Numerical Linear Algebra Program Assignment 3a Report

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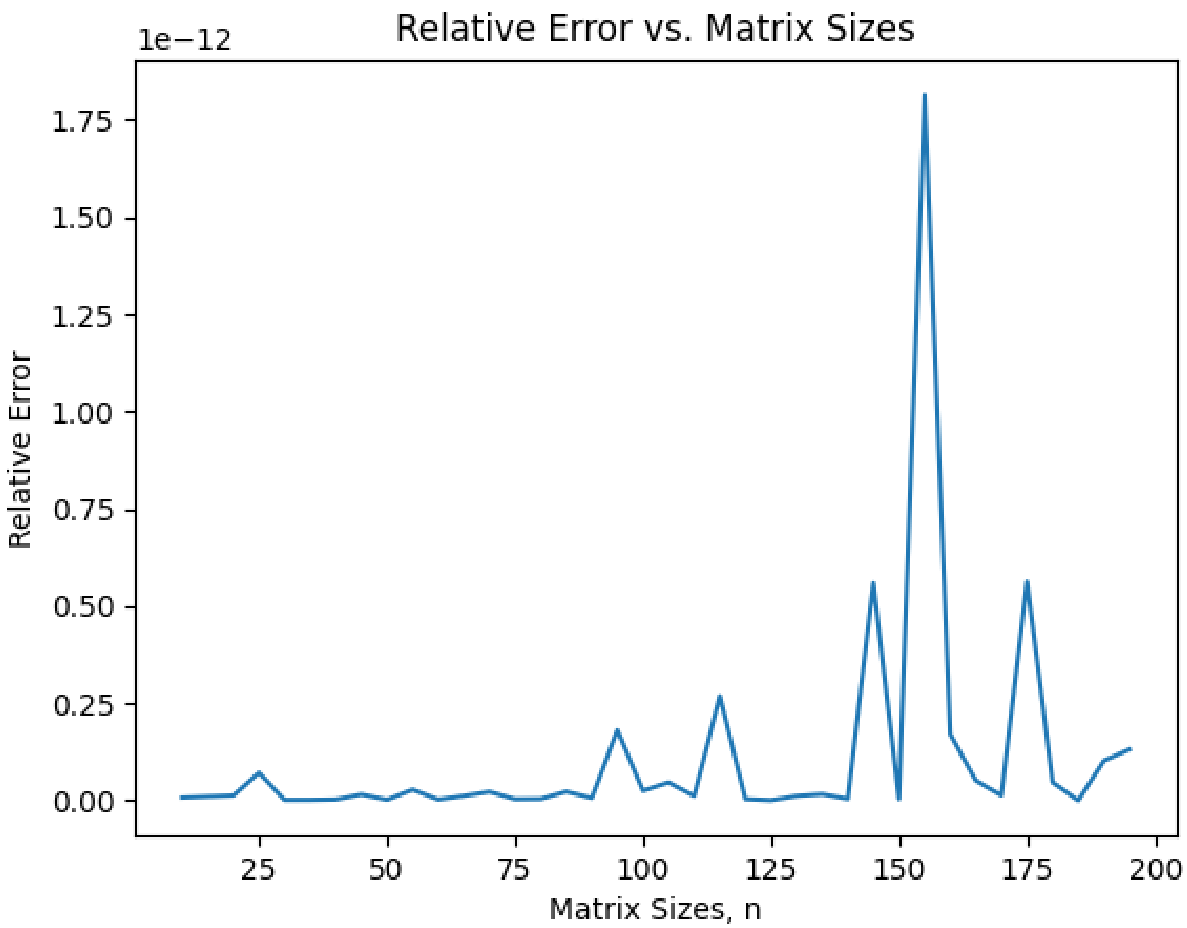
Due: March 29, 2024 @ 11:59PM

Task 1

Algorithm 1

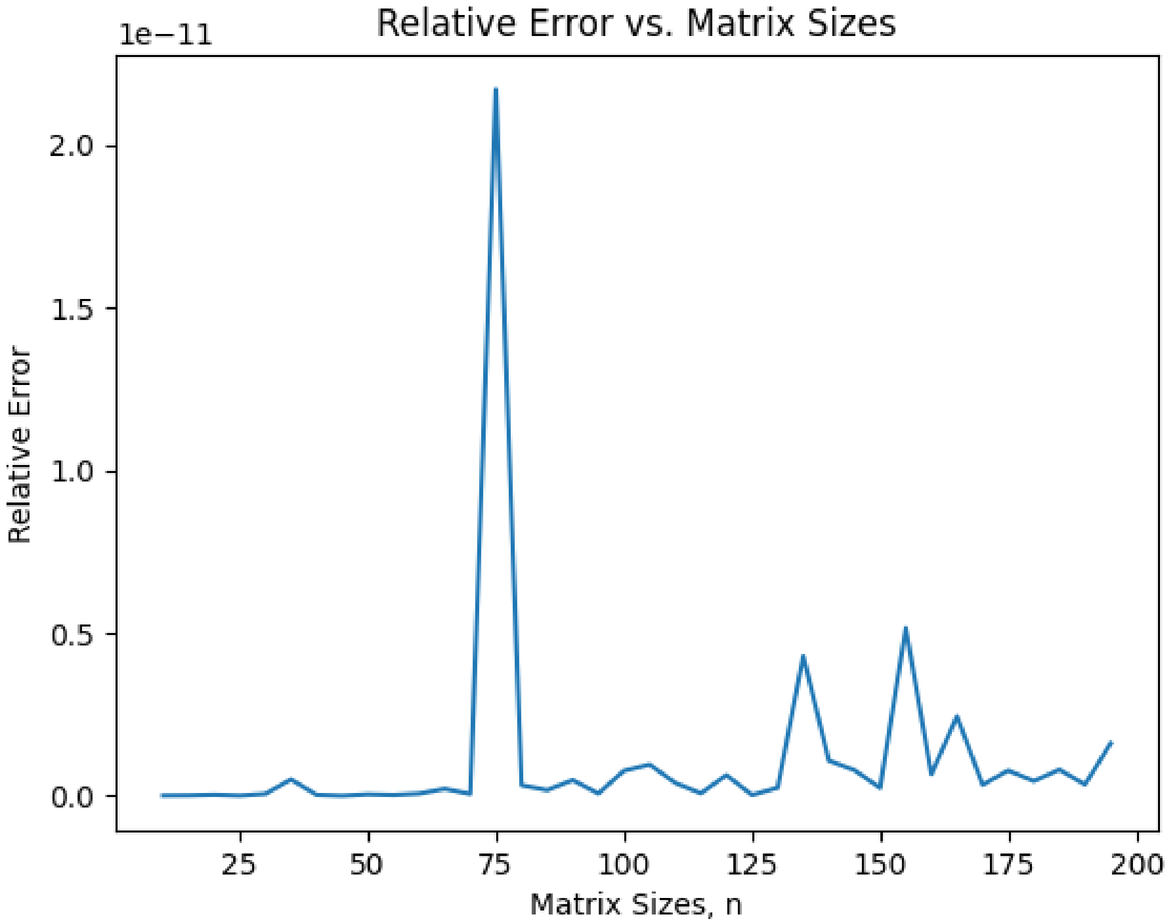
The purpose of Algorithm 1 is to solve the linear least squares using household reflectors to convert a matrix A to form R (an upper right triangular matrix). Upon doing this, we can multiply the same household reflectors against a vector b in order to find the solution to our problem.

We begin by testing our algorithm on square nonsingular matrices with sizes ranging from 10 to 200 showing relative error vs. matrix size, n. The relative error is compared against the np.linalg.lstsq method as "true x".



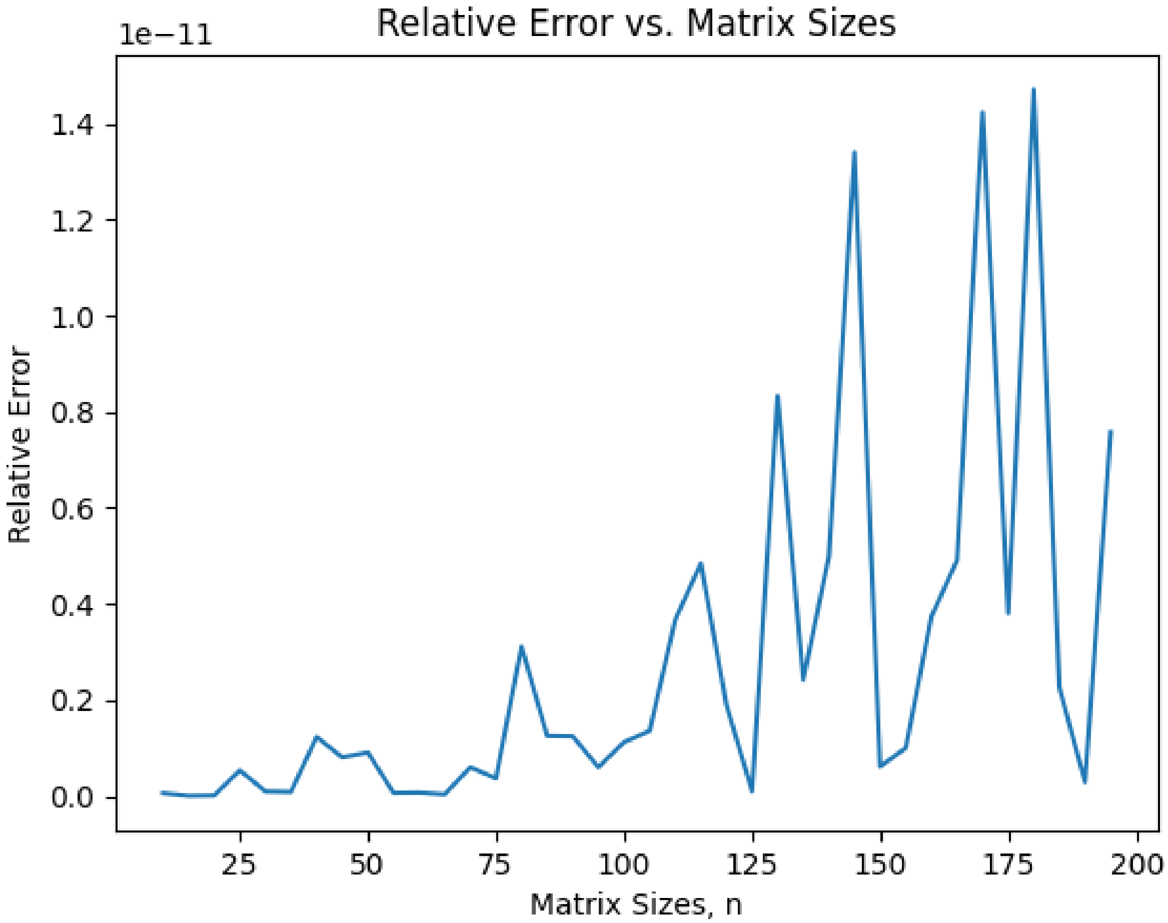
We see that as our matrix size increases, we start to get more and more unstable errors, which is expected.

We now test our algorithm on rectangular full column rank matrices A such that $n > k$ and $b \in \mathcal{R}(A)$.



We see that we have a few spikes, but for the most part our errors are low.

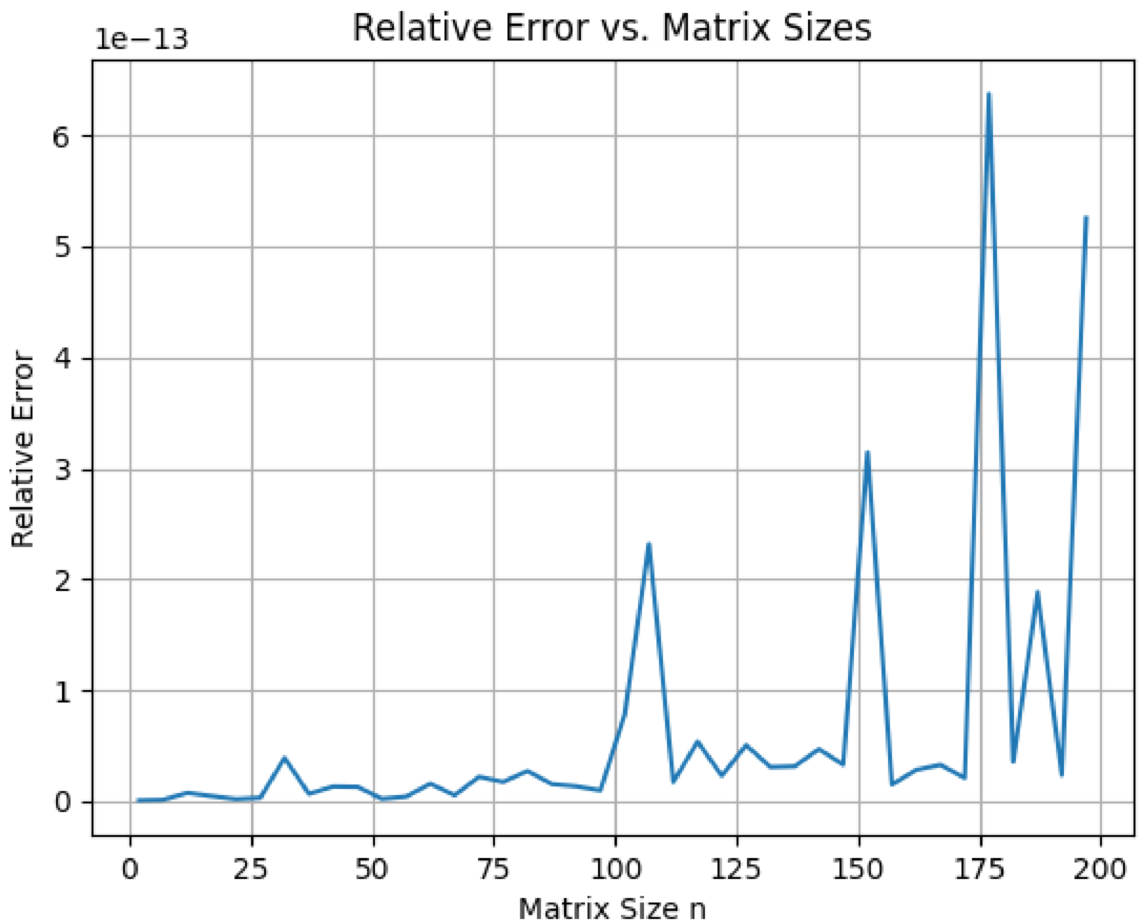
Finally, our last test is on rectangular full column rank matricies A such that $n > k$ and $b \notin \mathcal{R}(A)$ and $b = b_1 + b_2$ where $b_1 \in \mathcal{R}(A)$ and $b_2 \notin \mathcal{R}(A)$ that define a linear least squares problem with a nonzero residual $r_{\min} = b_2 = b - Ax_{\min}$.



We see that our results are similar to those from test 1, with the larger matrix sizes resulting in larger relative errors.

Algorithm 2

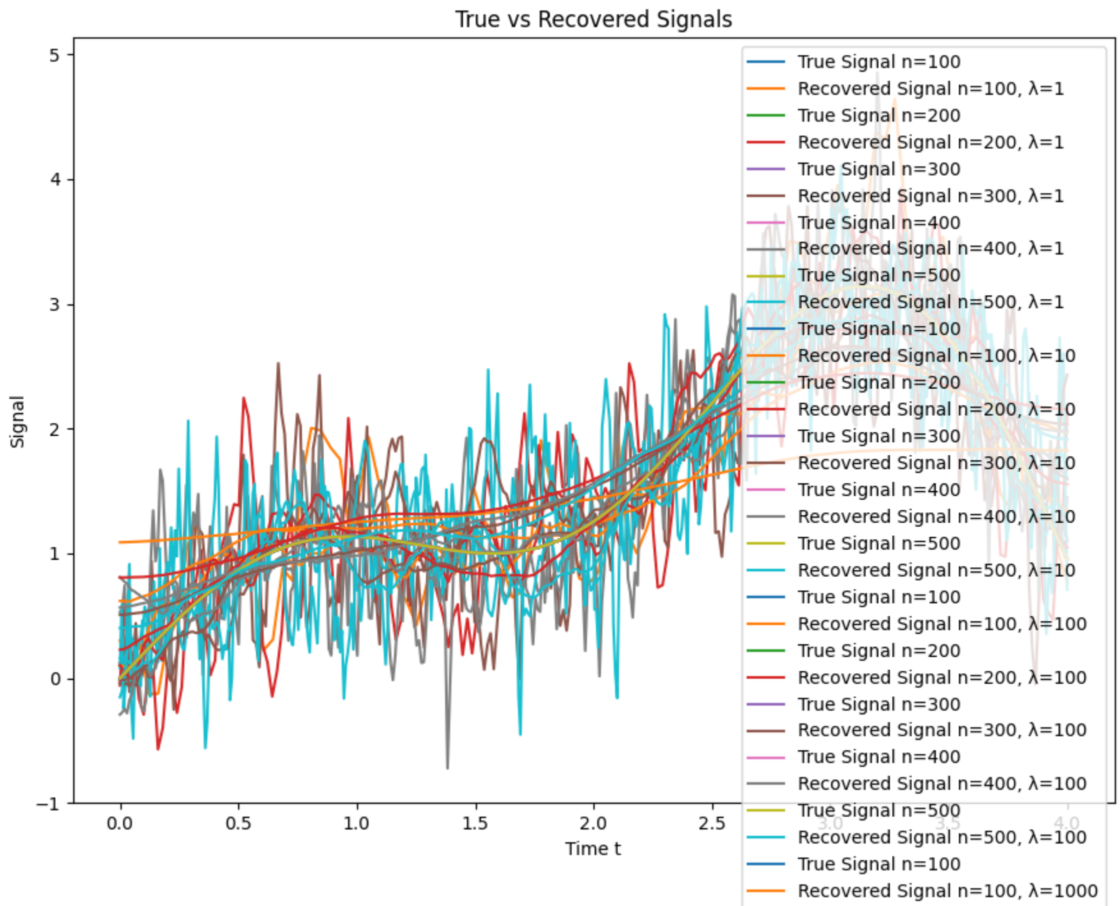
Our second algorithm is performing the incremental least squares approach to find x_{min}. We generate random matrices of size n=2 to n=200 and plot the relative errors errors below.



We see that similar to our other algorithm, our errors are starting to get unstable as the size of the matrix increases.

Task 2

Our second task is to consider the regularizes linear least squares described in the program assignment details. Below is the sinusoid showing all combinations of different λ and n values.



True vs Recovered Signals
Signal
Time t
True Signal n=100
Recovered Signal n=100, λ=1
True Signal n=200
Recovered Signal n=200, λ=1
True Signal n=300
Recovered Signal n=300, λ=1
True Signal n=400
Recovered Signal n=400, λ=1
True Signal n=500
Recovered Signal n=500, λ=1
True Signal n=100
Recovered Signal n=100, λ=10
True Signal n=200
Recovered Signal n=200, λ=10
True Signal n=300
Recovered Signal n=300, λ=10
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Recovered Signal n=500, λ=10
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Recovered Signal n=100, λ=100
True Signal n=200
Recovered Signal n=200, λ=100
True Signal n=300
Recovered Signal n=300, λ=100
True Signal n=400
Recovered Signal n=400, λ=100
True Signal n=500
Recovered Signal n=500, λ=100
True Signal n=100
Recovered Signal n=100, λ=1000