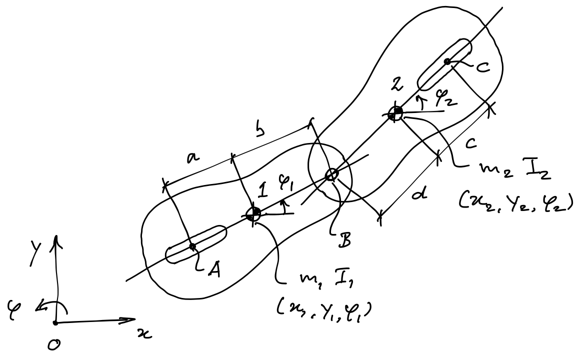
ME41055
Multibody Dynamics B

Spring Term 2019

Homework Assignment 8 (HW8)



The EzyRoller, shown in the top figure, is the ultimate riding machine for kids. The machine is propelled by an oscillating motion of the steering assembly, which is operated by the feet. For the proper operation watch the video[1]. A mechanical model for this machine is shown in the bottom figure. The model consists of two rigid bodies connected by a hinge in B. Each body has a rolling contact, for body 1 the wheel in A and for body 2 the wheel in C. The centre of mass location is denoted by the numbers 1 and 2. With the help of this model we like to demonstrate the operation of this machine in a number of steps.

The system consists of two rigid bodies and therefore has six coordinates. The system has 2 holonomic constraints, the hinge in B, and two nonholonmic constraints, the rolling contact condition in A and C. Therefore you have two constraints on six coordinates and four constraints on the six velocities. So the number of degrees of freedom in the coordinates space is four, whereas you only have two degrees of freedom in the velocity space. We will solve this problem by setting up the constraint equations of motion in DAE form and stabilise the constraints by means of the Coordinate Projection Method. Note that there is now a difference between the constraints on the coordinates and the constraints on the velocities.

Please address the following questions:

a. Formulate the holonomic constraint equations $C_k(x_i) = 0$ and derive the Jacobian matrix $C_{k,i}$ and the convective terms $h_k = C_{k,ij}\dot{x}_i\dot{x}_j$.

b. Formulate the non-holonomic constraint equations $S_{ki}\dot{x}_i = 0$ and derive the Jacobian matrix S_{ki} and the convective terms $h_k = S_{ki,j}\dot{x}_i\dot{x}_j$.

c. Derive the equations of motion for this system in DAE form and implement these in a Matlab function.

d. Formulate the Coordinate Projection Method for the constraints on the coordinates and for the constraints on the velocities and implement these in two Matlab functions.

e. First we like to determine the motion of the unpowered system by numerical integration of the equations of motion (use a fixed step RK4 method) for a short time, say 1 second, to demonstrate that the system is working properly. For dimensions of the EzyRoller we take: $a = 0.2, b = 0.6, c = 0.1, d = 0.1, m_1 = 46, I_1 = 10, m_2 = 1, I_2 = 0.005$ (SI units). For initial conditions on the coordinates take $(x_1, y_1, \varphi_1, x_2, y_2, \varphi_2) = (a, 0, 0, a + b, d, \pi/2)$ and for the velocities start with $\dot{x}_1 = 1$ and $\dot{\varphi}_1 = 0$. Make sure that all other initial velocities fulfill the constraints, what are they? Plot the path of the CoM of body 1 and body 2 in one figure and discuss the result.

Add and action-reaction torque in the hinge in B such that it resembles an oscillating torque as applied by the rider on the steering assembly. Take for the torque the following periodic function $M = M_0 \cos(\omega t)$, with $M_0 = 2$ [Nm] and $\omega = \pi$ [rad/s]. Start from rest with body 1 and body 2

[1] https://youtu.be/GMnJ9q1D4hU

aligned along the x-axis, $\varphi_1 = 0, \varphi_2 = \pi$ (point C between A and B). Determine the motion of the system by numerical integration of the constraint equations of motion for 60 seconds.

f. Plot the path of point A and C in one figure. Discuss the result.

g. Plot the speed ($\sqrt{\dot{x}^2 + \dot{y}^2}$) and angular velocity of the CoM of body 1 as a function of time in one figure. Discuss the result.

h. Determine the Kinetic energy of the system and plot it as a function of time. Discuss the result.

i. Plot the applied torque M and the relative angular velocity in hinge B between body 2 and body 1, defined as $\omega = \dot{\varphi}_2 - \dot{\varphi}_1$, as a function of time in one figure and discuss the result. Pay extra attention to what happens in the first few seconds and when the transient has died out.

j. Determine the work done by the applied torque as a function of time and plot this as a function of time in the same figure as the Kinetic Energy of the system. What do you notice? Can you explain this?