Multibody Dynamics B - Assignment 9

July 4, 2019

Short problem statement

In this assignment we are concerned with the rotary motion of a torque-free satellite in deep space, which results in no gravity force and no drag. We will determine the rotation matrix for a z-x-z rotation and use that to determine coordinates and angular velocities in both the inertial and body frame and Euler angles and their rate of change. Furthermore, we will determine the motion of the satellite for 60 seconds, through numerical integration using the Runge-Kutta 4th order, and look at how the Euler angles and angular velocities will change, both in the inertial and body frame.

0.1 a) Rotation matrix R

Now we want to determine the rotation matrix at t=0 which transforms the body fixed frame coordinates ${}^{B}x$ into the space fixed frame coordinates ${}^{N}x$ as in ${}^{N}x =^{N} R_B\ {}^{B}x$. The body fixed coordinate base vectors as expressed in the state space frame are given as ${}^{N}e_x = e_x' = (0.768, 0.024, 0.640)$, ${}^{N}e_y == e_y' = (-0.424, 0.768, 0.480)$ and ${}^{N}e_z = e_z' = (-0.480, -0.640, 0.600)$. Now we also know that the space fixed coordinate base vectors for the global system in `x-y-z` are

$$e_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad e_y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad e_x = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Now we can write the position vector in terms of both the state space frame and the body-fixed frame as in

$$\mathbf{p} = p_1 e_1 + p_2 e_2 + p_3 e_3$$
$$\mathbf{p} = p_1' e_1' + p_2' e_2' + p_3' e_3'$$

Now it is important to note that for a rigid body these coordinates are constant. Now if we want to keep track of the position of point P in the inertial frame from the body-fixed frame positions we take the expression of the inertial-frame coordinates and substitute for vector $\mathbf{p}$ the expression in terms of the body-fixed components, this means

$$p_i = (p_1' e_1' + p_2' e_2' + p_3' e_3')\mathbf{e}_i \qquad i = 1..3$$

which we can write as

$$p_i = (\mathbf{e}_i \mathbf{e}_j') p_j' \qquad i, j = 1...3$$

where $\mathbf{e}_i\mathbf{e}_j$ is the linear transformation which maps the coordinates of point P in terms of the body-fixed frame into the coordinates expressed in the initially fixed frame. This transformation matrix,$\mathbf{e}_i\mathbf{e}_j$, is the rotation matrix ${}^N\mathbf{R}_B$, so we have

$$p = Rp'$$

where

$$\begin{aligned} R &= e_i e_j' \\ &= \begin{bmatrix} e_x & e_y & e_x \end{bmatrix}^T \begin{bmatrix} e_x' & e_y' & e_x' \end{bmatrix} \\ &= \begin{bmatrix} 0.7680 & -0.4240 & -0.4800 \\ 0.0240 & 0.7680 & -0.6400 \\ 0.6400 & 0.4800 & 0.6000 \end{bmatrix} \end{aligned}$$

0.2 b) Euler angles

Now we determine the associated Euler angles (z-x-z) ϕ, θ and ψ at t=0 from the rotation matrix found in section **a)**. Now for a z-x-z rotation we have the following rotation matrices, which follow 1.) Rotation about z-axis by an angle ϕ 2.) Rotation about x-axis by an angle θ 3.) Rotation about z-axis by an angle ψ

$$R_\phi = \begin{bmatrix} cos(\phi) & -sin(\phi) & 0 \\ sin(\phi) & cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$R_\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\theta) & -sin(\theta) \\ 0 & sin(\theta) & cos(\theta) \end{bmatrix}$$
$$R_\psi = \begin{bmatrix} cos(\psi) & -sin(\psi) & 0 \\ sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now the rotation matrix for the Euler angles is the product of the three elementary rotation matrices in the following order

$$\begin{aligned} R &= R_\phi R_\theta R_\psi \\ &= \begin{bmatrix} cos(\phi)cos(\psi) - cos(\theta)sin(\phi)sin(\psi) & -cos(\phi)sin(\psi) - cos(\psi)cos(\theta)sin(\phi) & sin(\phi)sin(\theta) \\ cos(\psi)sin(\phi) + cos(\phi)cos(\theta)sin(\psi) & cos(\phi)cos(\psi)cos(\theta) - sin(\phi)sin(\psi) & -cos(\phi)sin(\theta) \\ sin(\psi)sin(\theta) & cos(\psi)sin(\theta) & cos(\theta) \end{bmatrix} \end{aligned}$$

Now we compare this rotation matrix to that which we found in section 0.1 and find the approptiate Euler angles. This gives us:

$$\begin{aligned} \theta &= cos^{-1}(R_{3,3}) = acos(0.6) = 0.9273rad \\ \phi &= sin^{-1}(\frac{R_{1,3}}{sin(\theta)}) = sin^{-1}(\frac{0.48}{sin(0.9273)}) = -0.6435rad \\ \psi &= sin^{-1}(\frac{R_{3,1}}{sin(\theta)}) = sin^{-1}(\frac{0.64}{sin(0.9273)}) = 0.9273rad \end{aligned}$$

0.3 c) Angular velocities

Now we determine the angular velocities at $t = 0$ in the body fixed frame. We are given the angular velocities in the state space frame at $t = 0$, they are ${}^N\omega = (7.67952, 0.23936, 6.40060)$ rad/s. We use equation 19.60 in the book which tells us that we can use the rotation matrix ${}^B\mathbf{R}_N$, where ${}^B\mathbf{R}_N = ({}^N\mathbf{R}_B)^{-1}$, to transform the angular velocities in the state space frame to the body-fixed frame, so we have

$$\begin{aligned} {}^B\omega &= {}^B\mathbf{R}_N\, {}^N\omega \\ &= \begin{bmatrix} 10 \\ 0 \\ 0.001 \end{bmatrix} \end{aligned}$$

0.4 d) Rate of change of the Euler angles

Now we determine the rate of change of the Euler angles $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ at $t = 0$. Now this can be determined using equation 19.61 in the book which tells us that

$$\begin{aligned} {}^B\omega &= {}^N\omega_\psi + {}^B\mathbf{R}_\psi\, {}^B\omega_\theta + {}^B\mathbf{R}_\psi\, {}^B\mathbf{R}_\theta\, {}^B\omega_\phi \\ &= \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + {}^B\mathbf{R}_\psi \begin{bmatrix} \dot{\theta} \\ 0 \\ 0 \end{bmatrix} + {}^B\mathbf{R}_\psi\, {}^B\mathbf{R}_\theta \begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix} \\ \begin{bmatrix} \omega'_x \\ \omega'_y \\ \omega'_z \end{bmatrix} &= \begin{bmatrix} sin(\psi)sin(\theta) & cos(\psi) & 0 \\ cos(\psi)sin(\theta) - sin(\psi) & 0 & \\ cos(\theta) & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \end{aligned}$$

So rearranging this equation, i.e. we multiply the inverse of the matrix through the equation, we get

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \frac{1}{sin(\theta)} \begin{bmatrix} sin(\psi) & cos(\psi) & 0 \\ cos(\psi)sin(\theta) & -sin(\psi)sin(\theta) & 0 \\ -sin(\psi)cos(\theta) & -cos(\psi)cos(\theta) & sin(\theta) \end{bmatrix} \begin{bmatrix} \omega'_x \\ \omega'_y \\ \omega'_z \end{bmatrix} \qquad = \begin{bmatrix} 10.0000 \\ 6.0000 \\ -5.9990 \end{bmatrix}$$

0.5 e) Mass moment of inertia

We determine the mass moment of inertia matrix ${}^B\mathbf{I}_C$ at the CoM in the body fixed frame, where the satelite is modeled as a rectangular box with mass $m = 60$ kg, and dimensions $l_x = 0.4$, $l_y = 1.2$ and $l_z = 0.3$ in the body fixed frame. We align the the principal axes of the body such that we get a diagonal mass moment of inertia, i.e. such that the object is symmetric around its principal axes. Furthermore we know that moment of inertia around the CoM of a object is

$1/12mL^2$. Now we use the form of equation 18.28 in the book and get:

$$I_{xx} = \frac{1}{12}m(l_y^2 + l_z^2) = 7.6500kgm^2$$
$$I_{yy} = \frac{1}{12}m(l_x^2 + l_z^2) = 1.2500kgm^2$$
$$I_{zz} = \frac{1}{12}m(l_x^2 + l_y^2) = 8.000kgm^2$$
$${}^B\mathbf{I}_C = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

0.6 f) Euler equations of motion and state equations

Now we write down the Euler equations of motion for the rigid body and the state equations $= \mathbf{f}(\mathbf{y})$ where the state variables $\mathbf{y}$ are the Euler angles and the angular velocities expressed in the body fixed frame. So we have

$$\mathbf{y} = \begin{bmatrix} \phi \\ \theta \\ \psi \\ \omega'_x \\ \omega'_y \\ \omega'_z \end{bmatrix}$$

Now the Euler equations of motion are, equation 19.66 in the book,

$$\sum {}^B M_C = {}^B\mathbf{I}_C\, {}^B\dot{\omega} + {}^B\omega \times ({}^B\mathbf{I}_C\, {}^B\omega)$$

By rearranging the equation we can solve for angular accelerations,

$${}^B\dot{\omega} = ({}^B\mathbf{I}_C)^{-1}\left(\sum {}^B M_C - {}^B\omega \times ({}^B\mathbf{I}_C\, {}^B\omega)\right) \tag{1}$$

Now since the satellite is in deep space and no gravity nor drag is exerted on the body the external moment applied on the body is zero, i.e. $M_C = 0$ so we have

$${}^B\dot{\omega} = ({}^B\mathbf{I}_C)^{-1}\, {}^B\omega \times ({}^B\mathbf{I}_C\, {}^B\omega)$$

Furthermore we can find the Euler angles as depicted in section 0.4. Thus the state equations are

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \frac{1}{sin(\theta)} \begin{bmatrix} sin(\psi) & cos(\psi) & 0 \\ cos(\psi)sin(\theta) & -sin(\psi)sin(\theta) & 0 \\ -sin(\psi)cos(\theta) & -cos(\psi)cos(\theta) & sin(\theta) \end{bmatrix} \begin{bmatrix} \omega'_x \\ \omega'_y \\ \omega'_z \end{bmatrix}$$

$$\begin{bmatrix} \dot{\omega}'_x \\ \dot{\omega}'_y \\ \dot{\omega}'_z \end{bmatrix} = \begin{bmatrix} \dfrac{I_{yy}\omega'_y\omega'_z - I_{zz}\omega'_y\omega'_z}{I_{xx}} \\ -\dfrac{I_{xx}\omega'_x\omega'_z - I_{zz}\omega'_x\omega'_z}{I_{yy}} \\ \dfrac{I_{xx}\omega'_x\omega'_y - I_{yy}\omega'_x\omega'_y}{I_{zz}} \end{bmatrix}$$

0.7 g) Motion of the satellite

Now we determine the motion of the satellite as a function of time by numerical integration of the state equations for 60 seconds. We plot the Euler angles as a function of time and the angular velocities expressed in the body fixed frame as a function of time.
The numerical integration method that we use is the Runge-Kutta 4th order method which follows the scheme

$$k_1 = f(t_n, y_n)$$

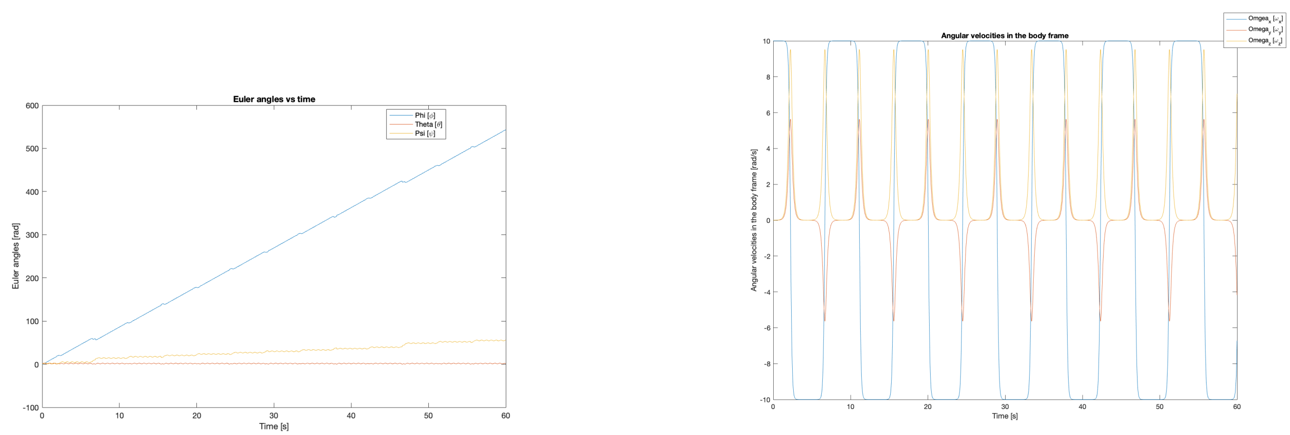
$$k_2 = f(t_n + h/2, y_n + h/2 * k_1)$$

$$k_3 = ff(t_n + h/2, y_n + h/2 * k_2)$$

$$k_4 = ff(t_n + h, y_n + h * k_3)$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

This method is used over 60 seconds with a time step of $h = 9.1553 * 10^{-4}$, the Matlab code for the numerical integration can be found in appendix A. Now the plots for the Euler angles and angular velocities can be seen in figures 1a and 1b.

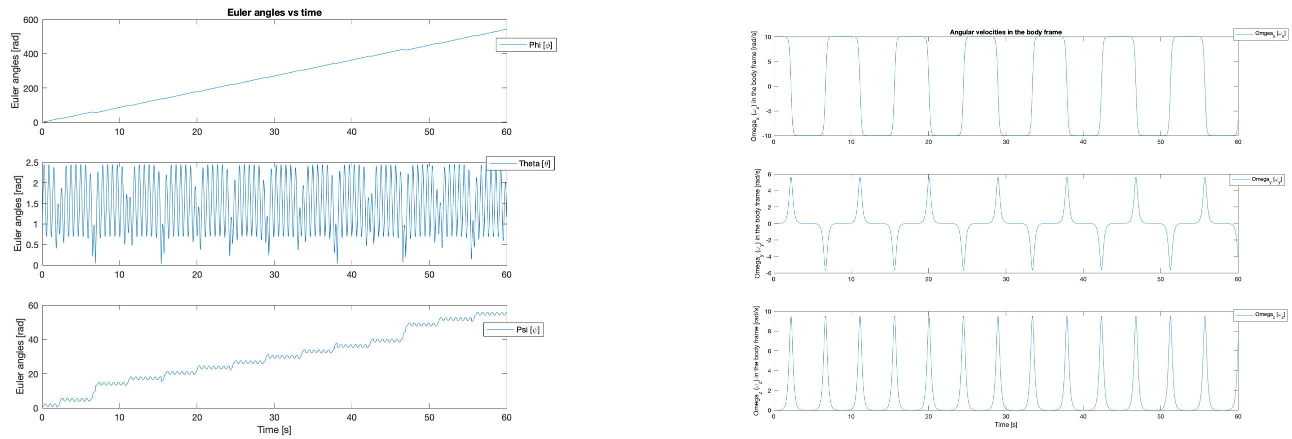


(a) Euler angles

(b) Angular velocities

Figure 1: Euler angles and angular velocities expressed in the body fixed frame as a function of time.

We have a closer look at each of the angles and angular velocities in separate graphs in figure 2 below.



(a) Euler angles **(b)** Angular velocities

Figure 2: Euler angles and angular velocities expressed in the body fixed frame as a function of time.

What we observe is that ϕ is constant and increases with time, meaning a constant increase in spinning around the z axis expressed in the body fixed frame. Furthermore we observe that θ wiggles around the 1.5 rad constantly however we observe that there are some disturbances periodically, about every 5 seconds. This wiggle in θ seems to affect the ψ angle as it also wiggles, we also observe the same disturbances, which results in a jump in the angle ψ. Now looking at the angular velocities, nothing special happens, we notice that they are periodic which is what we expect since there is no external force so nothing should disturb the angular velocities of the body, i.e. no acceleration of the total system is happening, they only shift with the rotation of the body, since the system has a constant velocity. Moreover, we see that when the ω_x decreases ω_y and ω_z increase and the same can be observed when ω_x increases then ω_y and ω_z decrease where ω_y and ω_z are in phase, this is expected since the body has a constant velocity. We should note that it is strange that there are disturbances about every 5 seconds.

0.8 h) Trajectory of a point p

We plot the Euler angles θ and ψ separately from ϕ to look better at what is happening at the disturbances. Furthermore, we plot the 3D trajectory of a point $p = (l_x/2, 0, 0)$ on the satellite through the span of 60 seconds.
Now we plot only the θ and ψ as a function of time, this can be seen in figure 3 below.

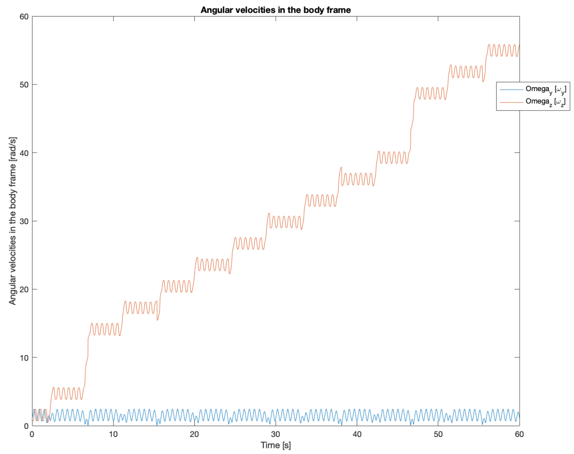


Figure 3: Euler angles θ and ψ as functions of time.

We notice that the wiggle in θ definitely seems to affect ψ, furthermore we notice that the disturbances are synced up. Looking further into this we notice that this happens when the angle is close to $\theta = 0 + k\pi$. Now if we look at the state equations we notice that what happens is that when $\theta = 0 + k\pi$, $sin(\theta) = 0$ the matrix

$$B = \frac{1}{sin(\theta)} \begin{bmatrix} sin(\psi) & cos(\psi) & 0 \\ cos(\psi)sin(\theta) & -sin(\psi)sin(\theta) & 0 \\ -sin(\psi)cos(\theta) & -cos(\psi)cos(\theta) & sin(\theta) \end{bmatrix}$$

is singular so we are unable to determine the time derivatives of the Euler angles from the angular speeds. This singular configuration is usually called ’`gimbal lock`. What happens is actually that we can not determine what $\dot{\phi}$ and $\dot{\psi}$ are as they are both rotating around the z-axis. We could avoid this by redefining the rotational matrix such that it rotates around each axis, like z-x-y rotations.

Now we also look at the trajectory of the point p on the satellite during the time-span of 60 seconds, this can be seen in figure 4 below.

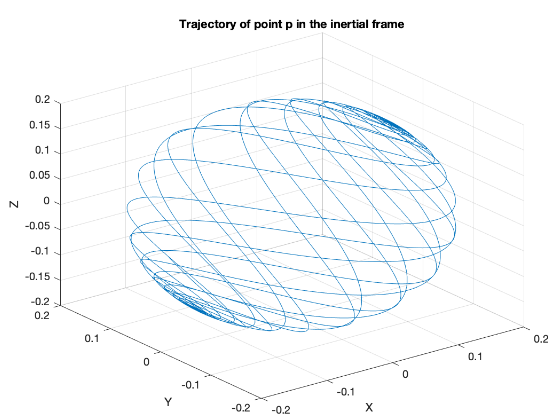


Figure 4: 3D trajectory of a point p on the satellite over a time-span of 60 seconds.

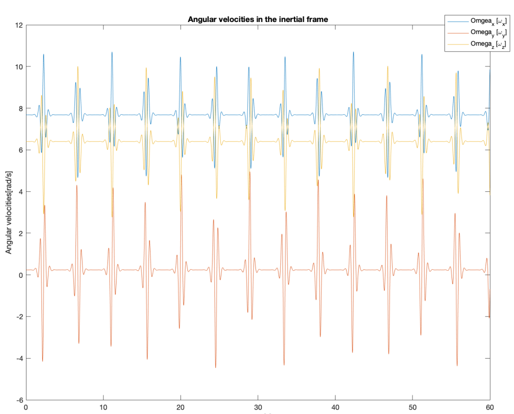
What we notice is that the trajectory resembles that of a sort of ellipsoid around the origin. What we also notice is that the satellite starts with small circles, than moves into a sort of bigger circles, which result in a sort of half-helix, which again moves into smaller circles. The satellite also moves according to how big the circles at the 'ends' are, when they are small, like in the beginning, the satellite moves farther in the y and z direction but when the end circles are bigger they move closer to the origin, less deviation in the y and z direction.

0.9 i) Angular velocities in the space fixes frame and angular momentum

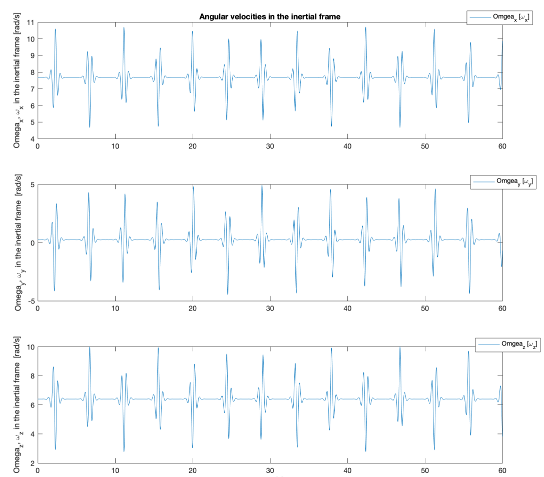
We plot the three components of the angular velocities expressed in the space fixed frame as a function of time and the angular momentum in the space fixed frame as a function of time. Now the angular velocities in the inertial frame can be calculated from the rotation matrix and the angular velocities in the body fixed frame from the equation

$$^{N}\omega = {}^{N}\mathbf{R}_{B}\,{}^{B}\omega$$

Now we have the rotation matrix, which can be seen in section 0.2 and the angular velocities in the body frame and the Euler angles at each time-step, like can be seen in section 0.7. This allows us to calculate the angular velocities in the state space frame for each time-step, using the equation above. These angular velocities can be seen in figure 5.



(a) Angular velocities together



(b) Angular velocities separately

Figure 5: Angular velocities in the state space frame as a function of time.

Looking at the angular velocities in the state space frame we see that they make sense. We notice that the angular velocities are completely out of phase as in ω_x is completely out of phase from ω_y and ω_z, which makes sense since there is no external force acting on the system and so the angular velocities should stay the same. It is also what you would expect that ω_z follows the phase of ω_y since it is affected by it' rotation, and this is in line with what we saw in section 0.7, where the angular velocities in the body frame for ω_y and ω_Z were working in phase and reacting to how the ω_x behaved, similarly to what can be seen here.

Now angular momentum expressed in the state space frame can be found by finding the angular momentum in the body frame and pre-multiplying that with the rotation matrix found in section 0.2. This can be described in equation form as

$$^{N}\mathbf{H}_{C} = {}^{N}\mathbf{R}_{B}\,{}^{B}\mathbf{I}_{C}\,{}^{B}\omega$$

where the rotation matrix and the angular velocities in the body frame are taking for each time-step. The angular momentum vector **H** expressed in the space fixed frame as a function of time can be seen in figure 6 below.

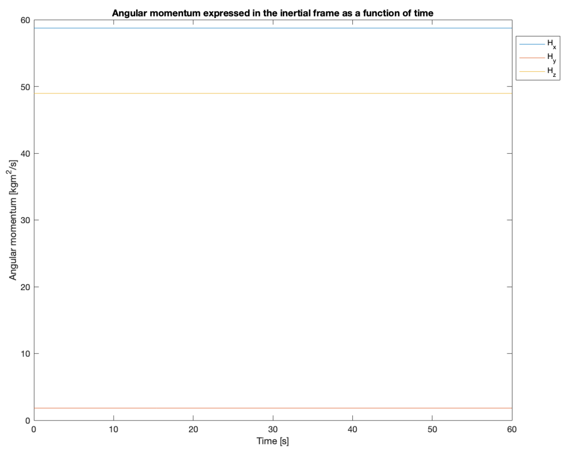


Figure 6: Angular momentum as a function of time.

This is what we expected since there is no external force, as there is no gravity and no drag, and thus no external momentum is applied and thus the momentum should be in the same direction and have the same value through the whole time-span of 60 seconds, actually the momentum should be same for any time-span as long as there is no external force applied.

0.10 i) Invariants

Now we determine three invariants which we can use to check our result.
The three invariants that I found were angular momentum, distance between 2 points on the object and the kinetic energy of the system, since there is no external force and thus the velocity is constant. First the angular momentum can be seen in figure 6 above, and obviously it can be seen that the angular momentum does not change with time.
Now we can look at the distance between two points on the satellite, this can be seen in figure 7 below.

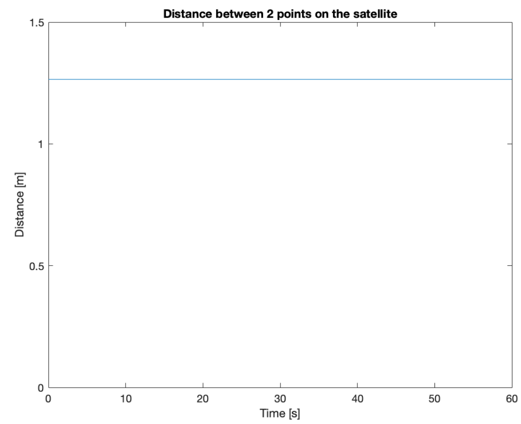


Figure 7: Distance between two points on the satellite as a function of time.

Like we can see the distance between two points on the satellite does not change, which is what we expected, this should of course always be true, since if this was not the case the object would be ripping apart or some other non-expected situations.
Now the kinetic energy of the system can be calculated as

$$T = \frac{1}{2}\,{}^{B}\omega^{T}\,{}^{B}\mathbf{I}_{C}\,{}^{B}\omega$$

where we take ${}^{B}\omega$ for each time-step. Now we look at the kinetic energy of the system as a function of time in figure 8 below.

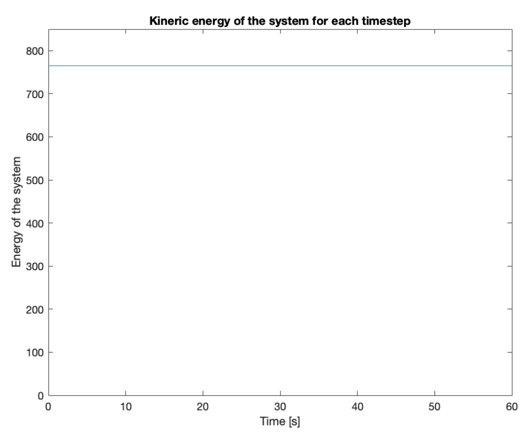


Figure 8: Kinetic energy of the system as a function of time.

We can see that the kinetic energy of the system is invariant of time. This is expected since there are no external forces acting on the system and thus the velocity never changes, since there is no acceleration, and thus the kinetic energy is the same at all times. We should note that if we look carefully at the kinetic energy it is not completely invariant, the same can be said about the angular momentum, due to the singularity problem we described in 0.8, i.e. there are small disturbances in the calculations due to θ being close to $0 + k\pi$. However these disturbances affect the momentum and energy so little that it can't be seen unless you look at decimal places at 10^{-6} for our step size in the numerical integration.

Appendix A

Matlab code

```
%% a) Find Rotation Matrix from the body to the inertial frame nRb
% Body frame
e_bx = [0.768;0.024;0.640];
e_by = [-0.424;0.768;0.480];
e_bz = [-0.480; -0.640; 0.600];
e_b = [e_bx, e_by, e_bz];

% Inertial frame
e_x = [1;0;0];
e_y = [0;1;0];
e_z = [0;0;1];
e = [e_x, e_y, e_z];

% Rotation matrix from body to inertial frame
R = e*e_b;

%% b) Determine the Euler angles  (z-x-z)
syms phi theta psi
ang = [phi; theta; psi];

% Find first the rotation marixes about z-x-z axes, i.e. phi,theta and psi,
% individually

R_phi = [cos(phi) -sin(phi) 0;          % Same rotation matrix as in 19.26 ...
    in the book
        sin(phi) cos(phi) 0;
        0 0 1];

R_theta = [1 0 0;                       % Same rotation matrix as in 19.28 ...
    in the book
           0 cos(theta) -sin(theta);
           0 sin(theta) cos(theta)];

R_psi = [cos(psi) -sin(psi) 0;          % Same rotation matrix as in 19.30 ...
    in the book
        sin(psi) cos(psi) 0;
        0 0 1];


% The final rotation matrix

R_Eul = R_phi*R_theta*R_psi;

matlabFunction(R_Eul,'File','Rhw9');

% Now we find the value of the angles

theta = acos(R(3,3));
phi = asin(R(1,3)/sin(theta));
psi = asin(R(3,1)/sin(theta));

```

```
Eul = [phi;theta;psi];

%% c) Finding the angular velocities in the body frame

% Angular velocities in the inertia frame at t=0
w0_i = [7.67952;0.23936;6.40060];

% Calculate angular velocities in body frame, using equation 19.60
w0_b = inv(R)*w0_i;


%% d) Find the rate of change of Euler angles
% we use equation 19.70 to determine the rate of change
B = [sin(psi)*sin(theta) cos(psi) 0;
    cos(psi)*sin(theta) -sin(psi) 0;
    cos(theta) 0 1];

dEul = inv(B)*w0_b;

%% e) Find the inertia matrix bIc in the body fixed frame.

m = 60;    % [kg]

l_x = 0.4;  % [m]
l_y = 1.2;  % [m]
l_z = 0.3;  % [m]

% Moment of inertia around the center of mass is 1/12*ml^2, and we use the
% inertia matrix form in equation 18.28, i.e. we align the principal axes of
% the body such that we get a diagonal mass moment of inertia, i.e. such
% that the object is symmetric around its pricipal axes.

bIx = 1/12*m*(l_y^2+l_z^2);
bIy = 1/12*m*(l_x^2+l_z^2);
bIz = 1/12*m*(l_x^2+l_y^2);

bIc = diag([bIx bIy bIz]);


%% f) Equation of motion
syms bwx bwy bwz Ixx Iyy Izz

wb = [bwx; bwy; bwz];
I = diag([Ixx Iyy Izz]);

dwb = inv(I)*(-cross(wb,(I*wb)));

%% g) Numerical Integration with the Runge-Kutta 4th order method

% setup
time=60;
nn=16;
N=2.^nn;
h=time./N;

t= 0;
```

```
y0 = [phi; theta; psi; w0_b];
ang = [phi; theta; psi; w0_b];
for j=1:N
    k1 = qa(y0);
    k2=qa(y0 + h/2*k1);
    k3=qa(y0 + h/2*k2);
    k4=qa(y0 + h*k3);
    qn= y0+1/6*h*(k1+2*k2+2*k3+k4);

    y0 = qn;
    q_n(j,:)=qn;
    t = t+h;
    T(j)=t;
end

%% Plots
q_plot = [ang';q_n];
T_plot = [0;T'];

figure(1)
plot(T_plot,q_plot(:,1),T_plot,q_plot(:,2),T_plot,q_plot(:,3));
xlabel('Time [s]')
ylabel('Euler angles [rad]')
legend('Phi [\phi]', 'Theta [\theta]', 'Psi [\psi]')
title('Euler angles vs time')

figure(2)
plot(T_plot,q_plot(:,4),T_plot,q_plot(:,5),T_plot,q_plot(:,6));
xlabel('Time [s]')
ylabel('Angular velocities in the body frame [rad/s]')
legend('Omgea_x [\omega_x]', 'Omega_y [\omega_y]', 'Omega_z [\omega_z]')
title('Angular velocities in the body frame')

figure(3)
subplot(311)
plot(T_plot,q_plot(:,1))
title('Euler angles vs time')
ylabel('Euler angles [rad]')
legend('Phi [\phi]')

subplot(312)
plot(T_plot,q_plot(:,2))
ylabel('Euler angles [rad]')
legend('Theta [\theta]')

subplot(313)
plot(T_plot,q_plot(:,3))
ylabel('Euler angles [rad]')
legend('Psi [\psi]')
xlabel('Time [s]')

figure(4)
subplot(311)
plot(T_plot,q_plot(:,4))
title('Angular velocities in the body frame')
ylabel('Omega_x (\omega_x) in the body frame [rad/s]')
```

```matlab
legend('Omgea_x [\omega_x]')

subplot(312)
plot(T_plot,q_plot(:,5))
ylabel('Omega_y (\omega_y) in the body frame [rad/s]')
legend( 'Omega_y [\omega_y]')

subplot(313)
plot(T_plot,q_plot(:,6))
ylabel('Omega_z (\omega_z) in the body frame [rad/s]')
legend('Omega_z [\omega_z]')
xlabel('Time [s]')

figure(9)
plot(T_plot,q_plot(:,2),T_plot,q_plot(:,3))
xlabel('Time [s]')
ylabel('Angular velocities in the body frame [rad/s]')
legend('Omega_y [\omega_y]', 'Omega_z [\omega_z]')
title('Angular velocities in the body frame')
%% h) 3D trajectory of point p
p = [(l_x)/2;0;0];

p_I = R_Euler(p,q_plot(:,1:3));

figure(5)
plot3(p_I(:,1),p_I(:,2),p_I(:,3))
grid on
xlabel('X')
ylabel('Y')
zlabel('Z')
title('Trajectory of point p in the inertial frame')

%% i) plot the angular velocities in the inertial frame

w_i = R_Euler_vel(q_plot);

figure(6)
plot(T_plot,w_i(:,1),T_plot,w_i(:,2),T_plot,w_i(:,3))
title('Angular velocities in the inertial frame')
ylabel('Angular velocities[rad/s]')
xlabel('Time [s]')
legend('Omgea_x [\omega_x]', 'Omega_y [\omega_y]', 'Omega_z [\omega_z]')


figure(7)
subplot(311)
plot(T_plot,w_i(:,1))
title('Angular velocities in the inertial frame')
ylabel('Omega_x, \omega_x in the inertial frame [rad/s]')
legend('Omgea_x [\omega_x]')

subplot(312)
plot(T_plot,w_i(:,2))
ylabel('Omega_y, \omega_y in the inertial frame  [rad/s]')
legend('Omgea_y [\omega_y]')

```

```matlab
subplot(313)
plot(T_plot,w_i(:,3))
ylabel('Omega_z, \omega_z in the inertial frame  [rad/s]')
legend('Omgea_z [\omega_z]')
xlabel('Time [s]')

%% i) Angular momentum vector in the inertial frame

momentum_i = R_Euler_mom(q_plot);

figure(8)
plot(T_plot,momentum_i(:,1),T_plot,momentum_i(:,2),T_plot,momentum_i(:,3))
ylabel('Angular momentum [kgm^2/s]')
xlabel('Time [s]')
title('Angular momentum expressed in the inertial frame as a function of time')
legend('H_x', 'H_y', 'H_z')

%% j) Invariants

% distance between two points on the satellite.
p1 = [0;l_y;0];
p2 = [l_x;0;0];

P_1 = R_Euler(p1,q_plot(:,1:3));
P_2 = R_Euler(p2,q_plot(:,1:3));

for i = 1:length(T_plot)
    dist(i,:) = norm(P_1(i,:)-P_2(i,:));
end

figure(10)
plot(T_plot,dist)
ylabel('Distance [m]')
xlabel('Time [s]')
title('Distance between 2 points on the satellite')
yaxis([0 1.5])

%% Kinetic energy of the system

T = R_Euler_KE(q_plot);

figure(11)
plot(T_plot,T)
xlabel('Time [s]')
ylabel('Energy of the system')
title('Kineric energy of the system for each timestep')
%yaxis([0 850])

%% Functions

function state_d = qa(y)
phi = y(1);
theta = y(2);
psi = y(3);

wb = y(4:6);
```

```
% Find the rate of change in angular velocities
m = 60;     % [kg]

l_x = 0.4;   % [m]
l_y = 1.2;   % [m]
l_z = 0.3;   % [m]


bIx = 1/12*m*(l_y^2+l_z^2);
bIy = 1/12*m*(l_x^2+l_z^2);
bIz = 1/12*m*(l_x^2+l_y^2);

bIc = diag([bIx bIy bIz]);

dwb = inv(bIc)*(-cross(wb,(bIc*wb)));

% Find the rate of change for Euler angles
B = [sin(psi)*sin(theta) cos(psi) 0;
     cos(psi)*sin(theta) -sin(psi) 0;
     cos(theta) 0 1];

dEul = inv(B)*wb;

state_d = [dEul;dwb];
end



function position = R_Euler(p,angles)

for i = 1:length(angles)
    phi = angles(i,1);
    theta = angles(i,2);
    psi = angles(i,3);

    R_Eul = Rhw9(phi,psi,theta);

    position(i,:) = R_Eul*p;
end

end

function angvel = R_Euler_vel(angles)

for i = 1:length(angles)
    phi = angles(i,1);
    theta = angles(i,2);
    psi = angles(i,3);

    wx = angles(i,4);
    wy = angles(i,5);
    wz = angles(i,6);

    wb = [wx;wy;wz];

```

```
    R_Eul = Rhw9(phi,psi,theta);

    angvel(i,:) = R_Eul*wb;
end

end

function momentum = R_Euler_mom(angles)

m = 60;     % [kg]

l_x = 0.4;   % [m]
l_y = 1.2;   % [m]
l_z = 0.3;   % [m]


bIx = 1/12*m*(l_y^2+l_z^2);
bIy = 1/12*m*(l_x^2+l_z^2);
bIz = 1/12*m*(l_x^2+l_y^2);

bIc = diag([bIx bIy bIz]);


for i = 1:length(angles)
    phi = angles(i,1);
    theta = angles(i,2);
    psi = angles(i,3);

    wx = angles(i,4);
    wy = angles(i,5);
    wz = angles(i,6);

    wb = [wx;wy;wz];

    R_Eul = Rhw9(phi,psi,theta);

    momentum(i,:) = R_Eul*bIc*wb;
end

end

function kenergy = R_Euler_KE(angles)

m = 60;     % [kg]

l_x = 0.4;   % [m]
l_y = 1.2;   % [m]
l_z = 0.3;   % [m]


bIx = 1/12*m*(l_y^2+l_z^2);
bIy = 1/12*m*(l_x^2+l_z^2);
bIz = 1/12*m*(l_x^2+l_y^2);

bIc = diag([bIx bIy bIz]);

```

```
for i = 1:length(angles)

    wx = angles(i,4);
    wy = angles(i,5);
    wz = angles(i,6);

    wb = [wx;wy;wz];

    kenergy(i,:) = wb'*bIc*wb;
end

end
```