Exercise 1

Equivalent logical expression $= (\neg a \wedge b) \vee (a \wedge \neg b)$
of the circuit

Applying Tseytin transformation,

$x \longleftrightarrow \neg a \wedge b : \left(\neg x \vee (\neg a \wedge b)\right) \wedge \left(\neg(\neg a \wedge b) \vee x\right)$

$\left((\neg x \vee \neg a) \wedge (\neg x \vee b)\right) \wedge (a \vee \neg b \vee x)$

$(\neg x \vee \neg a) \wedge (\neg x \vee b) \wedge (a \vee \neg b \vee x)$

$y \longleftrightarrow a \wedge \neg b : \left(\neg y \vee (a \wedge \neg b)\right) \wedge \left(\neg(a \wedge \neg b) \vee y\right)$

$\left((\neg y \vee a) \wedge (\neg y \vee \neg b)\right) \wedge (\neg a \vee b \vee y)$

$(\neg y \vee a) \wedge (\neg y \vee \neg b) \wedge (\neg a \vee b \vee y)$

$z \longleftrightarrow x \vee y : \left(\neg z \vee (x \vee y)\right) \wedge \left(\neg(x \vee y) \vee z\right)$

$(\neg z \vee x \vee y) \wedge \left((\neg x \wedge \neg y) \vee z\right)$

$(\neg z \vee x \vee y) \wedge (\neg x \vee z) \wedge (\neg y \vee z)$

$\therefore$ Equisatisfiable CNF expression :

$(\neg x \vee \neg a) \wedge (\neg x \vee b) \wedge (a \vee \neg b \vee x)$

$\wedge (\neg y \vee a) \wedge (\neg y \vee \neg b) \wedge (\neg a \vee b \vee y)$

$\wedge (\neg z \vee x \vee y) \wedge (\neg x \vee z) \wedge (\neg y \vee z)$

Exercise 2

$$(E \lor F) \land (\neg A \lor B) \land C$$

$$(\neg E \rightarrow F) \land (A \rightarrow B) \land (1 \rightarrow C)$$

C must be 1 for the CNF exp.
to be satisfiable.

Available cuts :   3 C_1 on the left hand side,
3 C_1 on the right hand side

$$3 \times 3 \times 1 = 9$$

∴ There are 9 satisfying assignments for this expression