Dynamic Windowing

Course 4, Module 6, Lesson 4

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

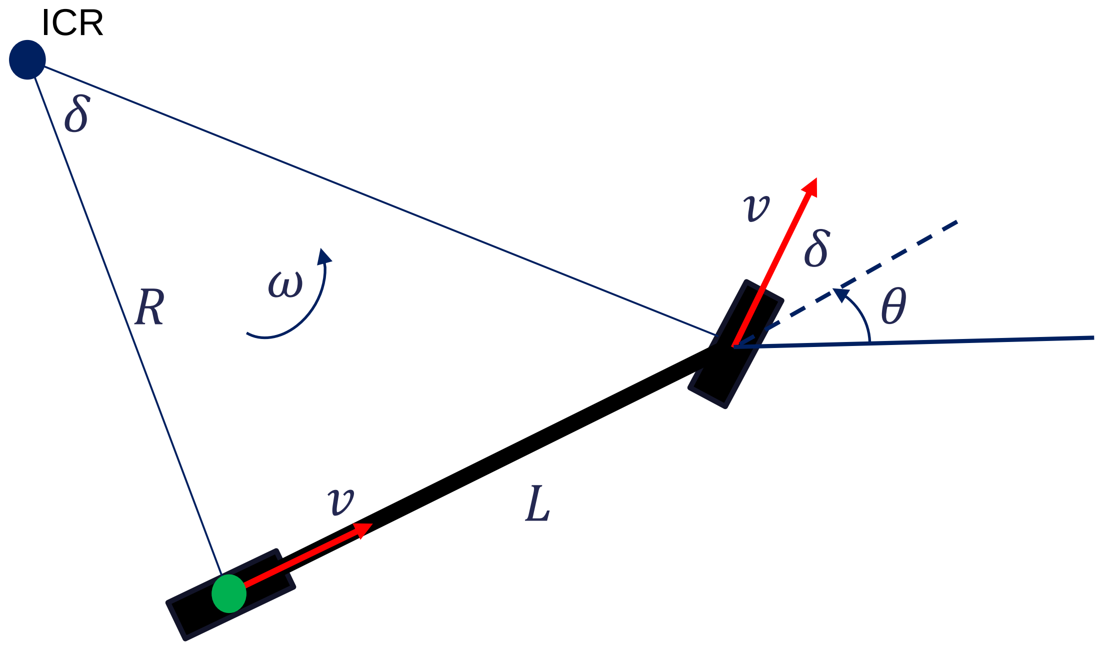
Learning Objectives

- Know how to add linear and angular acceleration constraints to the bicycle model
- Understand how these constraints impact our planner
- Handle these constraints in the planning process using dynamic windowing

Recall: Kinematic Bicycle Model

- Inputs are linear velocity and steering angle
- No consideration of higher-order terms

$$\dot{\theta} = \frac{v\tan(\delta)}{L}$$
$$\dot{x} = v\cos(\theta)$$
$$\dot{y} = v\sin(\theta)$$
$$\delta_{min} \leq \delta \leq \delta_{max}$$
$$v_{min} \leq v \leq v_{max}$$



Bicycle Model + Acceleration Constraints

- Higher order terms handled by adding constraints
- More comfort for passengers, but less maneuverability

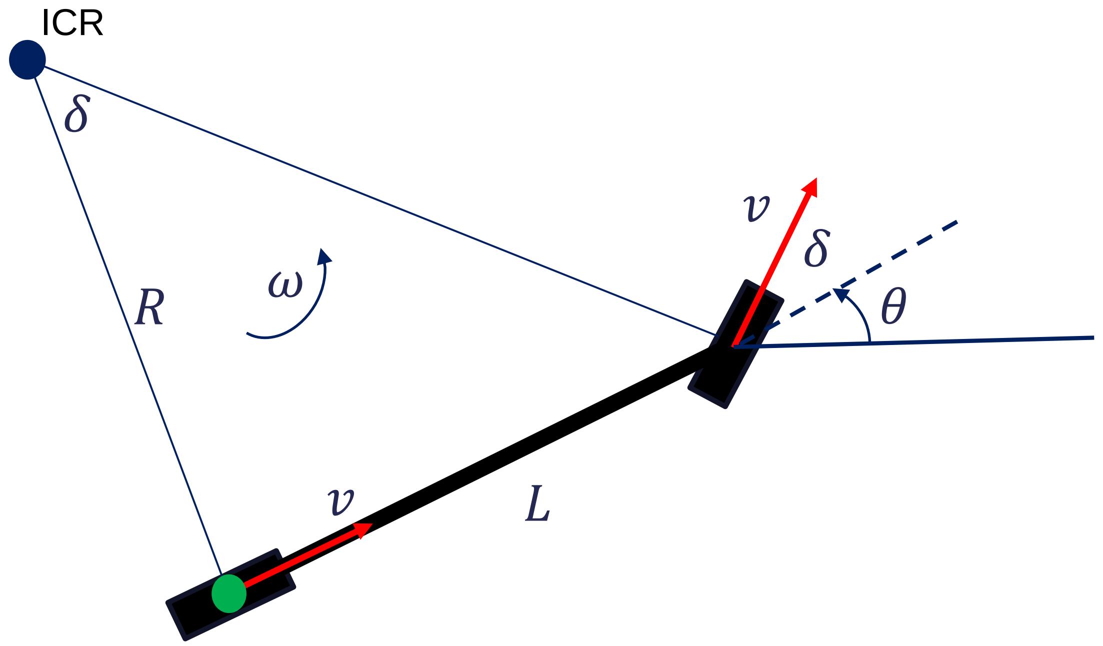
$$\dot{\theta} = \frac{v\tan(\delta)}{L}$$

$$\dot{x} = v\cos(\theta) \qquad \ddot{\theta}_{min} \leq \ddot{\theta} \leq \ddot{\theta}_{max}$$

$$\dot{y} = v\sin(\theta) \qquad \ddot{x}_{min} \leq \ddot{x} \leq \ddot{x}_{max}$$

$$\delta_{min} \leq \delta \leq \delta_{max}$$

$$v_{min} \leq v \leq v_{max}$$



Constraint in Terms of Steering Angle

- Angular acceleration constraint may prevent us from selecting certain maneuvers based on current angular velocity
- Change in steering angle between planning cycles is bounded
- Similar logic applies for changes in linear velocity inputs between planning cycles

$$\dot{\theta} = \frac{v \tan(\delta)}{L}$$

$$|\ddot{\theta}| = \left|\frac{\dot{\theta}_2 - \dot{\theta}_1}{\Delta t}\right|$$

$$\boxed{|\tan(\delta_2) - \tan(\delta_1)| \leq \frac{\ddot{\theta}_{max} L \Delta t}{v}}$$

Example

- Given current steering angle and the angular acceleration bound, which candidate trajectories do not violate that bound?

$v = 1\ m/s$

$\delta_1 = \frac{\pi}{8}$

$\delta_{min} = -\frac{\pi}{4}$

$\delta_{max} = \frac{\pi}{4}$

$\Delta t = 1\ s$

$L = 1\ m$

$|\ddot{\theta}| \leq 0.6\ s^{-2}$

Example

- Changing our steering angle to $-\frac{\pi}{8}$ or $-\frac{\pi}{4}$ violates our angular acceleration constraint

$$\boxed{|\tan(\delta_2) - \tan(\delta_1)| \le \frac{\ddot{\theta}_{max} L \Delta t}{v}}$$

$$|\tan(\pi/4) - \tan(\pi/8)| = 0.586 \le \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

$$|\tan(\pi/8) - \tan(\pi/8)| = 0 \le \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

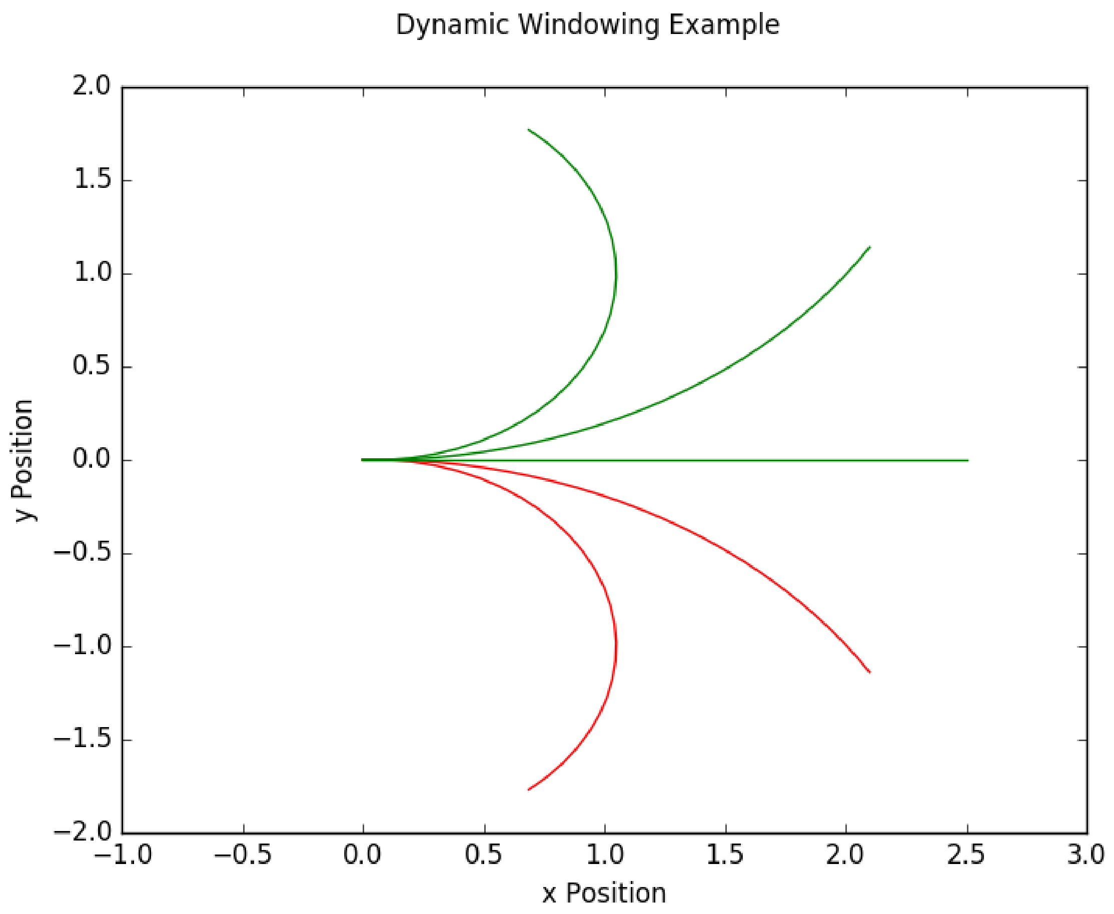
$$|\tan(0) - \tan(\pi/8)| = 0.414 \le \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

$$|\tan(-\pi/8) - \tan(\pi/8)| = 0.828 > \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

$$|\tan(-\pi/4) - \tan(\pi/8)| = 1.414 > \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

Comparing Trajectories

- Trajectories that exceed the angular acceleration constraint are coloured red
- Added constraints reduce manoeuvrability of the robot



Summary

- Introduced linear and angular acceleration constraints to our motion planning problem
- Discussed dynamic windowing and how it allows us to handle these new constraints in the trajectory rollout algorithm

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