Simulation-Based Inference: Time Series of a Coherent Signal

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Consider the following time series

$$s(t) = h(t) + n(t) \tag{1}$$

where $h(t)$ is a coherent signal and $n(t)$ is the noise. Each of them is characterized by

$$\langle n(t_I)n(t_J)\rangle = N_{IJ} = \delta_{IJ}\sigma^2 \tag{2}$$

and

$$h(t) = h_0\cos(2\pi f t + \varphi) \tag{3}$$

The index I denotes the discrete time index and N_{IJ} is the noise covariance matrix. The noise is assumed to be white, although this assumption can be easily relaxed. For the generation of simulated time series, we set $\sigma = 1$.

In this note, we generate time series according to the above characterization. We then perform a traditional Bayesian analysis for the estimation of the signal $h(t)$. We also use simulation-based inference technique for the parameter estimation, and compare the performance of two approaches.

1 Likelihood

For the traditional approach, we begin by constructing the likelihood function. The likelihood function is given by

$$p(D|\theta) = \frac{1}{\sqrt{|2\pi N|}}\exp\left[-\frac{1}{2}(s-h)^T\cdot N^{-1}\cdot(s-h)\right] \tag{4}$$

where we treat the time series as a column vector $s = (s(t_1),\, s(t_2)\cdots, s(t_{N_t}))^T$. The parameter is

$$\theta = \{\sigma, h_0, f, \varphi\}. \tag{5}$$

We impose flat uniform prior for each of these parameters. Then the posterior can be constructed as

$$p(\theta|D) = p(D|\theta)p(\theta). \tag{6}$$

If we model that the underlying noise is white, then the noise covariance matrix is diagonal. The likelihood function can then be simplified as

$$P(D|\theta) = \frac{1}{(2\pi)^{N_t/2}\sigma^{N_t}}\exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N_t}(s-h)^2\right]. \tag{7}$$