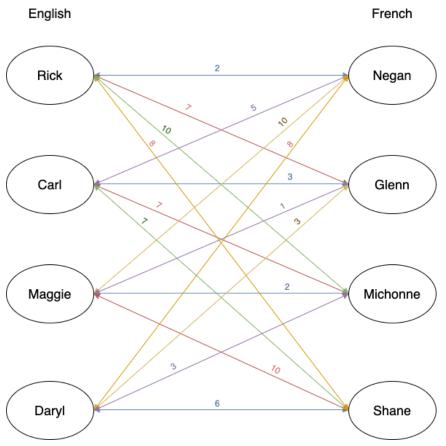
Homework 3 Advanced Analytics and Metaheuristics

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1. Team Building

(a) English and French. We give the English a positive weight and French a negative weight. We have the costs on the arcs. Here we slightly changed the model of the MCNFP by introducing a negative to the objective function. This cause the minimizer to return a maximum.



Here is our model file:

```
#Homework 3 Problem 1 Part A - Group 1
```

```
reset;
option solver cplex;
```

```
#define the sets and parameters
set Nodes;
set Arcs within {i in Nodes, j in Nodes};
param compatibilityScores{i in Nodes, j in Nodes} >= 0; #compatibility must be grea
#define decision variables
```

var networkFlow {Arcs} >= 0, <= 1;
var totalCompatibility;</pre>

#define objective function

```
maximize compatibilityPairs: sum {(i,j) in Arcs} compatibilityScores[i,j] * network
#define constraints
subject to one Pairing {i in Nodes}: sum {(i,j) in Arcs} network Flow[i,j] = 1; #only flow[i,j] flow[i,
subject \ to \ netZero \ \{i \ in \ Nodes\}: sum \ \{(i,j) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs\} \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs] \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs] \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs] \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs] \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs] \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs] \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs] \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs] \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs] \ networkFlow[i,j] \ - \ sum \ \{(j,i) \ in \ Arcs] \ networkFlow[i,j] \ networkFlow
#connect to the data file
data group1_HW3_p1a.dat;
#solve to maximize objective function
solve;
#print the results
printf "Total Compatibility Score: %f\n", totalCompatibility;
printf "Pairs:\n";
for {(i,j) in Arcs: networkFlow[i,j] > 0} {
                        printf "%s - %s\n", i, j;
/*
1: Rick
2: Negan
3: Carl
4: Glenn
5: Maggie
6: Michonne
7: Daryl
8: Shane
*/
Here is our data file:
data;
set Nodes := 1 2 3 4 5 6 7 8;
/*
1: Rick
2: Negan
3: Carl
4: Glenn
5: Maggie
6: Michonne
7: Daryl
8: Shane
*/
```

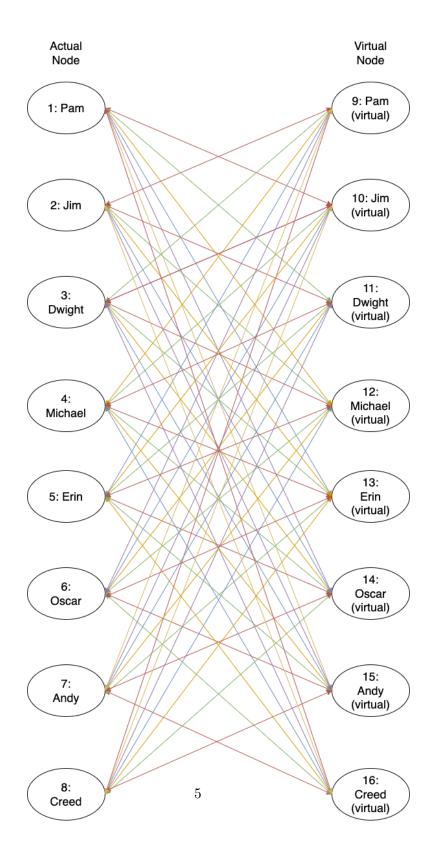
```
set Arcs :=
         (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (1, 7) (1, 8)
            (2, 3) (2, 4) (2, 5) (2, 6) (2, 7) (2, 8)
                    (3, 4) (3, 5) (3, 6) (3, 7) (3, 8)
                            (4, 5) (4, 6) (4, 7) (4, 8)
                                   (5, 6) (5, 7) (5, 8)
                                           (6, 7) (6, 8)
                                                   (7, 8);
                                         2
                                                   3
                                                                              6
                                                                                      7
param compatibilityScores:
                                                                     5
    1
                               0
                                         2
                                                  4
                                                            7
                                                                     4
                                                                              10
                                                                                      9
    2
                               2
                                        0
                                                  5
                                                             1
                                                                     10
                                                                              10
                                                                                      8
    3
                               4
                                        5
                                                  0
                                                             3
                                                                              7
                                                                                      2
                                                                     3
                               7
    4
                                        1
                                                  3
                                                             0
                                                                     1
                                                                              4
                                                                                      3
    5
                               4
                                        10
                                                  3
                                                             1
                                                                     0
                                                                              2
                                                                                      6
    6
                               10
                                        10
                                                  7
                                                             4
                                                                     2
                                                                              0
                                                                                      3
    7
                               9
                                        8
                                                  2
                                                             3
                                                                     6
                                                                                      0
                                                                              3
                               9
                                         4
                                                  7
                                                            9
                                                                     10
                                                                              2
                                                                                      6
```

Here is our output:

```
ampl: model group1_HW3_planj.mod
CPLEX 20.1.0.0: optimal solution; objective -32
2 dual simplex iterations (0 in phase I)
1 2
1 4
      1
1 6
      0
1 8
      0
3 2
      0
3 4
      0
3 6
      1
3 8
      0
5 2
      0
5 4
      0
5 6
      0
5 8
      1
7 2
      1
7 4
      0
7 6
      0
7 8
      0
```

For our example we see person 1 and 4 paired, 3 and 6 paired, etc. We see this as fully generalizable. This MCNFP is always balanced we have equal number of English and French so we always have even pairs with no left over in this model when solved.

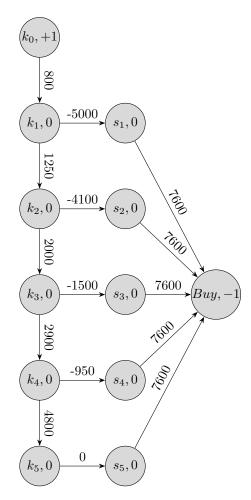
(b) Office Team Building



2. Outdoor Grilling

Below is our model diagram.

Maintain Sell



```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# 1[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
```

```
reset;
options solver cplex;
set NODES;
                                     # nodes in the network
                                     # arcs in the network
set ARCS within {NODES, NODES};
param b {NODES} default 0;
                                    # supply/demand for node i
param c {ARCS} default 0;
                                    # cost of one of flow on arc(i,j)
param 1 {ARCS} default 0;
                                    # lower bound on flow on arc(i,j)
param u {ARCS} default Infinity; # upper bound on flow on arc(i,j)
                                     # flow on arc (i,j)
var x {ARCS};
minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j]; #objective: minimize arc flow cost
# Flow Out(i) - Flow In(i) = b(i)
subject to flow_balance {i in NODES}:
sum\{j \text{ in NODES: } (i,j) \text{ in ARCS} \times [i,j] - sum\{j \text{ in NODES: } (j,i) \text{ in ARCS} \times [j,i] = b[i];
subject to capacity \{(i,j) \text{ in ARCS}\}: 1[i,j] \leq x[i,j] \leq u[i,j];
data group1_HW3_p2.dat;
solve;
display x;
Here is our data file:
set NODES := m0, m1, m2, m3, m4, m5,
                          s1, s2, s3, s4, s5,
                          b1;
set ARCS := (m0, m1), (m1, m2), (m2, m3), (m3, m4), (m4, m5),
                          (m1, s1), (m2, s2), (m3, s3), (m4, s4), (m5, s5),
                          (s1, b1), (s2, b1), (s3, b1), (s4, b1), (s5, b1);
param: b:=
        m0 1
        b1 -1;
                c 1 u:=
param:
```

```
[m0, m1]
                 800
[m1, m2]
                  1250
[m2, m3]
                  2000
[m3, m4]
                 2900
[m4, m5]
                 4800
[m1, s1]
                 -5000
[m2, s2]
                  -4100
[m3, s3]
                 -1500
[m4, s4]
                 -950
[m5, s5]
[s1, b1]
                 7600
[s2, b1]
                 7600
[s3, b1]
                 7600
[s4, b1]
                 7600
[s5, b1]
                 7600
```

Here is our output:

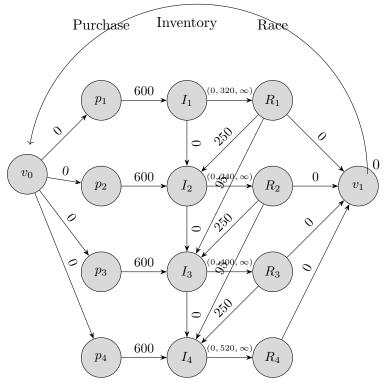
```
Console
AMPL
ampl: model group1_HW3_p2.mod
CPLEX 20.1.0.0: optimal solution; objective 3400
0 dual simplex iterations (0 in phase I)
x :=
m0 m1
m1 m2
       0
m1 s1
        1
m2 m3
        0
m2 s2
m3 m4
        0
m3 s3
m4 m5
        0
m4 s4
m5 s5
        0
s1 b1
s2 b1
s3 b1
        0
s4 b1
        0
s5 b1
ampl:
```

Buy a new one every year seems to be our cost effective method. It would cost us \$3400 each year to be a grill master.

3. Race Car Tires

Here is my flow model: All nodes have zero b costs are displayed on

arcs. If minimums are needed, the ordered triple represents (cost, lowerLimit, upperLimit). The v nodes are virtual to balance the flow.



```
# AMPL model for the Minimum Cost Network Flow Problem
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
# Parameters not specified in the data file will get their default values.
reset;
options solver cplex;
set NODES;
                                  # nodes in the network
set ARCS within {NODES, NODES};
                                  # arcs in the network
param b {NODES} default 0;
                                 # supply/demand for node i
param c {ARCS} default 0;
                                 # cost of one of flow on arc(i,j)
param 1 {ARCS} default 0;
                             # lower bound on flow on arc(i,j)
param u {ARCS} default Infinity; # upper bound on flow on arc(i,j)
```

```
var x {ARCS};
                                     # flow on arc (i,j)
minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j]; #objective: minimize arc flow cost
# Flow Out(i) - Flow In(i) = b(i)
subject to flow_balance {i in NODES}:
sum\{j \text{ in NODES: } (i,j) \text{ in ARCS} \times [i,j] - sum\{j \text{ in NODES: } (j,i) \text{ in ARCS} \times [j,i] = b[i];
subject to capacity \{(i,j) \text{ in ARCS}\}: 1[i,j] \leftarrow x[i,j] \leftarrow u[i,j];
data group1_HW3_p3.dat;
solve;
display x;
Here is our data file:
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015
#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
       default arc upper bounds are infinity
#
       default node requirements are 0
set NODES :=
                       v0, p1, p2,p3,p4, i1,i2,i3,i4,r1,r2,r3,r4,v1;
set ARCS := (v0,p1), (v0,p2), (v0,p3), (v0,p4), #start the flow
                          (p1,i1),(p2,i2),(p3,i3),(p4,i4), #purchase new tires each race
                          (i1,r1),(i2,r2),(i3,r3),(i4,r4), #move inventory to race
                          (r1,v1),(r2,v1),(r3,v1),(r4,v1), #move spent tires not fixed to
                          (i1,i2),(i2,i3),(i3,i4), #move unused inventory
                          (r1,i2),(r1,i3), #race 1 quick and slow fix
                          (r2,i3),(r2,i4), #race 2 quick and slow fix
                          (r3,i4), #race 3 quick fix
                          (v1,v0) #move from virtual to virtual to complete flow
```

c lu:=

param:

```
#purchase new tires each race
[p1,i1] 600
[p2,i2] 600
[p3,i3] 600
[p4,i4] 600
[i1,r1]
                  320 . #minimum tires needed each race
[i2,r2]
                  240 .
[i3,r3]
                  400 .
                 520 .
[i4,r4]
[r1,i2] 250
                         #quick fix
[r2,i3] 250
[r3,i4] 250
[r1,i3] 95
                      .#slowfix
[r2,i4] 95
```

Here is our output:

```
Console
                                                                                            2 🔳 🗟
AMPL
ampl: model group1_HW3_p3.mod
CPLEX 20.1.0.0: optimal solution; objective 490000
6 dual simplex iterations (0 in phase I) \times [*,*]
      i1
                                                               r2
                                                                                    v0
:
i1
                                                        320
i3
i4
                            0
                                                                      400
                                                                             520
р1
      320
p2
p3
p4
r1
r2
             40
                   280
                   120
                          120
                           400
v0
                                 320
                                        200
                                               0
                                                   0
                                                                                    520
v1
      ν1
r1
        0
r3
r4
;
      520
ampl:
```

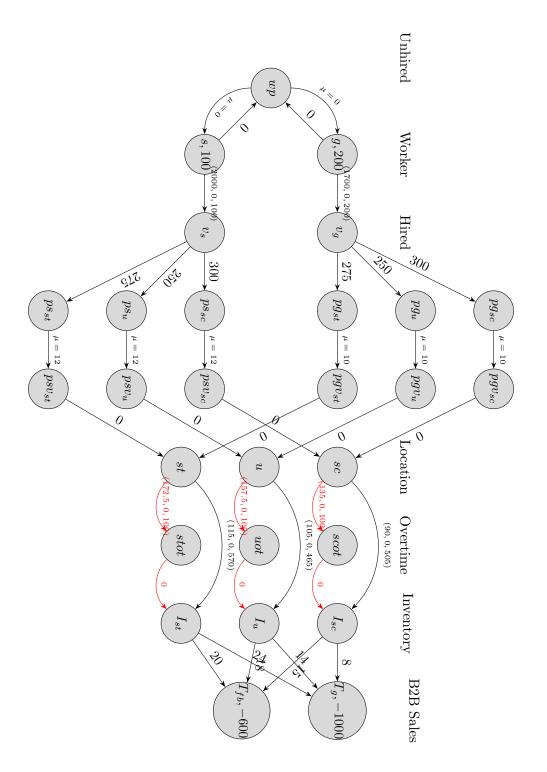
We look to be purchasing new tires for both the needs of the first two races, 320 and 200 respectively. This is the maximum number of tires needed. We use the normal service on 280 tires from the first race and quick on the other 40. In second race we use the normal service on 120 but quick fix 120. For the third race we quick fix all 400 tires used. We end up with exactly the number of tires needed in the fourth race. Total cost is \$490 000.

4. Dunder Mifflin

By far our most complicated model. Unhired workers will flow to the worker pool, wp and flow back out to the Workers with a weight of zero to maintain the flow. g and s represent the available generalist and specialists. If they are hired, they flow to the hired pool of each type. They are then transported to each plant at the respective cost. We then convert each worker into their number of products created with the weight. Finally we combine the number of possible products they can create into each location. Inventory is created with the regular cost but with limits on the maximums. Overtime is represented in red and uses a different virtual node for each location. Inventory is then shipped from each of the factories to the different businesses we serve, meeting the demand as represented inside the node. We attempted to make the flow circular but do to the difference in weights based on employment categories, could never get it to create a balanced flow.

This model provides labor for overtime but does not account for a premium the wage for the employee working outside of regular hours. We see this as an assumption that there is not a bump in pay for working second shift but there is a 50% increase in our overhead costs of production.

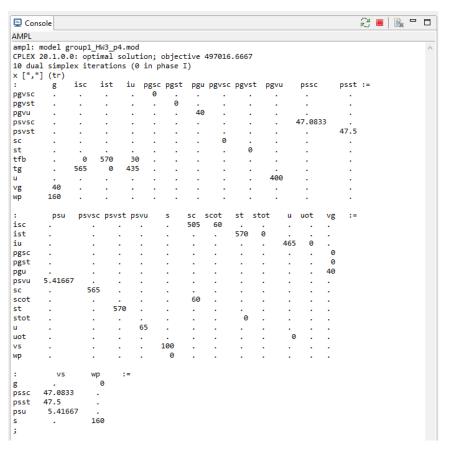




```
# AMPL model for the Minimum Cost Network Flow Problem
# By default, this model assumes that b[i] = 0, c[i,j] = 0, # 1[i,j] = 0 and u[i,j] = Infinity.
# Parameters not specified in the data file will get their default values.
options solver cplex;
set NODES;
                                           # nodes in the network
set ARCS within {NODES, NODES}; # arcs in the network
param b {NODES} default 0;
                                          # supply/demand for node i
param c {ARCS} default 0; # cost of one of flow on arc(i,j)
param 1 {ARCS} default 0; # lower bound on flow on arc(i,j)
param u {ARCS} default Infinity; # upper bound on flow on arc(i,j)
param mu {ARCS} default 1;
                                         # multiplier on arc(i,j) -- if one unit leaves i, mu[i,j] units arrive
var x {ARCS}:
                                          # flow on arc (i,j)
\label{eq:minimize} \mbox{minimize cost: sum} \{(\mbox{i,j}) \mbox{ in ARCS}\} \mbox{ c[i,j] * x[i,j]; } \mbox{\#objective: minimize arc flow cost}
# Flow Out(i) - Flow In(i) = b(i)
subject to flow_balance {i in NODES}:
sum\{j \text{ in NODES: } (i,j) \text{ in ARCS} \ x[i,j] - sum\{j \text{ in NODES: } (j,i) \text{ in ARCS} \ mu[j,i] * x[j,i] = b[i];
subject to capacity \{(i,j) \text{ in ARCS}\}: 1[i,j] \leftarrow x[i,j] \leftarrow u[i,j];
data group1_HW3_p4.dat;
solve;
display x;
Here is our data file:
#MCNFP Problem - data file for problem instance #Charles Nicholson, ISE 5113, 2015
#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
        default arc upper bounds are infinity
        default node requirements are 0
set NODES :=
                           #v0, v1, #virtual nodes at begining and end to get the flow going
                                        g, s, #general and specialist
                                         vg, vs, #virtual to get the cost of general and specialist
                                        pgsc, pgu, pgst, pssc, psu, psst, #shipping cost of each employee
                                        pgvsc, pgvu, pgvst, psvsc, psvu, psvst, #convert each employee to items
                                        sc, u, st, #workers (as items) now at the plants scot, uot, stot # overtime possible
                                        isc, iu, ist, #inventory at each plant
                                        tg, tfb, #transport goods to location wp; #unhired worker pool
set ARCS := (s,vs),(g,vg), #hire the workers
                               (s,wp), (g,wp), #unhired workers
                               (wp,s), (wp,g), #flow the unhired workers back to keep the balance
                               (vg,pgsc),(vg,pgu),(vg,pgst),(vs,psc),(vs,psu),(vs,pst), #move different workers to factories
                               (pgsc,pgvsc),(pgu,pgvu),(pgst,pgvst),(pssc,psvsc),(psu,psvu),(psst,psvst), #convert the workers into items
                               (pgvsc,sc),(psvsc,sc),(pgvu,u),(psvu,u),(pgvst,st),(psvst,st), #more production capacity to each factory (sc,isc),(u,iu),(st,ist), #create the products
                              (sc, scot), (u, uot), (st, stot), #overtime hours making products (scot, isc), (uot, iu), (stot, ist), #overtime products created go to inventory for free (isc,tg), (isc,tfb), (iu, tg), (iu,tfb), (ist,tg), (ist,tfb), #move the product from inventory to customer
param: b:=
          g 200
```

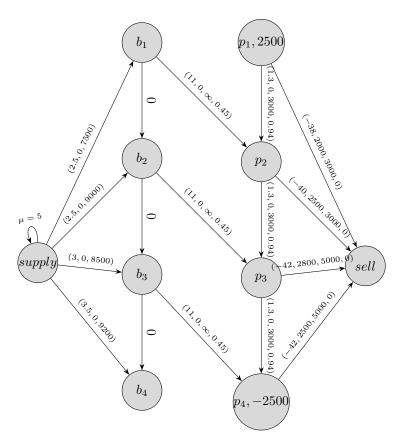
```
s 100
tg -1000
tfb -600;
                          c l u mu:=
[s,vs]
param:
                                                                                       2000
1700
                                                                                                                                    100
200
                                                                                                                                                            . #recruit workers
                              [g,vg]
[vg,pgsc]
                                                                              300
                                                                                                                                                               #move workers to factories
                                                                             250
275
300
                              [vg,pgu]
                              [vg,pgst]
[vs,pssc]
[vs,psu]
[vs,psst]
[pgsc,pgvsc]
                                                                             250
275
                                                                                                                                                           10 #convert workers to items
                             [pgu,pgvu]
[pgst,pgvst]
[pssc,psvsc]
[pssc,psvsc]
[psst,psvu]
[psst,psvst]
[sc,isc]
[u,iu]
[st, ist]
[sc, scot]
[u,uot]
[st,stot]
[isc, tf]
[isc, tf]
[iu, tg]
[iu, tf]
[iu, tg]
[iu, tf]
[ist,tf]
[ist,tfb]
[wp,s]
                                                                                                                                                                       10
                                                                                                                                                           10
                                                                                                                                                           12
                                                                                                                                                                       12
                                                                                                                               505
465
                                                                             90
                                                                                     105
                                                                                                                       100
100
100
                                                                          135
157.5
                                                                                                                                                 . #overtime possible
                                                                              172.5
                                                                              8
                                                                                                                                                                     14
18
24
20
                                                                                                                                                                       .
0
0
```

Here is my output:



Solution is non-integer in employees which is unfortunate but can be the case with the generalized network flow problems. All 100 specialists are hired, 40 generalists. Scranton does use 60 products of OT, none of the other plants do but they max out production at each.

5. Mud b Gone



```
# AMPL model for the Minimum Cost Network Flow Problem
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# 1[i,j] = 0 and u[i,j] = Infinity.
# Parameters not specified in the data file will get their default values.
options solver cplex;
option cplex_options 'sensitivity';
set NODES;
                                                                                                               # nodes in the network
set ARCS within {NODES, NODES}; # arcs in the network
param b {NODES} default 0;
                                                                                                              # supply/demand for node i
param c {ARCS} default 0;
param l {ARCS} default 0;
                                                                                                              # cost of one of flow on arc(i,j)
# lower bound on flow on arc(i,j)
                                                default infinity; # upper bound on flow on arc(i,j)
} default 1; # multiplier on arc(i,j) -- if one unit leaves i, mu[i,j] units arrive
param u {ARCS}
param mu {ARCS} default 1;
var x {ARCS};
                                                                                                              # flow on arc (i,j)
\label{eq:minimize} \mbox{minimize cost: sum} \{(\mbox{i,j}) \mbox{ in ARCS}\} \mbox{ c[i,j] * x[i,j]; $$ \#objective: minimize arc flow cost $$ (\mbox{in minimize cost: sum})$ and $$ (\mbox{in minimize cost: sum})$ arc flow cost $$ (
# Flow Out(i) - Flow In(i) = b(i)
```

```
subject to upcapacity \{(i,j) \text{ in ARCS}\}: x[i,j] \leftarrow u[i,j];
subject to lowcapacity \{(i,j) \text{ in ARCS}\}: l[i,j] \leftarrow x[i,j];
data group1_HW3_p5.dat;
solve;
display x;
display upcapacity, upcapacity.up, upcapacity.down;
display x.current, x.up, x.down;
Here is our data file:
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015
#use with MCNFP.txt model
\mbox{\tt\#note:} default arc costs and lower bounds are 0
        default arc upper bounds are infinity
        default node requirements are 0
set NODES := supply, #suppliers
                               b1,b2,b3,b4, #base for production
p1, p2, p3, p4, #product
sold #sold product
set ARCS := (supply,*) b1 b2 b3 b4 #base purchased from supplier
                             (*,sold) p1 p2 p3 p4 #product sold
(b1,p2),(b2,p3),(b3,p4), #base converted to product
                             (b1,b2),(b2,b3),(b3,b4),
(p1,p2),(p2,p3),(p3,p4)
                              (supply, supply)
         p1 2500
         p4 -2500;
                 c 1 u mu:=
param:
                    [supply,b1] 2.5
                                                                         . #buy new base
                    [supply,b2] 2.5 .
                                                   9000
                   [supply, b3] 3 . [supply,b4]
                                                  8500
                                                                   9200
                   [p1,sold] -38 2000
[p2,sold] -40 2500
[p3,sold] -42 2800
[p4,sold] -42 2500
                                                   3000
                                                                  0 #sell product
                                                   3000
                                                   5000
                                                                  ٥
                   [p1,p2] 1.3
[p2,p3] 1.3
                                                     3000
                                                                    .94 #store product till next month
                    [p3,p4] 1.3
                                                     3000
                    [b1,p2] 11
                                                                .45 #convert base into product. Assumption not too worry about max storage
                   [b2,p3] 11
[b3,p4] 11
                                                                .45
                                                                .45
                   [supply,supply] .
```

(a) We see a solution for our flow. We sell 2000, 2500, 4220 and 2500 in each of the respective periods. We remark on the infinite supply house obtained by creating a loop with a μ factor set to 5 but could be any value greater than 1.

```
Console
                                                                                                         윤 🔳 🗟
ampl: model group1 HW3 p5.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective -115840
4 dual simplex iterations (1 in phase I)
suffix up OUT;
suffix down OUT;
suffix current OUT;
x :=
b1
        р2
b3
                     4511.11
b2
                     2611.11
b2
                     9377.78
        рЗ
b3
b3
                    11111.1
        p4
р1
        p2
                      500
         sold
                     2000
p2
p2
        p3
sold
                     2500
        p4
sold
                     4220
р3
р4
        sold
                     2500
supply b1
                     7500
supply b2
supply b3
                     9000
                     8500
supply b4
supply supply
                     6250
```

(b) We next look at sensitivity of the capacity. We see a value of -\$5.40 in the report, supply to b2. We interpret this as having additional gallons of base will increase (recall minimizing) our income by \$5.40. Both the up and the down are reported as 0. We are unsure of how to interpret this as we clearly would use more of the base if it was available but are unsure why these values are all zero. We even broke apart the upper and lower limit thinking this was the issue but did not change the up and down on the shadow price. We do clearly see this as the bottle neck in our model.

```
upcapacity upcapacity.up upcapacity.down
b1
        b2
                                   0
b1
        p2
                     0
                                   0
                                                    0
b2
        b3
                     0
                                   0
                                                    0
b2
                                   0
                                                    0
        рЗ
                     0
b3
        b4
                     0
                                   0
                                                    0
b3
        p4
                                   0
                                                    0
p1
        p2
                     0
                                   0
                                                    0
р1
        sold
                     0
                                   0
                                                    0
p2
                                   0
                                                    0
       рЗ
                     0
p2
        sold
                     0
                                   0
                                                    0
                                                    0
       p4
                     0
                                   0
рЗ
рЗ
        sold
                                   0
                                                    0
        sold
                                                    0
p4
supply b1
                    -5.4
                                   0
                                                    0
supply b2
                    -5.4
                                   0
                                                    0
supply b3
                    -4.9
                                   0
                                                    0
supply b4
                     0
                                   0
                                                    0
supply supply
                     0
                                   0
                                                    0
```

(c) We see that the contribution to the cost is -\$40 on the flow from p2 to Sold. This value can be modified up to -\$42 and not change the model at all.

:		x.current	x.up	x.down	:=
b1	b2	0	0.0861702		
b1	p2	11	1e+20	10.9138	
b2	b3	0	1.82872	-3.19744e-15	
b2	р3	11	11	9.17128	
b3	b4	0	0	0	
b3	p4	11	12.8287	11	
p1	p2	1.3	1.48	-1e+20	
p1	sold	-38	1e+20	-38.18	
p2	р3	1.3	1e+20	-2.52	
	sold	-40	1e+20	-42	
р3	p4	1.3	1e+20	-2.52	
р3	sold	-42	-42	-1e+20	
p4	sold	-42	1e+20	-42	
supply	b1	2.5	7.9	-1e+20	
supply	b2	2.5	7.9	-1e+20	
supply	b3	3	7.9	-1e+20	
supply	b4	0	0	0	
supply	supply	0	19.6	-1e+20	
;					