Assn 3 Written assignment.

4-1. 1)

$H_i = W_{0,i} I_0 + W_{1,i} I_1 + \cdots + W_{n82,i} I_{n82} + W_{n83,i} I_{n83} + b_i$ ... ①.

$Z_i = \phi(H_i)$ ... ②. $\quad i: 0 \sim 127$.

$O_j = W'_{0,j} Z_0 + W'_{1,j} Z_1 + \cdots + W'_{127,j} Z_{127} + b'_j$ ... ③

$R_j = \phi(O_j)$ ... ④. $\quad j: 0 \sim 9$.

$MSE = \frac{1}{2} \sum_{k=0}^{1} (\hat{R}_k - R_k)^2$ ... ⑤ $\hat{R}_k = 0$ if label == k else 0.

a) $\frac{\partial MSE}{\partial W_{i,j}} = \frac{\partial MSE}{\partial Z_j} \cdot \frac{\partial Z_j}{\partial H_j} \cdot \frac{\partial H_j}{\partial W_{i,j}}$ ... ⑥.

i) Let's denote $MSE_k = \frac{1}{2} (\hat{R}_k - R_k)^2$ and $\therefore MSE = \sum_{k=0}^{9} MSE_k$.

Then $\frac{\partial MSE}{\partial Z_j} = \frac{\partial MSE_0}{\partial Z_j} + \frac{\partial MSE_1}{\partial Z_j} + \cdots + \frac{\partial MSE_9}{\partial Z_j}$

$= \frac{\partial MSE_0}{\partial R_0} \frac{\partial R_0}{\partial O_0} \frac{\partial O_0}{\partial Z_j} + \cdots + \frac{\partial MSE_9}{\partial R_9} \frac{\partial R_9}{\partial O_9} \frac{\partial O_9}{\partial Z_j}$

$= (\hat{R}_0 - R_0) \cdot R_0 (1 - R_0) \cdot W_{j,0} + \cdots +$

$(\hat{R}_9 - R_9) \cdot R_9 (1 - R_9) \cdot W_{j,9}$.

(by ⑥, ④, ③.)

$= \sum_{k=0}^{9} (\hat{R}_k - R_k) \cdot R_k (1 - R_k) W_{j,k}$.

ii) $\frac{\partial Z_j}{\partial H_j} = Z_j(1-Z_j)$ (by ②)

iii) $\frac{\partial H_j}{\partial W_{i,j}} = I_i$. (by ①).

$\therefore \frac{\partial MSE}{\partial W_{i,j}} = \sum_{k=0}^{q} (\hat{R}_k - R_k) \cdot R_k(1-R_k) W_{j,k} \cdot Z_j(1-Z_j) \cdot I_i$.

b) $\frac{\partial MSE}{\partial b_j} = \frac{\partial MSE}{\partial Z_j} \cdot \frac{\partial Z_j}{\partial H_j} \cdot \frac{\partial H_j}{\partial b_j}$

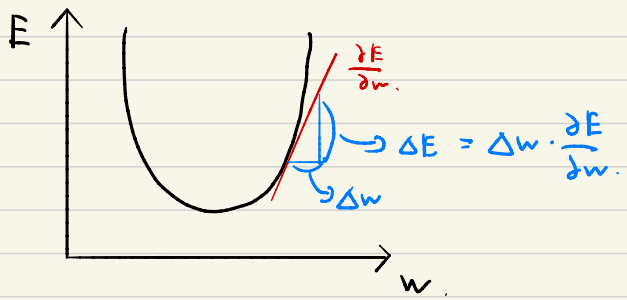
As $\frac{\partial H_j}{\partial b_j} = 1$ (by ①),

$\frac{\partial MSE}{\partial b_j} = \sum_{k=0}^{q} (\hat{R}_k - R_k) \cdot R_k(1-R_k) W_{j,k} \cdot Z_j(1-Z_j)$

(by a-i), a-ii)).

2) $w = w - \eta \frac{\partial MSE}{\partial w} = w + \Delta w$.

notice that $\Delta MSE = \frac{\partial E}{\partial w} \Delta w$.



so $\Delta w = -\eta \frac{\partial E}{\partial w}$,

$$\Delta E = \Delta w \cdot \frac{\partial E}{\partial w} = -\eta \left(\frac{\partial E}{\partial w}\right)^2 < 0.$$