**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer :-

We have a normal distribution with 45 and 8.0

Let X be the amount of time it takes to complete the rapairing of a customer’s car.

To finish in 1 hour you must have X<= 50 so the question is to find Pr(X > 50)

Pr(X>50) = 1 – Pr(X<=50)

Z = (X - )/ = (X -45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X<=50) = Pr(Z<=(50-45)/8.0) = Pr(Z <= 0.625) = 73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% i. e. 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer :-

We have a normal distribution with = 38 and = 6.

Let X be the number of employees.

So according to the question

1. Probability of employees greater than the age of 44 = Pr(X>44)

Pr(X>44) = 1 – Pr(X<=44)

Z = (X - )/ =(X-38)/6

Thus the question can be answered by using the normal table to find

Pr(X<=44) = Pr(Z<=(44-38)/6) = Pr(Z<=1) = 84.1345%

Probability that the employee will be greater than the age of 44 = 100-84.1345 = 15.86%

So the probability of number of employees between 38-44 years of age

= Pr(X<44) – 0.5 = 84.13450 = 34.1345%

Therefore, the statement that “ More employees at the processing center are older than 44 than between 34-44” is true

1. Probability of employees less than age of 30 = Pr(X<30)

Z = (X- )/ = (30-38)/6

Thus the question can be answered by using the normal table to find

Pr(X<= 30) = Pr(Z<=(30-38)/6) = Pr(Z<=-1.3333) = 9.12%

So the number of employees with probability 0.912 of them being under the age of 30

= 0.0912\*400 = 36.48 (or 36 employees)

Therefore the statement B of the question is also True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer :-

As we know that if X ~ *N*(μ1, σ12 ) and Y ~(μ2, σ22 ) are two independent random variables then X + Y ~ N(μ1+ μ2, σ12 + σ22 ) and X – Y ~ N(μ1- μ2, σ12 + σ22 )

Similarly, if Z = a\*X + b\*Y, where X and Y are as defined above, i.e. Z is linear combination of Z and Y , then Z~N(aμ1+ bμ2, a^2σ12 + a^2σ22 )

Therefore in the question

2X1~N(2 u , 4 σ2 ) and

X1+X2~N(μ+ μ, σ2 + σ2 ) ~ N(2μ, 2σ2)

2X1 – (X1 + X2) = N(4μ, 6σ2 )

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer :-

The probability of getting value between a and b should be 0.99

So the Probability of going wrong, or the probability outside the a and b area is 0.01 ( 1 – 0.99)

The Probability towards left from a = -0.005

The Probability towards right from b = +0.005

So since we have the Probabilities of a and b, we need to calculate X, the random variable at a and b which has got these Probabilities

By finding the Standard Normal Variable Z (Z Value), we need to calculate X Values

Z = (X – meu) / sigma

For probability 0.005 the Z – Value -2.57

Z \* sigma + meu = X

Z(-0.005)\*20+100 = 151.5

Z(+0.005)\*20+100 = 48.5

So option D is correct

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answer :-

1. Mean Profit is Rs. 540M

Standard Deviation is Rs. 225M

Range is Rs (99.0081 , 980.9918) in Millions

1. 5th Percentile of profit (in Million Rupees)) is 170.0
2. Making 1 Loss :- Confidence Interval :- 0.0477903522728147

Making 2 Loss :- Confidence Interval :- 0.040059156863817086