Jump Detection

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The jump time series is defined as

$$x_t = \frac{r_t}{f_t \sigma_t} \tag{1}$$

where $r_t = \ln(p_t/p_{t-1})$ is 1-minute return, f_t is an estimator of intraday periodicity, and σ_t is an estimator for the local volatility. The σ_t is defined as

$$\sigma_t^2 = \frac{\pi}{2K} \sum_{i=1}^{390} |r_{t-i}||r_{t-i+1}| \tag{2}$$

1 Wavelet

1.1 Continuous Wavelet

What is a wavelet? A function $\psi(x)$ is called a wavelet if it satisfies

$$\int_{-\infty}^{\infty} dx\, \psi(x) = 0 \tag{3}$$

$$\int_{-\infty}^{\infty} d\omega\, \frac{|\psi(\omega)|^2}{\omega} = C_\psi < 0 \tag{4}$$

Here C_ψ is called admissible constant. A wavelet family can be generated from the above mother wavelet by translation and dilation

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \psi\left(\frac{x-b}{a}\right) \tag{5}$$

The parameter a controls the dilation, while b controls the shift of the wavelet.

The *continuous wavelet transformation* of a function $f(x)$ is defined by

$$Wf(a,b) = \int_{-\infty}^{\infty} dx\, f(x) \psi_{a,b}(x) \tag{6}$$

while the inverse transformation is defined as

1.2 Discrete Wavelet

In the discrete case, a family of wavelets is constructed from the mother wavelet via

$$\psi_{j,k}(x) = 2^{-j/2} \psi(2^{-j}x - k) \tag{7}$$

for $j, k \in \mathbb{Z}$.