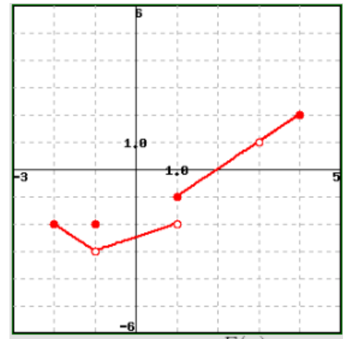
**Definition.** The <u>limit</u> of a function $f(x)$ as $x$ approaches $a$ is the value $L$ (if one exists) that $f(x)$ becomes very near as $x$ becomes very near to $a$. In this case, we write $\lim_{x\to a} f(x) = L$.

**Example 2.2.1.** Find each indicated limit for the function graphed.



The graph of $y = F(x)$.

(a) $\lim_{x\to 2} F(x)$

(b) $\lim_{x\to -1} F(x)$

(c) $\lim_{x\to 1} F(x)$

**Definition.** The <u>one-sided limit</u> (or right-handed limit) of a function $f(x)$ as $x$ approaches $a$ from the right is the value $L$ (if one exists) that $f(x)$ becomes very near as $x$, for values greater than $a$, becomes very near to $a$. In this case, we write $\lim_{x\to a^+} f(x) = L$.

Similarly, the one-sided limit (or left-handed limit) of a function $f(x)$ as $x$ approaches $a$ from the left is the value $L$ (if it exists) that $f(x)$ becomes very near as $x$, for values less than $a$, becomes very near to $a$. In this case, we write $\lim_{x\to a^-} f(x) = L$.

The "two-sided" limit given earlier is defined exactly when the one-sided limits exist and agree.

**Example 2.2.2.** For the function graphed in Example 2.2.1, find the indicated limits.

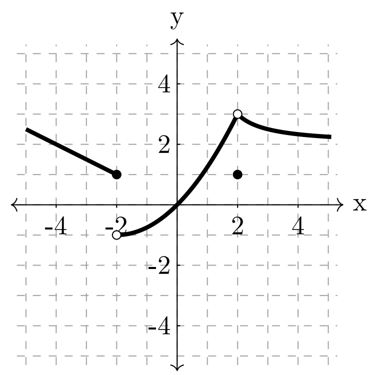
(a) $\lim_{x\to 4^-} F(x)$

(b) $\lim_{x\to 3^-} F(x)$

(c) $\lim_{x\to 3^+} F(x)$

(d) $\lim_{x\to 3} F(x)$

**Example 2.2.3.** Consider the graph of $y = f(x)$, shown below.




Compute each of the following limits. If a limit does not exist, write "DNE".

(a) $\lim_{x \to -2} f(x)$

(b) $\lim_{x \to 0} f(x)$

(c) $\lim_{x \to 2} f(x)$

**Example 2.2.4.** Consider the function

$$f(x) = \begin{cases} \sin(x) & \text{if } x < 0 \\ 3 & \text{if } x = 0 \\ x^2 & \text{if } 0 < x \leq 1 \\ 1 - x & \text{if } x > 1 \end{cases}$$

Find the values of $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1^-} f(x)$, if they exist. Sketch a graph of $y = f(x)$ and use it to determine all values of $a$ for which $\lim_{x \to a} f(x)$ does not exist.