$$g^{\nu}{}_{\alpha} g_{\beta\mu} - g^{\nu}{}_{\beta} g_{\alpha\mu}$$

$$\begin{aligned}
\Delta\omega^{\nu}{}_{\mu}\left(g^{\nu}{}_{\alpha}\gamma_{\beta} - g^{\nu}{}_{\beta}\gamma_{\alpha}\right) &= -\frac{i}{4}\left(\Delta\omega^{\nu}{}_{\mu} - \Delta\omega_{\mu}{}^{\nu}\right)[\gamma^{\nu}, \sigma_{\alpha\beta}] \\
&= -\frac{i}{2}\Delta\omega^{\nu}{}_{\mu}[\gamma^{\nu}, \sigma_{\alpha\beta}]
\end{aligned}$$

$$\Longrightarrow 2i\left(g^{\nu}{}_{\alpha}\gamma_{\beta} - g^{\nu}{}_{\beta}\gamma_{\alpha}\right) = [\gamma^{\nu}, \sigma_{\alpha\beta}]$$

$$\sigma_{\alpha\beta} = \frac{i}{2}[\gamma_{\alpha}, \gamma_{\beta}]$$

$$I^{\nu}{}_{\mu} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (I^{\nu}{}_{\mu})^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}
a^{\nu}{}_{\mu} &\approx g^{\nu}{}_{\mu} + \Delta\omega^{\nu}{}_{\mu} \\
&= g^{\nu}{}_{\mu} + \Delta\omega I^{\nu}{}_{\mu}
\end{aligned}$$

$$\begin{aligned}
(x^{\nu})' &\approx \left(g + \frac{\omega}{N}I\right)^{\nu}{}_{\alpha_1}\left(g + \frac{\omega}{N}I\right)^{\alpha_1}{}_{\alpha_2}\cdots\left(g + \frac{\omega}{N}I\right)^{\alpha_{N-1}}{}_{\alpha_N} x^{\alpha_N} = \lim_{N\to\infty}\left(g + \frac{\omega}{N}I\right)^{\nu}{}_{\alpha_1}\cdots\left(g + \frac{\omega}{N}I\right)^{\alpha_{N-1}}{}_{\alpha_N} x^{\alpha_N} \\
&= \lim_{N\to\infty}\left(g + \frac{\omega}{N}I\right)^{N\,\nu}{}_{\mu} x^{\alpha_N} = \left(e^{\omega I}\right)^{\nu}{}_{\mu} x^{\mu} \\
&= \left(\cosh\omega I + \sinh\omega I\right)^{\nu}{}_{\mu} x^{\mu}
\end{aligned}$$

$$(x^{\nu})' = \left(\mathbb{I} - I^2 + I^2\cosh\omega + I\cosh\omega\right)^{\nu}{}_{\mu} x^{\mu}$$

$$\begin{pmatrix} x^{0\prime} \\ x^{1\prime} \\ x^{2\prime} \\ x^{3\prime} \end{pmatrix} = \begin{pmatrix} \cosh\omega & -\sinh\omega & 0 & 0 \\ -\sinh\omega & \cosh\omega & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$\begin{aligned}
\psi'(x') = S\psi(x) &= \lim_{N\to\infty}\left(1 - \frac{i}{4}\frac{\omega}{N}\sigma_{\mu\nu}I_n{}^{\mu\nu}\right)^{N}\psi(x) \\
&= \exp\left(-\frac{i}{4}\omega\sigma_{\mu\nu}I_n{}^{\mu\nu}\right)\psi(x)
\end{aligned}$$

$$\psi'(x') = e^{-\frac{i}{4}\omega\left(\sigma_{01}I^{01} + \sigma_{10}I^{10}\right)}\psi(x) = e^{-\frac{i}{2}\omega\sigma_{01}}\psi(x)$$

$$I^{\nu}{}_{\mu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \psi'(x') = e^{-\frac{i}{2}\phi\sigma_{12}}\psi(x)$$

$$j^\mu(x) = c\psi^\dagger\gamma^0\gamma^\mu\psi,\;\; j^\mu{}'(x') = c\psi'^\dagger(x')\gamma^0\gamma^\mu\psi'(x')$$

$$\begin{aligned} j^\mu{}'(x') &= c\psi^\dagger(x)S^\dagger\gamma^0\gamma^\mu S\psi(x) \\ &= c\psi^\dagger(x)\gamma^0 S^{-1}\gamma^\mu S\psi(x) \\ &= ca^\mu{}_\nu\psi^\dagger(x)\gamma^0\gamma^\mu\psi(x) \\ &= ca^\mu{}_\nu j^\mu(x) \end{aligned}$$

$$a^\nu{}_\mu\gamma^\mu = S^{-1}(a)\gamma^\nu S(a) \Longrightarrow a^\nu{}_\mu\gamma^\mu = P^{-1}\gamma^\nu P$$

$$a^\nu{}_\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = g^{\nu\mu}$$

$$\begin{aligned} &a^\sigma{}_\nu a^\nu{}_\mu\gamma^\mu = P^{-1}a^\sigma{}_\nu\gamma^\nu P,\;\; a^\sigma{}_\nu\gamma^\nu = g^{\sigma\sigma}\gamma^\sigma \\ &\Longrightarrow Pg^\sigma{}_\mu\gamma^\mu P^{-1} = g^{\sigma\sigma}\gamma^\sigma \end{aligned}$$

$$P = e^{i\phi}\gamma^0 \Longrightarrow \psi'(x') = \psi(-x) = P = e^{i\phi}\gamma^0\psi(x)$$

$$\begin{aligned} &\Gamma^s = \mathbb{I},\;\; \Gamma^V_\mu = \gamma_\mu,\;\; \Gamma^T_{\mu\nu} = \sigma_{\mu\nu} \\ &\Gamma^P = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma^5 = \gamma_5,\;\; \Gamma^A_\mu\gamma_5\gamma_\mu \end{aligned}$$

$$\begin{aligned} U(I) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, U(C_3) = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}, U(C_3^2) = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}, \\ U(C_2) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, U(C_2') = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}, U(C_2'') = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix} \end{aligned}$$