Exercise 1

$ab \backslash cd$	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	0	0	1
10	1	0	0	1

$\to \neg a \wedge c$

$\to a \wedge \neg d$

(a) $(\neg a \wedge c) \vee (a \wedge \neg d)$

$ab \backslash cd$	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	0	0	1
10	1	0	0	1

$\to \neg a \wedge \neg c$

$\to a \wedge d$

(b) $\neg(\neg a \wedge \neg c) \wedge \neg(a \wedge d)$

so $(a \vee c) \wedge (\neg a \vee \neg d)$

Exercise 2

Karnaugh map for $r \leftrightarrow (a \vee b)$ would be

r \ ab	00	01	11	10
0	1	0	0	0
1	0	1	1	1

In order to make the DNF minimal,
we need to make the block as large as possible.
Since the size of the block needs to be $\frac{8}{2^n}$ $(n = 1 \text{ or } 2 \text{ or } 3)$,
situation above makes the DNF minimal.

$\boxed{1}$ $\quad \neg r \wedge \neg a \wedge \neg b$

$\boxed{1 \,|\, 1}$ $\quad r \wedge b$

$\boxed{1 \,|\, 1}$ $\quad r \wedge a$

Therefore, the DNF equivalent of the expression is

$$(\neg r \wedge \neg a \wedge \neg b) \vee (r \wedge b) \vee (r \wedge a)$$

Exercise 3

(a)

$a \vee \neg b$, $\neg a \vee \neg d$

$\Leftrightarrow (a \vee \neg b) \wedge (\neg a \vee \neg d)$

$\Leftrightarrow \neg(\neg a \wedge b) \wedge \neg(a \wedge d)$

For this syllogism to return 1, $(\neg a \wedge b)$ and $(a \wedge d)$ should be both 0.

Therefore possible pairs of a, b, and d are

a	b
0	0
1	1
1	0

a	d
0	0
0	1
1	0

ab \ cd	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	1	0	0	1
10	1	0	0	1

Zeros arising from $(a \vee \neg b)$

Zeros arising from $(\neg a \vee \neg d)$

(b) example of $\delta = (\neg a \wedge \neg b) \vee (a \wedge \neg d)$

In $a \vee \neg b, \neg a \vee \neg d \vDash \delta$,

0	$\vDash$	0	(o)
0	$\vDash$	1	(o)
1	$\vDash$	1	(o)
1	$\vDash$	0	(x)

if the antecedent is 0,
the succedent δ can be either 0 or 1

if the antecedent is 1,
the succedent δ must be 1.

Therefore, there are 2^8 different δ. (8 being the number of 0s in Karnaugh map)