Exercise 1)

$$\begin{array}{c} a, b \vDash \neg c \\ \hline\hline a \wedge b \vDash \neg c \\ \hline\hline c \vDash \neg(a \wedge b) \end{array} \qquad\qquad \begin{array}{c} a, b \vDash \neg c \\ \hline\hline c \vDash \neg a, \neg b \\ \hline\hline c \vDash \neg a \vee \neg b \end{array}$$

Both of these proofs involve only equivalences, and both of them starts from the same two premises.

$$\therefore \quad \begin{array}{c} c \vDash \neg(a \wedge b) \\ \hline\hline c \vDash \neg a \vee \neg b \end{array}$$

$$\begin{array}{c} \hline \neg a \vee \neg b \vDash \neg a \vee \neg b \\ \hline\hline \neg a \vee \neg b \vDash \neg(a \wedge b) \end{array} \text{ immediate} \qquad \begin{array}{c} \hline \neg(a \wedge b) \vDash \neg(a \wedge b) \\ \hline\hline \neg(a \wedge b) \vDash \neg a \vee \neg b \end{array} \text{ immediate,}$$

Interpreting $\vDash$ as $\subseteq$,

$$\neg a \vee \neg b \subseteq \neg(a \wedge b)$$

$$\neg(a \wedge b) \subseteq \neg a \vee \neg b$$

$$\therefore \ \neg(a \vee b) = \neg a \wedge \neg b$$

Exercise 2)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{x \vDash y, x}\mathrm{I} \quad \dfrac{}{x, y \vDash y, x}}{x \vee y, x \vDash y, x}\vee L \quad \dfrac{\dfrac{}{z, x \vDash y, x} \quad \dfrac{}{z, y \vDash y, x}}{x \vee y, z \vDash y, x}\vee L}{x \vee y, x \vee z \vDash y, x}\vee L \quad \dfrac{\dfrac{\dfrac{}{x \vDash z, x} \quad \dfrac{}{x, y \vDash z, x}}{x \vee y, x \vDash z, x}\vee L \quad \dfrac{\dfrac{}{z, x \vDash z, x} \quad \dfrac{}{z, y \vDash z, x}}{x \vee y, z \vDash z, x}\vee L}{x \vee y, x \vee z \vDash z, x}\vee L}{x \vee y, x \vee z \vDash x, y \wedge z}\wedge R$$

∴ Conclusion can be derived from immediate rule,

it is universally valid.

Exercise 3)

$$\dfrac{}{x, y \models x, z}\ \mathrm{I} \qquad \dfrac{}{x, y \models y, z} \qquad \dfrac{}{y, z \models x, z} \qquad \dfrac{}{y, z \models y, z}\ \mathrm{I}$$

$$\dfrac{x, y \models x, z \qquad x, y \models y, z}{x, y \models x \wedge y, z}\ \wedge R \qquad \dfrac{y, z \models x, z \qquad y, z \models y, z}{y, z \models x \wedge y, z}\ \wedge R$$

$$\dfrac{x, y \models x \wedge y, z}{y, \neg z, x \models x \wedge y}\ \neg L \qquad \dfrac{y, z \models x \wedge y, z}{y, \neg z, z \models x \wedge y}\ \neg L$$

$$\dfrac{y, \neg z, x \models x \wedge y \qquad y, \neg z, z \models x \wedge y}{x \vee z, y, \neg z \models x \wedge y}\ \vee L$$

$$\dfrac{x \vee z, y, \neg z \models x \wedge y}{x \vee z, y \wedge \neg z \models x \wedge y}\ \wedge L$$

$$\dfrac{x \vee z, y \wedge \neg z \models x \wedge y}{(x \vee z) \wedge (y \wedge \neg z) \models x \wedge y}\ \wedge L$$

$$\dfrac{(x \vee z) \wedge (y \wedge \neg z) \models x \wedge y}{\neg(\neg(x \vee z) \vee (\neg y \vee z)) \models x \wedge y}\ \text{De Morgan's Law}$$

$$\dfrac{\neg(\neg(x \vee z) \vee (\neg y \vee z)) \models x \wedge y}{\models (x \wedge y), (\neg(x \vee z) \vee (\neg y \vee z))}\ \neg R$$

$$\dfrac{\models (x \wedge y), (\neg(x \vee z) \vee (\neg y \vee z))}{\models (x \wedge y) \vee (\neg(x \vee z) \vee (\neg y \vee z))}\ \vee R$$

∴ This sequent is universally valid,
it is a tautology.