$$g^{\nu}{}_{\alpha} g_{\beta\mu} - g^{\nu}{}_{\beta} g_{\alpha\mu}$$

$$\begin{aligned}\Delta\omega^{\nu}{}_{\mu}\left(g^{\nu}{}_{\alpha}\gamma_{\beta} - g^{\nu}{}_{\beta}\gamma_{\alpha}\right) &= -\frac{i}{4}\left(\Delta\omega^{\nu}{}_{\mu} - \Delta\omega_{\mu}{}^{\nu}\right)[\gamma^{\nu}, \sigma_{\alpha\beta}] \\ &= -\frac{i}{2}\Delta\omega^{\nu}{}_{\mu}[\gamma^{\nu}, \sigma_{\alpha\beta}]\end{aligned}$$

$$\Longrightarrow 2i\left(g^{\nu}{}_{\alpha}\gamma_{\beta} - g^{\nu}{}_{\beta}\gamma_{\alpha}\right) = [\gamma^{\nu}, \sigma_{\alpha\beta}]$$

$$\sigma_{\alpha\beta} = \frac{i}{2}[\gamma_{\alpha}, \gamma_{\beta}]$$

$$I^{\nu}{}_{\mu} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (I^{\nu}{}_{\mu})^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}a^{\nu}{}_{\mu} &\approx g^{\nu}{}_{\mu} + \Delta\omega^{\nu}{}_{\mu} \\ &= g^{\nu}{}_{\mu} + \Delta\omega I^{\nu}{}_{\mu}\end{aligned}$$

$$\begin{aligned}(x^{\nu})' &\approx \left(g + \frac{\omega}{N}I\right)^{\nu}{}_{\alpha_1}\left(g + \frac{\omega}{N}I\right)^{\alpha_1}{}_{\alpha_2}\cdots\left(g + \frac{\omega}{N}I\right)^{\alpha_{N-1}}{}_{\alpha_N} x^{\alpha_N} = \lim_{N\to\infty}\left(g + \frac{\omega}{N}I\right)^{\nu}{}_{\alpha_1}\cdots\left(g + \frac{\omega}{N}I\right)^{\alpha_{N-1}}{}_{\alpha_N} x^{\alpha_N} \\ &= \lim_{N\to\infty}\left(g + \frac{\omega}{N}I\right)^{N\,\nu}{}_{\mu} x^{\alpha_N} = \left(e^{\omega I}\right)^{\nu}{}_{\mu} x^{\mu} \\ &= \left(\cosh\omega I + \sinh\omega I\right)^{\nu}{}_{\mu} x^{\mu}\end{aligned}$$

$$(x^{\nu})' = \left(\mathbb{I} - I^2 + I^2\cosh\omega + I\cosh\omega\right)^{\nu}{}_{\mu} x^{\mu}$$

$$\begin{pmatrix} x^{0\prime} \\ x^{1\prime} \\ x^{2\prime} \\ x^{3\prime} \end{pmatrix} = \begin{pmatrix} \cosh\omega & -\sinh\omega & 0 & 0 \\ -\sinh\omega & \cosh\omega & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$\begin{aligned}\psi'(x') = S\psi(x) &= \lim_{N\to\infty}\left(1 - \frac{i}{4}\frac{\omega}{N}\sigma_{\mu\nu}I_n{}^{\mu\nu}\right)^N \psi(x) \\ &= \exp\left(-\frac{i}{4}\omega\sigma_{\mu\nu}I_n{}^{\mu\nu}\right)\psi(x)\end{aligned}$$

$$\psi'(x') = e^{-\frac{i}{4}\omega\left(\sigma_{01}I^{01} + \sigma_{10}I^{10}\right)}\psi(x) = e^{-\frac{i}{2}\omega\sigma_{01}}\psi(x)$$

$$I^{\nu}{}_{\mu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \psi'(x') = e^{-\frac{i}{2}\phi\sigma_{12}}\psi(x)$$

$$j^{\mu}(x) = c\psi^{\dagger}\gamma^{0}\gamma^{\mu}\psi,\;\; j^{\mu\prime}(x') = c\psi'^{\dagger}(x')\gamma^{0}\gamma^{\mu}\psi'(x')$$

$$\begin{aligned} j^{\mu\prime}(x') &= c\psi^{\dagger}(x)S^{\dagger}\gamma^{0}\gamma^{\mu}S\psi(x) \\ &= c\psi^{\dagger}(x)\gamma^{0}S^{-1}\gamma^{\mu}S\psi(x) \\ &= ca^{\mu}{}_{\nu}\psi^{\dagger}(x)\gamma^{0}\gamma^{\mu}\psi(x) \\ &= ca^{\mu}{}_{\nu}j^{\mu}(x) \end{aligned}$$

$$a^{\nu}{}_{\mu}\gamma^{\mu} = S^{-1}(a)\gamma^{\nu}S(a) \Longrightarrow a^{\nu}{}_{\mu}\gamma^{\mu} = P^{-1}\gamma^{\nu}P$$

$$a^{\nu}{}_{\mu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = g^{\nu\mu}$$

$$\begin{aligned} &a^{\sigma}{}_{\nu}a^{\nu}{}_{\mu}\gamma^{\mu} = P^{-1}a^{\sigma}{}_{\nu}\gamma^{\nu}P,\;\; a^{\sigma}{}_{\nu}\gamma^{\nu} = g^{\sigma\sigma}\gamma^{\sigma} \\ &\Longrightarrow Pg^{\sigma}{}_{\mu}\gamma^{\mu}P^{-1} = g^{\sigma\sigma}\gamma^{\sigma} \end{aligned}$$

$$P = e^{i\phi}\gamma^{0} \Longrightarrow \psi'(x') = \psi(-x) = P = e^{i\phi}\gamma^{0}\psi(x)$$

$$\begin{aligned} &\Gamma^{s} = \mathbb{I},\;\; \Gamma^{V}_{\mu} = \gamma_{\mu},\;\; \Gamma^{T}_{\mu\nu} = \sigma_{\mu\nu} \\ &\Gamma^{P} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} = \gamma^{5} = \gamma_{5},\;\; \Gamma^{A}_{\mu}\gamma_{5}\gamma_{\mu} \end{aligned}$$

$$\begin{aligned} U(I) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, U(C_3) = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}, U(C_3^2) = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}, \\ U(C_2) &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, U(C_2') = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}, U(C_2'') = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix} \end{aligned}$$

$$\begin{aligned} U(t) &= \left\langle \mathbf{x} \left| e^{-i(\mathbf{p}^2/2m)t} \right| \mathbf{x_0} \right\rangle \\ &= \int \frac{d^3p}{(2\pi)^3} \left\langle \mathbf{x} \left| e^{-i(\mathbf{p}^2/2m)t} \right| \mathbf{p} \right\rangle \langle \mathbf{p}|\mathbf{x_0}\rangle \\ &= \frac{1}{(2\pi)^3}\int d^3p e^{-i(\mathbf{p}^2/2m)t}\cdot e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x_0})} \\ &= \left(\frac{m}{2\pi it}\right)^{\frac{3}{2}} e^{im(\mathbf{x}-\mathbf{x_0}{}^2)/2t}. \end{aligned}$$

$$\begin{aligned} U(t) &= \left\langle \mathbf{x} \left| e^{-it\sqrt{\mathbf{p}^2+m^2}} \right| \mathbf{x_0} \right\rangle \\ &= \frac{1}{(2\pi)^3}\int d^3p e^{-it\sqrt{\mathbf{p}^2+m^2}}\cdot e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x_0})} \\ &= \frac{1}{2\pi^2|\mathbf{x}-\mathbf{x_0}|}\int_0^{\infty} dp\, p\, \sin(p|\mathbf{x}-\mathbf{x_0}|)e^{-it\sqrt{p^2+m^2}} \end{aligned}$$

$$\lim_{\delta z\to 0}\frac{\delta f}{\delta z} = \lim_{\delta y\to 0}\frac{\delta u + i\delta v}{i\delta y} = -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \tag{1}$$

$$\lim_{\delta z\to 0}\frac{\delta f}{\delta z}=\lim_{\delta x\to 0}\frac{\delta u+i\delta v}{\delta x}=\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x} \tag{2}$$

$$\int_{z_1}^{z_2} f(z)\,dz=\int_{x_1,y_1}^{x_2,y_2}[u(x,y)+iv(x,y)][dx+idy]=\int_{x_1,y_1}^{x_2,y_2}[u(x,y)dx-v(x,y)dy]+i\int_{x_1,y_1}^{x_2,y_2}[v(x,y)dx+u(x,y)dy] \tag{3}$$

$$\oint_C f(z)\,dz=0. \tag{4}$$

$$\begin{aligned}\alpha\Delta\mathcal{L}&=\frac{\partial\mathcal{L}}{\partial\phi}(\alpha\Delta\phi)+\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\partial_\mu(\alpha\Delta\phi)\\&=\alpha\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\phi\right)+\alpha\left[\frac{\partial\mathcal{L}}{\partial\phi}-\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\right]\Delta\phi\end{aligned}$$

$$\alpha\partial_\mu\mathcal{J}^\mu(x)=\alpha\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\phi\right)$$

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\phi-\mathcal{J}^\mu(x)\right)=0$$

$$j^\mu(x)=\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\phi-\mathcal{J}^\mu(x)$$

$$\delta\mathcal{L}=\frac{\partial\mathcal{L}}{\partial\phi}i\alpha_a t^a\phi+\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}i\alpha_a t^a\partial_\mu\phi=0$$

$$\frac{\partial\mathcal{L}}{\partial\phi}=\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}$$

$$\alpha_a\partial_\mu\left(i\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}t^a\phi\right)=0$$

$$J^a_\mu=i\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}t^a\phi,\ \ \partial^\mu J^a_\mu=0$$