

# Rewarding the Pull: Plotting Ultimate Frisbee Pull Outcomes in Phase Space

Hugh Coleman, James Crosnoe

## Rationale

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To obtain this advantage, high-level defenses exploit ‘the pull’ or the kickoff that starts each point. The pull is perhaps the “most strategically important” plays in a game of Ultimate, and can provide a defensive advantage by trapping an opposing team in their endzone (Westerfield, 2018). One unique method for experimenting with pull outcomes involves the use of Phase Space. The Phase Space of a system offers a neat visualization of “all possible physical outcomes” as influenced by a set of specific parameters (2001). Phase Spaces have already been put to use in visualizing outcomes for changing trajectories in the release of a disc. According to Crowther & Potts’ simulation plotting flight distance against angle of release predicts that launch angles of 10° and 20° maximize flight range and duration respectively, while an initial roll angle of 6° returns the tightest flight path (2007). These results offer a helpful baseline for players looking to increase distance. However, the pull parameters involved could be manipulated further, with respect to a specific defensive point-of-attack in the opponent’s end zone.

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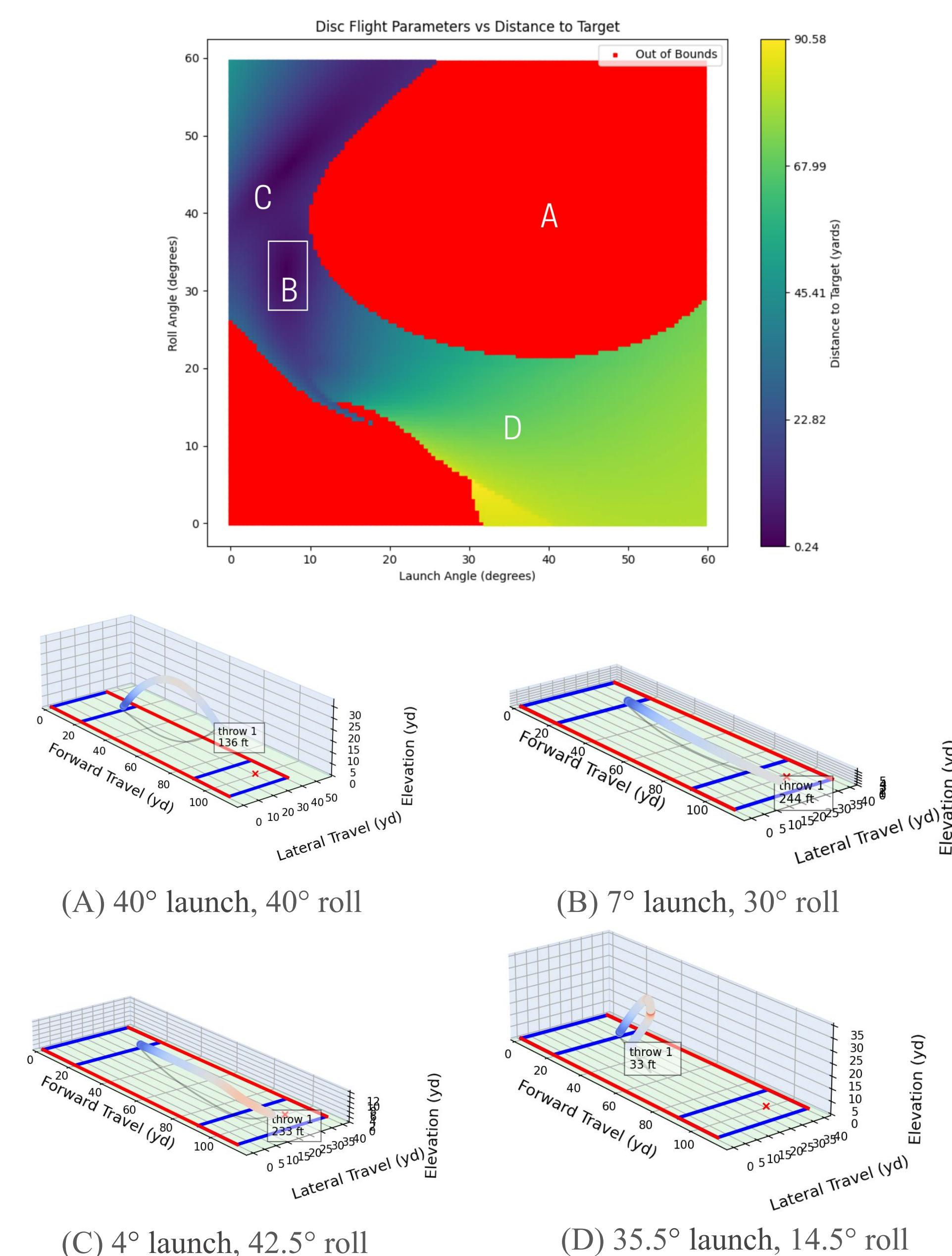
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## Findings

From these results we generated a phase space diagram. A red mark represents that the disc was out of bounds, resulting in a brick. The rest are a gradient, from 90 yards away from the target to 0.24 yards away.



Our phase space graph has several markings. The box represents the optimal throwing parameters for a pull landing closest to the target location. The box is bounded by launch angle between 4.5° and 9.5°, and roll angles between 27.5° and 36.5°. If the thrower can throw within these bounds, their pull will land within approximately 10 yards of the target.

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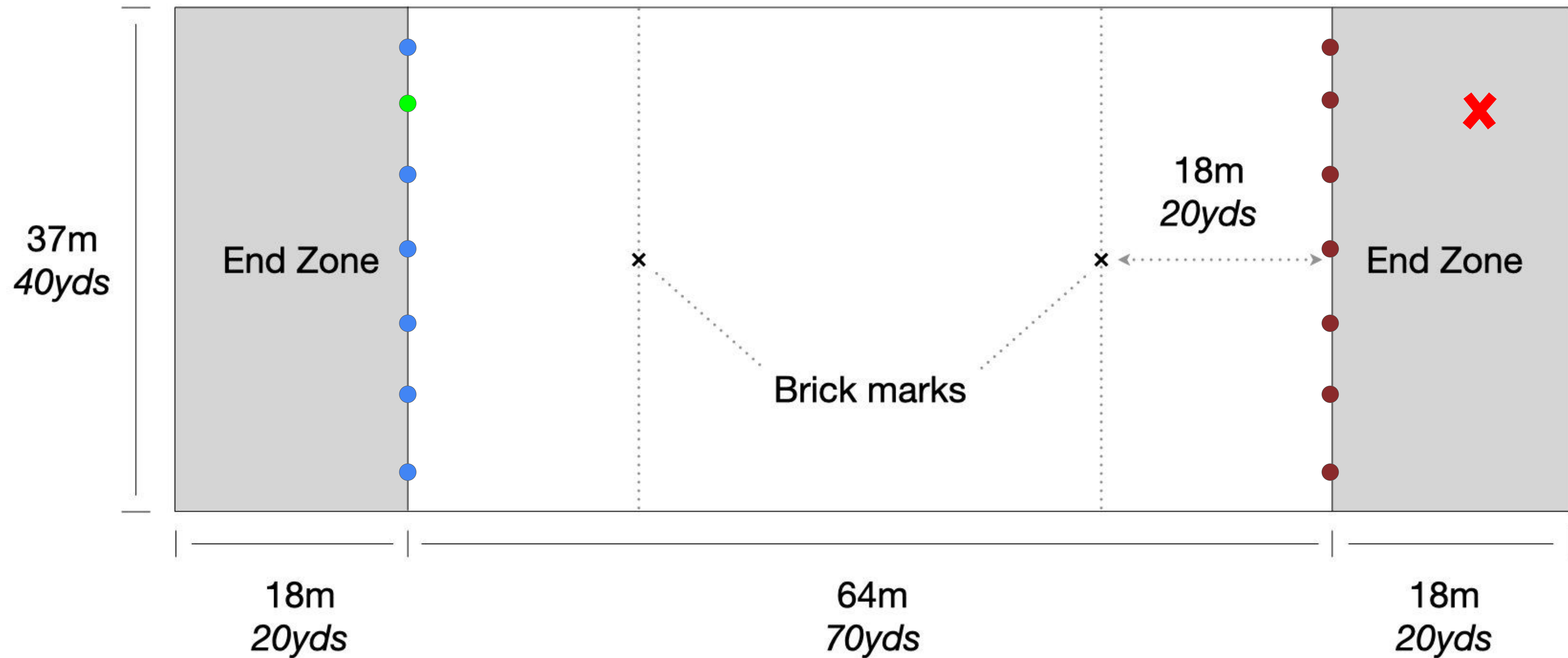
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# Ultimate Frisbee Field Dimensions





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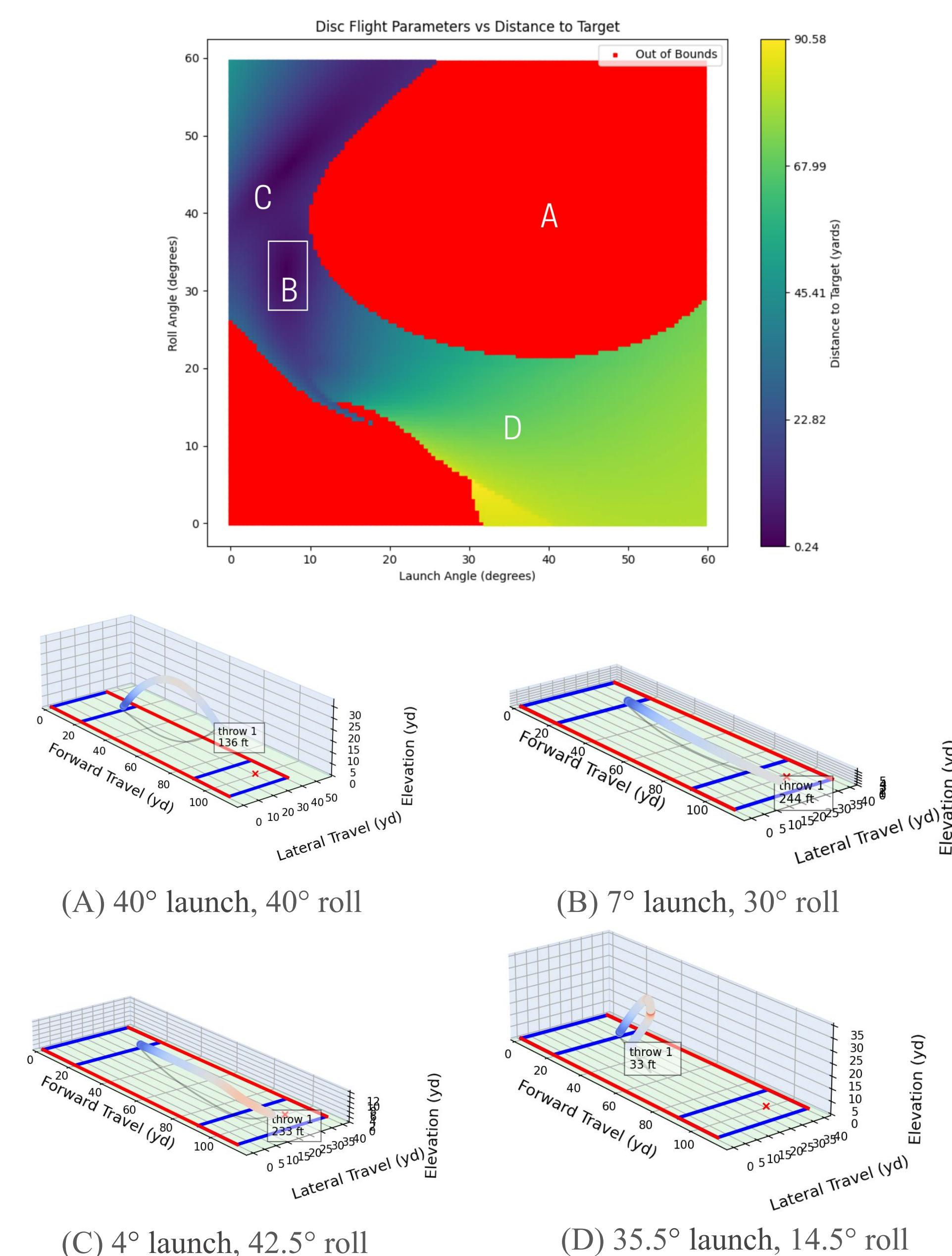
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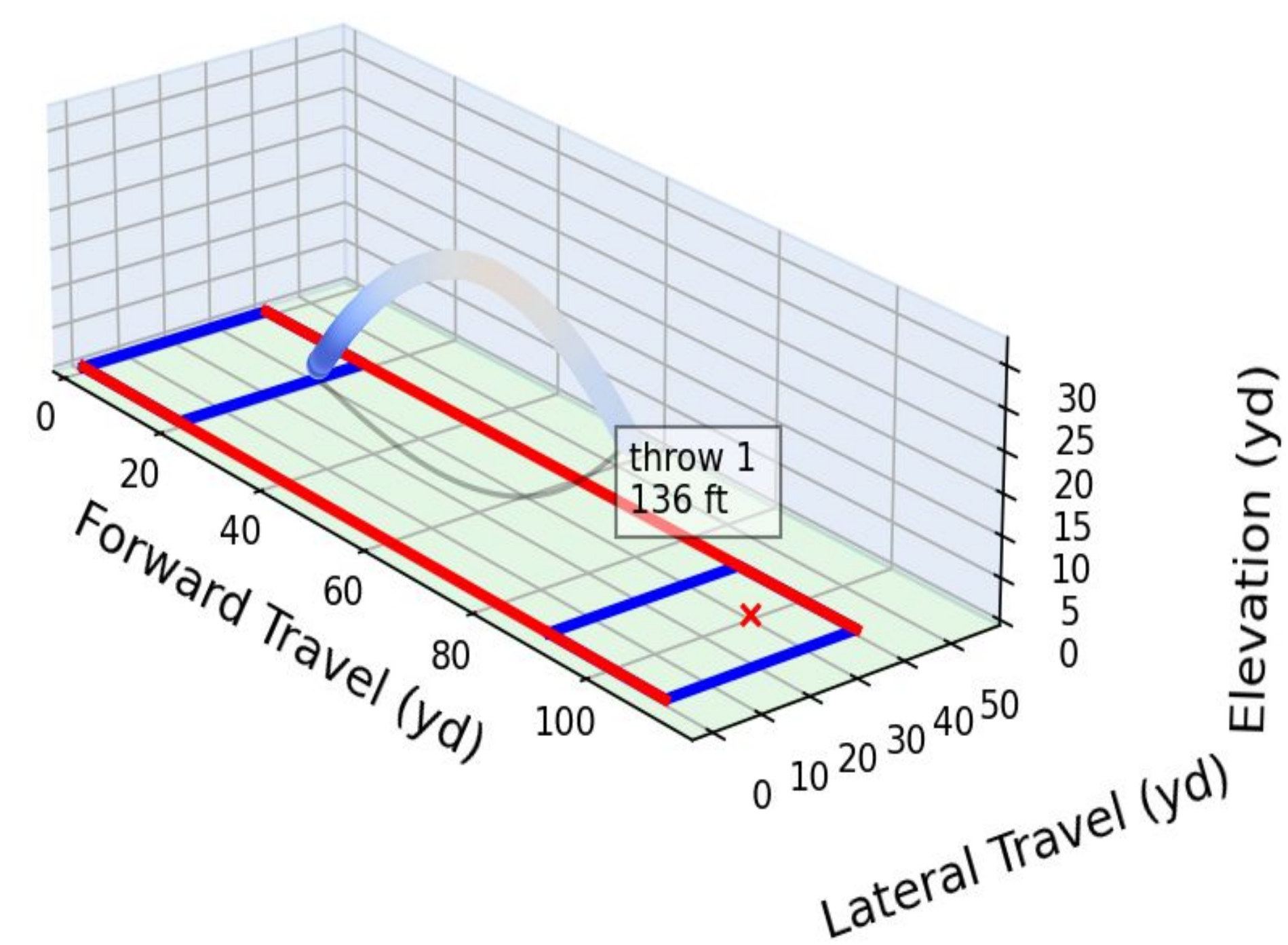
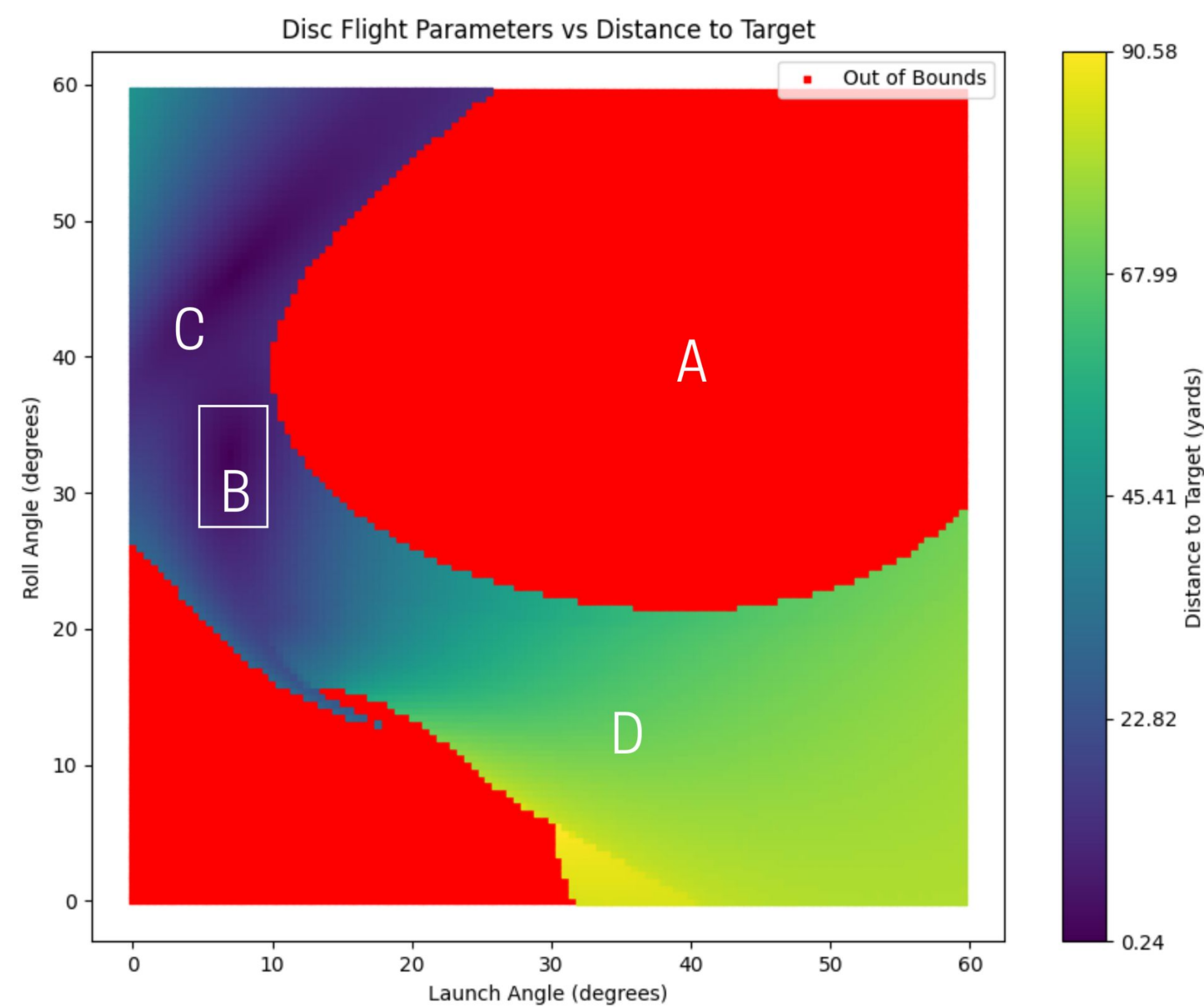
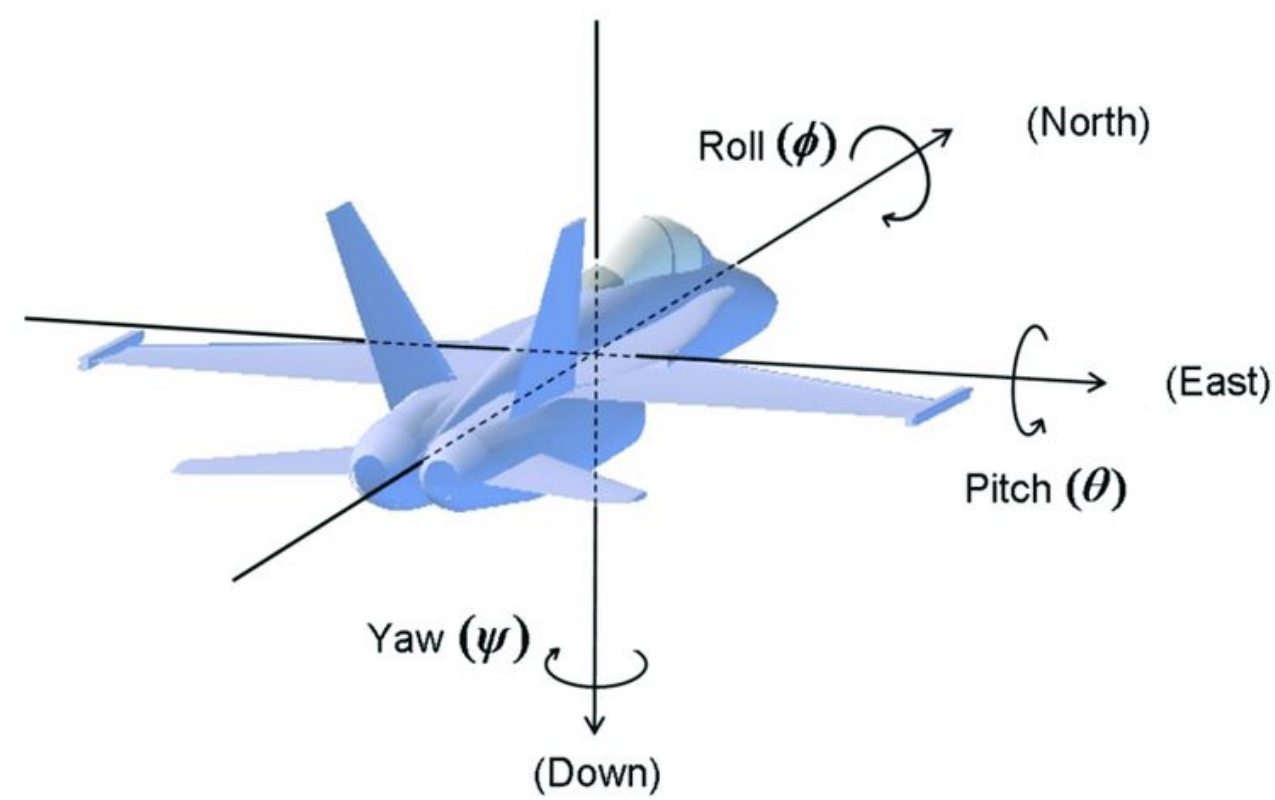
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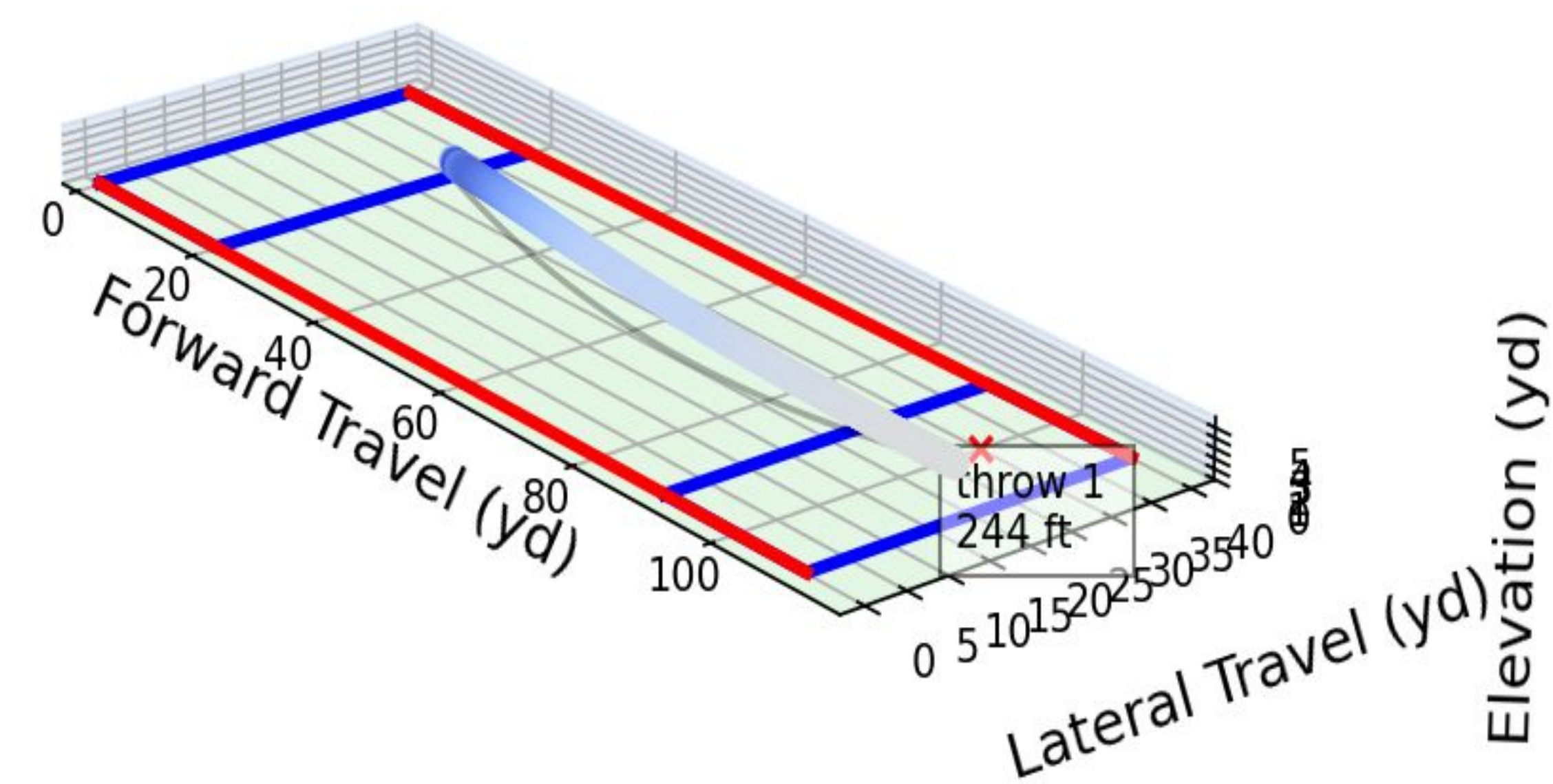
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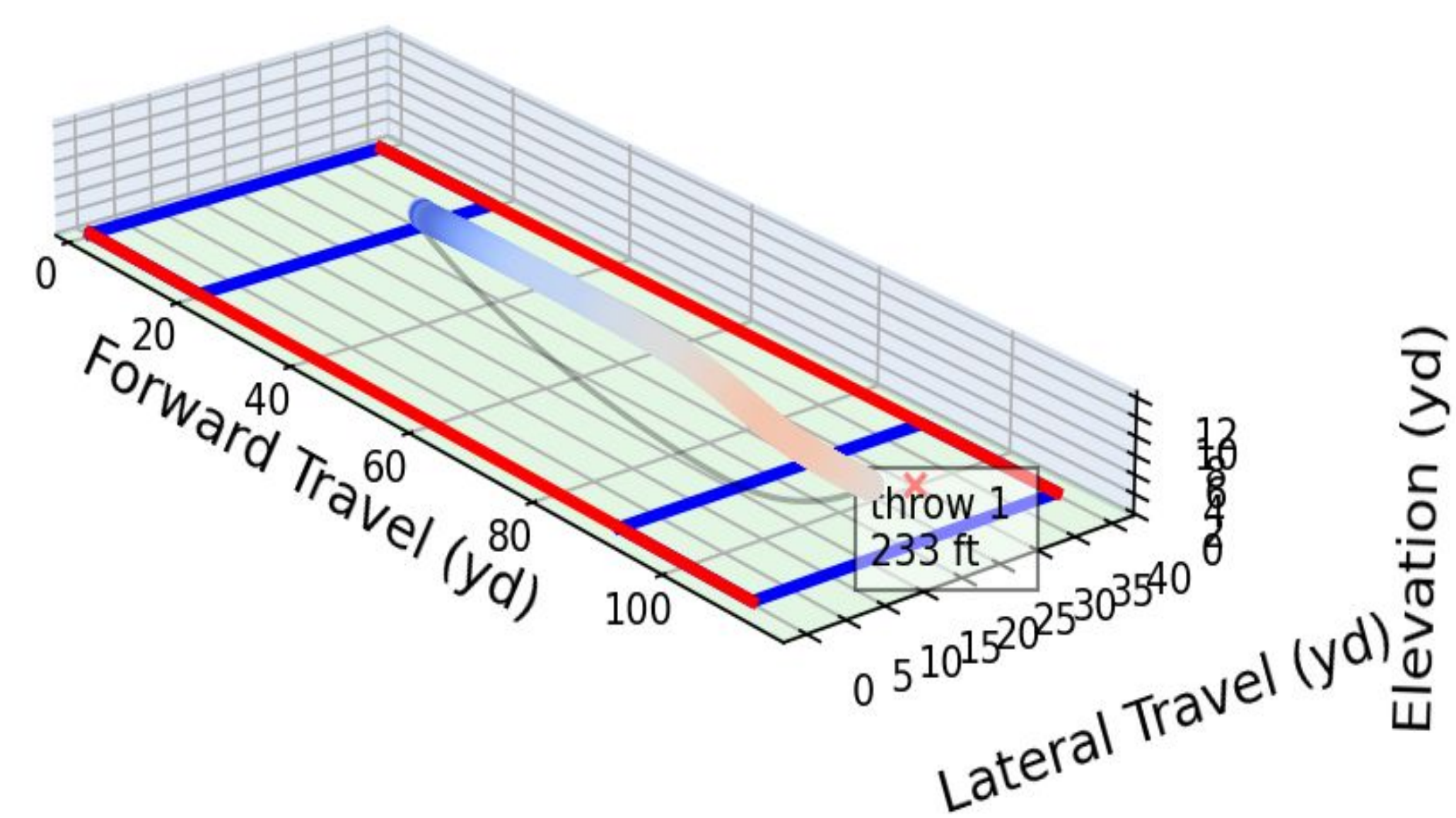
# Supplemental Images



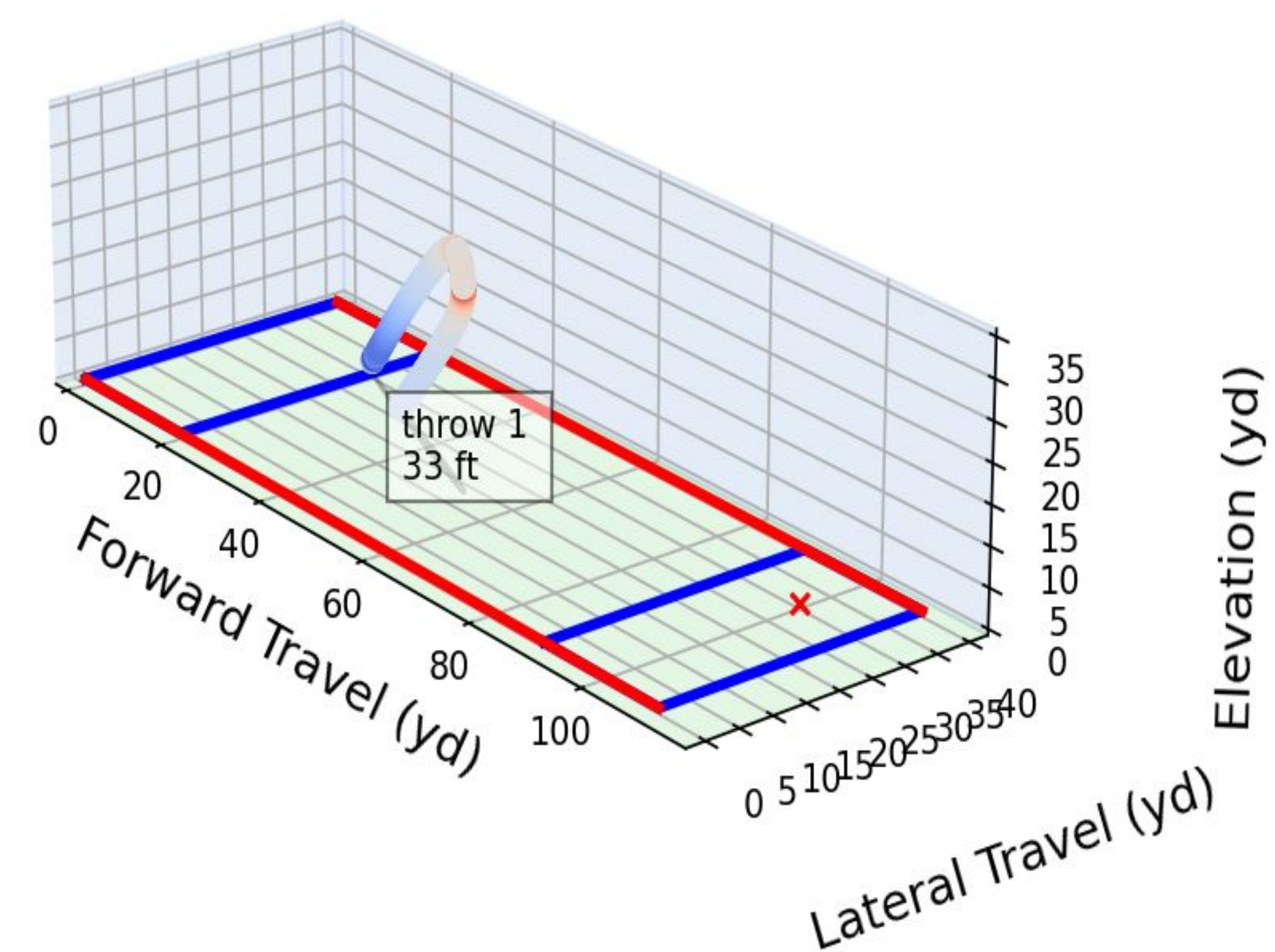
(A) 40° launch, 40° roll



(B) 7° launch, 30° roll



(C) 4° launch, 42.5° roll



(D) 35.5° launch, 14.5° roll



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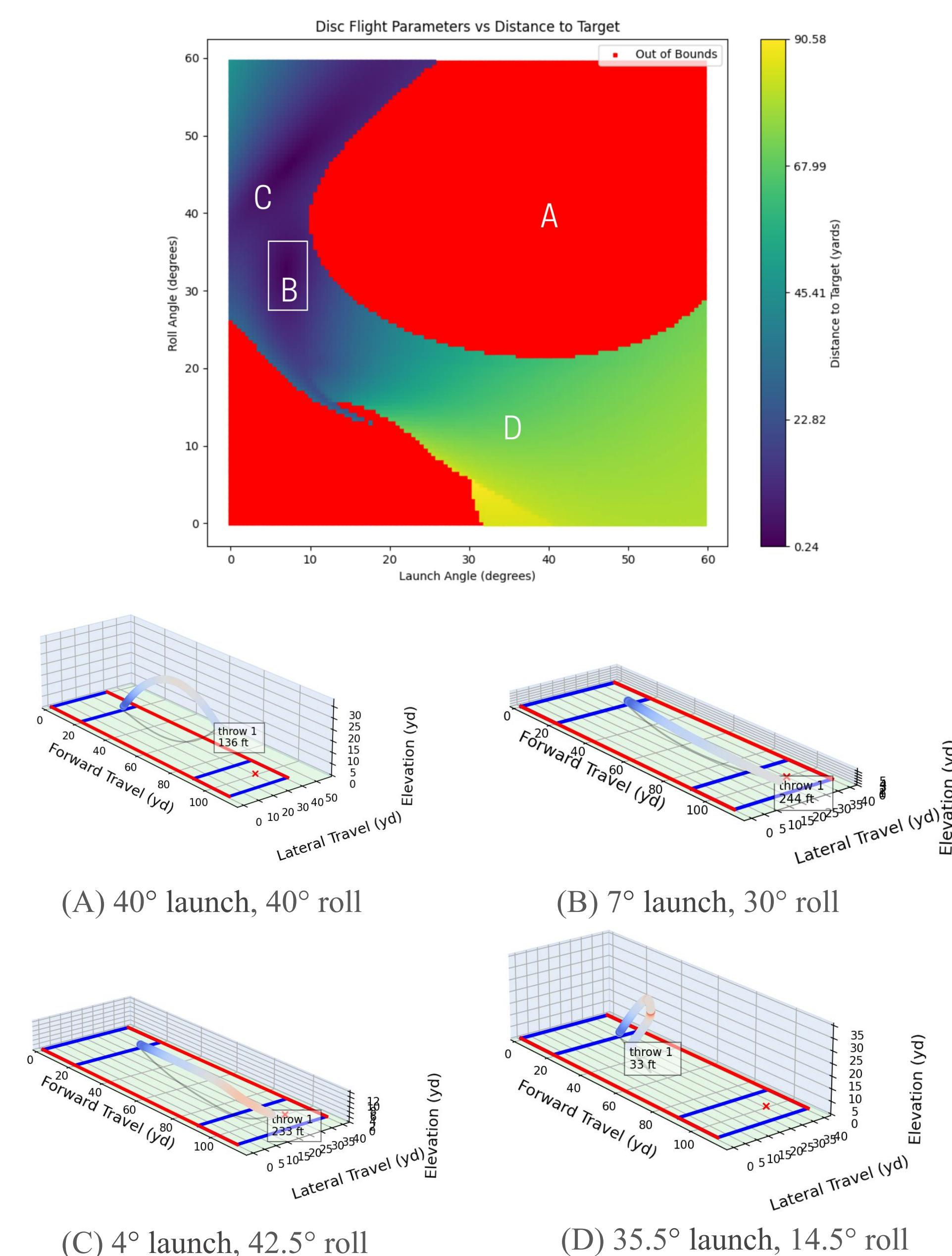
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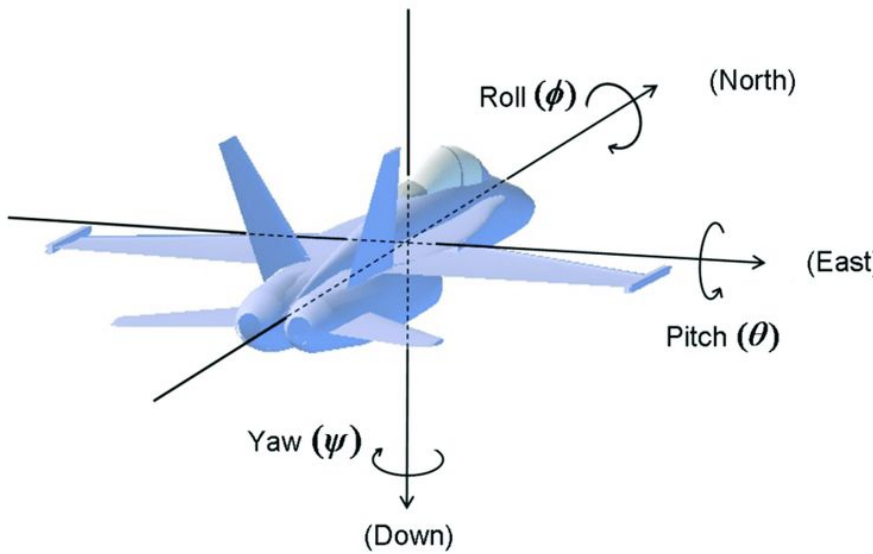
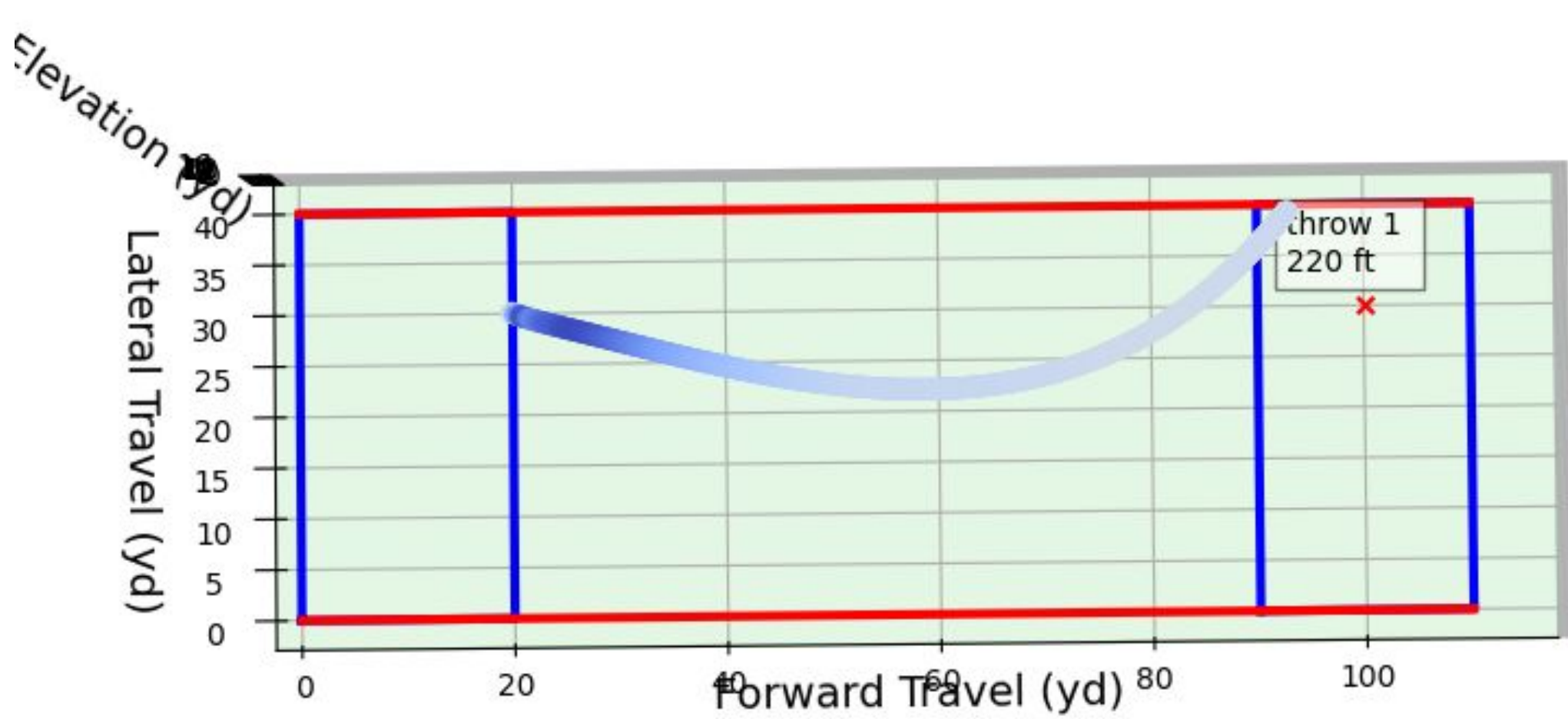
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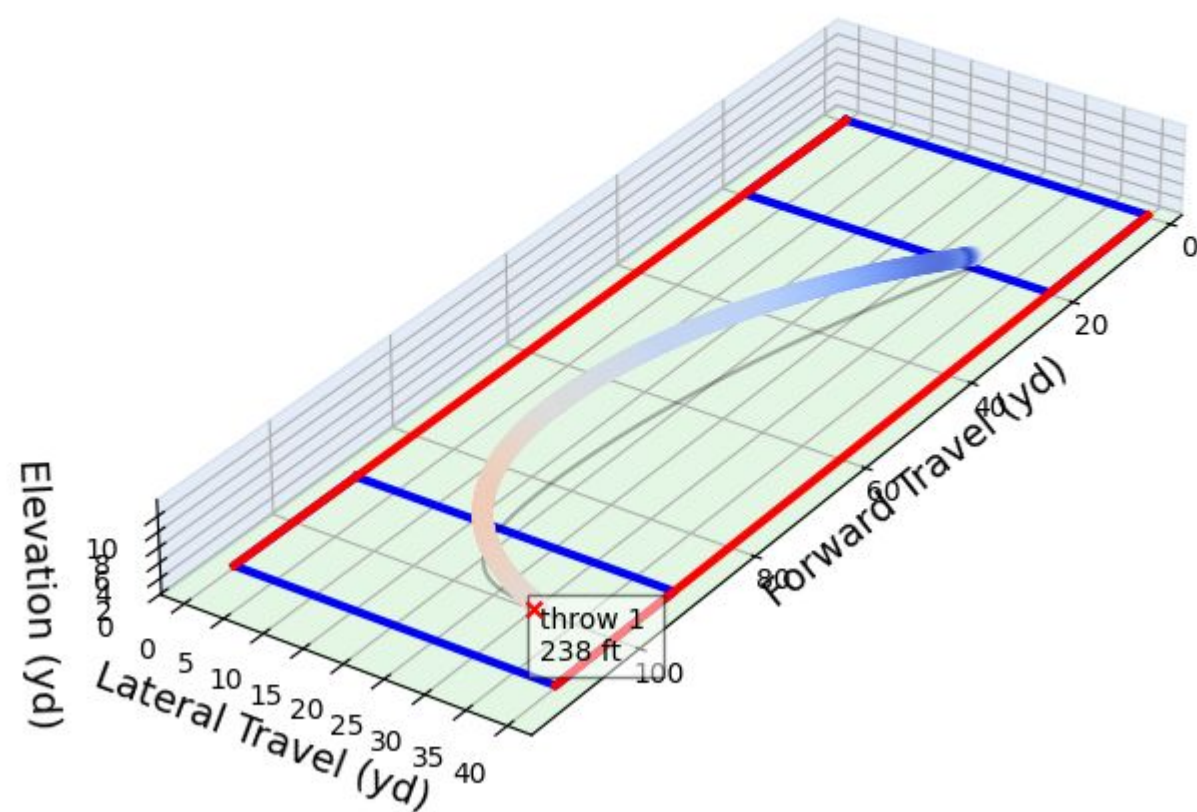


# Hugh's Images (OLD)

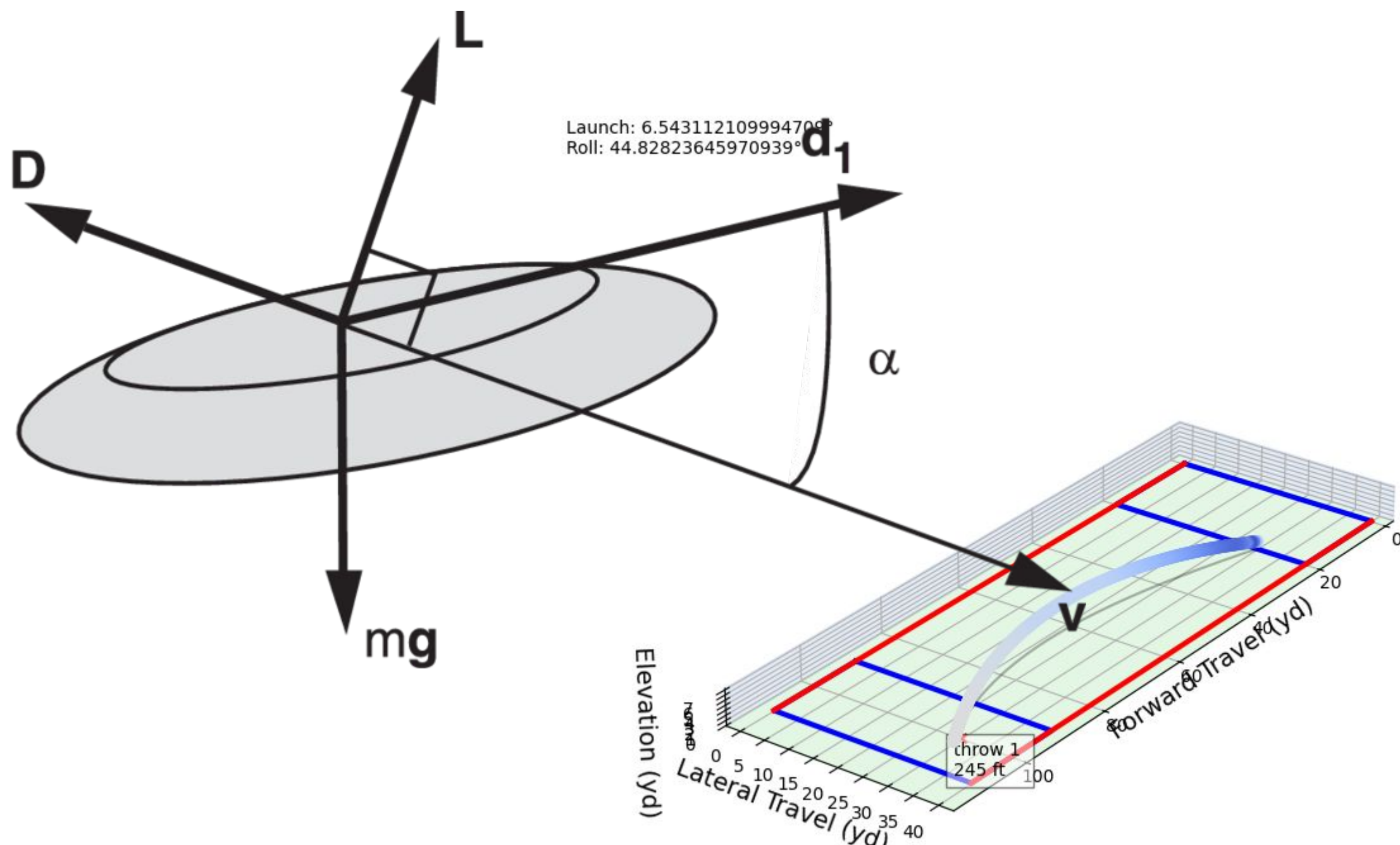
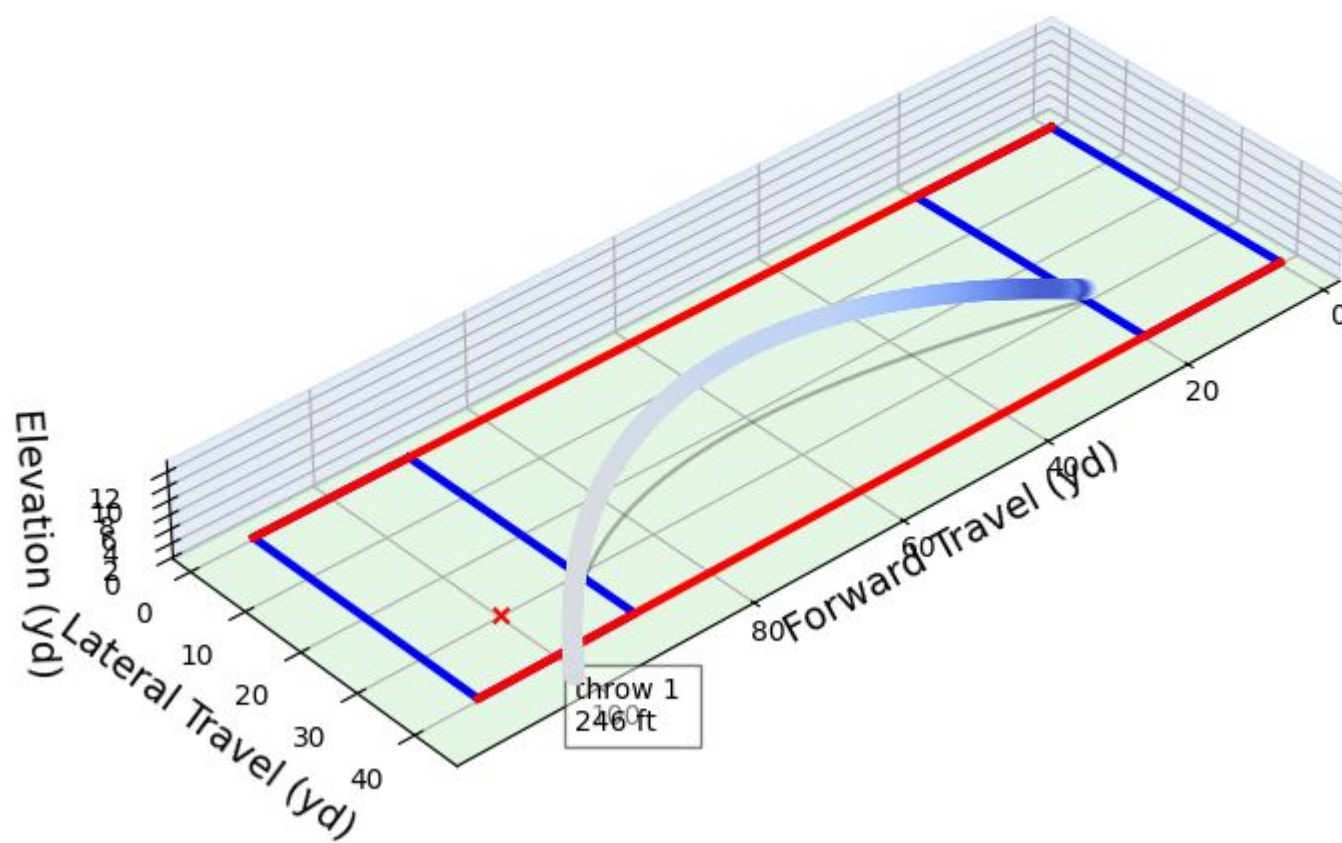
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Launch: 7.083913934426228°  
Roll: 32.701420079260245°



Launch: 16.061224219989423°  
Roll: 48.0845112285337°



Analysis/Findings

have chart w/ box

have row of sub images of pulls

under all of the images

talk about the best ranges

drew box

dot in red means out of bounds

“target”

