Implementation of an Alcohol Tolerance Prediction Model

Using Differential Equations and Laplace Transforms

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Outline

Background: Blood Alcohol Level Model

- Two-compartment model: stomach A(t) and blood B(t)
- First-order kinetic model:

$$\frac{dA}{dt} = -k_1 A(t), \quad A(0) = A_0 \tag{1}$$

$$\frac{dB}{dt} = k_1 A(t) - k_2 B(t), \quad B(0) = 0 \tag{2}$$

- k_1: Absorption rate from stomach to bloodstream
- k_2: Elimination rate from bloodstream
- A_0: Initial alcohol amount in stomach

Motivation

Alcohol-related Statistics in South Korea (2019-2023)

- Average 42 drunk driving incidents daily
- 75,950 alcohol-related traffic accidents
- 1,161 fatalities and 122,566 injuries
- Peak incidents: Thursday-Friday nights (10 PM - midnight)

Goal

Develop a mathematical algorithm to estimate alcohol tolerance without actual consumption

Problem Statement

Limitations of Classical Models

- Assume constant absorption (k_1) and elimination (k_2) rates
- Fail to capture non-local memory effects
- Cannot model dynamic variation in elimination rates

Our Solution

Integration of:

- Non-local memory effects using fractional calculus
- Dynamic elimination rate variation
- More realistic BAC predictions

Caputo Fractional Derivative

Definition

The Caputo fractional derivative of order α is:

$$ {}^{C}D_0^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau $$

Fractional BAC Model

$$ {}^{C}D_0^{\alpha} A(t) = -k_1 A(t), \quad A(0) = A_0 \tag{3} $$

$$ {}^{C}D_0^{\beta} B(t) = k_1 A(t) - k_2 B(t), \quad B(0) = 0 \tag{4} $$

where $0 < \alpha, \beta \leq 1$

Solution Using Laplace Transform

Stomach Alcohol Concentration

$$A(t) = A_0 E_\alpha(-k_1 t^\alpha)$$

where $E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+1)}$ is the Mittag-Leffler function

Blood Alcohol Concentration

$$B(t) = k_1 A_0 t^{\beta-1} E^{(2)}_{\alpha,\beta}(-k_1 t^\alpha, -k_2 t^\beta)$$

where $E^{(2)}_{\alpha,\beta}(x, y) = \sum_{m,n\geq 0} \frac{x^m y^n}{\Gamma(\alpha m+\beta n+1)}$

Tolerance Definition

Key Thresholds

- Intoxication threshold: $B(t_i) = 0.08\%$ (legal limit)
- Recovery threshold: $B(t_f) = 0.01\%$ (safe level)

Tolerance Time

$$\Delta T = t_f - t_i$$

Time duration from intoxication to recovery

Data Preprocessing

- Weight: m (kg)
- Total Body Water ratio: r (Male: 0.68, Female: 0.55)
- Initial concentration: $A_0 = \frac{V \times (ABV/100) \times \rho_{EtOH}}{r \times m}$

Web Application Architecture

Backend

- Python Flask server
- NumPy and SciPy for computation
- Mittag-Leffler function implementation

Frontend

- HTML5, CSS3, JavaScript
- Dynamic visualization with Matplotlib
- Korean language support

Input Parameters

- Gender (Male/Female)
- Age (19-100 years)
- Body weight (30-200kg)
- Alcohol type and volume
- Drinking start time

Model Selection

- Classical Model
- Fractional Model

Core Algorithm

Initial Concentration Calculation

$$A_0 = \frac{\text{Volume}(mL) \times \frac{\text{Alcohol\%}}{100} \times \rho_{ethanol}}{\text{TBW ratio} \times \text{Body Weight}(kg)}$$

where $\rho_{ethanol} = 0.789$ g/mL

Age-Adjusted TBW Ratio

$$\text{TBW}_{Male} = 0.68 - (Age - 25) \times 0.001 \quad (5)$$

$$\text{TBW}_{Female} = 0.55 - (Age - 25) \times 0.001 \quad (6)$$

Model Parameters

Standard Parameters Used

$$k_1 = 0.8 \text{ h}^{-1} \text{ (absorption rate)} \quad (7)$$

$$k_2 = 1.0 \text{ h}^{-1} \text{ (elimination rate)} \quad (8)$$

$$\alpha = 0.8 \text{ (fractional order for absorption)} \quad (9)$$

$$\beta = 0.9 \text{ (fractional order for elimination)} \quad (10)$$

Test Scenario

- Subject: 25-year-old male, 70kg
- Alcohol: 360mL soju (17% ABV)
- Expected peak BAC: 150-170 mg/100mL

Model Comparison Results

Metric	Classical	Fractional
Peak BAC (mg/100mL)	158.2	162.3
Time to Peak (h)	1.2	1.4
Legal threshold recovery (h)	1.8	2.1
Safe to drive (h)	4.2	4.9
Full recovery (h)	14.8	16.6

Key Finding

Fractional model predicts longer impairment periods, providing better safety margins

Key Findings

BAC Time-Course Analysis

- Peak BAC: 0.02-0.10 g/100mL (varies by gender, weight, alcohol type)
- Recovery times: 2-8 hours (classical) vs 3-10 hours (fractional)
- Proper scaling with body weight

Gender Differences

- Females show higher peak BAC (lower TBW: 55% vs 68%)
- Longer recovery times across all weight ranges

Memory Effect

- Non-exponential decay patterns
- Slower initial decline, prolonged tail
- More realistic metabolic representation

Advantages of Fractional Model

Theoretical Advantages

- **Physiological Realism**: Memory effects better represent complex metabolism
- **Individual Variability**: Adjustable fractional orders (α, β)
- **Mathematical Flexibility**: Reduces to classical when $\alpha = \beta = 1$

Practical Advantages

- **Prolonged Effects**: Better prediction of extended impairment
- **Safety Applications**: More conservative estimates for driving safety
- **Individual Calibration**: Potential for personalized parameters

Limitations and Future Work

Current Limitations

- Parameters require empirical determination
- Higher computational overhead
- Limited experimental validation with real BAC data

Future Research Directions

- Parameter estimation from individual BAC measurements
- Integration with wearable sensor data
- Population-based parameter distributions
- Real-time model adaptation algorithms

Conclusions

Model Validity

Both models produced physiologically reasonable BAC predictions across diverse scenarios

Fractional Model Superiority

- More realistic absorption/elimination kinetics
- Captures memory effects in alcohol metabolism
- Better prediction of prolonged impairment periods

Safety Implications

- More conservative estimates for driving safety
- Better cognitive impairment duration prediction
- Improved risk assessment for alcohol-related activities

Final Recommendations

1. **Adopt Fractional Models** for critical safety applications
2. **Individual Calibration** when possible using measured BAC data
3. **Conservative Estimates** for safety applications
4. **Further Validation** with controlled studies and real-world measurements

Impact

The fractional calculus approach represents a significant advancement in BAC modeling, offering improved physiological realism and practical utility for both research and applied contexts.

Questions?

Thank you for your attention!

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