CPDA SP18 Assignment #1

Code

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Hide

```
library(matlab)
```

```
Attaching package: 'matlab'

The following object is masked from 'package:stats':

    reshape

The following objects are masked from 'package:utils':

    find, fix

The following object is masked from 'package:base':

    sum
```

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```
library(pracma)
```

```
package 'pracma' was built under R version 3.4.3
Attaching package: 'pracma'

The following objects are masked from 'package:matlab':

    ceil, eye, factors, fliplr, flipud, hilb, isempty,
    isprime, linspace, logspace, magic, meshgrid, mod,
    ndims, nextpow2, numel, ones, pascal, pow2, primes,
    rem, repmat, rosser, rot90, size, std, strcmp, tic,
    toc, vander, zeros
```

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```
library("matrixcalc")
```

Consider the following Matrix:

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```
A <- matrix(c(8,-1,2,-1,2,-2,2,-2,3),3,3,byrow = TRUE)
print(A)
```

```
     [,1] [,2] [,3]
[1,]    8   -1    2
[2,]   -1    2   -2
[3,]    2   -2    3
```

a)Compute the determinant and rank of the Matrix A. What can you say about the rank of the matrix from the determinant?

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```
det(A)
```

```
[1] 13
```

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```
qr(A)$rank
```

```
[1] 3
```

Since the determinant(13) is not zero, we know the matrix is not sigular. In this case, A has full rank. We can confirm rank by putting A in reduced row echelon form, showing 3 non-zero pivots.

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```
rref(A)
```

```
     [,1] [,2] [,3]
[1,]    1    0    0
[2,]    0    1    0
[3,]    0    0    1
```

b) Write a code to calculate the eigenvalue-eigenvector pairs of the matrix A.

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```
A.eigen <- eigen(A)
A.eigen
```

```
eigen() decomposition
$values
[1] 9.0847569 3.5072388 0.4080043

$vectors
           [,1]       [,2]        [,3]
[1,]  0.8989580 -0.4326343 -0.06857203
[2,] -0.2318077 -0.6026902  0.76356379
[3,]  0.3716716  0.6705162  0.64208116
```

c) Calculate the angle between the eigenvectors corresponding to the two largest eigenvalue in degrees. (2 points)

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```
degs<-function(rads) {
  round(rads*180/pi, digits = 0)
}
theta <- function(v1, v2){
  degs(acos(dot(v1, v2) / (sqrt(sum(v1^2)) * sqrt(sum(v2^2)))))
}
theta(A.eigen$vectors[,1], A.eigen$vectors[,2])
```

```
[1] 90
```

d) Is the matrix A positive definite? (2 points)

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```
A.eigen
```

```
eigen() decomposition
$values
[1] 9.0847569 3.5072388 0.4080043

$vectors
           [,1]       [,2]        [,3]
[1,]  0.8989580 -0.4326343 -0.06857203
[2,] -0.2318077 -0.6026902  0.76356379
[3,]  0.3716716  0.6705162  0.64208116
```

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```
is.positive.definite(A)
```

```
[1] TRUE
```

All the eigen$values are > 0, so A is positive-definite.

e) Let T be the matrix constructed by stacking the eigenvectors of A as its columns. Calculate T−1 A T . (2 points)

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```
calculation.e <- function(v, t){
  tinv <- inv(t)
  prd <- tinv %*% v
  prd %*% t
}
T <- A.eigen$vectors
calculation.e(A, T)
```

```
             [,1]         [,2]         [,3]
[1,] 9.084757e+00 8.881784e-16 1.332268e-15
[2,] 9.992007e-16 3.507239e+00 3.552714e-15
[3,] 3.608225e-16 2.581269e-15 4.080043e-01
```

Using the matrix mult operator yields the same result:

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```
T <- A.eigen$vectors
inv(T) %*% A %*% T
```

```
              [,1]         [,2]         [,3]
[1,] 9.084757e+00 8.881784e-16 1.332268e-15
[2,] 9.992007e-16 3.507239e+00 3.552714e-15
[3,] 3.608225e-16 2.581269e-15 4.080043e-01
```

f) Find the trace of the matrix A. (2 points)

By definition, the trace is the sum of the diagonals.

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```
sum(diag(A))
```

```
[1] 13
```

g) Find A^2 . (2 points)

A^2 is also A dot A

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```
A %*% A
```

```
     [,1] [,2] [,3]
[1,]   69  -14   24
[2,]  -14    9  -12
[3,]   24  -12   17
```