範例: Feedback Linearization　　P0

考慮一系統

$$\begin{cases}\dot{x}_1 = x_2 + x_2^3 \\ \dot{x}_2 = u\end{cases}$$

當 output $y = x_1$ 時，令 $z_1 = y$，$z_2 = \dot{y}$

則
$$\begin{cases}\dot{z}_1 = z_2 \\ \dot{z}_2 = \dot{x}_2 + 3x_2^2\dot{x}_2 = (1+3x_2^2)u.\end{cases}$$

希望 (z_1, z_2) 的系統成為線性的系統

而 eigenvalue 在 $-1+2j$，$-1-2j$.

則 $(\lambda+1-2j)(\lambda+1+2j) = \underline{\lambda^2+2\lambda+5=0}$.

即 $\begin{pmatrix}\dot{z}_1 \\ \dot{z}_2\end{pmatrix} = \begin{bmatrix}0 & 1 \\ -5 & -2\end{bmatrix}\begin{bmatrix}z_1 \\ z_2\end{bmatrix}$ 才是所需要的線性系統

所以 $u = \left(\frac{1}{1+3x_2^2}\right)(-5z_1 - 2z_2)$

$$\underline{u = \left(\frac{1}{1+3x_2^2}\right)\left(-5x_1 - 2x_2 - 2x_2^3\right)}$$

本範例之程式如下:

```
delt=0.0001;
totTime = 10 ;
totalStep = totTime/delt ;
```

```
tarr= [0 : 1 : totalStep]*delt;
uarr = tarr*0;   % 0 array
yarr= uarr;      % 0 array
x1arr=uarr;     x2arr=uarr;
z1arr=uarr;     z2arr=uarr;
x1prev =  2;  % ªì©lÈ
x2prev = -1;  % ªì©lÈ
x1arr(1) = x1prev ;     % time = 0;
x2arr(1) = x2prev ;
z1prev = x1prev ;
z2prev = x2prev + x2prev*x2prev*x2prev ;
z1arr(1) = z1prev ;
z2arr(1) = z2prev ;
for i = 1 :  totalStep
    tmp1 = 1 + 3*x2prev*x2prev ;
    u =  (-5*z1prev - 2*z2prev)/tmp1;
    x1cur = x1prev + (x2prev + x2prev*x2prev*x2prev)*delt ;
    x2cur = x2prev + u*delt;
    z1cur  = x1prev ;
    z2cur  = x2cur + x2cur*x2cur*x2cur ;
    x1arr(i+1) = x1cur ;
    x2arr(i+1) = x2cur ;
    z1arr(i+1) = z1cur ;
    z2arr(i+1) = z2cur ;
    uarr(i+1) = u ;
    yarr(i+1) = x1cur;
    x1prev = x1cur;
    x2prev = x2cur;
    z1prev = z1cur;
    z2prev = z2cur;
end
figure(1);  plot(tarr,x1arr,'r',tarr,x2arr,'b');
figure(2);  plot(tarr,z1arr,'r',tarr,z2arr,'b');
figure(3);  plot(x1arr, x2arr,'r');
figure(4);  plot(z1arr, z2arr,'b');
figure(5);  plot(tarr, yarr,'b');
```

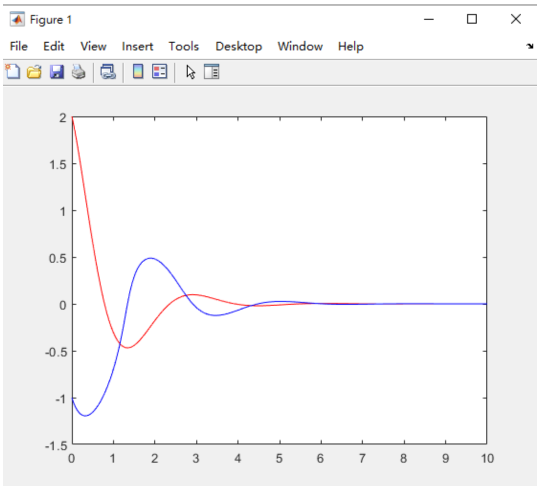
Figure 1
File Edit View Insert Tools Desktop Window Help
2
1.5
1
0.5
0
-0.5
-1
-1.5
0 1 2 3 4 5 6 7 8 9 10

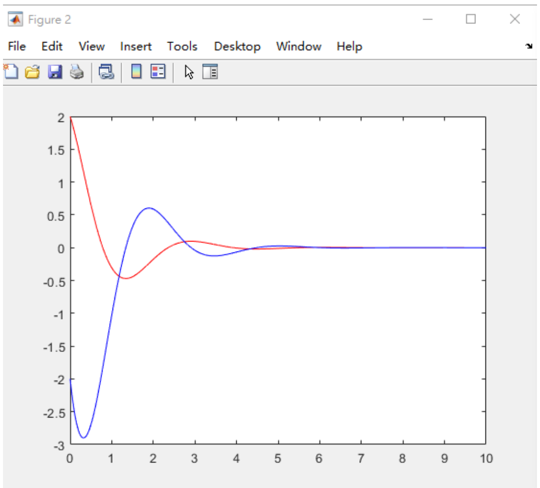
Figure 2
File Edit View Insert Tools Desktop Window Help
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0 1 2 3 4 5 6 7 8 9 10

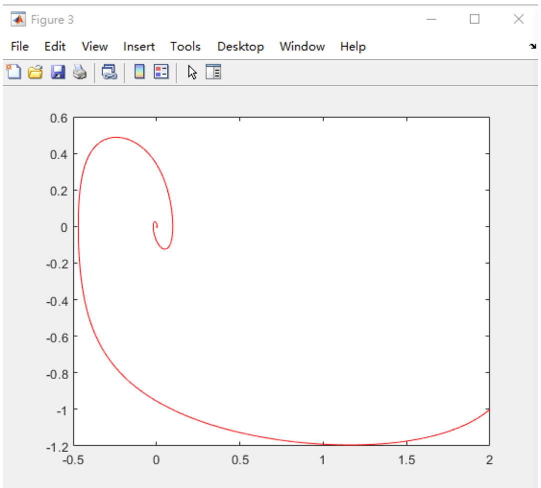
Figure 3
File Edit View Insert Tools Desktop Window Help
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-0.5 0 0.5 1 1.5 2

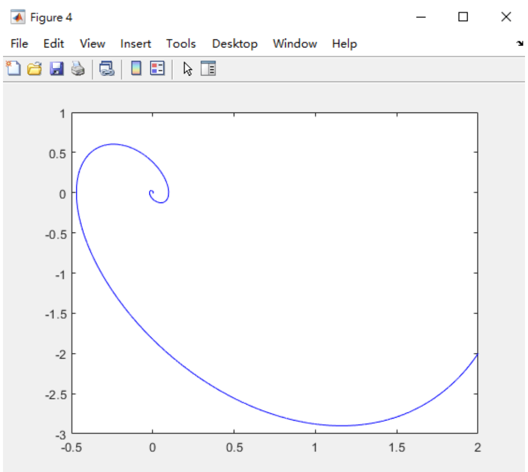
Figure 4
File Edit View Insert Tools Desktop Window Help
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-0.5 0 0.5 1 1.5 2

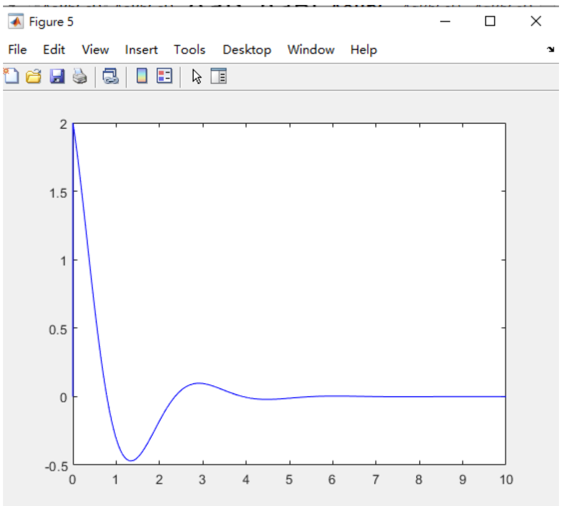
Figure 5
File Edit View Insert Tools Desktop Window Help
2
1.5
1
0.5
0
-0.5
0 1 2 3 4 5 6 7 8 9 10

作業

將你的學號相加 = 10*a + b

1. 利用 Feedback Linearization 的方法，設計一控制器使下面系統穩定，且如一線性系統 [eigenvalue 在 $-a \pm jb$，及 $-a$ (如果有第3個eigenvalue)]

(甲)

$$\begin{cases} \dot{x}_1 = x_1^2 + x_2 \\ \dot{x}_2 = -x_1 + u \end{cases}$$

(a) 讓 $z_1 = x_1$, $z_2 = \dot{z}_1$，則上述系統會變成

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = \alpha(x) + \beta(x) u \end{cases}$$

找出 $\alpha(x) = ?$ 及 $\beta(x) = ?$

(b) 設計一 F.B. Linearization 控制器使得

$$\begin{cases} \dot{z}_1 = \\ \dot{z}_2 = \end{cases} \begin{pmatrix} z_2 \\ +k_1 z_1 + k_2 z_2 \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix}}_{A} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \text{ 即 } \dot{z} = Az$$

其中A的 eigenvalues 為 $-a \pm jb$.

(c) 寫 matlab 程式模擬結果，畫出 (x_1, x_2) 的 phase portrait，也畫出 (z_1, z_2) 的 phase portrait

(乙).

$$\dot{x}_1 = x_2 + x_1^3$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = u.$$

讓 output $y = x_1$ ($z_1 = y$, $z_2 = \dot{y}$, $z_3 = \ddot{y}$)
(設計 FB control 使系統 (以3度). 利用 Matlab 模擬

(丙)

$$\dot{x}_1 = \frac{1}{2}x_1^3 + \theta_1 x_2$$
$$\dot{x}_2 = \theta_2 x_1 x_2 + u.$$

當 θ_i 有誤差時, 即 ($\theta_1 = 7 \pm 2$, $\theta_2 = 2 \pm 0.5$) 時, 設計者不知道誤差是多少, 所以只能以標準的值來做設計模擬, (即 $\theta_1 = 7$, $\theta_2 = 2$). 設計 $u = \text{------}$。
但是在 Matlab 模擬時, 必須使用真正的 θ_1, θ_2, (在本題中使用 $\theta_1 = 9$, $\theta_2 = 1.5$)
($\theta_1 = 7+2$) ($\theta_2 = 2-0.5$)

(i) 利用 Matlab. 模擬當沒有誤差時的情況 ($\theta_1 = 7$, $\theta_2 = 2$)

(ii) 〃 〃 〃 當存在誤差 〃 〃 〃. ($\theta_1 = 9$, $\theta_2 = 1.5$)

這兩個 Case 的 u 都是一樣是以沒有誤差時在做