Properties. Assume that for the functions $f(x)$ and $g(x)$, $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = K$. Also assume that c and $p > 0$ are constant.

$$\lim_{x\to a} x = a \qquad \lim_{x\to a} c = c$$

$$\lim_{x\to a} [f(x) + g(x)] = L + K \qquad \lim_{x\to a} [f(x) - g(x)] = L - K$$

$$\lim_{x\to a} [f(x) \cdot g(x)] = L \cdot K \qquad \lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{K} \text{ as long as } K \neq 0$$

$$\lim_{x\to a} [c \cdot f(x)] = c \cdot L$$

$$\lim_{x\to a} [f(x)]^p = L^p \text{ as long as } L^p \text{ exists}$$

Note: Here we allow for the expression a to represent $\pm\infty$ (see Section 2.5).

Example 2.3.1. Assume that we know $\lim_{t\to 2} f(t) = -3$, $\lim_{t\to -1} f(t) = 1$, $\lim_{t\to 2} g(t) = -2$, and $\lim_{t\to -1} g(t) = 0$. Find each of the folllowing limits, if possible.

(a) $\lim_{t\to 2} [f(t) - 3g(t)]$

(b) $\lim_{t\to 2} [g(t)]^3$

(c) $\lim_{t\to -1} \dfrac{f(t)}{g(t)}$

Note. Given any rational (or polynomial) function f for which $f(a)$ is defined, $\lim_{x\to a} f(x) = f(a)$.

Note. If $f(x) = g(x)$, except possibly at $x = a$, then $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$ (assuming the limits exist).

Example 2.3.2. Find the indicated limit using limit laws and/or algebra.

(a) $\lim_{x\to 2} \sqrt{\dfrac{2x^2+1}{3x-2}}$

(b) $\lim_{x\to 1} (x^2 + x - 3)^4$

(c) $\lim_{x\to 3} \dfrac{x^3 - 9x}{3 - x}$

(d) $\displaystyle\lim_{x\to 0}\frac{\sqrt{x^2+1}-1}{x}$

(e) $\displaystyle\lim_{t\to 4}\frac{t-\sqrt{3t+4}}{4-t}$

Example 2.3.3. Let $f(x)=\dfrac{x^2-1}{|x-1|}$. Does $\lim\limits_{x\to 1} f(x)$ exist?

Properties. If $f(x)\le g(x)$ when x is near a (except possibly at a) and the limits of f and g both exists as x approaches a, then $\lim\limits_{x\to a} f(x)\le \lim\limits_{x\to a} g(x)$.

Theorem (The Squeeze Theorem)**.** If $f(x)\le g(x)\le h(x)$ when x is near a (except possibly at a), and $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} h(x)=L$, then $\lim\limits_{x\to a} g(x)=L$.

Example 2.3.4. If $2x\le g(x)\le x^4-x^2+2$ for all x, evaluate $\lim\limits_{x\to 1} g(x)$.