Dijkstra's Shortest Path Search

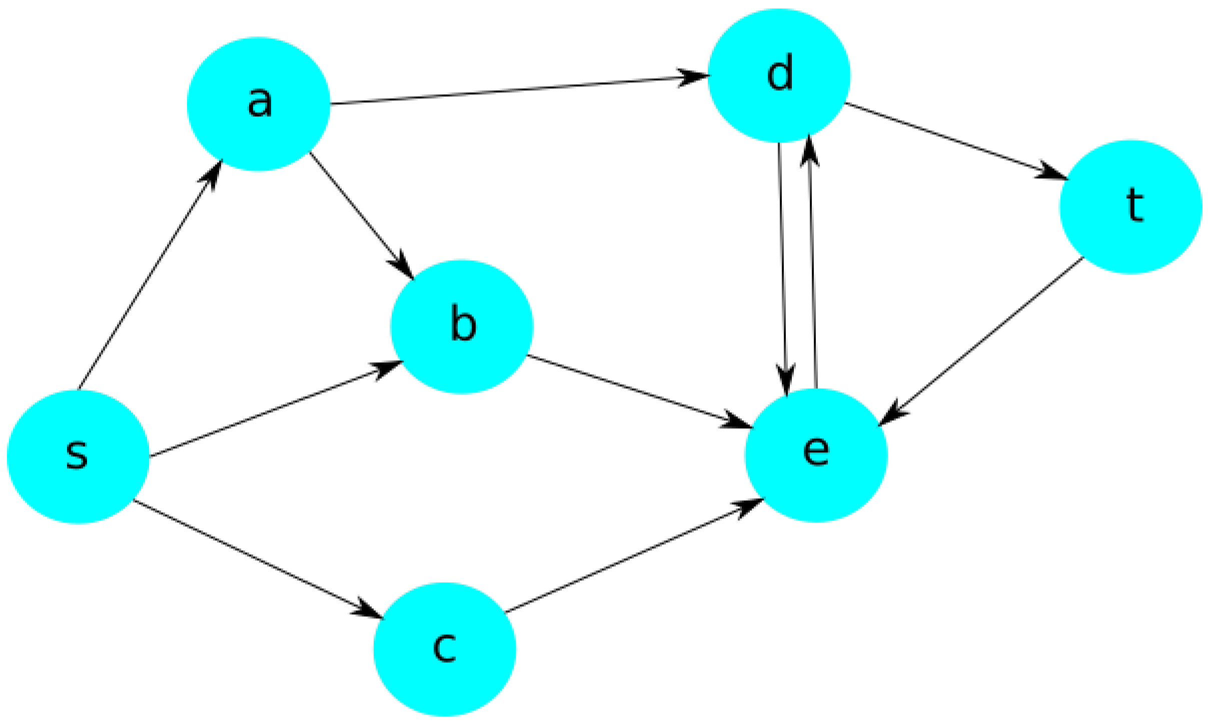
Course 4, Module 3, Lesson 2

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

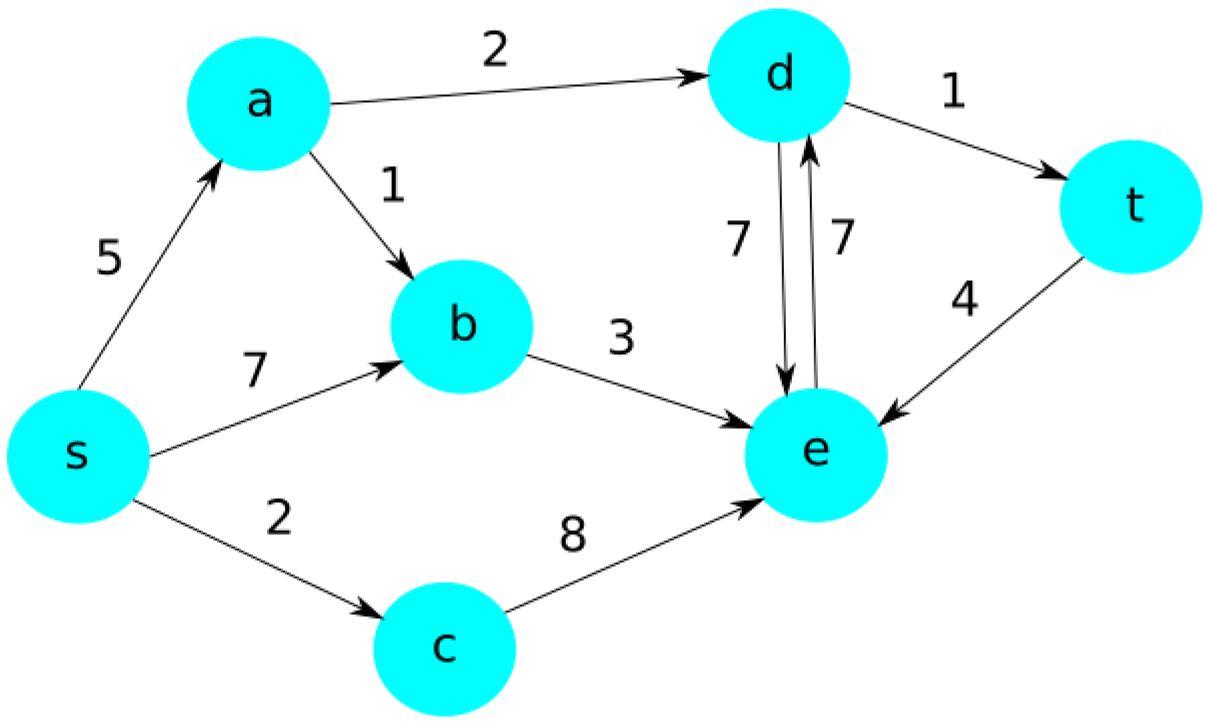
Learning Objectives

- Understand the difference between weighted and unweighted graphs
- Recognize the value of weighted graphs to the mission planning problem
- Be able to implement Dijkstra’s algorithm in a mission planning context to find the shortest path to a destination in a graph

Unweighted Graph



Weighted Graph



Dijkstra's Algorithm

Algorithm Dijkstra's(G,s,t)

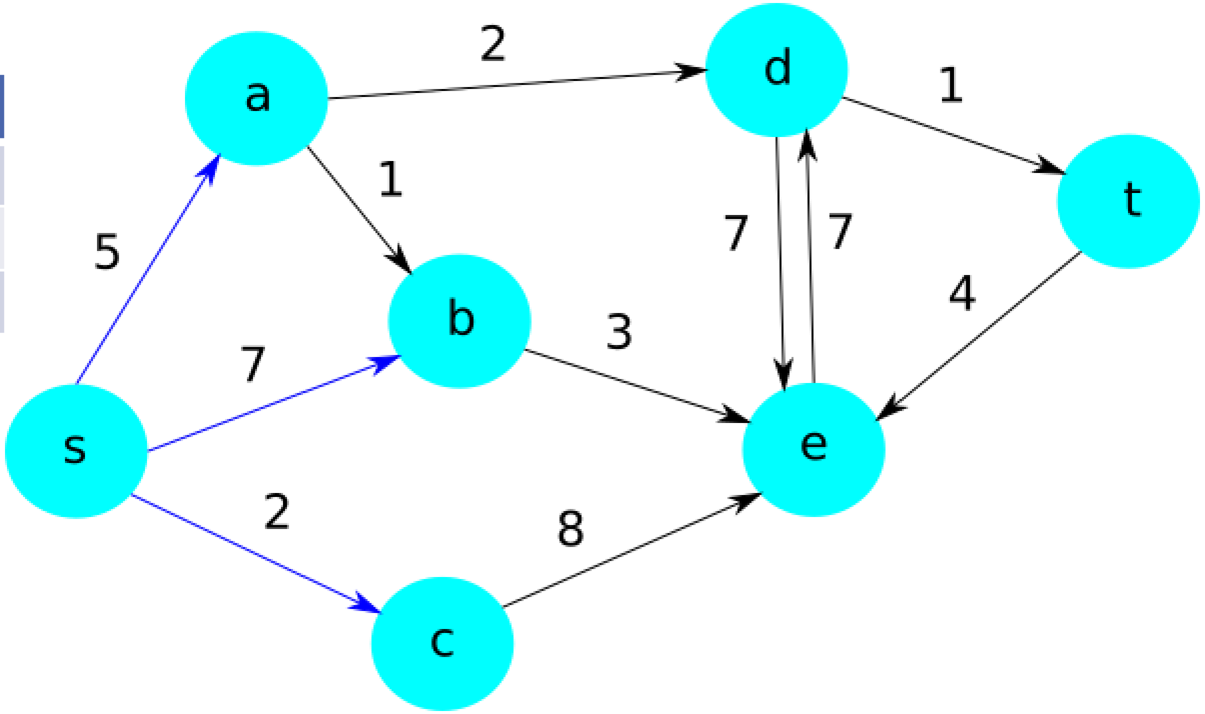
```
open ← MinHeap()
closed ← Set()
predecessors ← Dict()
open.push(s, 0)
while !open.isEmpty() do
  u, uCost ← open.pop()
  if isGoal(u) then
    return extractPath(u, predecessors)
  for all v ∈ u.successors()
    if v ∈ closed then
      continue
    uvCost ← edgeCost(G, u, v)
    if v ∈ open then
      if uCost + uvCost < open[v] then
        open[v] ← uCost + uvCost
        predecessors[v] ← u
    else
      open.push(v, uCost + uvCost)
      predecessors[v] ← u
  closed.add(u)
```

Example - Processing s

Open Min Heap:

Node	Cost to vertex
c	2
a	5
b	7

Closed Set: s

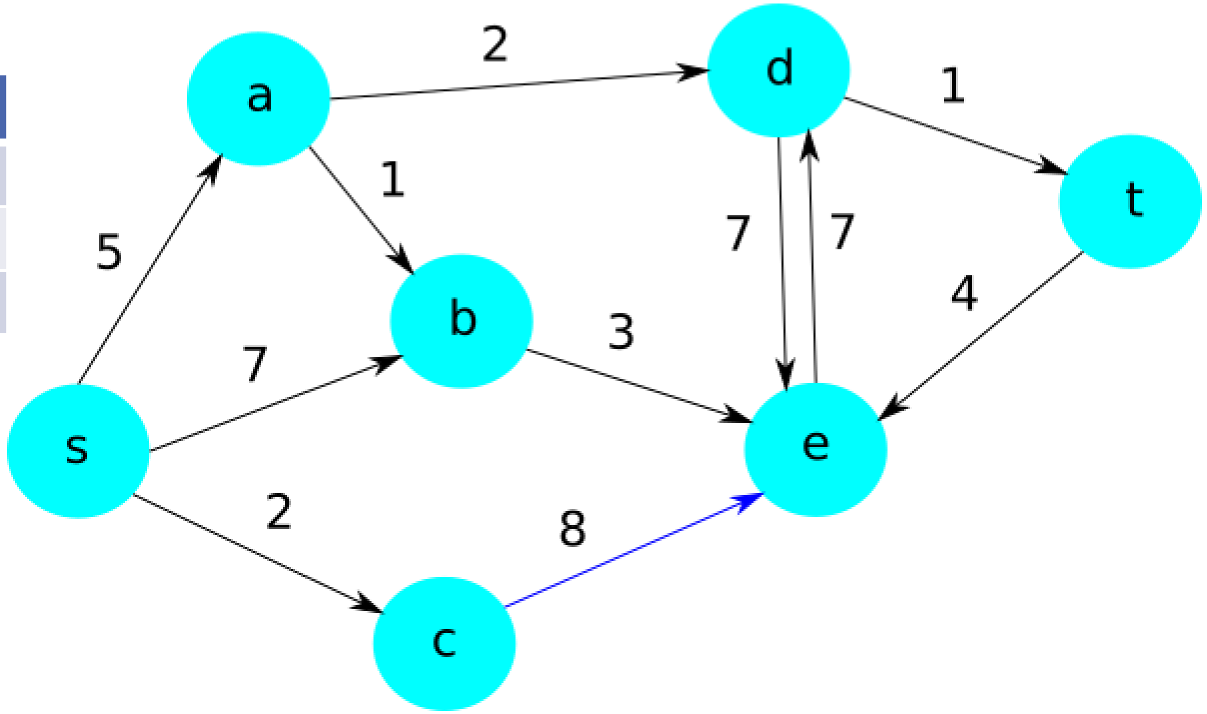


Example - Processing c

Open Min Heap:

Node	Cost to vertex
a	5
b	7
e	10

Closed Set: s
c

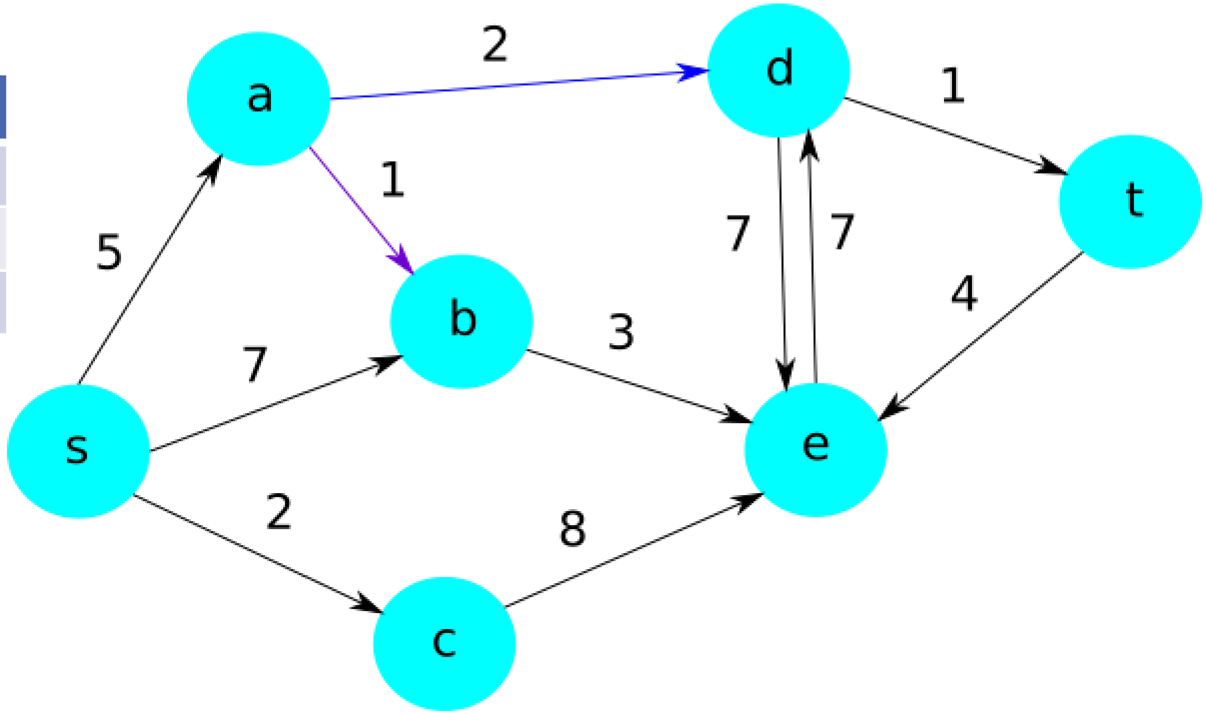


Example - Processing a

Open Min Heap:

Node	Cost to go
b	6
d	7
e	10

Closed Set: s
c
a

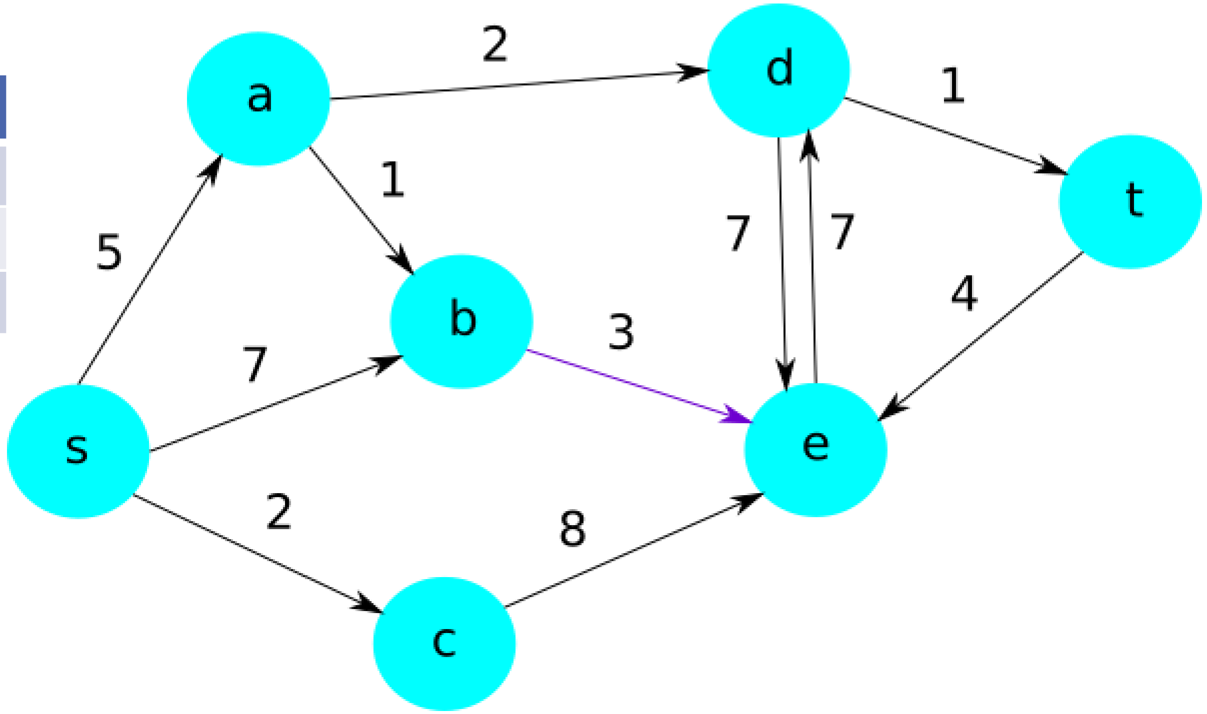


Example - Processing b

Open Min Heap:

Node	Cost to go
d	7
e	9

Closed Set: s
c
a
b

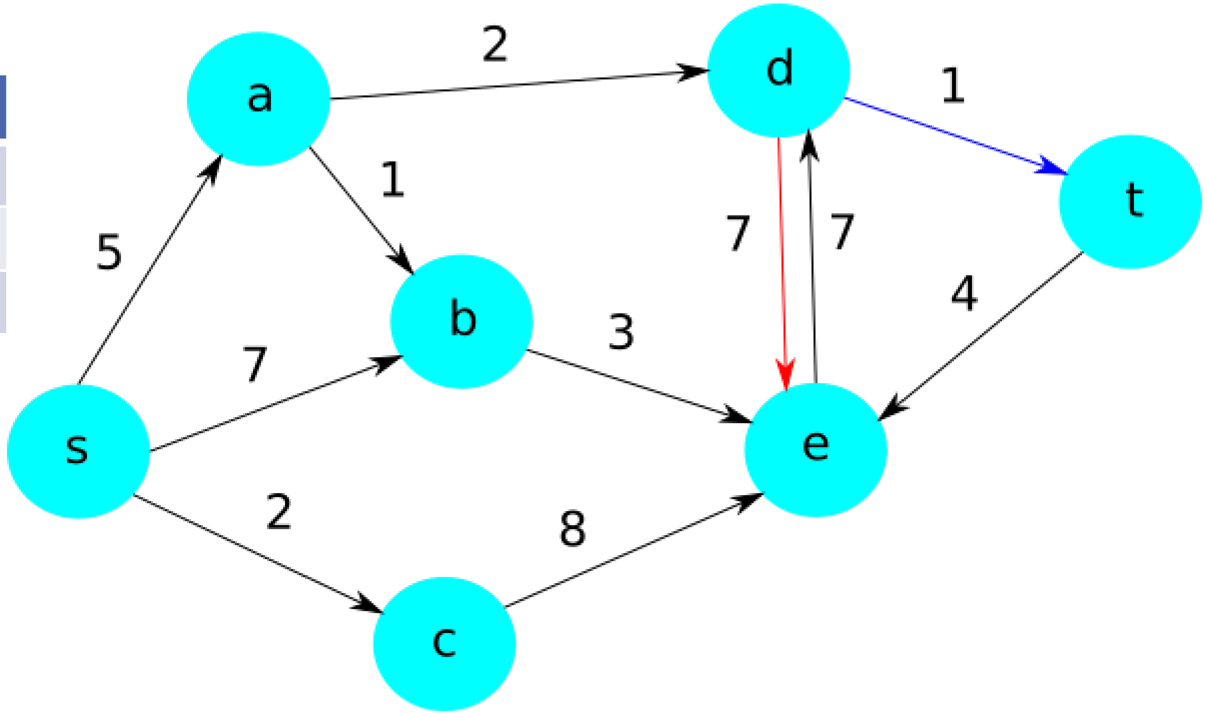


Example - Processing d

Open Min Heap:

Node	Cost to go
t	8
e	9

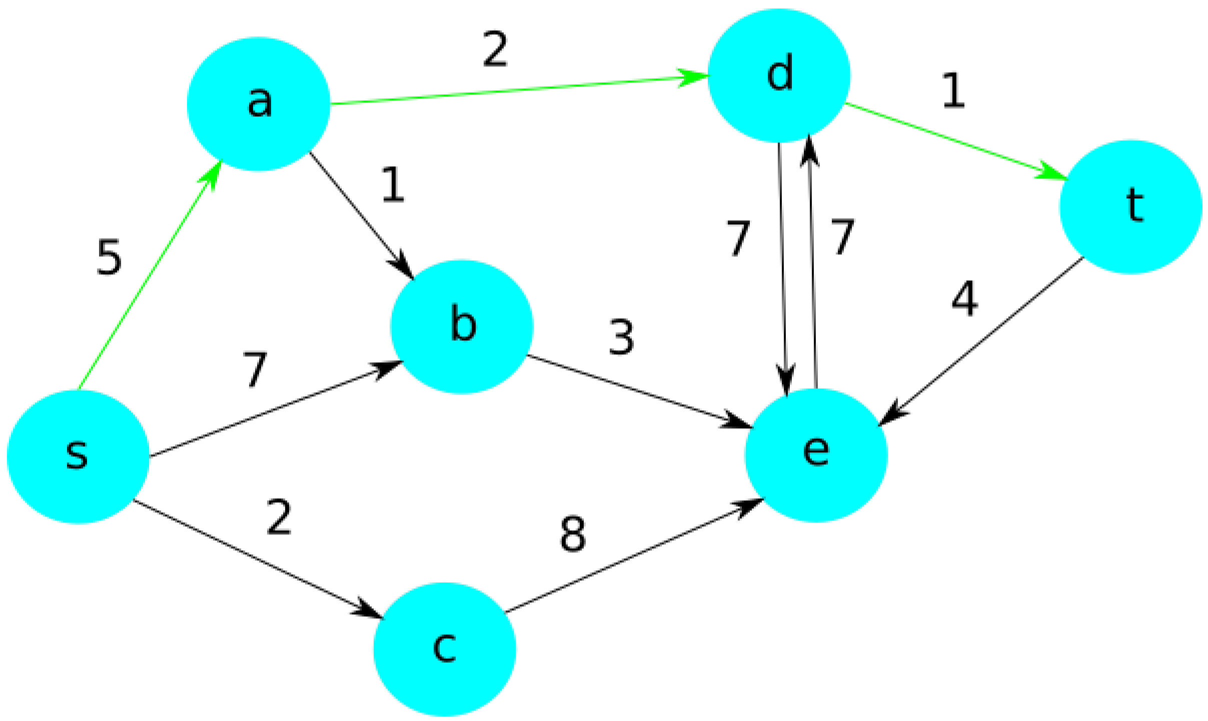
Closed Set: s
c
a
b
d



Example - Optimal Path

Final path: s
a
d
t

Closed Set: s
c
a
b
d



Search on a Map

- Example - map of Berkeley, California
  - 2,097 vertices
  - 5,740 edges

- Example - map of New York City, New York
  - 54,837 vertices
  - 140,497 edges

Summary

- Introduced the concept of a weighted graph
- Developed the use case of a weighted graph for mission planning
- Introduced Dijkstra’s algorithm for searching weighted graphs for the shortest path to a destination

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