Homework 3 Advanced Analytics and Metaheuristics

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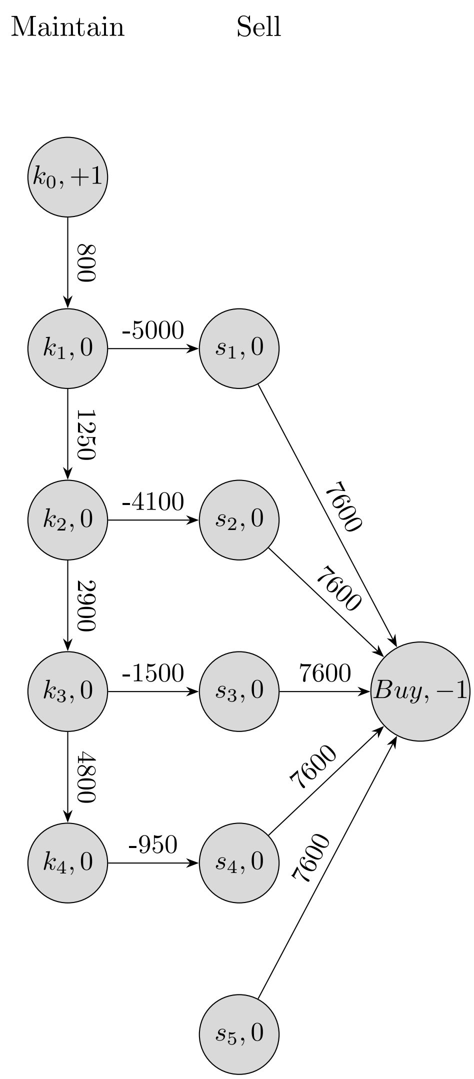
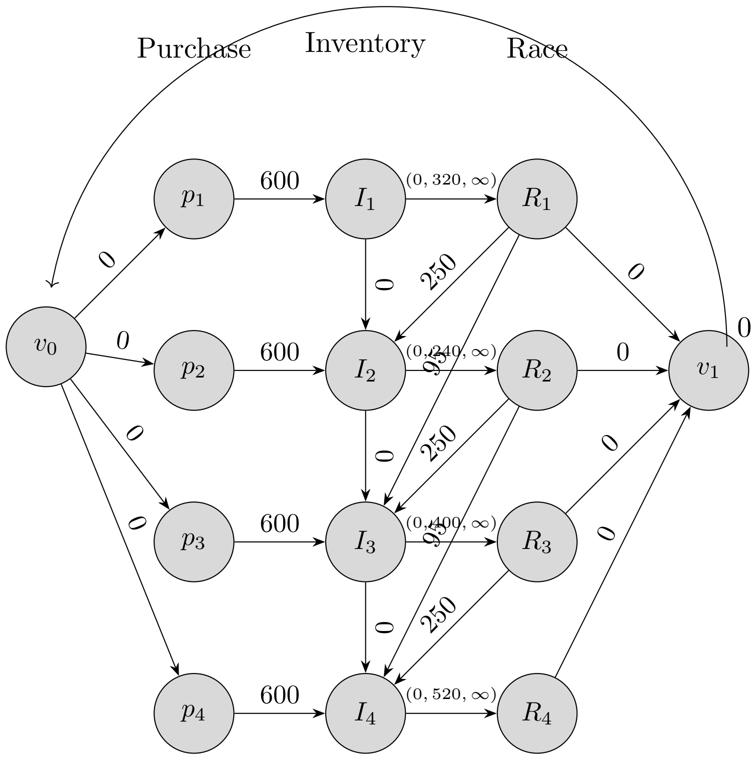
February 21, 2024

1. Team Building
   (a)
   (b)
2. Outdoor Grilling



3. Race Car Tires

   Here is my flow model: All nodes have zero b costs are displayed on arcs. If minimums are needed, the ordered triple represents $(cost, lowerLimit, upperLimit)$. The v nodes are virtual to balance the flow.



Here is our model file:

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;

set NODES;                        # nodes in the network
set ARCS within {NODES, NODES};   # arcs in the network

param b {NODES} default 0;        # supply/demand for node i
param c {ARCS}  default 0;        # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;        # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)

var x {ARCS};                     # flow on arc (i,j)
```

```
minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];

data group1_HW3_p3.dat;

solve;

display x;
```

Here is our data file:

```
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015

#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
#      default arc upper bounds are infinity
#      default node requirements are 0


set NODES :=          v0, p1, p2,p3,p4, i1,i2,i3,i4,r1,r2,r3,r4,v1 ;

set ARCS := (v0,p1),(v0,p2),(v0,p3),(v0,p4), #start the flow
                        (p1,i1),(p2,i2),(p3,i3),(p4,i4), #purchase new tires each race
                        (i1,r1),(i2,r2),(i3,r3),(i4,r4), #move inventory to race
                        (r1,v1),(r2,v1),(r3,v1),(r4,v1), #move spent tires not fixed to
                        (i1,i2),(i2,i3),(i3,i4), #move unused inventory
                        (r1,i2),(r1,i3), #race 1 quick and slow fix
                        (r2,i3),(r2,i4), #race 2 quick and slow fix
                        (r3,i4), #race 3 quick fix
                        (v1,v0) #move from virtual to virtual to complete flow
                        ;



param:        c  l u :=
                [p1,i1] 600  . . #purchase new tires each race
                [p2,i2] 600  . .
```

```
          [p3,i3] 600   . .
          [p4,i4] 600   . .
          [i1,r1] .         320 . #minimum tires needed each race
          [i2,r2] .         240 .
          [i3,r3] .         400 .
          [i4,r4] .         520 .
          [r1,i2] 250 .          . #quick fix
          [r2,i3] 250 .         .
          [r3,i4] 250 .         .
          [r1,i3] 95  .         .#slowfix
          [r2,i4] 95        .         .
;
```

Here is our output:

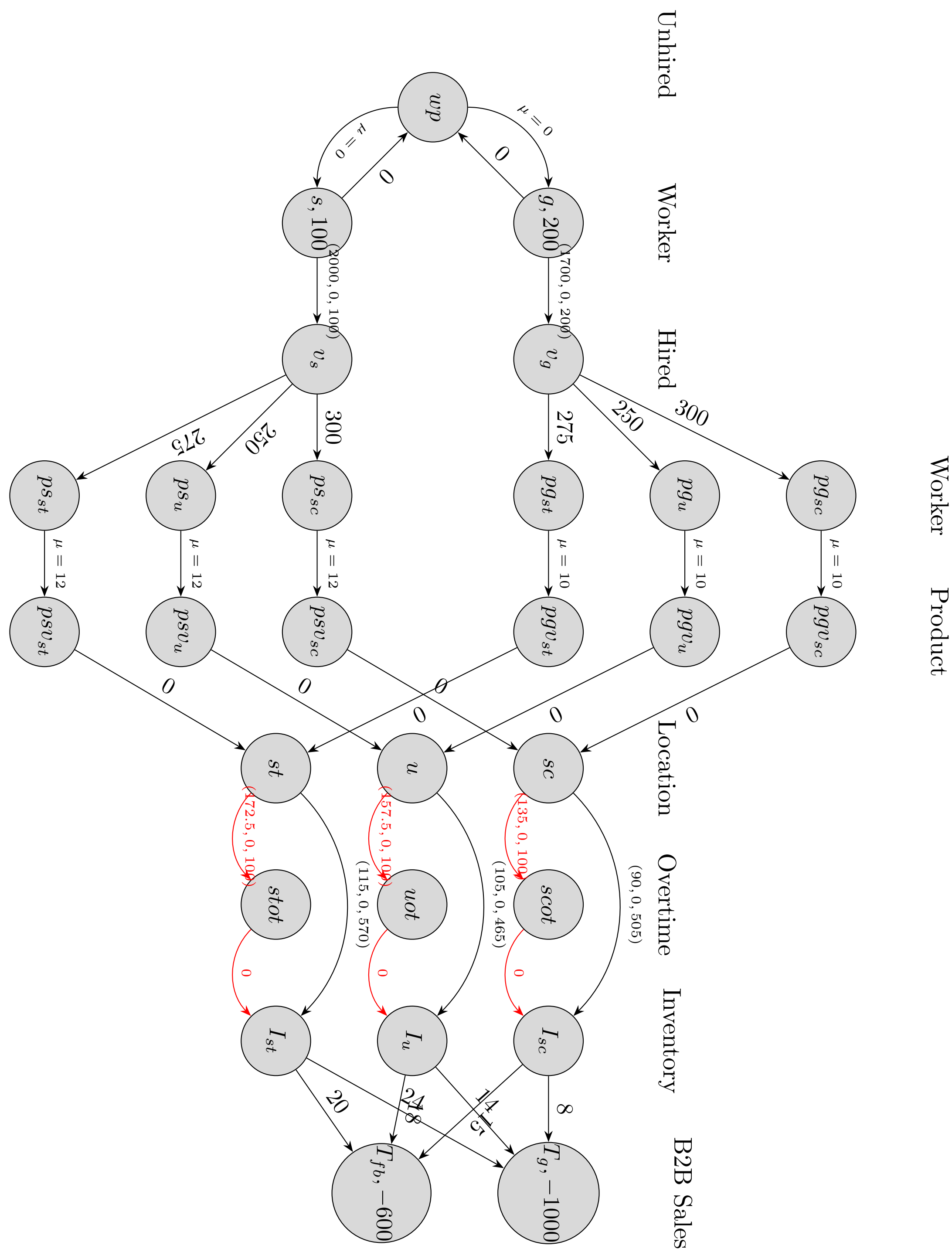
```
Console
AMPL
ampl: model group1_HW3_p3.mod
CPLEX 20.1.0.0: optimal solution; objective 490000
6 dual simplex iterations (0 in phase I)
x [*,*]
:    i1    i2    i3    i4    p1    p2   p3  p4   r1    r2    r3    r4    v0  :=
i1    .     0     .     .     .     .    .   .   320    .     .     .     .
i2    .     .     0     .     .     .    .   .    .    240    .     .     .
i3    .     .     .     0     .     .    .   .    .     .    400    .     .
i4    .     .     .     .     .     .    .   .    .     .     .    520    .
p1   320    .     .     .     .     .    .   .    .     .     .     .     .
p2    .    200    .     .     .     .    .   .    .     .     .     .     .
p3    .     .     0     .     .     .    .   .    .     .     .     .     .
p4    .     .     .     0     .     .    .   .    .     .     .     .     .
r1    .    40    280    .     .     .    .   .    .     .     .     .     .
r2    .     .    120   120    .     .    .   .    .     .     .     .     .
r3    .     .     .    400    .     .    .   .    .     .     .     .     .
v0    .     .     .     .    320   200   0   0    .     .     .     .     .
v1    .     .     .     .     .     .    .   .    .     .     .     .    520

:    v1    :=
r1    0
r2    0
r3    0
r4   520
;

ampl: |
```

We look to be purchasing new tires for both the needs of the first two races, 320 and 200 respectively. This is the maximum number of tires needed. We use the normal service on 280 tires from the first race and quick on the other 40. In second race we use the normal service on 120 but quick fix 120. For the third race we quick fix all 400 tires used. We end up with exactly the number of tires needed in the fourth race. Total cost is \$490 000.

4. Dunder Mifflin

By far our most complicated model. Unhired workers will flow to the worker pool, wp and flow back out to the Workers with a weight of zero to maintain the flow. g and s represent the available generalist and specialists. If they are hired, they flow to the hired pool of each type. They are then transported to each plant at the respective cost. We then convert each worker into their number of products created with the weight. Finally we combine the number of possible products they can create into each location. Inventory is created with the regular cost but with limits on the maximums. Overtime is represented in red and uses a different virtual node for each location. Inventory is then shipped from each of the factories to the different businesses we serve, meeting the demand as represented inside the node. We attempted to make the flow circular but do to the difference in weights based on employment categories, could never get it to create a balanced flow.

This model provides labor for overtime but does not account for a premium the wage for the employee working outside of regular hours. We see this as an assumption that there is not a bump in pay for working second shift but there is a 50% increase in our overhead costs of production.



Unhired
Worker
Hired
Worker
Product
Location
Overtime
Inventory
B2B Sales
up
$s, 100$
$g, 200$
v_s
v_g
ps_{st}
ps_u
ps_{sc}
pg_{st}
pg_u
pg_{sc}
psv_{st}
psv_u
psv_{sc}
pgv_{st}
pgv_u
pgv_{sc}
st
u
sc
$stot$
uot
$scot$
I_{st}
I_u
I_{sc}
$T_{fb}, -600$
$T_g, -1000$
$\mu = 0$
$\mu = 12$
$\mu = 10$
$(2000, 0, 100)$
$(1700, 0, 200)$
$(172.5, 0, 100)$
$(157.5, 0, 100)$
$(135, 0, 100)$
$(115, 0, 570)$
$(105, 0, 465)$
$(90, 0, 505)$
300
250
275
0
20
24
18
14
15
8

Here is our model file:

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;

set NODES;                        # nodes in the network
set ARCS within {NODES, NODES};   # arcs in the network

param b {NODES} default 0;        # supply/demand for node i
param c {ARCS}  default 0;        # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;        # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)
param mu {ARCS} default 1;       # multiplier on arc(i,j) -- if one unit leaves i, mu[i,j] units arrive

var x {ARCS};                     # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} mu[j,i] * x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];

data group1_HW3_p4.dat;

solve;

display x;
```

Here is our data file:

```
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015

#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
#      default arc upper bounds are infinity
#      default node requirements are 0


set NODES :=          #v0, v1, #virtual nodes at begining and end to get the flow going
                                g, s, #general and specialist
                                vg, vs, #virtual to get the cost of general and specialist
                                pgsc, pgu, pgst, pssc, psu, psst, #shipping cost of each employee
                                pgvsc, pgvu, pgvst, psvsc, psvu, psvst, #convert each employee to items
                                sc, u, st, #workers (as items) now at the plants
                                scot, uot, stot # overtime possible
                                isc, iu, ist, #inventory at each plant
                                tg, tfb, #transport goods to location
                                wp; #unhired worker pool

set ARCS := (s,vs),(g,vg), #hire the workers
                        (s,wp), (g,wp), #unhired workers
                        (wp,s), (wp,g), #flow the unhired workers back to keep the balance
                        (vg,pgsc),(vg,pgu),(vg,pgst),(vs,pssc),(vs,psu),(vs,psst), #move different workers to factories
                        (pgsc,pgvsc),(pgu,pgvu),(pgst,pgvst),(pssc,psvsc),(psu,psvu),(psst,psvst), #convert the workers into items
                        (pgvsc,sc),(psvsc,sc),(pgvu,u),(psvu,u),(pgvst,st),(psvst,st), #more production capacity to each factory
                        (sc,isc),(u,iu),(st,ist), #create the products
                        (sc, scot), (u, uot), (st,stot), #overtime hours making products
                        (scot, isc), (uot,iu), (stot, ist), #overtime products created go to inventory for free
                        (isc,tg), (isc,tfb), (iu, tg), (iu,tfb), (ist,tg), (ist,tfb), #move the product from inventory to customer
                        ;

param: b:=
        g 200
```

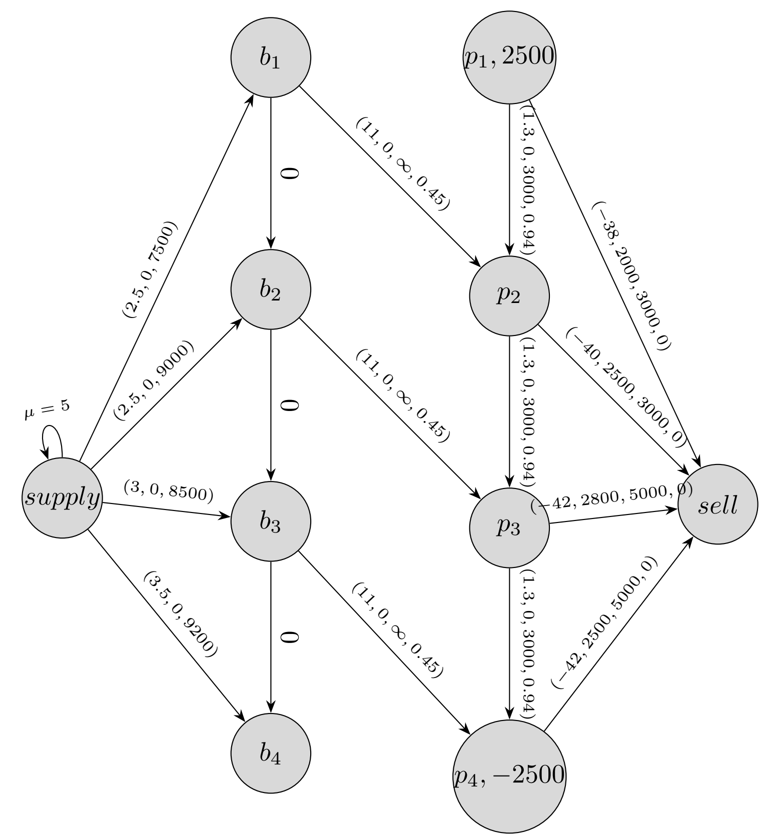
```
        s 100
        tg -1000
        tfb -600;

param:        c  l u mu:=
                [s,vs]                        2000         .         100         . #recruit workers
                [g,vg]                        1700         .         200         .
                [vg,pgsc]                300          .         .                  . #move workers to factories
                [vg,pgu]                250          .         .                  .
                [vg,pgst]               275          .         .                  .
                [vs,pssc]                300          .         .                  .
                [vs,psu]                250          .         .                  .
                [vs,psst]                275          .         .                  .
                [pgsc,pgvsc]        .                  .         .                  10 #convert workers to items
                [pgu,pgvu]                .                  .         .                  10
                [pgst,pgvst]        .                  .         .                  10
                [pssc,psvsc]        .                  .         .                  12
                [psu,psvu]                .                  .         .                  12
                [psst,psvst]        .                  .         .                  12
                [sc,isc]                90                  .         505         . #create the products
                [u,iu]                        105         .         465         .
                [st, ist]                115         .         570         .
                [sc, scot]                135         .         100         . #overtime possible
                [u,uot]                157.5         .         100         .
                [st,stot]                172.5         .         100         .
                [isc, tg]                8                  .         .                  . #move product to customer
                [isc, tfb]                15                  .         .                  . ################################check me
                [iu, tg]                14                  .         .                  .
                [iu, tfb]                18                  .         .                  .
                [ist, tg]                24                  .         .                  .
                [ist,tfb]                20                  .         .                  .
                [wp,s]                        .                  .         .                  0
                [wp,g]                        .                  .         .                  0
;
```

Here is my output:

```
Console
AMPL
ampl: model group1_HW3_p4.mod
CPLEX 20.1.0.0: optimal solution; objective 497016.6667
10 dual simplex iterations (0 in phase I)
x [*,*] (tr)
:         g    isc   ist    iu  pgsc pgst  pgu pgvsc pgvst  pgvu     pssc     psst :=
pgvsc     .     .     .     .     0    .    .     .     .     .        .        .
pgvst     .     .     .     .     .    0    .     .     .     .        .        .
pgvu      .     .     .     .     .    .   40     .     .     .        .        .
psvsc     .     .     .     .     .    .    .     .     .     .  47.0833        .
psvst     .     .     .     .     .    .    .     .     .     .        .     47.5
sc        .     .     .     .     .    .    .     0     .     .        .        .
st        .     .     .     .     .    .    .     .     0     .        .        .
tfb       .     0   570    30     .    .    .     .     .     .        .        .
tg        .   565     0   435     .    .    .     .     .     .        .        .
u         .     .     .     .     .    .    .     .     .   400        .        .
vg       40     .     .     .     .    .    .     .     .     .        .        .
wp      160     .     .     .     .    .    .     .     .     .        .        .

:         psu    psvsc psvst psvu    s    sc  scot   st  stot    u  uot   vg   :=
isc        .        .     .     .    .   505   60    .     .     .    .    .
ist        .        .     .     .    .     .    .  570     0     .    .    .
iu         .        .     .     .    .     .    .    .     .   465    0    .
pgsc       .        .     .     .    .     .    .    .     .     .    .    0
pgst       .        .     .     .    .     .    .    .     .     .    .    0
pgu        .        .     .     .    .     .    .    .     .     .    .   40
psvu   5.41667      .     .     .    .     .    .    .     .     .    .    .
sc         .      565     .     .    .     .    .    .     .     .    .    .
scot       .        .     .     .    .    60    .    .     .     .    .    .
st         .        .   570     .    .     .    .    .     .     .    .    .
stot       .        .     .     .    .     .    .    0     .     .    .    .
u          .        .     .    65    .     .    .    .     .     .    .    .
uot        .        .     .     .    .     .    .    .     .     0    .    .
vs         .        .     .     .  100     .    .    .     .     .    .    .
wp         .        .     .     .    0     .    .    .     .     .    .    .

:         vs       wp     :=
g          .        0
pssc   47.0833      .
psst   47.5         .
psu     5.41667     .
s          .      160
;
```

Solution is non-integer in employees which is unfortunate but can be the case with the generalized network flow problems. All 100 specialists are hired, 40 generalists. Scranton does use 60 products of OT, none of the other plants do but they max out production at each.

5. Mud b Gone



Here is our model file:

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.
reset;

options solver cplex;
option cplex_options 'sensitivity';

set NODES;                        # nodes in the network
set ARCS within {NODES, NODES};   # arcs in the network

param b {NODES} default 0;        # supply/demand for node i
param c {ARCS}  default 0;        # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;        # lower bound on flow on arc(i,j)
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)
param mu {ARCS} default 1;       # multiplier on arc(i,j) -- if one unit leaves i, mu[i,j] units arrive

var x {ARCS};                     # flow on arc (i,j)

minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost

# Flow Out(i) - Flow In(i) = b(i)

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} mu[j,i] * x[j,i] = b[i];
```

```
subject to upcapacity {(i,j) in ARCS}: x[i,j] <= u[i,j];

subject to lowcapacity {(i,j) in ARCS}: l[i,j] <= x[i,j];

data group1_HW3_p5.dat;

solve;

display x;

display upcapacity,upcapacity.up, upcapacity.down;

display x.current, x.up, x.down;
```

Here is our data file:

```
#MCNFP Problem - data file for problem instance
#Charles Nicholson, ISE 5113, 2015

#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
#       default arc upper bounds are infinity
#       default node requirements are 0


set NODES :=  supply, #suppliers
                         b1,b2,b3,b4, #base for production
                         p1, p2, p3, p4, #product
                         sold #sold product
;

set ARCS := (supply,*) b1 b2 b3 b4 #base purchased from supplier
                       (*,sold)   p1 p2 p3 p4 #product sold
                       (b1,p2),(b2,p3),(b3,p4), #base converted to product
                       (b1,b2),(b2,b3),(b3,b4),
                       (p1,p2),(p2,p3),(p3,p4)
                       (supply,supply)
                       ;

param: b:=
        p1 2500
        p4 -2500;

param:        c  l u mu:=
                [supply,b1] 2.5        .        7500        . #buy new base
                [supply,b2] 2.5 .         9000        .
                [supply, b3] 3 .         8500        .
                [supply,b4]        3.5        .        9200        .

                [p1,sold] -38 2000        3000        0 #sell product
                [p2,sold] -40 2500        3000        0
                [p3,sold] -42 2800        5000        0
                [p4,sold] -42 2500        5000        0

                [p1,p2] 1.3        .        3000        .94 #store product till next month
                [p2,p3] 1.3        .        3000        .94
                [p3,p4] 1.3        .        3000        .94

                [b1,p2] 11        .        .        .45 #convert base into product.  Assumption not too worry about max storage
                [b2,p3] 11        .        .        .45
                [b3,p4] 11        .        .        .45

                [supply,supply] .        .        .        5
;
```

(a) We see a solution for our flow. We sell 2000, 2500, 4220 and 2500 in each of the respective periods. We remark on the infinite supply house obtained by creating a loop with a μ factor set to 5 but could be any value greater than 1.

```
Console
AMPL
ampl: model group1_HW3_p5.mod
CPLEX 20.1.0.0: sensitivity
CPLEX 20.1.0.0: optimal solution; objective -115840
4 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
x :=
b1      b2         2988.89
b1      p2         4511.11
b2      b3         2611.11
b2      p3         9377.78
b3      b4            0
b3      p4        11111.1
p1      p2          500
p1      sold       2000
p2      p3            0
p2      sold       2500
p3      p4            0
p3      sold       4220
p4      sold       2500
supply b1          7500
supply b2          9000
supply b3          8500
supply b4             0
supply supply      6250
;
```

(b) We next look at sensitivity of the capacity. We see a value of -\$5.40 in the report, supply to b2. We interpret this as having additional gallons of base will increase (recall minimizing) our income by \$5.40. Both the up and the down are reported as 0. We are unsure of how to interpret this as we clearly would use more of the base if it was available but are unsure why these values are all zero. We even broke apart the upper and lower limit thinking this was the issue but did not change the up and down on the shadow price. We do clearly see this as the bottle neck in our model.

```
:              upcapacity upcapacity.up upcapacity.down    :=
b1      b2          0            0              0
b1      p2          0            0              0
b2      b3          0            0              0
b2      p3          0            0              0
b3      b4          0            0              0
b3      p4          0            0              0
p1      p2          0            0              0
p1      sold        0            0              0
p2      p3          0            0              0
p2      sold        0            0              0
p3      p4          0            0              0
p3      sold        0            0              0
p4      sold        0            0              0
supply  b1         -5.4          0              0
supply  b2         -5.4          0              0
supply  b3         -4.9          0              0
supply  b4          0            0              0
supply  supply      0            0              0
;
```

(c) We see that the contribution to the cost is -$40 on the flow from p2 to Sold. This value can be modified up to -$42 and not change the model at all.

```
:              x.current       x.up            x.down         :=
b1      b2          0         0.0861702    -1e+20
b1      p2         11      1e+20              10.9138
b2      b3          0         1.82872          -3.19744e-15
b2      p3         11        11                 9.17128
b3      b4          0         0                 0
b3      p4         11        12.8287           11
p1      p2          1.3       1.48          -1e+20
p1      sold      -38      1e+20              -38.18
p2      p3          1.3    1e+20               -2.52
p2      sold      -40      1e+20              -42
p3      p4          1.3    1e+20               -2.52
p3      sold      -42        -42            -1e+20
p4      sold      -42      1e+20              -42
supply  b1          2.5       7.9           -1e+20
supply  b2          2.5       7.9           -1e+20
supply  b3          3         7.9           -1e+20
supply  b4          0         0                 0
supply  supply      0        19.6           -1e+20
;
```