# ME41055
# Multibody Dynamics B

Spring Term 2019

## Homework Assignment 10 (HW10)

A simple mechanical model of the human (left) arm consists of two rigid bodies connected by three hinges. The inertial frame $\mathcal{N}$ is, seen from a human perspective looking straight ahead North, the $z$-axis up, the $x$-axis North, and the $y$-axis West. The arm is an open loop structure with, starting from the torso (the origin $O$), a hinge with an angle $\alpha$ about the $y$-axis, a hinge with an angle $\beta$ about the $x$-axis, the upper arm (body 1) with length $d = 0.3$ m in the minus $z$-direction, a hinge with an angle $\gamma$ about the $y$-axis, and finally the forearm including the hand (body 2) with length $e = 0.375$ m in the plus $x$-direction. The location of the imaginary hand at the endpoint is now $\boldsymbol{r}_H = (e, 0, -d)$ with the angles $\alpha$, $\beta$, and $\gamma$ all equal to zero. The CoM of the upper arm is located at a distance $d/2$ from the shoulder, and it has a mass of $m_1 = 1.9$ kg, whereas the CoM of the forearm (including the hand) is located at a distance $4e/10$ from the elbow, and it has a mass of $m_2 = 1.5$ kg. The body fixed frame of the upper arm is denoted by $\mathcal{U}$ whereas the body fixed frame of the forearm is denoted by $\mathcal{F}$. In the depicted configuration from above both frames coincided with the inertial frame $\mathcal{N}$. The mass moments of inertia of the upper arm at the CoM are ${}^{\mathcal{U}}(I_{xx}, I_{yy}, I_{zz})_1 = (0.015, 0.014, 0.002)$ kgm$^2$, whereas those of the forearm are ${}^{\mathcal{F}}(I_{xx}, I_{yy}, I_{zz})_2 = (0.001, 0.019, 0.019)$ kgm$^2$. These are principal values about the principal axes, so there are no off-diagonal terms.([1]) We assume gravity to work in the minus $z$-direction with a field strength of $g = 9.81$ N/kg.

In this assignment we will derive the equations of motion for the system expressed in terms of the independent generalized coordinates $\boldsymbol{q} = (\alpha, \beta, \gamma)$ and their time derivatives, using the TMT-method.

a. Make a sketch of the model, use cans-in-series to depict the hinges, and mark out the origin $O$ including the inertial frame, the elbow location $E$, the location of the hand $H$, and the locations of the CoMs.

b. Write down the expressions for the CoM coordinates of the two bodies, $x_i = (x_1, y_1, z_1, x_2, y_2, z_2)$, expressed in terms of the generalised coordinates $\boldsymbol{q}$ and system parameters, $x_i = T_i(q_j)$.

c. Determine for the initial configuration (all angles zero) the three hinge torques, which are the three generalized forces $\boldsymbol{Q} = (M_\alpha, M_\beta, M_\gamma)$, necessary to maintain this configuration (static equilibrium). Check this with a pencil and paper calculation.

d. Write down the expressions for the angular velocities of the two bodies expressed in their individual body fixed frames as a function of the generalized coordinates, their time derivatives, and the system parameters. Write them in a linear-in-speed form $\omega_i = B_{ij}(q_k)\dot{q}_j$.

e. Now derive the equations of motion for the arm in terms of the generalised coordinates $\boldsymbol{q}$, their time derivatives and the system parameters using the TMT-method. Remember that you have to transform both the Newton as well as the Euler equations of motion for the two bodies. Do not write the resulting equations of motion out term-by-term but just write down the symbols and steps to form these equations of motion.

f. Code these equations of motion in a Matlab program and check your result from the calculation of some simple configurations where you can predict the resulting accelerations of generalised coordinates.

g. For the initial configuration (all angles zero) calculate the three by three reduced generalised mass-matrix $\bar{\boldsymbol{M}}$.

[1] All anthropomorphic data is taken from: Chandler, R.F., Clauser, C.E., McConville, J.T., Reynolds, H.M. and Young, J.W., 1975. Investigation of inertial properties of the human body (No. AMRL-TR-74-137). Air Force Aerospace Medical Research Lab Wright-Patterson AFB OH.

h. Now picture a ball catch posture given by $(\alpha, \beta, \gamma) = (-70°, 70°, -30°)$. Determine the three hinge torques necessary to maintain this posture (static equilibrium).

i. Check your result by means of a forward dynamic analysis of the system for a time period of 5 seconds (copy the static equilibrium torques from item h and paste them as applied torques in the equations of motion). Clearly, after initially being at rest, the arms start moving. Explain why this happens.

j. What happens when you start your 5 second simulation from a different equilibrium posture, for instance from $(\alpha, \beta, \gamma) = (30°, 20°, 60°)$? Explain the results. (First calculate the static equilibrium torques to maintain this new posture and then copy and paste these in the equations of motion for the forward dynamic analysis).