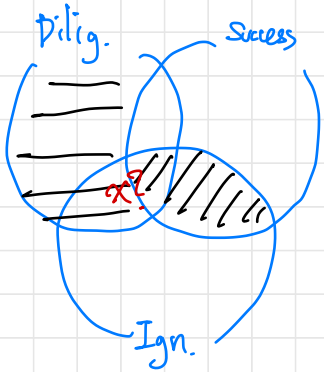
Exercise 1)

1. $\dfrac{a \models b \quad c \models \neg b}{a \not\models \neg c}$

universe = students
a = diligent students
b = being successful
c = ignorant students



Not sound.

Counter example: Assume there is a universe consisted of one diligent student.
According to the syllogism, this student is successful.
Since there is no ignorant student in the universe,
the second assumption can be said to be true.
However, since there is one diligent student in the universe but no ignorant student,
the conclusion "Some diligent students are ignorant" is false.
Therefore, this argument is not sound, because sound argument should be true
in every universe.

2.

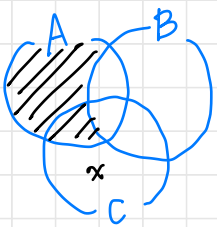
$$\frac{a \vDash b \quad c \nvDash b}{c \nvDash a}$$

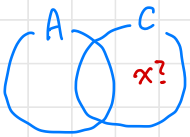
universe = animals

a = eagle

b = able to fly

c = pig





Sound.

Deriving from Barbara

$$\frac{a \vDash b \quad b \vDash c}{a \vDash c} \Rightarrow \frac{b \vDash c \quad a \nvDash c}{a \nvDash b} \Rightarrow \begin{matrix} a \to c \\ b \to a \\ c \to b \end{matrix} \quad \frac{a \vDash b \quad c \nvDash b}{c \nvDash a}$$

Exercise 2)

If $\not\vDash$ appears only once in a syllogism, it can't be derived from Barbara, because you need two $\not\vDash$s to make two $\vDash$ by switching one of the assumptions & a conclusion. Hence, it is always unsound if $\not\vDash$ occurs only once in any syllogism.

The number of occurrences of $\not\vDash$ in th n should be the same as that in the conclusion

The number of occurrences of $\neg$ should be even for a syllogism to be sound

$$\frac{a \vDash \neg b \quad b \vDash c}{c \not\vDash a}$$

a = animals
b = unicorns
c = horses

The number of $\not\vDash$ in assumption is different from that in conclusion.
Plus, the number of $\neg$ is not even.
Therefore, it is not sound.

Exercise 3)

Barbara $\dfrac{a \vDash b \quad b \vDash c}{a \vDash c}$

Substitute $a \to c$, $b \to a$, $c \to b$ $\Rightarrow$ $\dfrac{c \vDash a \quad a \vDash b}{c \vDash b}$

$\Rightarrow$ $\dfrac{\dfrac{c \vDash a \quad a \vDash b}{c \vDash b} \quad \dfrac{}{c \not\vDash \neg c}}{c \not\vDash \neg b}$

$\Rightarrow \therefore$ with the existential assumption,

$$\frac{c \vDash a \quad a \vDash b}{c \not\vDash \neg b}$$

Exercise 5)

1. $\text{isBig}, \text{isAmber} \not\models \text{isThickBorder}$
2. $\text{isSmall} \not\models \neg\,\text{isDisc}$
3. $\text{isSmall}, \text{isSquare} \models \neg\,\text{isAmber}$