Machine Learning Tarea #4

Herman Jaramillo Villegas
Universidad de Medellín

March 15, 2024

1. Show that the inverse of A^*A, for A, an $m \times n$ matrix, exists if A has linearly independent columns.

   Proof We use the theorem that says, if B is a square matrix and $Bx = 0$, implies that $x = 0$, then B is non-singular.

$$\begin{aligned} A^*Ax = 0 \Rightarrow x^*A^*Ax &= \|Ax\|^2 = 0 \\ &\Rightarrow Ax = 0 \\ &\Rightarrow x = 0. \end{aligned}$$

   Since $Ax = \sum_{i=1}^n x_i A_i$, with A_i the i^{th} column of A, and, by hypothesis, the columns of A are linearly independent.

2. Show that the smallest vector such that $2x + 3y = 5$ is

$$u = \begin{pmatrix} 0.76923 \\ 1.15385 \end{pmatrix}$$

   Solution: The smallest vector that solves $ax+by = c$ is located in the null space of the matrix (a, b). This null space is simple find reversing the order of the components and changing the sign of any of them. That is, $\mathcal{N}(A) = (b, -a)$. That null space is given by the line through the vector $(3, -2)$ in this problem. Just solve the system

$$\begin{aligned} 2x + 3y &= 5 \\ 3x - 2y &= 0 \end{aligned}$$

The solution is

$$u = \begin{pmatrix} 0.76923 \\ 1.15385 \end{pmatrix}$$

3. Find the pseudo-inverse of the matrix $A = (2 \quad 3)$. **Hint:** Recall that the pseudoinverse A^+ should satisfy $A^+ b = u$. b can be any non-zero number in $\mathbb{R}$, you can choose $b = 5$ to fit the equation on the previous point but that is not necessary. Do not use SVD, just solve linear equations.

Solution: The system $A^+ b = u$ (recall that is A is 2×1, then A^+ is 1×2.

$$\begin{pmatrix} a_1^+ \\ a_2^+ \end{pmatrix} 5 = \begin{pmatrix} \frac{10}{13} \\ \frac{15}{13} \end{pmatrix}$$

Clearly from here the $a^+ = (10/13)/5 = 2/13$ and $a_2^+/ = (15/13)/5 = 3/13$ so

$$A^+ = \begin{pmatrix} \frac{2}{13} \\ \frac{3}{13} \end{pmatrix}.$$