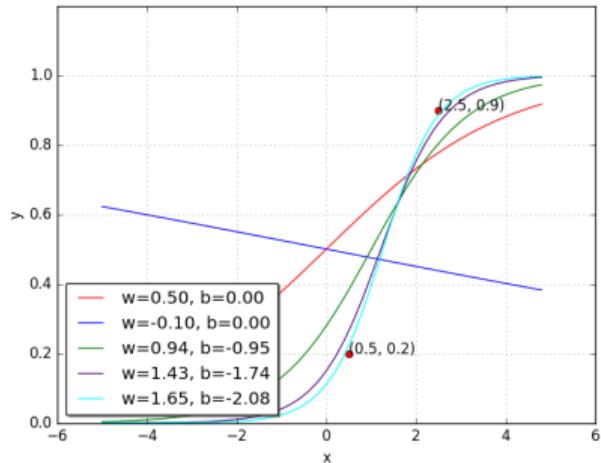
Let us try some other values of w , b



| w | b | $\mathcal{L}(w, b)$ |
|-------|-------|---------------------|
| 0.50 | 0.00 | 0.0730 |
| -0.10 | 0.00 | 0.1481 |
| 0.94 | -0.94 | 0.0214 |
| 1.42 | -1.73 | 0.0028 |

Let us keep going in this direction, *i.e.*, increase w and decrease b

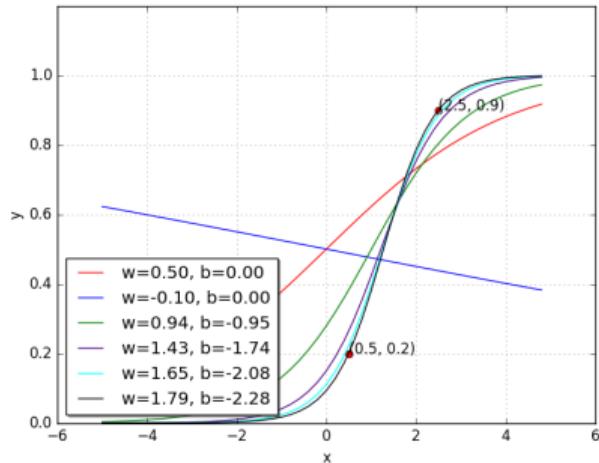
Let us try some other values of w , b



| w | b | $\mathcal{L}(w, b)$ |
|-------|-------|---------------------|
| 0.50 | 0.00 | 0.0730 |
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| 0.94 | -0.94 | 0.0214 |
| 1.42 | -1.73 | 0.0028 |
| 1.65 | -2.08 | 0.0003 |

Let us keep going in this direction, *i.e.*, increase w and decrease b

Let us try some other values of w , b



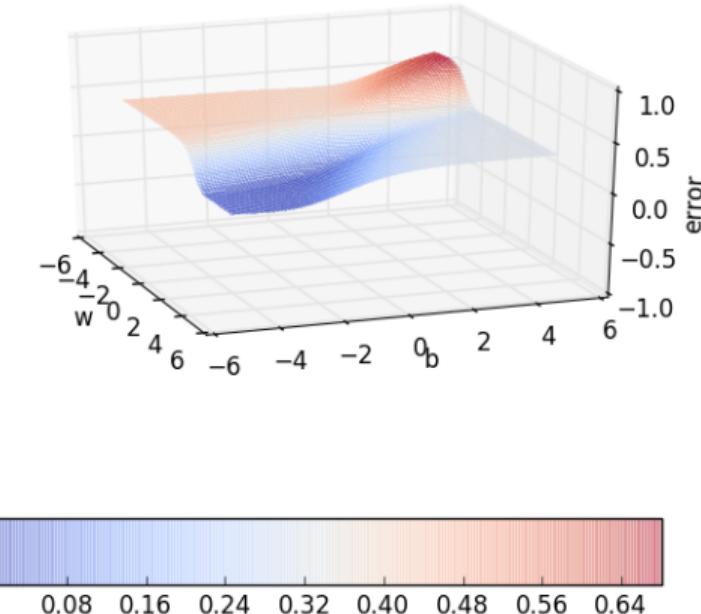
| w | b | $\mathcal{L}(w, b)$ |
|-------|-------|---------------------|
| 0.50 | 0.00 | 0.0730 |
| -0.10 | 0.00 | 0.1481 |
| 0.94 | -0.94 | 0.0214 |
| 1.42 | -1.73 | 0.0028 |
| 1.65 | -2.08 | 0.0003 |
| 1.78 | -2.27 | 0.0000 |

With some guess work and intuition we were able to find the right values for w and b

Let us look at something better than our “guess work” algorithm....

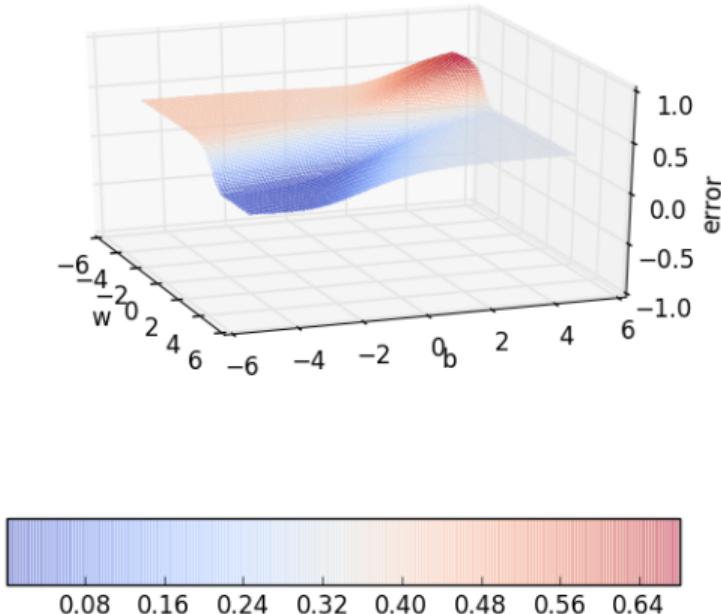
- Since we have only 2 points and 2 parameters (w , b) we can easily plot $\mathcal{L}(w, b)$ for different values of (w, b) and pick the one where $\mathcal{L}(w, b)$ is minimum

Random search on error surface



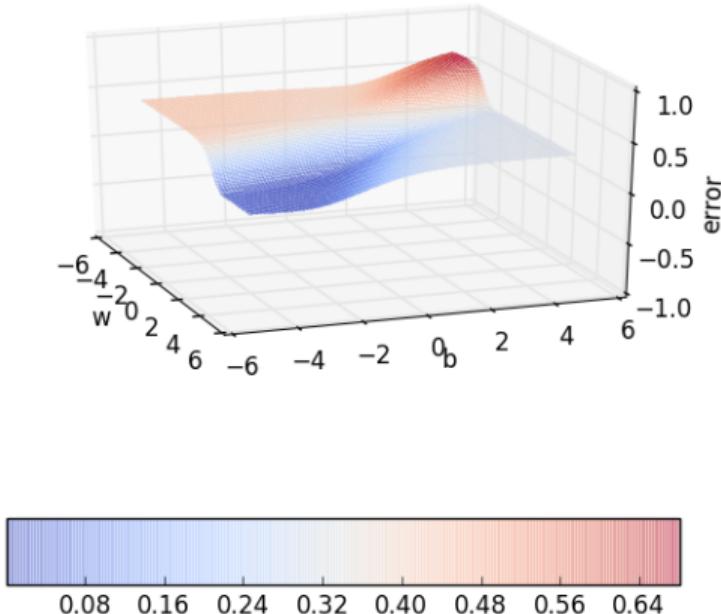
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Random search on error surface



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- But of course this becomes intractable once you have many more data points and many more parameters !!

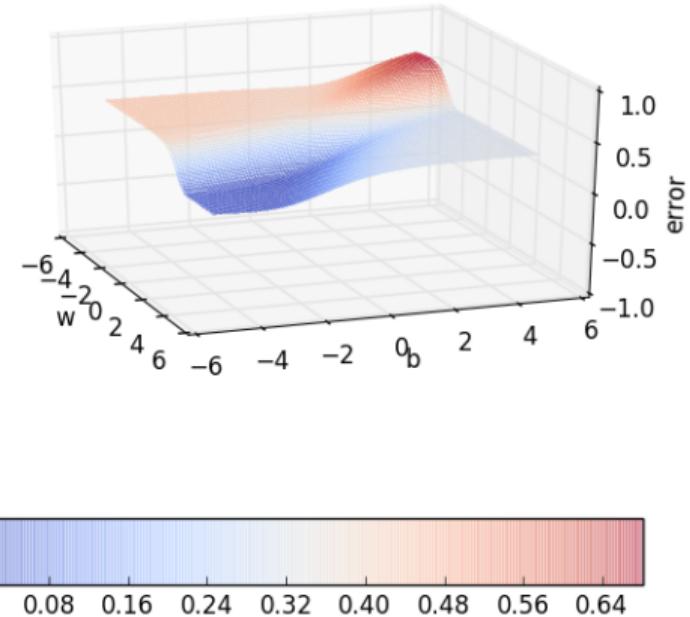
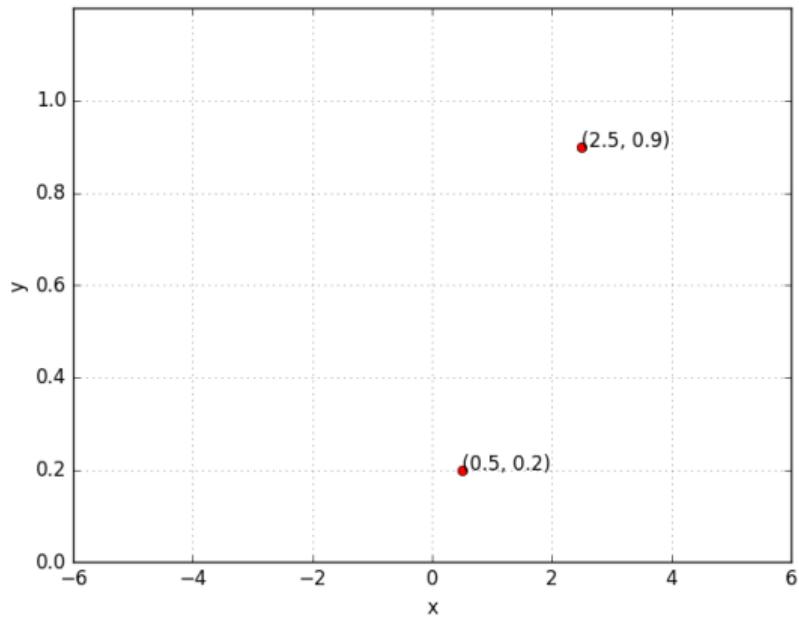
Random search on error surface

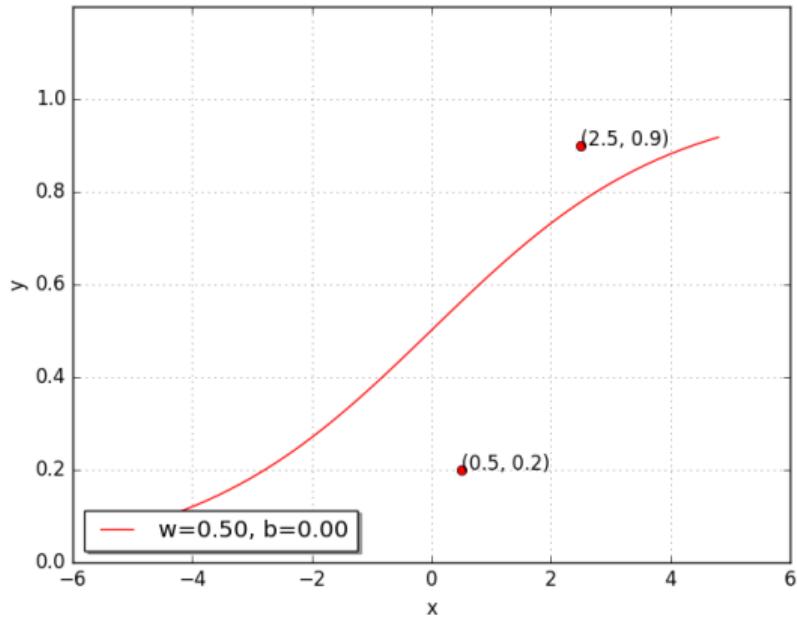


- Since we have only 2 points and 2 parameters (w, b) we can easily plot $\mathcal{L}(w, b)$ for different values of (w, b) and pick the one where $\mathcal{L}(w, b)$ is minimum
- But of course this becomes intractable once you have many more data points and many more parameters !!
- Further, even here we have plotted the error surface only for a small range of (w, b) [from $(-6, 6)$ and not from $(-\infty, \infty)$]

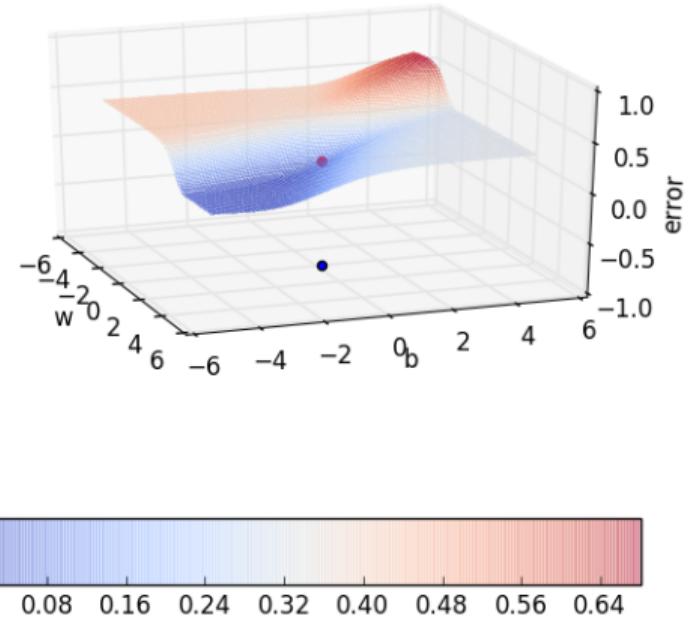
Let us look at the geometric interpretation of our “guess work” algorithm in terms of this error surface

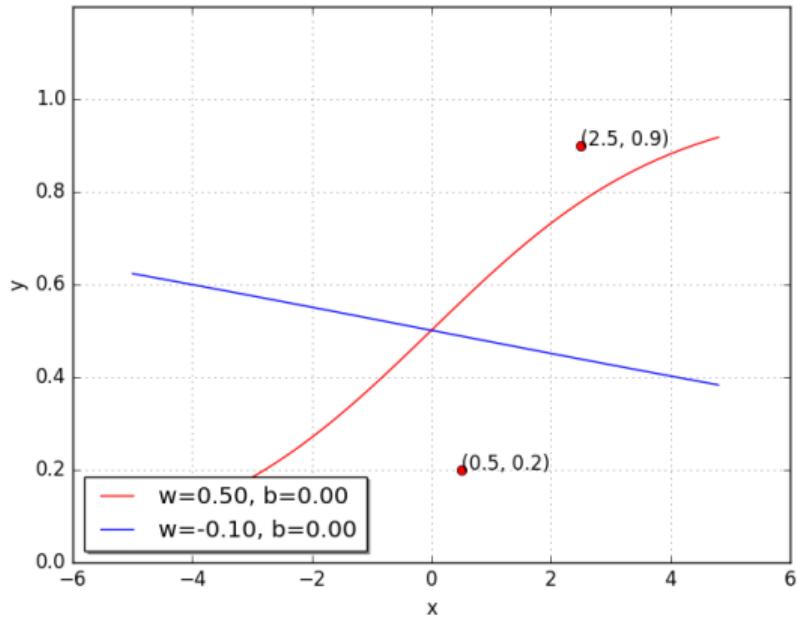
Random search on error surface



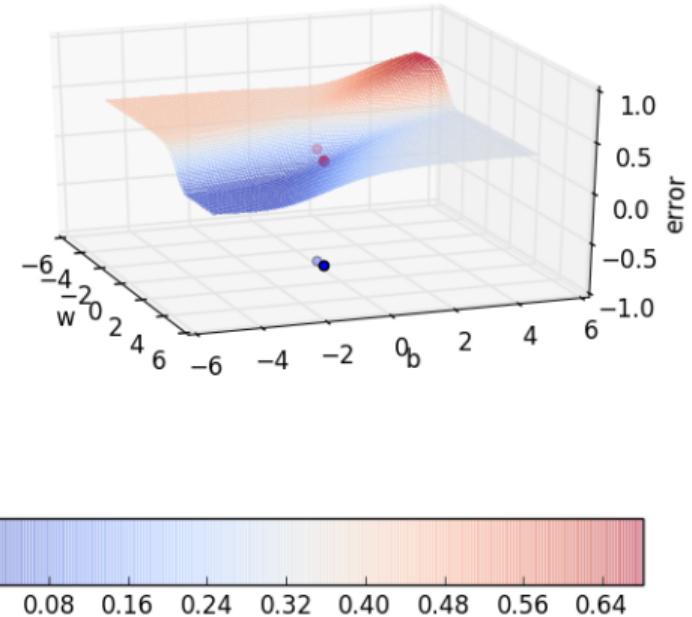


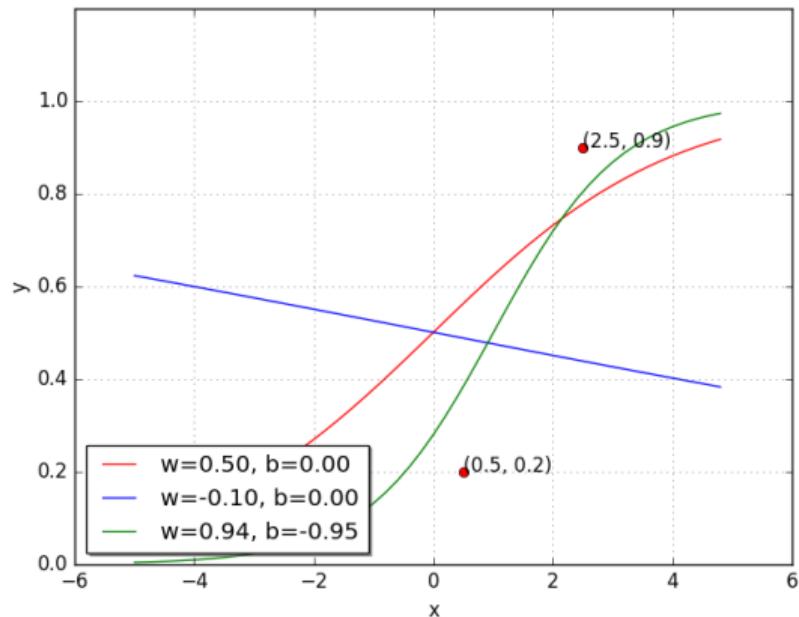
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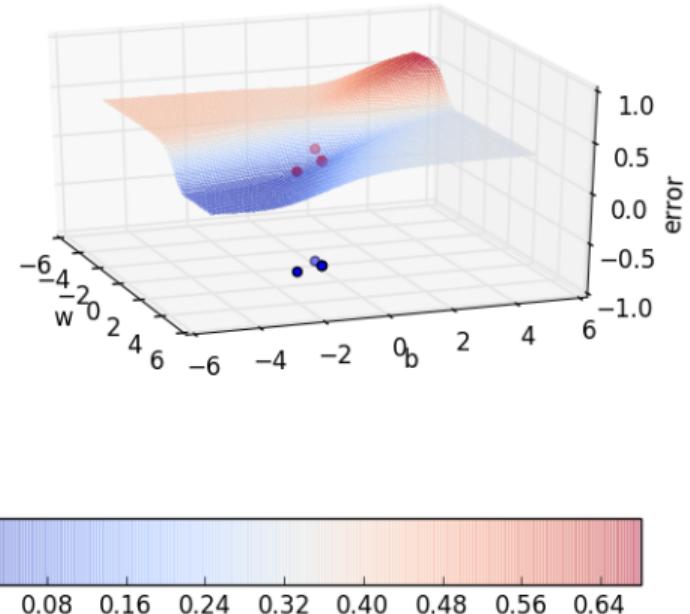


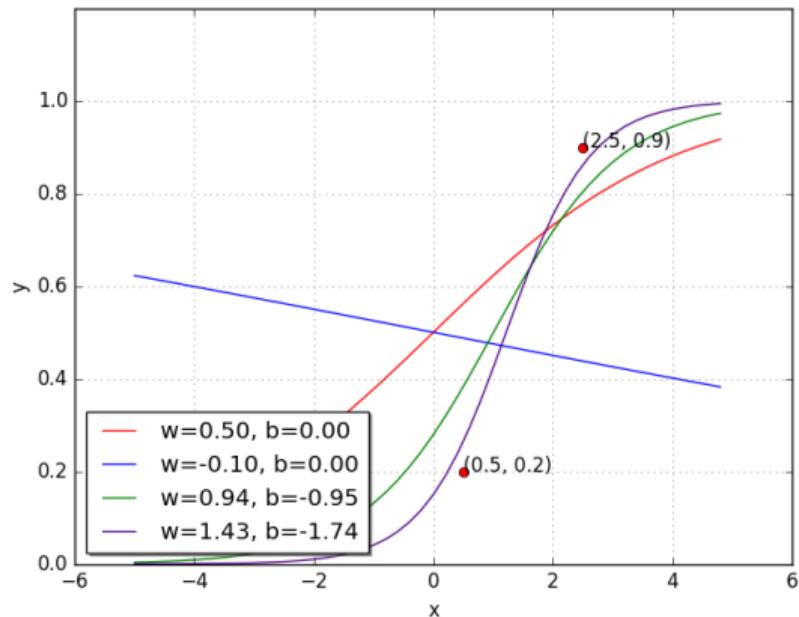
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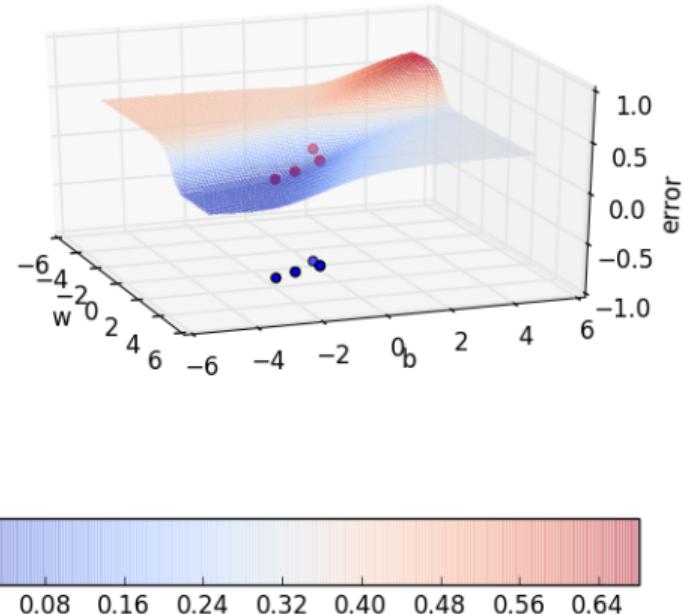


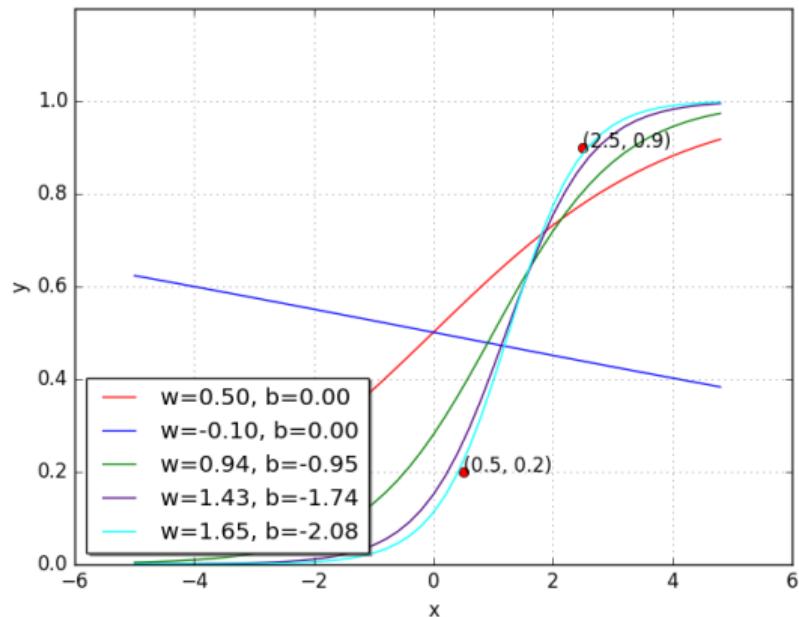
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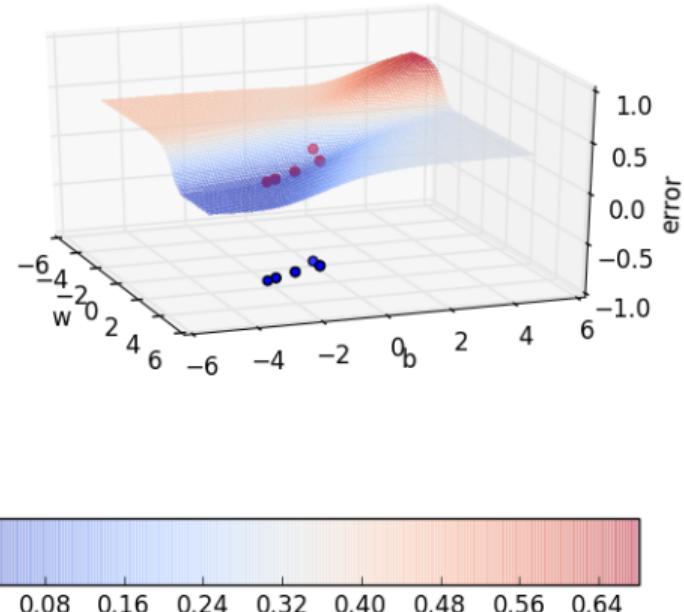


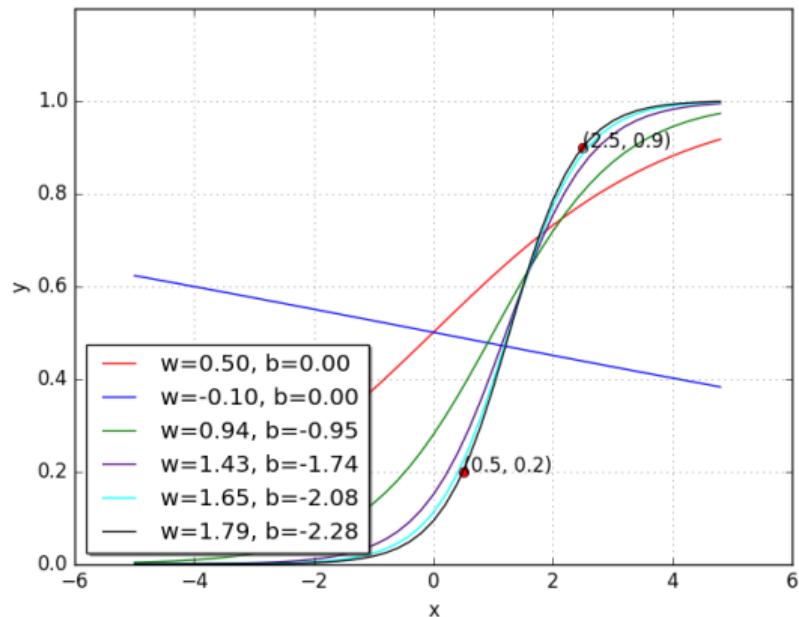
Random search on error surface



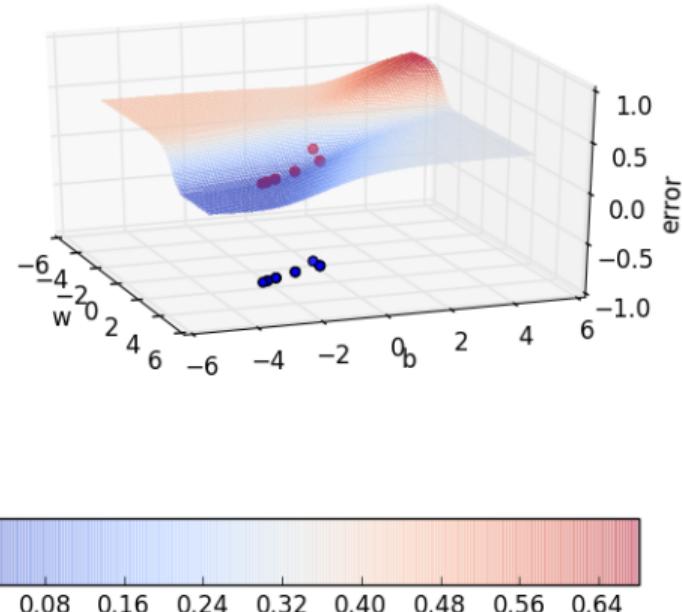


Random search on error surface





Random search on error surface



Now let us see if there is a more efficient and principled way of doing this

Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!

vector of parameters,
say, randomly initialized

$$\theta = [w, b]$$

vector of parameters,
say, randomly initial-
ized

$$\theta = [w, b]$$

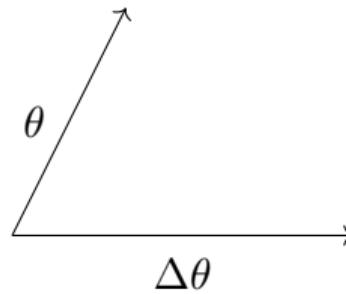
$$\Delta\theta = [\Delta w, \Delta b]$$

change in the
values of w, b

vector of parameters,
say, randomly initial-
ized

$$\theta = [w, b]$$
$$\Delta\theta = [\Delta w, \Delta b]$$

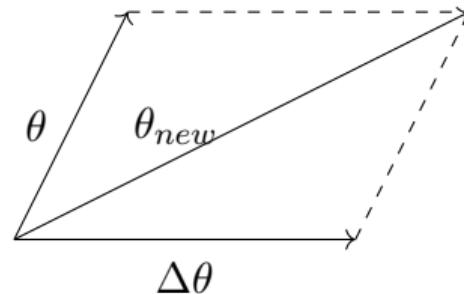
change in the
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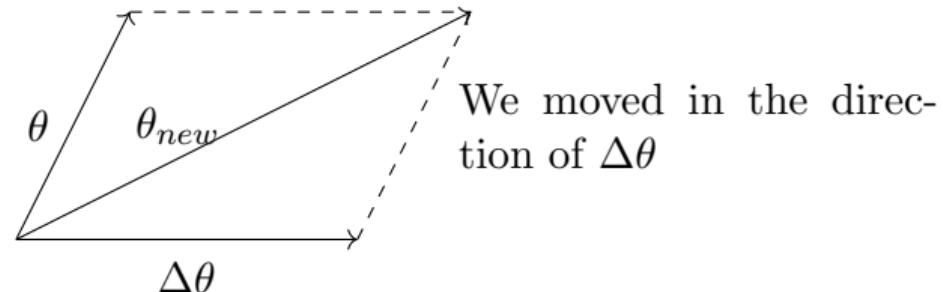
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vector of parameters,
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change in the
values of w, b

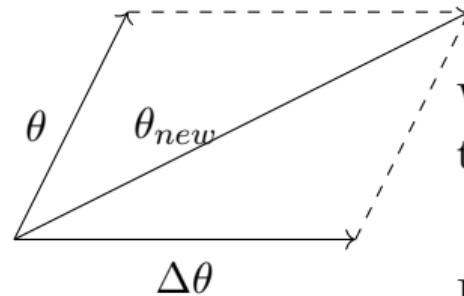


vector of parameters,
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$$\theta = [w, b]$$

change in the
values of w, b

$$\Delta\theta = [\Delta w, \Delta b]$$



We moved in the direction of $\Delta\theta$

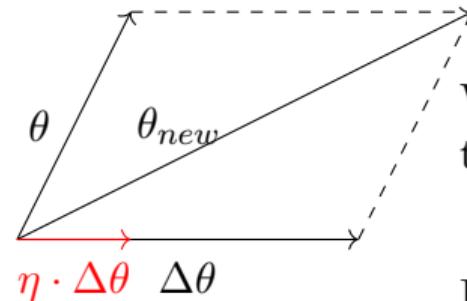
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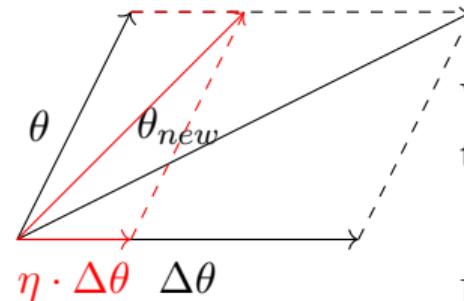
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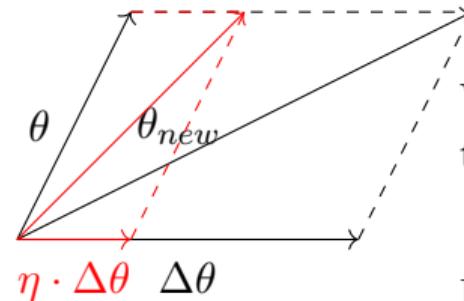
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$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$



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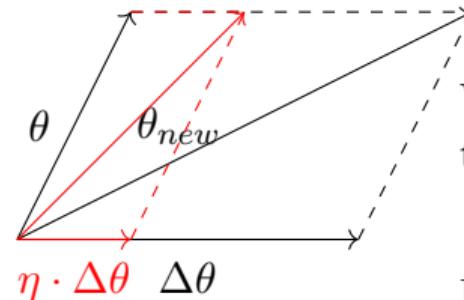
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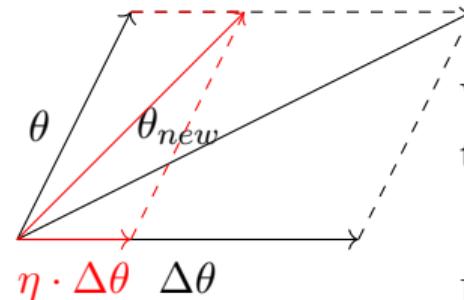
Question: What is the right $\Delta\theta$ to use?
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The answer comes from Taylor series

For ease of notation, let $\Delta\theta = u$, then from Taylor series, we have,

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Note that the move (ηu) would be favorable only if,

$$\mathcal{L}(\theta + \eta u) - \mathcal{L}(\theta) < 0 [\text{i.e., if the new loss is less than the previous loss}]$$

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This implies,

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Okay, so we have,

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But, what is the range of $u^T \nabla \mathcal{L}(\theta)$?

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multiply throughout by $k = \|u\| * \|\nabla \mathcal{L}(\theta)\|$

$$-k \leq k * \cos(\beta) = u^T \nabla \mathcal{L}(\theta) \leq k$$

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multiply throughout by $k = \|u\| * \|\nabla \mathcal{L}(\theta)\|$

$$-k \leq k * \cos(\beta) = u^T \nabla \mathcal{L}(\theta) \leq k$$

Thus, $\mathcal{L}(\theta + \eta u) - \mathcal{L}(\theta) = u^T \nabla \mathcal{L}(\theta) = k * \cos(\beta)$ will be most negative when $\cos(\beta) = -1$ i.e., when β is 180°

Gradient Descent Rule

- The direction u that we intend to move in should be at 180° w.r.t. the gradient

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Parameter Update Equations

$$w_{t+1} = w_t - \eta \nabla w_t$$

$$b_{t+1} = b_t - \eta \nabla b_t$$

where, $\nabla w_t = \frac{\partial \mathcal{L}(w, b)}{\partial w}$ at $w = w_t, b = b_t$, $\nabla b = \frac{\partial \mathcal{L}(w, b)}{\partial b}$ at $w = w_t, b = b_t$

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So we now have a more principled way of moving in the w - b plane than our “guess work” algorithm

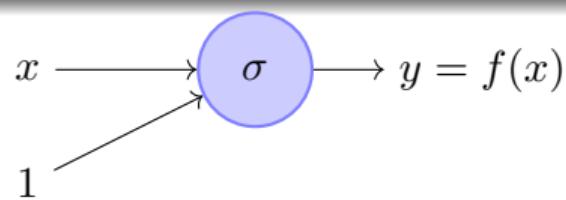
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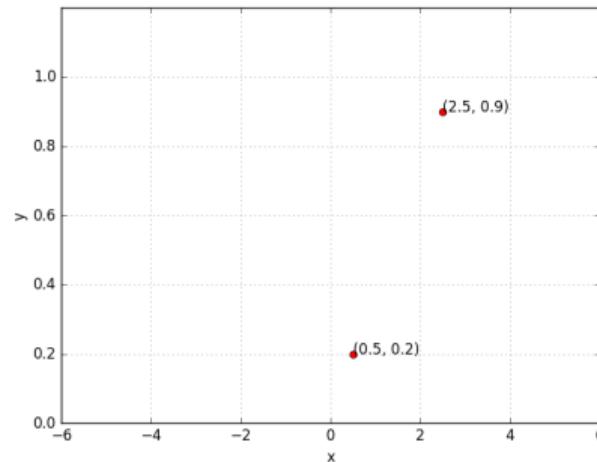
Algorithm: gradient_descent()

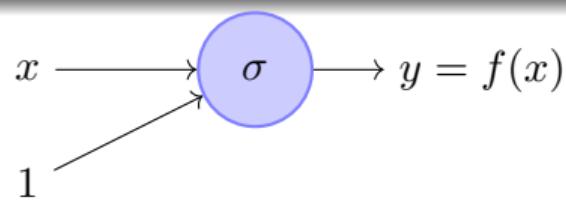
```
t ← 0;  
max_iterations ← 1000;  
while  $t < max\_iterations$  do  
    |  $w_{t+1} \leftarrow w_t - \eta \nabla w_t;$   
    |  $b_{t+1} \leftarrow b_t - \eta \nabla b_t;$   
end
```

- To see this algorithm in practice let's first derive ∇w and ∇b for our toy neural network



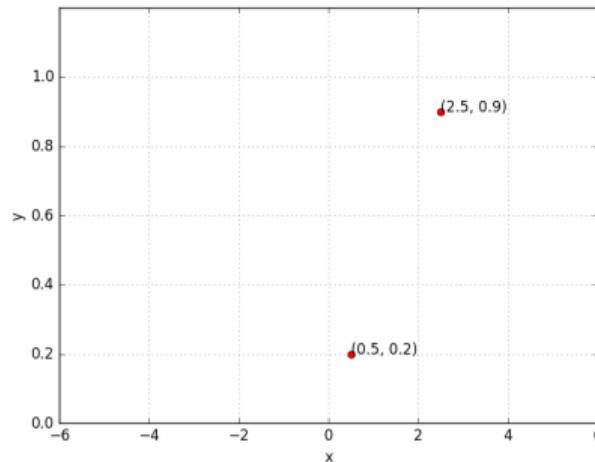
$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

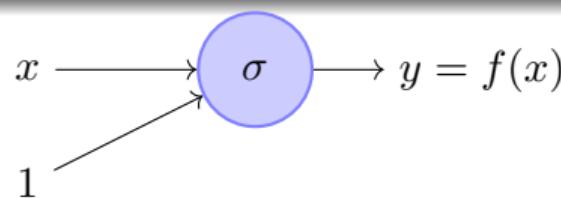




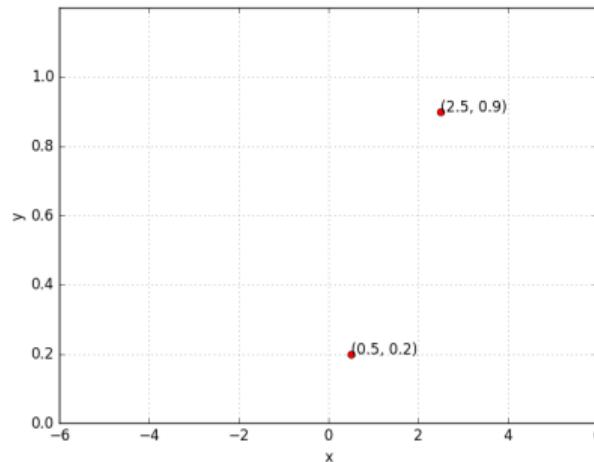
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Let's assume there is only 1 point to fit
 (x, y)



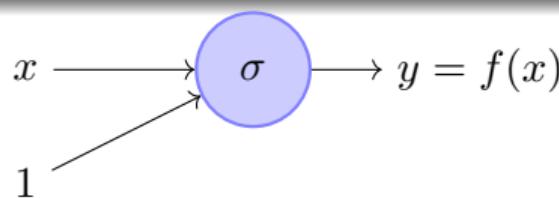


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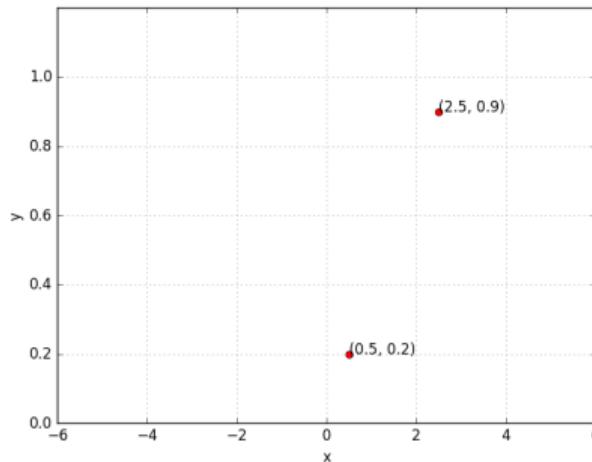


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$$\mathcal{L}(w, b) = \frac{1}{2} * (f(x) - y)^2$$



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Let's assume there is only 1 point to fit
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$$\begin{aligned}\mathcal{L}(w, b) &= \frac{1}{2} * (f(x) - y)^2 \\ \nabla w &= \frac{\partial \mathcal{L}(w, b)}{\partial w} = \frac{\partial}{\partial w} \left[\frac{1}{2} * (f(x) - y)^2 \right]\end{aligned}$$

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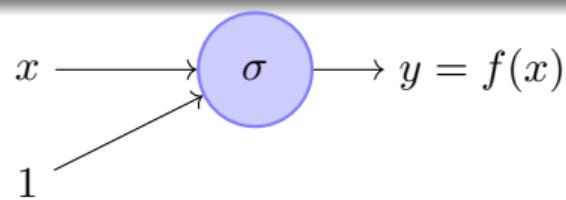
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 &= \frac{1}{(1 + e^{-(wx+b)})} * \frac{e^{-(wx+b)}}{(1 + e^{-(wx+b)})} * (x) \\
 &= f(x) * (1 - f(x)) * x
 \end{aligned}$$

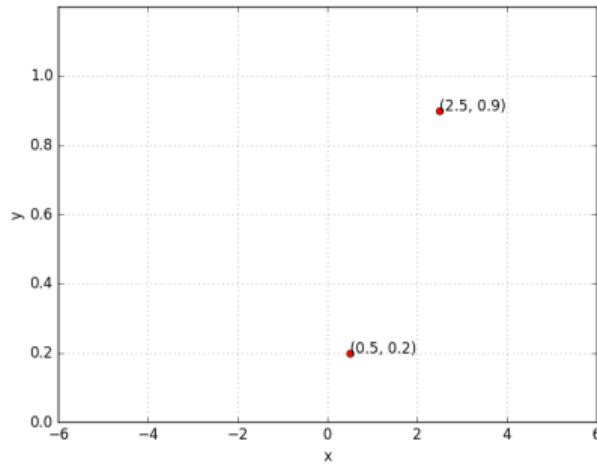
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\end{aligned}$$

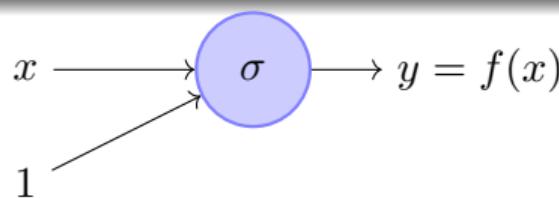
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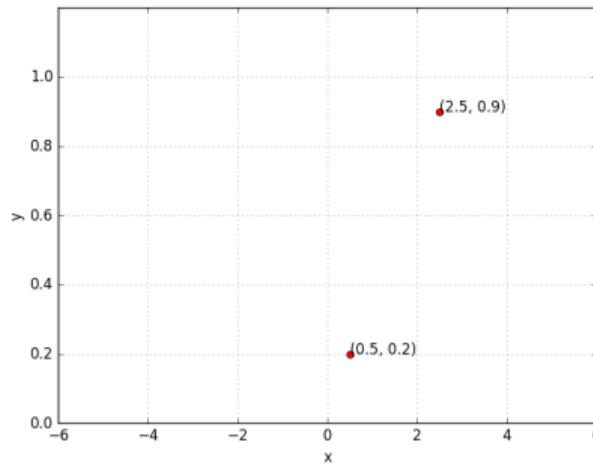
$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

So if there is only 1 point (x, y) , we have,



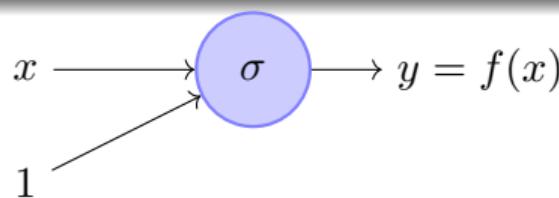


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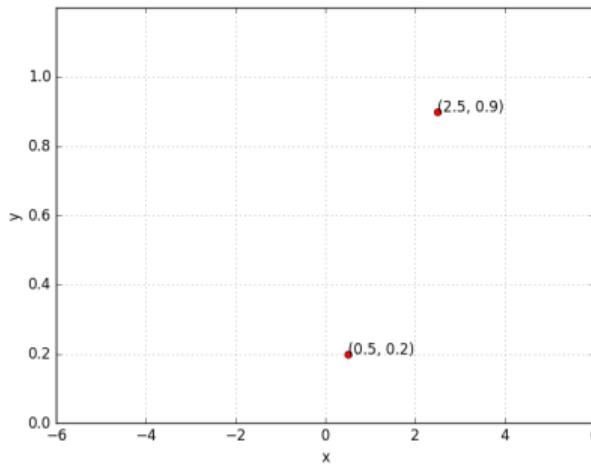


So if there is only 1 point (x, y) , we have,

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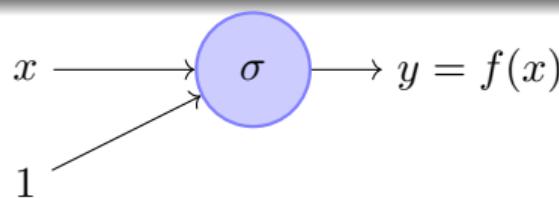
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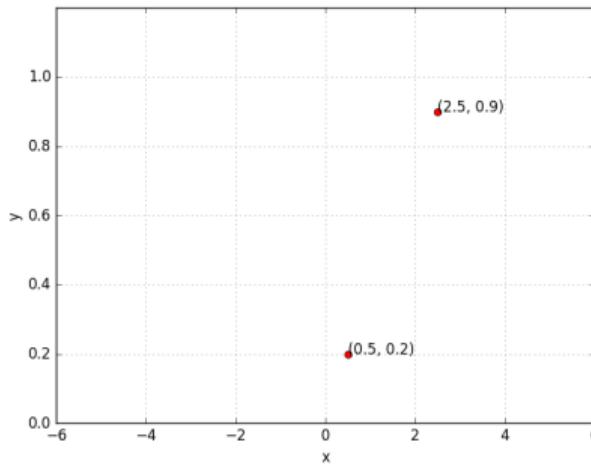
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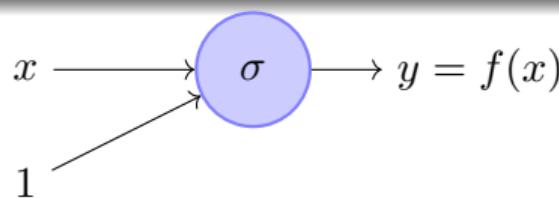


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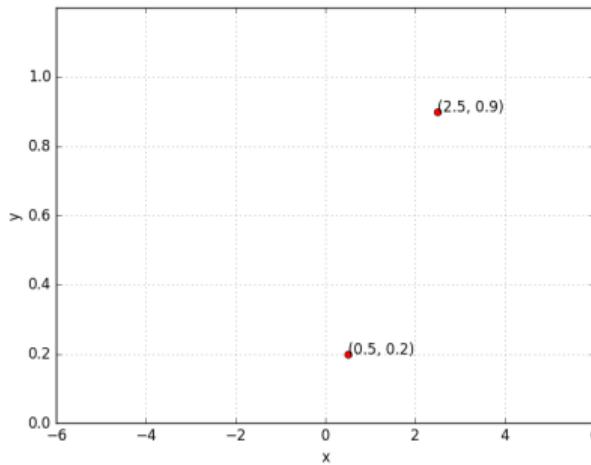
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For two points,

$$\nabla w = \sum_{i=1}^2 (f(x_i) - y_i) * f(x_i) * (1 - f(x_i)) * x_i$$



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For two points,

$$\nabla w = \sum_{i=1}^2 (f(x_i) - y_i) * f(x_i) * (1 - f(x_i)) * x_i$$

$$\nabla b = \sum_{i=1}^2 (f(x_i) - y_i) * f(x_i) * (1 - f(x_i))$$

```
X = [0.5, 2.5]  
Y = [0.2, 0.9]
```

```
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def f(w,b,x) : #sigmoid with parameters w,b
    return 1.0 / (1.0 + np.exp(-(w*x + b)))
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def f(w,b,x) : #sigmoid with parameters w,b
    return 1.0 / (1.0 + np.exp(-(w*x + b)))

def error (w, b) :
    err = 0.0
    for x,y in zip(X,Y) :
        fx = f(w,b,x)
        err += 0.5 * (fx - y) ** 2
    return err
```

```

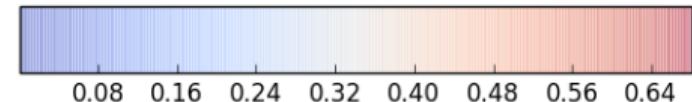
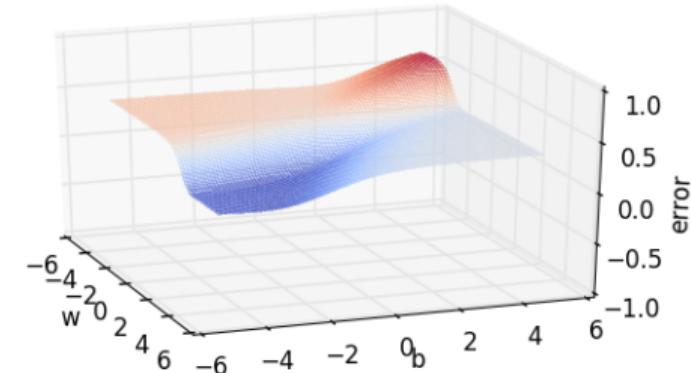
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Random search on error surface



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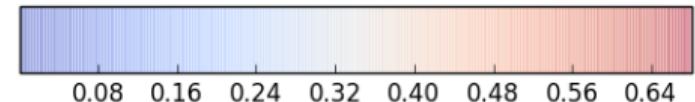
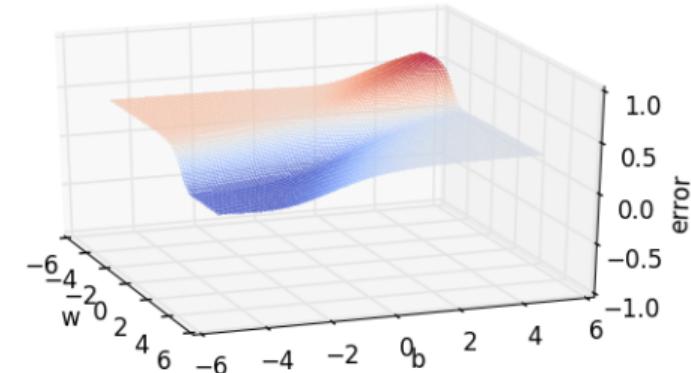
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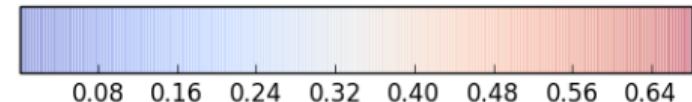
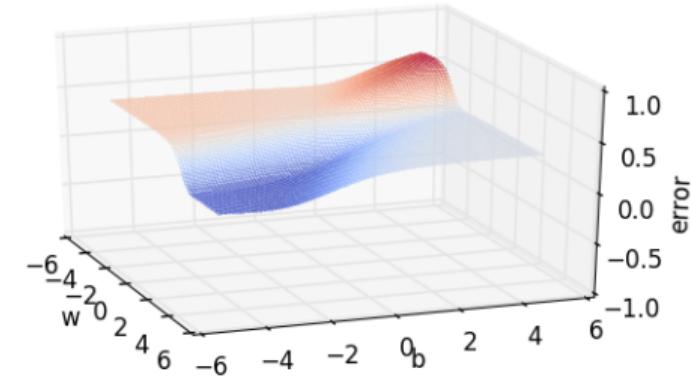
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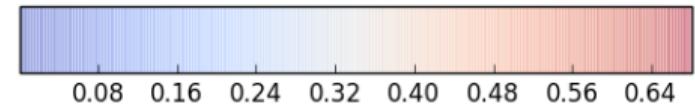
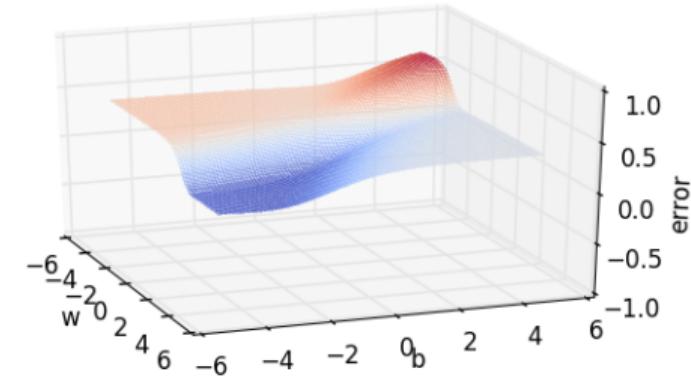
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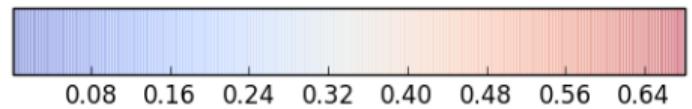
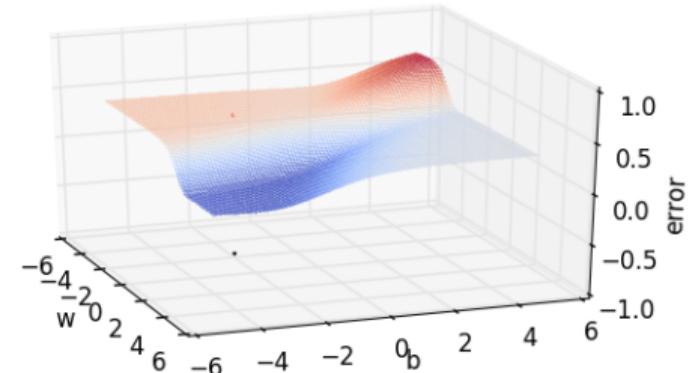
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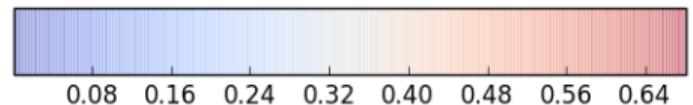
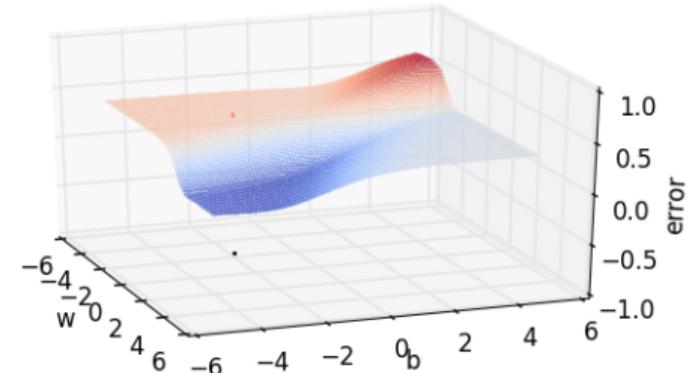
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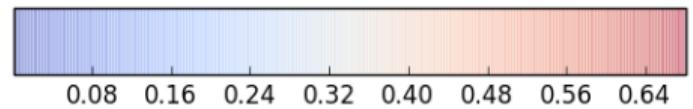
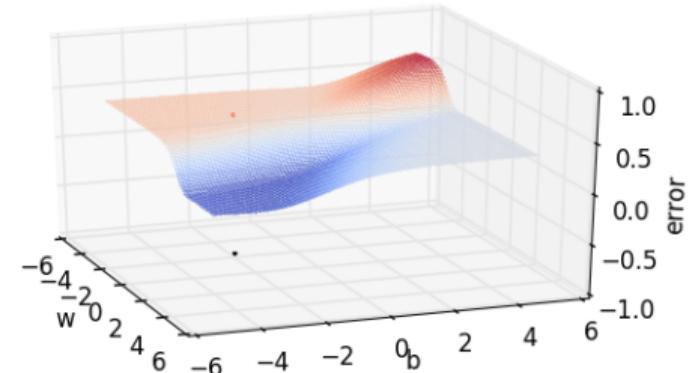
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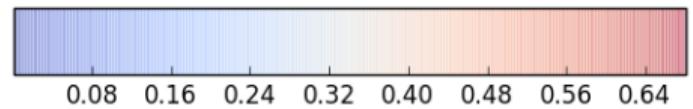
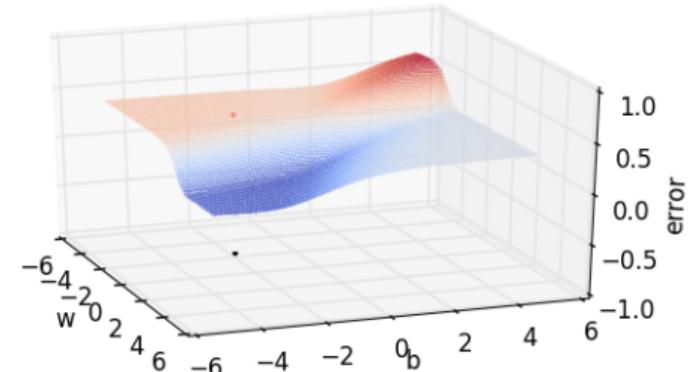
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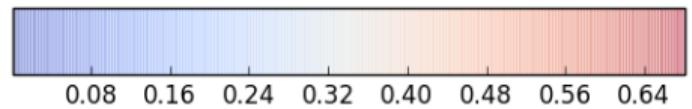
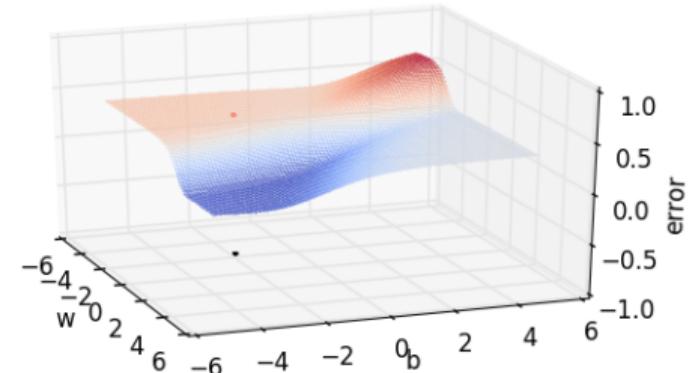
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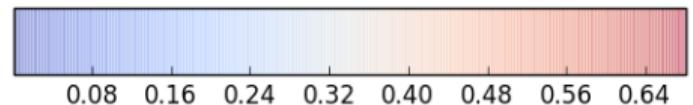
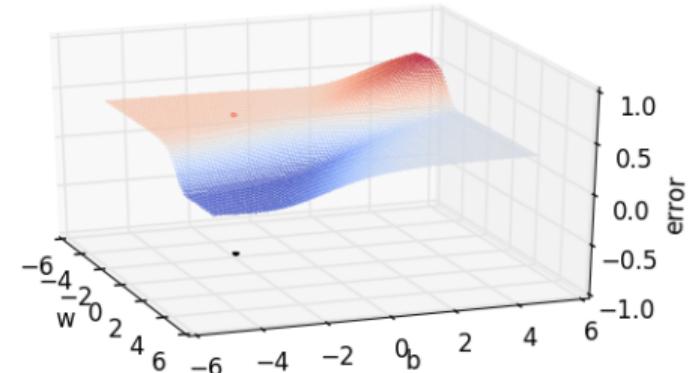
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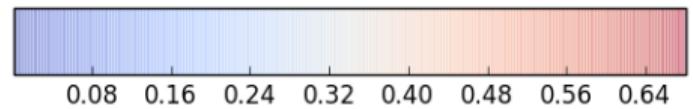
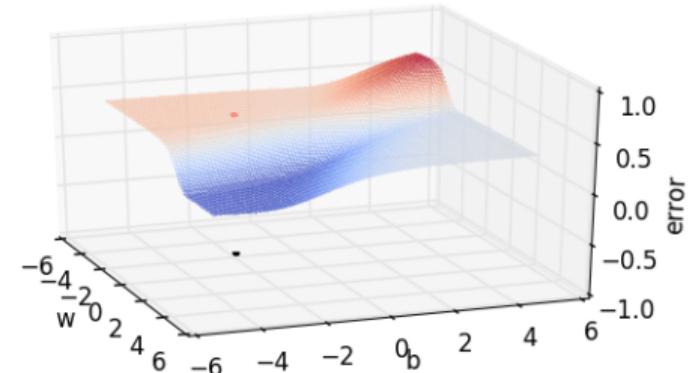
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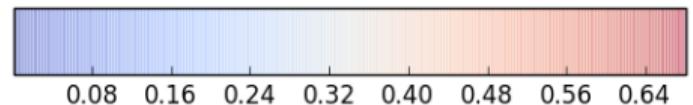
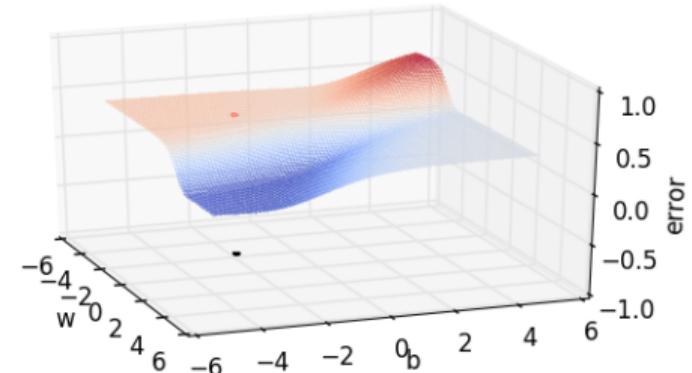
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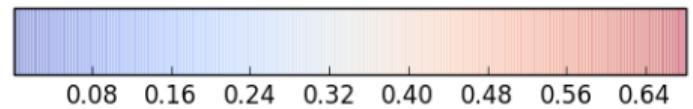
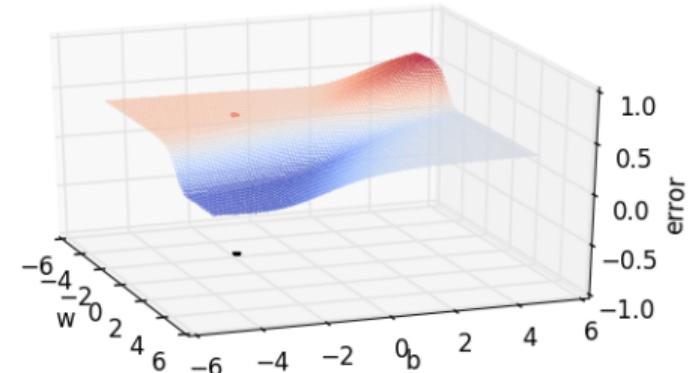
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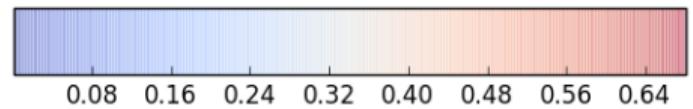
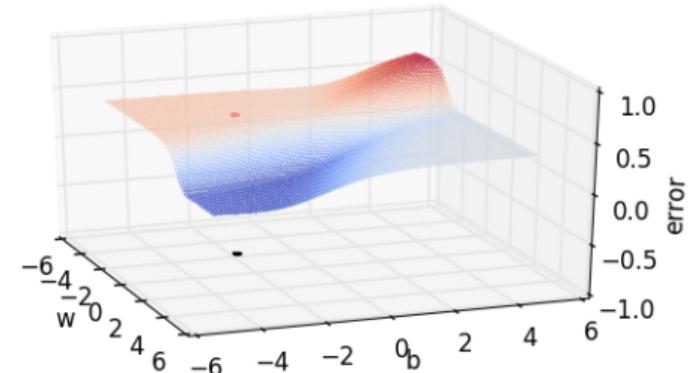
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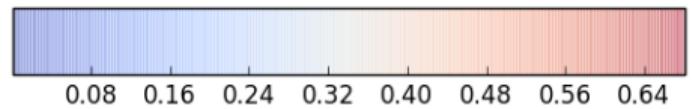
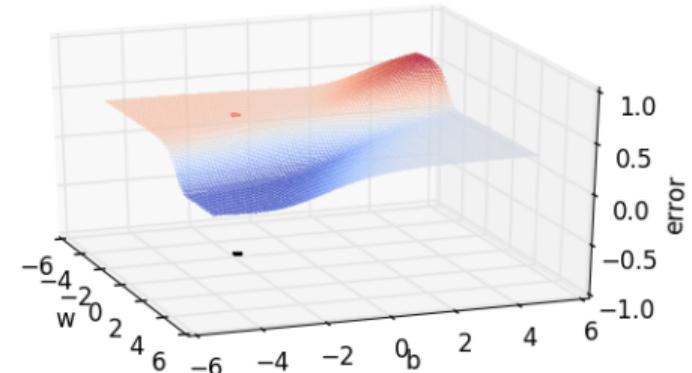
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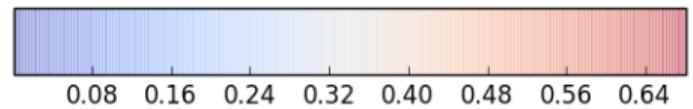
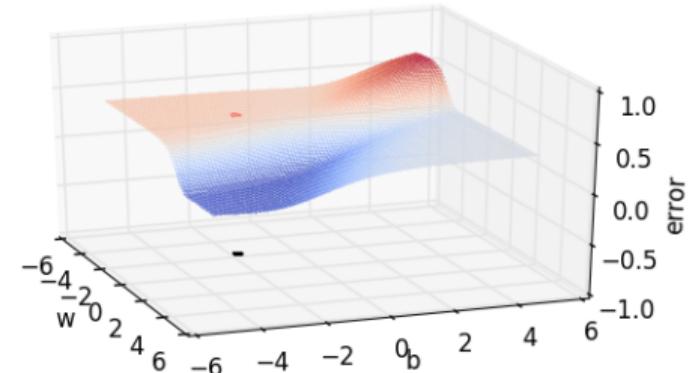
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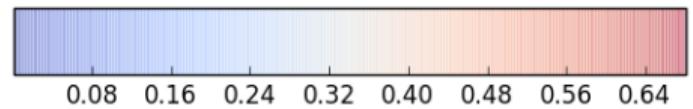
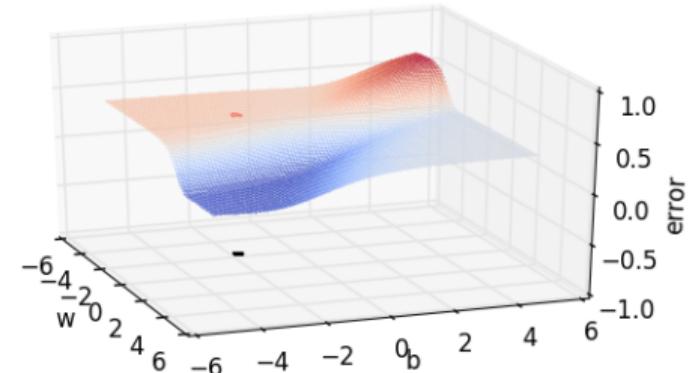
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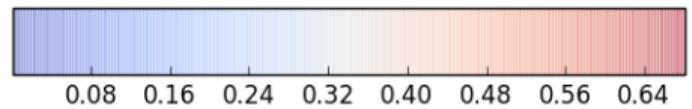
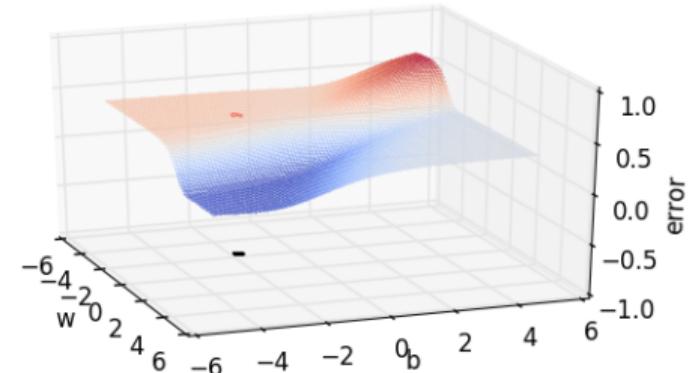
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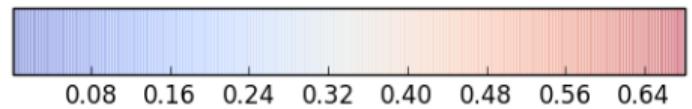
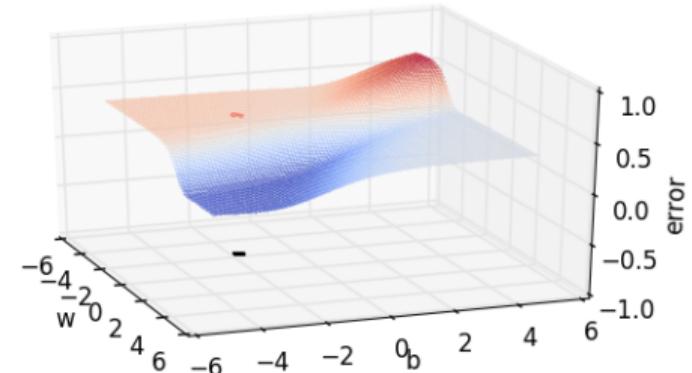
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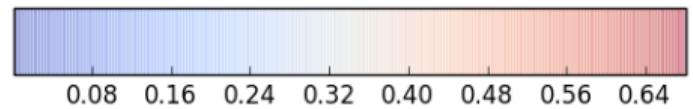
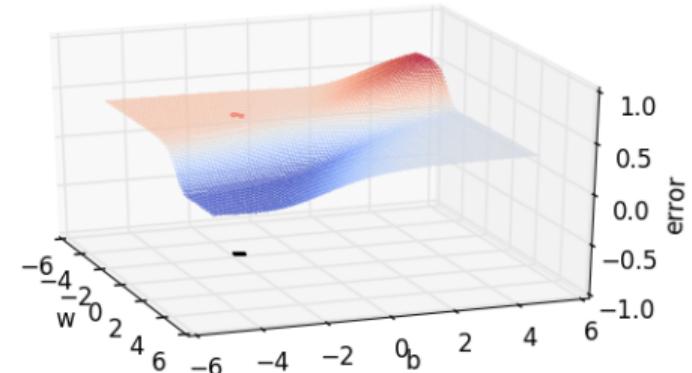
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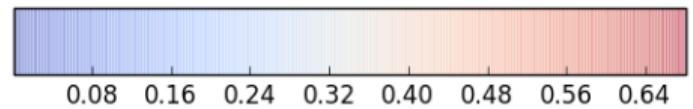
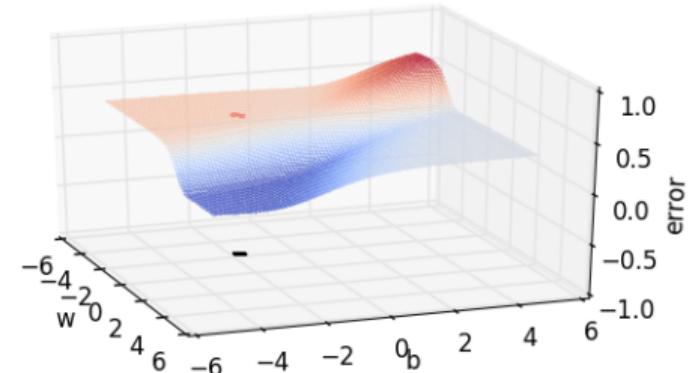
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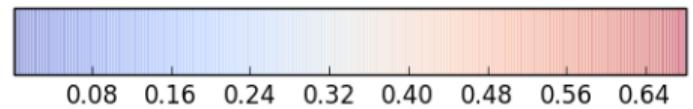
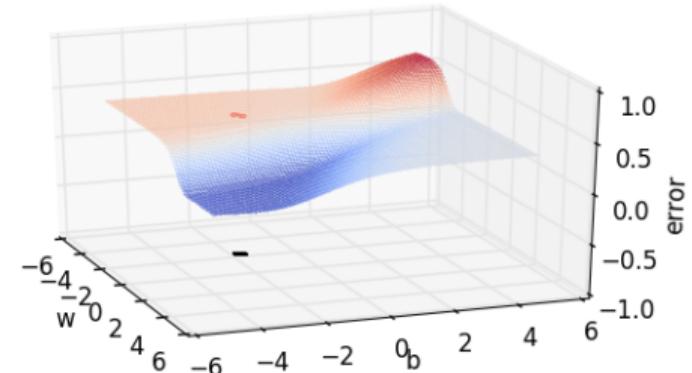
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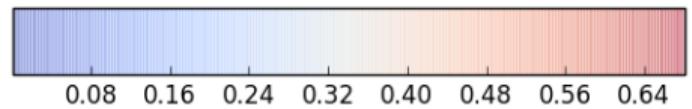
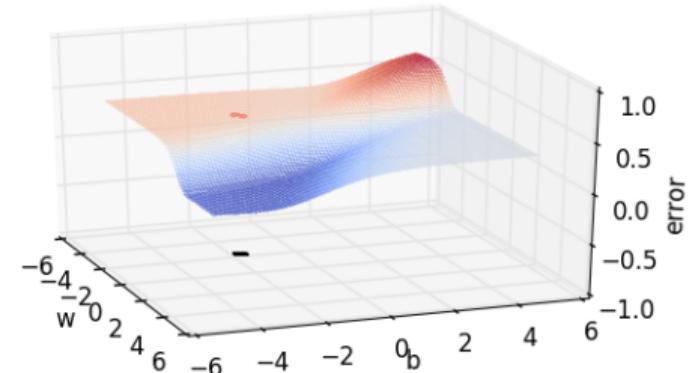
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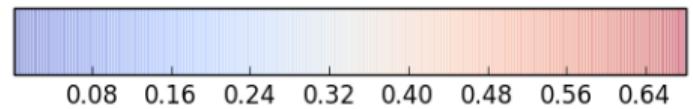
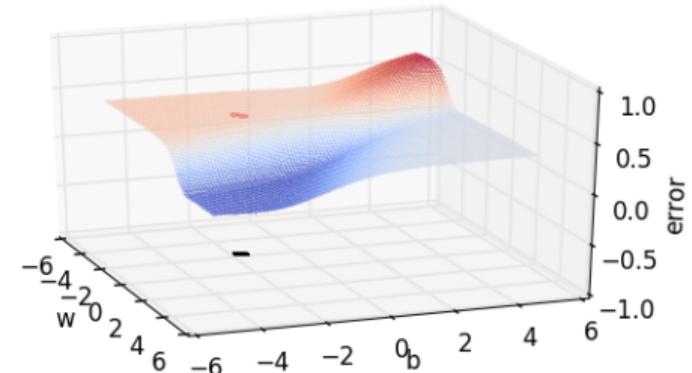
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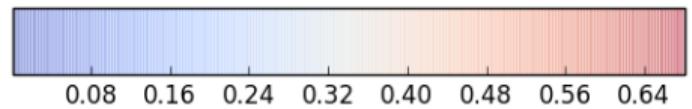
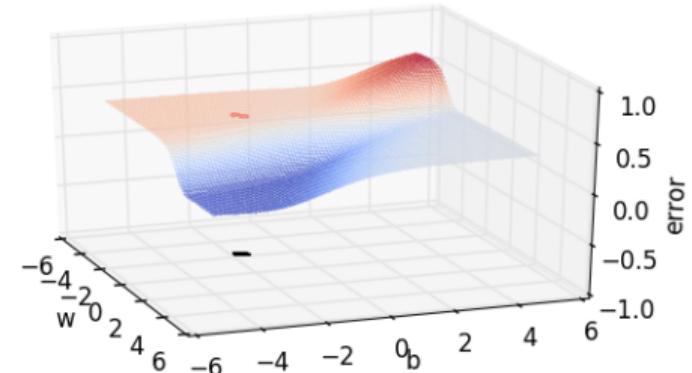
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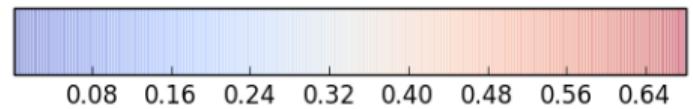
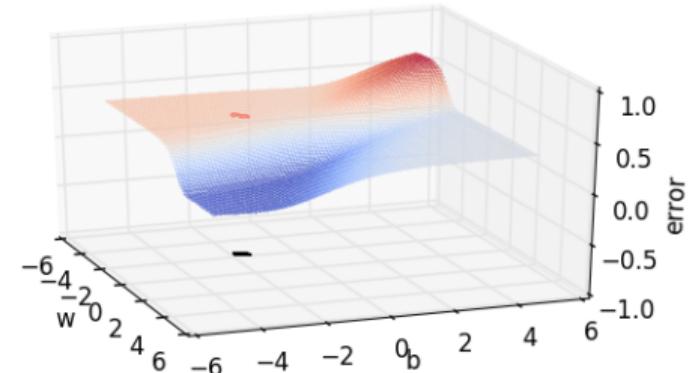
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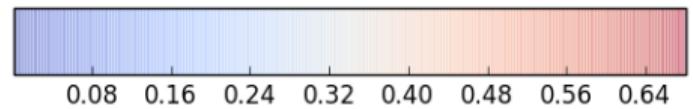
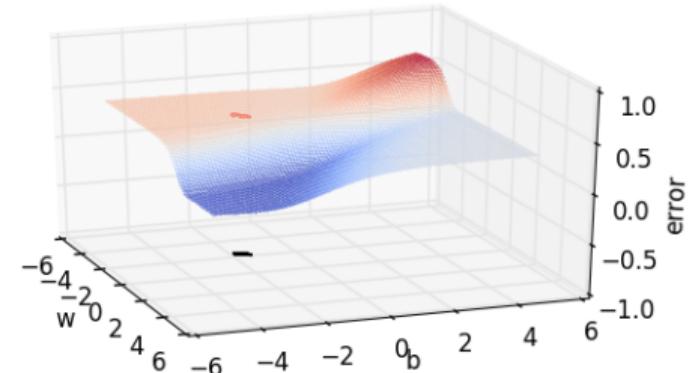
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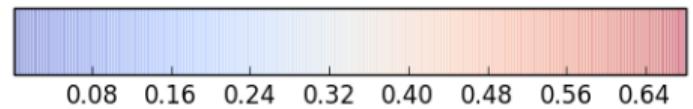
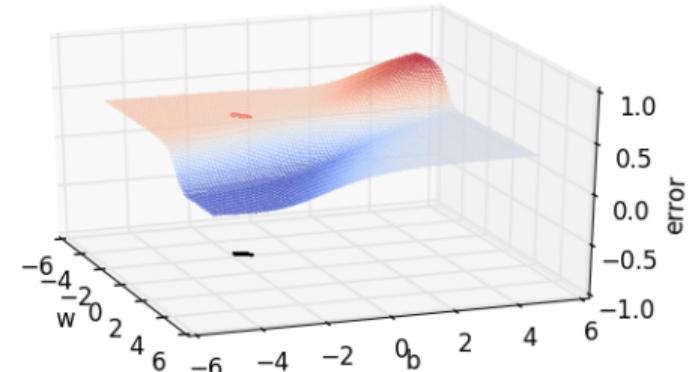
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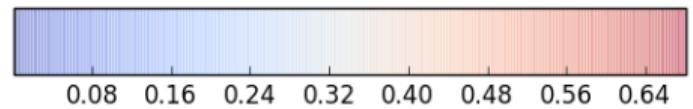
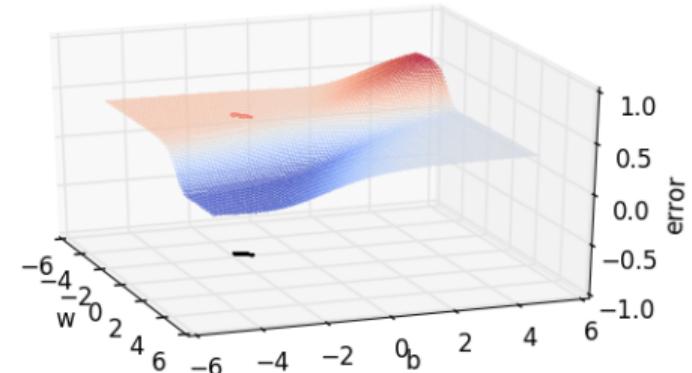
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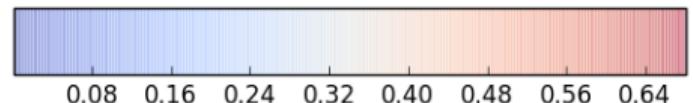
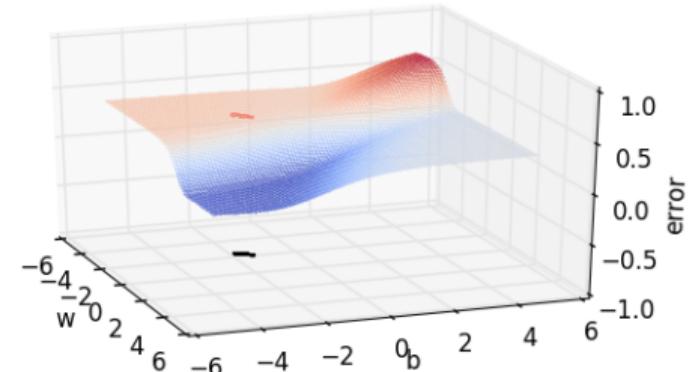
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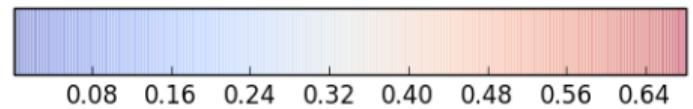
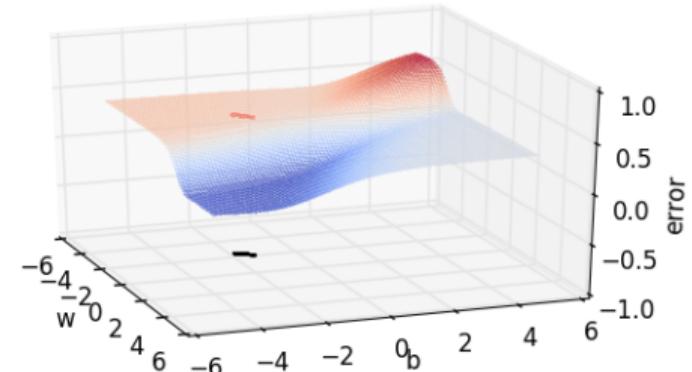
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Y = [0.2, 0.9]

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def error (w, b) :
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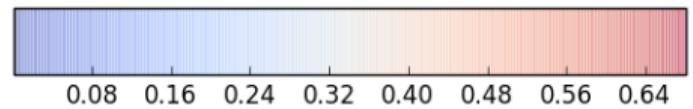
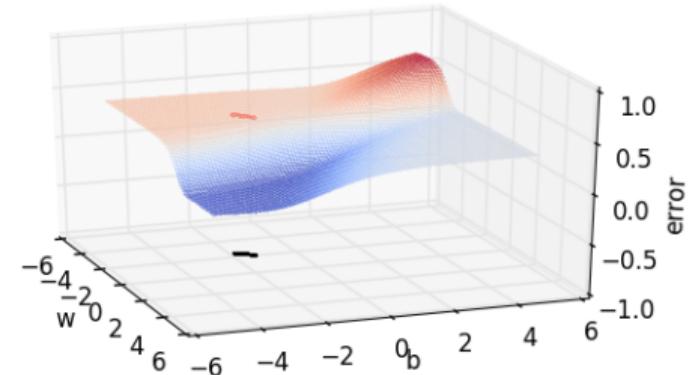
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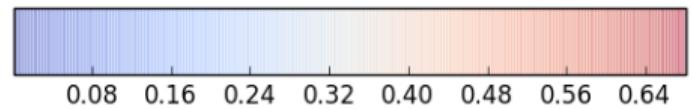
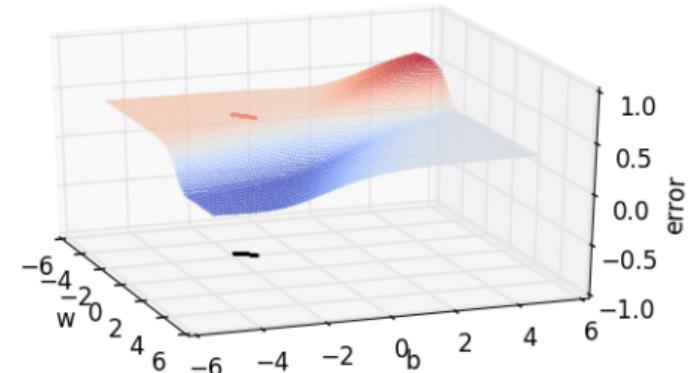
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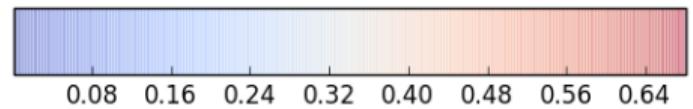
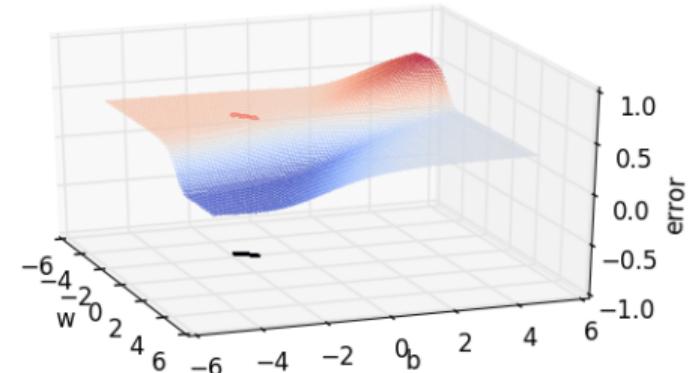
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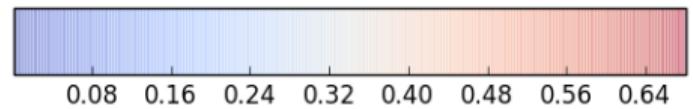
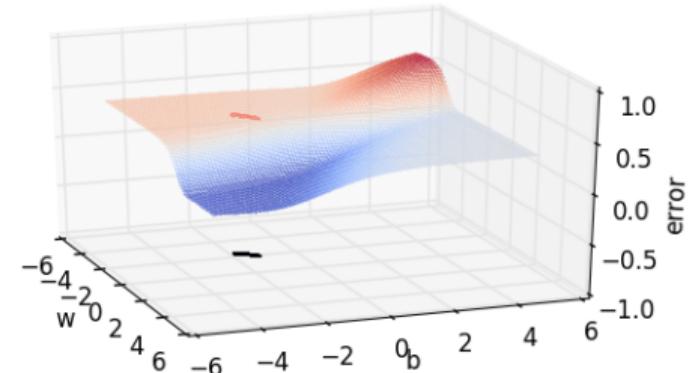
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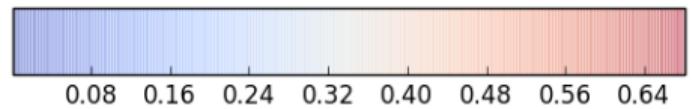
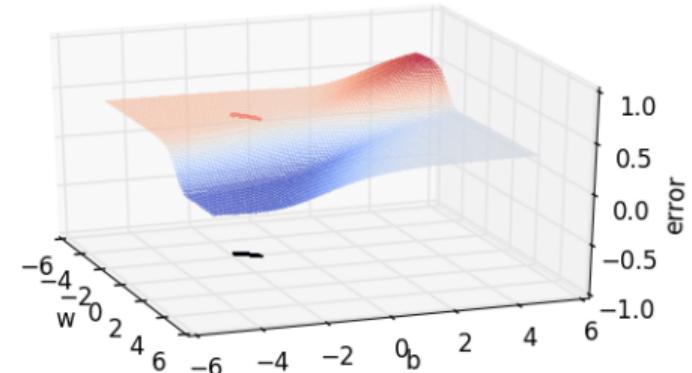
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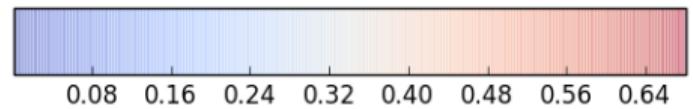
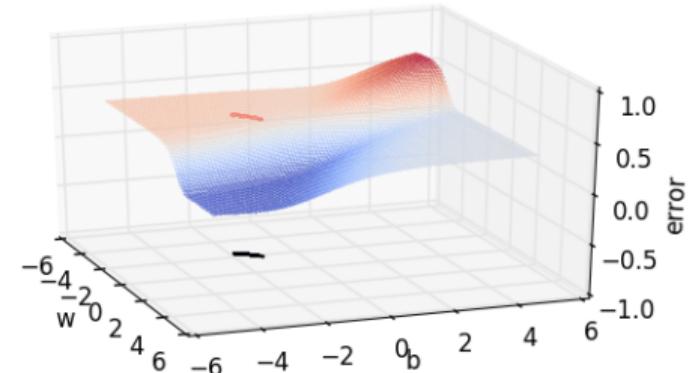
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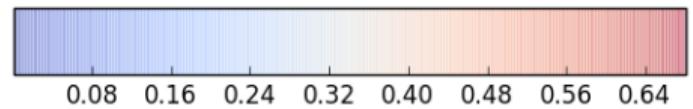
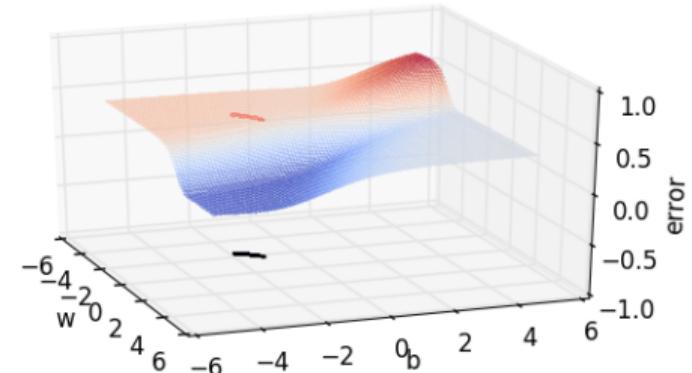
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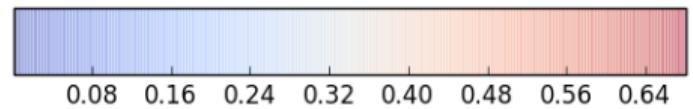
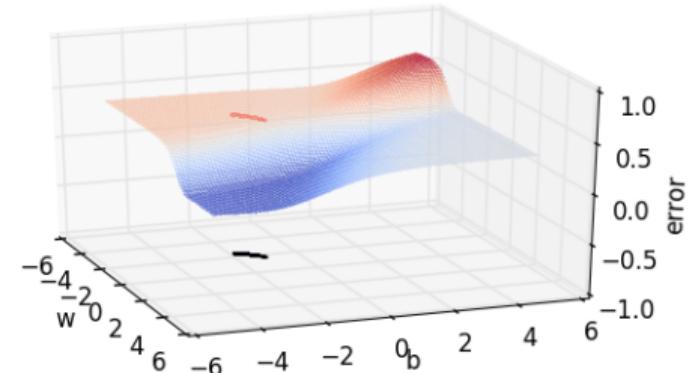
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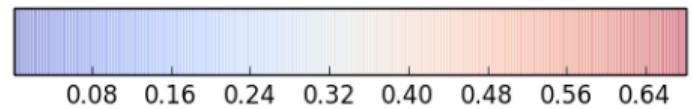
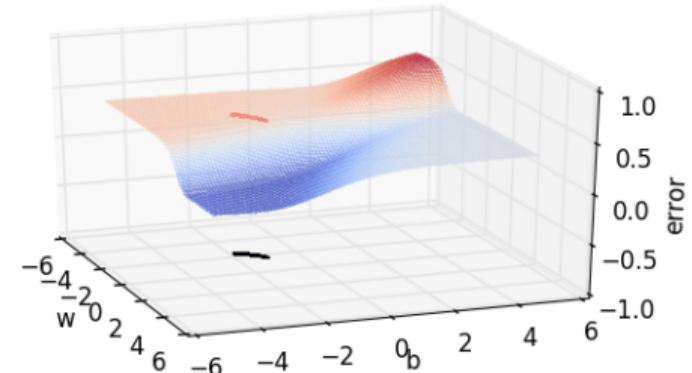
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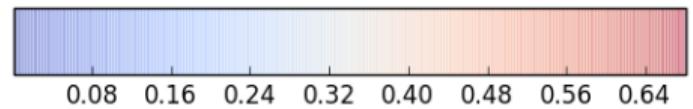
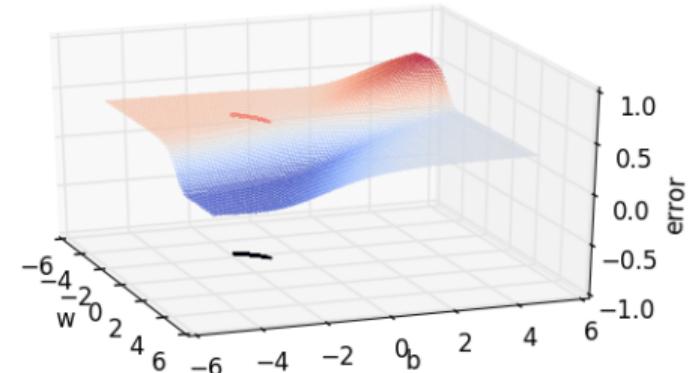
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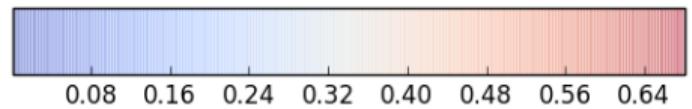
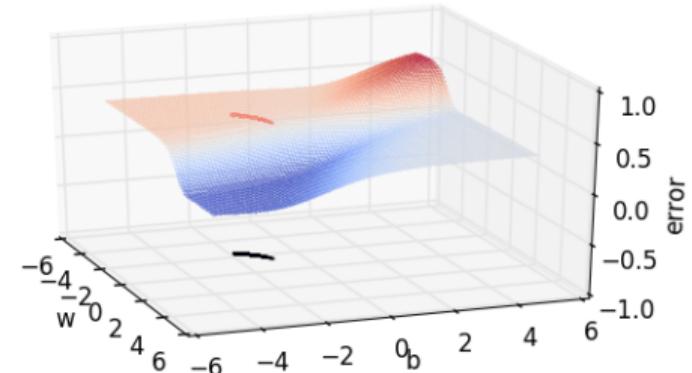
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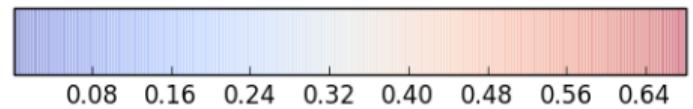
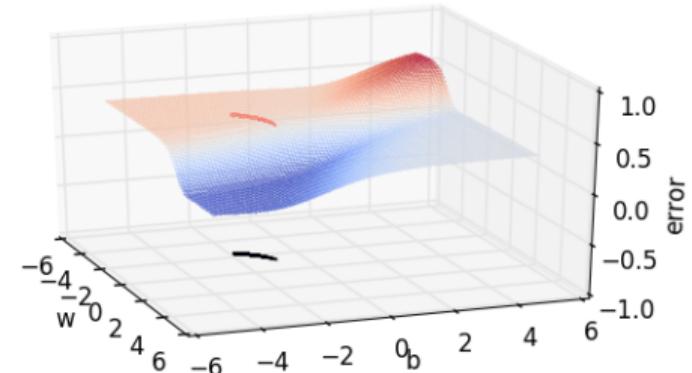
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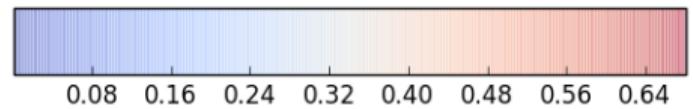
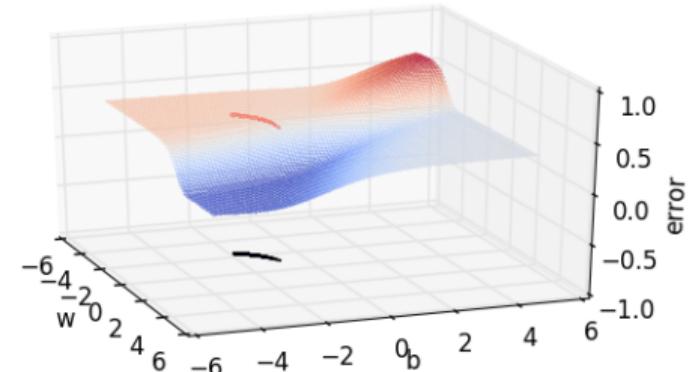
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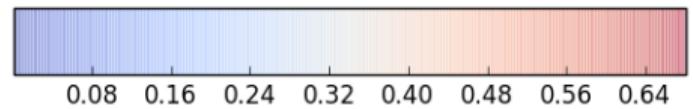
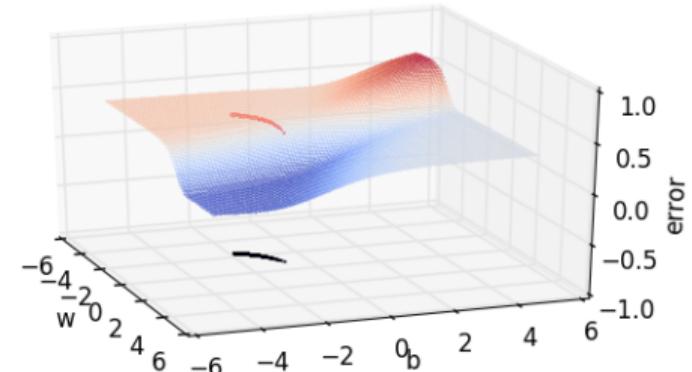
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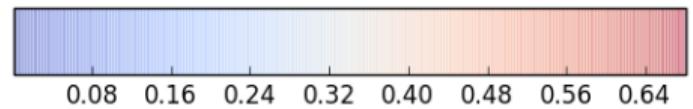
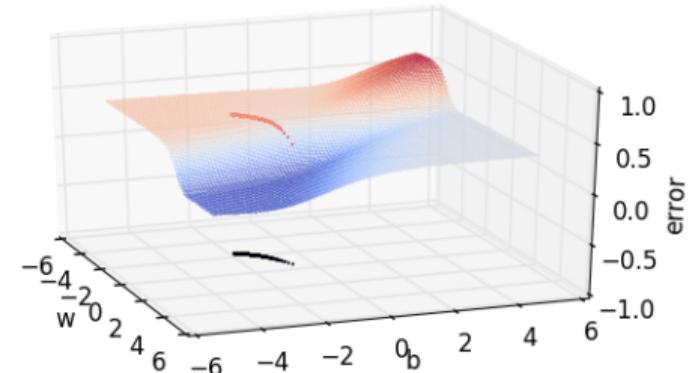
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[X = [0.5, 2.5]
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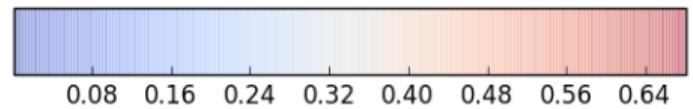
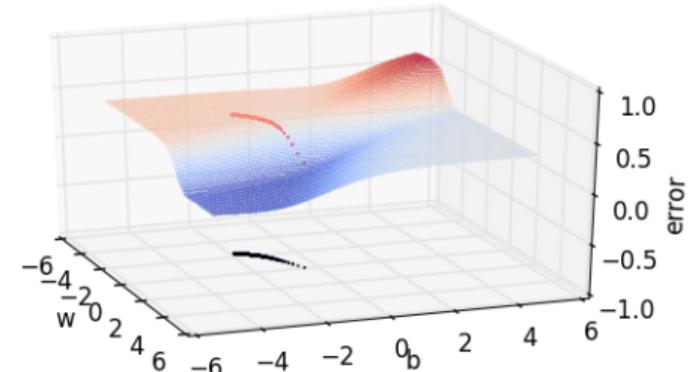
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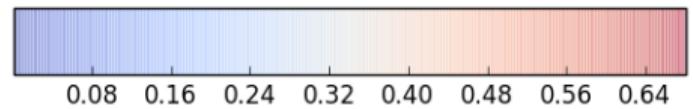
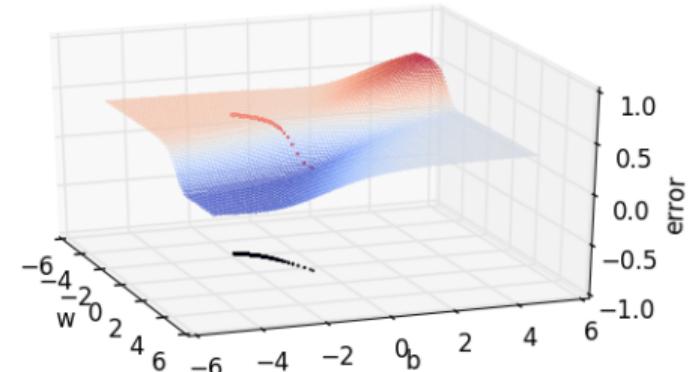
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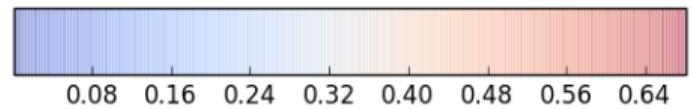
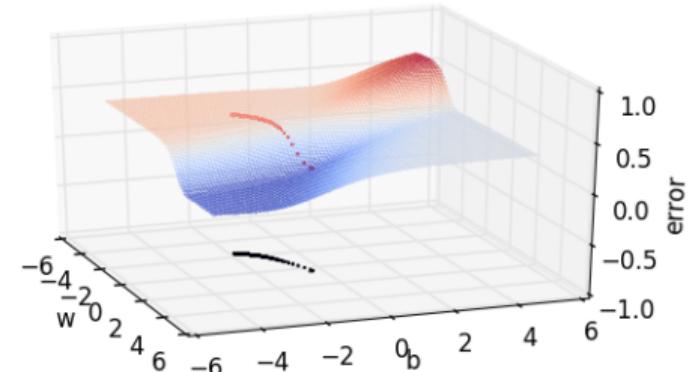
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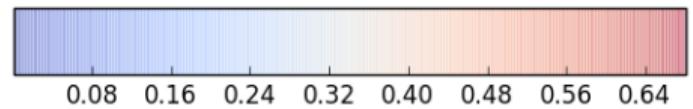
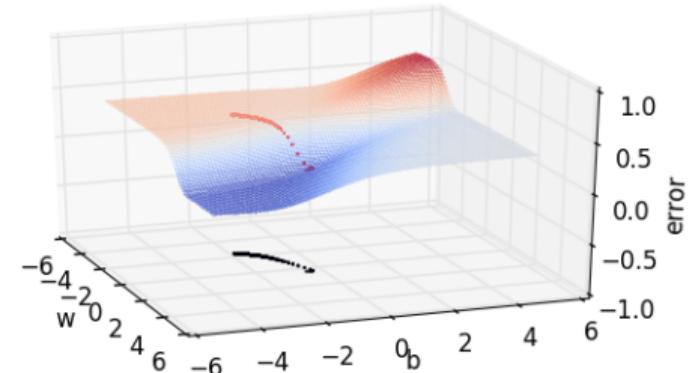
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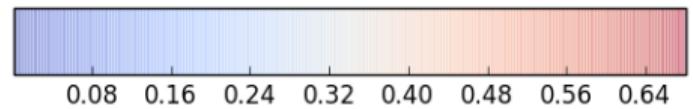
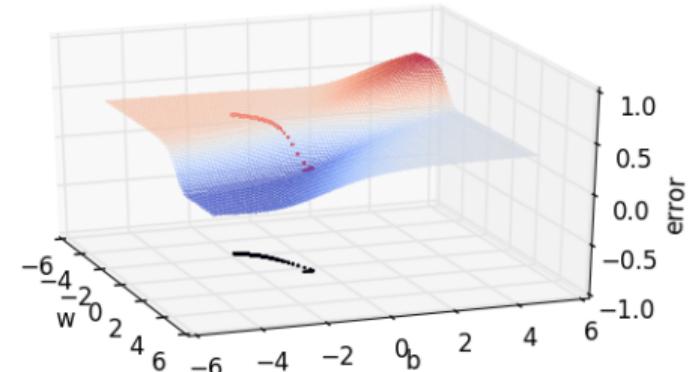
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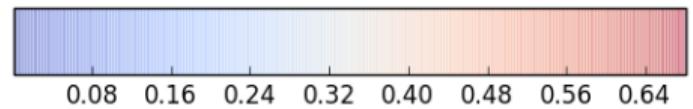
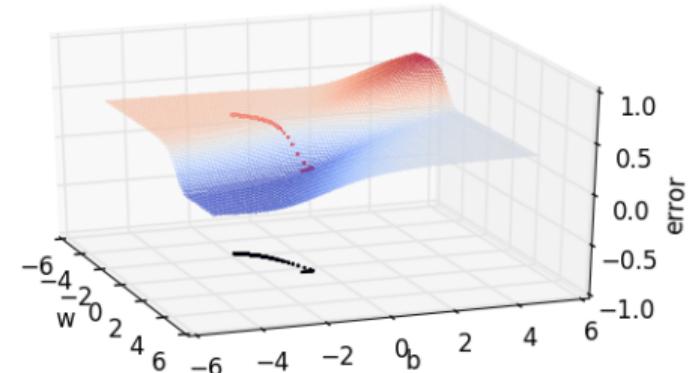
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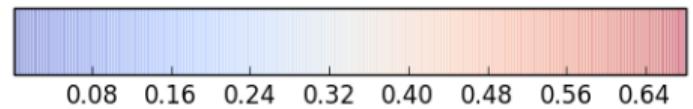
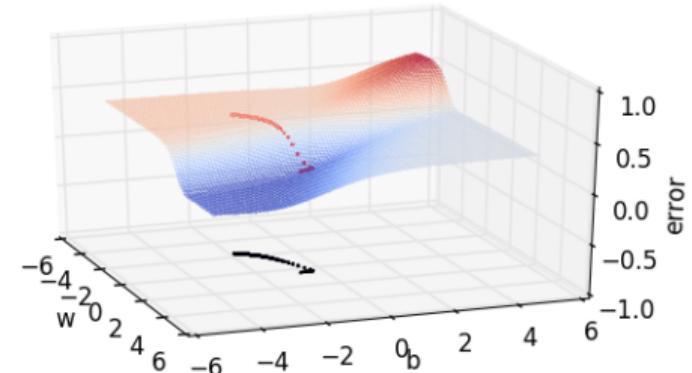
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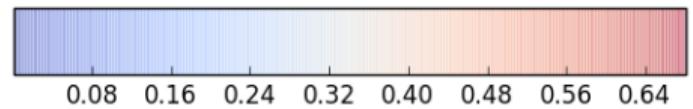
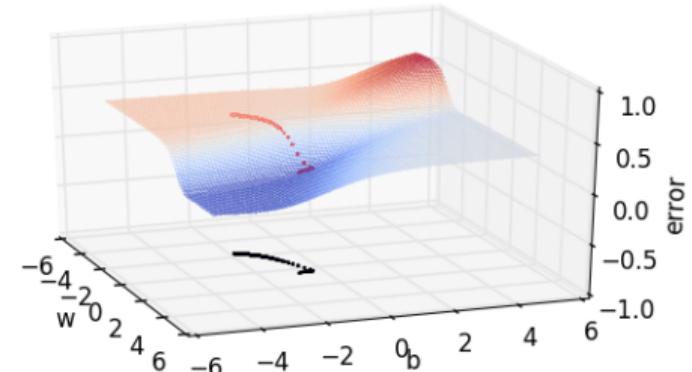
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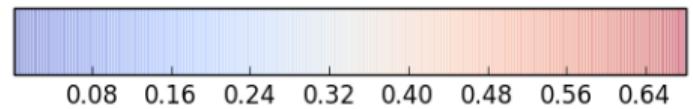
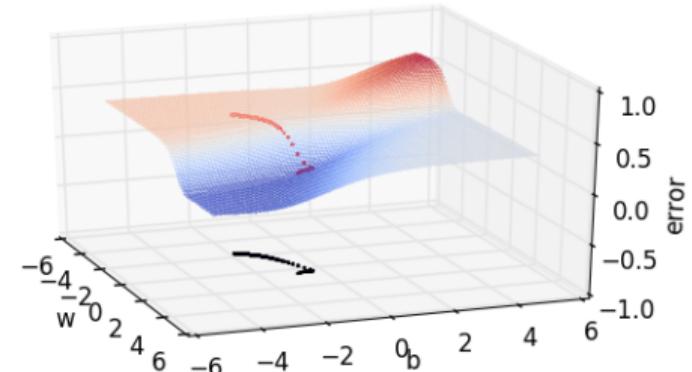
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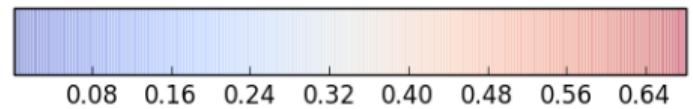
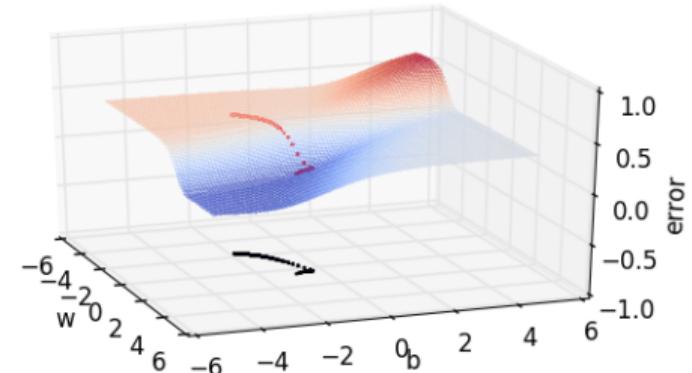
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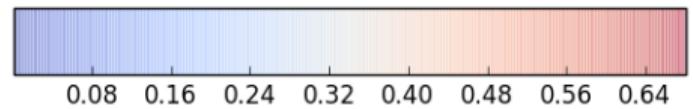
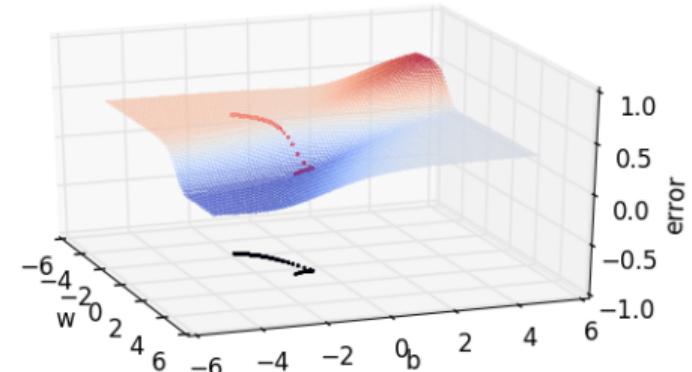
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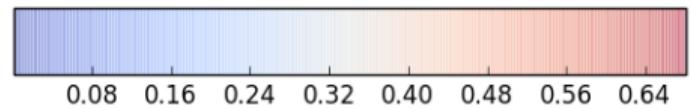
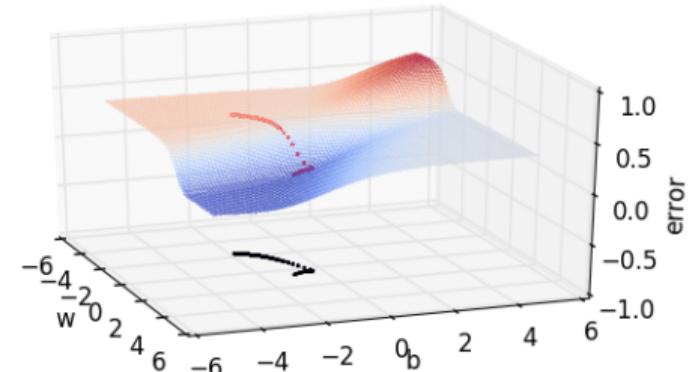
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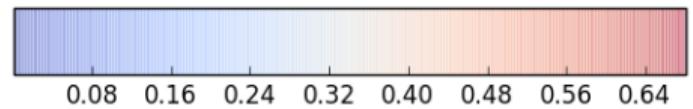
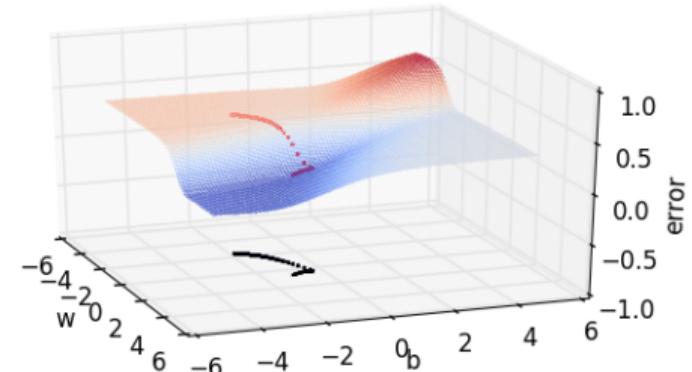
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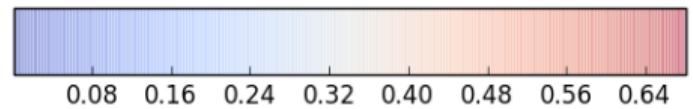
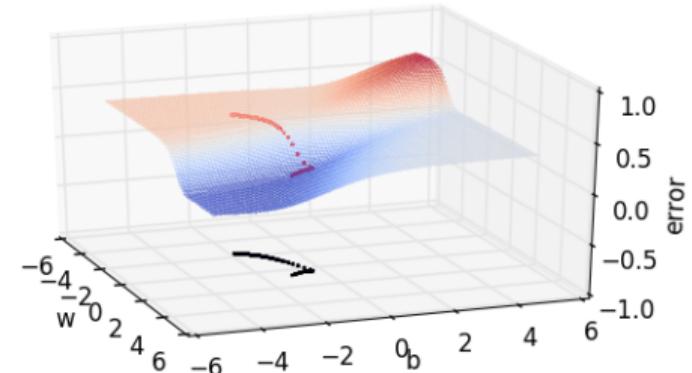
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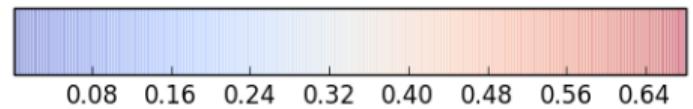
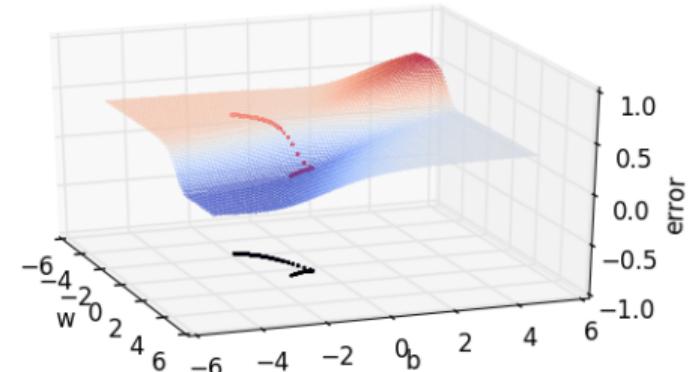
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Gradient descent on the error surface



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[X = [0.5, 2.5]
Y = [0.2, 0.9]

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    return 1.0 / (1.0 + np.exp(-(w*x + b)))

def error (w, b) :
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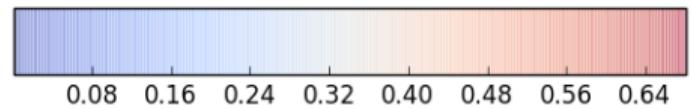
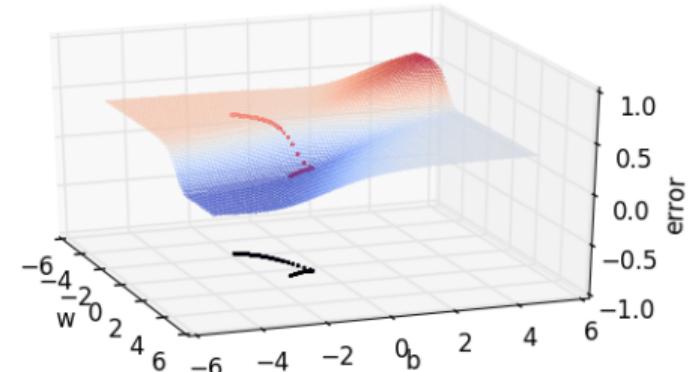
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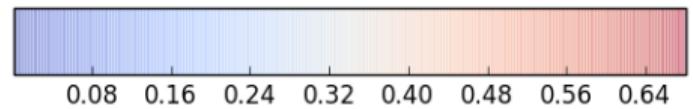
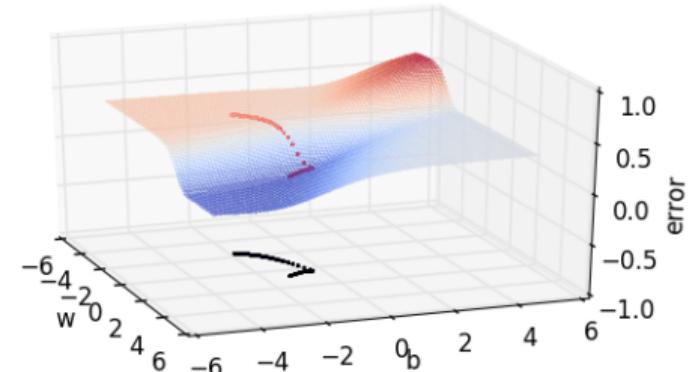
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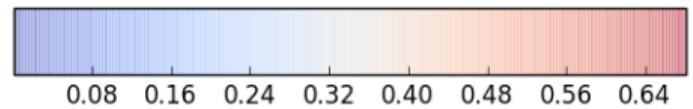
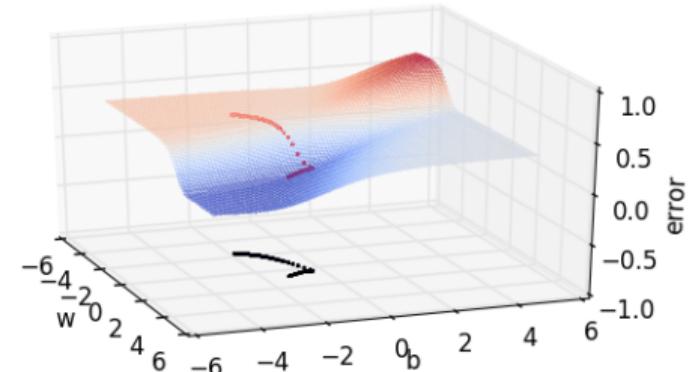
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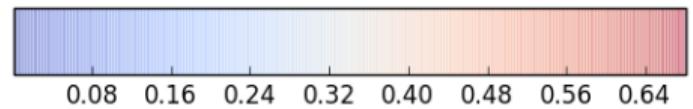
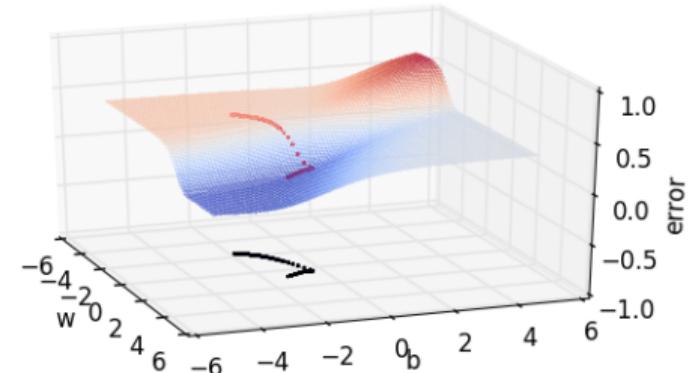
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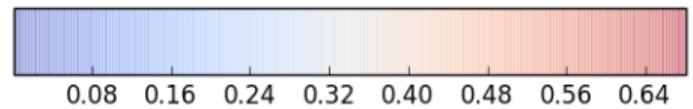
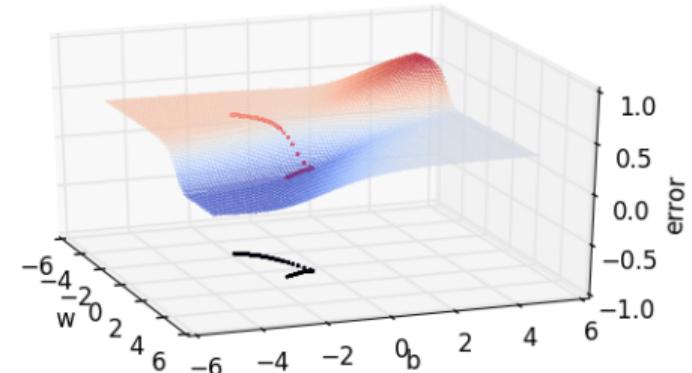
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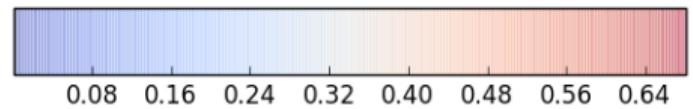
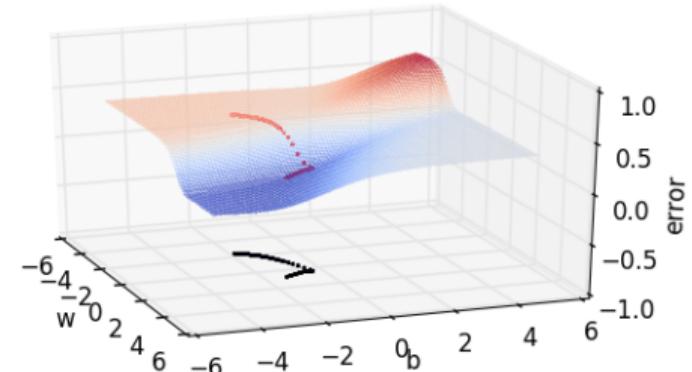
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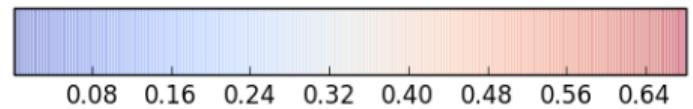
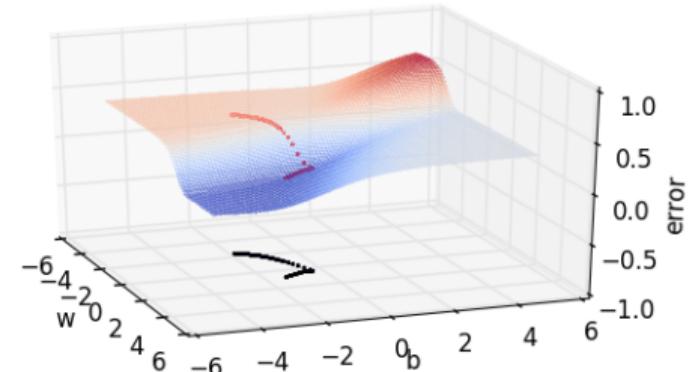
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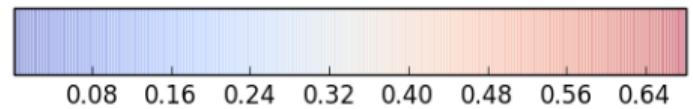
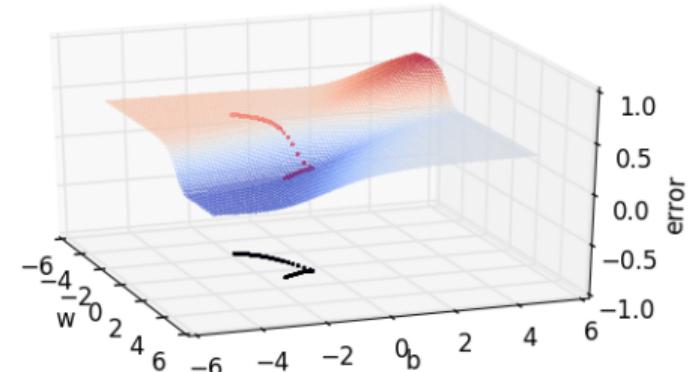
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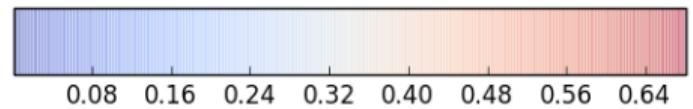
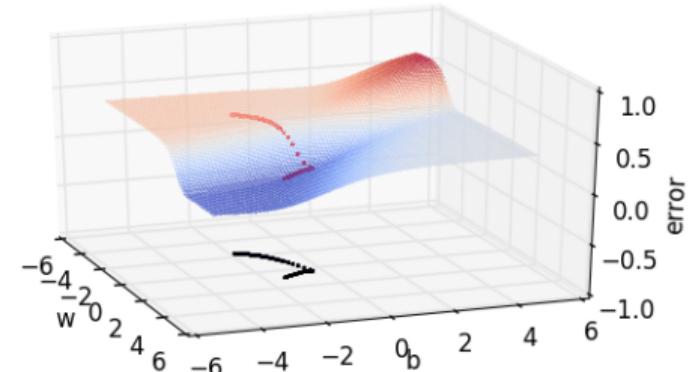
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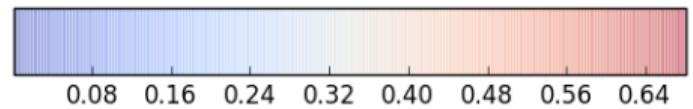
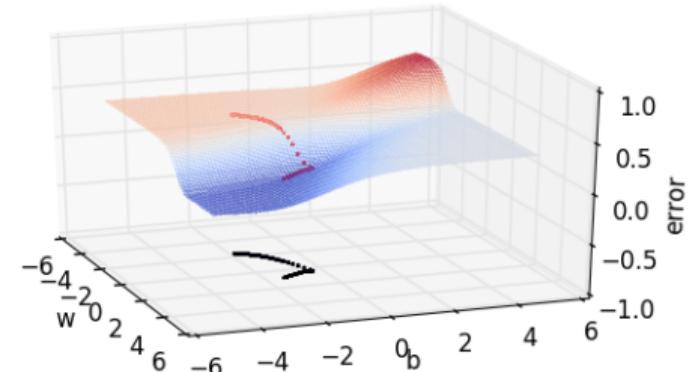
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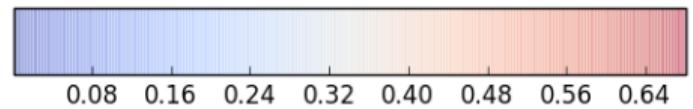
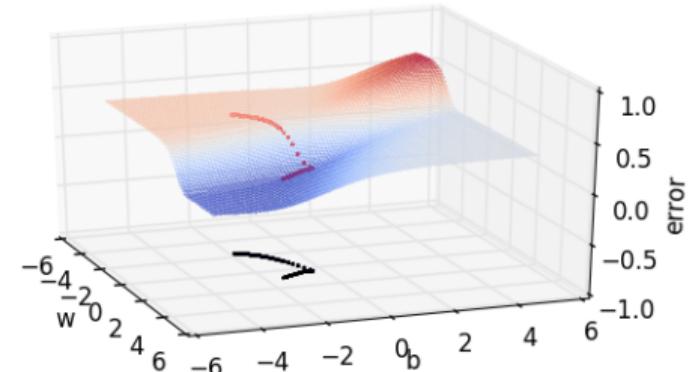
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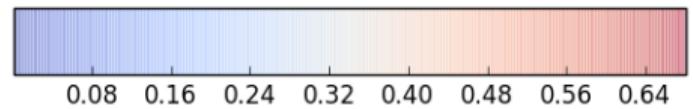
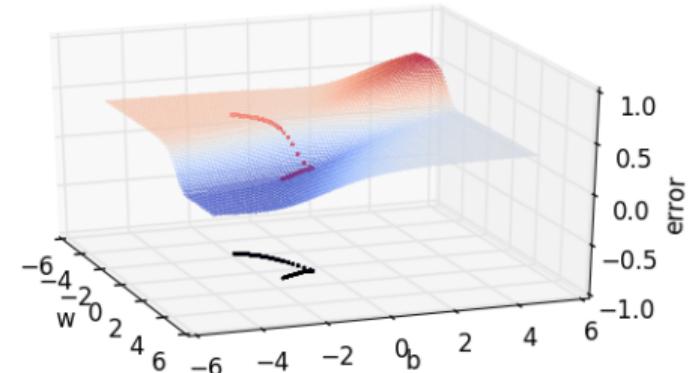
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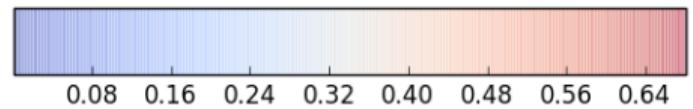
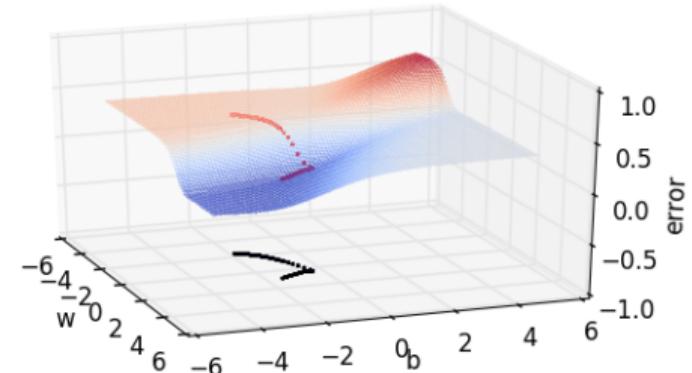
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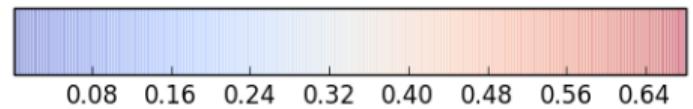
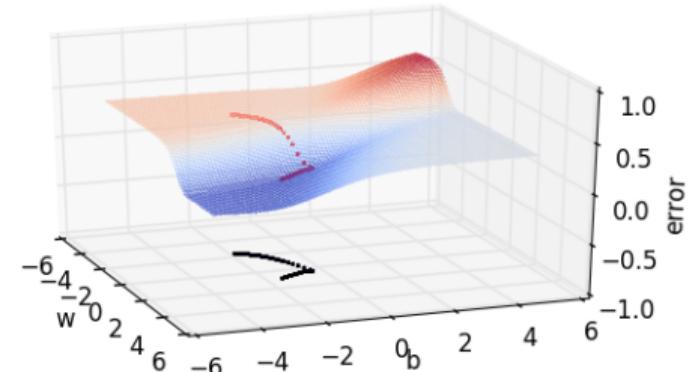
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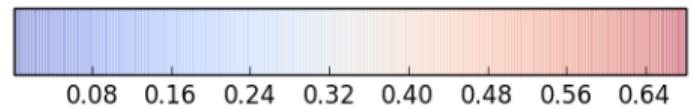
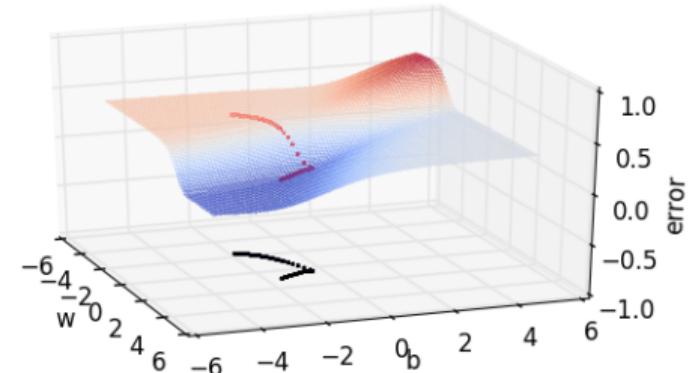
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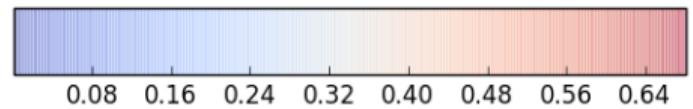
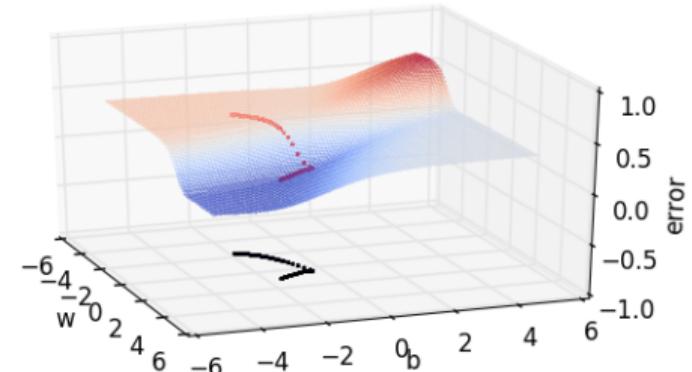
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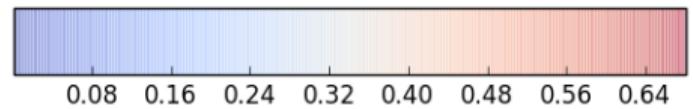
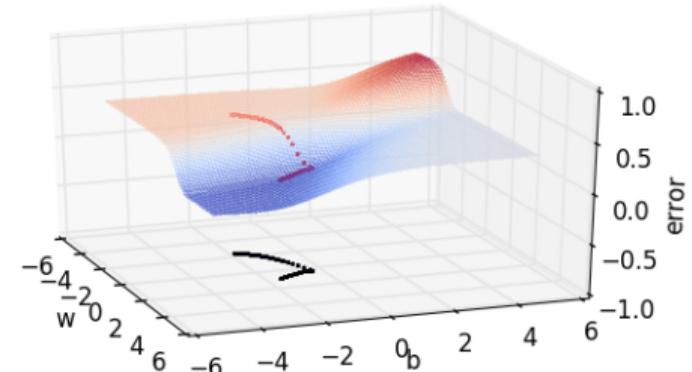
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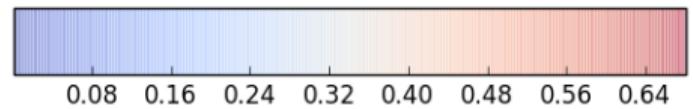
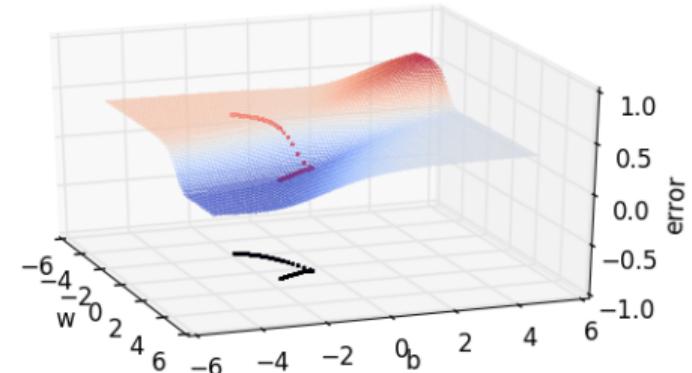
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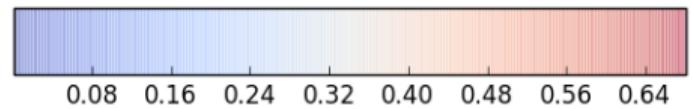
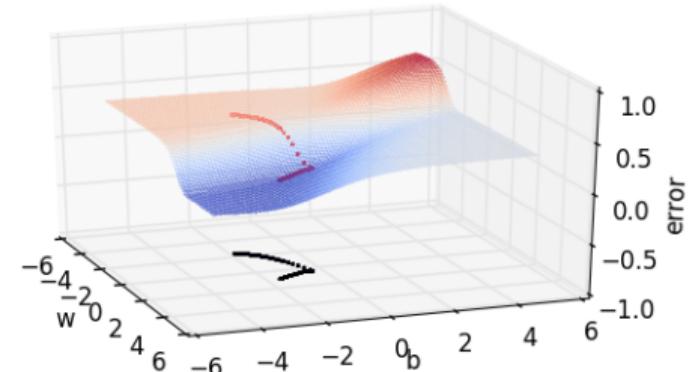
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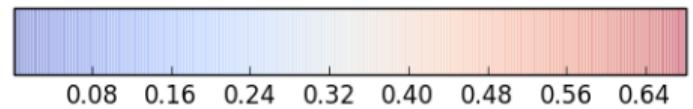
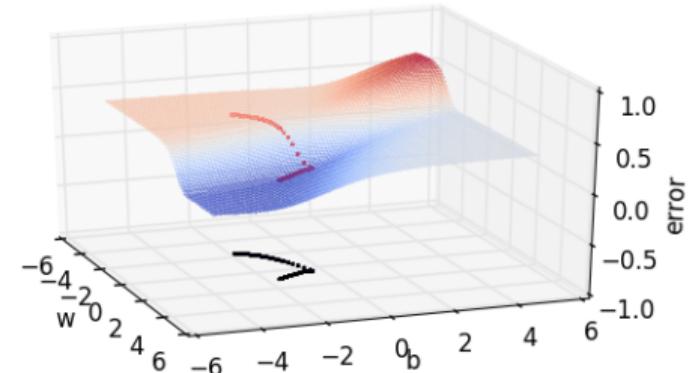
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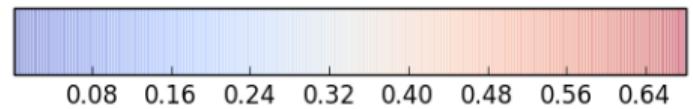
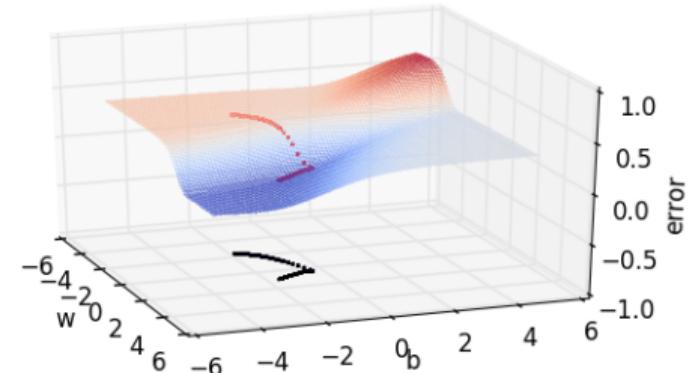
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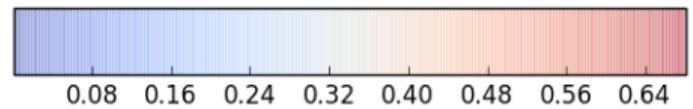
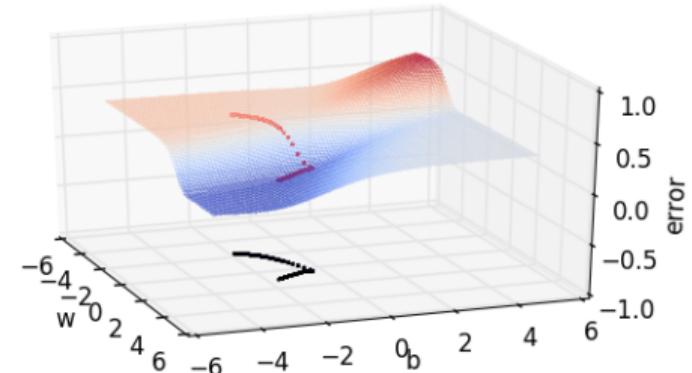
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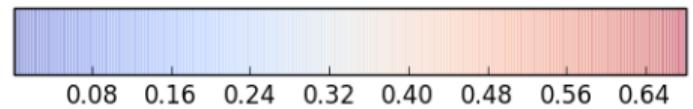
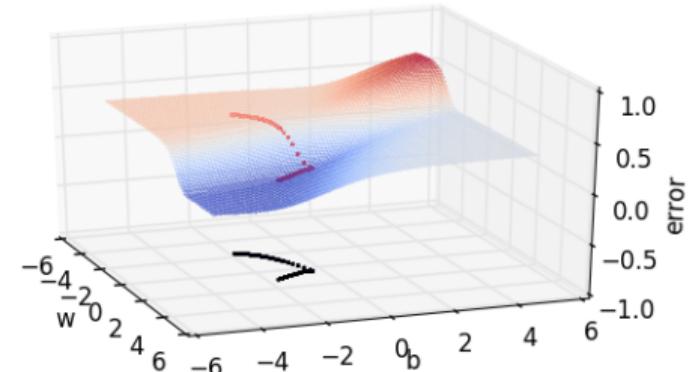
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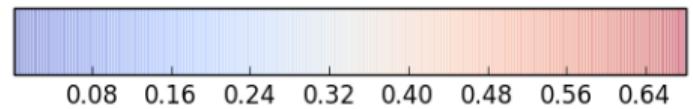
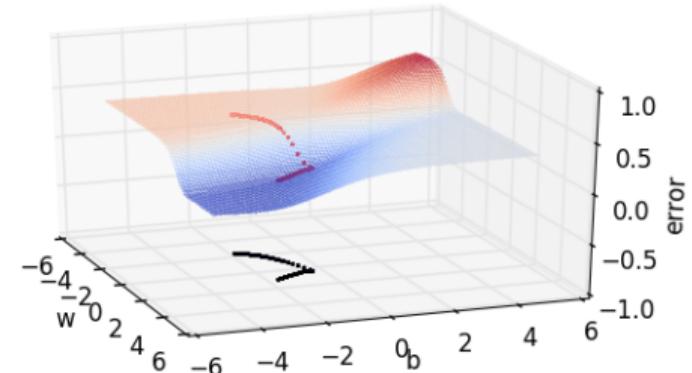
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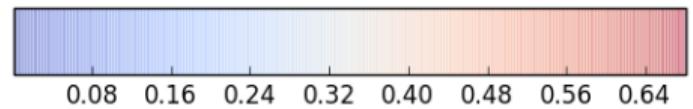
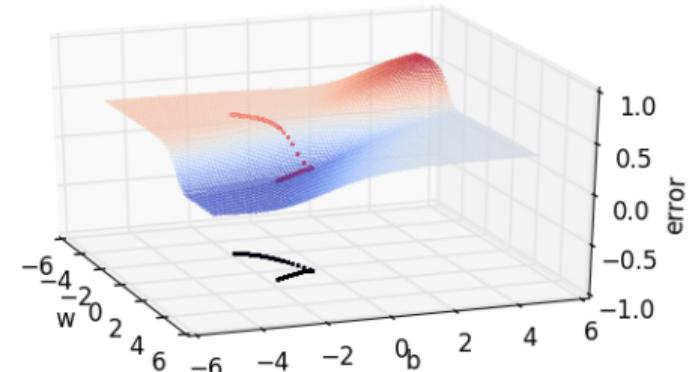
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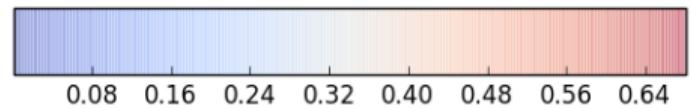
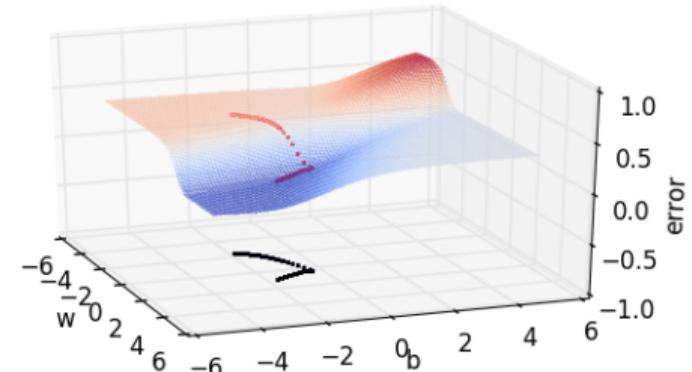
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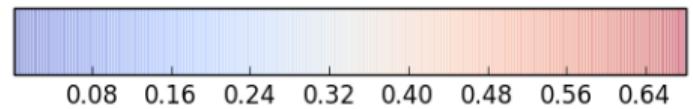
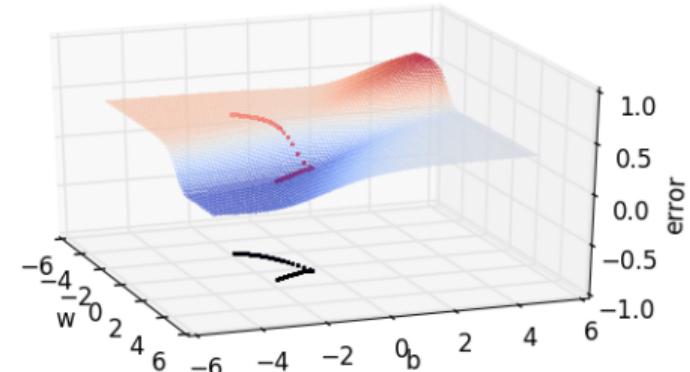
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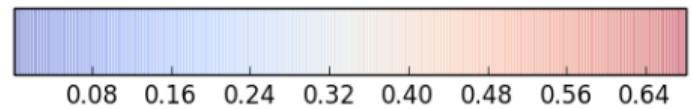
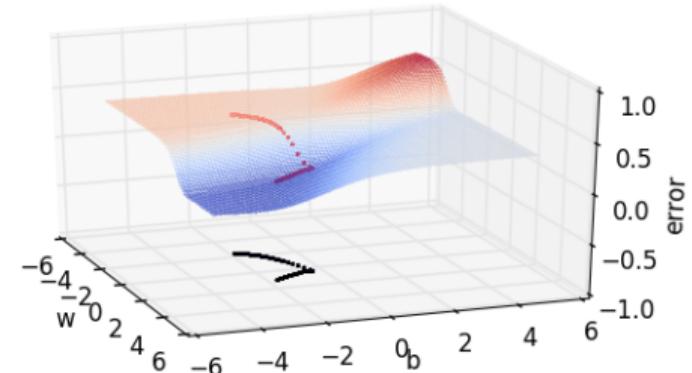
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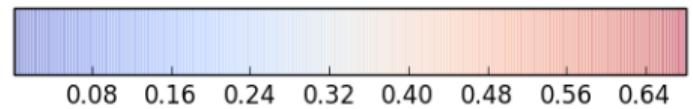
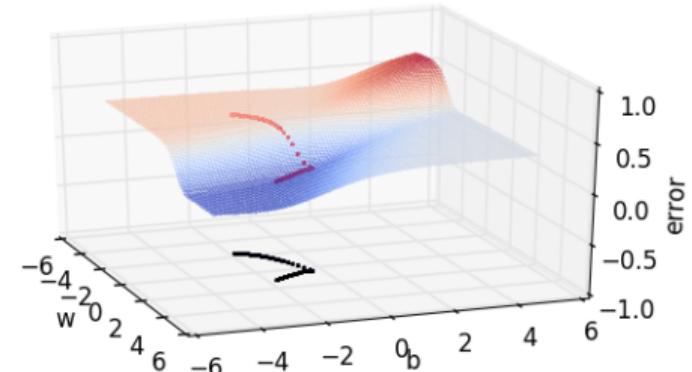
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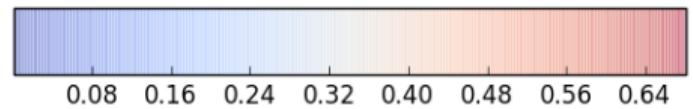
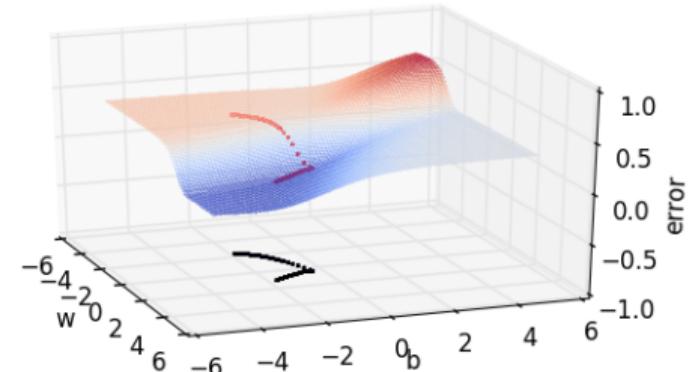
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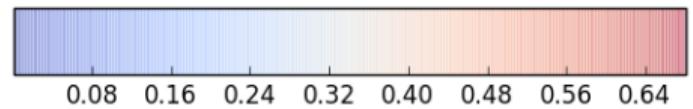
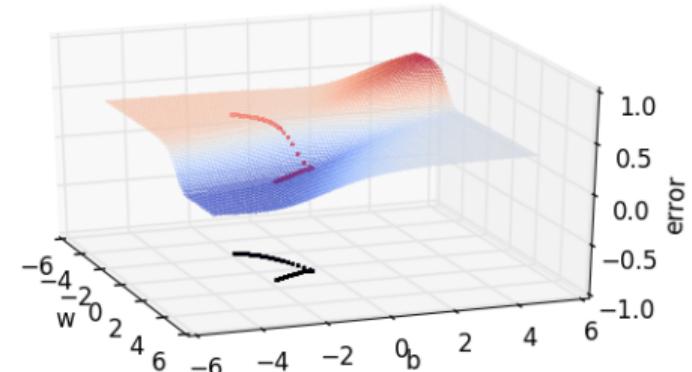
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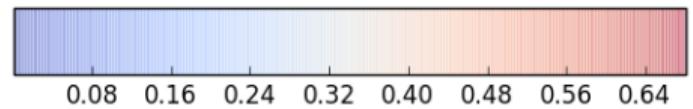
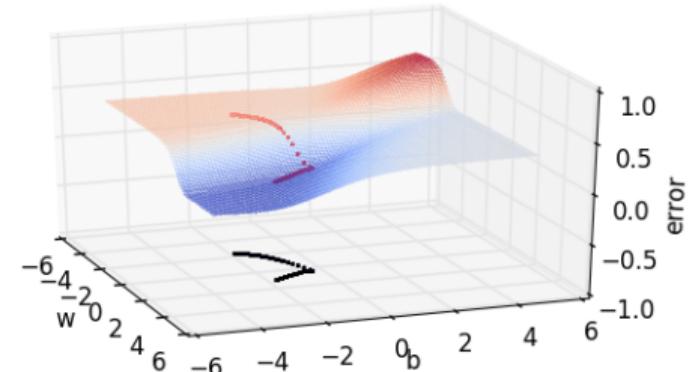
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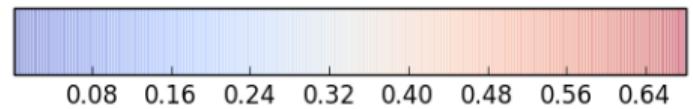
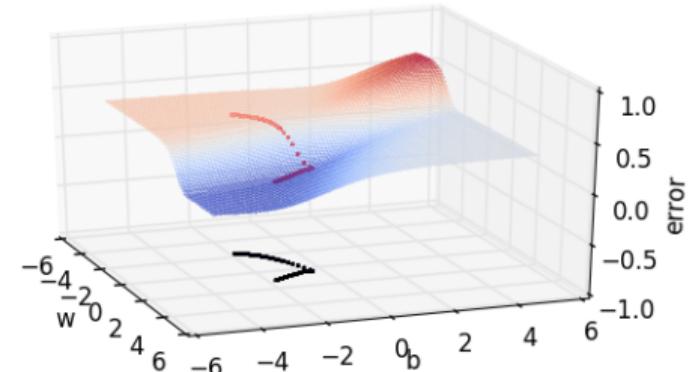
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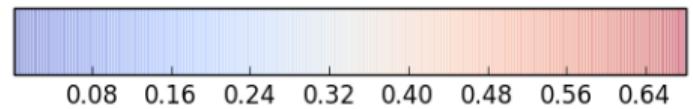
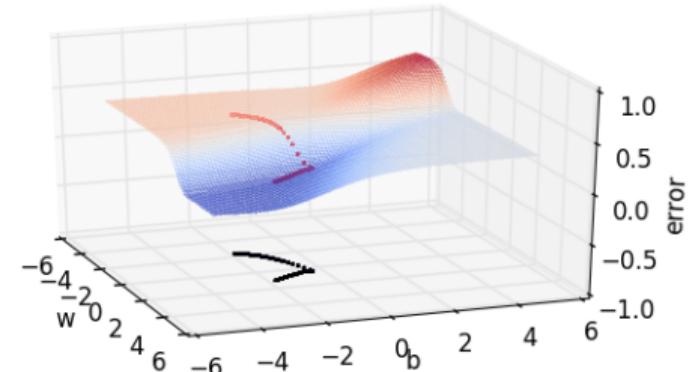
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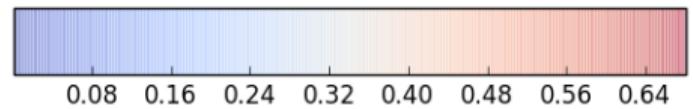
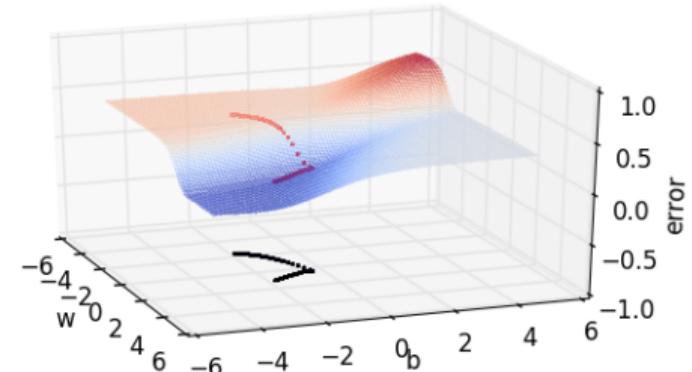
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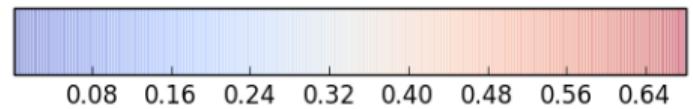
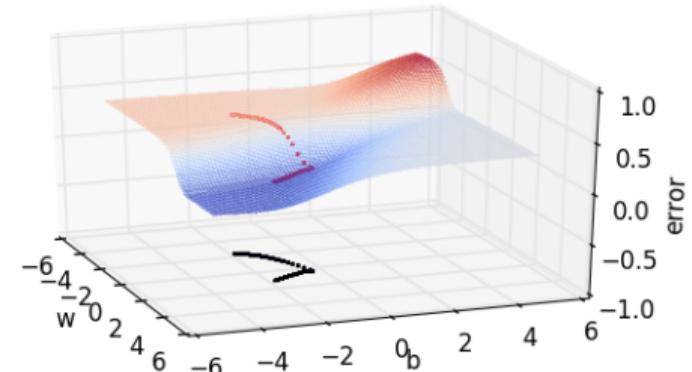
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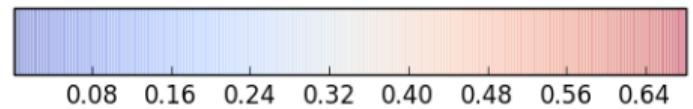
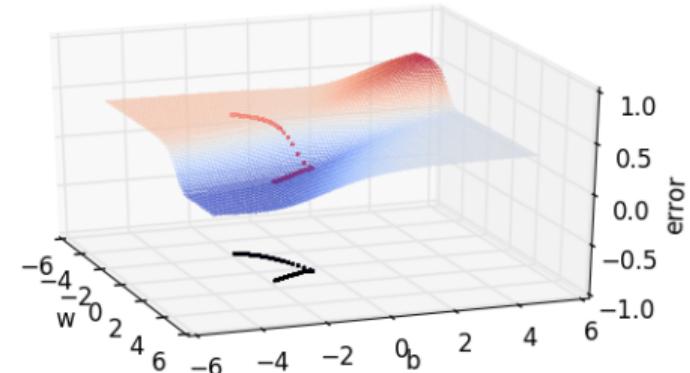
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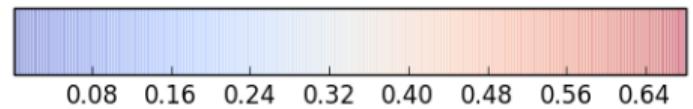
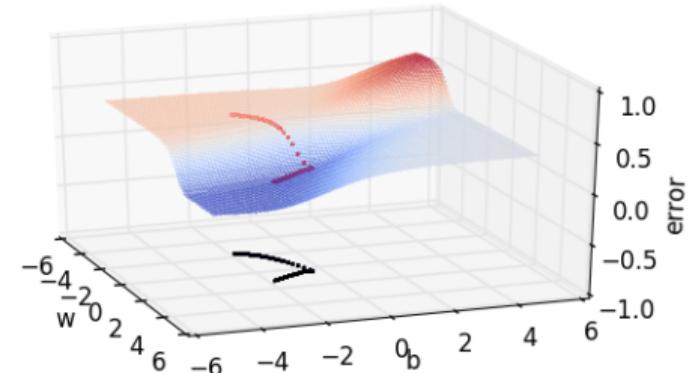
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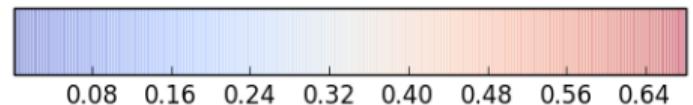
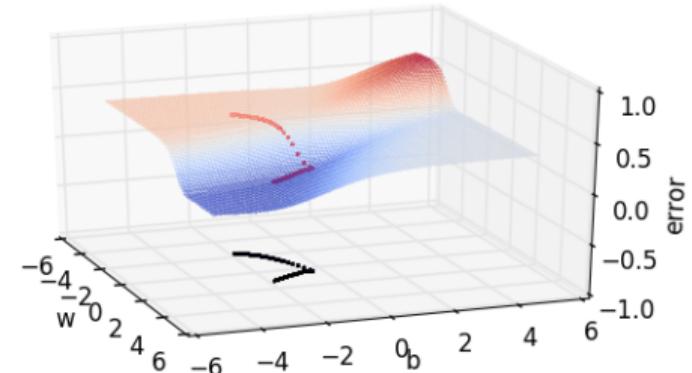
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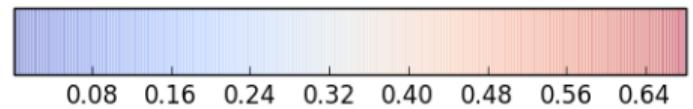
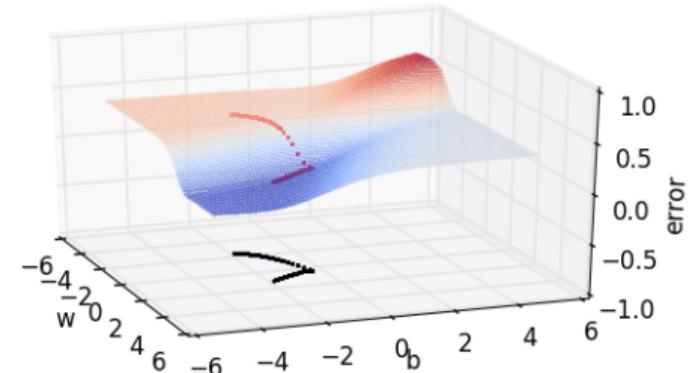
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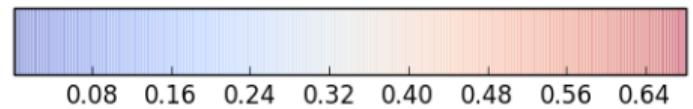
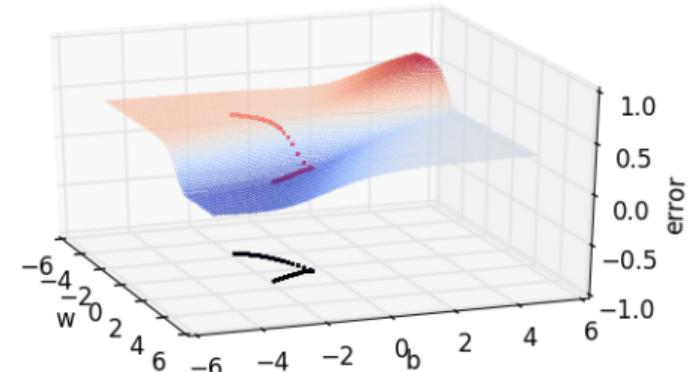
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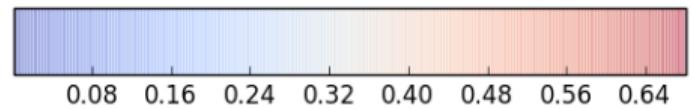
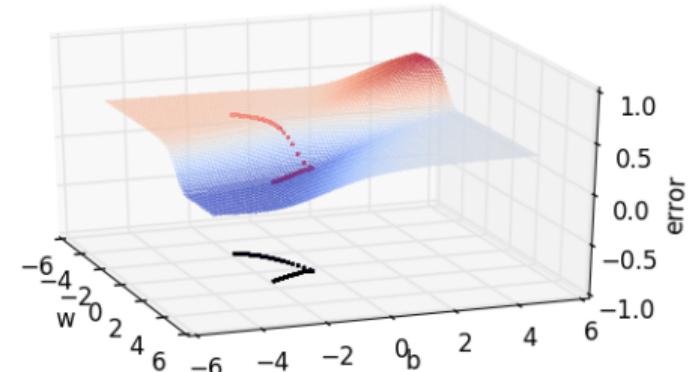
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