ME41055
Multibody Dynamics B

Spring Term 2019

Homework Assignment 6 (HW6)

In this assignment we will study the performance of the following four numerical integration methods applied to the double pendulum problem from homework assignment 1:

1. Euler.
2. Heun.
3. A Runge-Kutta 3^{rd} order.
4. Classical Runge-Kutta 4^{th} order.

Use the equations of motion expressed in terms of independent generalized coordinates as derived in homework assignment 4. The initial conditions are both bars vertically up at zero speed. We assume a gravitational field operating in the *horizontal* direction with a field strength of $g = 9.81$ N/kg.

Our assignment is to determine the angle, in radians, of both bars with respect to the horizontal axis after 3.0 seconds with a maximal absolute error of 10^{-6} rad.

To find that angle we have to determine the accordingly maximum constant step size for the specific numerical integration method applied. We will use the error estimate method based on the method-inherent truncation error and the round-off error due to the finite precision as described in the course book, Chapter 15.3 entitled "Error Estimates".

a. Determine, for the four different methods, the solutions at $T = 3$ seconds for a number of successive step size, where you could use a step refinement like $h = T/2^n$. Start with a small enough step size, for instance $n = 5$. Tabulate the angles and their error estimates as a function of the step size for all four methods. Be sure to use enough digits (at least 10) to show convergence. In some cases it may not be feasible to find such a maximum step size. Either the method used is unstable or the step size becomes so small that it is impractical to reach the end point at $T = 3.0$ seconds. If so, please do not spend too much time on that, but state your case clearly! Discuss your results.

b. Make for each angle a plot, where you plot in one figure the $\log_{10}$(estimated error) versus the $\log_{10}$(step size) for all four methods. Indicate in this plot the various maximum step sizes by drawing a horizontal line at estimated error $= 10^{-6}$. Check the method specific exponent p of the global truncation error for the four methods by determining the average slope of the lines in the log-log plot.

c. Finally use the three ODE solvers `ode23`, `ode45`, and `ode113` from Matlab. Set the error tolerance `RelTol` and `AbsTol` such that you get the same final accuracy (that is global error!) as above and integrate the equations of motion for 3.0 seconds. Now Compare the angles of both bars at $T = 3.0$ sec with the results from above, and determine the average step size and the total number of function evaluations (calls to the differential equation $\dot{y} = f(t, y)$) as used in the three methods. How do these step sizes and total number of function calls compare to the ones used in the four methods from above?