# Motion Planning Constraints

Course 4, Module 1, Lesson 2

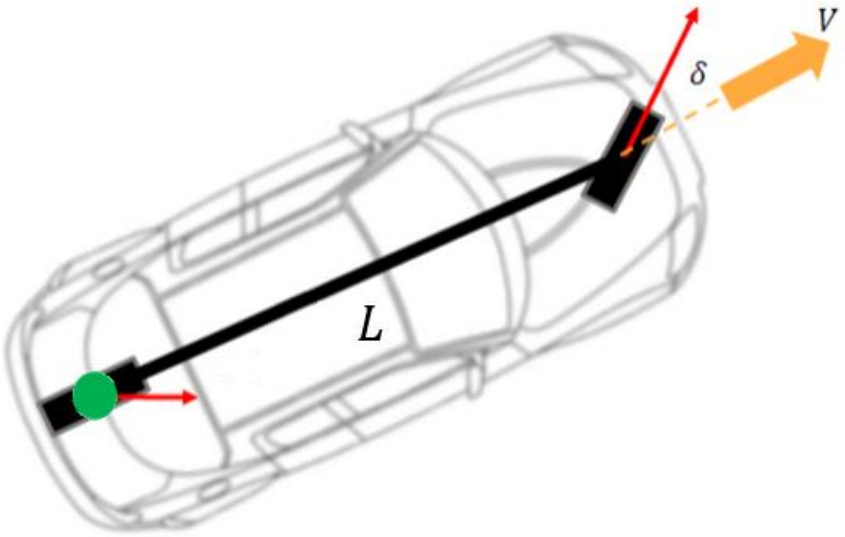
UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

# Learning Objectives

- Understand how the vehicle’s kinematics and dynamics constrain the motion planning problem
- Understand how static and dynamic obstacles affect motion planning
- Understand the impact of regulatory elements on motion planning

# Bicycle Model

- Kinematics simplified to bicycle model (as discussed in Course 1)



# Bicycle Model

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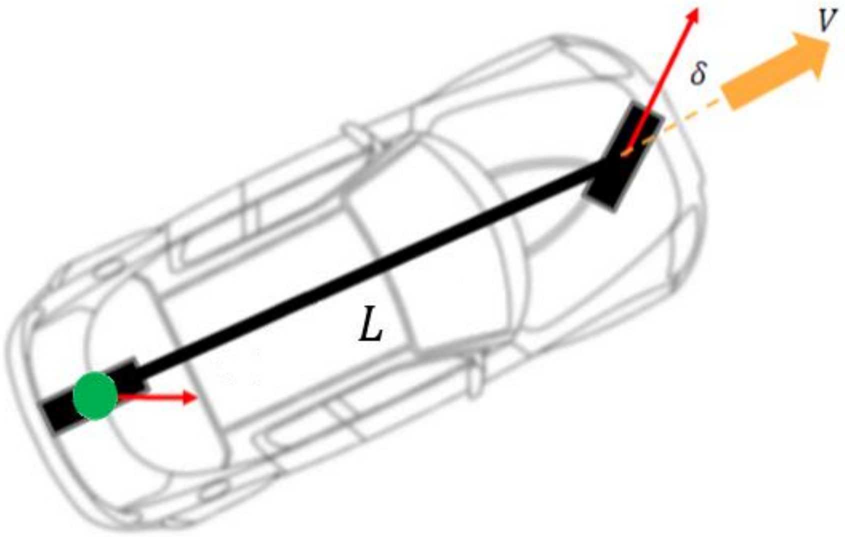
- Bicycle model imposes curvature constraint on our path planning process

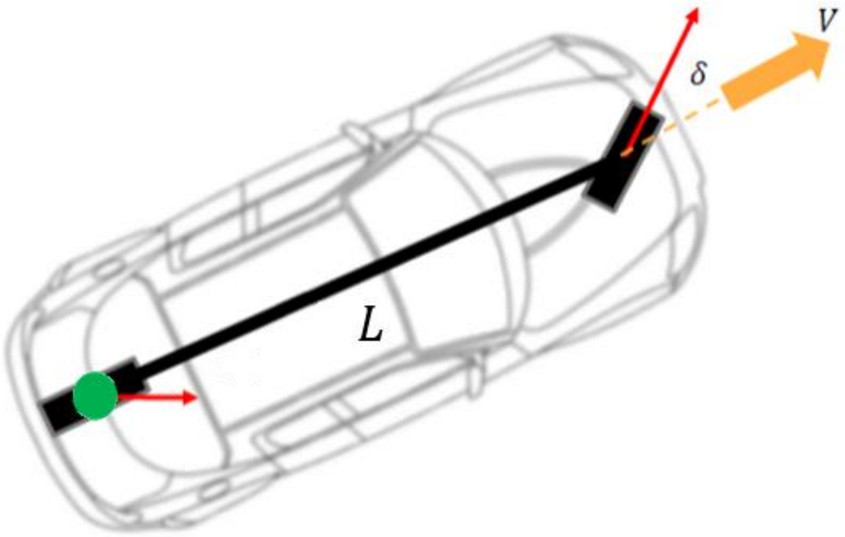



$$\dot{\theta} = \frac{V\tan(\delta)}{L}$$

$$|\kappa| \leq \kappa_{max}$$

# Bicycle Model

- Kinematics simplified to bicycle model (as discussed in Course 1)
- Bicycle model imposes curvature constraint on our path planning process
- Curvature constraint is non-holonomic
  - Non-holonomic constraints reduce the directions a mobile robot can travel at any point
  - Makes motion planning challenging

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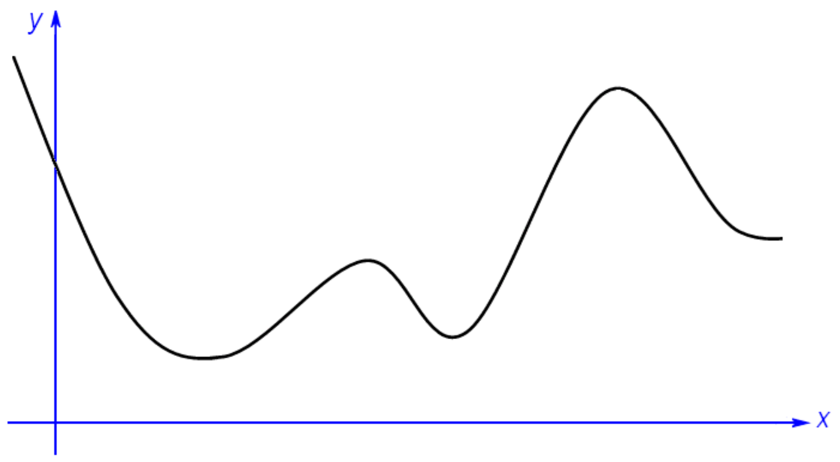
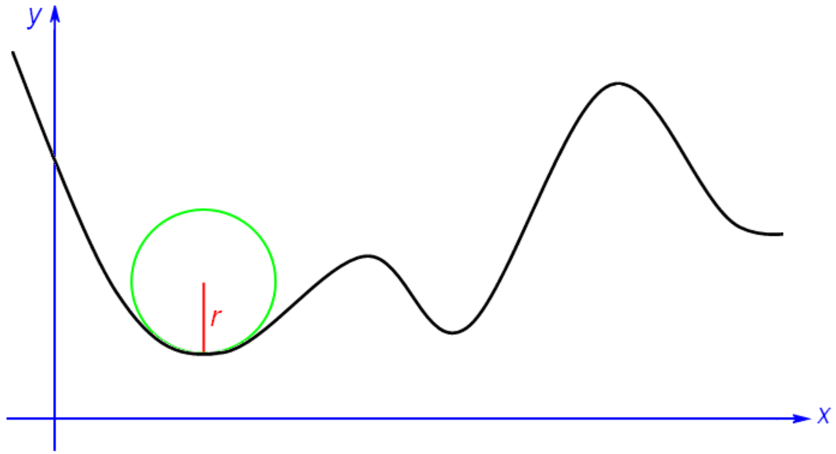
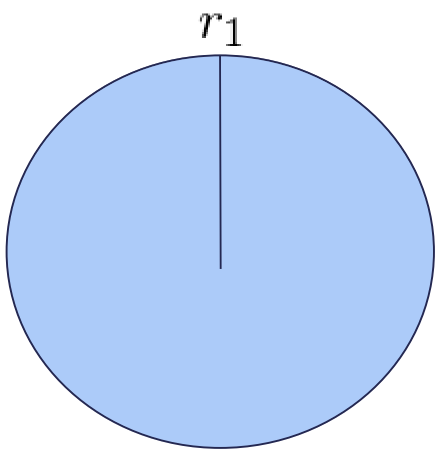
# Curvature



# Curvature

$$\kappa = \frac{1}{r}$$



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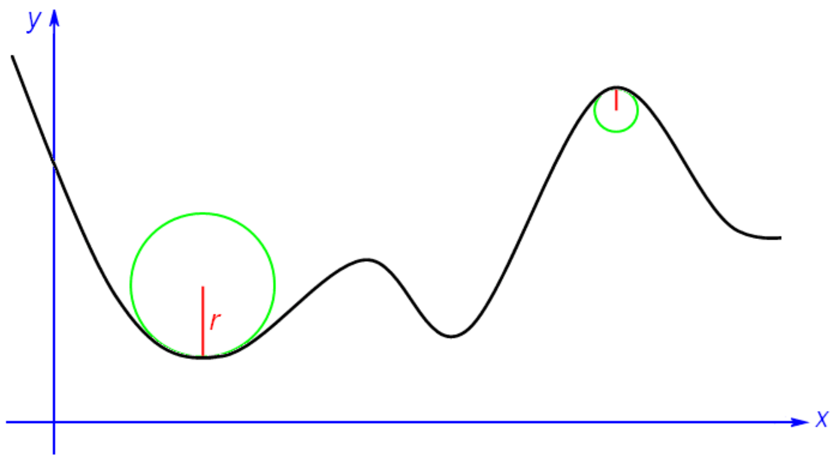


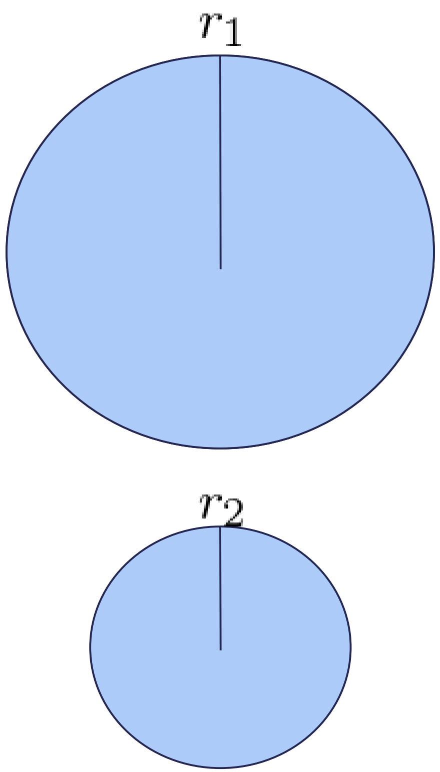


$r_2 < r_1$
$\kappa_2 > \kappa_1$

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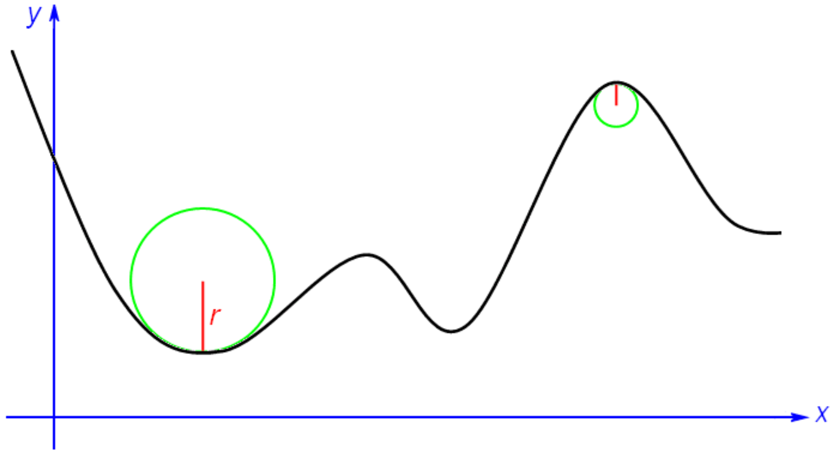
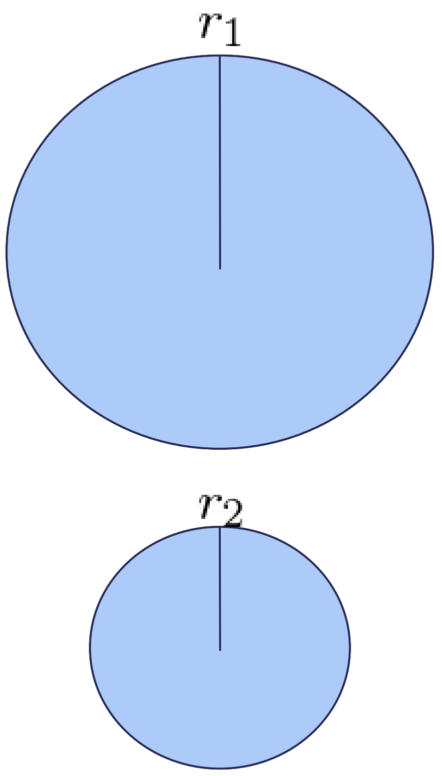


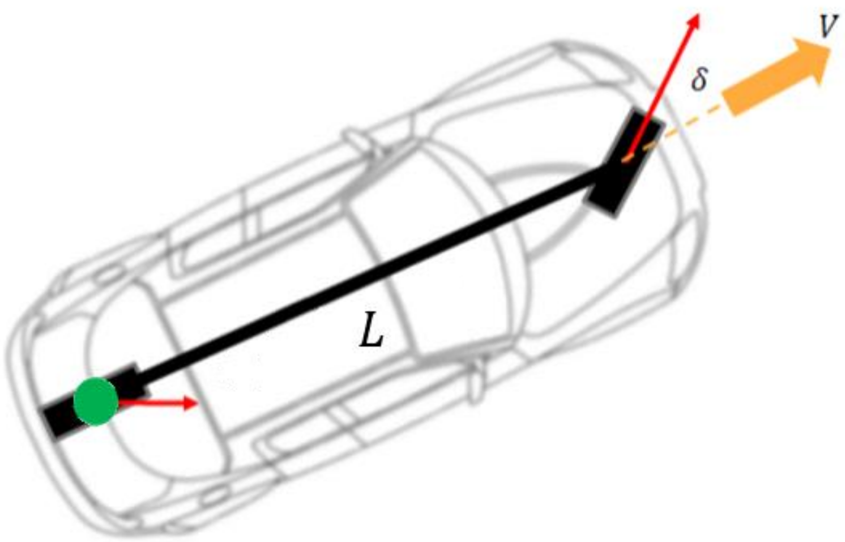

$$\kappa = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}}$$

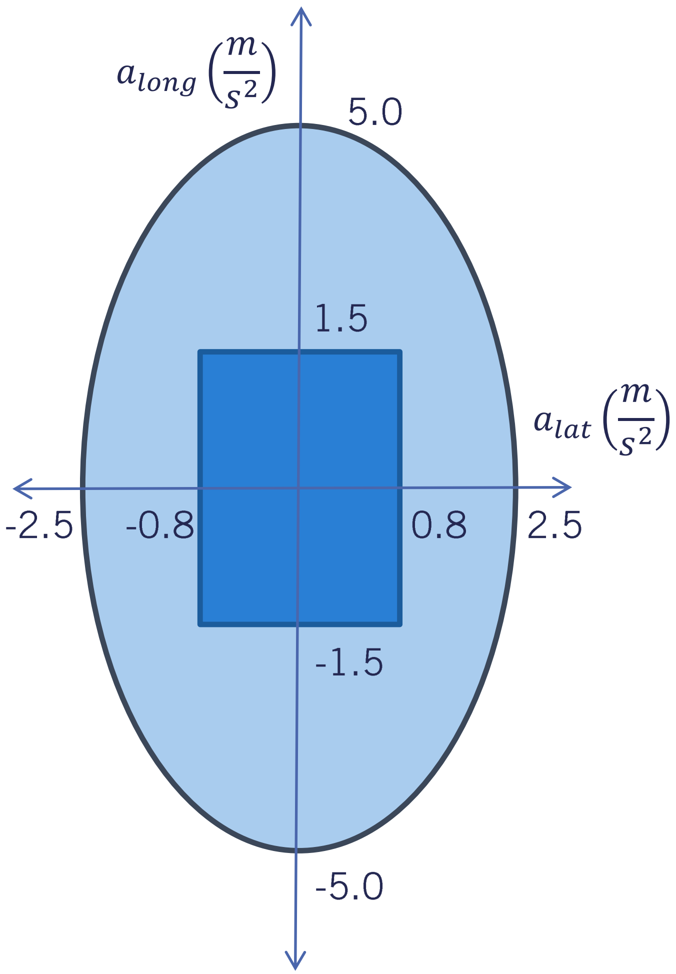
$r_2 < r_1$

$\kappa_2 > \kappa_1$

# Vehicle Dynamics

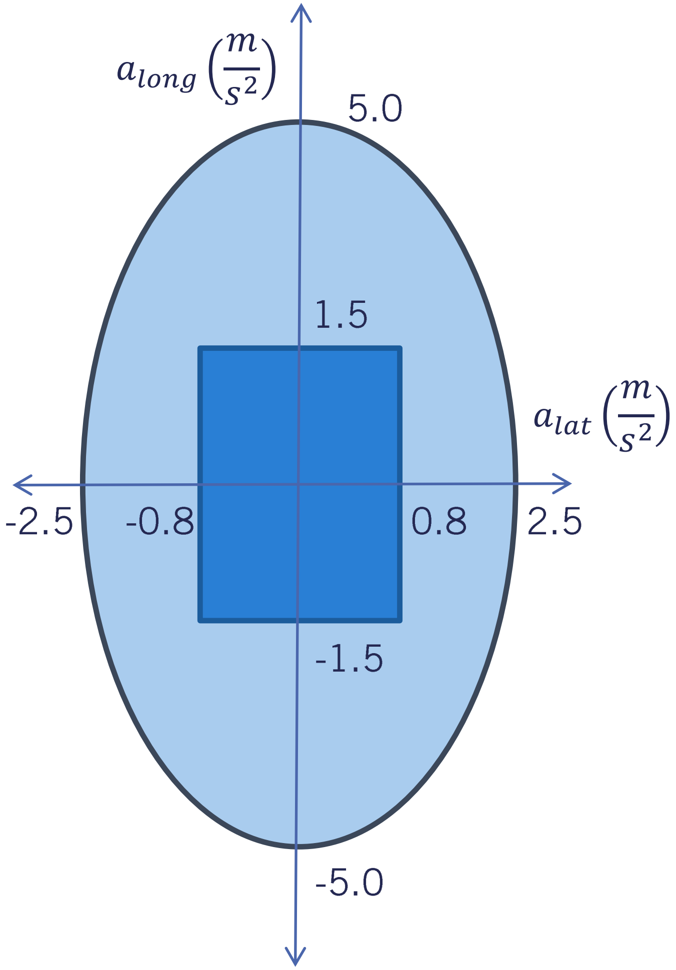
- Recall: friction ellipse denotes maximum magnitude of tire forces before stability loss



# Vehicle Dynamics

- Recall: friction ellipse denotes maximum magnitude of tire forces before stability loss
- Friction forces are extreme limit; more useful constraint is accelerations tolerable by passengers
  - Given by "comfort rectangle" range of lateral and longitudinal accelerations



# Dynamics and Curvature

- Friction limits and comfort restrict lateral acceleration
  - Lateral acceleration is a function of instantaneous turning radius of path and velocity

$$a_{lat} = \frac{v^2}{r}, \qquad a_{lat} \leq a_{lat_{max}}$$

# Dynamics and Curvature

- Friction limits and comfort restrict lateral acceleration
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- Recall: instantaneous curvature is inverse of instantaneous turning radius

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# Dynamics and Curvature

- Friction limits and comfort restrict lateral acceleration
  - Lateral acceleration is a function of instantaneous turning radius of path and velocity
- Recall: instantaneous curvature is inverse of instantaneous turning radius
- Substituting, velocity is constrained by path curvature and lateral acceleration

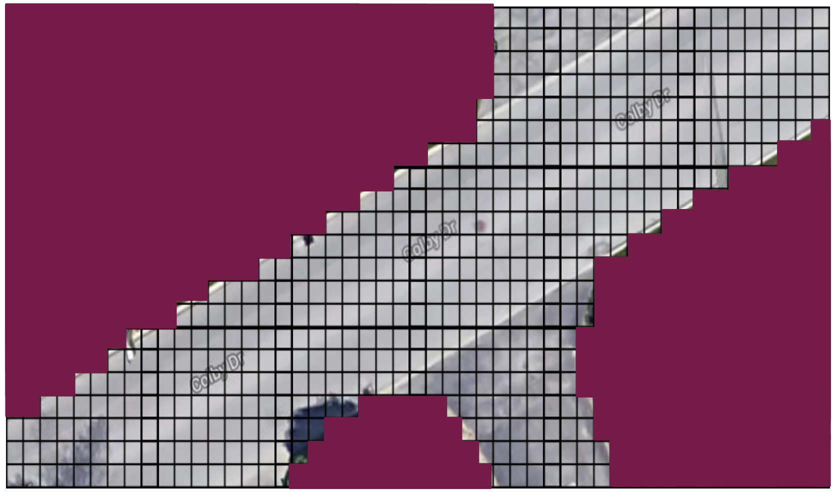
$$a_{lat} = \frac{v^2}{r}, \qquad a_{lat} \leq a_{lat_{max}}$$

$$\kappa = \frac{1}{r}$$

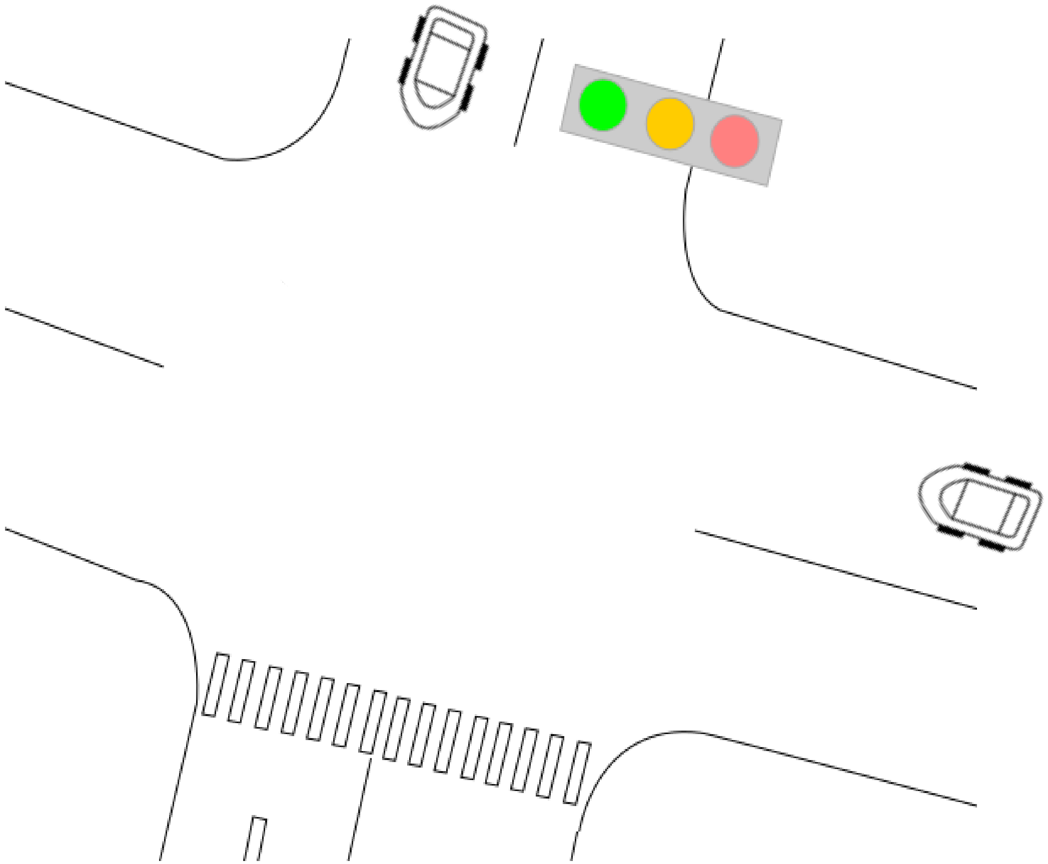
$$v^2 \leq \frac{a_{lat_{max}}}{\kappa}$$

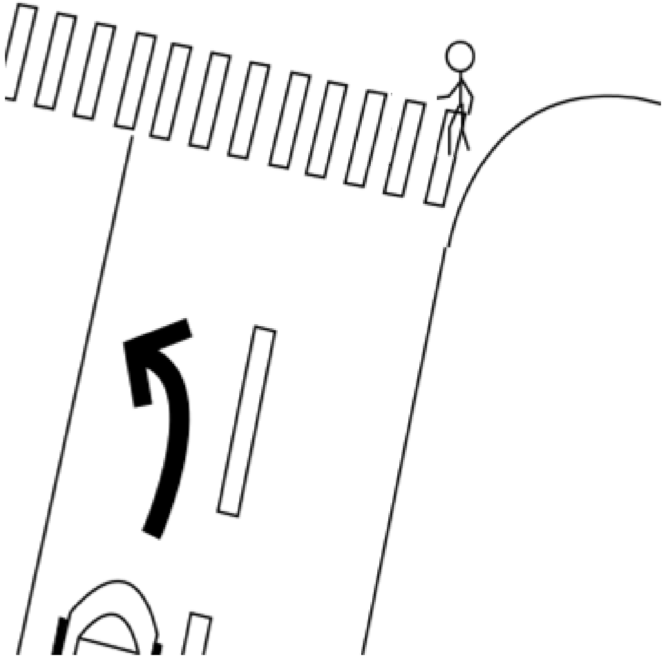
# Static Obstacles

- Static obstacles block portions of workspace
  - Occupancy grid encoding stores obstacle locations
- Static obstacle constraints satisfied by performing collision checking
  - Can check for collisions using the swath of the vehicle's path
  - Can also check for closest obstacle along ego vehicle's path



# Dynamic Obstacles





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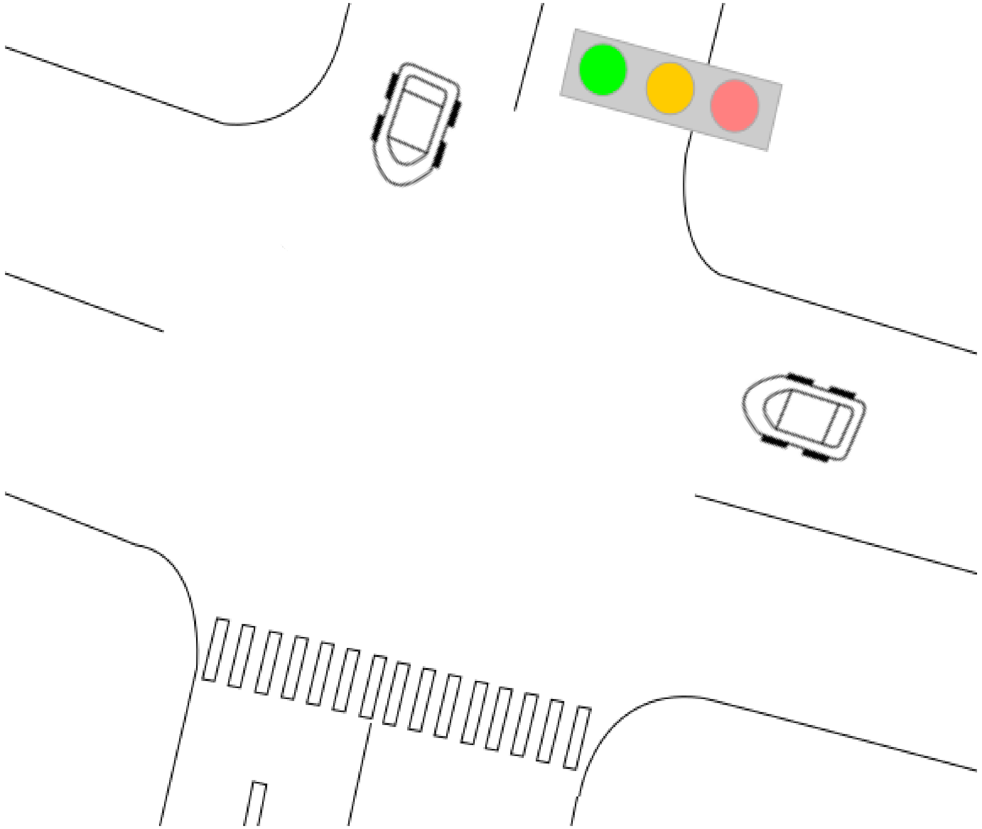

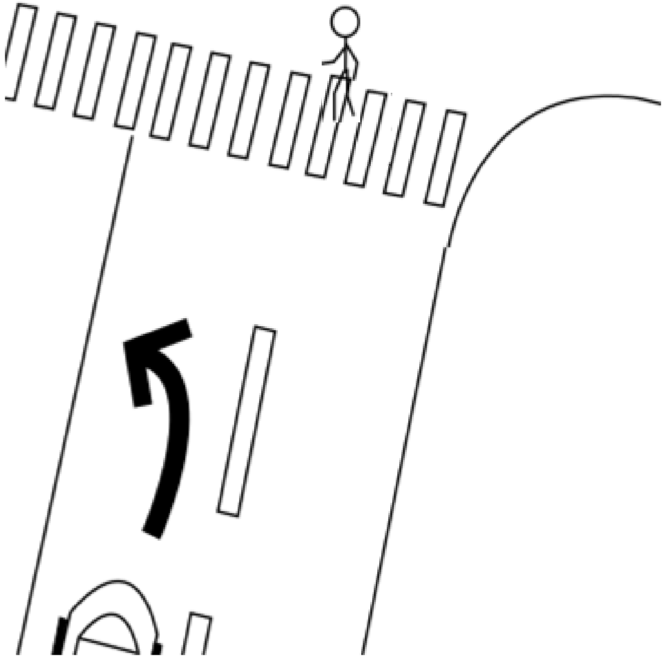

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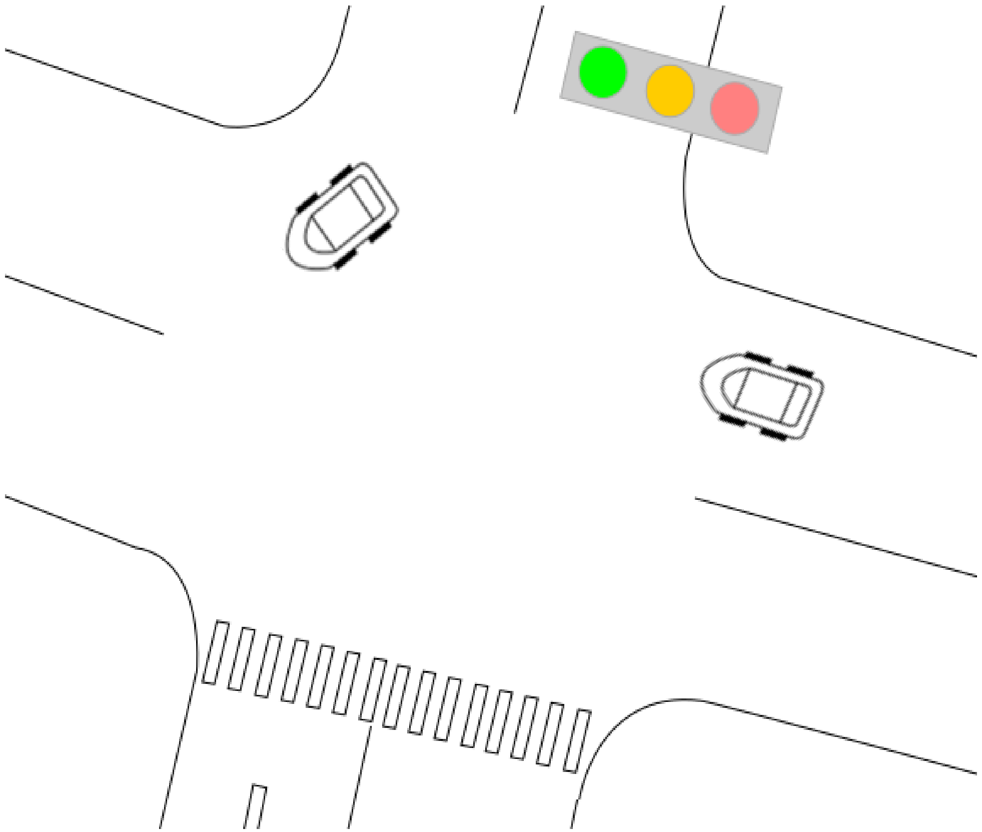

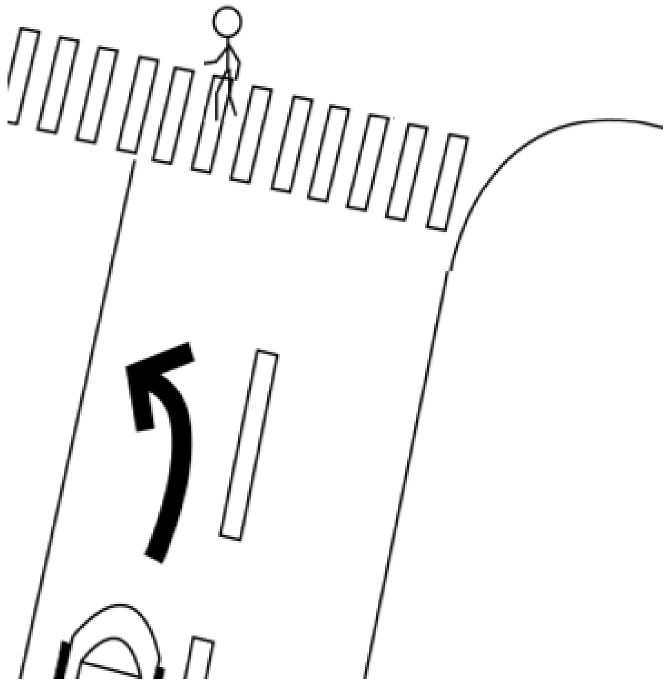

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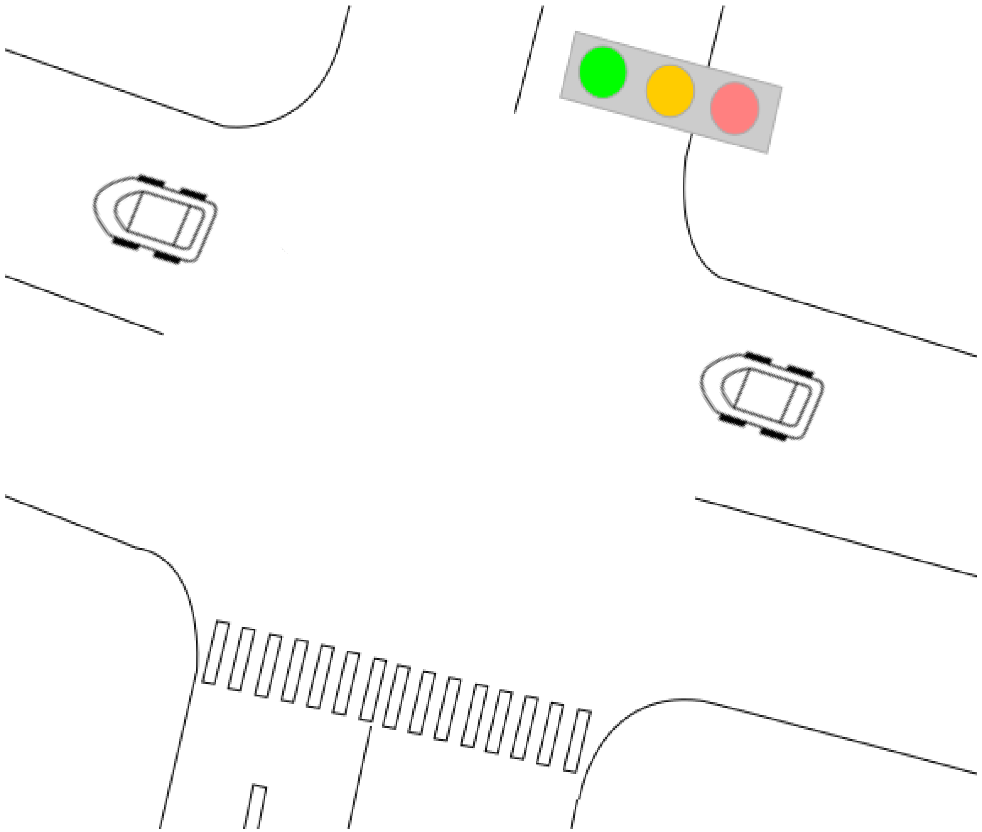

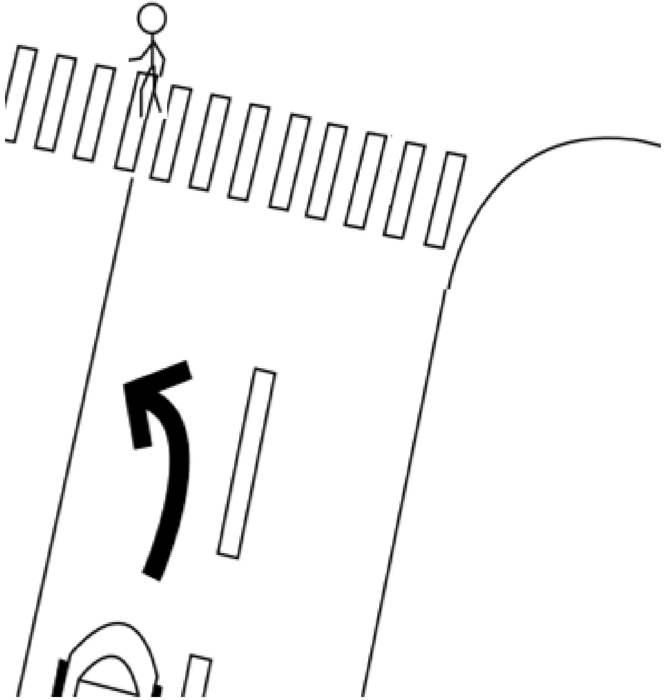

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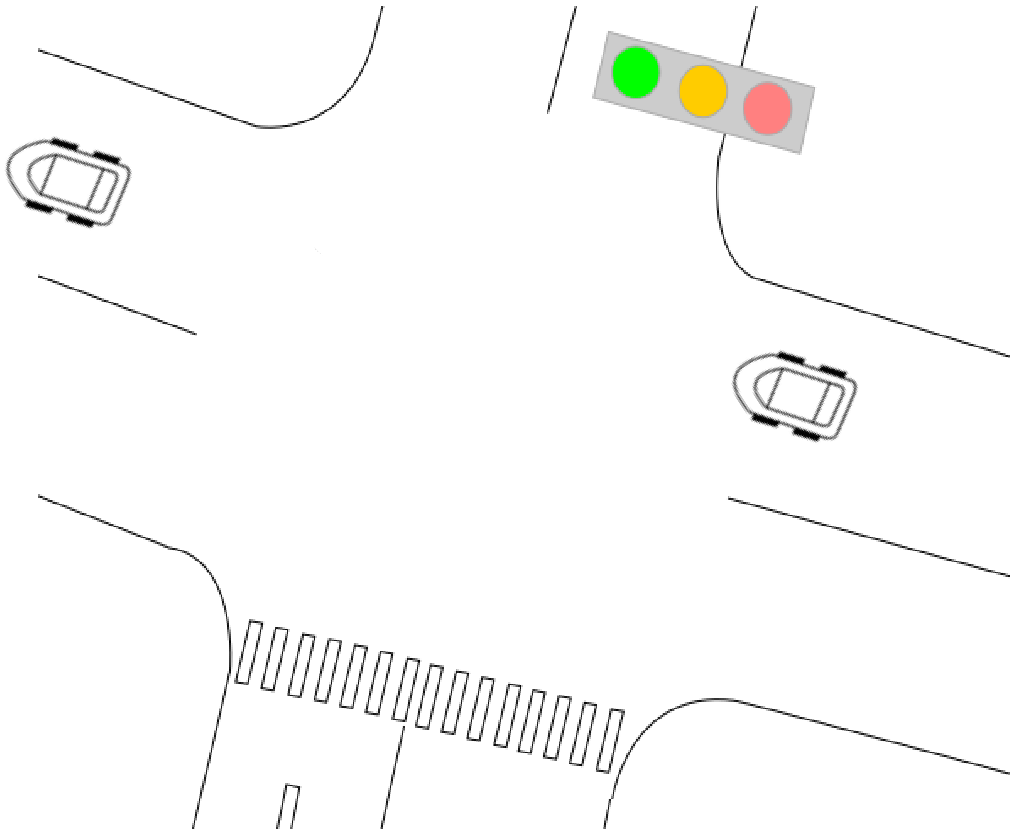

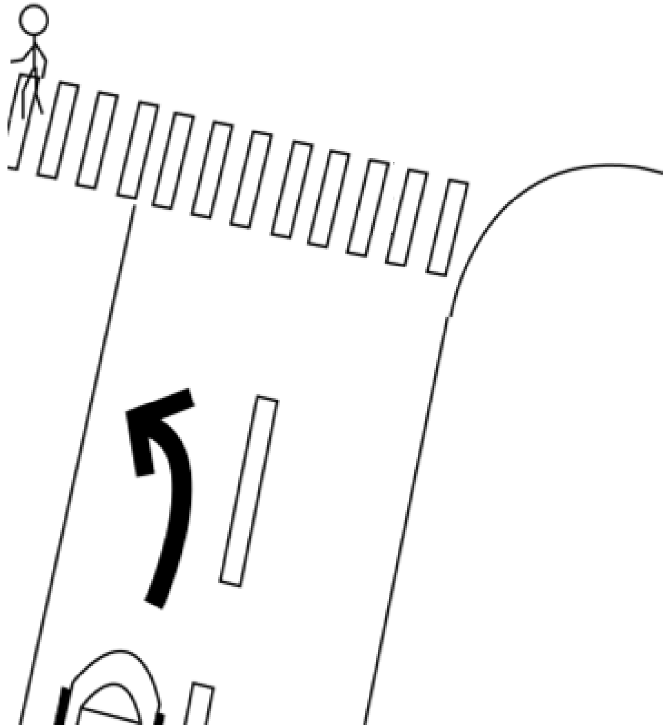

# Rules of the Road and Regulatory Elements

- Lane constraints restrict path locations
- Signs, traffic lights influence vehicle behaviour


# Summary

- Explained the kinematic and dynamic constraints present in the motion planning problem, and how path curvature affects them
- Discussed how static and dynamic obstacles impact our planning process
- Discussed how regulatory elements impact our driving behaviour