

$$g^\nu{}_\alpha g_{\beta\mu} - g^\nu{}_\beta g_{\alpha\mu}$$

$$\begin{aligned}\Delta\omega^\nu{}_\mu (g^\nu{}_\alpha\gamma_\beta - g^\nu{}_\beta\gamma_\alpha) &= -\frac{i}{4}(\Delta\omega^\nu{}_\mu - \Delta\omega_\mu{}^\nu)[\gamma^\nu, \sigma_{\alpha\beta}] \\ &= -\frac{i}{2}\Delta\omega^\nu{}_\mu [\gamma^\nu, \sigma_{\alpha\beta}]\end{aligned}$$

$$\implies 2i\left(g^\nu{}_\alpha\gamma_\beta - g^\nu{}_\beta\gamma_\alpha\right) = [\gamma^\nu, \sigma_{\alpha\beta}]$$

$$\sigma_{\alpha\beta} = \frac{i}{2} [\gamma_\alpha, \gamma_\beta]$$

$$I^\nu{}_\mu = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (I^\nu{}_\mu)^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}a^\nu{}_\mu &\approx g^\nu{}_\mu + \Delta\omega^\nu{}_\mu \\ &= g^\nu{}_\mu + \Delta\omega I^\nu{}_\mu\end{aligned}$$

$$\begin{aligned}(x^\nu)' &\approx \left(g + \frac{\omega}{N}I\right)_{\alpha_1}^\nu \left(g + \frac{\omega}{N}I\right)_{\alpha_2}^{\alpha_1} \cdots \left(g + \frac{\omega}{N}I\right)_{\alpha_N}^{\alpha_{N-1}} x^{\alpha_N} = \lim_{N \rightarrow \infty} \left(g + \frac{\omega}{N}I\right)_{\alpha_1}^\nu \cdots \left(g + \frac{\omega}{N}I\right)_{\alpha_N}^{\alpha_{N-1}} x^{\alpha_N} \\ &= \lim_{N \rightarrow \infty} \left(g + \frac{\omega}{N}I\right)_{\mu}^{N^\nu} x^{\alpha_N} = (e^{\omega I})_{\mu}^\nu x^\mu \\ &= (\cosh \omega I + \sinh \omega I)_{\mu}^\nu x^\mu\end{aligned}$$

$$(x^\nu)' = (\mathbb{I} - I^2 + I^2 \cosh \omega + I \cosh \omega)_{\mu}^\nu x^\mu$$

$$\begin{pmatrix} x^{0'} \\ x^{1'} \\ x^{2'} \\ x^{3'} \end{pmatrix} = \begin{pmatrix} \cosh \omega & -\sinh \omega & 0 & 0 \\ -\sinh \omega & \cosh \omega & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$\begin{aligned}\psi'(x') &= S\psi(x) = \lim_{N \rightarrow \infty} \left(1 - \frac{i}{4} \frac{\omega}{N} \sigma_{\mu\nu} I_n{}^{\mu\nu}\right)^N \psi(x) \\ &= \exp\left(-\frac{i}{4} \omega \sigma_{\mu\nu} I_n{}^{\mu\nu}\right) \psi(x)\end{aligned}$$

$$\psi'(x') = e^{-\frac{i}{4}\omega(\sigma_{01}I^{01}+\sigma_{10}I^{10})}\psi(x) = e^{-\frac{i}{2}\omega\sigma_{01}}\psi(x)$$

$$I^\nu{}_\mu = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \psi'(x') = e^{-\frac{i}{2}\phi\sigma_{12}}\psi(x)$$

$$j^\mu(x) = c\psi^\dagger\gamma^0\gamma^\mu\psi, \quad j^{\mu'}(x') = c\psi'^\dagger(x')\gamma^0\gamma^\mu\psi'(x')$$

$$\begin{aligned} j^{\mu'}(x') &= c\psi^\dagger(x)S^\dagger\gamma^0\gamma^\mu S\psi(x) \\ &= c\psi^\dagger(x)\gamma^0S^{-1}\gamma^\mu S\psi(x) \\ &= ca^\mu{}_\nu\psi^\dagger(x)\gamma^0\gamma^\mu\psi(x) \\ &= ca^\mu{}_\nu j^\mu(x) \end{aligned}$$

$$a^\nu{}_\mu\gamma^\mu = S^{-1}(a)\gamma^\nu S(a) \implies a^\nu{}_\mu\gamma^\mu = P^{-1}\gamma^\nu P$$

$$a^\nu{}_\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = g^{\nu\mu}$$

$$\begin{aligned} a^\sigma{}_\nu a^\nu{}_\mu \gamma^\mu &= P^{-1} a^\sigma{}_\nu \gamma^\nu P, \quad a^\sigma{}_\nu \gamma^\nu = g^{\sigma\sigma} \gamma^\sigma \\ \implies P g^\sigma{}_\mu \gamma^\mu P^{-1} &= g^{\sigma\sigma} \gamma^\sigma \end{aligned}$$

$$P = e^{i\phi}\gamma^0 \implies \psi'(x') = \psi(-x) = P = e^{i\phi}\gamma^0\psi(x)$$

$$\begin{aligned} \Gamma^S &= \mathbb{I}, \quad \Gamma_\mu^V = \gamma_\mu, \quad \Gamma_{\mu\nu}^T = \sigma_{\mu\nu} \\ \Gamma^P &= i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma^5 = \gamma_5, \quad \Gamma_\mu^A \gamma_5 \gamma_\mu \end{aligned}$$

$$\begin{aligned} U(I) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, U(C_3) = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}, U(C_3^2) = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}, \\ U(C_2) &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, U(C_2') = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}, U(C_2'') = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix} \end{aligned}$$

$$\begin{aligned} U(t) &= \left\langle \mathbf{x} \left| e^{-i(\mathbf{p}^2/2m)t} \right| \mathbf{x}_0 \right\rangle \\ &= \int \frac{d^3p}{(2\pi)^3} \left\langle \mathbf{x} \left| e^{-i(\mathbf{p}^2/2m)t} \right| \mathbf{p} \right\rangle \langle \mathbf{p} | \mathbf{x}_0 \rangle \\ &= \frac{1}{(2\pi)^3} \int d^3p e^{-i(\mathbf{p}^2/2m)t} \cdot e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}_0)} \\ &= \left(\frac{m}{2\pi i t} \right)^{\frac{3}{2}} e^{im(\mathbf{x}-\mathbf{x}_0)^2/2t}. \end{aligned}$$

$$\begin{aligned} U(t) &= \left\langle \mathbf{x} \left| e^{-it\sqrt{\mathbf{p}^2+m^2}} \right| \mathbf{x}_0 \right\rangle \\ &= \frac{1}{(2\pi)^3} \int d^3p e^{-it\sqrt{\mathbf{p}^2+m^2}} \cdot e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}_0)} \\ &= \frac{1}{2\pi^2|\mathbf{x}-\mathbf{x}_0|} \int_0^\infty dp p \sin(p|\mathbf{x}-\mathbf{x}_0|) e^{-it\sqrt{p^2+m^2}} \end{aligned}$$

$$\lim_{\delta z \rightarrow 0} \frac{\delta f}{\delta z} = \lim_{\delta y \rightarrow 0} \frac{\delta u + i\delta v}{i\delta y} = -i \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \quad (1)$$

$$\lim_{\delta z \rightarrow 0} \frac{\delta f}{\delta z} = \lim_{\delta x \rightarrow 0} \frac{\delta u + i\delta v}{\delta x} = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} \quad (2)$$

$$\int_{z_1}^{z_2} f(z) dz = \int_{x_1, y_1}^{x_2, y_2} [u(x, y) + iv(x, y)][dx + idy] = \int_{x_1, y_1}^{x_2, y_2} [u(x, y)dx - v(x, y)dy] + i \int_{x_1, y_1}^{x_2, y_2} [v(x, y)dx + u(x, y)dy] \quad (3)$$

$$\oint_C f(z) dz = 0. \quad (4)$$