708 Microeconomics II

HW 15% { Feb ~~[illegible]~~ 2nd

3 MIDTERMS 20%

Final 25%, Replace lowest midterm

31% A 31% AB 31% B

learn @ UW

economic theory

拉丁语 经济人

① Homo Economics desire to max

choices constrains

every people knows

② Markets

③ Game Theory

Constrains Maximization

Preference

Means & Efficiency

benefits → preferce.

cost → constrains

chapter 1

$\boxed{\text{cost-benefit analysis \& optimization.}}$

→ actions <≡ policy>

→ discrete → in (choice) extensive margin

↘ out

→ variable choice [0, 1] intensive margin

① Margin Analysis (Intensive)

(a) Activity → $x \geq 0$.

return. → $r(x) = 8x - x^2$

$r(x) = 16 - (x-4)^2$

$r'(x) = 8 - 2x = 0$ marginal benefit = marginal cost

(b) If $x > 0$, pay fee $F > 0$

$x^* = 4$ ∀fees, if $x^* > 0$

if $F \le 16$, choose $x^* = 4$

Global optimanity

is Corner solution

Interior solution

Constrained optimation

Bang-Per-Buck Principle

buy each good up to the point where its marginal utility relative to what it costs.

2 activity $x + y = 4$, $x, y \ge 0$

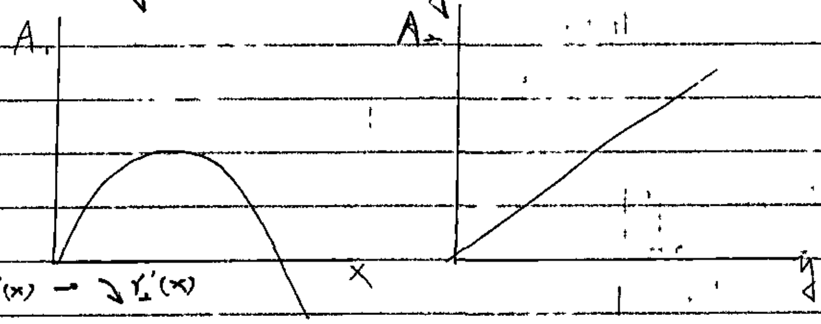
Opportunity cost

↓

shadow cost (indirectly seen)

Activity 1, return $8x - x^2$ → marginal return $r_1'(x) = 8 - 2x$

Activity 2, return $2y$ → ...... $r_2'(x) = 2$



$\nearrow r_1'(x) \to \searrow r_2'(x)$

$8 - 2x^* = 2$

$x^* = 3$ $y^* = 1$

diminished marginal return

$x + y = 1$

If $x^* > 0$, $\Rightarrow 8 - 2x^* \ge 2$

$y^* > 0$, $2 \ge 8 - 2x^*$. ✗ No

$x^* = 1$, $y^* = 0$ corner solution

↑ static optimation

Dynamic →

· Bellman's Principle of Optimality (1953)

An optimal policy has the property that whatever the initial state & decision, the remaining decisions must constitute an optimal policy.

Zermelo's → 1903 backward induction algorithm

Selten 1965 → subgame perfect equilibrium

Wage Search Problem.

→ Finite Horizon (n periods)

→ wages $W \sim U(a,b) \equiv U(100, 200)$ $a>0$

→ wages are independent

→ search cost $c < \frac{a+b}{2}$ (assume)

→ reservation wage $R_n = a$

→ $R_{n-1} = -c$ (not paying search cost) $+ \frac{a+b}{2} > 0.$

(assume $-c + \frac{a+b}{2} > a$)

$$R_k = -c + P_r(W \geq R_{k+1}) \cdot E(W \mid W > R_{k+1}) + P_r(W < R_{k+1}) \cdot \boxed{R_{k+1}}$$

$[a \quad R_{k+1} \quad b]$

So $R_k = -c + \frac{b - R_{k+1}}{b-a} \cdot \frac{b + R_{k+1}}{2} + \frac{R_{k+1} - a}{b-a} \cdot R_{k+1}$

$$= -c + \frac{b^2 - R_{k+1}^2}{2(b-a)} + \frac{2R_{k+1}^2 - 2aR_{k+1}}{2(b-a)}$$

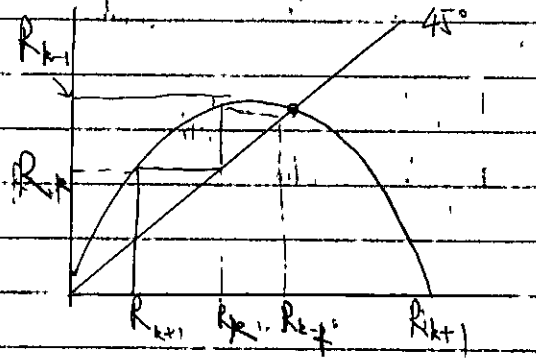
$$= -c + \frac{b^2 - 2aR_{k+1} + R_{k+1}^2}{2(b-a)}$$

$$= -c + \frac{(b - R_{k+1})^2}{2(b-a)} + R_{k+1}$$

$$R_k - R_{k+1} = -c + \frac{(b - R_{k+1})^2}{2(b-a)} \geq 0.$$

iff $R_{k+1} \leq b - \sqrt{2c(b-a)}$

$$f(x) = -c + \frac{(b-x)^2}{2(b-a)} + x$$



$n = \infty \leftarrow$ simplest

stationary $\bar{R}$

$$c = \frac{(b-\bar{R})^2}{2(b-a)}$$

$\bar{R} = b.$ exercise

Producer theory

Chapter 1

Producer Theory

$Q = f(X_1, \ldots X_n)$

$Q = f(K, L)$

das Kapital

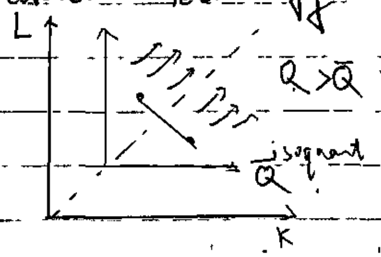
① fixed properties $f(K, L) = \min(aK, (1-\alpha)L)$

e.g. $f(K, L) = \min(K, L)$

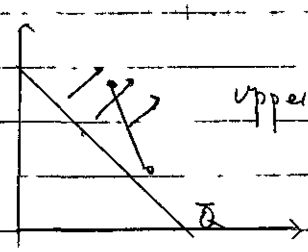
② $f(K, L) = aK + (1-a)L$

③ $f(K, L) = K^{a} L^{1-a}$

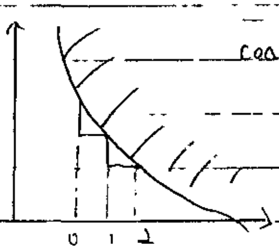
① convex Technology



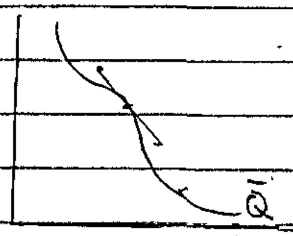
$Q > \bar{Q}$ "upper contour set" is convex



upper contour set is convex



convex upper contour set



must be global → optimal

What is the isoquant slope?

$\bar{Q} = f(K, L)$

Differentiates. $0 = f_K \frac{dK}{dL}\Big|_{\bar{Q}} + f_L$

rate of technical substitution

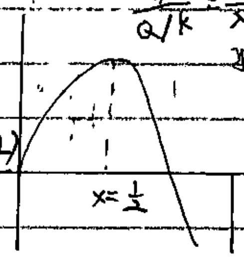
RTS

$\frac{K}{\ell}$ $\quad \frac{dK}{dL}\Big|_Q = -\frac{f_L}{f_K} < 0$ if $f_L, f_K > 0$

$\frac{Q/\lambda}{Q/K} = \frac{K}{\lambda}$

$y = 5 + x - x^2$

$$\bar{Q} \equiv f(K(L), L)$$
$$0 \equiv f_K K'(L) + f_L$$
$$0 = f_{KK} K'(L)^2 + f_{KL} K'(L) + f_K K''(L) + f_{LK} K'(L) + f_{LL}$$

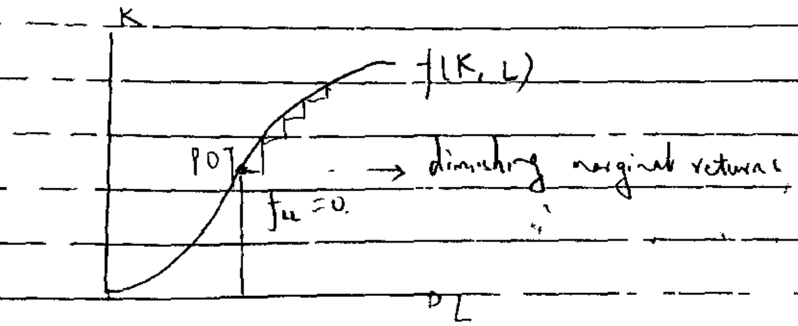


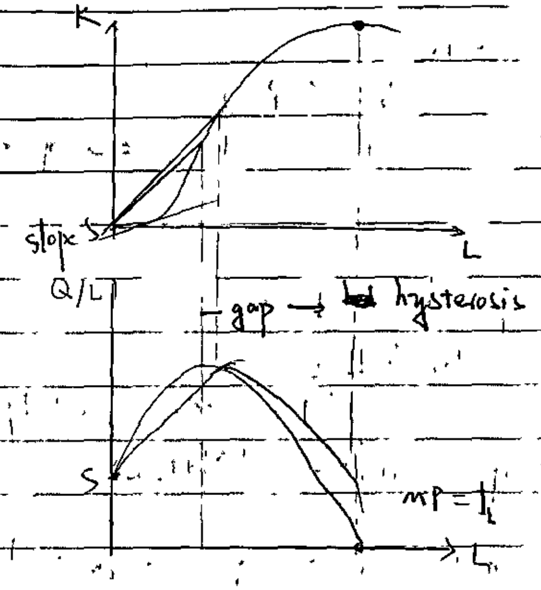
$$K''(L) = \frac{f_K^2 f_{LL} - 2 f_K f_L f_{LK} + f_L^2 f_{KK}}{f_K^3} > 0$$

CHECK UNITS $\quad \frac{Q^3 K^{-2} \lambda^{-2}}{Q^3 K^{-3}} = \frac{K}{\lambda^2}$

Property 2: Law of ~~Diminishing~~ eventual Diminishing Returns

Law of Variable Proportions





$$MP = \frac{\partial f}{\partial L} = f_L$$

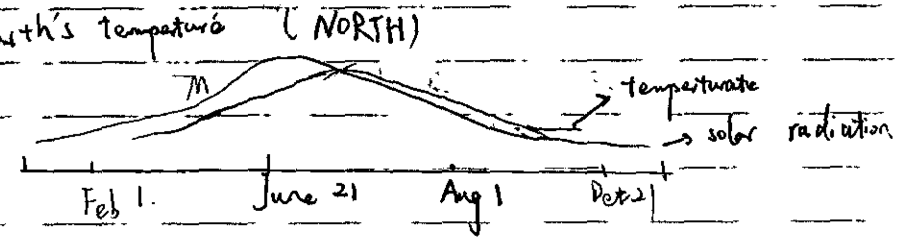
$$AP = \frac{f}{L} = \frac{Q}{L}$$

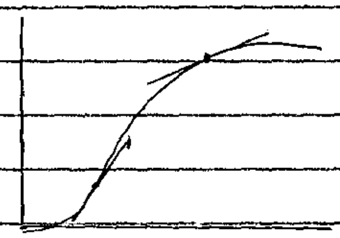
Prove: AP = MP at L=0.

Hint: Q/L

hysterosis → delayed effect

e.g. earth's tempature (NORTH)





Property 3 Returns to Scale

$f(tK, tL) = t\, f(K, L)$

constant returns to scale (CRS)

$f(tK, tL) > t f(K, L)$ if $t > 1$ increasing return to scale

Eulers Theorem

If f is homogenius of degree 1 $(\equiv$

then $f \equiv K f_K + L f_L$

$1 \to \alpha f \equiv K f_K + L f_L$

F is homogeneous α if $f(tK, tL) = t^{\alpha} f(K, L)$

e.g. f $\quad g(x) = x^{\alpha} \Rightarrow g'(x) = \alpha x^{\alpha-1}$

$\Rightarrow x g'(x) = \alpha x^{\alpha}$

$f = K f_K + L f_L$

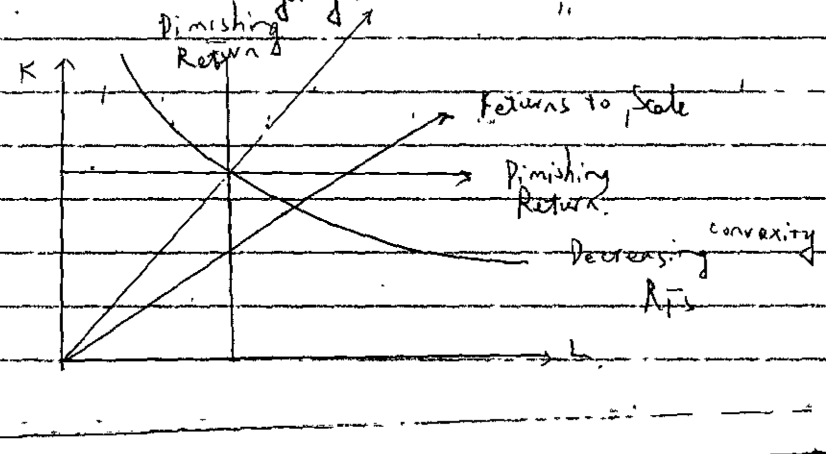
$w = f_L$

rental $= r = f_K$

$GNP = f = Kr + Lw$

$1 = \frac{Kr}{GNP} + \frac{Lw}{GNP}$ → labor share

share of income going to K



Infinite Horizon / Wage Search

① Obviously, a reservation wage is optimal

$[\underline{w}, 1] \leftarrow$ acceptable wage

So at $\underline{w}$, you are indifferent!

$\underline{w} = E($ best possible continuation $)$ Bellman

$= -c + Pr(\text{wage is too low})(\underline{w})$

$+ Pr(\text{wage} \geq \underline{w}) \underbrace{E(w)}_{\frac{\underline{w}+1}{2}}$

$= -c + \underline{w} \cdot \underline{w} + (1-\underline{w}) \cdot (1+\underline{w})/2$

② finite horizon (last chapter!)

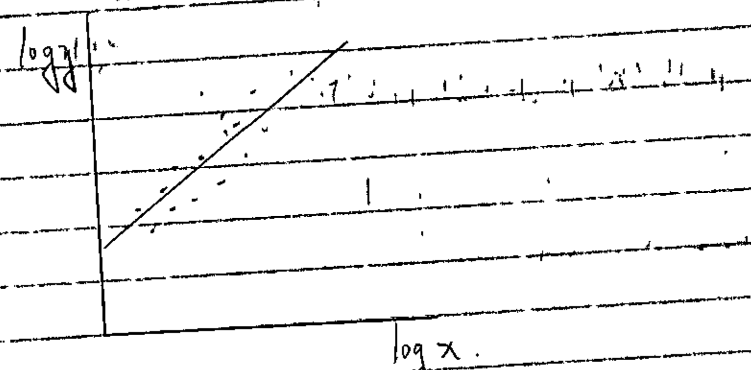
solve $w_n, w_{n-1}, w_{n-2}, \cdots$

Chapter 2

Elasticity → the elasticity of $y(x)$ in x is the ratio of the proportionate change in $y(x)$ induced by a given proportionate change in x

$$\mathcal{E}_x(y) = \frac{\%\text{age change in } y}{\%\text{age change in } x} = \frac{dy/y}{dx/x} =$$

$$\frac{d(\log y)}{d(\log x)} = \frac{dy}{dx}\cdot\frac{x}{y} \approx \frac{\Delta y/y}{\Delta x/x}$$



1% increase in x → $\mathcal{E}_x(y)$ % increase in y, approximately.

$$\mathcal{E}_x(y(x, z(x)) = \frac{d\log(yz)}{d\log(x)} = \frac{d\log y}{d\log x} + \frac{d\log z}{d\log x}$$

$$= \mathcal{E}_x(y) + \mathcal{E}_x(z)$$

Demand $p(q)$ has elasticity -2

Revenue $q \cdot p(q)$ has elasticity $= -2+1 = -1$

$\uparrow{+1}\ \downarrow{-2}$

$\mathcal{E}_x(\phi(y(x)))$

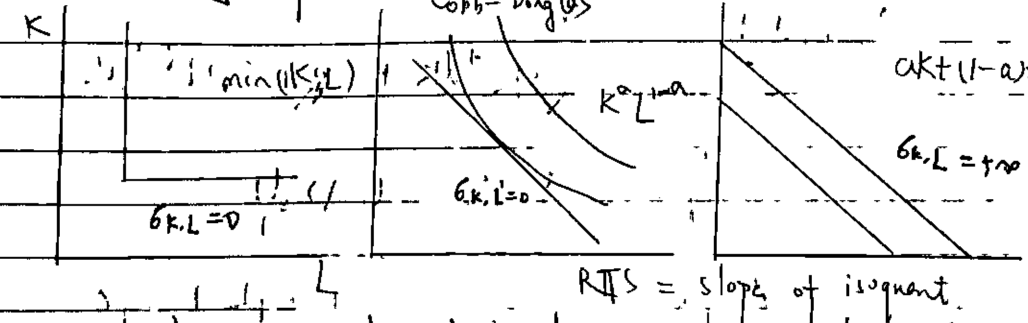
use chain rule of differentiate

$$\frac{d}{dx}\phi(y(x)) = \phi'(y)\, y'(x)$$

$$\frac{x}{\phi}\frac{d}{dx}\phi(y(x)) = \underbrace{\frac{y\,\phi'(y)}{\phi}}_{\mathcal{E}_y(\phi)}\ \underbrace{y'(x)\frac{x}{y}}_{\mathcal{E}_x(y)}$$

$$\varepsilon_x(\phi(y(x))) = \varepsilon_y(\phi(y)) \cdot \varepsilon_x(y(x))$$

Elasticity of Substitution



RTS = slope of isoquant

how fast the slope changes?

$$= \frac{f_L}{f_K} = \frac{(1-a)K^a L^{-a}}{aK^{a-1}L^{1-a}} = \frac{(1-a)K}{aL}$$

$$\sigma_{KL} = \varepsilon_{RTS}(K/L)$$

e.g. Cobb-Douglas

$$\sigma_{KL} = \frac{d(K/L)}{d(RTS)} \cdot \frac{RTS}{(K/L)}$$

$$= \frac{d(K/L)}{d\left(\frac{(1-a)K}{aL}\right)} \cdot \frac{\frac{(1-a)K}{aL}}{K/L} = 1$$

When is the elasticity of substitution constant?

Answers: $Q = f(K,L) = [\delta K^\rho + (1-\delta)L^\rho]^{\varepsilon/\rho}$

has returns to scale ε and elasticity of substitution $\sigma = \frac{1}{1-\rho}$

$$f(tK,tL) = t^\varepsilon f(K,L)$$

constant returns to scale $\varepsilon = 1$

$\sigma = \infty \to \rho \to 1$ $\quad f(K,L) = \delta K + (1-\delta)L$

$\sigma = 0 \to \rho \to -\infty$ $\quad \lim_{\rho\to-\infty} f(K,L) = \lim_{\rho\to-\infty} K[\delta \cdot 1 + (1-\delta)(L/K)^\rho]^{1/\rho} = K \quad L>K$

$L \quad L<K$

$= \min(K,L)$

$\sigma = 1 \rightarrow \rho = 0$, try $\lim \rho \to 0$

$$Q^\rho = \delta K^\rho + (1-\delta) L^\rho$$

by infinitate

$$\cancel{\rho} Q^{\rho-1} dQ = \cancel{\rho} \delta K^{\rho-1} dK + \cancel{\rho}(1-\delta) L^{\rho-1} dL$$

$$\frac{dQ}{Q} = \delta \frac{dK}{K} + (1-\delta) \frac{dL}{L}$$

$$d\log Q = \delta \log K + (1-\delta) \log L + \log c$$

$$Q = c K^\delta L^{1-\delta}$$

Cost Minimization → for a convex production technology

Inputs $z = (z_1, \dots, z_n)$

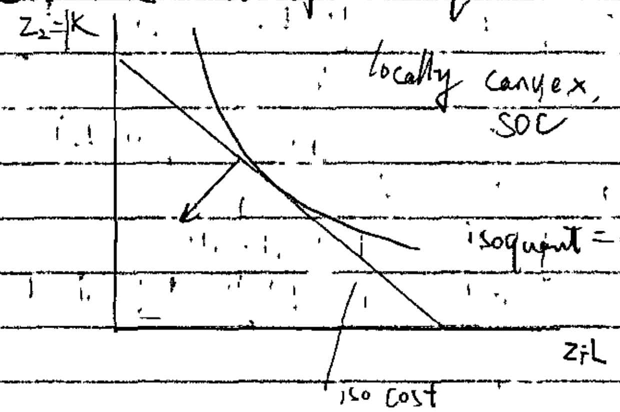
input prices $w = (w_1, \dots, w_n)$

Firm solves $\min\limits_{z \in \mathbb{R}^n} w \cdot z$ s.t. $f(z) = \underset{\text{given}}{\underline{q}}$

minimum is $C(w, q)$

Lagrangian

$$\mathcal{L} = -w \cdot z + \lambda (f(z) - q) \quad \geq 0 \quad f(z) \geq q$$



FOC in z_i $\quad \lambda \frac{\partial f}{\partial z} = w_i \quad \forall i = 1 \dots n$

If $n=2$ $RTS_{L,K} = \frac{\partial f}{\partial z_L} \Big/ \frac{\partial f}{\partial z_K} = \frac{w_L}{w_K}$

Properties of $c(w,q)$ cost function

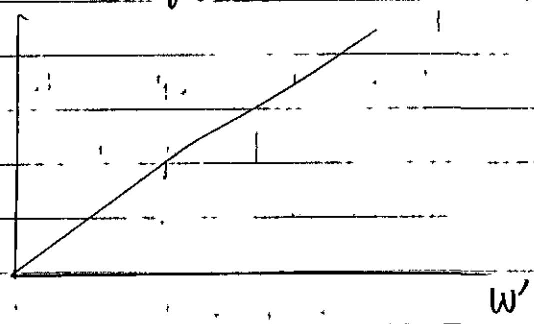
① $c(w,q)$ is homogeneous of degree 1 in w.

② Nondecreasing in q and w

③ Concave in w

If firm does not react to input price changes.

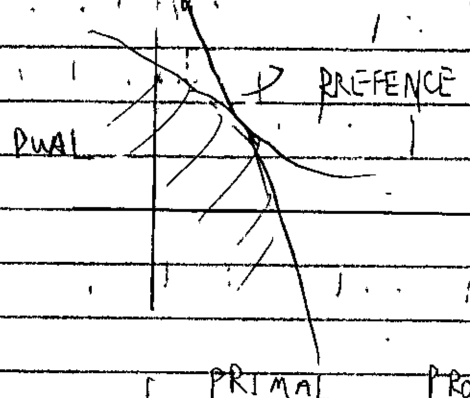
$C(w,q) = w \cdot z^0$ (w^0) $= z'(w')$



Cost Function

$c(w,q) = \min_z w \cdot z \quad s.t. \quad f(z) = q$

solves $z(w,q)$ = conditional function demand

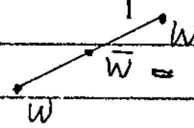


PRIMAL PROBLEM

① $c(w,q)$ is CRS in w

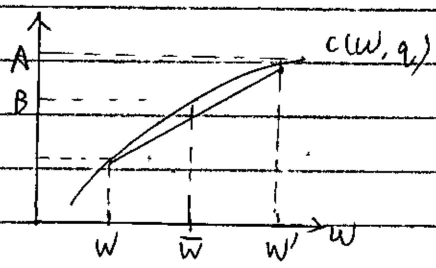
② $\uparrow$ in q, w

③ concave in w



$c(\bar{w},q) \geq \alpha c(w',q) + (1-\alpha) c(w,q)$

$\bar{w} = \alpha w' + (1-\alpha) w$



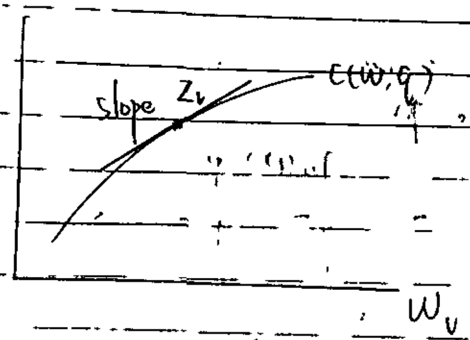
Proof: $c(\bar{w},q) = \bar{w} \cdot z(\bar{w},q)$

$= (\alpha w + (1-\alpha)w') \, z(\bar{w},q)$

$= \alpha \underbrace{\boxed{w \cdot z(\bar{w},q)}}_{\substack{\geq \\ c(w,q)}} + (1-\alpha) \underbrace{w' z(\bar{w},q)}_{\substack{\geq \\ c(w',q)}}$

④ Shepards Lemma

$\frac{dc}{dw_i}$ ~~$c(w,q) = z_i(w,q)$~~



ENVELOPE Theorem

$c(w,q) = z^*(w,q) \cdot w$

$z_1 = 100$

$c(w,q) = w \cdot z(w,q) \leq w \cdot z(w+\Delta w, q)$ ①

$c(w+\Delta w, q) = (w+\Delta w) \cdot z(w+\Delta w, q)$

$\leq (w+\Delta w)\, z(w,q)$ ②

$\Delta w \cdot z(w,q) = (w+\Delta w) \cdot z - w \cdot z$

$\geq c(w+\Delta w, q) - c(w,q)$

$\Delta w \cdot z(w+\Delta w, q) \leq c(w+\Delta w, q) - c(w,q)$

let $\Delta w = (0, 0, \dots, \Delta w_1, 0, 0 \dots)$

$z(w+\Delta w, q) \leq \dfrac{c(w+\Delta w, q) - c(w,q)}{\Delta w_1}$

$z(w,q) \geq \dfrac{c(w+\Delta w, q) - c(w,q)}{\Delta w_1}$

concave function are continuous

a. everywhere differentiable

⑤ Conditional factor demands $z(w,q)$

slope down

$\dfrac{\partial z_i}{\partial w_i} = \dfrac{\partial^2 c(w,q)}{\partial w_i} \leq 0$

PROFIT Maximization

for a competitive firm

profit function $\pi(p, w)$

$$= \max_{z,q} = p \cdot q - w \cdot z \quad s.t. \ q = f(z)$$

$q(p, w)$ = supply function

$z(p, w)$ = demand for factor

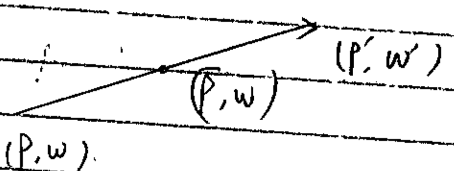
/derivable

Properties of profit function $\pi(p,w)$

① $\pi(p, w)$ is homogeneous of degree one in (p, w)

② π is non decreasing in p and non increasing in w.

③ $\pi(p, w)$ is convex in prices



Pf: $(\bar{p}, \bar{w}) = \alpha(p, w) + (1-\alpha) \cdot (\bar{p}', \bar{w}')$

so $\pi(\bar{p}, \bar{w}) = \bar{p}\bar{q} - \bar{w} \cdot \bar{z}$

$$= (\alpha p + (1-\alpha) p') \bar{q} - (\alpha w + (1-\alpha) w') \bar{z}$$

$$= \alpha (p\bar{q} - w\bar{z}) + (1-\alpha)(p'\bar{q}) - w'\bar{z})$$

$$\leq \alpha \pi(p, w) + (1-\alpha) \pi(p', w')$$

③ Hotelling's Lemma

$\frac{\partial \pi}{\partial p_l} = y_l(p, w)$ is firm supply of out l.

$\frac{\partial \pi}{\partial w_l} = -z_l(p, w)$ is (minus) firm demand for z_l

Pf. sketch: $\pi(p, w) = p \cdot y(p, w) - w \cdot z(p, w)$

By Envelope Theorem, $\frac{\partial \pi}{\partial p_l} = y_l(p, w) - 0$

$$\frac{\partial \pi}{\partial w_i} = 0 - z_i(p, w)$$

① Law of supply

$\Delta p \cdot \Delta q \geq 0$

Law of Factor Demand

$\Delta w \cdot \Delta z \leq 0$

Proof: If π is twice differentiable

$$\therefore \frac{\partial q_i}{\partial p_i} = \frac{\partial^2 \pi}{\partial p_i^2} \geq 0$$

$$\frac{\partial z_i}{\partial w_i} = -\frac{\partial^2 \pi}{\partial w^2} \leq 0$$

Consumer Theory

→ ordinal → $x \succ y$ → "x is strictly preferred to"

$x \succsim y$ → "x is weakly preferred to"

$x \sim y$ → "x is indifferent to y"

$\leftrightarrow x \succsim y$ & $y \succsim x$

Axiom 1 $\succsim$ is complete

$\therefore x \succsim y$ or $y \succsim x$ for $\forall x, y \in \underline{X}$ ← domain of choices

2. is transitive i.e. $x \succsim y$ and $y \succsim z \Rightarrow x \succsim z$

Also $\succ$ is transitive

$x \succ y$ & $y \succ z \Rightarrow x \succ z$

is anti asymmetric.

$x \prec y \Rightarrow \neg y \succ x$

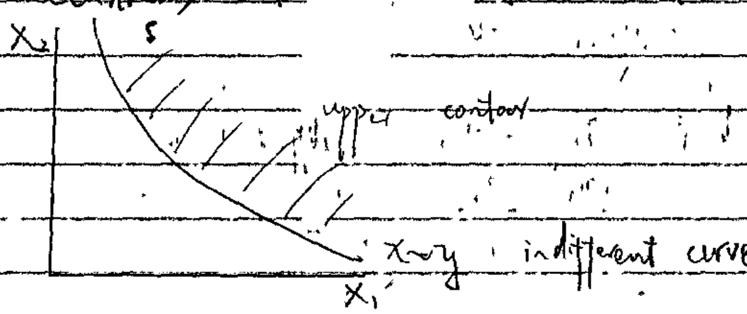
A.3 $\succsim$ is continous

i.e. if $x_n \succsim y_n$ $\forall n = 1, \ldots, n$

and $x_n \to \bar{x}$, $y_n \to \bar{y}$

then $\bar{x} \succsim \bar{y}$

More technically, $\{x \in X \mid x \succsim y\}$ is closed



Choice Theory

complete	$\succ$ anti systemetric
transitive	transitive
continuous	continous

$\cdot\, x$ $\quad$ $x \succ y$

$\cdot\, x'$

$\cdot\, y$ $\quad$ $x' \precsim y$ cannot happen for x' close enough to x

Assume space of consumption bundles $X \subseteq \mathbb{R}^n$

→ monotonic preference: $x_k > y_k$ $\forall k$ $\Rightarrow x \succ y$

strongly

"Goods are "good""

"more is better"

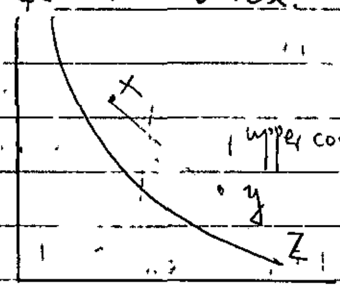
→ local non-satiation:

$\forall x$ $\exists$ y in close to x $(\forall \varepsilon > 0)$ with $y \succ x$

⇒ rules out "bliss point"

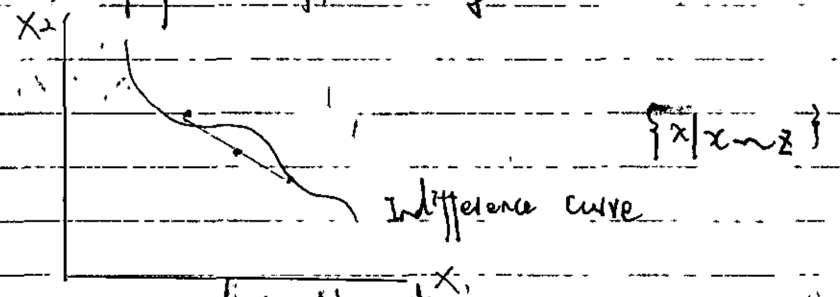
strongly monotonic → monotonic → local non-satiation

Preferences are convex



If $x \succeq z$ and $y \succeq z \Rightarrow \alpha x + (1-\alpha) y \succeq z$

→ preference for variety



don't like diversity

→ non-addictive preference

people → extreme.

Utility Theory

Jeremy Bentham

N actors

$\binom{N}{2} = \frac{N(N-1)}{2}$ utility = index numbers

$$u(x) \geq u(y) \iff x \succeq y$$

Does a utility function exist?

① If preference are over a $\boxed{\text{finite}}$ bundle X then $\succeq$ admit a utility repretation

Pf: Sort X as $x_n \succeq x_{n-1} \dots \succeq x_{\dots}$ in a bubble sort

Pick $u(x_1) = 0$ $\quad u(x_2) = 0$ if $x_2 \sim x_1$ etc.

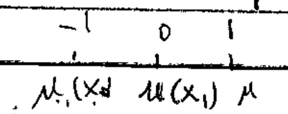
$\qquad\qquad\qquad\qquad 1$ if $x_2 \succ x_1$

② If X is countable infinite, then $\succeq$ admit a utility representation

$u(x_1) = 0 \quad u(x_2) = -1$ if $x_1 > x_2$

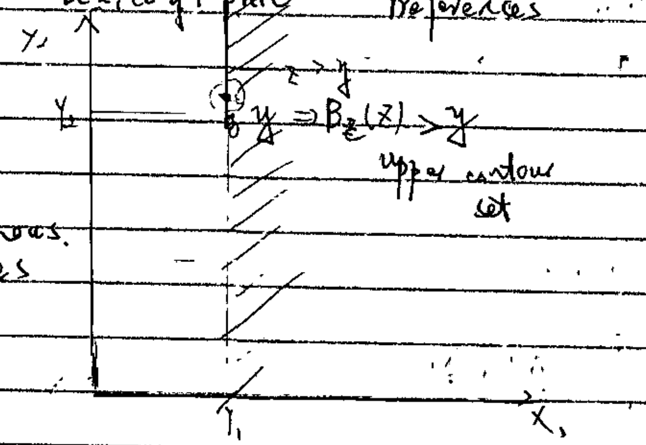
$\qquad\qquad\qquad\qquad 0 \quad x_1 \sim x_2$

$\qquad\qquad\qquad\qquad 1 \quad x_1 < x_2$



An uncountable set might not allow a utility function representation

Lexicographic Preferences



Not continuous Preferences

$X = \mathbb{R}^2$

$x = (x_1, x_2)$

$x \succ y$ if $x_1 > y_1$

or $x_1 \leq y_1$, $x_2 > y_2$

$x \sim y$ if $x_i = y_i$ for $i = 1, 2$

Debreu's Theorem

If $(X, \succsim)$ obey completeness, transitivity & continuity

$\exists$ $u\ X \to \mathbb{R}$ that represents $\succsim$

1. i.e., $u(x) \geq u(y)$ iff $x \succsim y$

$\rightarrow f(u(x)) = v(x) \geq v(y) = f(u(y))$

Further, any other utility function $V = f(u)$, where f is an increasing function, also represents $\succsim$

e.g. if $u(x) = \sqrt{xy}$ is utility for money, then

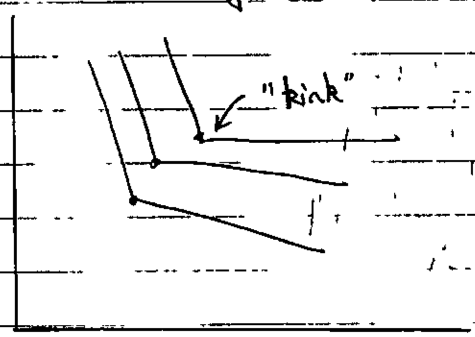
$V(x) = x^{3/4}$ is also a utility for money

$\succsim$ are complete, transitive, continuous and monotonic, then $\exists$ indifference curves

"smooth Preference"

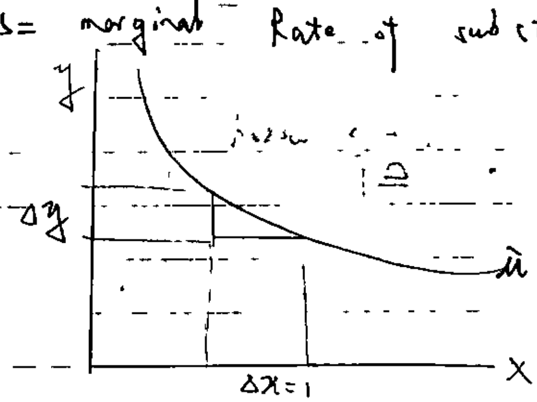
Differentiable utility

u_x u_y exist & are continuous



TOTREP

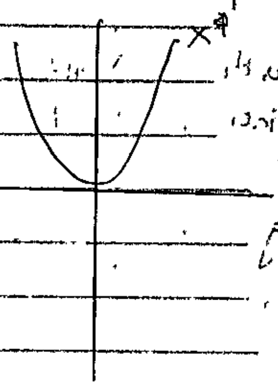
MRS = marginal Rate of substitution ~ RTS



$$MRS \equiv -\frac{dy}{dx}\Big|_{\bar{u}} = \frac{u_x}{u_y}$$

MRS does not depend on the ordinal representation of u.

$$u(x,y) = \sqrt{xy}$$
$$V(x,y) = u^4(x,y) = x^2y^2$$



$$MRT\ u = \frac{u_x}{u_y} = \frac{(y/x)^{1/2} \cdot 1/2}{(x/y)^{1/2} \cdot 1/2} = \frac{y}{x}$$

$$MRS^u = \frac{V_x}{V_y} = \frac{2xy^2}{2yx^2} = \frac{y}{x}$$

$$V = f(u)$$

$$MRT^u = \frac{f'(u)\,u_x}{f'(u)\,u_y} = \frac{u_x}{u_y}$$

$$V(x,y) = \log x + \log y$$
$$u(x,y) = x^{\alpha}y^{\beta}$$

Revealed Preference. "RATIONAL" → "CHOICES"
CHOICES → RATIONATTY

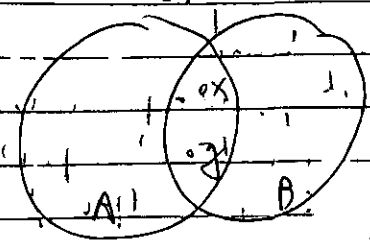
Choice Function
A map from X → subset of X
$C(B) \subseteq B$

Weak Axiom of Revealed Preferences (WARP)

A choice function C obeys:

if $x, y \in A \cap B$, $x \in C(A)$ and $y \in C(B)$

then $x \in C(B)$ $y \in C(A)$



If $C(\cdot)$ is point valued, then WARP says

if $x, y \in A \cap B$, $C(A) = x$, $C(B) = y$

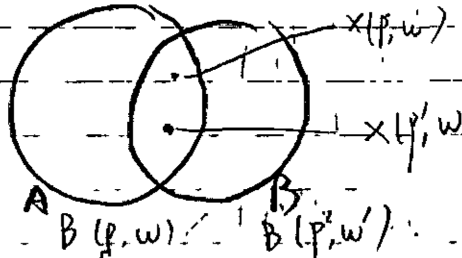
then $x = y$

2.5 Revealed Preference

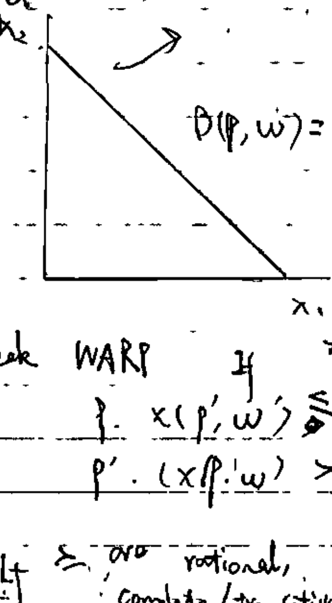
WARP

$x \in C(B)$

$y \in C(A)$



Budget Sets



$B(p, w) = \{x | R^n | p \cdot x \le w\}$

weak WARP If $x(p', w') \neq x(p, w)$, if

$p \cdot x(p', w') \le w$ then

$p' \cdot (x(p, w)) > w'$

x RP x

$\Rightarrow$ Not x SRP x'

strictly

If $\succsim$ are rational, the

complete / transitive

continuous

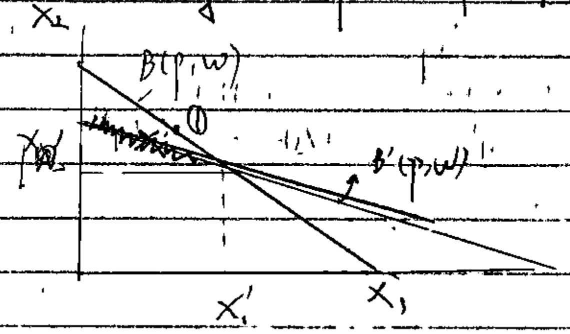
If $\succsim$ are strictly convex, then p??? value. WARP holds

Law of Compensated demand

If (p', w') change, $w' = p' \cdot x(p, w)$

then $(p'-p)[x(p', w') - x(p, w)] \le 0$

Strictly < 0 if $x(p, w) \ne x(p', w')$



Proof: $p' x(p', w') = w'$ $\quad p\, x(p', w) = w$

Assume $p' \cdot x(p, w) = w' = p\, x(p', w')$

$p'[x(p', w') - x(p, w)] = 0$ ①

$p' \Delta x = 0$

$x' \; RP \; x \quad$ not $x' \; SRP \; x'$

$x \ne x' \quad \Rightarrow p\, x(p', w') \ge w = p\, x(p, w)$ ②

think of $RP \Rightarrow \succsim$

$x' \succsim x$

not $x' \; RP \; x$

②−①

$\Rightarrow (p - p')(x(p', w') - x(p, w)) \ge 0$

WARP $\quad$ If $x \succsim y$

not $y \succsim x$

IGNORES TRANSITIVITY

SARP (Strong Axiom of RP)

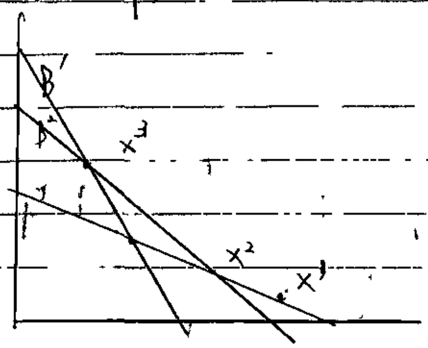
For any $(p^1, w^1), (p^2, w^2), \ldots, (p^N, w^N)$

and choice $x(p^{n+1}, w^{n+1}) \ne x(p^n, w^n) \quad \forall n \le N-1$

we have $p^n \cdot x(p^{n+1}, w^{n+1}) \leq w^n \quad \forall n \leq N-1$

$\Rightarrow p^N \cdot x(p^{n+1}, w^{n+1}) \leq w^n \quad \forall n \leq N-1$

$\Rightarrow p^N x(p^1, w^1) > w^N$



$x^1 RP\, x^2 RP\, x^3$

$\Rightarrow$ Not $x^3 RP\, x^1$

Example:

1 $p^1 = (1, 1, 2)$ $x_1 = (1, 0, 0) \leftarrow w_1 = 1$

2 $p^2 = (2, 1, 1)$ $x_2 = (0, 1, 0) \leftarrow w_2 = 1$

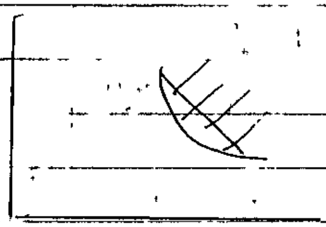
3 $p^3 = (1, 2, 2)$ $x_3 = (0, 0, 1) \leftarrow w^3 = 2$

$x_1\ RP\ x_2$
$x_2\ RP\ x_3$
$x_3\ RP\ x_1$

Violate SARP

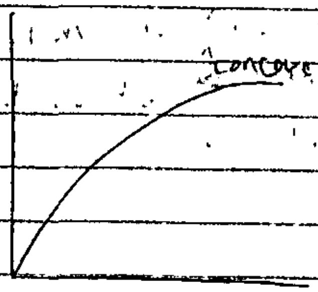
Then SARP $\Rightarrow$ UTILITY MAXIMIZATION

$\succsim$ convex



quasiconcavity

concavity If $u(\alpha x + (1-\alpha) y) \geq \alpha u(x) + (1-\alpha) u(y)$



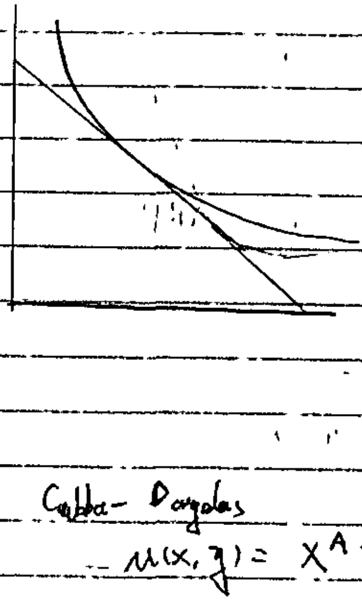
concavity is not qualified property.

concavity $\Rightarrow$ Q concavity.
$\not\Leftarrow$

$u(\alpha x + (1-\alpha) \geq \min(u(x), u(y))$

$\geq$

strictly quasiconcave



Assume quasiconcave & differentiatity utility function

$\max_x u(x)$ s.t $p \cdot x \leq w$

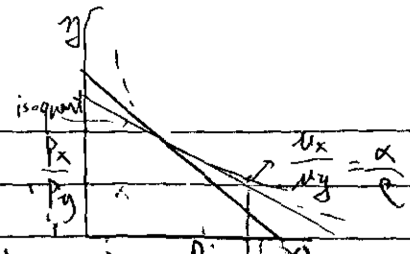
$\to$ unique x if utility fuction is strictly quasiconcave

e.g Cobb-Douglas

$u(x, y) = x^A y^\beta$

FOC $\quad x(p.w) = \frac{w}{p_x} \cdot \frac{\alpha}{\alpha+\beta}$

$y(p.w) = \frac{w}{p_y} \cdot \frac{\beta}{\alpha+\beta}$

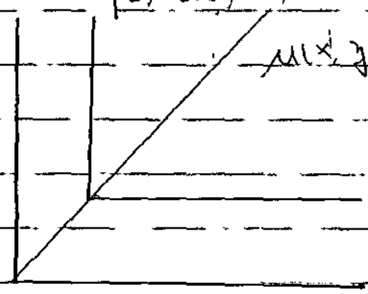


PEFECT

$$u(x,y) = \alpha x + \beta y$$

$$x(p,w) = \begin{cases} \frac{w}{p_x} & \text{if } \frac{p_x}{p_y} < \frac{\alpha}{\beta} \\ \text{Budget line} & > \\ 0 & \end{cases}$$

PEFECT Complements

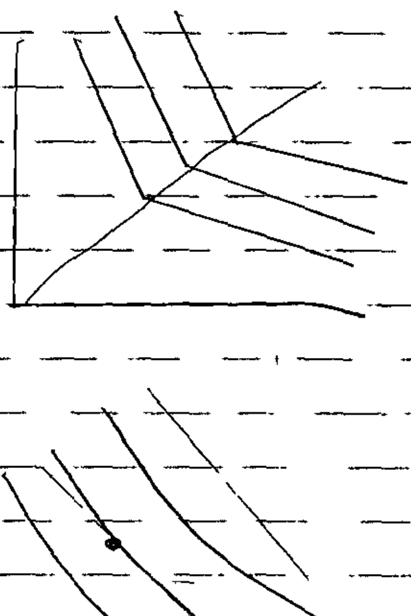


$$u(x,y) = \min(\alpha x, \beta y)$$

$$x(p,w) =$$

$$\frac{x(p,w)}{y(p,w)} = \frac{\alpha}{\beta}$$

$$x p_x + y p_y = w$$



$$\mathcal{L} = u(x,y) + \lambda(w - x p_x - y p_y)$$

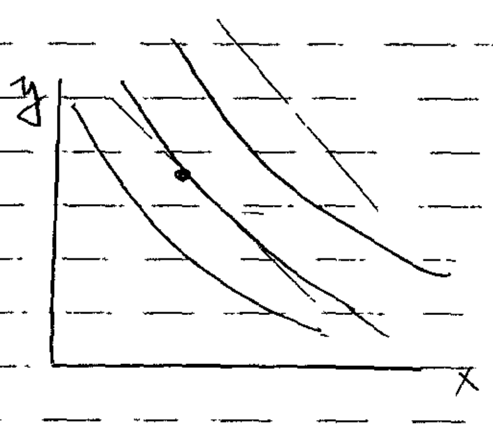
FOC

interior solution

$$\frac{u_x}{u_y} = \frac{p_x}{p_y}$$

$$\lambda = \frac{u_x}{p_x} = \frac{u_y}{p_y}$$

$$\rightarrow \frac{u_x}{p_x} - \frac{u_y}{p_x}$$



corner solution

$$\mathcal{L}_x = u_x - \lambda p_x \le 0 \quad x \ge 0$$

$$x(u_x - \lambda p_x) = 0$$

$$\mathcal{L}_y = u_y - \lambda p_y \le 0 \quad y \ge 0$$

$$y(u_y - \lambda p_y) = 0$$

So if $y = 0$ then, $u_y \le \lambda p_y$

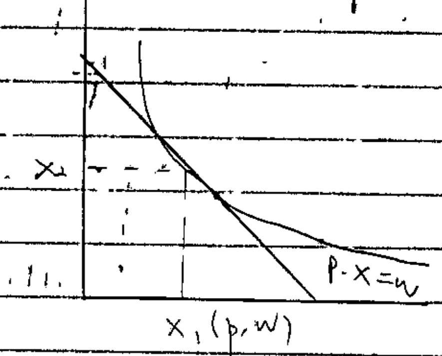
$x > 0$ $u_x = \lambda p_x$

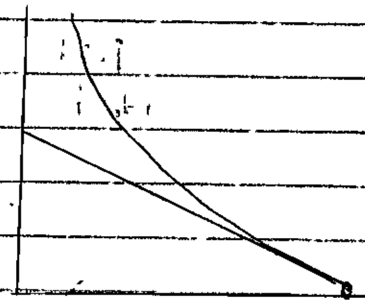
$MRS = \frac{u_x}{u_y} > \frac{p_x}{p_y}$

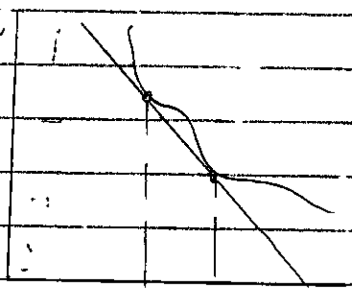
2. 10th.

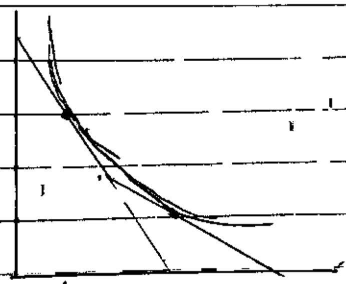
Exam Thursday

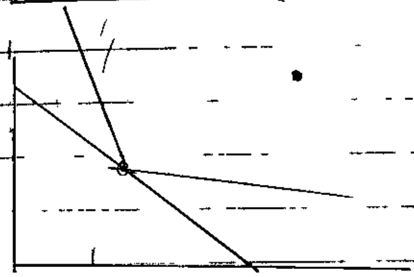
max $u(x)$ s.t. $p \cdot x \le w$











Sloution② is called the Marshallian demand curve.

$\boxed{x(p,w)}$

Comparitive statics

Recall from producer theory

$\vec{x}(tp, tw) = \vec{x}(p, w)$

Changes in income. demand ~~superior~~

x is normal if $x(p,w) \uparrow w$

scalar inferior if $x \downarrow w$

neutral if $x(p,w)$ is constant in w

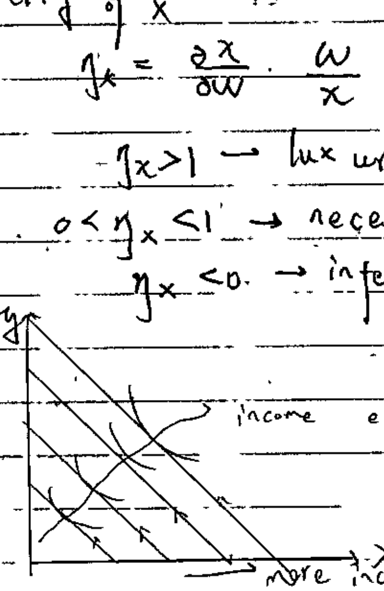
Income elasticity of x

$$\eta_x = \frac{\partial x}{\partial w} \cdot \frac{w}{x}$$

$\eta_x > 1 \to$ luxury

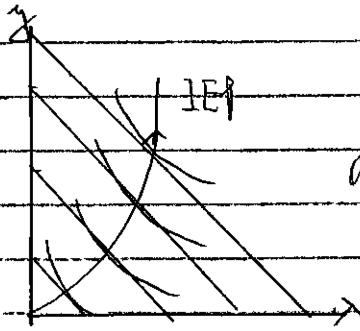
$0 < \eta_x < 1 \to$ necessity

$\eta_x < 0 \to$ inferior



extreme cases:

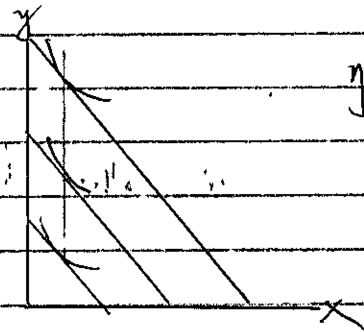
①



$0 < \eta_x < 1$

$P_x X + P_y y = W$

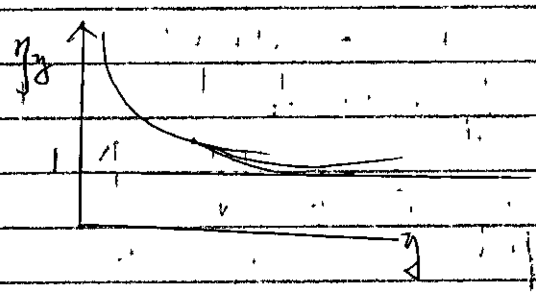
②



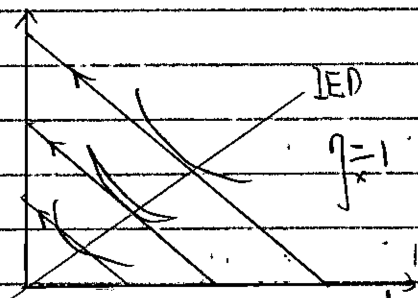
$\eta_x > 1$

$\eta_x = 0$

vertically parallel indifference curve

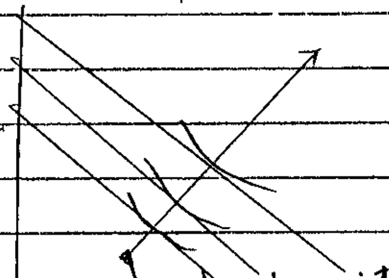


③



$\eta_x = 1$ homothetic preference

a lot of individuals act as one



quasi-homothetic preference

substant point

CES utility

e.g one of CES family

Cobb Douglas utility

$u(x,y) = x^{\alpha} y^{1-\alpha}$

FOC $MRS = \frac{\alpha}{1-\alpha} \frac{y}{x} = \frac{P_x}{P_y}$

Demand: $x = \frac{\alpha w}{P_x}$

$y = \frac{(1-\alpha) w}{P_y}$

APC_x = average propensity to consume of x

$= \frac{x P_x}{w}$

MPC_x = marginal propensity

$= P_x \cdot \frac{\partial x}{\partial w}$

For Cobb-Douglas : $APC_x = \alpha = MPC_x$

$\eta_x = 1$

e.g: quasi - homothetic preference Stone - Geary preference

$u(\vec{x}) = \sum_{i=1}^{n} \alpha_i \log (x_i - \bar{x}_i)$

vector ↑ substantence level

assume where $\sum_{i=1}^{n} \alpha_i = 1$

FOC $\overset{MRS}{\frac{\alpha_i}{x_i - \bar{x}_i}} = \lambda p_i$ ☺

$x_i = \bar{x}_i + \alpha_i \frac{1}{\lambda p_i}$

Solve for λ_i plug ☺ into budget constraint.

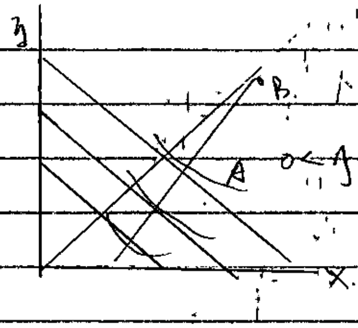
$\sum x_j p_j = w$

$\Longrightarrow \sum \left(\bar{x}_j + \frac{1}{\lambda p_j}\right) p_j = w$

$\frac{1}{\lambda} = w - \sum p_j \vec{x}_j$

$x_i^* = \bar{x}_i + \frac{\alpha_i}{p_i} \left(w - \sum \bar{x}_j p_j\right)$

$$\frac{MPC_x}{APC_x} = \eta_x$$



Comparative Statics for Income change
(2 goods) Smooth interior solution

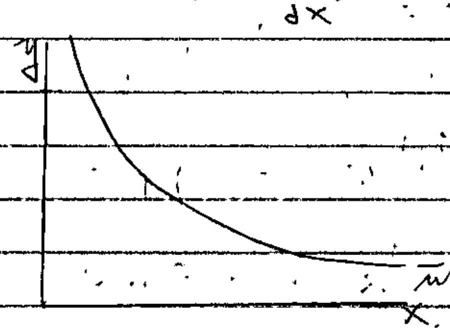
optimality $MRS(x, y) = \frac{P_x}{P_y}$ = slope of budget line

Constraints $xP_x + yP_y \equiv w$

$$\frac{\partial MRS}{\partial x}\frac{\partial x}{\partial w} + \frac{\partial MRS}{\partial y}\frac{\partial y}{\partial w} = 0$$

$$P_x \frac{\partial x}{\partial w} + P_y \frac{\partial y}{\partial w} = 1$$

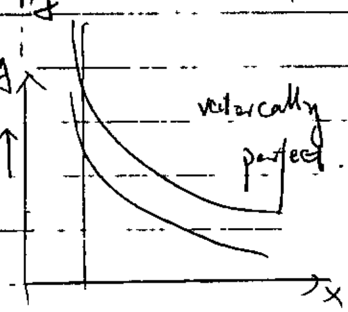
$$\frac{\partial x}{\partial w} = \frac{-\frac{\partial MRS}{\partial y}\cdot\frac{1}{P_y}}{\frac{\partial MRS}{\partial x} - \frac{P_x}{P_y}\frac{\partial MRS}{\partial y}}$$



But $\frac{dMRS}{dx}\Big|_{\bar{u}} = \frac{\partial MRS}{\partial y}\underbrace{\frac{dy}{dx}}_{-\frac{P_x}{P_y}} + \underbrace{\frac{\partial MRS}{\partial x} < 0}_{\text{convex MRS}}$

when is $\frac{\partial x}{\partial w} = 0$

① $\frac{\partial MRS}{\partial y} = 0$

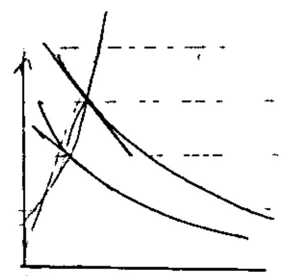


② denominator $= \infty$

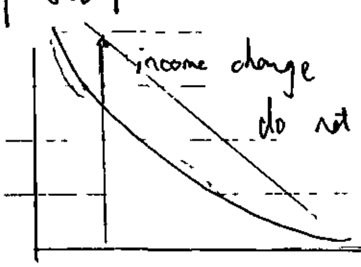
(ruled out by saying $MU_y > 0$

when is $\frac{\partial x}{\partial w} > 0$.

$\frac{\partial MRS}{\partial y} > 0$



when is $\left|\frac{\partial x}{\partial w}\right|$ larger? (large γ_x)



So consider price change $\quad \frac{\partial MRS}{\partial x}\frac{\partial x}{\partial P_x} + \frac{\partial MRS}{\partial y}\frac{\partial y}{\partial P_x} = \frac{1}{P_y}$

$$P_x\frac{\partial x}{\partial P_x} + P_y\frac{\partial y}{\partial P_x} = -X$$

$$\frac{\partial x}{\partial p_x} = \frac{1}{p_y}\left(\frac{\partial MRS}{\partial x} - \frac{p_x}{p_y}\frac{\partial MRS}{\partial y}\right)^{-1} + x\,\frac{\partial MRS}{\partial y}\,\frac{1}{p_y}\left(\frac{\partial MRS}{\partial x}\right.$$

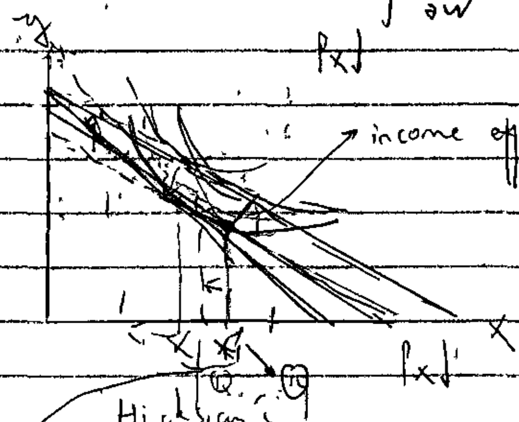
$$\left. - \frac{p_x}{p_y}\frac{\partial MRS}{\partial y}\right)^{-1}$$

substitution Effect

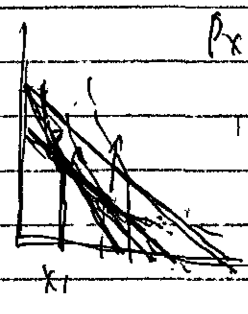
$$= (-) + (?)$$

Income effect

$-x\frac{\partial x}{\partial w}$ latter plugging our earlier expression of $\frac{\partial x}{\partial w}$



$p_x \downarrow$



Hicksian's

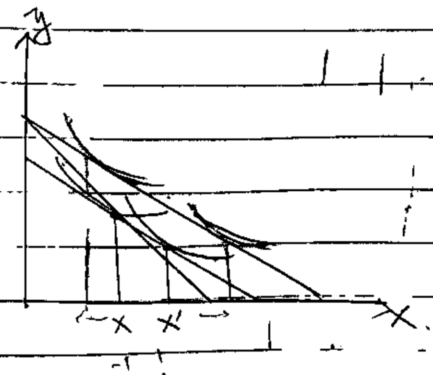
substitute effect

decompensation of income substitute effects

$\Delta P \Delta x > 0$

law of demand fails

$p_x \downarrow$ $x \downarrow$ "good"

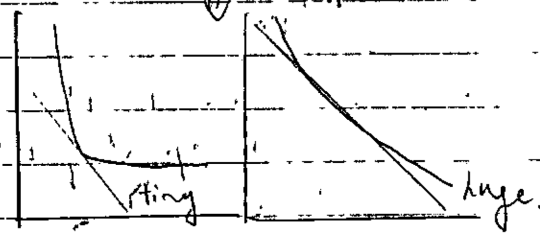


2.11th

Last time with two goods, how curved are indifferent curve

$$\frac{\partial x}{\partial p_x} = \frac{1}{p_y}\left(\frac{\partial MRS}{\partial x} - \frac{p_x}{p_y}\frac{\partial MRS}{\partial y}\right)^{-1} - x\frac{\partial x}{\partial w}$$

substitution effect — income effect



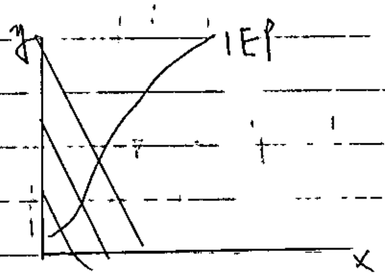
⊖ normal goods

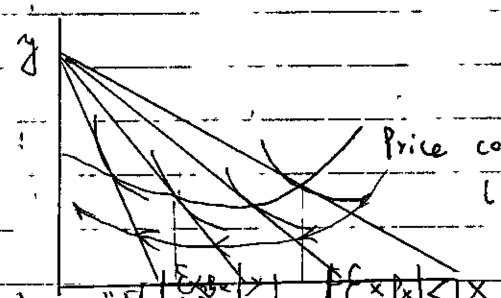
⊕ inferior goods

⊖

Slutsky Equation

($\exists$ law of demand)





show "the elastic curve" is horizontal for Cobb - Douglas function. /utility

Total Expenditure = $TE = x p_x$

$$ME = \frac{\partial}{\partial p_x} TE = x + p_x \frac{\partial x}{\partial p_x} \Rightarrow x\left(1 + \frac{p_x}{x}\frac{\partial x}{\partial p_x}\right) = x(1 + \mathcal{E}_{xp_x})$$

so as $p_x \uparrow$, $TE \uparrow$ if $\mathcal{E}_{xp_x} > -1$

or $|\mathcal{E}_{xp_x}| < 1$

TE $\downarrow$ if $|\varepsilon_{x p_x}| > 1$

TE on Y $\downarrow$ as $P_x \uparrow$
TE on X $\uparrow$ as $P_x \uparrow$

Cobb-Douglas function $\varepsilon_{x p_x} = -1$

Duality for Consumer Theory

Expenditure Function

$e(p, u) = \min p \cdot x$ s.t. $u(x) \geq u$

target utility

Solution: $x^h(p, u) = \arg\min \{ p x \mid u(x) \geq u \}$ (Hicksian)

$\Rightarrow e(p, u) = p \cdot x^h(p, u)$

Properties of Expenditure Function

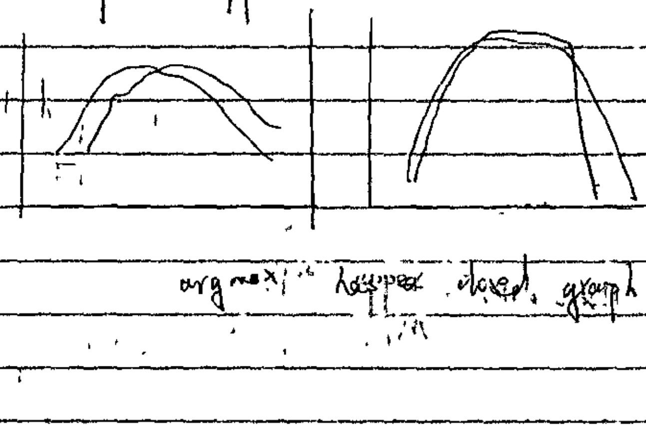
① $e(p, u)$ is homogeneous of degree 1 in p (constant returns to scale)

$e(\alpha p, u) = \alpha p \cdot x^h(p, u)$

$x^h(p, u)$ is ..... 0 in p, i.e. $x^h(\alpha p, u) = x^h(p, u)$ $\forall \alpha > 0$

② $e(p, u)$ is continuous in (p, u) when $p_i > 0$ $\forall$

Proof: applies (Berge's) Theorem of the Maximum

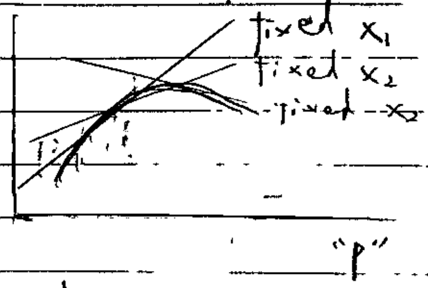


argmax is hemi upper closed graph

③ $e(p,u)$ is strictly increasing in u, p

$\{x \mid u(x) \geq u\}$ $\{x \mid u(x) \geq u_1\}$ $u > u_1$

④ $e(p,u)$ is concave in p



Proof, Exactly as with a cost function

④ Shephard's Lemma

$$\frac{\partial e}{\partial p_i} = x_i^h(p,u)$$

Proof: Exactly as with a cost function)
(apply Envelope Theorem)

Corollary Hicksian demands slope down.

Proof: $\frac{\partial x_i^h}{\partial p_i} = \frac{\partial}{\partial p_i}\left(\frac{\partial e}{\partial p_i}\right) \leq 0$ by concavity.
if the derivatives exist.

Corollary Cross price effects are equal

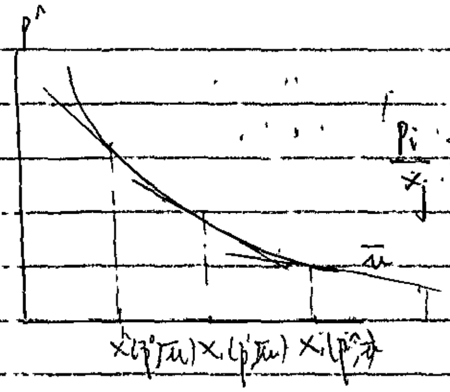
$$\frac{\partial x_i^h}{\partial p_j} = \frac{\partial x_j^h}{\partial p_i}$$

Proof: $\frac{\partial^2 e}{\partial p_j \partial p_i} = \frac{\partial^2 e}{\partial p_i \partial p_j}$ by Young's Theorem

Generalized Slutsky Equation:

Let $x_j^m(p,w)$ be the Marshallian (ordinary) demand for good j.

$$\frac{\partial x_j^m}{\partial p_j} =$$



$$\frac{p_i}{x_j}\frac{\partial x_j^n}{\partial p_i} = \frac{\partial x^h}{\partial p_i}\frac{p_i}{x_j} - \frac{p_i x_i}{w}\frac{\partial x_j^n}{\partial w}\frac{w}{x_j}$$

where $x^n(p,w) = x^h(p,u)$

Application $\quad \varepsilon^n_{x_i,p_j} = \varepsilon^h_{x_j,p_i} - \overset{\text{share}}{s_i}\,\eta_{x_j}$

Proof of Generalized Slutsky

$$x_j^h(p, u) \equiv x_j^m(p, \overbrace{e(p(p,u))}^{w})$$

$$\frac{\partial x_j}{\partial p_i} = \frac{\partial x^m}{\partial p_i} + \frac{\partial e}{\partial p_i}\frac{\partial x_j^m}{\partial w}$$

$$= \frac{\partial x_j^n}{\partial p_i} + x_j\frac{\partial x_j^m}{\partial w}$$

$$\Rightarrow \frac{\partial x_j^n}{\partial p_i} = \frac{\partial x_j^h}{\partial p_i} - x_j\frac{\partial x_j^m}{\partial w}$$

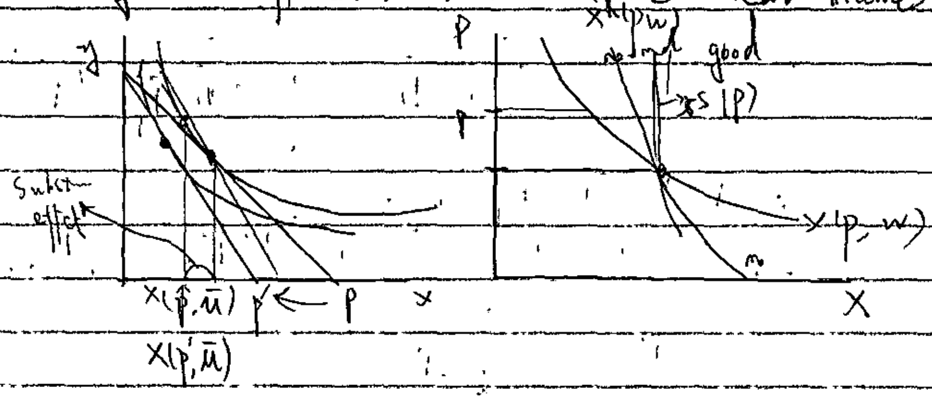
S J

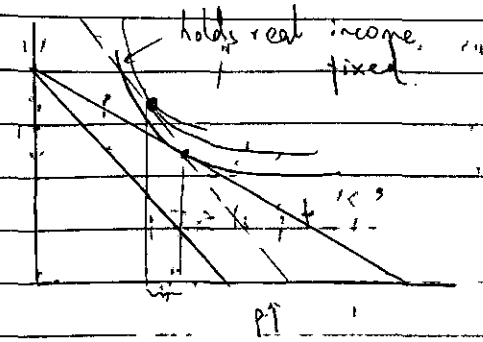
Three Demand Curve

① Marshallian (Ordinary Demand)

② Hicksian demand (fixed utility)

③ Slutsky - compensated demand (fixed real income)

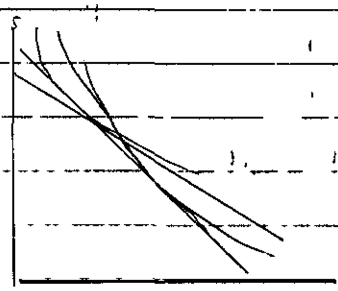




Price Indexes

Lasperes Index (CPI) $= \dfrac{p' \cdot x}{p x}$

Ideal Price Index $= \dfrac{E(p', u(x))}{p \cdot x} = \dfrac{E(p', u(x))}{E(p, u(x))}$



Indirect Utility Function

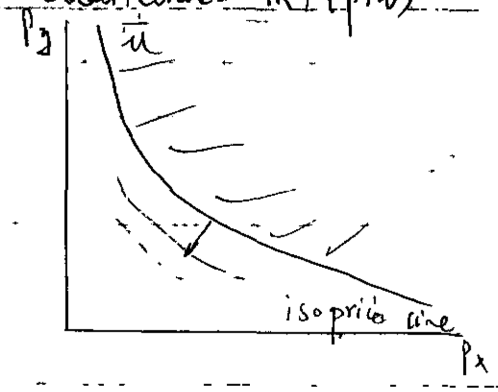
$V(p,w) = \max U(x)$ s.t. $p \cdot x \leq w$

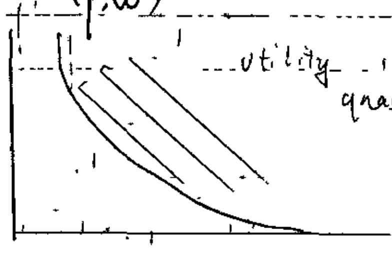
Properties:

① Non decreasing in w, non increasing in p

② homogeneous of degree 0 in (p,w)

③ Quasi convex in (p,w)



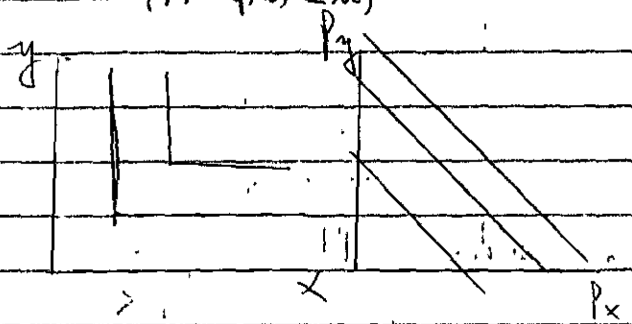


e.g. Cobb-Douglas utility $u(x,y) = x^{\alpha} y^{1-\alpha}$

$v(p_x, p_y, w) =$

i.e. lower contour sets are convex.

$\{p \mid v(p,w) \le \bar{u}\}$



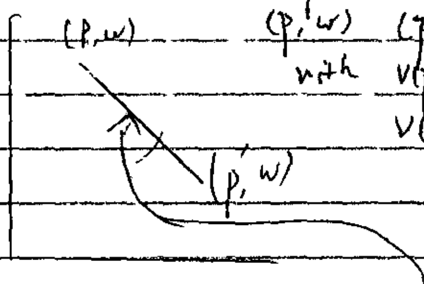
Proof of Quasiconvexity

Take two points in lower contour set.

(p,w) (p',w')

with $v(p,w) \le \bar{u}$

$v(p',w') \le \bar{u}$



$\bar{p} = \alpha p + (1-\alpha) p'$

$\bar{w} = \alpha w + (1-\alpha) w'$

$p \cdot x(p,w) \le w$

$p' x(p',w') \le w'$

$\bar{p} \cdot x(\bar{p},\bar{w}) \le \bar{w}$

$\hookrightarrow \alpha p + (1-\alpha) p' \; x(\bar{p},\bar{w}) \le \alpha w + (1-\alpha) w'$

$\Rightarrow \alpha \left[p \, x(\bar{p},\bar{w}) - w \right] + (1-\alpha) \left[p' x(\bar{p},\bar{w}) - w' \right] \le 0$

Either $p \cdot x(\bar{p},\bar{w}) \le w$

or $p' \cdot x(\bar{p},\bar{w}) \le w'$

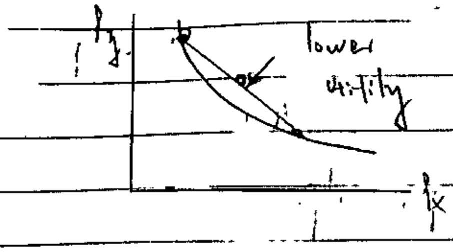
or both

case ① $x(\bar{p},\bar{w})$ is affordable at (p,w)

$\Rightarrow u(x(\bar{p}, \bar{w})) = v(\bar{p}, \bar{w}) \le v(p, w)$

case ② similar

so: $v(\bar{p}, \bar{w}) \le \max\{v(p,w), v(p',w')\}$



⑤ $v(p,w)$ is continuous in (p,w)

Proof: (Theorem of Maximum)

⑥ Roy's Identity

$$x_i^m(p,w) = \frac{\dfrac{-\partial v(p,w)}{\partial p_i}}{\dfrac{\partial v(p,w)}{\partial w}} \qquad \frac{u / \$/x_1}{u}$$

marginal utility of money 2

\$ 1 ↑ p $p_i \Rightarrow p_i \Rightarrow$

utility fall 4 units

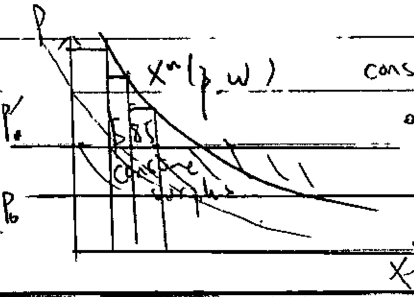
\$2p, w

Proof: $v(p, \overset{w}{e(p,u)}) \equiv u$ $\qquad (p,u) \equiv x^m(p,w)$

Differentiate in p_i $\qquad \equiv x_i^h = x_i^m$ $\qquad \boxed{x^h(p,u) \equiv x^m(p, E(p,u))}$

$$\frac{\partial v(p,w)}{\partial p_i} + \frac{\partial v(p,w)}{\partial w} \cdot \frac{\partial e}{\partial p_i} = 0$$

$$x_i^m(p,w) = \frac{\dfrac{-\partial v}{\partial p_i}}{\partial v / \partial w}$$



consumer surplus

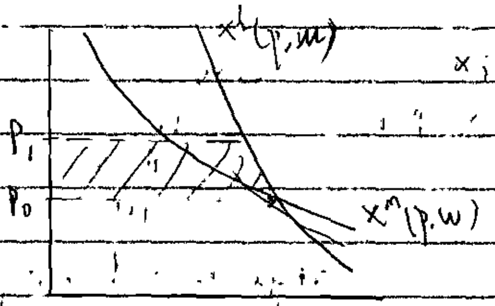
overestimate willingness to pay for a normal good

Compensating Variation

what is someone willing to pay for a price change.

$CV = e(p_1, u_0) - w$ where $w = e(p_0, u_0)$

$$CV = e(p_1, u_0) - e(p_0, u_0)$$
$$= \int_{p_0}^{p_1} \frac{\partial e}{\partial p}(p, w)\, dp$$
$$= \int_{p_0}^{p_1} x^h(p, u)\, dp$$



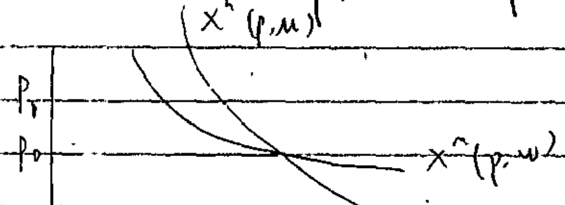
x is a normal good

for a price rise, and a normal good,
the loss in consumer surplus
< true CV

for price rise

The equivalent variation is the income change that makes you just as well as if a price change happened.

$$EV = e(p_0, u_1) - w$$
$$= e(p_0, u_1) - e(p_1, u_1)$$



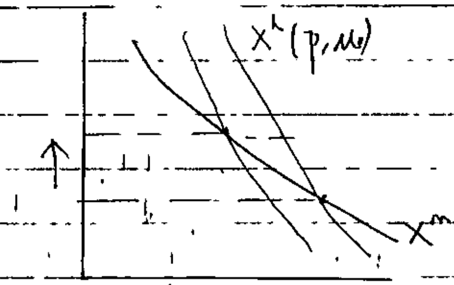
2. 24th

(WTP) Compensating Variation.

$$CV = \underbrace{e(p_1, u_0)}_{\text{new income needed to get old utility}} - e(p_0, u_0) = \int_{p_0}^{p_1} x^h(p, u_0)\,dp$$

(WTA) Equivalent Variation.

$$EV = e(p_1, u_1) - e(p_0, u_1) = \int_{p_0}^{p_1} \frac{\partial e}{\partial p}(p, u_0)\,dp$$
$$= \int_{p_0}^{p_1} x^h(p, u_1)$$



x normal good

Slutsky Equ.

$$\frac{\partial x_j^m}{\partial p_i} = \frac{\partial x_j^h}{\partial p_i} - \frac{\partial x_j^m}{\partial w} x_j^m$$

complement / substitute.

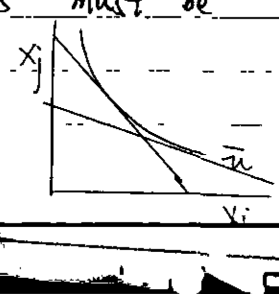
gross $\quad \dfrac{\partial x^m_{\text{tennis balls}}}{\partial x^m_{\text{rackets}}} < 0 \quad , \quad \dfrac{\partial x^m_{\text{tea}}}{\partial p^m_{\text{coffee}}} > 0 \quad$ but $\quad \dfrac{\partial x^m_{\text{coffee}}}{\partial p_{\text{tea}}} < 0?$

equality of cross effects for Hicksian goods

$$\frac{\partial x_i^h}{\partial p_j} = \frac{\partial}{\partial p_j} \frac{\partial e}{\partial p_i} = \frac{\partial}{\partial p_i} \frac{\partial e}{\partial p_j} = \frac{\partial x_j^h}{\partial p_i}$$

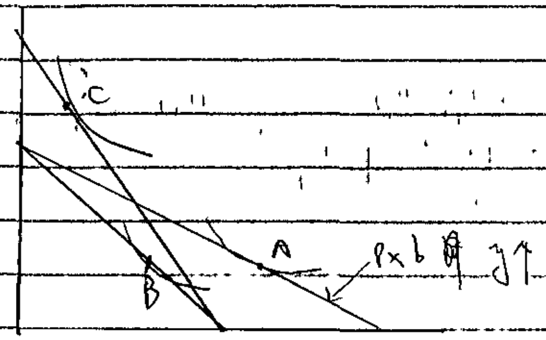
→ Goods i & j are Hicks-Allen substitutes if $\dfrac{\partial x_i^h}{\partial p_j} > 0$

In a two good world, both goods must be Hicks-Allen substitute.



$$\frac{\partial x_j^m}{\partial p_i} = \frac{\partial x_j^h}{\partial p_i} - \frac{\partial x_j}{\partial w} x_i^m \quad \oplus$$

$$\frac{\partial x_i^m}{\partial p_j} = \frac{\partial x_i^h}{\partial p_j} - \frac{\partial x_i}{\partial w} x_j^m \quad \ominus$$



Labor Economics

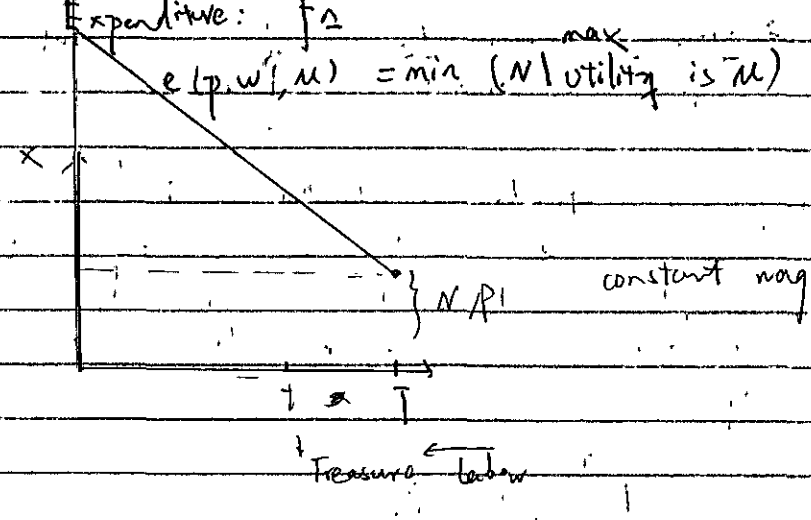
$\max u(l, x)$ s.t. $px \le w(T - l) + N$

leisure, other goods, wage, labor, non-labor income, Total hours = 24

$px + wl \le wT + N$ "full income"

Expenditure: fn

$e(p, w, \bar{u}) = \min (N \mid \text{utility is } \bar{u})$ max



Shepard's lemma $\quad L = T - t$

$$\frac{\partial e}{\partial w} = -L = t^h - T$$

$$\max u(x, T-L) \quad s.t \quad px - wL \le N$$

$$\max_{x,z} u(x, T+z) \quad s.t \quad px + wz \le N$$

$$\frac{\partial e}{\partial w} = z = -L$$

Income & Substitution Effects of a wage increase.

$$\underset{\text{labor}}{L} = T - \underset{\text{Leisure}}{t}$$

Define $t^m(p, w, N)$, $t^h(p, w, N)$ as usual

$$L^m(p, w, N) = T - t^m$$

$$L^h(p, w, N) = T - t^h$$

$$L^h(p, w, u) = L^m(p, w, \underbrace{e(p, w, u)}_{N})$$

Differentiate

$$\frac{\partial L^h}{\partial w} = \frac{\partial L^m}{\partial w} + \frac{\partial L^m}{\partial N} \cdot \frac{\partial e}{\partial w}$$

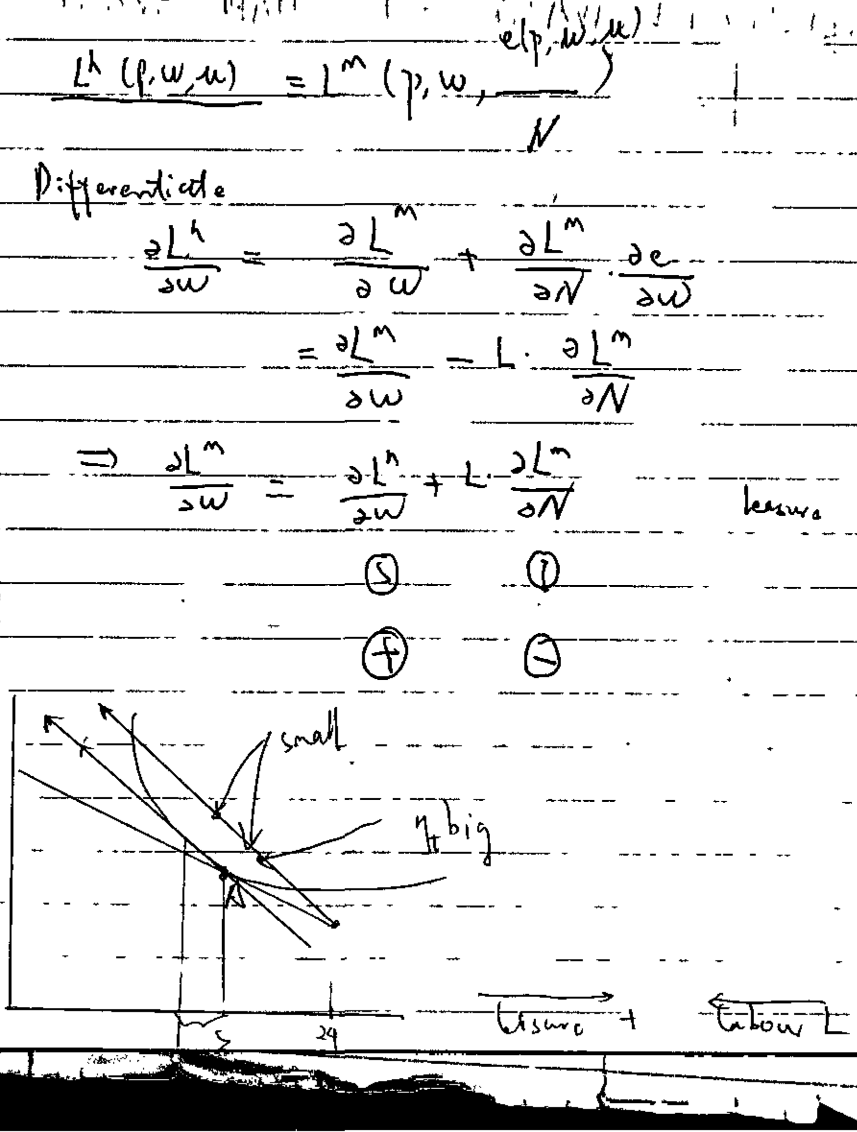
$$= \frac{\partial L^m}{\partial w} - L \cdot \frac{\partial L^m}{\partial N}$$

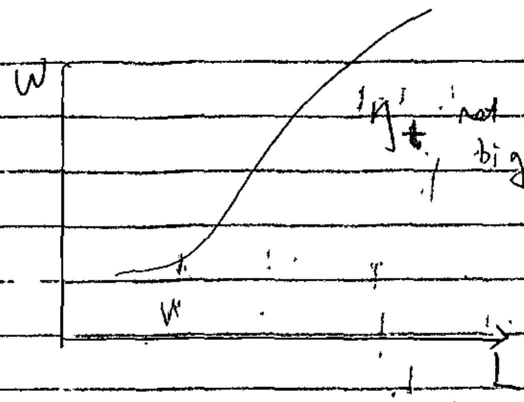
$$\Rightarrow \frac{\partial L^m}{\partial w} = \frac{\partial L^h}{\partial w} + L \cdot \frac{\partial L^m}{\partial N}$$

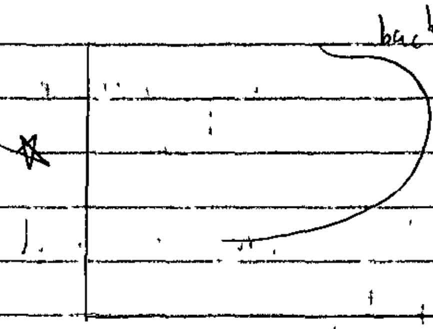
leisure is normal good

S $\quad$ I

+ $\quad$ −

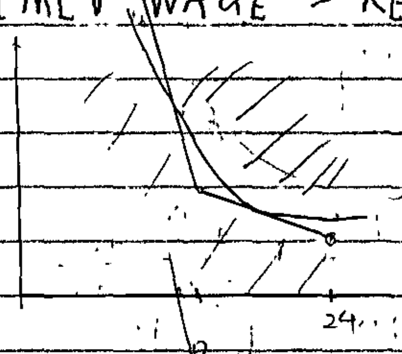


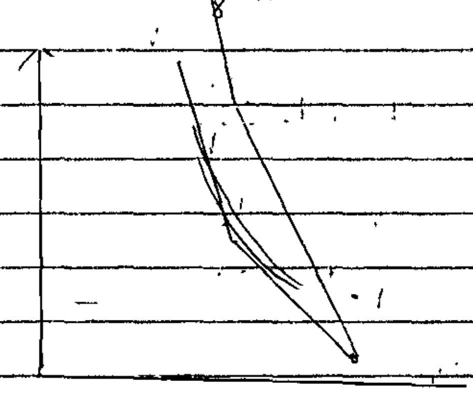




backward bending labor supply curve.

OVERTIMED WAGE > REGUAR WAGE





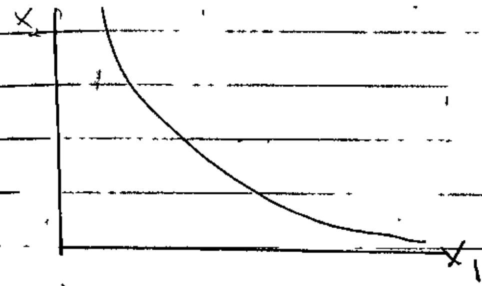
W' = Wot > W

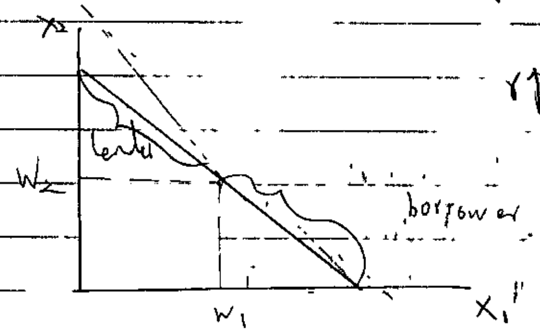
regular wage ↑
work less

Fisher Model of Two Period ~~Consumption~~ Consumption

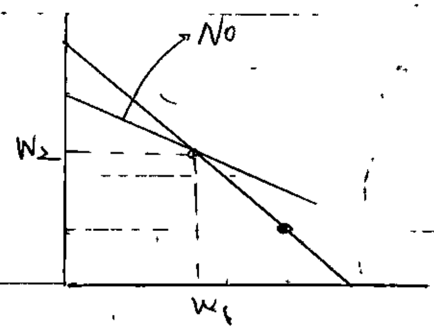
$u(x_1, x_2)$ s.t. $x_1 + \frac{x_2}{1+r} = w_1 + \frac{w_2}{1+r}$

$(1+\frac{1}{n})^n \to e$



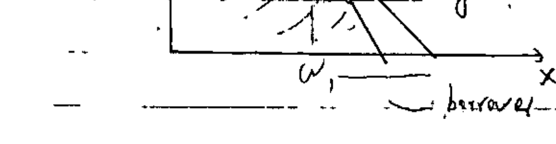


$r\uparrow$



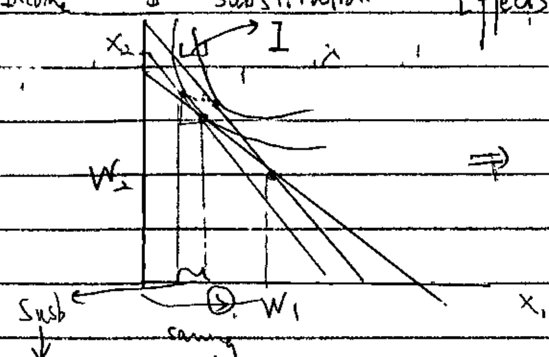
$r\downarrow$

2.26th



Fisher diagram Borrowers remain borrowers when interest rate rises.

Income & Substitution Effects



real
$r\uparrow \iff$ income $\uparrow$
(if saver)
$\Rightarrow$ income effect $\uparrow$

Susb
encourage future lender
discourage now

Choice Under Uncertainty
$\rightarrow$ Expected Utility
Bernoulli 伯努利

1	$\frac{1}{2}$
2	$\frac{1}{4}$
2^2	$\frac{1}{2^3}$
$\vdots$	$\vdots$

$\frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \dots + \frac{1}{8}\cdot 2^2$

Utility for money

$u(w)$
e.g. $u(x) = \sqrt{x}$

$$\sum_{k=1}^{\infty} 2^{-k}\cdot u(2^k) = \sum_{k=1}^{\infty} 2^{-k}\cdot 2^{k/2} = \sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{2}}\right)^k = \frac{\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}-1}$$

$u'(\text{GAMBLE}) = \frac{1}{\sqrt{2}-1}$

"GAMBLE" $= (\quad)^2$

Von Neuman / Morgenstern
1944 Games & Economics behavior

Prizes $X = \{x_1, \dots, x_n\}$

Lottery $L = (p_1, \dots, p_N)$

Space of lotteries $\mathcal{L} = \{ L = (p_1, \dots, p_N) \in \mathbb{R}^N_+,\ p_1 + \dots + p_N = 1 \}$

Compound lottery

$\rightarrow L = (\alpha_1, \dots, \alpha_K)$

where α_i is the chance of winning lottery L_k
where $L_k = (p_1^k, \dots, p_N^k)$
is a lottery over X

Ultimately, prize x_j is won with chance $p_j = \sum_{i=1}^{k} \alpha_i p_j^i$ ⊛

Consequentialism Axiom

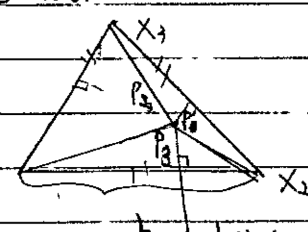
Any compound lottery L is formally equivalent to the reduced form lottery on X with change ⊛

We want preference $\succsim$ on $\mathcal{L}$

$\succsim$ ① complete $L_1 \succsim L_2$ or $L_2 \succsim L_1$

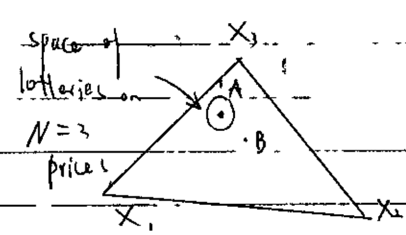
② transitive $L_1 \succsim L_2$ & $L_2 \succsim L_3 \Rightarrow L_1 \succsim L_3$

③ continous



area of $\Delta = \frac{b p_3}{2} + \frac{b p_2}{2} + \frac{b p_1}{2}$

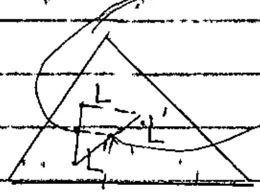
$= \frac{b}{2}(p_1 + p_2 + p_3)$
1



By Debreus Theo, ∃utility function $U(L)$

Instead, call $\succsim$ on $\mathcal{L}$ continuous if $\forall L, L', L'' \in \mathcal{L}$, the sets
$\{\alpha \in [0,1] \mid \alpha L + (1-\alpha) L'' \succsim L'\}$
and $\{ \qquad \precsim \}$
are closed sets in $[0,1]$

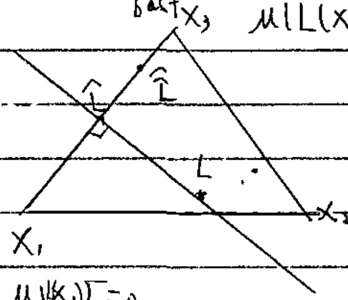
Independence Axiom (IA)
If $L \succsim L'$
Then $\alpha L + (1-\alpha) L'' \succsim \alpha L' + (1-\alpha) L''$
eggs > hamburger
eggs + ketchup hamburger + ketchup



$L \sim L'$
i.e. $\alpha = 1/2$

IA $\Rightarrow$ If $L \sim L'$
then $\alpha L + (1-\alpha) L'' \sim$
$\alpha L' + (1-\alpha) L''$

Indifference curves are linear & parallel in $\mathcal{L}$



$u(L(x_3)) = 1$
$\hat{L} \sim L$
project $\mathcal{L}$ on to the (x_1, x_3) side.
choice utility $u(\hat{L})$

$u(L(x_1)) = 0$.

Pick $u(\hat{L}) = P_r(x_3 \mid \hat{L})$

vNM Expected Utility Theo.
Continuous preference $\succsim$ on $\mathcal{L}$
that obeys the IA can be represented
by a utility function U that use
linear in probabilities

$u(L) = \sum_{i=1}^{n} p_i u_i$, and

any other $au+b$ also works the vector $u = (u_1, \dots, u_N)$ is unique

up to a positive affine transformation.

for $a > 0$, $b \in \mathbb{R}$

3.3 2015

vNM Expected Utility Theorem.

A continuous preference relation $\succsim$ on $\mathcal{L}$ that obeys IA can be represented by a utility function that is linear in probability

$\therefore u(L) = \sum_{i=1}^{n} p_i u_i$. The Bernoulli utility function $u = (u_1, \dots, u_N)$ is unique up to a positive affinite function, in any

$v = au+b$, with if $a > 0$

vNM utility

Bernoulli utility

IA $u_i = u(x_i)$. The function u is unique, up to a positive affinite transformation.

Allais (1953)

→ test IA

$x_1 = \$0$ $x_2 = \$10M$ $x_3 = \$50M$

→ L_A → $10M for sure

L_B → 10% of $50M, 89% of $10M, 1% of $0

model choice $L_A > L_B$

→ K_A → 11% of $10M

89% of $0

K_B → 10% of $50M, 90% of 0

model → $K_B > K_A$

① show this violates IA ← you

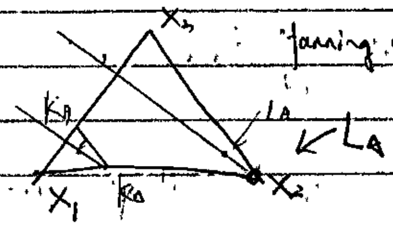
② Show this violate EU Theorem

$\boxed{L}$ $1 \times u_2 > 0.1 u_3 + 0.89 u_2 + 0.01 u_1 \to 0.11 u_2 - 0.1 u_3 - 0.01 u_1 > 0$

$\boxed{K}$ $0.11 u_2 + 0.89 u_1 < 0.1 u_3 + 0.9 u_1$

$\to 0.11 u_2 - 0.1 u_3 - 0.01 u_1 < 0$

certainty effect



value of life
risky activity

→ probability $p > 0$ at bad outcome (die).

Premium $\pi = \$200$ good bad

$u(\omega) = (1-p)\, u(\omega + \pi) + p\, u(\omega + \pi - L)$

Bernoulli: u concave so $u' > 0 > u''$

Assume u is linear i.e. $u(x) \equiv x$

$\omega = (1-p)(\omega + \pi) + p(\omega + \pi - L)$

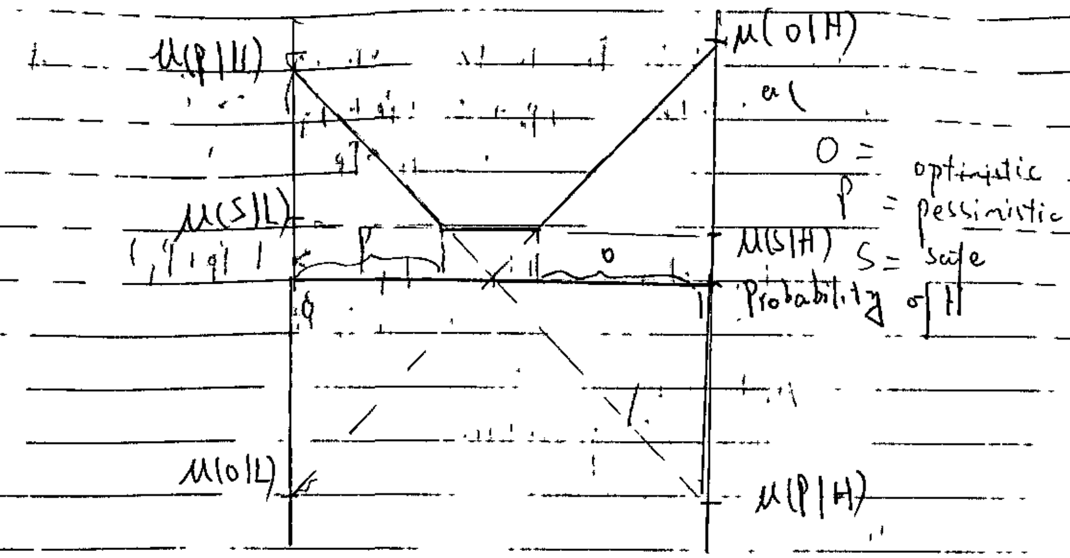
$\omega = \omega + \pi - pL \rightsquigarrow L = \pi/p$

Choice in Discrete Cases with Risk

	H	T
H	1, 0	0, 1
T	0, 1	1, 0

→ payoffs are in utils (Bernoulli) (IA holds)

two "states" of the world L, H



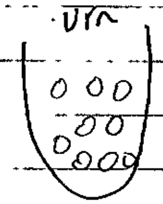
Pascal

Subjective expected utility
Savage (1954) Foundations of Statistics

Under many axioms including IA,
the decision maker will act "as if" he is maximizing his EU for some utility function $u(x)$
and some probability $p(x)$,
where $p(x)$ is unique, but $u(x)$ is only unique up to an affine transformation.

Daniel Ellsberg (1961)

Urn



100 red balls, 200 yellow or blue balls

L_A: \$1000 if red
\$0 else

L_B: \$1000 if blue, \$0 else

$L_A > L_B$ $R = 100$ $Y + B = 200$

K_A: \$1000 if red or yellow, 0 else

$K_B > K_A$ K_B: \$1000 if blue or yellow, 0 else subjective beliefs P_R P_B P_Y utility 1: if win, or not

$L_A > L_B$ $1 \cdot P_R + 0 \cdot (P_B + P_Y) > 1 \cdot P_B + 0$

i.e. $P_R > P_B$

$K_B > K_A$ $1(P_A + P_Y) \leq 1(P_B + P_Y)$

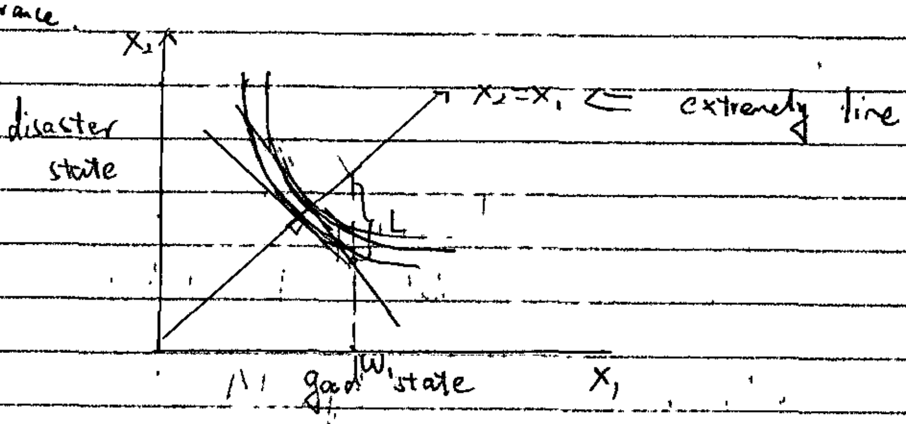
i.e. $P_R < P_B$

Ambiguity

unknown probabilities

3.10th. Stochastic dominance

Insurance



Insurance pays q in the event of disaster. (Loss $= L > 0$)

wealth $W - pq$ (premium) no disaster

$W - L + q - pq$ disaster (chance $\pi > 0$)

$\max E(u) = \max \pi\, u(W - L + q - pq) + (1-\pi)\, u(W - pq)$

$0 \leq q \leq W$

FOC: $\pi (1-p) u'(W - L + q - pq) = p (1-\pi) u'(W - pq)$

↳ fair insurance

$\frac{p}{1-p} = \frac{\pi}{1-\pi}$

$\therefore \quad u'(w-L+q-pq) = u'(w-pq)$

$u''<0$

$\cancel{w-L+q-pq = w-pq}$

$q=L$ full insurance

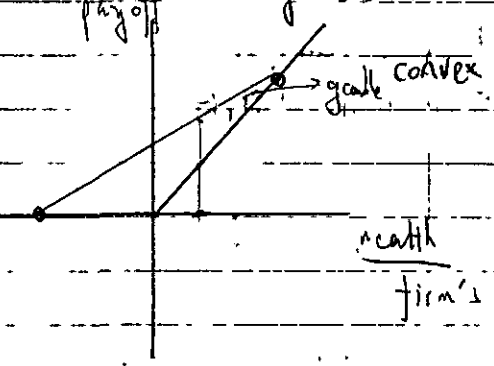
not fair $\quad \dfrac{p}{1-p} > \dfrac{\pi}{1-\pi}$

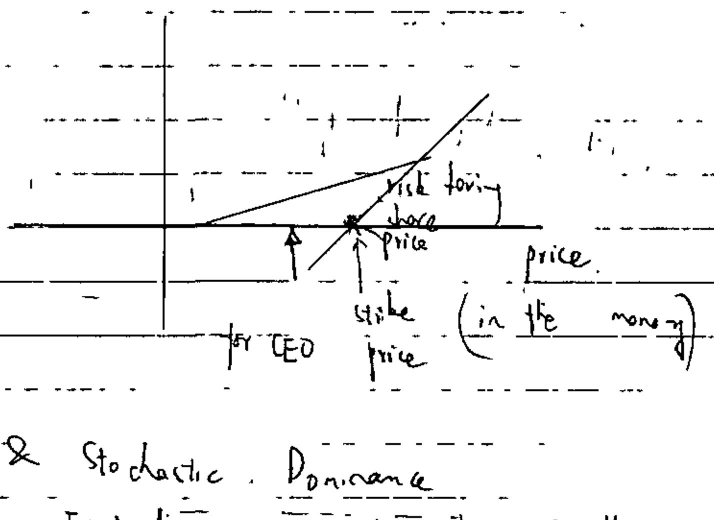
$u'(w-L+q-pq) > u'(w-pq)$

$w-L+q-pq < w-pq$

$q<L$

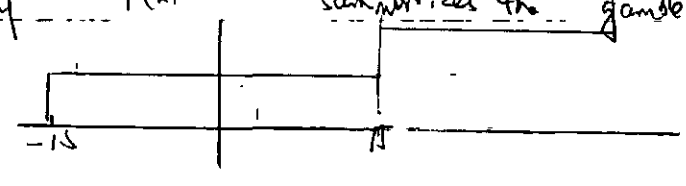
Bankruptcy & Risk-Loving Behavior





Risk & Stochastic Dominance

→ cdf $F(x)$ summarizes the gamble



F dominates G in the sense of First Order stochastic dominances if any increasing utility function has a higher expenditure with F than G

$u(x)$ ↑ing function (weakly ↑ing)

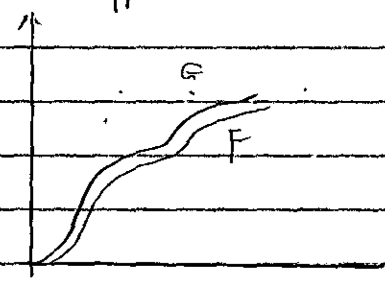
$$E_F u = \int_R u(x) F'(x)\, dx \quad \text{(if differential)}$$
$$= \int u(x)\, dF(x) \quad \text{(more generally)}$$

We want $E_F u \geq E_G u$ ∀ utility $u(x)$ non-decreasing

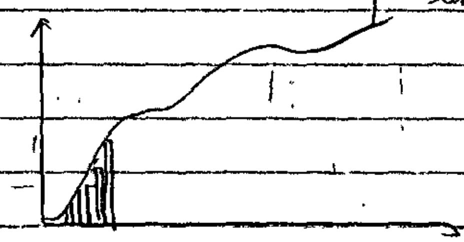
$\boxed{F \geq_1 G}$ if $F(x) \leq G(x)$ $\forall x$

First Ranking Theorem

F FSD G iff $F \geq_1 G$



Proof Every weakly ↑ function that is ≥0 can be written as the the limit of sums of step of fns.



$$s(x) = \begin{cases} 0 & \text{if } x \leq a \\ b & \text{if } x \geq a \end{cases}$$

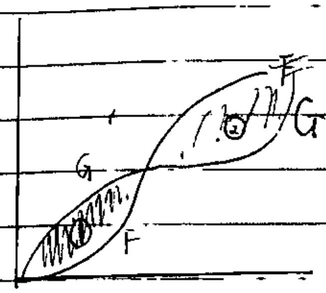
$$E_F\, s(x) = Pr(x \geq a)\, b = [1 - F(a)]\, b$$
$$E_G\, s(x) = [1 - G(a)]\, b$$

$F(a) \le G(a)$

if

write any utility $u(x)$ as a limit of sums of step fns

if



$F \geq_2 G$

② < ①

Second Ranking Theorem

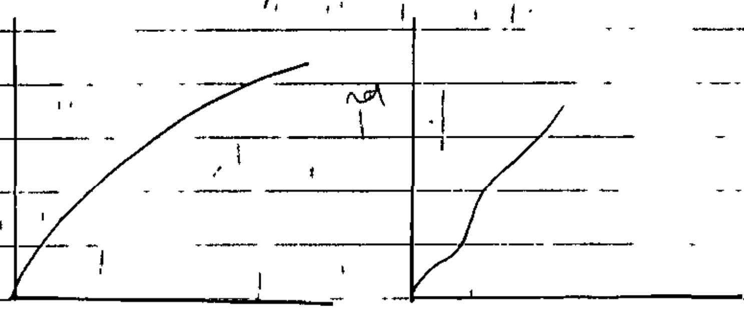
Every weakly ↑ing and concave utility fn has a higher expectation under F than G iff $F \succeq_2 G$

i.e. $\int_0^x F(t)\,dt \le \int_0^x G(t)\,dt \quad \forall x$

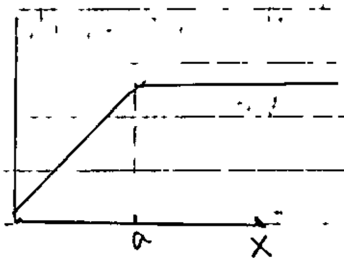
$\geq_1$ and $\geq_2$ are very partial orders

$F \geq_1 G \Rightarrow F \geq_2 G$

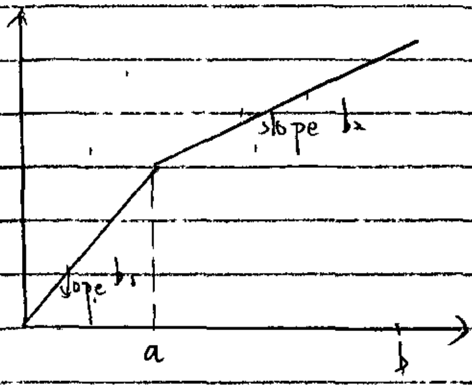
$\not\Leftarrow$

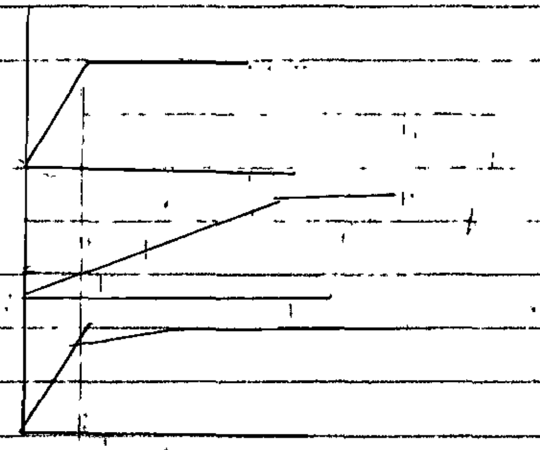


min (a, x)



$(b_1+b_2)\min(x, a) + b_2 \min(x-a, 0)$





$E_F \min(a, x)$

$$= \int_0^a x\, dF(x) + a\int_a^\infty dF(x)$$

$$= xF(x)\Big|_0^a - \int_0^a F(x)\,dx + a(1-F(a))$$

$$= a - \int_0^a F(x)\,dx$$

for $E_F \min(a, x) \geq E_G \min(b, x)$ iff

$$\int_0^a F(x)\,dx \leq \int_0^a G(x)\,dx$$

for $E_F\, u \geq E_G\, u$

$\forall\, u$ concave ↑ing

iff $\int_0^a F(x)\,dx \leq \int_0^a G(x)\,dx$

$\forall a$

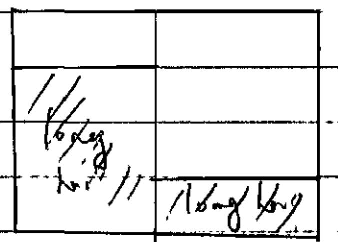
Bayes Rule

$Pr(\text{Woman} \mid \text{long hair}) \rightarrow$ prior belief $\cdot$ $Pr(\text{woman}) = 50\%$

$\rightarrow$ likelihoods of data

75% of woman have long hair.

15% of man.



10,000 M | 10000 W

9000 long haired people

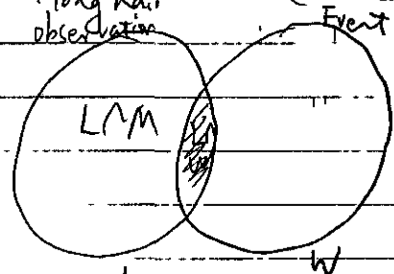
5/6 are woman

$$Pr(W \mid LH) = \frac{Pr(LH \mid W)\, Pr(W)}{Pr(LH)}$$

$$= \frac{Pr(LH \,\&\, W)}{Pr(LH)}$$

3.12th, 2015

Bayes Rule



"clearly" evidence observation

$$Pr(W \mid L) = \frac{Pr(W \wedge L)}{Pr(L)} = \frac{Pr(L \mid W)\, Pr(W)}{Pr(L)}$$

$$\frac{Pr(L \mid W)\, Pr(W)}{Pr(L \wedge W) + Pr(L \wedge M)} = \frac{Pr(L \mid W)\, Pr(W)}{Pr(L \mid W) Pr(W) + P(L \mid M) Pr(M)}$$

$$Pr(M \mid L) = \frac{Pr(L \mid M)\, Pr(M)}{Pr(L \mid W)\, Pr(W) + Pr(L \mid M)\, Pr(M)}$$

→ Reformulation of Bayes Rules via "odds"

Posterior Odds of W, M

$$\frac{Pr(W|L)}{Pr(M|L)} = \frac{Pr(L|W)}{Pr(L|M)} \cdot \frac{Pr(W)}{Pr(M)}$$

↑ Likelood Ratio. ↑ "odds" Prior odds

Long hair wearing pants

n pieces of independent

$$\frac{Pr(W|\text{information})}{Pr(M|\text{information})} = \frac{Pr(\text{info}1|W)}{Pr(\text{info}1|M)} + \frac{Pr(\text{info}\,n|W)}{Pr(\text{info}\,n|M)} \cdot \frac{Pr(W)}{Pr(M)}$$

Medical school

$$Pr(D) = \frac{1}{1000}$$

positive "test"

Prior odds against D: $\frac{999}{1} \approx 1000$

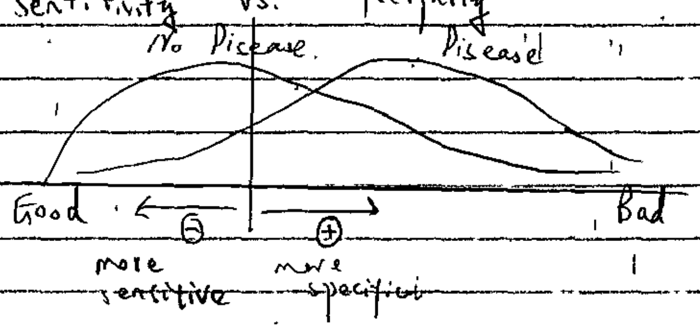
false + chance of 5%

$$\frac{Pr(+|D)}{Pr(+|\sim D)} = \frac{95\%}{5\%} \approx 20, \quad \frac{Pr(+|\sim D)}{Pr(+|D)} \approx \frac{1}{20}$$

$$\frac{Pr(\sim D)}{Pr(D)} = 1000 \quad \times \quad = 50$$

$$\frac{Pr(\sim D|+)}{Pr(D|+)} - 50$$

Sentitivity vs. Specificity

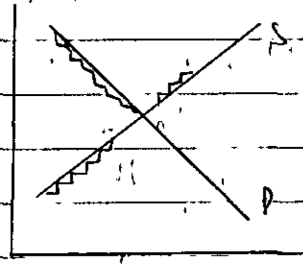


Market vs. Exchange : Paradigm . Thomas

Double auction with Unit Demands

buyers → valuation / WTP = V

seller's → cost / WTA = c



preference → quasilinear →

payoff (of buyer) = $\begin{cases} V + \text{money} & \text{if own good} \\ \text{money} & \text{if not} \end{cases}$

(of seller) = $\begin{cases} c + \text{money} & \text{if own good} \\ \text{money} & \text{if not} \end{cases}$

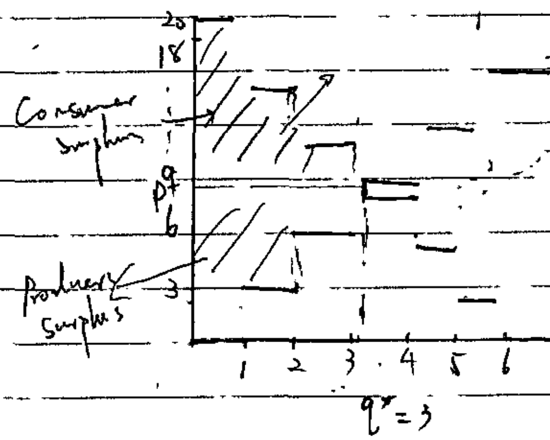
Heterogeneous B buyers & S sellers

$v_1 > v_2 > \cdots > v_B$

$c_1 < c_2 < \cdots \; c_S$

e.g. Buyers (20, 16, 12) 8, 4 ↗ 10

Sellers (3, 6, 9) 11, 12, 15



$9 \le p^* ? \le 12$.
10 11

The market clearing price p^* given the quantity $q^* = k$ solves:

① $c_k \le p^* \le V_k$

② $V_{k+1} \le p^* \le c_{k+1}$

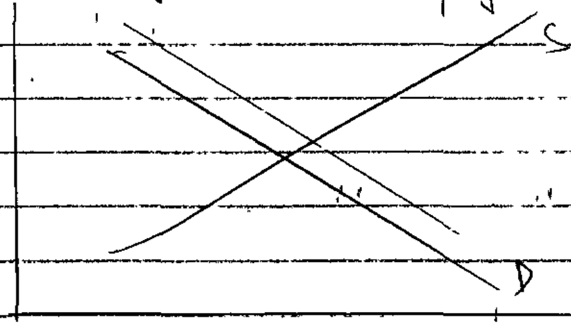
First WELFARE Theorem

Any market equilibrium is efficient

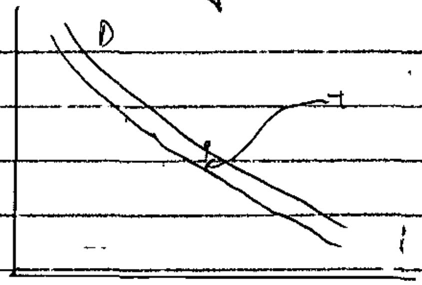
i.e It's impossible to make anyone better off without hurting someone else.

The allocation is immune to side payments

Hetergeneity → source of gains from trade



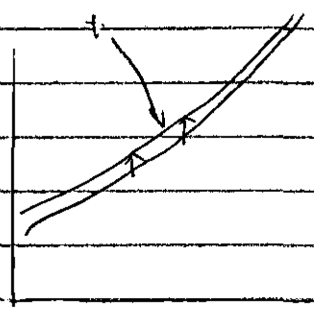
Tax / subsidy incidence.



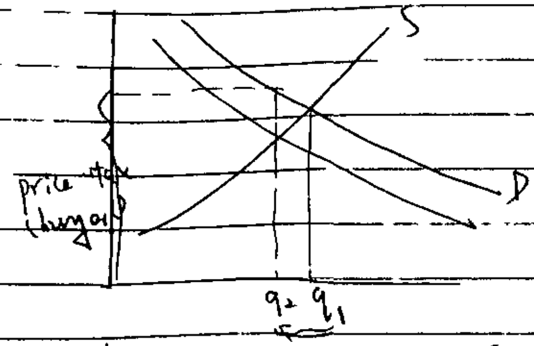
specific tax

advalrem tax.

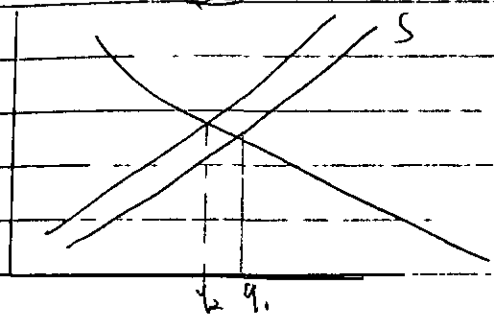
~~such as sale tax~~



V+T 從價稅



Incidence
Irrelevance.



Econ 708

3.26th

Supply

$C(q) = 1 + q^2$

extreme margin — Short Run: $p = c'(q) = 2q$

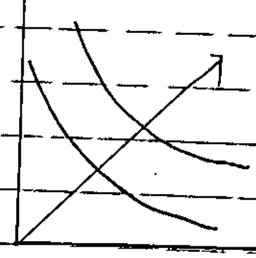
$p \geq AC_{SR} = q$ ✓

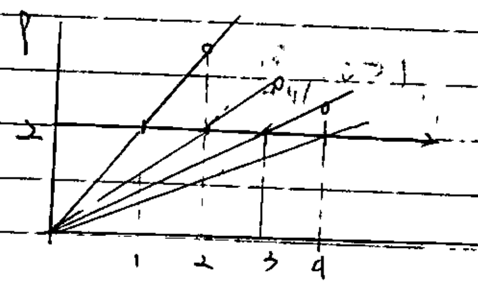
intensive margin — Long Run: $p = c'(q) = 2q$

$p \geq AC_{LR} = q + \frac{1}{q}$

$\Rightarrow q \geq 1$

$Q = nq = n(p/2)$





Heterogeneity

$C(q) = n + q^2 \qquad n = 1, 2, 3, \ldots$

SHORT RUN: $p = c'(q) = 2q$ ✓

$p \geq AC(q)_{SR} = q$

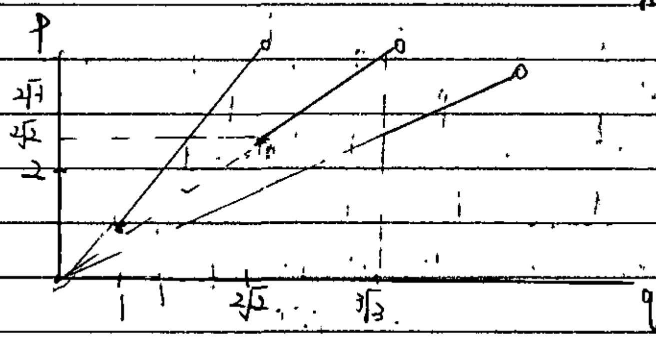
Long RUN: $p = 2q$

$p \geq q + \frac{n}{q}$

zero profits: $p < q + \frac{n}{q}$ & $p_n = \cancel{2q}$

$$2q_n = q_n + \frac{n}{q_n} \longrightarrow q_n^2 = n \rightarrow q_n = \sqrt{n}$$

$$p = 2\sqrt{n}$$



$q_2 = \sqrt{2}$

$p = 2q$. same p for both $\rightarrow$ same q

$q_3 = \sqrt{3}$

General Equilibrium

N consumers $i \in N = \{1, 2, \ldots, N\}$

L goods $l \in \mathcal{L} = \{1, \ldots, L\}$

$p = (p^1, \cdots, p^L) \in \mathbb{R}^L_+$

$x_i = (x_i^1, \ldots, x_i^L) \in \mathbb{R}^L_+$

$x = (x_1, \cdots, x_N) \in \mathbb{R}^{LN}_+$

exchange economy

utility $u^i : \mathbb{R}^L_+ \longrightarrow \mathbb{R}$

endowment $w_i = (w_i^1 \ldots w_i^L) \in \mathbb{R}^L_+$

economy

$\mathcal{E} = \{\{u^i, w_i\}_{i=1}^N\}$

Every maximizes utility s.t budget constraint

$$\max_{x_i \in \mathbb{R}^L_+} u^i(x_i) \quad \underbrace{p \cdot x_i}_{\text{cost of bundle } x_i} \leq \underbrace{p \cdot w_i}_{\text{value of endowment}}$$

$B(p, w_i)$

An allocation, $x \in \mathbb{R}_+^{NL}$ is feasible if $\sum_{i=1}^{N} x_i^l \le \sum_{i=1}^{N} w_i^l \ \forall l$

A feasible allocation is Pareto efficient (PE) if $\nexists x'$ st

$u_i(x_i') \ge u_i(x_i) \quad \forall i$

and $u_j(x_j') > u_j(x_j)$ for some j

Competitive Equilibrium (CE)

A CE for $\mathcal{E}$ is (p, x) with $p \in \mathbb{R}_+^L$

s.t. ⊛ and "market clear" ⊛ (with equality)

Welfare Theorems

First: Any CE (p, x) of $\mathcal{E}$ yields a P.O allocation x,

Second: Any P.O. allocation x yields a CE (p, x) for some p,

provided preferences are convex.

Proof of First Welfare Theorem (1951, Arrow - Debreu)

1776 Adam Smith "Wealth of Nations"

"Invisible hand"

Proof: By ⊛, assume $\exists x$ feasible that is a CE but NOT a P.O

so $\exists \hat{x}$ s.t. $u^i(\hat{x}_i) \ge u^i(x_i) \quad \forall i \in N$

$u^j(\hat{x}_j) > u^j(x_j)$ some $j \in N$

By revealed preference

$p \cdot \hat{x}_i \ge p \cdot x_i \quad \forall i \in N \quad i \ne j$

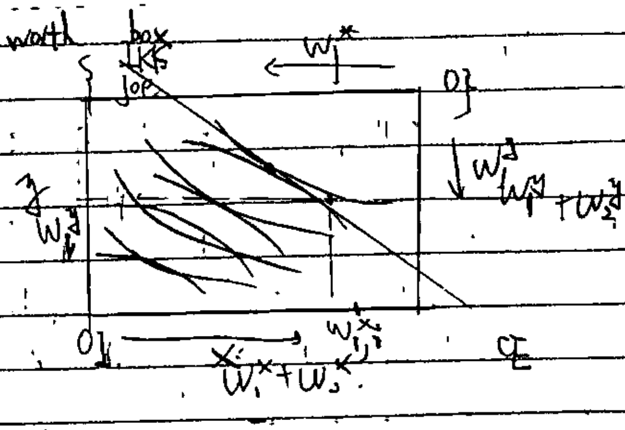
$p \cdot \hat{x}_j > p \, x_j$

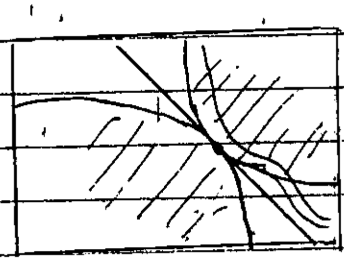
Sum: $p \cdot \sum_{i \in N} \hat{x}_i > p \sum_{i \in N} x_i$

$\rightarrow p \cdot \sum_{i \in N} w_i > p \cdot \sum_{i = N} w_i$ #

Edgeworth box

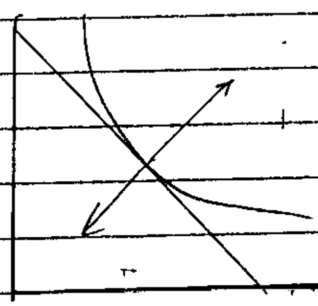
$N=2$



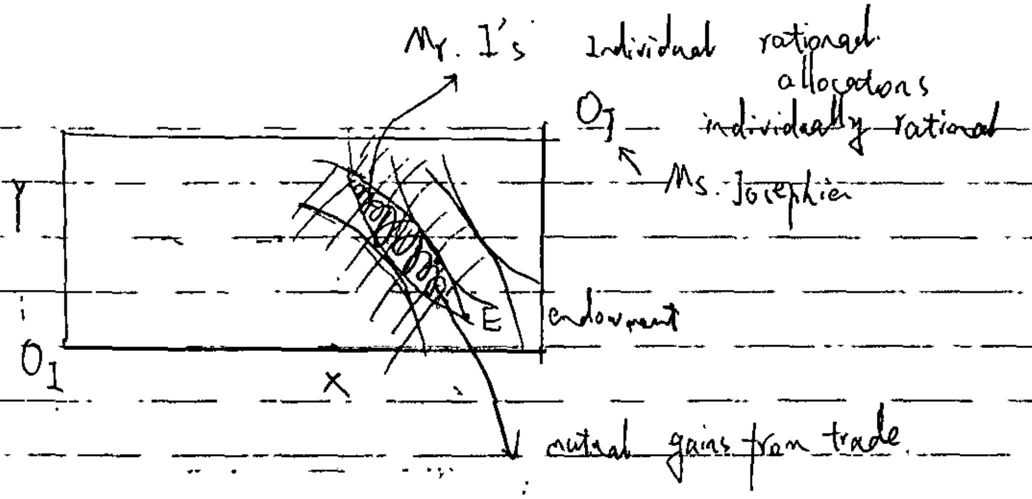


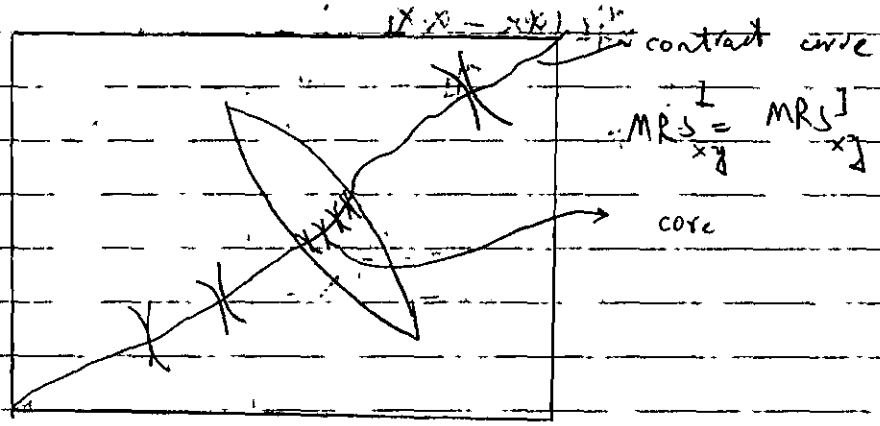
Separating Hyperplane theorem

Any two convex sets, possibly touching at a single point, can be separated by a plane with each set on opposite sides of the plane.



4.7th General Equilibrium





e.g. Cobb-Douglas

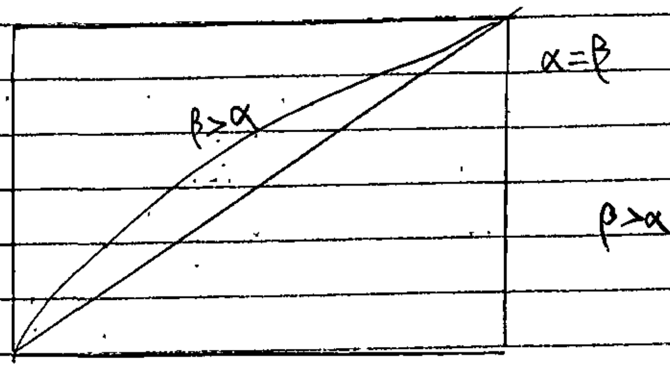
$$U_I(x,y) = x^\alpha y$$
$$U_J(x,y) = x^\beta y$$

$$MRS^I_{xy} = \frac{\partial u_I}{\partial x} \Big/ \frac{\partial u_I}{\partial y} = \frac{\alpha y_I}{x_I}$$

$$MRS^J_{xy} = \frac{\partial u_J}{\partial x} \Big/ \frac{\partial u_J}{\partial y} = \frac{\beta y_J}{x_J}$$

$$x_I + x_J = \bar{x} \text{ (constant)}$$
$$y_I + x_J = \bar{y} \text{ (total amount of } y\text{)}$$

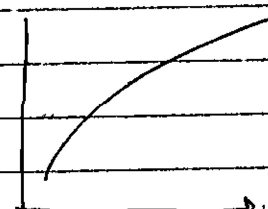
$$\leadsto \frac{\alpha y_I}{x_I} = \frac{\beta(\bar{y} - y_I)}{\bar{x} - x_I}$$

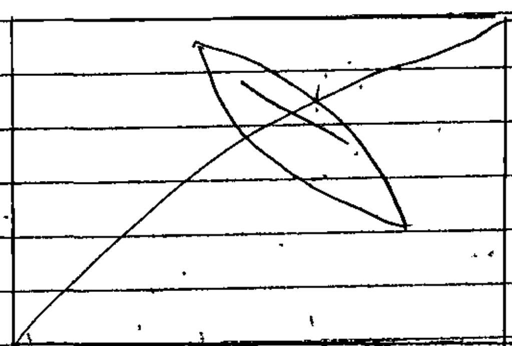


$$y_1(\alpha\bar{x} - \alpha x_1 + \beta x_1) = \beta \bar{y} x_1$$

$$y_1 = \frac{\beta \bar{y} x_1}{\alpha \bar{x} + (\beta - \alpha) x_1}$$

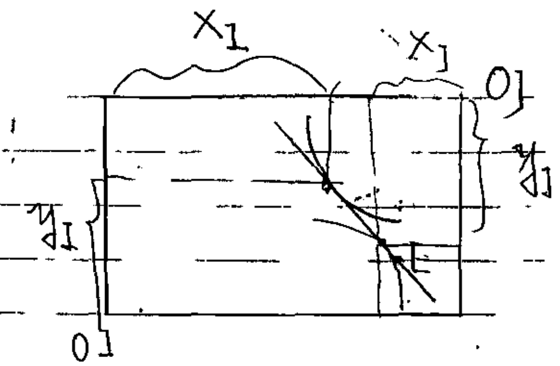
$$\frac{x}{\theta + x} = 1 - \frac{\#}{\# + x}$$





To solve for a CE, we could use the 2nd Welfare Theorem.

Solve for CE
using Walras' Law



Just clear the X market

(excess demand) $ED_x^I(p) = x_I(p) - \bar{x}_I$

$ED_x^J(p) = x_J(p) - \bar{x}_J$

p_x = price of x
p_y = price of y

$p = \frac{p_y}{p_x}$

X is "numeraire"

$$x_I(p) = \frac{\alpha}{\alpha+1}(\bar{x}_I + p\bar{y}_I)$$

$$x_J(p) = \frac{\beta}{\beta+1}(\bar{x}_J + p\bar{y}_J)$$

$\rightarrow ED_x(p) = x_I(p) - \bar{x}_I + x_J(p) - \bar{x}_J$

$= \frac{\alpha}{\alpha+1}(\bar{x}_I + p\bar{y}_I) - \bar{x}_I + \frac{\beta}{\beta+1}(\bar{x}_J + p\bar{y}_J) - \bar{x}_J = 0$

Walras' Law $\quad x_I(p) + p \cdot y_I(p) = \bar{x}_I + p\bar{y}_I$

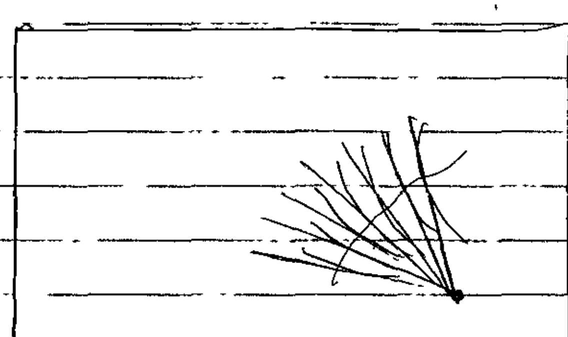
$\quad x_J(p) + p \cdot y_J(p) = \bar{x}_J + p \cdot \bar{y}_J$

$$[x_I(p) + x_J(p) - \bar{x}_I - \bar{x}_J] + p[y_I(p) + y_J(p) - \bar{y}_I - \bar{y}_J] =$$

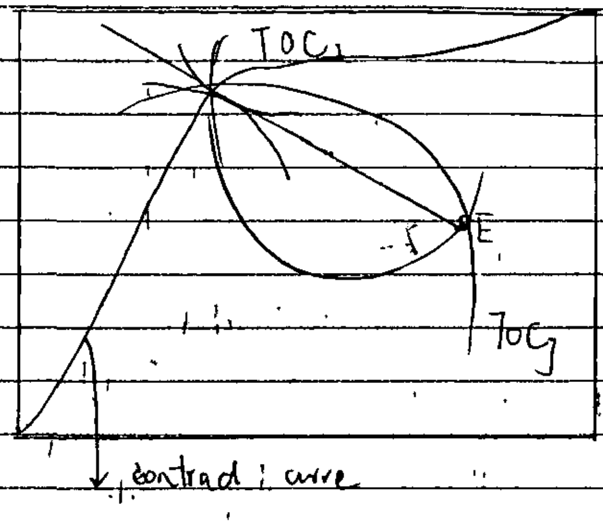
$$ED_x(p) + p\, ED_y(p) = 0$$

$p\left[\frac{\alpha}{\alpha+1}\bar{y}_I + \frac{\beta}{\beta+1}\bar{y}_J\right] = \frac{\bar{x}_I}{\alpha+1} + \frac{\bar{x}_J}{\beta+1}$

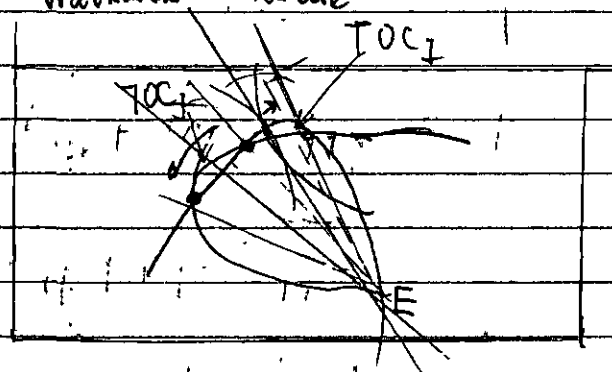
$\uparrow p$



(Trade) offer curve preference $\alpha\uparrow \; \frac{\alpha}{\alpha+1}\uparrow$



Walrasian Unstable



$ED_y < 0$ excess supply of y

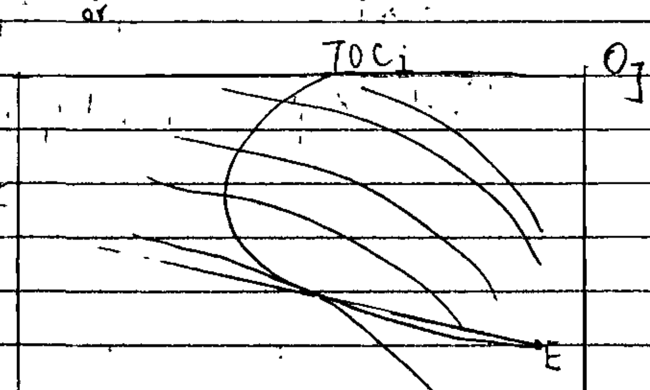
$P \downarrow$

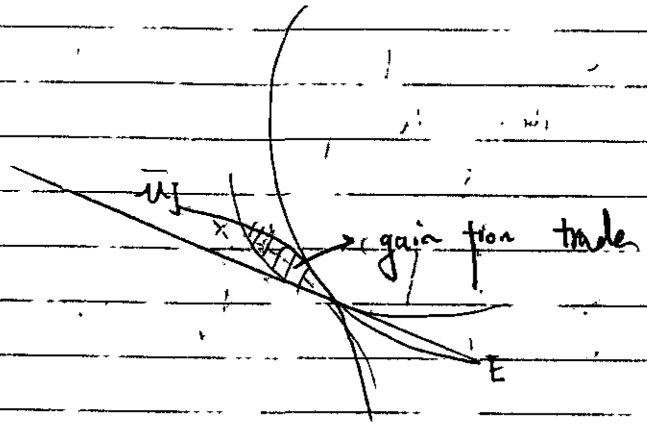
/ Units of P: x per y. Price line steep

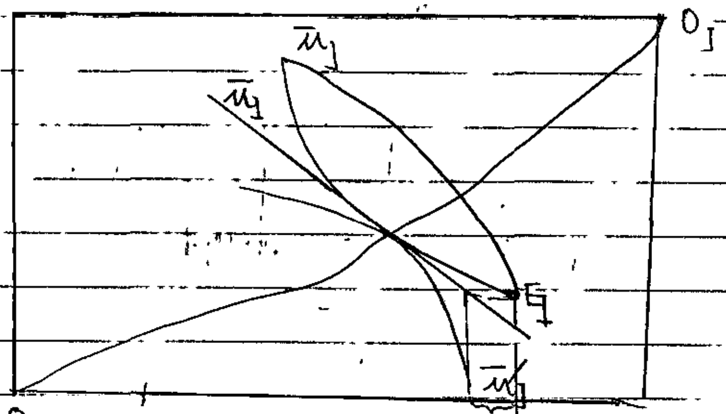
or

If I can set Price

Monopoly



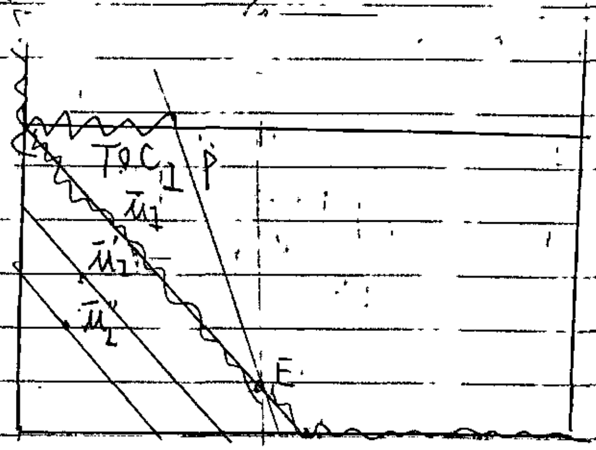




highest welfare she would get

fee

perfect substitute:



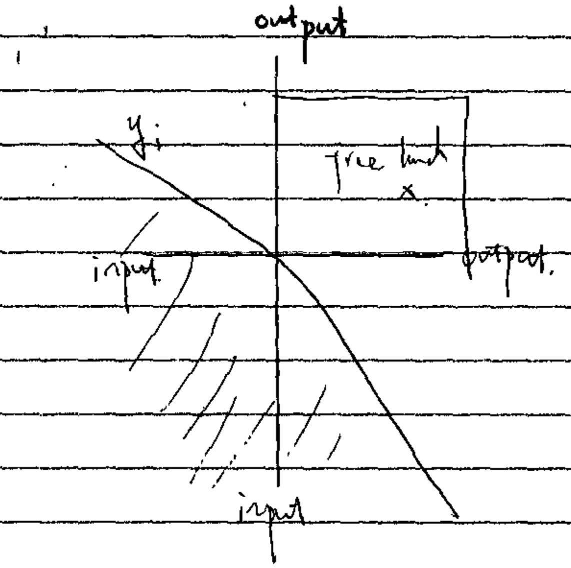
April 9th

Trade offer curves

e.g. Cobb - Douglas $u(x, y) = x^{\alpha} y^{1-\alpha}$.

$$\left.\begin{array}{l} MRS_{xy} = \dfrac{(1-\alpha)x}{\alpha y} \\ p = \dfrac{\bar{x} - x}{y - \bar{y}} \end{array}\right\} \Rightarrow \frac{(1-\alpha)x}{\alpha y} = \frac{\bar{x} - x}{y - \bar{y}}$$

Arrow - Debreu



x : consumption bundle

y : production set.

General Equilibrium

(x, y, p, e, θ)

x vector of possible cons. bundles $\in \mathbb{R}_+^{NL}$

(N) y : vector of all M firms productions $\in \mathbb{R}^{ML}$

θ : ownership shares of firms $\in \mathbb{R}_+^{MN}$

e : endowment vector $\in \mathbb{R}_+^{NL}$ p

p : price vector $\in \mathbb{R}_+^{L}$

let $y = y(-z, q)$

factors of production → inputs w — price of inputs

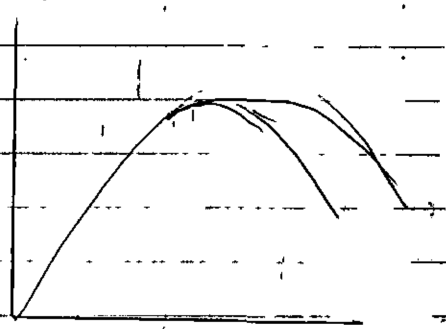
outputs — p : price of output

pareto efficiency (x, y)
It is impossible to make any consumer better off without hurting another.

Competitive eqm (x, y, p, e, θ)

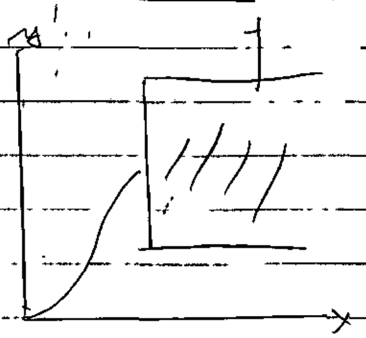
- Consumers maximize utility given (p, e, θ).
- firms maximize profits given p

First welfare Theorem
Converse holds provided that consumers have convex preferences and firms have convex production set (technology)



any maximizer set changes upper hemi continuously in a parameter.

Kakutani (1949)



Any uhc function f on a compact set has a fixed point $x \in f(x)$

Robinson Cruse Economy
case 1. CRTs Technology (constant return to scale)
$u(x, T) = a \log x + (1-\alpha) \log T$ (convex preference)
coconut leisure

Robinson In C : $f(L) = aL$ (convex technology)

Pareto maximum.

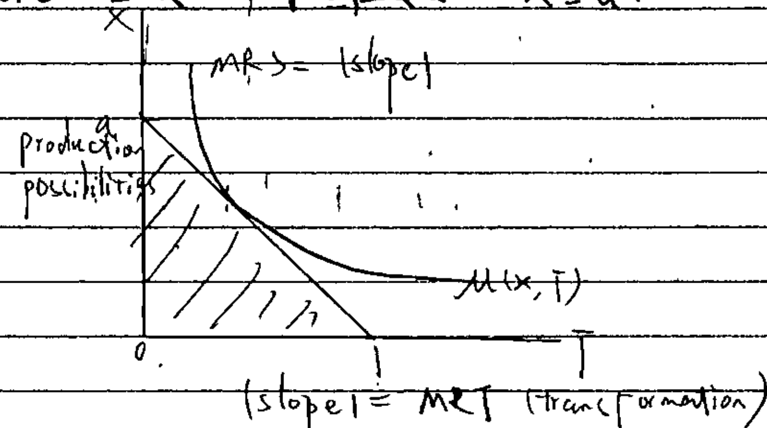
max $a\log x + (1-a)\log T$ endowment: no x, $T=1$

s.t. $x \le aL$

$L + T = 1$

$$\mathcal{L} = a\log x + (1-a)\log T + \lambda(aL - x) + \mu(1-T-L)$$

F.O.C $L = \alpha$, $T^* = 1-a$ $x^* = a^2$



Robinson's maximization as a consumer

max $a\log x + (1-\alpha)\log T$

s.t. $px + wT \le p\cdot 0 + w\cdot 1$

$(\bar{x})$ $(\bar{T})$

$\Rightarrow x^* = \frac{a}{p}w$ $T^* = \frac{(1-\alpha)w}{w} = 1-\alpha$

$\Rightarrow L^* = a$

Robinson's Inc profit $p = MC = \frac{w}{a}$ $W/p = a$

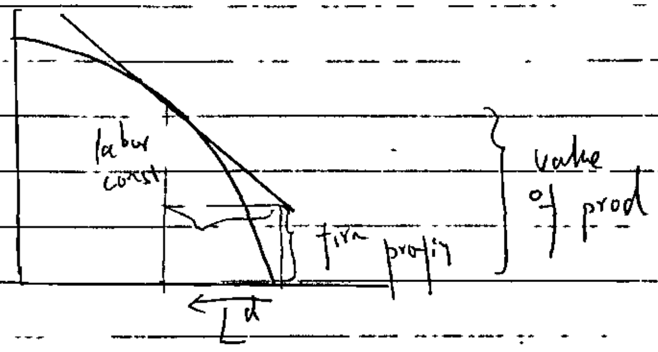
Then $x^* = \frac{aw}{p} = a^2$

Case 2. Decreasing Return to scale Technology

$f(L) = \sqrt{L}$

Robinson Inc max $p\sqrt{L} - wL$

$L^d = \frac{1}{4}\left(\frac{p}{w}\right)^2$



New budget constraint for Robinson substitue $L^d = \frac{1}{4}\left(\frac{p}{w}\right)^2$ into

$$px + wT \le p \cdot 0 + w \cdot 1 + \text{profits} \quad \left(p\sqrt{L} - wL,\ \text{profit} = \frac{p^2}{4w}\right)$$

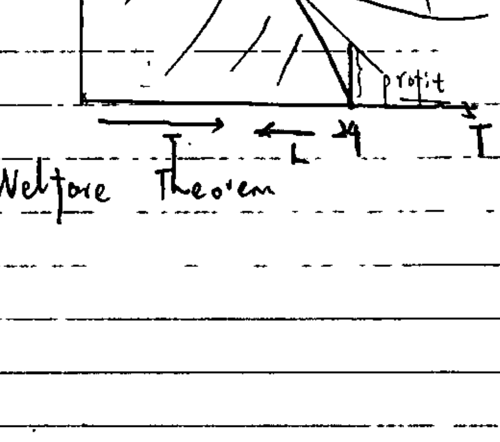
$$x^* = \frac{a}{p}\left(w + \frac{p^2}{4w}\right) = \frac{aw}{p} + \frac{ap}{4w}$$

$$T^* = \frac{1-a}{w}\left(w + \frac{p^2}{4w}\right)$$

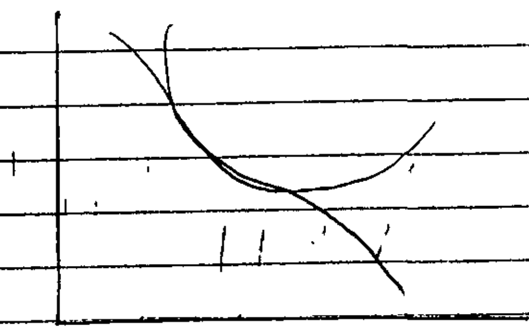
$$\Rightarrow x^* = \sqrt{\frac{a}{2-a}} \quad L^* = \frac{a}{2-a} \quad T^* = \frac{2(1-a)}{2-a}$$

$$w^*/p^* = \sqrt{2-a/4a}$$

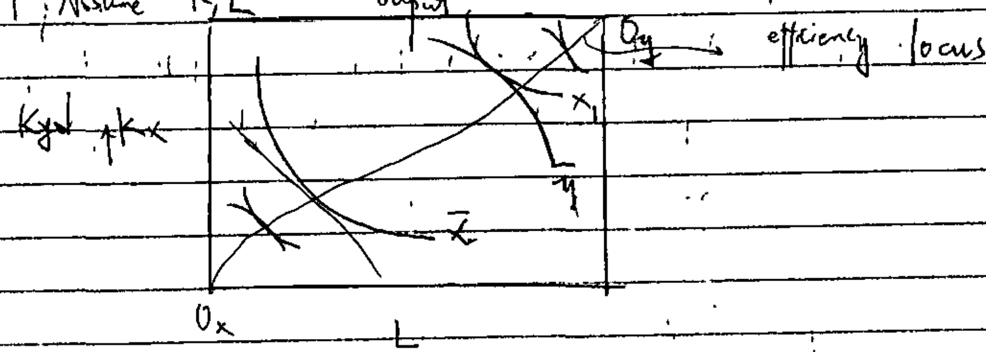
Robinson Crusoe Economy



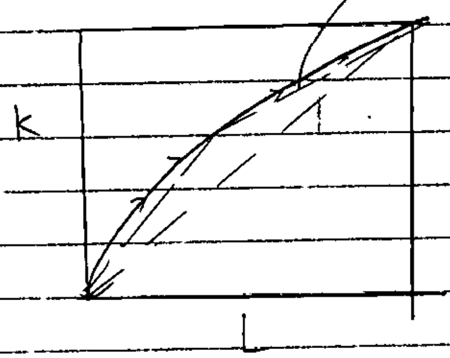
2nd Welfare Theorem



1. Assume K, L — output



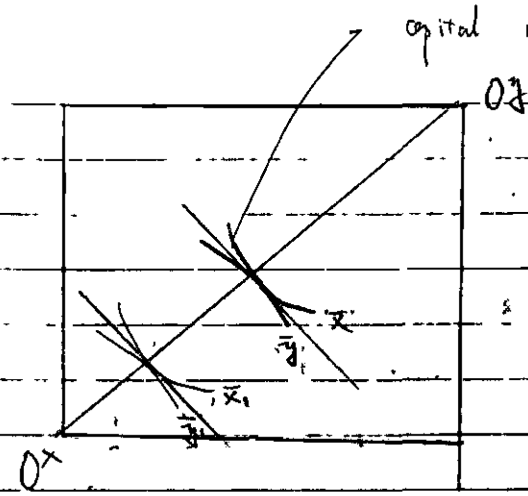
$\rightarrow L_x$
$\leftarrow L_y$ $\left(\frac{K}{L}\right)_x > \left(\frac{K}{L}\right)_y$



$\left(\frac{K}{L}\right)_x \downarrow$, $\left(\frac{K}{L}\right)_y \downarrow$

x is the capital-intensive industry

capital intensity reversal



impossible if x & y have CRS homothetic

e.g. $x = f(K_x, L_x) = \sqrt{K_x L_x}$

$f(tK, tL) = t f(K, L)$ CRS

$y = g(K_y, L_y) = K_y^{1/4} L_y^{3/4}$

$$RTS^x_{LK} = \frac{f_L}{f_K} = \frac{K_x}{L_x}$$

$$RTS^y_{LK} = \frac{3K_y}{L_y}$$

efficiency locus

$$\frac{K_x}{L_x} = \frac{3K_y}{L_y} = \frac{3(\bar{K} - K_x)}{\bar{L} - L_x}$$

$$\rightarrow K_x = \frac{3\bar{K} L_x}{\bar{L} + L_x} \quad \uparrow L_x$$

Pareto Efficiency in the Economy

① Efficient production max

$MRS_{xy} = MRT_{x,y}$ $\forall$ good x, y — slope is MRT of PPF marginal rate of transformation

② Efficient production

$RTS_x = RTS_y$, $\forall$ industries, x, y

③ Efficient consumption

$MRS_{xy}^I = MRS^J_{x,y}$ $\forall$ goods x, y $\forall$ consumers I, J

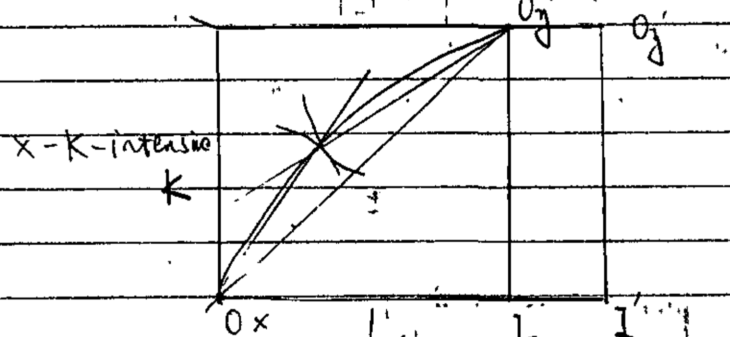
"dual" → Rybczynski Theorem QUANTIES
→ Stolper Samuelson Theorem 1941 PRICES

Rybczynski Theorem

Assume convex CRS technologies x, y

given K, L At a fixed price ratio $\frac{P_x}{P_y}$, an increase in the supply of total labor

$\bar{L}$, raises the output of the L - intensive good and reduces the output of the K-intensive good.



CLAIM: $\left(\frac{K}{L}\right)_x$ and $\left(\frac{K}{L}\right)_y$ same after the $\bar{L}$ increase.

Proof $W_{factor} = VMP_{factor} = P \cdot MP_{factor}$

CRS of f & g. so $f(K, L) = L f\left(\frac{K}{L}, 1\right)$

homogeneous of degree 1 (Euler's Theorem)

e.g. Cobb - Douglas $= \sqrt{KL} = f(K, L)$

$f_K = \frac{1}{2}\left(\frac{L}{K}\right)^{\frac{1}{2}}$

$f_K(tK, tL) = f_K(K, L)$

→ f_K, f_L, g_K, g_L are homogeneous of degree 0

so $f_K(K, L) = L^0 f_K\left(\frac{K}{L}, 1\right) = f\left(\frac{K}{L}, 1\right)$

$P_y g_L\left(\frac{K}{L}, 1\right) = w = P_x MP_L^x = P_x f_L(K, L) = P_x f_L\left(\frac{K}{L}, 1\right)$

$P_y g_K\left(\frac{K}{L}, 1\right) = V = P_x f_K\left(\frac{K}{L}, 1\right)$

$\frac{P_x}{P_y}$ is constant $\rightarrow$ $\frac{w}{v} = \frac{g_L(\frac{K}{L},1)}{g_K(\frac{K}{L},1)}$

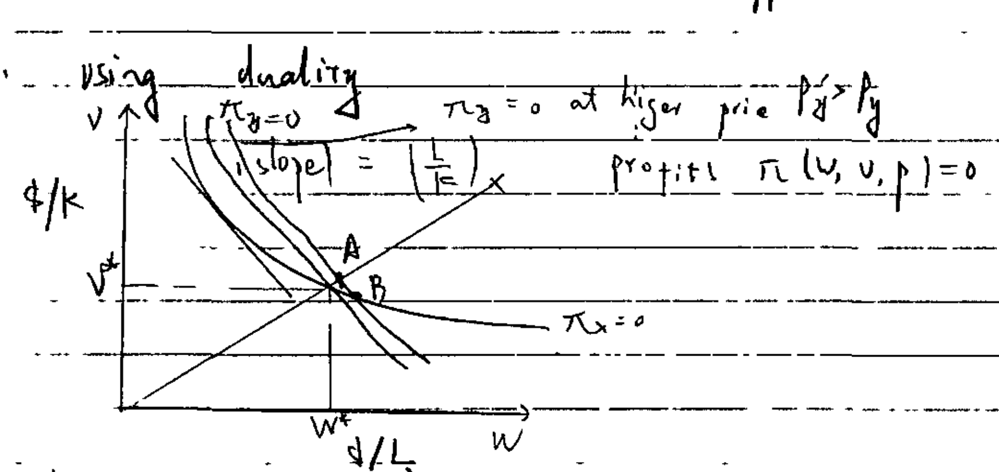
fixed if $\frac{P_x}{P_y}$ fixed.

Stopper Samuelson Theorem

Assume CRS & convex technologies x, y

A rise ~~with~~ in the price of the L-intensive good raises the wage - rental ratio $\frac{w}{v}$, so much that laborers are unambiguously better off.

Proof. using duality



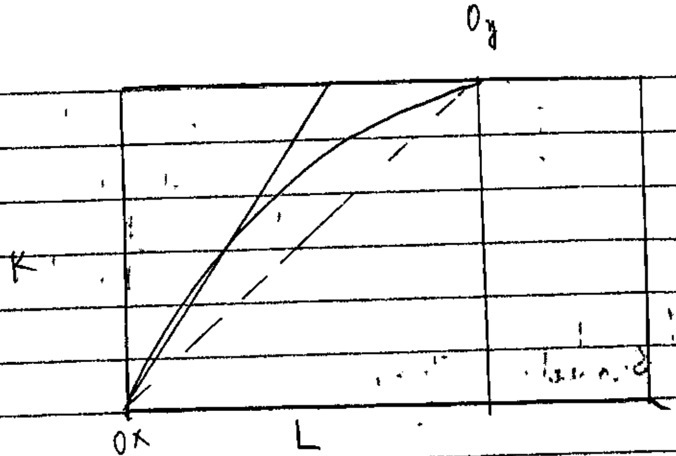
At A, $\frac{w}{v}$ is same as at beginning, and w, v are each higher by factor $\frac{P_x}{P_y}$.

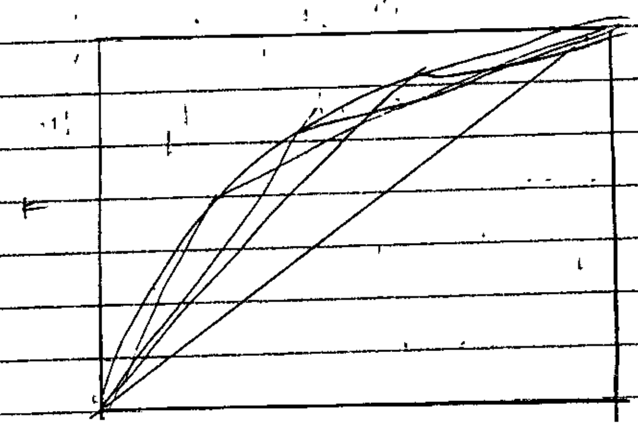
$w_B > w_A$, $v_B < v_A$

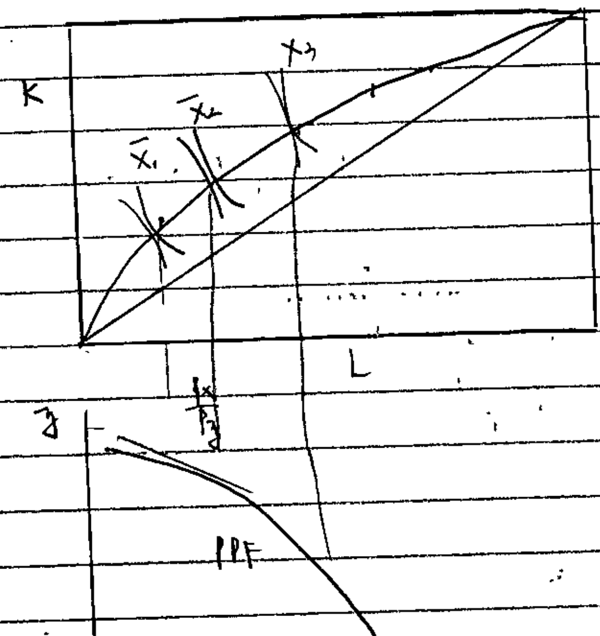
$v_B > \frac{P_y'}{P_y}$ (original wage)

4.16th

Rybczynski Theorem







input space

output space

$\rightarrow \frac{K}{L} \downarrow \left(\frac{w}{r}, b\right)$

$\rightarrow \frac{p_x}{p_y}, p$

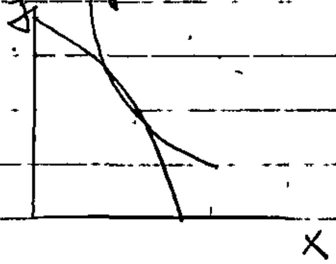
$\leftarrow$ Stolper Samuelson Theorem

Heckscher – Ohlin

Production – Possibility Frontiers

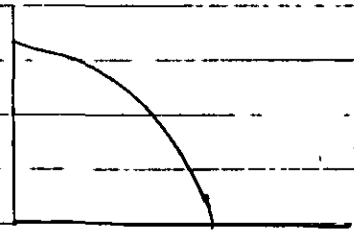
$x = a\sqrt{L_x}$ $\quad y = b\sqrt{L_y}$

$L_x + L_y = \bar{L} \longrightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$



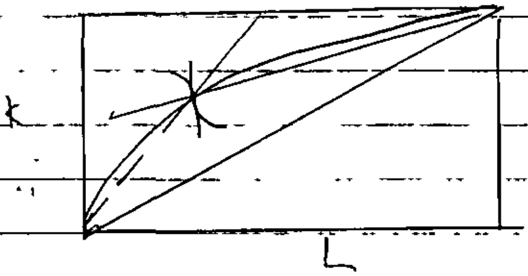
$x = aL_x$ $\quad y = b\sqrt{L_y}$

$L_x + L_y = \bar{L} \sim \frac{x}{a} + \frac{y^2}{b^2} = 1$



$x = \sqrt{L_x K_x}$ $\quad Y = L_x^{2/3} K_x^{1/3}$

K - intensive $\quad$ L - intensive



General Equilibrium with Uncertainty

states of world $s \in \{1, \dots, S\}$

$S = 2$ in example. chance π_i of states

N consumers

L commodities $\rightarrow L = 1$ ("wealth")

Recall vNen Theorem

The individual maximizes $\sum \pi_j u_i(x_i)$

where $u' > 0 > u''$ (risk aversion)

FOC $\lambda = \dfrac{\pi_i u_i(x_i^i)}{p}$

$\uparrow$

marginal utility of

when p_i = price of x_i

Fundamental Theorem of Risk Theorem

(in macro, this is Euler Theorem)

2015 April 23rd.

Free market Failures

1. externalities $\rightarrow$ Pigou (1920, Economics of Welfare)
2. Public goods
3. Market power (Big Traders)

Public Goods

CONGESTION → ROADS

→ TRAGEDY OF THE COMMONS

RIVAL WIFI e.g FISHING

NONRIVAL OLS

information

A CONSTANT RETURNS TO SCALE

$$Q_A(X_A) = X_A \qquad x = \text{hours of fishing}$$

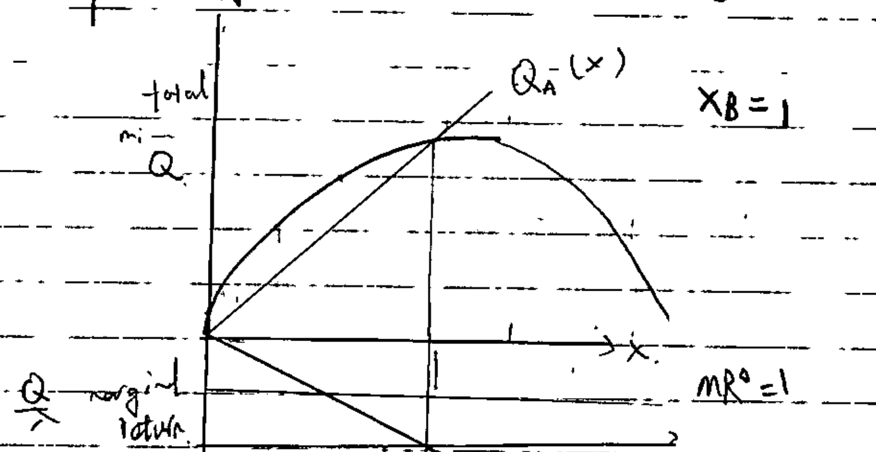
B diminishing RETURNS

$$Q_B(X_B) = 2X_B - X_B^2$$

arbitrage $AR_A = \frac{Q_A(X_A)}{X_A} = 1$

$$AR_B = \frac{Q_B(X_B)}{X_B} = 2 - X_B$$

Private Optimum $AR^A = AR^B \qquad 1 = 2 - X_B$



Social Optimum

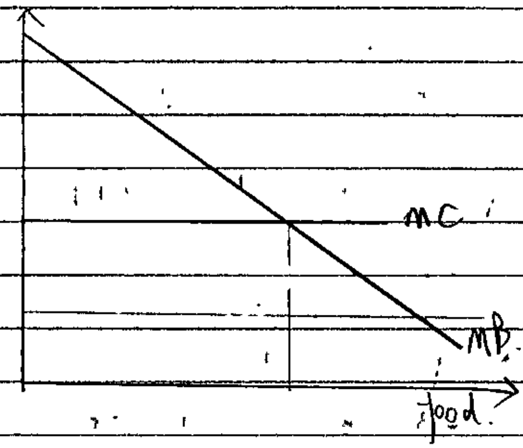
extensive margin

$$MR^A = MR^B$$

(Maximize total fish caught)

$1 = 2 - 2x_B$

$x_B = 1/2$



PURE PUBLIC Good.

① INDIVISIBLE

utility $u(q, w)$ $q \in \{0, 1\}$

$q = \begin{cases} 1 & \text{public good} \\ 0 & \text{no} \end{cases}$ $i = 1, 2.$

public good costs C

Govt extract g_i from M_r $i = 1, 2$

So $g_1 + g_2 \geq C$

efficiency demands build if

$\exists\ g_1, g_2$ s.t.

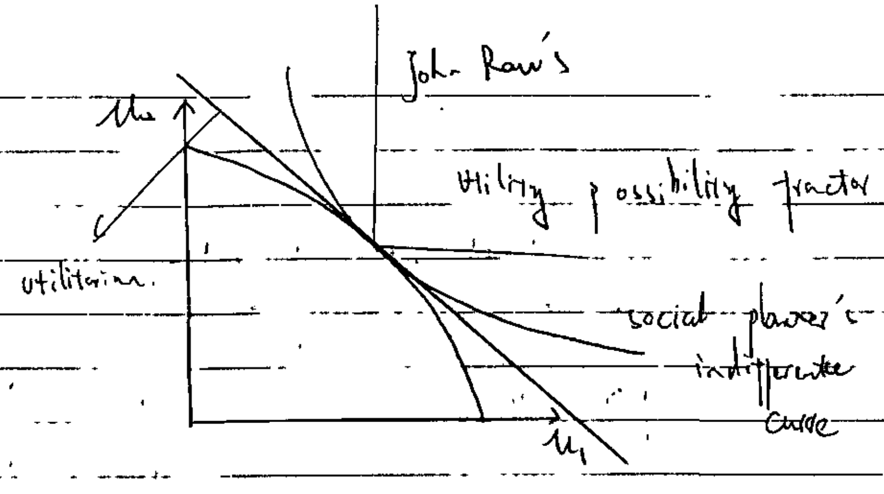
$u_1(1, w_1 - g_1) \geq u_1(0, w_1)$

$u_2(1, w_2 - g_2) \geq u_2(0, w_2)$

② CONTINOUS Public ~~fod~~ Good

$q \in [0, \infty)$

differentiate utility



max $\quad u_1(q_1,$

Also simplify utility
$$V_i(q, w) = u_i(q_i) + w$$
[quasi - linear preference]
$$\mathcal{L} = u_1 + u_2 - g_1 - g_2$$
$$q = g_1 + g_2 \qquad \max_{g_1, g_2} \; u_1(g_1+g_2) + W_1 - g_1 + u_2(g_1+g_2) + W_2 - g_2$$
FOC $\quad u_1'(g) - 1 = 0$
$$u_2'(g) - 1 = 0$$
$$g = g_1 + g_2$$

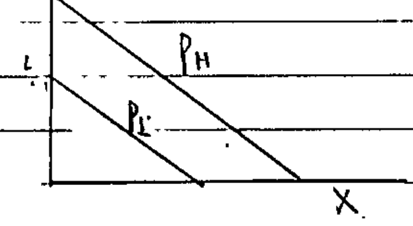
$$MSB = u_1'(g) + u_2'(g) = 1 = MSC$$
Samuelson condition (1954)

Peak Load Pricing

Peak $\quad P_H = H - X_H \qquad H > L$

off Peak $\quad P_L = L - X_L$

demand-side.



operating costs b per user

capacity costs β per user.

Find optimal prices p^*_H, p^*_L, capacity x^*

CS = customer surplus

$= \frac{1}{2} x_L^2 + \frac{1}{2} x_H^2$

PS = producer's surplus $= (p_H - b)\, x_H$

$+ (p_L - b)\, x_L - \beta \bar{x}$

where $\max(x_L, H_H) \le \bar{x}$

$\mathcal{L} = CS + PS + \lambda^H(\bar{x} - x_H) + \lambda^L(\bar{x} - x_L)$

$p_H = H - x_H$, $p_L = L - x_L$

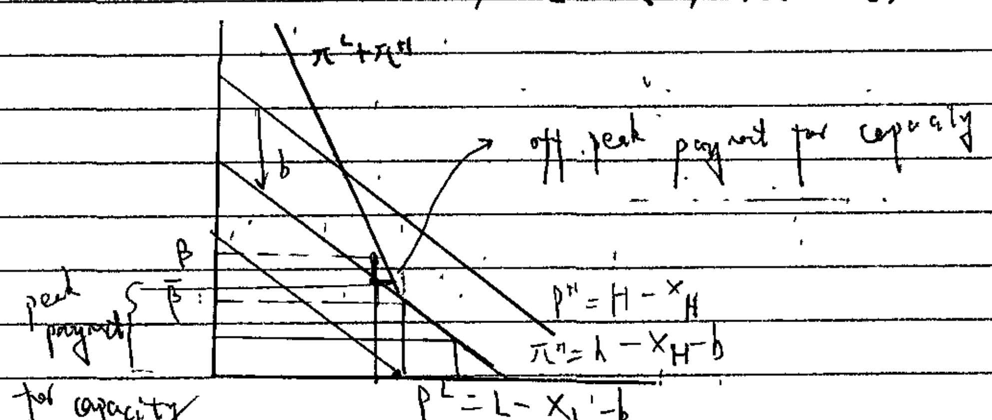
FOC (x_H): $H - x_H - b = \lambda^H$

(x_L): $L - x_L - b = \lambda^L$ complementary slackness

$(\bar{x})$: $\lambda^H + \lambda^L = \beta$

(λ^H): $\lambda^H \ge 0$, $x_H \le \bar{x}$, $\lambda^H(\bar{x} - \lambda_H) = 0$

(λ^L): $\lambda^L \ge 0$, $x_L \le \bar{x}$, $\lambda^L(\bar{x} - x_L) = 0$



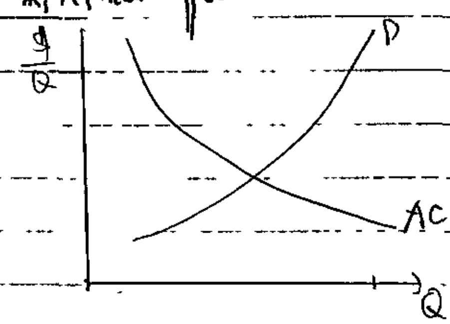
samuelson condition.

peak pays 100% of capacity costs

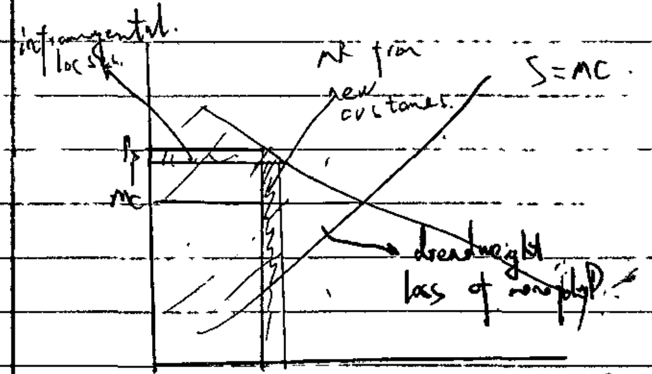
Market Power

no more price taking
→ monopoly → ① legal ~~act~~ mandate
  ↳ franchise.
  ↳ electric (nature & monopoly)
oligopoly → game theory

minimum efficient scale



② technical reason
③ criminal enterprice



$$\pi^m(q) = \underbrace{P(q)q}_{TR} - TC(q)$$

$$\pi_m'(q) = \underbrace{P(q)}_{\substack{\text{marginal gain from}\\ \text{more customers}}} + \underbrace{qP'(q)}_{\substack{\text{losses on inframarginal}\\ \text{customers}}} - MC(q)$$

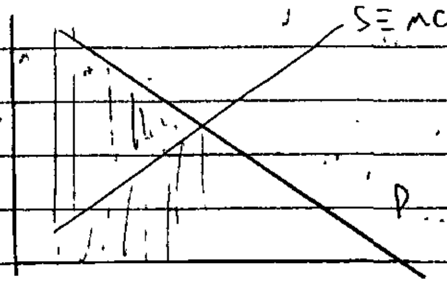
$$P(q)\left[1 + q\frac{P'(q)}{P(q)}\right] = MC. \rightarrow P(q)\left[1 + \frac{1}{\varepsilon}\right] = MC$$

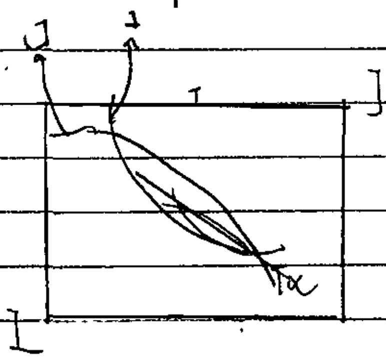
$$\varepsilon = \frac{dq}{dp}\frac{p}{q} < 0 \qquad \rightarrow P(q) = \frac{MC}{1 - \frac{1}{|\varepsilon|}}$$

inverse elasticity rule.

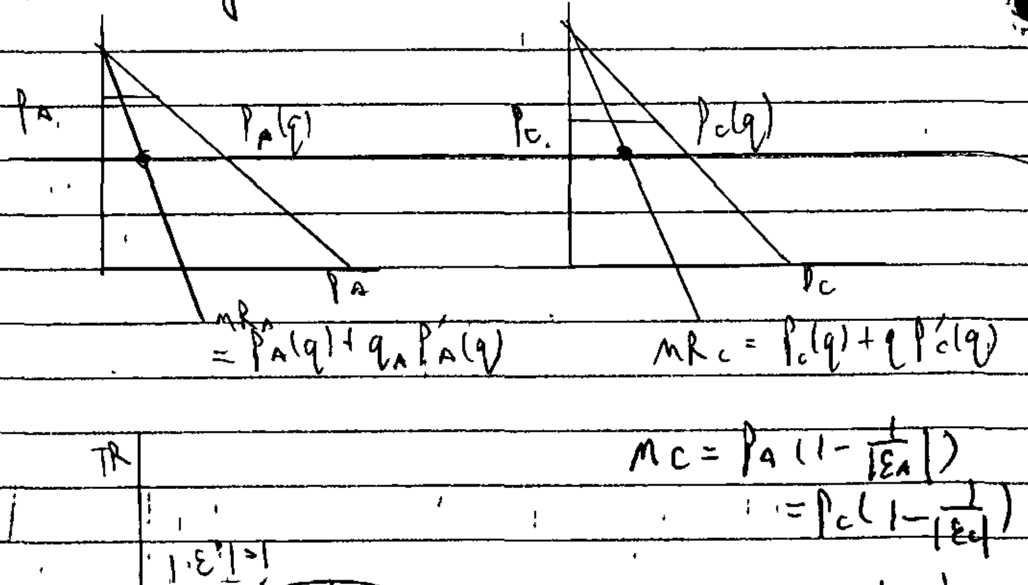
special case, $|\varepsilon| = \infty$

Price Discrimination.

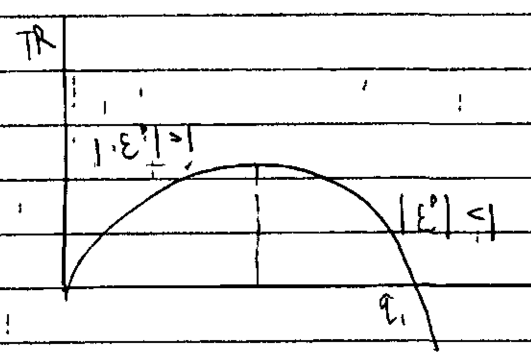




Second-degree Price Discrimination



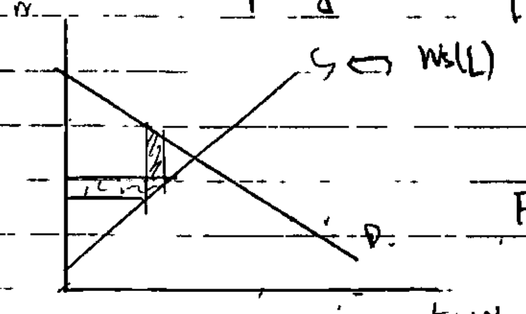
$MR_A = P_A(q) + q_A P_A'(q)$ $\quad MR_C = P_C(q) + q P_C'(q)$



$$MC = P_A\left(1 - \frac{1}{|\varepsilon_A|}\right) = P_C\left(1 - \frac{1}{|\varepsilon_C|}\right)$$

$$\frac{P_A}{P_C} = \frac{1 - \frac{1}{|\varepsilon_C|}}{1 - \frac{1}{|\varepsilon_A|}} = \frac{1 - \frac{1}{3}}{1 - \frac{1}{2}} = \frac{4}{3}$$

Monopsony (monopoly in input market)



$\pi(L) = P\underbrace{f(L)}_{q} - L$

FOC $\quad Pf'(L) - W_s(L) - LW_s'(L) = 0$

$W_s(L)\left[1 + \frac{LW_s'(L)}{W_s(L)}\right] = P$

$$W_s(L) = \frac{Pf'(L)}{1 + \frac{LW_s'(L)}{W_s(L)}} = \frac{Pf'(L)}{1 + \frac{1}{\varepsilon^s_{L,W}}}$$

Oligopoly

① cartel → set price as if it is a monopoly

set quantity as a multi plant monopolist

$MR = MC(q_1) = MC(q_2) \cdots = MC(q_n)$

Claim ∃ incentive to cheat on your cartel share

$\pi(q_1, \dots, q_n) = P(q_1 + \cdots + q_n)\underbrace{(q_1, \dots q_n)}$
$- \Sigma\, TC(q_i)$

FOC for q_i $\quad \underbrace{P + QP'(Q)}_{MR\ \ominus} - MC(q_i) = 0.$

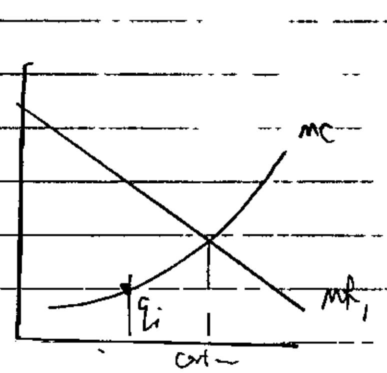
for $i = 1, 2, \dots, n$; $\quad MR \longrightarrow MR_i$

FOC for cheating firm i

$P + \underset{\ominus}{q_i P'(Q)} = MC(q_i)$

Cartel demands.

$MC(q_i) = MR < MR_i$



Cournot (1849)

Cournot Duopoly

Each Firm maximizes his profits
taking as given all other quantities

$\max_{q_i} \left(q_i P(q_1 + \dots q_n) - TC(q_i)\right)$

FOC. $q_i P'(Q) + P(Q) = MC(q_i)$

$\forall i = 1, \dots, n$

Duopoly $MC = c > 0$. (<1) constant, for firm 1, 2

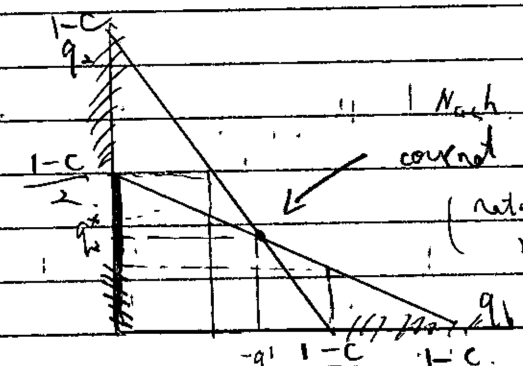
$P(q) = 1 - Q$

Firm 1 $c = q(-1) + (1 - q_1 - q_2)$

Firm 2 $c_2 = 1 - q_1 - 2q_2$

1's best Reply function. $q_1 = \frac{1 - c - q_2}{2}$

2's best Reply function. $q_2 = \frac{1 - c - q_1}{2}$



Nash
cournot outcome
(natural best reply function)

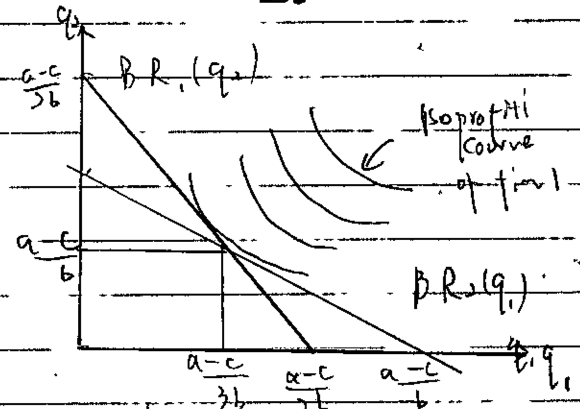
Cournot Duopoly, Revisited

$MC = c > 0$

linear demand $p = a - bQ$, $Q = q_1 + q_2$

profits $\pi_i(q_i, q_{-i}) = (a - b(q_i + q_{-i}))q_i - cq_i$

FOC $q_i = \frac{a - bq_j - c}{2b} = BR_i(q_j)$



BERTRAND Price Competition

price competition $\rightarrow$ $p_1 = p_2 = c$

profits $= 0$

stackelberg

firm 1 choose q_1 the firm 2 choose q_2

$q_2 = \frac{q - bq_1 - c}{2b}$

firm 1 maximize$_{q_1}$ $\left[a - b\left(q_1 + \frac{a - c - bq_1}{2b}\right)\right]q_1 - q_1$

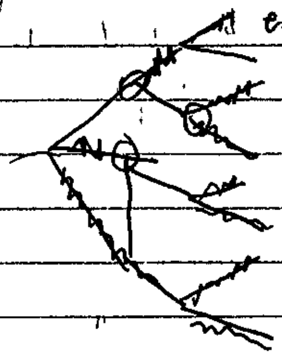
FOC $q_1^* = \frac{a-c}{2b}$ $q_2^* = \frac{a-c}{4b}$

TIMING Zermelo (1913)

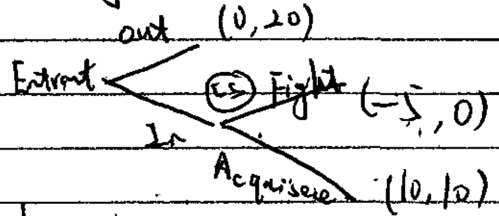
Theorem In a finite game of perfect info (sequential moves)

there is a pure strategy Nash equilibrium obtained by backward induction, if no player has the same payoff in any endgame (nodes) then that equilibrium is unique.

Backward Induction Algorithm



Entry Deterrence



CS – chain store

STARATEGIES (E) Out

In

(CS) F

(CS) A

		F	A	
(E)	In	(−5,0)	(10,10)	$10-15\delta$
	Out	(0,20)	(0,20)	0
		δ	$1-\delta$	

NE 1 (In, A)

NE 2 (Out, $Pr(F) \geq 2/3$)

$0 \geq 10 - 15\delta$

$\delta \geq 2/3$

REFINEMENTS OF NASH EQ

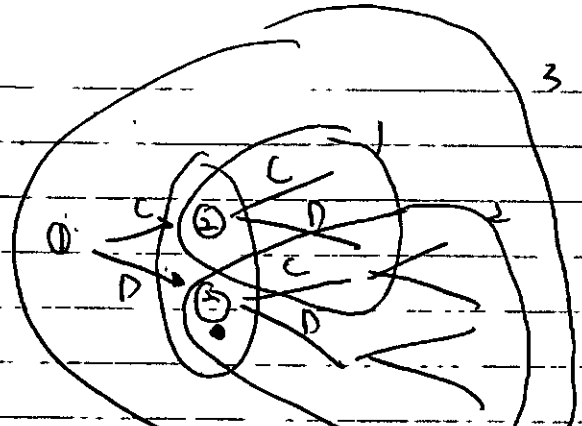
Selten (1965)

3

Divide game into subgame

e.g PD

	C	D
C		
D		



There are 3 subgame

An equilibrium is subgame perfect if it indudes a Nash equilibrium in every subgame

Incomplete Information. Harsanyi (1967)

Vickrey (1962.5)

Introduce Nature into games a player, not payoff ind with prespecified strategies

e.g sealed bid first price auction

2 bidders i; $i=1,2$ each have a value V_i for good

payoff of i if he bids p & wins is $V_i - p$

Nature choose $V_1, V_2 \sim U(1,2)$

cdf $F(V) = V-1$ on $[1,2]$

[APPLICATION OF MIXED STRATEGY NASH EQUILIBRIUM]

Free Rider Problem

1946 Kity Ganovese

Payoffs of n players

$1 \rightarrow$ no call, we saved

$1-c \rightarrow$ called, saved

$0 \rightarrow$ no call, not saved

call with chance $6 \in [0,1]$

mixed $0<\sigma<1$

call not call

1 $1-c$

$\underbrace{1-c}_{\text{pay off if call}} = (1-\sigma)^{n-1}\cdot 0 + \left(1-(1-\sigma)^{n-1}\right)\cdot 1$

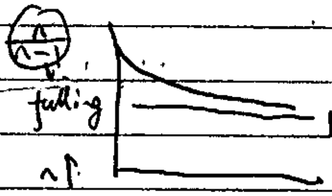
$(1-\sigma)^{n-1} = c$

$1-\sigma = c^{\frac{1}{n-1}}$

$\sigma = 1-c^{\frac{1}{n-1}}$

Probability no call is

$(1-\sigma)^{n} = c^{\frac{n}{n-1}}$



c^{x} $x \geq 1$

Any mixed strategy can be "purified"

Assume calling cost is randomly drawn from $U[c, c+\varepsilon]$ $\varepsilon>0$ tiny

$\exists$ cutoff $\bar{c} \in (c, c+\varepsilon)$

and if cost $> \bar{c}$, don't call

$\leq \bar{c}$, call

$y = x^{\alpha}$ $x>1$

P1

P1

α	W
0.5 – 55	0.2 – 240
0.5 – 100	0.8 – 0
C = 15	C = 20

$i/r = 10\%$

Alternative

① only α

② only W

③ α then W

④ W then α

⑤ α, W

① $V(\alpha) = -15 + \frac{1}{1.1}(0.5\times 55 + 0.5\times 100) \approx 55.5$

② $V(W) = -20 + \frac{1}{1.1^2}(0.2\times 240 + 0.8\times 0) \approx 19.7$

③ If $\alpha = 100$: $V(W|\alpha=100) = -20 + \frac{1}{1.1^2}(0.2\times 240 + 0.8\times 100) \approx 85.7$

<100

If $\alpha = 55$: $V(W|\alpha=55) = -20 + \frac{1}{1.1^2}(0.2\times 240 + 0.8\times 55) \approx 56 > 55$

$V(\overset{\alpha}{W}) = -15 + \frac{1}{1.1}(0.5\times 100 + 0.5(\cancel{V(W)} + \frac{1}{1.1^2}\ \ ) \approx 55.9$

④ $V(W \to \alpha) = -20 + \frac{1}{1.1^2}(0.2\times 240 + 0.8(-15 + \frac{1}{1.1}(0.5\times 55 + 0.5\times 100)))$

≈ 56.3

⑤ $V(W, \alpha) = -35 + \frac{1}{1.1}(0.5\times 100 + 0.5(\frac{1}{1.1}(0.2\times 240 + 0.8\times 55))$

≈ 48.5

P2 $P \sim [a, b]$

cost $= C$, payoff $= v$

looking for reservation price $\bar{p}$

$[a, \bar{p}] \rightarrow$ buy

$V(p) = \bar{v} - p$

$W = -c + E \max\{V(p), W\}$

$= -c + \int_a^b \max\{V(p), W\} \, dF(p)$

$W = V(\bar{p}) = \bar{v} - \bar{p}$

$\bar{v} - \bar{p} = -c + \int_a^{\bar{p}} (\bar{v} - p) \, dF(p) + \int_{\bar{p}}^b (\bar{v} - \bar{p}) \, dF(p)$

$= -c + \int_a^b v \, dF(p) - \int_a^{\bar{p}} p \, dF(p) - \bar{p} \int_{\bar{p}}^b dF(p)$

$\cancel{v} - \bar{p} = -c + \cancel{v} - \int_a^{\bar{p}} p \, dF(p) - \bar{p}\,(F(b) - F(\bar{p}))$

$\bar{p} \, F(\bar{p}) = C + \int_a^{\bar{p}} p \, dF(p)$

P3: College 1: $1, a_1$

College 2: $\mu < 1, a_2 > a_1$

cost : C

Alternatives:

① only C_1

② only C_2

③ both

④ ϕ

Preference: $X > Y$

$C_1 > C_2 \iff a_1 - c > a_2 \mu - c \iff a_2 < \frac{a_1}{\mu}$

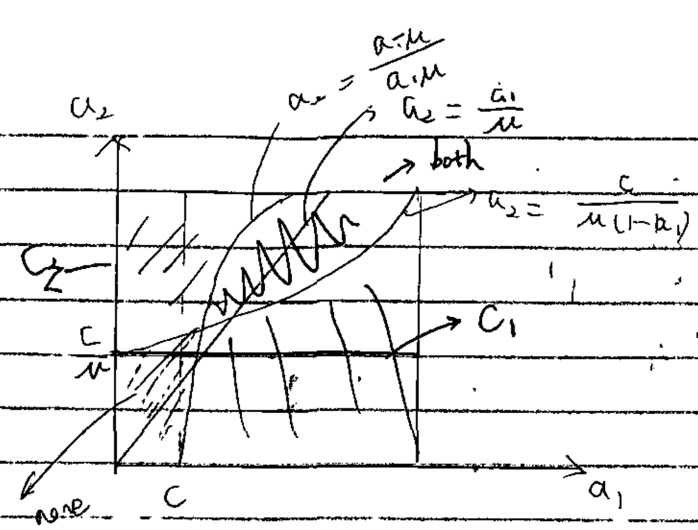
$C_1 > \text{both} \iff a_1 - c > a_1 + (1 - a_1) a_2 \mu - 2c \iff a_2 < \frac{c}{\mu(1-a_1)}$

$C_1 > \phi \iff a_1 - c > 0 \iff a_1 > c.$

$C_2 > \phi \iff a_2 \mu - c > 0 \iff a_2 > \frac{c}{\mu}$

$C_2 > \text{both} \iff a_2 \mu - c > a_1 + (1 - a_1) a_2 \mu - 2c \iff a_2 > \frac{a_1 - c}{a_1 \mu}$

$\phi > \text{both} \iff a_1 + (1 - a_1) a_2 \mu - 2c < 0 \iff a_2 < \frac{2c - a_1}{\mu(1-a_1)}$



P4: $x \geq 0$

$MC = x^3 - 6x^2 + 11x$

$MB = 6 \quad FC = 3$

$\Rightarrow MC = MB$

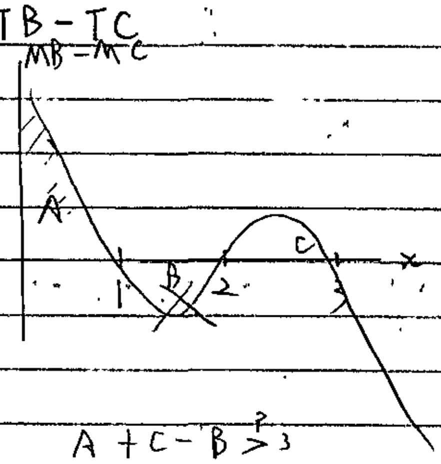
$6 - x^3 + 6x^2 - 11x = 0$

$(x-1)(x-2)(x-3) = 0$

$1 \quad 2 \quad 3 \quad \underline{\underline{0}}$

$NB = TB - TC$

$= MB - MC$



$A + C - B \overset{?}{>} 3$

$x = 0 : 0$

$x = 1 : \quad NB = -0.75,$

$x = 3 \quad NB = -0.75$

$x = 2 : \quad NB = -1$

Dis 2

P1 $y = \min\{2x_1 + x_2 ; 2x_2 + x_1\}$

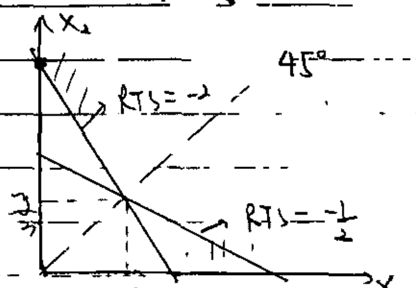
conditional demand

$x_1(w, y)$ $x_2(w, y)$

cost $c(w, y)$

[min $w \cdot x$ s.t. $y = \bar{y}$

$RTS = -\frac{w_1}{w_2}$

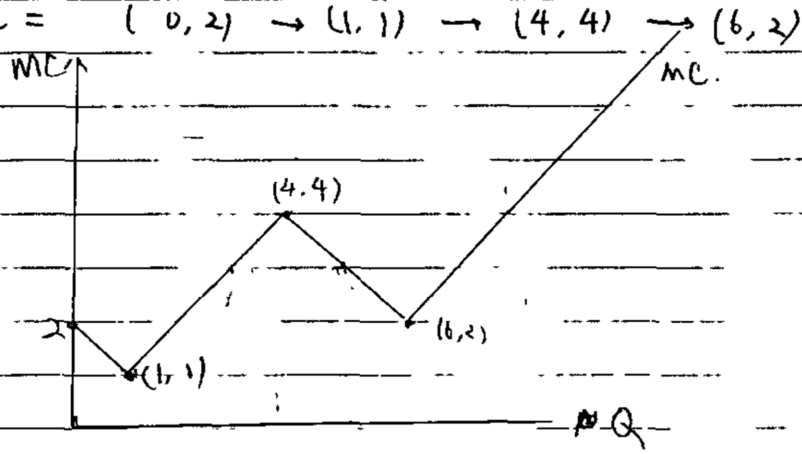


$\frac{w_1}{w_2} > 2$	$\frac{w_1}{w_2} = 2$	$\frac{w_1}{w_2} \in (\frac{1}{2}, 2)$	$\frac{w_1}{w_2} = \frac{1}{2}$	$\frac{w_1}{w_2} < \frac{1}{2}$
$x_1 = 0$	$x_1 \in [0, \frac{y}{3}]$	$x_1 = \frac{y}{3}$	$x_1 \in [\frac{y}{3}, y)$	$x_1 = y$
$x_2 = y$	$x_2 = y - 2x_1$	$x_2 = \frac{y}{3}$	$x_2 = \frac{y - x_1}{2}$	$x_2 = 0$
$c = w_2 y$	$c = w_1 x_1 + w_2 x_2$	$c = \frac{1}{3}(w_1 + w_2)$	$c = w_1 x_1 + w_2 x_2$	$c = w_1 y$
	$= 2w_2 x_1 + w_2 x_2$		$= w_1 x_1 + 2w_2 x_2$	
	$= w_2 y$		$= w_1 y_1$	

$y = 2x_1 + x_2 = 2x_2 + x_1$

P2 $MB = P$

$MC = (0, 2) \to (1, 1) \to (4, 4) \to (6, 2)$



optimal $P = MB = MC$ whenever $MC \uparrow$

① $P \geq 4$ last sequent

② $P \in (2,4)$ $P \geq 3$ last sequent

$P < 3$ 45°

$P = 2$

③ $P \in (1,2)$ 45° or $Q = 0$ ($P > \sqrt{2}$)

$P = 1$

④ $P < 1$ $Q = 0$

P3. $f = \min\{xy; z\}$

a) optimal $xy = z = q$

$z(w, q) = q$

$xy = q$

$\min\ w_x x + w_y y$ s.t. $xy = q$

$x(w, q) = \left(\frac{w_y}{w_x}\right)^{\frac{1}{2}} q^{\frac{1}{2}}$

$y(w, q) = \left(\frac{w_x}{w_y}\right)^{\frac{1}{2}} q^{\frac{1}{2}}$

$$c(w, q) = w_x \left(\frac{w_y}{w_x}\right)^{\frac{1}{2}} q^{\frac{1}{2}} + w_y \left(\frac{w_x}{w_y}\right)^{\frac{1}{2}} q^{\frac{1}{2}} + w_z q$$

P4: $c_1 = q_1^2$ $c_2 = 2q_2^2$ $q_1 = \frac{2q}{3}$

$MC_1 = MC_2$ $q_2 = \frac{q}{3}$

$2q_1 = 4q_2$

$q_1 = 2q_2$

$C(q) = \left(\frac{2q}{3}\right)^2 + 2\left(\frac{q}{3}\right)^2 = \frac{2}{3}q^2$

supply: $MB = MC$

$P = \frac{4}{3}q$

Dis 3

① $X = \{(x, y, z)\}$

$\mathbb{B} = \{ \{x, y\}, \{y, z\}, \{x, z\}, X \}$

$\downarrow x \quad \downarrow y \quad \downarrow z$

$C\{x\} = \{x\}$ $\quad \times \quad$ $C\{\{x, z\}\} = \{z\}$

$C\{X\} = \{y\}$ $\quad \times \quad$ $C\{(x, y)\} = \{x\}$

$C\{x\} = \{z\}$ $\quad \times \quad$ $C\{(y, z)\} = \{z\}$

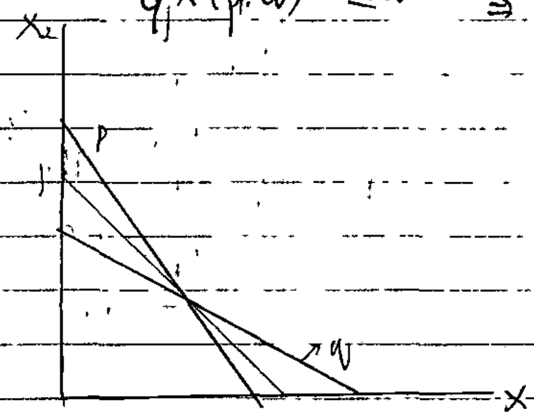
② p, q $\quad \bar{p} = \frac{1}{2}p + \frac{1}{2}q$

$\bar{p} \cdot x(\bar{p}, w) = \frac{1}{2} p \cdot x(\bar{p}, w) + \frac{1}{2} q \; x(\bar{p}, w)$

$\leq W$

$p \cdot x(\bar{p}, w) \leq w \Rightarrow x(p, w) \succ x(\bar{p}, w)$

$q \cdot x(\bar{p}, w) \leq w \Rightarrow x(q, w) \succ x(\bar{p}, w)$



③ flat $\quad x+y \quad (1, 0) \Rightarrow W = 1$

circular .. $\quad x^2+y^2 \quad (\frac{2}{3}, \frac{1}{3}) \Rightarrow W = \frac{5}{9}$

$(1, 0) \succ (\frac{2}{3}, \frac{1}{3})$

WARP holds $\not\Rightarrow$ rationality

SARP $\Rightarrow$ rationality

P4 suppose $u = X \to \mathbb{R}$

$X \subseteq \mathbb{R}^2$

$x_1 \in \mathbb{R} : (x_1, 1) \succ (x_1, 0)$

$\iff u(x_1, 1) > u(x_1, 0)$

$\exists$ rational $r(x_1)$

$x_1' \in \mathbb{R}$ $u(x_1', 1) = u(x_1, 1)$

$x_1' = x_1$

WLOG $x_1' > x_1$

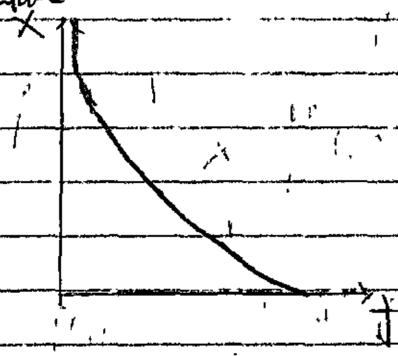
$u(x_1', 0) > u(x_1, 1)$

$u(x_1', 1) > \underbrace{u(x_1', 0)}_{r(x_1')} > \underbrace{u(x_1, 1) > u(x_1, 0)}_{r(x_1)}$

$r(x_1') \neq r(x_1)$

$r : \underset{\text{uncountable}}{\mathbb{R}} \to \underset{\text{countable}}{\text{rational}}$ is 1-1 $\times$

⑤ $u = x + \log(f)$



$\frac{du}{df} = \frac{dx}{df} + \frac{1}{f} = 0$ $\frac{dx}{df} = -\frac{1}{f}$

$\frac{MU_f}{P_f} = \frac{MU_x}{P_x}$

$\frac{\frac{1}{f}}{P_f} = \frac{1}{1}$

$\frac{1}{f} = P_f$ $f^*(P_f, y) = \frac{1}{P_f}$

$x^*(P_x, y) = y - 1$

$P_f \times \frac{1}{f}$

Problem 1

$P, L \in (0,1)$ Airbags ? $\Rightarrow L^* \uparrow$

$\frac{\partial L^m}{\partial P_D} < 0$?

$\frac{\partial L^m}{\partial P_D} = \frac{\partial L^H}{\partial P_D} + \frac{\partial L^m}{\partial W} \frac{D^m}{+1} < \xi$

+

necessary normal

$u = LP$ $T = t_{care} + t_{crazy}$

$D^* = \frac{T\theta}{2}$ $L^* = \frac{T}{2}$ $t_{care} = L$

Airbag X $t_{crazy}\ \theta = D$

A $T = L + \theta$

$u = L + \sqrt{D}$

$D^* = \frac{\theta^2}{4}$ $L = T - \frac{\theta^2}{4}$

Airbag X

Problem 2

$v(P, w) = \frac{w}{P_1 + P_2}$

(a) Roy's Identity

$x_i = -\frac{\frac{\partial v}{\partial P_i}}{\frac{\partial v}{\partial w}}$

$\frac{\partial v}{\partial P_1} = \frac{\partial v}{\partial P_2} = \frac{-w}{(P_1 + P_2)^2}$

$\frac{\partial v}{\partial w} = \frac{1}{P_1 + P_2}$

$x_1(P, w) = \frac{w}{P_1 + P_2}$

$x_2(P, w) = \frac{w}{P_1 + P_2}$

b) $e(p,u)$

$$v(p, e(p,u)) = u$$

$$\frac{e(p,u)}{p_1+p_2} = u \Rightarrow e(p,u) = u(p_1+p_2)$$

c) $u(x_1, x_2) = \min\{x_1, x_2\}$

Problem 3

$$V(x_1,x_2) = x_1^3 x_2^3$$

$$(p_1, p_2) = (2,2)$$

$$(p_1', p_2') = (1,2)$$

(a)

$$x_1^m(p,w) = \frac{w}{2p_1} = \frac{w}{4}$$

$$x_2^m(p,w) = \frac{w}{2p_2} = \frac{w}{4}$$

$$u = \left(\frac{w}{4}\right)^6$$

$$x_1^m(p',w) = \frac{w}{2}$$

$$x_2^m(p',w) = \frac{w}{4}$$

Hicksian $x_i^h(p,u)$

$$x_2 p_2 = x_1 p_1 \ldots$$

$$\frac{MV_i}{p_1} = \text{constant}$$

$$x_2 = \frac{x_1 p_1}{x_2}$$

$$u = x_1^3 \left(\frac{x_1 p_1}{p_2}\right)^3$$

$$x_1^h = u^{\frac{1}{6}} \left(\frac{p_2}{p_1}\right)^{\frac{1}{2}} \Rightarrow x_1^h(p', u) = \frac{w\sqrt{2}}{4}$$

$$x_2^h = u^{\frac{1}{6}} \left(\frac{p_1}{p_2}\right)^{\frac{1}{2}} \Rightarrow x_2^h(p', u) = \frac{w}{4\sqrt{2}}$$

$$x_1^s = x_1^m(p', p' x_1^m(p, w))$$

$$x_1^s(p') = \frac{p_1' \frac{w}{2p_1} + p_2' \frac{w}{2p_2}}{2p_1'}$$

$$x_2^s(p') = \frac{p_1' \frac{w}{2p} + p_2' \frac{w}{2p_2}}{2p_2'}$$

P4

$U = \min\{x, 2y\}$ I

Mashallian

$$\begin{cases} x = 2y \\ I = P_x X + P_y Y \end{cases}$$

$$x^m = \frac{2I}{2P_x + P_y}$$

$$y^m = \frac{I}{2P_x + P_y}$$

Hicksian $x = 2y = u$

$x^h = u$

$y^h = \frac{u}{2}$

$e(p, u) = u\left(P_x + \frac{P_y}{2}\right)$

b) A

$$EV = e(P_p, U_p) - e(P_m, U_p)$$ (U_p: U_{pairs})

$= 48\left(1 + \frac{3}{2}\right) - 48\left(1 + \frac{1}{2}\right) = 48$

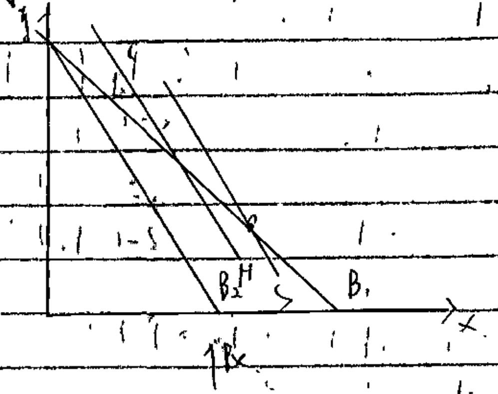
B

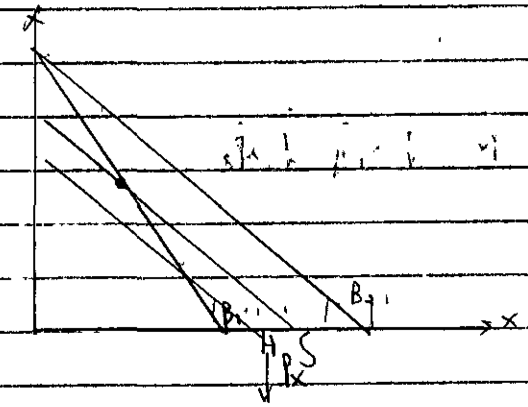
$$CV = e(P_p, U_m) - e(P_m, U_m)$$ (U_m: $U_{madison}$)

$= 80\left(1 + \frac{3}{2}\right) - 80\left(1 + \frac{1}{2}\right) = 80$

$U_{madison} = \frac{240}{3} = 80$

normal good $x^h \le x^s$





Problem 1

$$\sum_{j \neq i} \varepsilon_{ji} \ge 0$$

$$x_j^h(p_1, \dots, p_j, \dots p_n, u)$$

$\Rightarrow$ homogeneous of degree 0 in p.

$$p_1 \frac{\partial x_i}{\partial p_1} + p_2 \frac{\partial x_i}{\partial p_2} + \dots p_i \frac{\partial x_i}{\partial p_i} = 0$$

$$\frac{p_1}{x_i}\frac{\partial x_i}{\partial p_1} + \dots + \frac{p_i}{x_i}\frac{\partial x_i}{\partial p_i} + \dots \frac{p_n}{x_n}\frac{x_i}{\partial p_n} = 0$$

$$\varepsilon_{i1}^h + \dots \varepsilon_{ii}^h + \varepsilon_{in}^h = 0$$

if MRS diminishing

$$\varepsilon_{ii}^h \le 0$$

$$\sum_{j \neq i} \varepsilon_{ij}^h \ge 0$$

$u = -\frac{1}{x} - \frac{1}{y} - \frac{1}{z}$

s.t. $P_x x + P_y y + P_z z = w$

$L = -\frac{1}{x} - \frac{1}{y} - \frac{1}{z} + \lambda (w - P_x x - P_y y - P_z z)$

FOC
$$\frac{1}{x^2} = \lambda P_x \qquad x = (\lambda P_x)^{-\frac{1}{2}}$$
$$\frac{1}{y^2} = \lambda P_y \quad \Rightarrow \quad y = (\lambda P_y)^{-\frac{1}{2}}$$
$$\frac{1}{z^2} = \lambda P_z \qquad z = (\lambda P_z)^{-\frac{1}{2}}$$

$$P_x (\lambda P_x)^{-\frac{1}{2}} + P_y (\lambda P_y)^{-\frac{1}{2}} + P_z (\lambda P_z)^{-\frac{1}{2}} = w$$
$$\lambda = \left(\frac{P_x^{\frac{1}{2}} + P_y^{\frac{1}{2}} + P_z^{\frac{1}{2}}}{w} \right)^2$$

$$x = \frac{w}{P_x + \sqrt{P_x P_y} + \sqrt{P_x P_z}}$$

$$y = \frac{w}{P_y + \sqrt{P_y P_x} + \sqrt{P_y P_z}}$$

$$z = \frac{w}{P_z + \sqrt{P_x P_z} + \sqrt{P_y P_z}}$$

a) $\frac{\partial x}{\partial P_y} \leq 0 \qquad \frac{\partial x}{\partial P_z} \leq 0$

b) Hick - ~~Allen~~ ~~substitute~~ ?

$\frac{\partial x^h}{\partial P_y} =$

$\varepsilon_{xy} + \varepsilon_{xz} \geq 0$

$+ \quad +$

$\frac{\partial x^h}{\partial P_z}$

Q3 $\max_{c,l} \ u(c, l)$

$pc \le w(T-l)(1-\tau)$

$\varepsilon_{Lw} \Rightarrow \frac{\partial R}{\partial \tau} > 0$

$R = Lw\tau$

$c^*(w', p),\ l^*(w', p) \Rightarrow L^*(w', p)$

$w' = (1-\tau)w$

$$\frac{\partial R}{\partial \tau} = w\left(L^* + \tau \frac{\partial l^*}{\partial \tau}\right)$$

$$= w\left(L^* + \tau \frac{\partial L^*}{\partial w'} \cdot \frac{\partial w'}{\partial \tau}\right)$$

$$= w\left(L^* + \tau \cdot \underbrace{\frac{\partial l^*}{\partial w'} \cdot \frac{w'}{l^*}}_{\varepsilon_{Lw'}} \cdot \frac{l^*}{w'} \frac{\partial w'}{\partial \tau}\right)$$

$$= w\left(l^* + \tau\, \varepsilon_{Lw'} \frac{l^*}{(1-\tau)w} (-w)\right)$$

$$= wl^*\left(1 - \frac{\tau}{1-\tau} \varepsilon_{Lw'}\right) \geq 0$$

$$1 - \frac{\tau}{1-\tau} \varepsilon \geq 0$$

$$\varepsilon \leq \frac{1-\tau}{\tau}$$

Q4

	a	b	c
p_1	1	0	0
p_2	0	$\frac{1}{2}$	$\frac{1}{2}$
p_3	$\frac{1}{2}$	$\frac{1}{2}$	0
p_4	0	$\frac{3}{4}$	$\frac{1}{4}$

$p_1 \sim p_2$

$p_3 \succ p_4$

Way ① Expected utility

$U_a = \frac{1}{2} u_b + \frac{1}{2} u_c$

$\frac{1}{2} u_a = \frac{1}{4} u_b + \frac{1}{4} u_c$

$\frac{1}{2} u_a + \frac{1}{2} u_b = \frac{3}{4} u_b + \frac{1}{4} u_c$ ※

② IA

$q = (0, 1, 0)$ → possibility

$\frac{1}{2}\underbrace{P_1 + \frac{1}{2}q}_{P_3} \sim \frac{1}{2}\underbrace{P_2 + \frac{1}{2}q}_{4}$

P5. $P_1 \sim P_2$

Either $\frac{P_1+P_2}{2} \prec P_1$

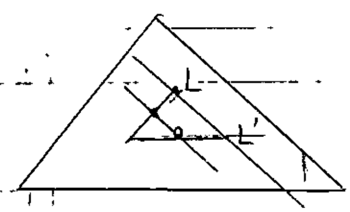
or $\frac{P_1+P_2}{2} \prec P_2$ convex

$L \sim L'$ $u(L) = u(L')$

$\frac{L+L'}{2}$ $\frac{1}{2}u(L) + \frac{1}{2}u(L') = u(L)$

② IA $L \sim L'$

$\alpha L + (1-\alpha)L' \sim L'$



708 Disc 6

P1 bush $P_r(\text{caught}) = p$

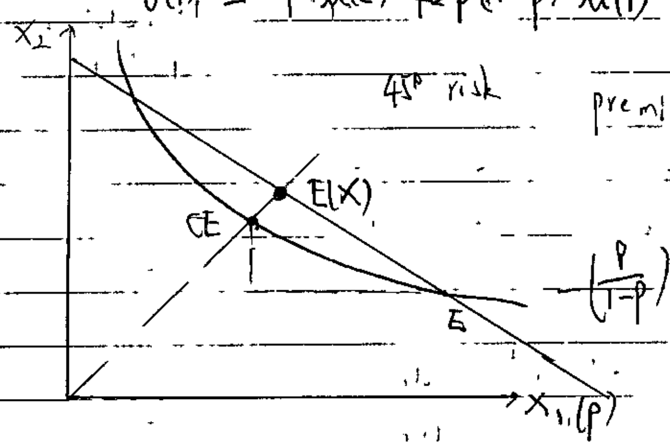
hand $P_r(\text{caught}) = 1$

$U_H(1) = U_B(2)$

$U(1) = p^2 u(2) + 2p(1-p)u(1) + (1-p)^2 u(0)$

P2

45° risk premium π

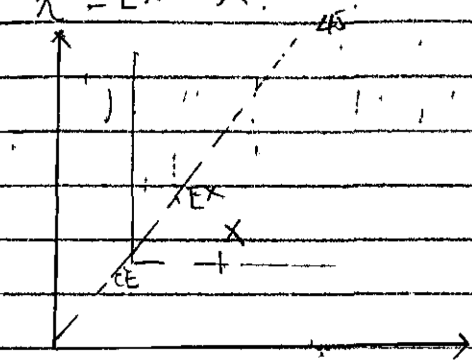


certainty equivalent ($\tilde{x}$)

risk aversion

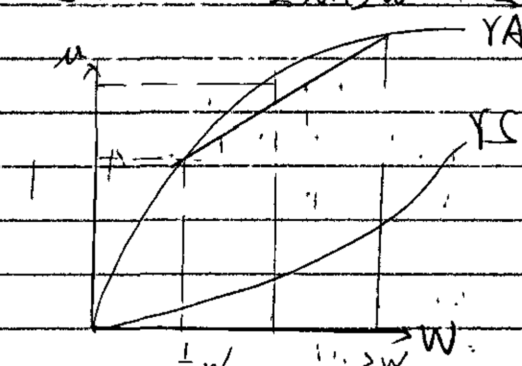
$$u(E(x)) > E(u(x))$$

$$\pi = Ex - \tilde{x}$$



$$EW = \frac{1}{2}\left(\frac{1}{2}w\right) + \frac{1}{2}(2w) = \frac{5w}{4}$$

$$E(u(w)) = \frac{1}{2}u\left(\frac{1}{2}w\right) + \frac{1}{2}u(2w)$$



$$\pi = \quad u\left(\frac{5w}{4} - \pi\right) = E(u(w))$$

$$\frac{5w}{4} - \pi = u^{-1}(E(u))$$

$$\pi = \frac{5w}{4} - \underbrace{u^{-1}(E(u))}_{CE}$$

Problem 3

$u'' < 0$ $\qquad \pi = \Pr(\text{theft})$

a) $\max \; \pi u(w - L + q - pq) + (1-\pi)u(w - pq)$

$0 \le q \le w$

b) FOC $\quad \pi(1-p)u'(w - L + q - pq) - p(1-\pi)u'(w - pq) = 0$

interior solution $q=0$

$>0 \quad q=0$

$<0 \quad q=w$

c) $p=\pi$ full insurance

$$u'(w-L+q-pq) = u'(w-pq)$$

$$w-L+q-pq = w-pq$$

$>0 \quad q=0$

$<0 \quad q=w$

d) $p>\pi$

$$u(x) = \frac{x^{1-\rho}}{1-\rho}$$

$$u'(x) = x^{-\rho}$$

$$\pi(1-p)(w-L+q-pq)^{-\rho} = (1-\pi)\,p\,(w-pq)^{-\rho}$$

$$\left(\frac{w-L+q-pq}{w-pq}\right)^{-\rho} = \frac{(1-\pi)p}{\pi(1-p)}$$

$$\frac{w-pq}{w-L+q-pq} = \left(\frac{(1-\pi)p}{\pi(1-p)}\right)^{\frac{1}{\rho}} = A > 1$$

$$w-pq = Aw - AL + A(1-p)q$$

$$q^* = \frac{w-Aw+AL}{p+A(1-p)} \qquad \frac{dq^*}{dw} = \frac{1-A}{p+A(1-q)} < 0$$

$$u''(x) = \rho x^{-\rho-1}$$

$$-\frac{u''(x)}{u'(x)} = \frac{\rho}{x}$$

Dis c 8

P1

	a	b	b-a
A	1	5	B-A
B	7	10	1

Matches

Ab, Ba

$$\begin{cases} A+b=5 \\ B+a=7 \end{cases}$$

$$\begin{cases} A+b\geq 1 \\ B+a\geq 10 \end{cases}$$

$3 \leq b-a \leq 4$

$5 \leq B-A \leq 6$

P2 $x, y \sim [0,1]$

$f(x,y) = xy^a, \quad a \geq 1$

a) $f_{xy} \begin{cases} \geq 0 & \text{PAM (complements)} \\ \leq 0 & \text{NAM (substitutes} \end{cases}$

$f_{xy} = ay^{a-1} > 0$

b) $v(x)$ — men outside option $=0$

$w(y)$ — woman

$f(x,y) - v(x) - w(y)$

max. when $x=y$

FOC. $f_x = v'(x) \Rightarrow v'(x) = x^a$

$f_y = w'(y) \quad w'(y) = ay^a$

$v(x) = \frac{1}{a+1} x^{a+1} + b$;

$w(y) = \frac{a}{a+1} ay^{a+1} + c$

$f(x,y) - v(x) - w(y) = 0$

$b + c = 0.$

$v(x) \geq 0 \quad \forall x$

$w(y) \geq 0 \quad \forall y$

$\Rightarrow b \geq 0 \quad c \geq 0$

$\therefore b = c = 0$

c) men outside $-D$

$b + c = 0$ $\quad b \geq -D$

P3 $\quad C_k(q) = k^2 + q + q^2$

S.R $\quad p = MC$ $\Rightarrow p = 1 + 2q$

LR $\quad p \geq AC_k = \frac{k^2}{q} + 1 + q$

Equilibriums $\quad p = MC = AC_k$

$1 + 2q = \frac{k^2}{q} + 1 + q$

$q = k \quad p_{entry} = 1 + 2k$

$k = 1 \quad p = 3$

4. θ: quality $\quad FC = 1 \quad q = 2\theta x$

$VC = x^2 \quad p = 1 \quad \tau$

$M(\theta) = \theta^{-\beta} \quad \beta > 2$

(a) $\pi = 2\theta\, x(1-\tau) - x^2 - 1$

choose x

F.O.C $\Rightarrow 2\theta(1-\tau) = 2x$

$x = \theta(1-\tau)$

marginal θ

$\pi(\theta) = 0$

$2\theta^2 (1-\tau)^2 - \theta^2 (1-\tau)^2 - 1 = 0$

$\theta^2 (1-\tau)^2 = 1$

$\theta = \frac{1}{1-\tau}$

$q = 2\theta^2(1-\tau)$

$\int_{\frac{1}{1-\tau}}^{\infty} 2\theta^2(1-\tau)\,(-M'(\theta))\,d\theta$

$=$

$$= \int_{\frac{1}{1-\tau}}^{\infty} 2\theta^2(1-\tau)\ \beta\theta^{-\beta-1}\, d\theta$$

$$= \int_{\frac{1}{1-\tau}}^{\infty} 2\beta(1-\tau)\ \theta^{1-\beta}\, d\theta$$

$$= \frac{2\beta(1-\tau)}{2-\beta}\ \theta^{2-\beta} \Big|_{\frac{1}{1-\tau}}^{\infty}$$

$$= \frac{2\beta(1-\tau)}{2-\beta}\ \left(0 - \left(\frac{1}{1-\tau}\right)^{2-\beta}\right)$$

$$Q = \frac{2\beta}{\beta-2}(1-\tau)^{\beta-1}$$

b) $\tau \uparrow \quad \theta = \frac{1}{1-\tau} \uparrow$

$q = 2\theta^2(1-\tau) \downarrow$

c) Tax Rev $\frac{2\beta}{\beta-2}(1-\tau)^{\beta-1}\tau$

$\max_{\tau}\ (1-\tau)^{\beta-1}\tau$

$(1-\tau)^{\beta-1} = \tau(1-\tau)^{\beta-2}\cdot(\beta-1)$

$\tau = \frac{1}{\beta}$

d) $\beta \uparrow \quad Q\,?$

① $\frac{\partial Q}{\partial \beta}$ (fails)

② $Q = \frac{2\beta}{\beta-2}(1-\tau)^{\beta-1}$

$$= \underset{\downarrow}{\left(2 + \frac{4}{\beta-2}\right)}\ \underset{\downarrow}{\overset{(0,1)}{(1-\tau)}}^{\beta-1}$$

③ $\beta\uparrow \Rightarrow$ less heterogeneity $\Rightarrow Q\downarrow$

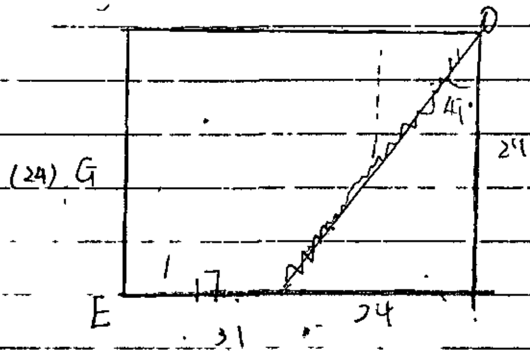
intuition

4.10.

Problem 3

$W^A = (\overset{L}{15}, \overset{G}{10})$ $G+L$

$W^B = (26, 14)$ $\min\{G, L\}$



Problem 4

$U^V = xy$ $(4, 12)$ $W_x = 16$ $p_x = 1$

$U^A = xy^2$ $(12, 6)$ $W_y = 18$ $p_y = p$

1) $x^A = \frac{I^A}{3} = \frac{12+6p}{3} = 4+2p$

$y^A = \frac{2}{3}\frac{I^A}{p} = \frac{2(12+6p)}{3p} = \frac{8+4p}{p}$

2) $x^V = 12-2p$ $y^V = 14 - \frac{8}{p}$

3) $\max_p (12-2p)(14 - \frac{8}{p}) = 168+16 - 28p - \frac{96}{p}$

FOC $28 = \frac{96}{p^2}$ $p = 2\sqrt{\frac{6}{7}}$

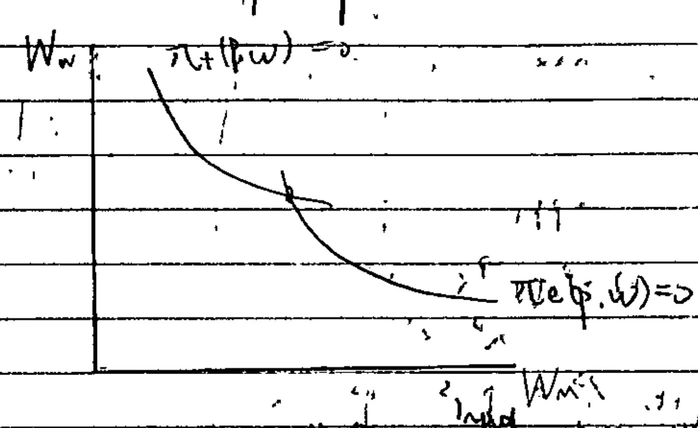
$MRS^V = \frac{y^V}{x^V} = \frac{14-8/p}{12-2p}$

4.17th.

P1. CRS production M, W

$\uparrow M$

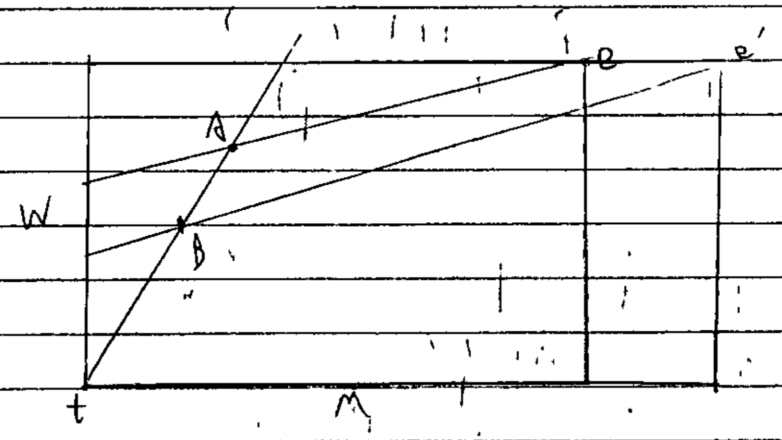
a) Equilibrium output price



$M\uparrow \Rightarrow M_e\uparrow$, input price unchanged

$\left(\frac{M}{W}\right)_e$ $\left(\frac{M}{\mu}\right)_t$ $\Rightarrow$ output price unchanged

b.c)



theory: $M_t\downarrow$, $W_t\downarrow$, $T\downarrow$

empirical: $M_e\uparrow$, $W_e\uparrow$, $\Phi\uparrow$

P2 $PPF = x^2+2y=300$

$u = xy$

a) $MRT_{xy} = MRS_{xy} = p$

$\frac{2x}{2} = \frac{y}{x} \Rightarrow y^* = x^2$

$3x^2 = 300 \quad x = 10 \quad y = 100 \quad p = 10$

b) Firm's optimization:

$MRT = p \Rightarrow x^s = p$

$y^s = \frac{300 - p^2}{2}$

Consumer: $\max\ xy \quad s.t.\ (p+t)x + y = px^s + y^s + T$

$+ x^D$

$\frac{y}{x} = p + t \qquad y = (p+t)x$

$2(p+t)x^D = px^s + y^s + tx^D$

$x^D = x^s$

$2(p+t)x^s = px^s + y^s + tx^s$

$(p+t)x^s = y^s$

$(p+t)p = 150 - \frac{p^2}{2}$

$3p^2 + 2tp - 300 = 0$

$p = \frac{-2t + \sqrt{4t^2 + 4900}}{6}$

$p = \frac{1}{3}(\sqrt{t^2 + 900} - t)$

$\frac{dp}{dt} = \frac{1}{3}\left(\frac{t}{\sqrt{t^2+900}} - 1\right) < 0$

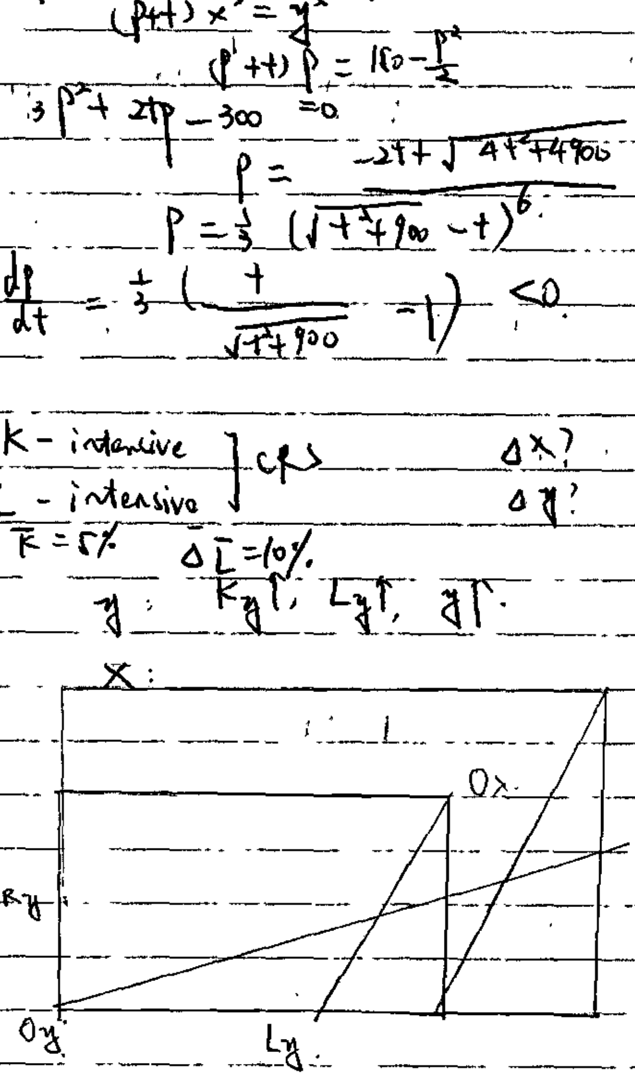
P3

x: K-intensive, y: L-intensive (CRS)　Δx? Δy?

$\Delta \bar{K} = 5\%$ $\quad \Delta \bar{L} = 10\%$

y: $K_y \uparrow$, $L_y \uparrow$, $y \uparrow$

X:



$$x = f(K_x, L_x) = L_x f\left(\frac{K_x}{L_x}\right)$$

$$\frac{x'-x}{x} = \frac{L_x'-L_x}{L_x}$$

$$\frac{K_x}{L_x} = a > b = \frac{K_y}{L_y}$$

$$(\bar{K} - K_x) = b(\bar{L} - L_x)$$

$$L_x = \frac{\bar{K} - \bar{L}b}{a-b}$$

$$\left.\begin{aligned} \bar{L}' &= (1+\delta_L)\bar{L} \\ \bar{K}' &= (1+\delta_K)\bar{K} \end{aligned}\right] \delta_L > \delta_K$$

$$L_x' = \frac{(1+\delta_K)\bar{K} - b(1+\delta_L)\bar{L}}{a-b}$$

$$\frac{L_x'-L_x}{L_x} = \frac{\bar{K}\delta_K - \bar{L}\delta_L b}{\bar{K}-\bar{L}b} < \frac{\bar{K}\delta_K - \bar{L}\delta_K b}{\bar{K}-\bar{L}b}$$

$$= \delta_K = 1\%$$

$$\frac{y'-y}{y} = \frac{L_y'-L_y}{L_y} > \delta_L$$

P4 State 1 $\bar{x}_1^A = 3$ $\bar{x}_1^B = 2$ $\left(\frac{2}{3}\right)$

State 2 $\bar{x}_2^A = 1$ $\bar{x}_2^B = 2$ $\left(\frac{1}{3}\right)$

Ann: $u(x) = \log x$

Bob: $v(x) = x$

a) $u^A = \frac{2}{3}\log x_1 + \frac{1}{3}\log x_2 = x_1^2 x_2$

$u^B = \frac{2}{3}x_1 + \frac{1}{3}x_2 = 2x_1 + x_2$

b) $MRS^A = MRS^B$

$$\frac{2x_1^A x_2^A}{x_1^{A2}} = 2 = 1$$

$$x_1^A = x_2^A$$

$$p(\bar{X}_1^A - X_1^A) = (X_2^A - \bar{X}_2^A)$$

$$2(3 - X_1^A) = X_1^A - 1$$

$$X_1^A = X_2^A = \frac{7}{3}$$

$$X_1^B = \frac{8}{3} \quad X_2^B = \frac{2}{3}$$

c) $v(x) = \sqrt{x}$

$$-\frac{v''(x)}{v'(x)} = \frac{1}{2x}$$

$$-\frac{u''(x)}{u'(x)} = \frac{1}{x} \text{ (more R.A.)}$$

d) Ann: $\frac{2x_1^A x_2^A}{x_1^{A2}} = p \quad 2x_2^A = px_1^A$

$$px_1^A + x_2^A = 3p + 1$$

$$x_2^A = \frac{3p+1}{3}$$

$$X_1^A = 2\,\frac{3p+1}{3} \cdot \frac{2}{p}$$

$$X_1^B = 5 - X_1^A = 3 - \frac{2}{3p}$$

$$X_2^B = 3 - X_2^A = \frac{8}{3} - p$$

$$\max_p \; 2x_1^B + x_2^B$$

$$2\left(3 - \frac{2}{3p}\right) + \frac{8}{3} - p \quad p = \frac{2}{\sqrt{3}}$$

Pis 11

1. $\pi = 10x - \frac{1}{2}x^2$

externality Ax

a) max $\pi = 10x - \frac{1}{2}x^2$

$x = 10.$

b) $t = ?$

① socially optimal

max $10x - \frac{1}{2}x^2 - Ax$

$x = 10 - A$

$x^* = 10 - E(A) = 8.5$

② firms profit w/ tax

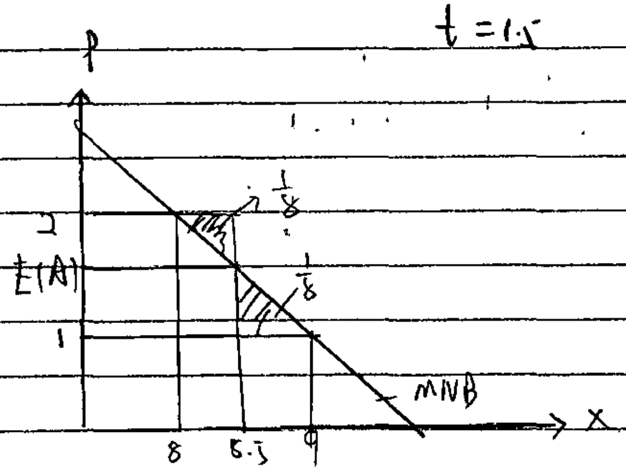
$\pi = 10x - \frac{1}{2}x^2 - tx$

$x = 10x - \frac{1}{2}x^2 - tx$

$x = 10 - t = 8.5$

$t = 1.5$

③



$E(DWL) = \frac{1}{8}$

c) 10 permits to firm

price of permit = Marginal $= A$

production = $10 - A$

efficient

d) Two people: $\frac{Ax}{2}$

price of permit $= \frac{A}{2}$

production $= 10 - \frac{A}{2}$

Problem 2

Samuelson Condition

Quasilinear $u_1'(g) + u_2'(g) = 1$

Generally: N agents $G = f(g_1 + \dots + g_N)$

$$\sum_{i=1}^{N} MRS^i_{x,g} = MRT_g$$

$$\max_{g_1, g_2} \lambda_1 u_1(f(g_1+g_2), w_1 - g_1) + \lambda_2 u_2(f(g_1+g_2), w_2 - g_2)$$

FOC (g_1) $\lambda_1 u_{1g} f_g - \lambda_1 u_{1x} + \lambda_2 u_{2g} f_g = 0$

Problem 3

Problem 4

$\theta > 0$. $q(\theta) = \theta - \frac{p}{2}$ $c(q) = \theta q$

a) $\pi = p(\theta - \frac{p}{2}) - \theta(\theta - \frac{p}{2})$

c) Bell $\xrightarrow{r(\theta)}$ Luke $\xrightarrow{p(\theta)}$ consumers

Bertrand Paradox

① Asymmetric MC

② heterogeneous products

③ capacity constraint (—)

P3 Sequential Bertrand.

2 firms, $MC=0$

$Q_i = 1-2p_i+p_j$

a) Simultaneous BR

Firm 1: $\max_{p_1} p_1(1-2p_1+p_2)$

FOC. $1-4p_1+p_2=0$

$p_1 = \frac{1+p_2}{4}$

$p_2 = \frac{1+p_1}{4}$ $\quad p_2=\frac{1}{3}$ $\quad p_1=\frac{1}{3}$

$p_2 = \frac{1+\frac{1+p_2}{4}}{4}$ $\quad \pi_1 = \frac{2}{9}$ $\quad \pi_2=\frac{2}{9}$

b) Sequential Firm 1 picks p_1

Firm 2 sees p_1, pick p_2

Backward Induction

2nd stage $p_2 = \frac{1+p_1}{4}$

1st stage $\max_{p_1} p_1\left(1-2p_1+\frac{1+p_1}{4}\right)$

FOC $1-4p_1+\frac{1}{4}+\frac{1}{2}p_1=0$

$p_1 = \frac{5}{14}$ $\quad p_2 = \frac{19}{56}$

c) $\pi = \left(\frac{700}{56^2}, \frac{722}{56^2}\right)$

$p_2(1-p_2+(\underline{p_1-p_2}))$

Problem 4

seller $\xrightarrow{x}$ buyer $\xrightarrow{p}$ seller $\xrightarrow{\text{refuse}}$ $-x, 0$

$\xrightarrow{\text{accepts}}$ $p-(x+1-\sqrt{x}),\ 1-p$

a) Backword Induction

③ Accept iff $p-(x+1-\sqrt{x}) \geq -x$

$p \geq 1-\sqrt{x}$.

② buyer wants seller to accept

pick lowest price $p = 1-\sqrt{x}$

① Sellers knows he will get $-X$

pick $X=0$.

b) Cooperative game,

$\max\ p-x-1+\sqrt{x}+1-p$

$= \sqrt{x}-X$

$x=\frac{1}{4}$, jointly get $\frac{1}{4}$

Problem 5 Bargaining

P_1 offer $\xrightarrow{(x,1-x)}$ P_2 $\xrightarrow{\text{counter } (y, 1-y)}$ P_1 $\xrightarrow{\text{reject}}$ $(0,0)$

P_2 $\xrightarrow{\text{accept}}$ $(x, 1-x)$

P_1 $\xrightarrow{\text{accept}}$ $(\delta_1 y, \delta_2(1-y))$

2nd round

P_1 accept iff $\delta_1 y \geq 0$.

P_2 counter offer $(0,1)$

1st round

P_1 wants P_2 to accept

lowest $1-x$ s.t. $1-x \geq \delta_2$

SPE: P_1 offers $(\delta_2, 1-\delta_2)$

P_2 accepts

2 $\delta_2 \uparrow$ offer P_2 more in 1st round

$\delta_1 \uparrow$: No affect