The Kinematic Bicycle Model

Course 1, Module 3, Lesson 2

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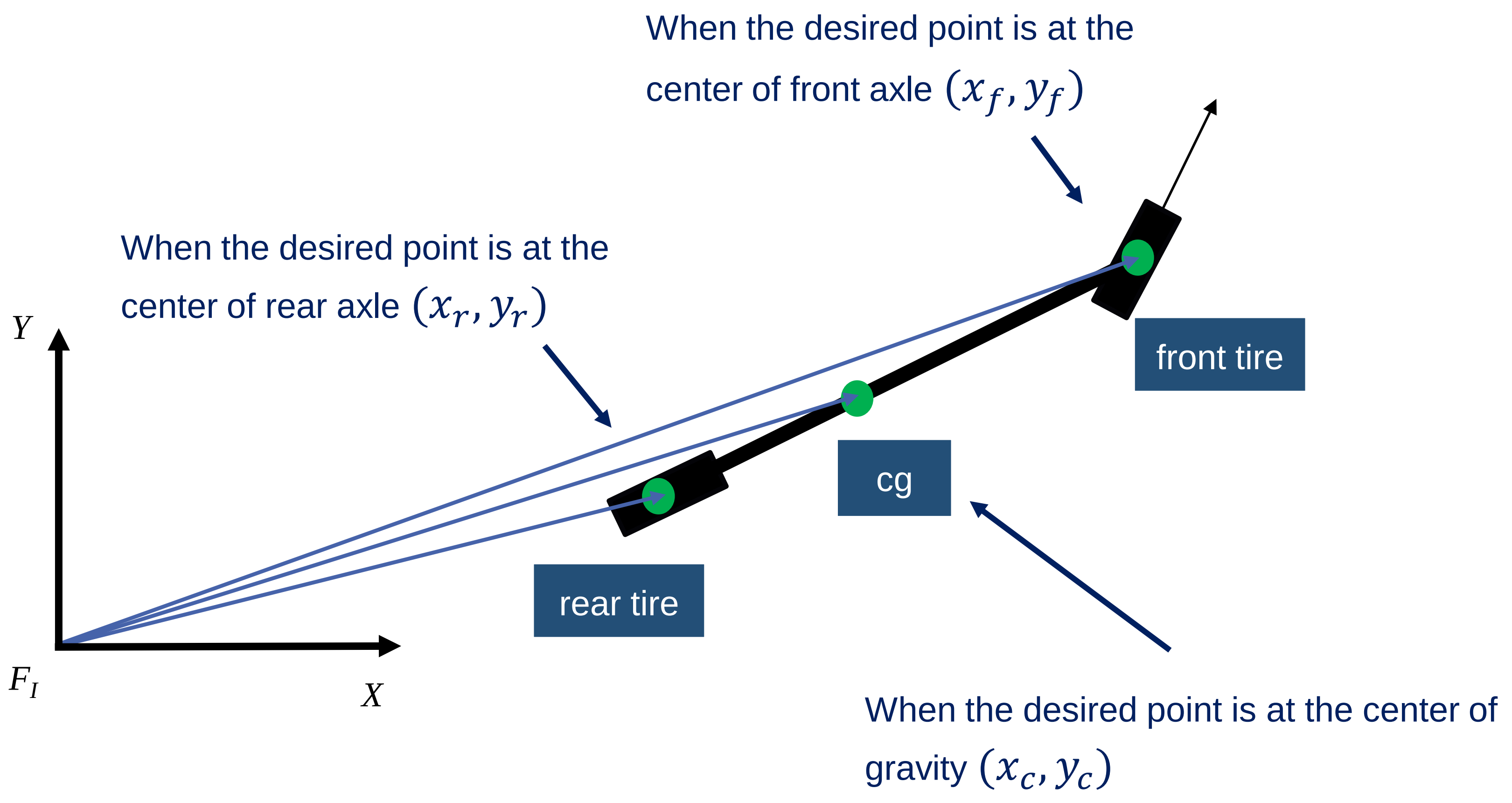
In this video…

- Learn about slip angle
- Develop the kinematic bicycle model

Bicycle Kinematic Model

- 2D bicycle model (simplified car model)
- Front wheel steering



Two-Wheeled Robot Kinematic Model

- Rear Wheel Reference Point
  - Apply Instantaneous Center of Rotation (ICR)

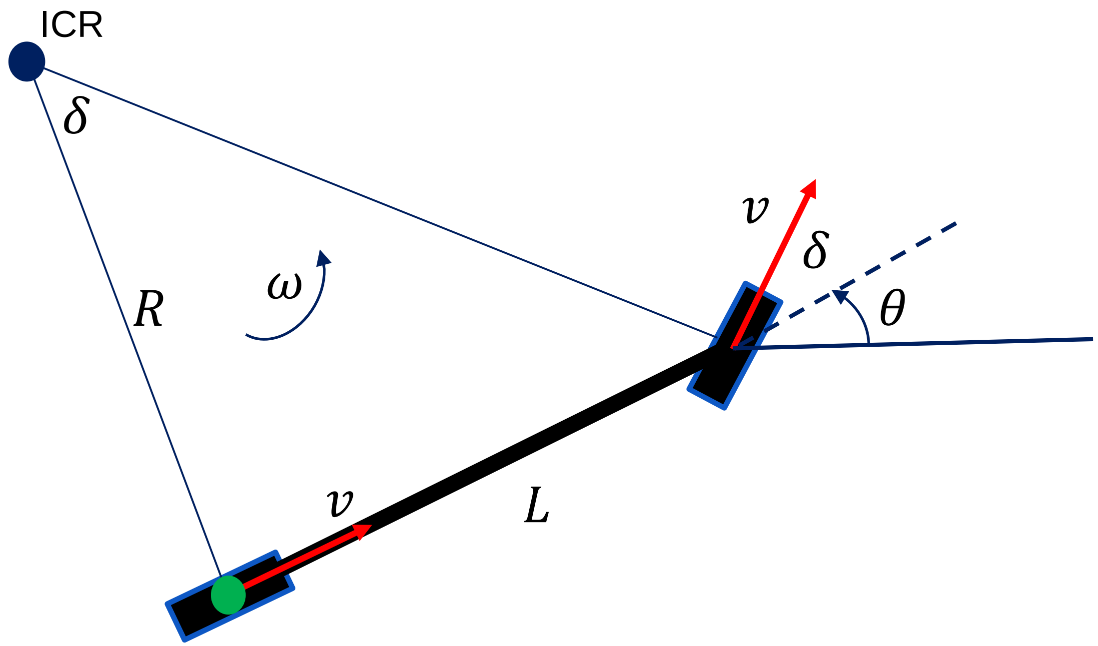
$$\dot{\theta} = \omega = \frac{v}{R}$$

  - Similar triangles

$$\tan\delta = \frac{L}{R}$$

  - Rotation rate equation

$$\dot{\theta} = \omega = \frac{v}{R} = \frac{v\tan\delta}{L}$$



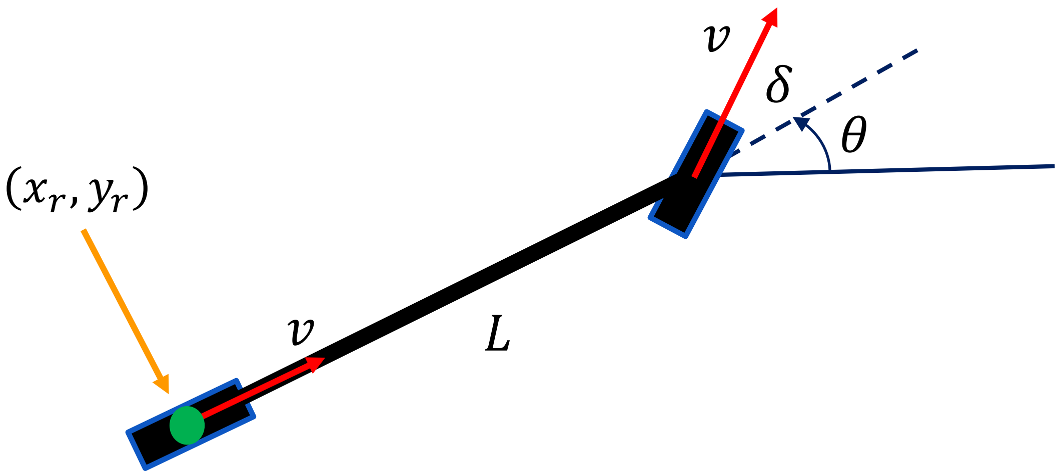
Rear Axle Bicycle Model

- If the desired point is at the center of the rear axle

$$\dot{x}_r = v\cos\theta$$

$$\dot{y}_r = v\sin\theta$$

$$\dot{\theta} = \frac{v\tan\delta}{L}$$



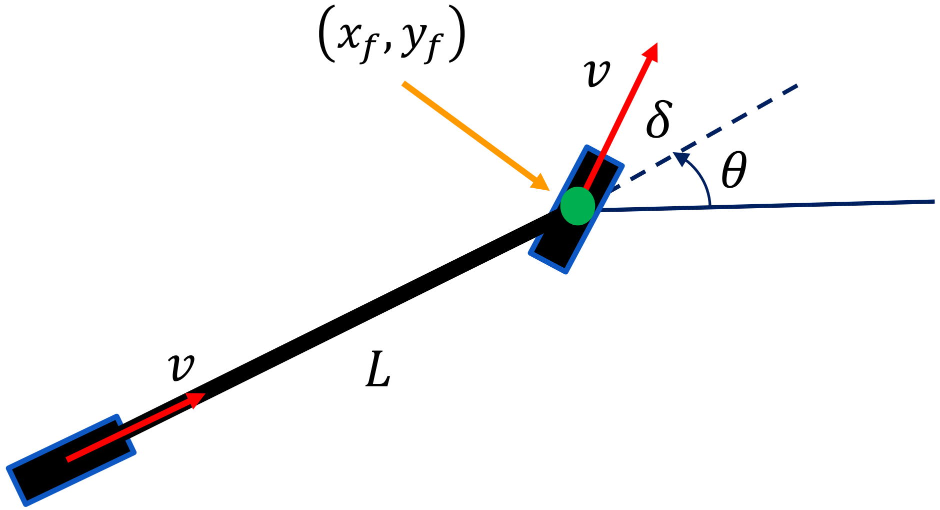
Bicycle Kinematic Model

- If the desired point is at the center of the front axle

$$\dot{x}_f = v\cos(\theta + \delta)$$

$$\dot{y}_f = v\sin(\theta + \delta)$$

$$\dot{\theta} = \frac{v\sin\delta}{L}$$



Bicycle Kinematic Model

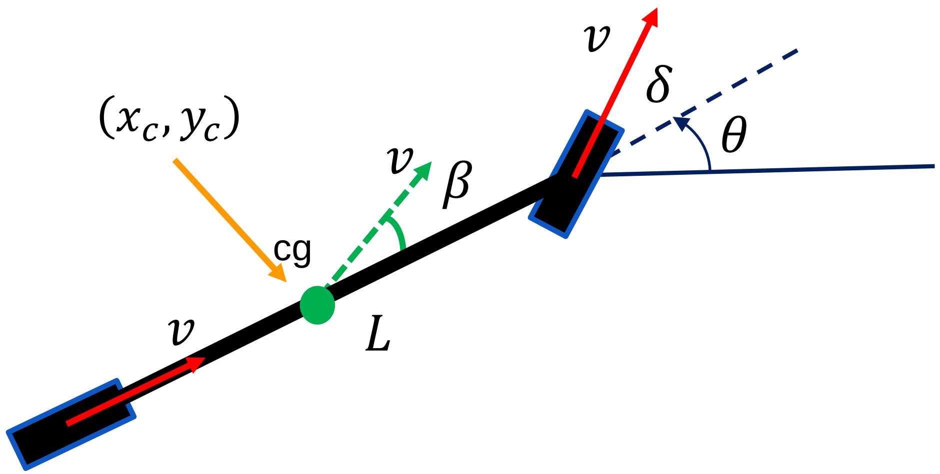
- If the desired point is at the center of the gravity (cg)

$$\dot{x}_c = v\cos(\theta + \beta)$$

$$\dot{y}_c = v\sin(\theta + \beta)$$

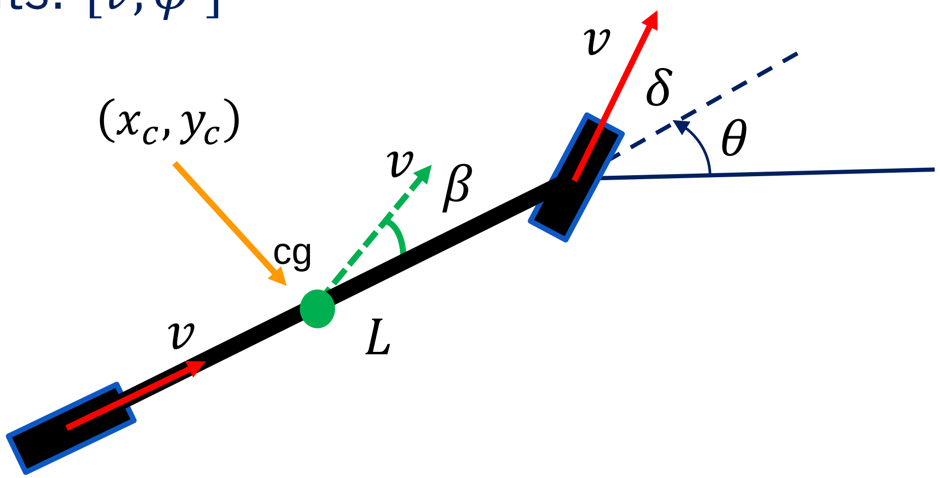
$$\dot{\theta} = \frac{v\cos\beta\tan\delta}{L}$$

$$\beta = \tan^{-1}\left(\frac{l_r\tan\delta}{L}\right)$$



State-space Representation

- Modify CG kinematic bicycle model to use steering rate input
  - State: $[x, y, \theta, \delta]^T$ Inputs: $[v, \varphi\ ]^T$



$$\dot{x}_c = v\cos(\theta + \beta)$$

$$\dot{y}_c = v\sin(\theta + \beta)$$

$$\dot{\theta} = \frac{v\cos\beta\tan\delta}{L}$$

$$\dot{\delta} = \varphi$$

Modified Input: rate of change of steering angle

Summary

- What we have learned from this lesson:
  - Formulation of a kinematic bicycle model
  - State-space representation.

- What is next?
  - We will explore the Dynamics in 2D and how to define a vehicle model as a 2D dynamic system.