

Definition. A function $f(x)$ is said to be continuous at a point a if

$$\lim_{x \rightarrow a} f(x) = f(a)$$

Notice that this definition implicitly requires three things for f to be continuous at a : $f(a)$ and $\lim_{x \rightarrow a} f(x)$ are both defined and are equal to each other.

We say that f is continuous on an interval if it is continuous at every point in the interval. If f is not continuous at a , we say that f is discontinuous or that f has a discontinuity at a .

Note. Polynomials are continuous for all real numbers. Rational functions are continuous on their domain (but could have discontinuities outside of their domain). Other functions that are continuous at every numbers in their domains include: root, trigonometric, inverse trigonometric, exponential, and logarithmic functions.

Example 2.4.1. Is the function

$$f(x) = \begin{cases} 4 - x & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$$

continuous on $(-\infty, \infty)$?

Example 2.4.2. Find a value of b such that

$$g(x) = \begin{cases} 2bx & \text{if } x \leq -1 \\ 3x^2 + x + b & \text{if } x > -1 \end{cases}$$

is continuous for all real numbers.

Properties. If f is a continuous function and $\lim_{x \rightarrow a} g(x)$ exists and is in the domain of f , then

$$\lim_{x \rightarrow a} f(g(x)) = f\left(\lim_{x \rightarrow a} g(x)\right).$$

Example 2.4.3. Find the indicated limit: $\lim_{x \rightarrow 1} \ln\left(\frac{x^2 - 1}{x - 1}\right)$

Exercise 2.4.4. Recall the function from the Section 2.2 Handout:

$$f(x) = \begin{cases} \sin(x) & \text{if } x < 0 \\ 3 & \text{if } x = 0 \\ x^2 & \text{if } 0 < x \leq 1 \\ 1 - x & \text{if } x > 1 \end{cases}$$

Sketch a graph of this function and use the graph to find all values a for which f is not continuous at a .