ME41055
Multibody Dynamics B

Spring Term 2019

Homework Assignment 9 (HW9)

Consider the rotary motion of a torque-free satellite in deep space (no gravity, no drag). At a certain instant in time, which we call $t = 0$, the orientation of the body fixed orthonormal coordinate frame $\mathcal{B}$ expressed in a space fixed frame $\mathcal{N}$ is given by ${}^{\mathcal{N}}\boldsymbol{e}_x = (0.768, 0.024, 0.640)$, ${}^{\mathcal{N}}\boldsymbol{e}_y = (-0.424, 0.768, 0.480)$, and ${}^{\mathcal{N}}\boldsymbol{e}_z = (-0.480, -0.640, 0.600)$.

a. Determine for $t = 0$ the rotation matrix ${}^{\mathcal{N}}\mathbf{R}_{\mathcal{B}}$ which transforms the body fixed frame coordinates ${}^{\mathcal{B}}\boldsymbol{x}$ into the space fixed frame coordinates ${}^{\mathcal{N}}\boldsymbol{x}$ as in ${}^{\mathcal{N}}\boldsymbol{x} = {}^{\mathcal{N}}\mathbf{R}_{\mathcal{B}}\,{}^{\mathcal{B}}\boldsymbol{x}$.

b. Determine for $t = 0$ from this ${}^{\mathcal{N}}\mathbf{R}_{\mathcal{B}}$ the associated Euler angles (zxz) ϕ, θ, and ψ.

The initial angular velocities at $t = 0$ expressed in the global reference frame $\mathcal{N}$ are given by ${}^{\mathcal{N}}\boldsymbol{\omega} = (7.67952, 0.23936, 6.40060)$ rad/s.

c. Determine for $t = 0$ the angular velocities ${}^{\mathcal{B}}\boldsymbol{\omega}$ expressed in the body fixed frame $\mathcal{B}$.

d. Determine for $t = 0$ the rate of change of the Euler angles: $(\dot{\phi}, \dot{\theta}, \dot{\psi})$.

The satellite is modelled by a rectangular box with mass $m = 60$ kg and dimensions $l_x = 0.4$, $l_y = 1.2$ and $l_z = 0.3$ in the body fixed frame.

e. Determine the mass moment of inertia matrix ${}^{\mathcal{B}}\boldsymbol{I}_C$ at the CoM in the body fixed frame $\mathcal{B}$.

Next we want to investigate the motion of the satellite.

f. Write down the Euler equations of motion for the rigid body and the state equations $\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y})$. Use as state variables $\mathbf{y}$ the Euler angles (ϕ, θ, ψ) and the angular velocities ${}^{\mathcal{B}}\boldsymbol{\omega}$ expressed in the body fixed frame $\mathcal{B}$.

g. Show the motion of the satellite as a function of time by numerical integration of state equations for 60 seconds. Plot in one figure the Euler angles (ϕ, θ, ψ) as a function of time and in a second figure the three components of the angular velocities ${}^{\mathcal{B}}\boldsymbol{\omega}$ expressed in the body fixed frame $\mathcal{B}$ as a function of time. Discuss the results.

h. The first Euler angle grows linearly with time, therefore plot the other two Euler angles, θ and ψ, as a function of time in one figure. To see what really happens, make a second figure with a 3D plot of the trajectory of point $p = (l_x/2, 0, 0)$ of the body for t=0..60 s. Discuss the results. What happens around $\theta = 0 \pm k\pi$, $k = 0 \ldots n$?

i. Next, plot in one figure the three components of the angular velocities ${}^{\mathcal{N}}\boldsymbol{\omega}$ expressed in the space fixed frame $\mathcal{N}$ as a function of time, and plot in a second figure the three components of angular momentum vector ${}^{\mathcal{N}}\boldsymbol{H}_C$ expressed in the space fixed frame $\mathcal{N}$ as a function of time. Discuss the results.

j. Which invariants (invariant with respect to time) can you use to check your time series results? Plot these invariants as a function of time. Are they really invariant? Explain.