DPP sign conventions

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Abstract

Here, P.Pobylitsa clarifies the relation between DPP and Bochum.

I. MINKOWSKI SPACE

Free fermion statistical weight

$$\exp\left[i\int d^4x\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi\right], \qquad \bar{\psi} = \psi^+\gamma_0 \tag{1}$$

The field ψ annihilates quarks and creates antiquarks.

Baryon charge

$$B = \frac{1}{N_c} : \int d^3x\bar{\psi}\gamma_0\psi : \tag{2}$$

$$\begin{aligned} \gamma_5 \; &= \; \gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \left\{\begin{array}{l} \text{Berestetsky, Lifshitz, Pitayevsky 80} \\ \text{Okun 81} \end{array}\right\} \\ &= \; -\left\{\begin{array}{l} \text{Bjorken, Drell} \\ \text{Itzykson, Zuber} \\ \text{Bochum} \end{array}\right\} \end{aligned} \tag{3}$$

In the limit $m \to 0$

$$\psi_L = \frac{1+\gamma_5}{2}\psi \left\{\begin{array}{l} \text{annihilates the left quark (helicity –1/2)} \\ \text{creates the right antiquark (helicity +1/2)} \end{array}\right. \tag{4}$$

$$\psi_R = \frac{1-\gamma_5}{2}\psi \left\{\begin{array}{l} \text{annihilates the right quark (helicity +1/2)} \\ \text{creates the left antiquark (helicity –1/2)} \end{array}\right. \tag{5}$$

Quark condensate:

$$sign\langle\bar{\psi}\psi\rangle = -sign\ m \tag{6}$$

Effective quark mass: $sign\ M(p) = sign\ m.$

$$\bar{\psi}_L = \psi_L^+\gamma_0, \qquad \bar{\psi}_R = \psi_R^+\gamma_0 \tag{7}$$

Phase matrix of the condensate

$$U_{ij} = const\ Tr_{spin}\langle\psi_{Ri}\bar{\psi}_{Lj}\rangle, \qquad const > 0 \tag{8}$$

Statistical weight of the effective chiral action

$$\exp\left[i\int d^4x\bar{\psi}(i\gamma^\mu\partial_\mu - MU^{\gamma_5})\psi\right] \tag{9}$$

Hamiltonian

$$H = -i\gamma_0\gamma^k\partial_k + M\gamma_0 U^{\gamma_5} = i\gamma_0\gamma_k\partial_k + M\gamma_0 U^{\gamma_5} \tag{10}$$

Electromagnetic covariant derivative

$$\nabla_\mu(A) = \partial_\mu + iQA_\mu \tag{11}$$

Lagrangian with the electromagnetic field

$$\bar\psi[\gamma^\mu(i\partial_\mu - QA_\mu) - m]\psi, \tag{12}$$

Here Q is the charge matrix of *quarks* (the field ψ annihilates quarks and creates the anti-quarks):

$$Q = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} \tag{13}$$

The vector potential $A^\mu = (\phi, \mathbf{A})$ is the same as in Landau II:

$$\begin{aligned} \mathbf{E} &= -\nabla\phi - \frac{\partial}{\partial t}\mathbf{A} \\ \mathbf{H} &= \mathrm{rot}\mathbf{A}, \qquad \text{etc.} \end{aligned} \tag{14}$$

II. EUCLIDIZATION

$$x^0_M = -ix^4_E, \qquad x^k_M = x^k_E, \qquad d^4x_M = -id^4x_E, \tag{15}$$

$$\partial_{0M} = i\partial_{4E}, \qquad \partial_{kM} = \partial_{kE} \tag{16}$$

$$\gamma^0_M = \gamma_{4E}, \qquad \gamma^k_M = i\gamma_{kE} \tag{17}$$

$$\not\partial_M = \gamma^\mu_M\partial_{\mu M} = i\gamma_{\mu E}\partial_{\mu E} = i\not\partial_E \tag{18}$$

$$\bar\psi_M = -i\psi^+_E \tag{19}$$

$$A^0_M = -iA^4_E, \qquad A^k_M = A^k_E \tag{20}$$

$$\nabla(A)^0_M = i\nabla(A)^4_E, \qquad \nabla(A)^k_M = -\nabla(A)^k_E \tag{21}$$

III. EUCLIDEAN SPACE

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4, \qquad \gamma_\mu^+ = \gamma_\mu \tag{22}$$

$$\epsilon_{1234} = +1 \tag{23}$$

$$Sp(\gamma_5\gamma_\lambda\gamma_\mu\gamma_\nu\gamma_\rho) = 4\epsilon_{\lambda\mu\nu\rho} \tag{24}$$

$$\gamma_k = \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix} \qquad \gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \tag{25}$$

Statistical weight

$$\exp\left[\int d^4x\psi^+(i\partial\!\!\!/ + iMU^{\gamma_5})\psi\right] \tag{26}$$

Hamiltonian

$$H = \gamma_4\gamma_k\partial_k + M\gamma_4U^{\gamma_5} \tag{27}$$

Electromagnetic covariant derivative

$$\nabla_\mu(A) = \partial_\mu - iQA_\mu \tag{28}$$

Lagrangian with the electromagnetic field

$$\psi^+[\gamma_\mu(i\partial_\mu + QA_\mu) + im]\psi, \tag{29}$$

Baryon charge

$$B = -\frac{i}{N_c} : \int d^3x\psi^+\gamma_4\psi : \tag{30}$$

$$\begin{aligned} \langle B\rangle &= \frac{\int \mathcal{D}\psi\mathcal{D}\psi^+ \left(-\frac{i}{N_c}\int d^3x\psi^+\gamma_4\psi\right)\exp\left[\int d^4x\psi^+(i\partial\!\!\!/ + iMU^{\gamma_5})\psi\right]}{\int \mathcal{D}\psi\mathcal{D}\psi^+ \exp\left[\int d^4x\psi^+(i\partial\!\!\!/ + iMU^{\gamma_5})\psi\right]} \\ &= Tr\theta(-H) \end{aligned} \tag{31}$$

Long wave expansion for the baryon charge

$$\begin{aligned} Tr\ \ [\theta(-H) - \theta(-H_0)] &= -\frac{1}{24\pi^2}\epsilon_{ijk}\int d^3xTr\left\{(U^+\partial_iU)(U^+\partial_jU)(U^+\partial_kU)\right\} \\ = \frac{1}{\pi}\left(\frac{sin2P}{2} - P\right)\Big|_0^\infty & \quad \left(= \frac{P(0)}{\pi}, \quad \text{if} \quad P(0) = \pi n,\ P(\infty) = 0\right) \end{aligned} \tag{32}$$

Here

$$U = \exp[iP(\vec{n}\vec{\tau})] \tag{33}$$

$$\epsilon_{123} = +1 \tag{34}$$

IV. CONNECTION TO THE BOCHUM GROUP

The different sign of the γ_5 matrix

$$\gamma_5^{DPP} = -\gamma_5^{Bochum} \tag{35}$$

is compensated by the different sign of the profile function

$$P(r)^{DPP} = -P(r)^{Bochum} \tag{36}$$

so both DPP and Bochum have the same Hamiltonian

$$H = \gamma_4\gamma_k\partial_k + M\gamma_4 U^{\gamma_5} \tag{37}$$

but people in Bochum prefer to write Hamiltonian in the form

$$H^{Bochum} = \frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + g\beta(\sigma + i\vec{\tau}\vec{\pi}\gamma_5) \tag{38}$$

where

$$\beta = \gamma_4 \tag{39}$$

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix} \tag{40}$$

Another source of inconsistencies is different spherical functions

$$Y_{lm}^{DPP} = Y_{lm}^{Landau} = i^l Y_{lm}^{Bochum} \tag{41}$$

This phase factor penetrates into all angular wave functions

$$\Omega_{JLJ_3}^{DPP} = i^l|ljm\rangle^{Bochum} \tag{42}$$

$$\Xi_{KK_3JL}^{DPP} = i^l|ljGM\rangle^{Bochum} \tag{43}$$

where

$$L^{DPP} = l^{Bochum} \tag{44}$$

$$J^{DPP} = j^{Bochum} \tag{45}$$

$$J_3^{DPP} = m^{Bochum} \tag{46}$$

$$K^{DPP} = G^{Bochum} \tag{47}$$

$$K_3^{DPP} = M^{Bochum} \tag{48}$$

V. HEDGEHOG ANSATZ

$$U = \exp[iP(r)(\vec{n}\vec{\tau})] = \cos P(r) + i(\vec{n}\vec{\tau})\sin P(r) \tag{49}$$

$$\begin{aligned} L_k &\equiv iU^+\partial_k U \\ &= -n_k(\vec{n}\vec{\tau})\left(P' - \frac{\sin 2P}{2r}\right) - \tau_k\frac{\sin 2P}{2r} - \epsilon_{klm}\tau_l n_m\frac{\sin^2 P}{r} \end{aligned} \tag{50}$$

$$L_k^2 = P'^2 + 2\frac{\sin^2 P}{r^2} \tag{51}$$

$$\begin{aligned} -i[L_k, L_l] &= \frac{\sin 2P}{r}P'\epsilon_{klm}\tau_m + 2\epsilon_{klm}n_m(\vec{n}\vec{\tau})\left(\frac{\sin^2 P}{r^2} - \frac{\sin 2P}{2r}P'\right) \\ &+ 2P'\frac{\sin^2 P}{r}(\tau_k n_l - \tau_l n_k) \end{aligned} \tag{52}$$

$$\partial_k L_k = (\vec{n}\vec{\tau})\left(\frac{\sin 2P}{r^2} - \frac{2}{r}P' - P''\right) \tag{53}$$

$$Tr\left([L_k, L_l]^2\right) = 16\left(2P'^2\frac{\sin^2 P}{r^2} + \frac{\sin^4 P}{r^4}\right) \tag{54}$$