# MATH5470 Assigment 3

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## 1 Q1, ISLR

### (a)

For $x < \xi$, we have $(x-\xi)_+^3 = 0$. Then if $f(x) = f_1(x)$, the corresponding factor of $x$ is equal, i.e.,

$$a_1 = \beta_0, b_1 = \beta_1, c_1 = \beta_2, d_1 = \beta_3. \tag{1}$$

### (b)

For $x < \xi$, we have $(x-\xi)_+^3 = x^3 - 3\xi x^2 + 3\xi^2 x - \xi^3$. Then if $f(x) = f_2(x)$, for any $x$, we have

$$a_2 + b_2 x + c_2 x^2 + d_2 x^3 = (\beta_0 - \beta_4\xi^3) + (\beta_1 + 3\beta_4\xi^2)x + (\beta_2 - 3\beta_4\xi)x^2 + (\beta_3 + \beta_4)x^3. \tag{2}$$

Then we have

$$a_2 = \beta_0 - \beta_4\xi^3, b_2 = \beta_1 + 3\beta_4\xi^2, c_2 = \beta_2 - 3\beta_4\xi, d_2 = \beta_3 + \beta_4. \tag{3}$$

### (c)

$$\begin{aligned} f_1(\xi) &= a_1 + b_1\xi + c_1\xi^2 + d_1\xi^3 & (4)\\ &= \beta_0 + \beta_1\xi + \beta_2\xi^2 + \beta_3\xi^3 & (5) \end{aligned}$$

$$\begin{aligned} f_2(\xi) &= a_2 + b_2\xi + c_2\xi^2 + d_2\xi^3 & (6)\\ &= (\beta_0 - \beta_4\xi^3) + (\beta_1 + 3\beta_4\xi^2)\xi + (\beta_2 - 3\beta_4\xi)\xi^2 + (\beta_3 + \beta_4)\xi^3 & (7)\\ &= \beta_0 + \beta_1\xi + \beta_2\xi^2 + \beta_3\xi^3 & (8) \end{aligned}$$

Thus, we have $f_1(\xi) = f_2(\xi)$.

### (d)

First, we have

$$\begin{aligned} f_1''(x) &= 2c_1 + 6d_1 x & (9)\\ f_2''(x) &= 2c_2 + 6d_2 x & (10) \end{aligned}$$

Then we can see

$$\begin{aligned} f_1''(\xi) &= 2c_1 + 6d_1\xi && (11)\\ &= 2\beta_2 + 6\beta_3\xi && (12) \end{aligned}$$

$$\begin{aligned} f_2''(\xi) &= 2c_2 + 6d_2\xi && (13)\\ &= 2(\beta_2 - 3\beta_4\xi) + 6(\beta_3 + \beta_4)\xi && (14)\\ &= 2\beta_2 + 6\beta_3\xi. && (15) \end{aligned}$$

Thus, we have $f_1''(\xi) = f_2''(\xi)$.

# 2 Q2, ISLR

## (a)

When $\lambda = \infty, m = 0$, the first term has no effect and $g(x) = 0$. Thus, $\hat{g} = 0$.

## (b)

When $\lambda = \infty, m = 1$, the first term has no effect and $g'(x) =$. Thus, $\hat{g}$ is a constant.

## (c)

When $\lambda = \infty, m = 2$, the first term has no effect and $g''(x) = 0$. Thus, $\hat{g}$ is a linear function such as $ax + b$.

## (d)

When $\lambda = \infty, m = 3$, the first term has no effect and $g'''(x) = 0$. Thus, $\hat{g}$ is a quadratic function such as $ax^2 + bx + c$.

## (e)

When $\lambda = 0, m = 3$, the second term has no effect and this is a least square problem. Thus, $\hat{g}$ could be any form.

# 3 Q3, ISLR

The model can be rewritten as

$$f(x) = \begin{cases} 1 + x, & x < 1 \\ -2x^2 + 5x - 1, & x \geq 1 \end{cases} \quad (16)$$

The intercepts for first and second parts are respectively 1 and -1; the slopes for first and second parts are respectively 1 and 5; the convexity for first and second parts are respectively 0 and -2.

The curve is shown as Fig.1.

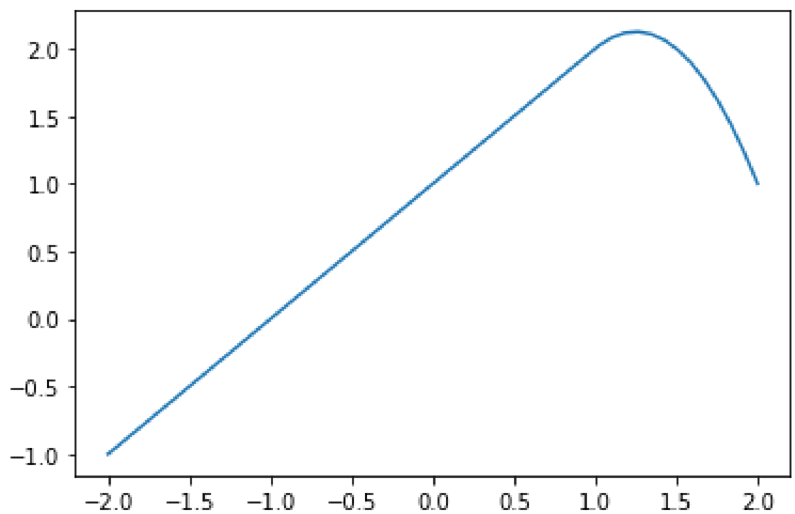


Figure 1: Curve for Q3

# 4 Q6, ISLR

## (a)

After performing polynomial regression with the degree from 1 to 10, we show the cross-validation scores in Fig.2. It can be found that the result of the optimal degree is 4.

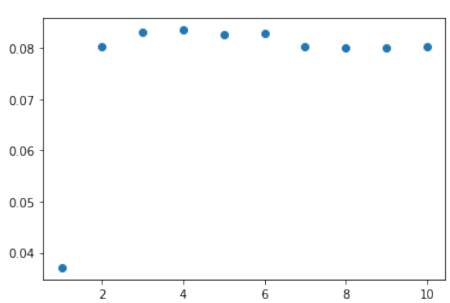


Figure 2: Cross-validation scores

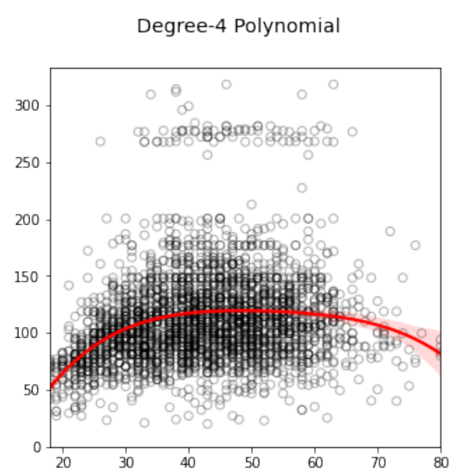


Figure 3: Fitting curve and data

Using ANOVA, we list the p-value of different models in Table 1. We find that 4 is also the optimal degree.

The figure of resulting fit and original data is shown in Fig.3.

| Degree | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| error | 2.21e-32 | 1.67e-3 | 5.09e-2 | 3.69e-1 | 1.61e-1 |

Table 1: p-value in ANOVA

### (b)

According to Fig.4, the optimal cut is 8. And the result figure is shown in Fig.5.

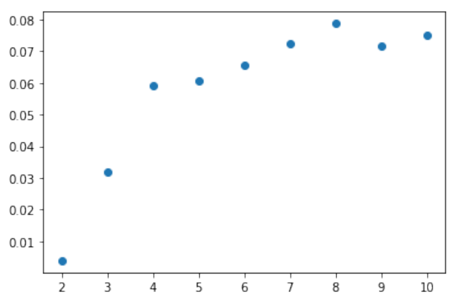


Figure 4: Cross-validation scores

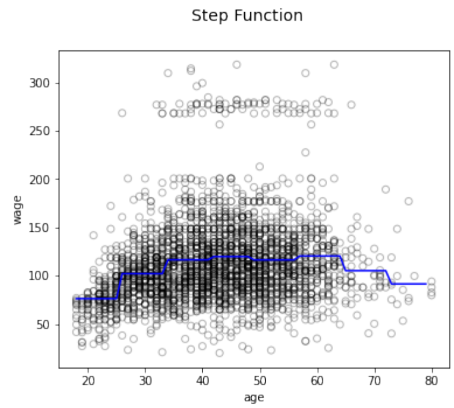


Figure 5: Fitting curve and data

## 5 Q8, ISLR

We explore the Auto data set here. First, we show the relevance between different factors in Fig.6.

Then we only consider two factors here, i.e., horsepower and displacement to respectively see their effects on the miles per gallon of different models of cars. We use natural splines to fit the data. We plot the cross-validation mean-square-error to find the optimal degree for the factor, and then fit data into the model.

For horsepower, we see the optimal degree is 3 in Fig.7 and corresponding fitting result is Fig.8.

For horsepower, we see the optimal degree is 4 in Fig.9 and corresponding fitting result is Fig.10.

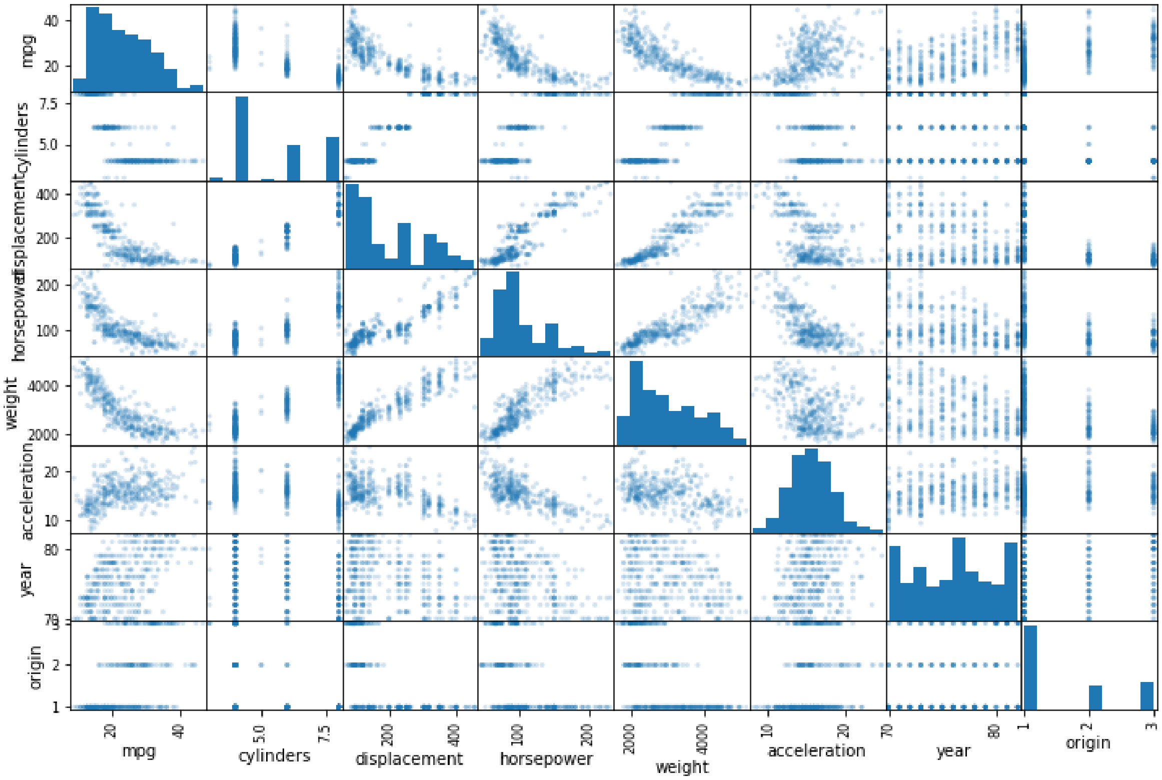


Figure 6: Relevance among factors

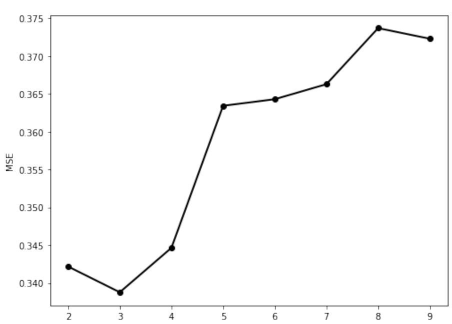


Figure 7: Cross-validation MSE

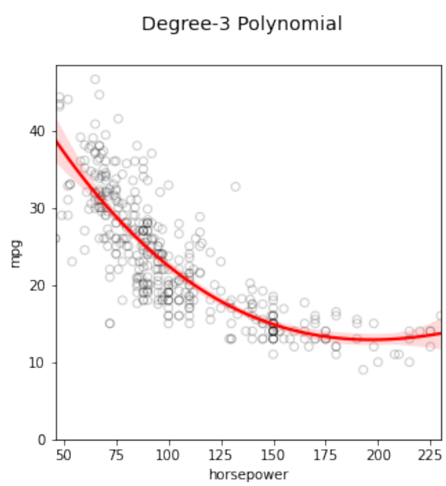


Figure 8: Fitting curve and data

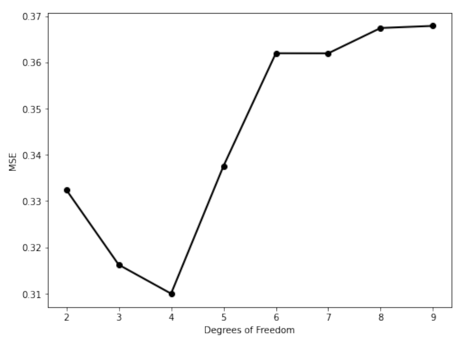


Figure 9: Cross-validation MSE

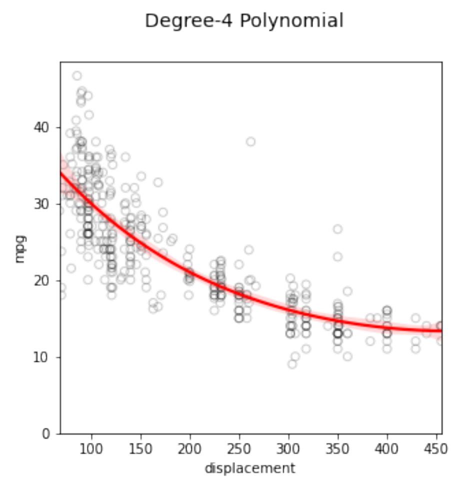


Figure 10: Fitting curve and data