

Populating Occupancy Grids from LIDAR Scan Data

Course 4, Module 2, Lesson 2 – Part 1



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Learning Objectives

- Issue with the Bayesian Probability Update
- Present a solution utilizing log odds
- Bayesian log odds update derivation

Bayesian Update Of The Occupancy Grid - Summary

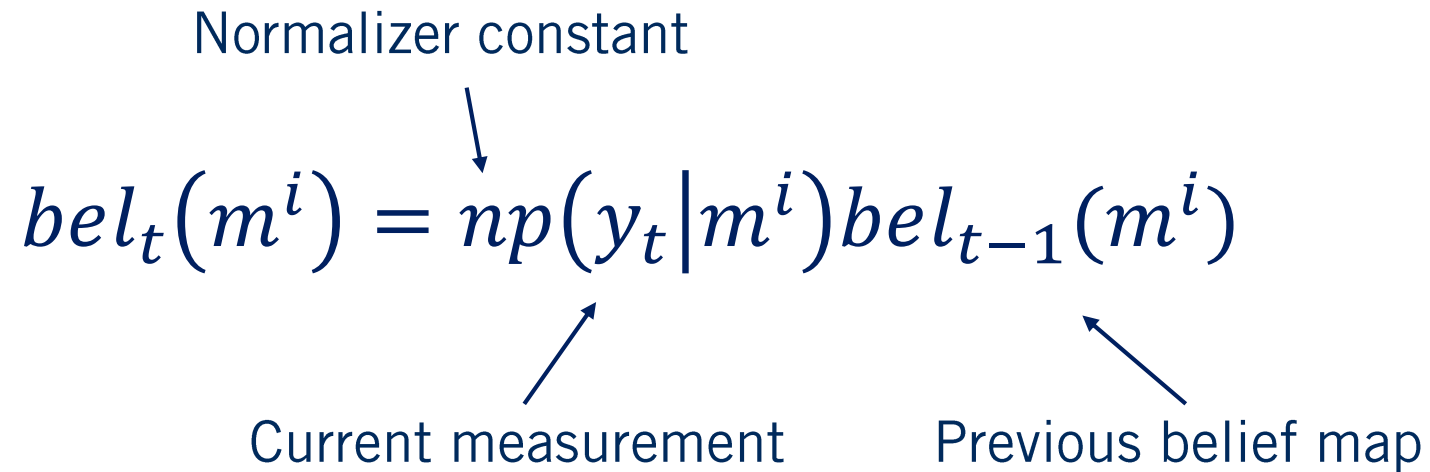
- Bayes' theorem is applied at each update step for each cell

Normalizer constant

$$bel_t(m^i) = np(y_t|m^i)bel_{t-1}(m^i)$$

Current measurement

Previous belief map

The diagram illustrates the Bayesian update equation for an occupancy grid. It features the equation $bel_t(m^i) = np(y_t|m^i)bel_{t-1}(m^i)$ in a large, dark blue font. Three labels are positioned around the equation with arrows pointing to specific parts: 'Normalizer constant' points to the np term, 'Current measurement' points to the y_t term, and 'Previous belief map' points to the bel_{t-1} term.

- There's a problem!

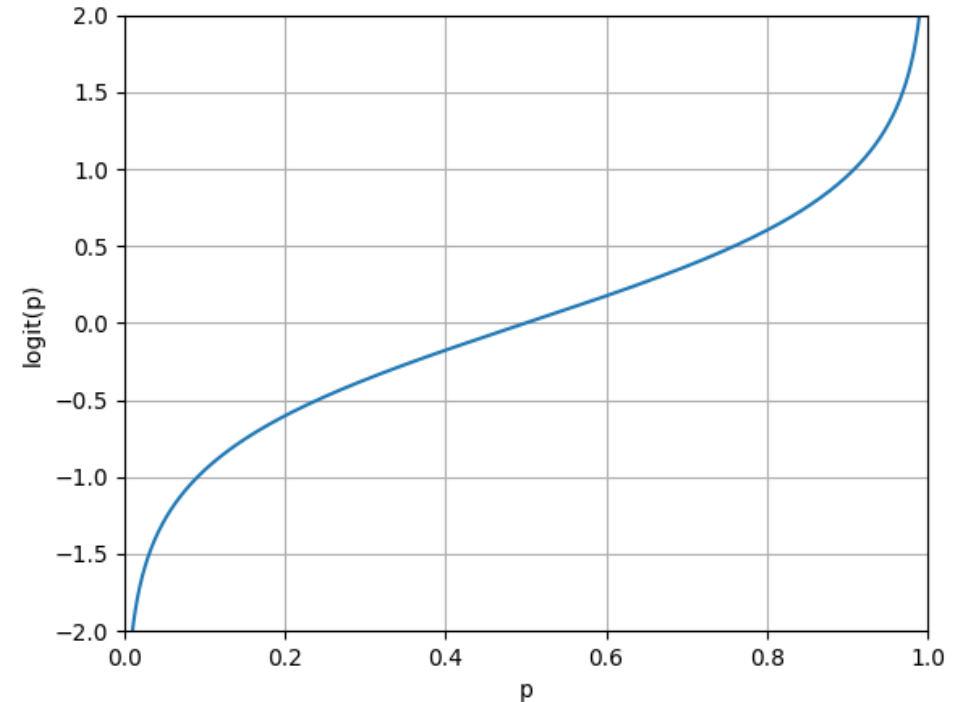
Issue With Standard Bayesian Update

- Update a single unoccupied grid cell

$$\underset{0.000000008}{\underset{|}{bel_t(m)}} = np(y_t|m) \underset{0.000012}{\underset{|}{bel_{t-1}(m)}} \underset{0.000638}{\searrow}$$

- Multiplication of numbers close to zero is hard for computers
- Store the log odds ratio rather than probability

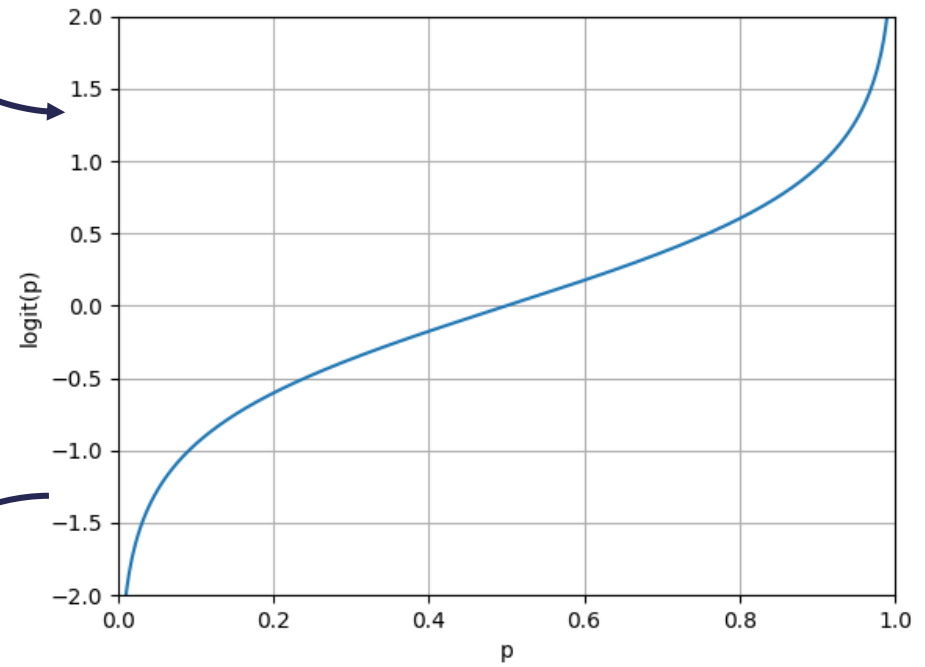
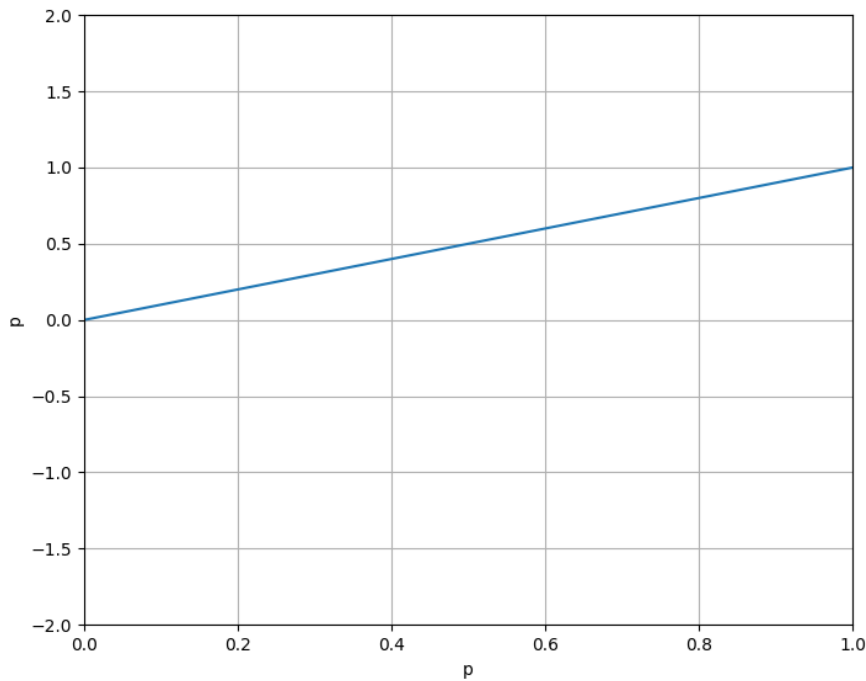
$$bel_t(m) \rightarrow (-\infty, \infty)$$



$$\log\left(\frac{p}{1-p}\right)$$

Conversion

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$



$$p = \frac{e^{\text{logit}(p)}}{1 + e^{\text{logit}(p)}}$$

Bayesian Log Odds Single Cell Update Derivation

- Applying Bayes' rule:

$$p(m^i | y_{1:t}) = \frac{p(y_t | y_{1:t-1}, m^i) p(m^i | y_{1:t-1})}{p(y_t | y_{1:t-1})}$$

Current map cell

Sensor measurement for given cell

Pulling out current measurement y_t from past measurements $y_{1:t-1}$

- Applying the Markov assumption:

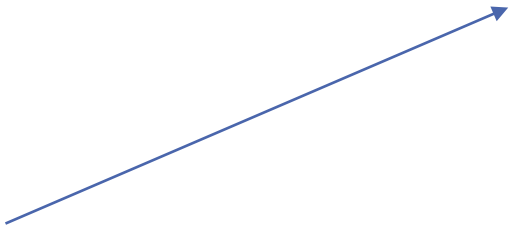
$$p(m^i | y_{1:t}) = \frac{p(y_t | m^i) p(m^i | y_{1:t-1})}{p(y_t | y_{1:t-1})}$$

Pulling out current measurement y_t from past measurements $y_{1:t-1}$

Bayesian Log Odds Single Cell Update Derivation

$$p(m^i|y_{1:t}) = \frac{p(y_t|m^i)p(m^i|y_{1:t-1})}{p(y_t|y_{1:t-1})}$$

- Applying Bayes' rule to measurement model: $\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$

$$p(y_t|m^i) = \frac{p(m^i|y_t)p(y_t)}{p(m^i)}$$


- Yields:

$$p(m^i|y_{1:t}) = \frac{p(m^i|y_t)p(y_t)p(m^i|y_{1:t-1})}{p(m^i)p(y_t|y_{1:t-1})}$$

Bayesian Log Odds Single Cell Update Derivation

- Denominator: $1 - p$

$$p(\neg m^i | y_{1:t}) = 1 - p(m^i | y_{1:t}) = \frac{p(\neg m^i | y_t) p(y_t) p(m^i | y_{1:t-1})}{p(\neg m^i) p(y_t | y_{1:t-1})}$$

- Logit function

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right) \quad \frac{p(m^i | y_{1:t})}{p(\neg m^i | y_{1:t})} = \frac{\frac{p(m^i | y_t) p(y_t) p(m^i | y_{1:t-1})}{p(m^i) p(y_t | y_{1:t-1})}}{\frac{p(\neg m^i | y_t) p(y_t) p(m^i | y_{1:t-1})}{p(\neg m^i) p(y_t | y_{1:t-1})}}$$

Bayesian Log Odds Single Cell Update Derivation

- Simplifying like terms results in:

$$\frac{p(m^i|y_{1:t})}{p(\neg m^i|y_{1:t})} = \frac{p(m^i|y_t)p(\neg m^i)p(m^i|y_{1:t-1})}{p(\neg m^i|y_t)p(m^i)p(\neg m^i|y_{1:t-1})}$$

- Can rewrite by taking $\neg p$ to $1 - p$:

$$\frac{p(m^i|y_{1:t})}{p(\neg m^i|y_{1:t})} = \frac{p(m^i|y_t)(1 - p(m^i))p(m^i|y_{1:t-1})}{(1 - p(m^i|y_t))p(m^i)(1 - p(m^i|y_{1:t-1}))}$$

- Finally, taking the log:

$$\text{logit}(p(m^i|y_{1:t})) = \text{logit}(p(m^i|y_t)) + \text{logit}(p(m^i|y_{1:t-1})) - \text{logit}(p(m^i))$$

$$l_{t,i} = \text{logit}(p(m^i|y_t)) + l_{t-1,i} - l_{0,i}$$

Bayesian log odds Update

Inverse Measurement Model

Previous belief

Initial belief



The diagram shows the equation $l_{t,i} = \text{logit}(p(m^i|y_t)) + l_{t-1,i} - l_{0,i}$. Three labels are positioned above the equation with arrows pointing to specific terms: 'Inverse Measurement Model' points to $\text{logit}(p(m^i|y_t))$, 'Previous belief' points to $l_{t-1,i}$, and 'Initial belief' points to $l_{0,i}$.

$$l_{t,i} = \text{logit}(p(m^i|y_t)) + l_{t-1,i} - l_{0,i}$$

- Numerically stable
- Computationally efficient

Summary

- Identified issue with the Bayesian probability update
- Presented a solution utilizing log odds
- Bayesian log odds update derivation