CSCI-4100 Assignment 7
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1. (500) Classifying Handwritten Digits: 1 vs. 5
Linear regression followed by pocket algorithm improvement is used in this problem.
Math definition for feature 1 (normalized y-axis asymmetry):

$$\frac{1}{256}\sum_{i=0}^{15}\sum_{j=0}^{7}|pix[i][j]-pix[i][15-j]|$$

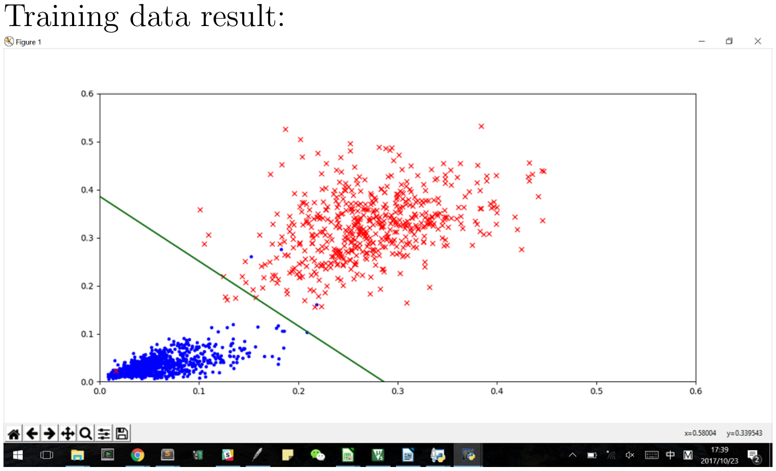
Math definition for feature 2 (normalized x-axis asymmetry):

$$\frac{1}{256}\sum_{i=0}^{7}\sum_{j=0}^{15}|pix[i][j]-pix[15-i][j]|$$

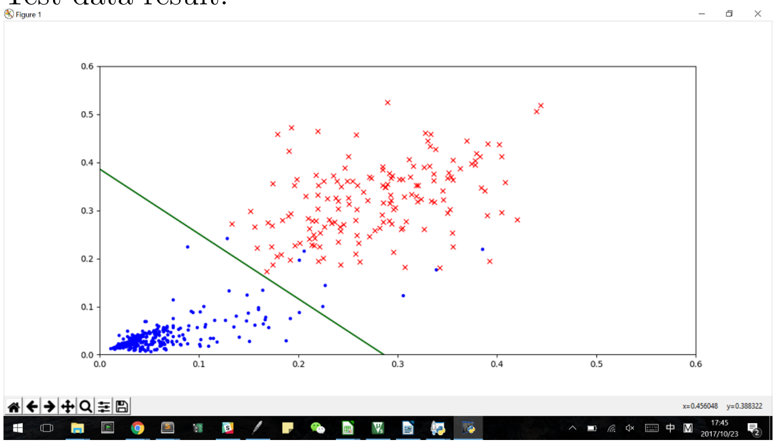
Normalization is required since original feature values are too large and will make linear regression result too small to play a role.
For all the plots below, feature 1 is horizontal axis, and feature 2 is vertical axis.
(a)
The training output $[g_0...g_d]$ is

$$[1.2215, -4.2678, -3.1657]$$

Training data result:



Test data result:



(b)
Error Calculation: The error calculation is done in the software, the result is given here:

$$E_{in} = 0.0045$$
$$E_{test} = 0.0189$$

(c)
Error bound calculation:

$$E_{out} \leq E_{in} + \sqrt{\frac{8}{1561} ln(\frac{4 \times ((1561 \times 2)^3 + 1)}{0.05})} = 0.0045 + 0.3823 = 0.3868$$

$$E_{out} \leq E_{test} + \sqrt{\frac{1}{2 \times 424} ln(\frac{1 \times 2}{0.05})} = 0.0189 + 0.0660 = 0.0849$$

The bound based on E_{test} is a better bound.

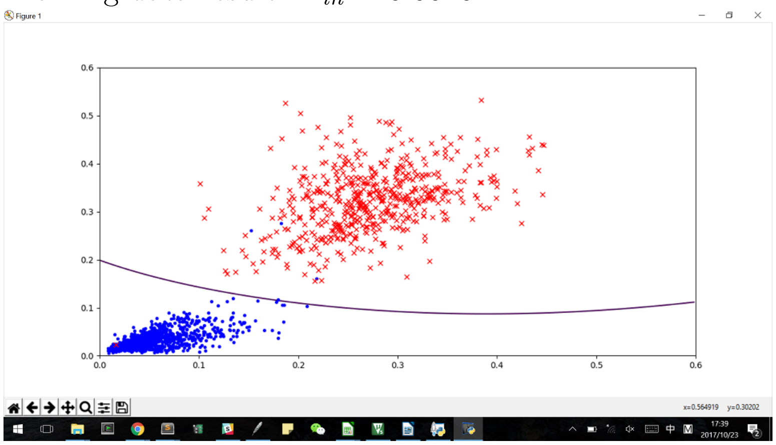
(d)
The 3rd-order transform is performed as following:

$$[x_0, x_1, x_2] \longrightarrow [x_0, x_1, x_2, x_1^2, x_2^2, x_1x_2, x_1^3, x_2^3, x_1^2x_2, x_1x_2^2]$$

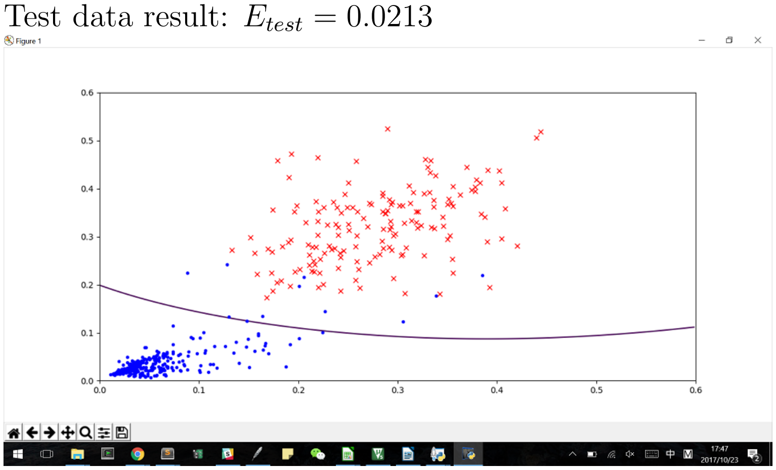
The training output $[g_0 ... g_d]$ is

$$[1.0567, 2.8568, -4.4757, -10.6157, -4.8360, -36.8246, 18.9741, 3.1472, -0.3295, 83.0706]$$

Training data result: $E_{in} = 0.0026$



Test data result: $E_{test} = 0.0213$



Error bound calculation:

$$E_{out} \leq E_{in} + \sqrt{\frac{8}{1561} ln(\frac{4 \times ((1561 \times 2)^{10} + 1)}{0.05})} = 0.0026 + 0.6594 = 0.6620$$

$$E_{out} \leq E_{test} + \sqrt{\frac{1}{2 \times 424} ln(\frac{1 \times 2}{0.05})} = 0.0213 + 0.0660 = 0.0873$$

The bound based on E_{test} is a better bound.

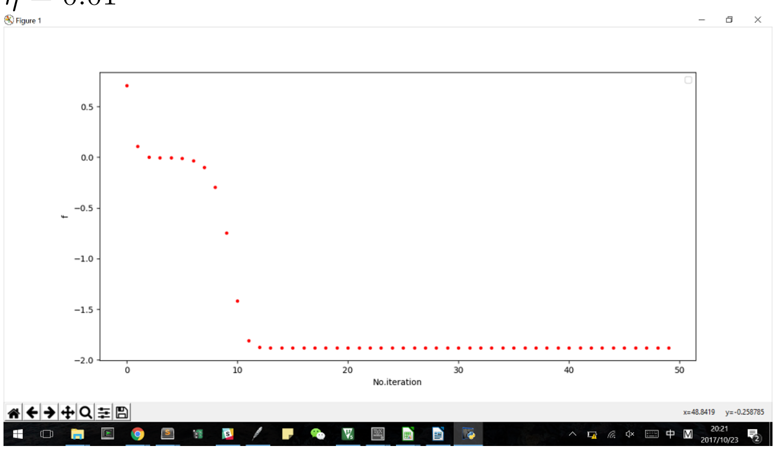
(e) I would like to deliver the 2D-perceptron model instead of the 3rd-order-transform one. First of all, the 2D model has a better error bar; data is nearly separable in its original form. Second, the 3rd-order-transform model is much more complicated than the 2D model ($d_{vc} = 3$ vs $d_{vc} = 10$), and a over complicated model is likely to fit more noise.

2. (200) Gradient Descent on a "Simple" Function
(a) Original Function: $f(x) = x^2 + 2y^2 + 2sin(2\Pi x)sin(2\Pi y)$

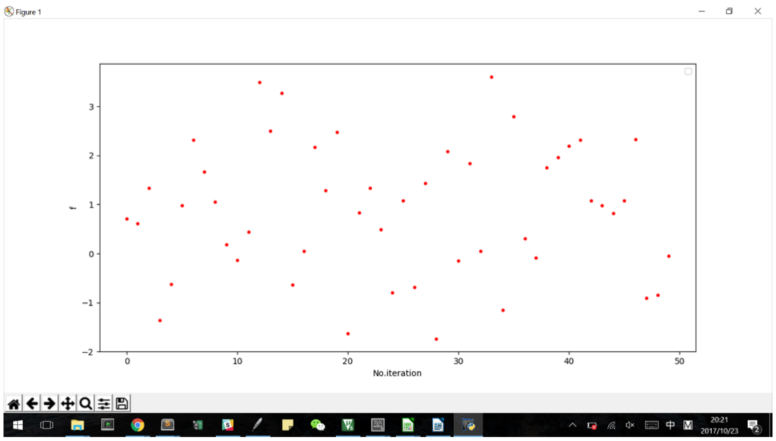
$$\frac{df(x)}{dx} = 2x + 4\Pi cos(2\Pi x)sin(2\Pi y)$$

$$\frac{df(x)}{dy} = 4y + 4\Pi sin(2\Pi x)cos(2\Pi y)$$

Gradient Decent result:
$\eta = 0.01$



$\eta = 0.1$
With $\eta = 0.1$ the algorithm results in serious bouncing and cannot find the local minimum.



(b) Table:

(x,y,n)	x_final	y_final	minimum
(0.1, 0.1, 0.01)	0.243804969365	-0.237925821486	-1.87668723808
(0.1, 0.1, 0.1)	0.348357019225	-0.069582260188	-0.0446299777336
(1, 1, 0.01)	1.21807030131	0.712811950602	0.0851684974051
(1, 1, 0.1)	0.198459835146	0.457823614804	0.489676970193
(-0.5, -0.5, 0.01)	-0.731377460414	-0.237855362902	-1.38905623599
(-0.5, -0.5, 0.1)	0.591800710459	-0.125032321141	-0.723177059966
(-1, -1, 0.01)	-1.21807030131	-0.712811950602	0.0851684974051
(-1, -1, 0.1)	-0.198459835146	-0.457823614804	0.489676970193

It is observed that different starting location gives different local minimum; it is hard to find a global minimum in such condition.

3. (300) Problem 3.16 in LFD

(a)

$$cost(accept) = P[correct] * 0 + P[intruder] * C_a = (1 - g(x)) \times C_a$$

$$cost(reject) = P[correct] * C_r + P[intruder] * 0 = g(x) \times C_r$$

(b) It's the trade off between accept and reject. When $cost(accept) \leq cost(reject)$ we will accept the person:

$$(1 - g(x)) \times C_a \leq g(x) \times C_r$$

$$C_a - g(x) \times C_a \leq g(x) \times C_r$$
$$C_a \leq g(x) \times (C_r + C_a)$$
$$g(x) \geq \frac{C_a}{C_r + C_a}$$
$$k = \frac{C_a}{C_r + C_a}$$

(c)
Supermarket: $C_a = 1$ $C_r = 10$

$$k = \frac{1}{10 + 1} = 0.0909$$

CIA: $C_a = 1000$ $C_r = 1$

$$k = \frac{1000}{1000 + 1} = 0.9990$$

The intuition is that the supermarket has a high cost of false reject, so it will accept the person at a relatively low threshold. CIA has a high cost of false accept, so they accept the person only when they are super sure that it's the correct person, and thus has a high threshold.