De-noising of 3D and fractal images using polynomial thresholding

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Abstract:
The primary goal of this project is to compare the efficacy of different denoising algorithms on 3D biomedical images and fractals using 3D wavelet transform. The principle mechanism of denoising is to the mitigation of Gaussian noise of different variances using polynomial thresholding [2].

In project, the focus is to study the effects by applying the same algorithms on noisy fractal images. For observatory control experiment purposes, denoising is done on both 3D biomedical images and 3D solids. Box counting dimension was used as a performance metric to determine how close the denoised image was to the original image.

Conclusions are drawn from the trials and observation.

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1. Introduction

In the case of polynomial thresholding, instead of directly using the transform coefficients themselves, different polynomial coefficients are generated depending on the threshold. The polynomial coefficients estimated in the minimization of Mean square error provides a solution which is deemed to be optimal.This mechanism[3] is applied to 3D biomedical images, 3D fractals and 3D solids.
We also demonstrate the use of the 3D transform to denoise the 2D images ,depending on what was considered as a third dimension, we considered the cases of bit plane and a set of gray levels.

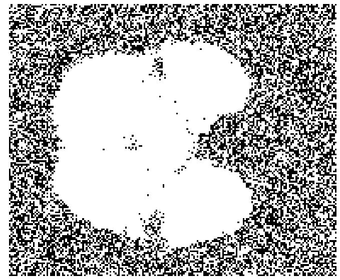
2. 3D Wavelet Transform

The forward operations of the 3D wavelet transform is performed via 2 steps
[1] 2D wavelet transform slice-wise
[2] 1D wavelet transform "into the slice"

The inverse of the 3D wavelet transform does the reverse operations of the aforementioned.

For the 3D image denoising, for instances on a selected number of slices, say N slices, the 2D wavelet transform on a slice-wise basis would give 4 sets of sub band coefficients for each image, amounting to 4N sets of subband coefficents,. This connotes that the optimal set of polynomial coefficients under the thresholding will occur 4N passes. In a simplistic view, with a first level of decomposition, there will be only 8 sets of sub-band coefficients, regardless of the size of the image, the determination of the polynomial coefficients with thresholding occurs 8 times.

The following illustrates the results of the 3D denoising on 3D noisy brain scan images. The 2D slice-wise images shows that the original CATSCAN is well mixed with Gaussian noise.

 Fig 2a: Orignal CATSCAN image	 Fig 2b: CATSCAN image corruputed by Gaussian noise

The same observation is conducted on 3D on both MATLAB and ImageJ for control purposes. The following series is seen on MATLAB.

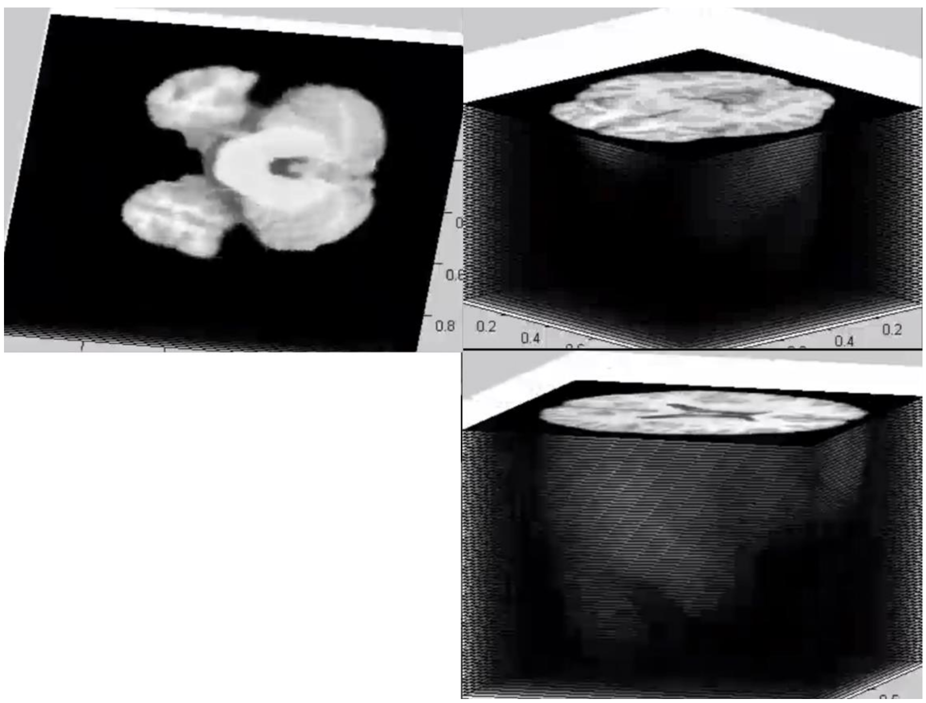


Fig 2c: Original CATSCAN 3D image (*observed under MATLAB*)

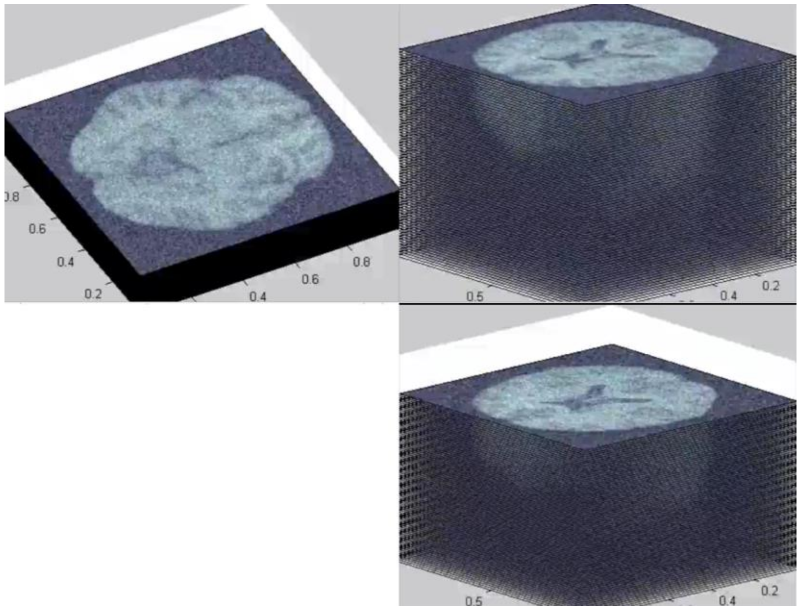


Fig 2d: Noisy CATSCAN 3D image (*observed under MATLAB*)

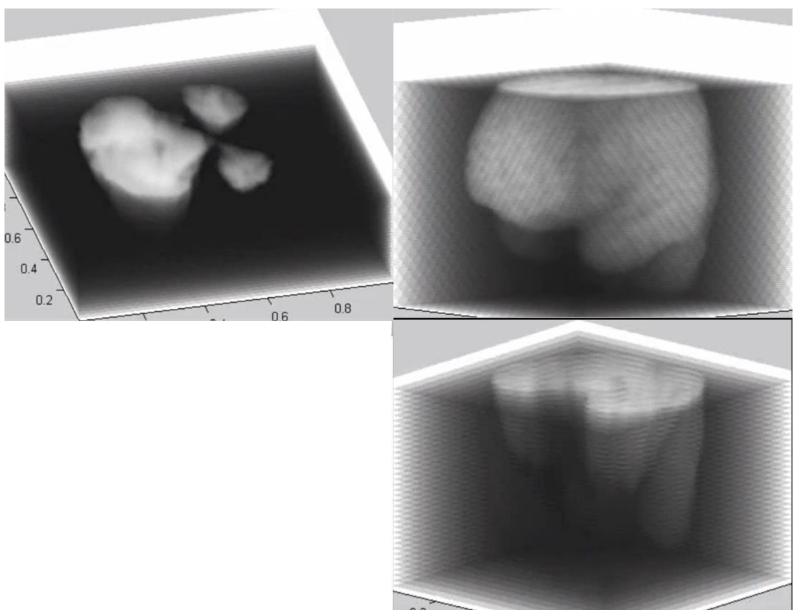


Fig 2e: Denoised CATSCAN 3D image (*observed under MATLAB*)

The next following series is seen on ImageJ.

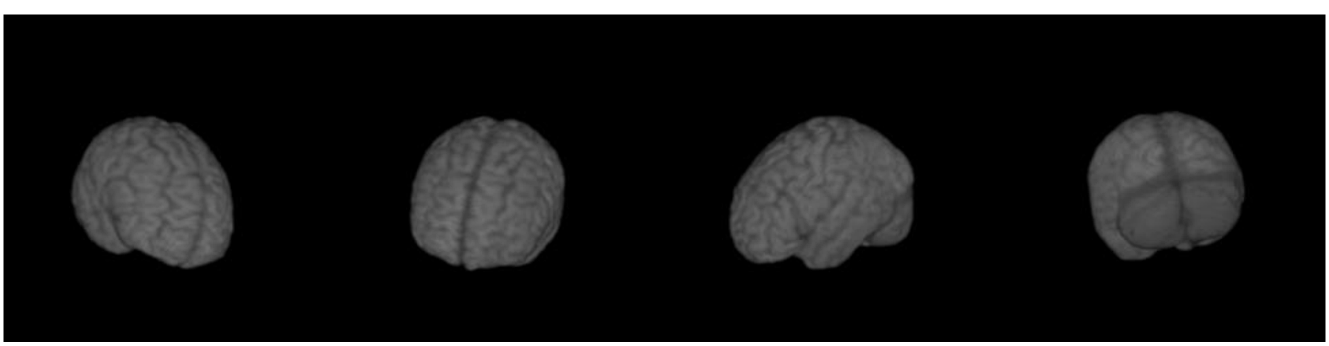


Fig 2f: Original (Round view) - (*observed under ImageJ*)

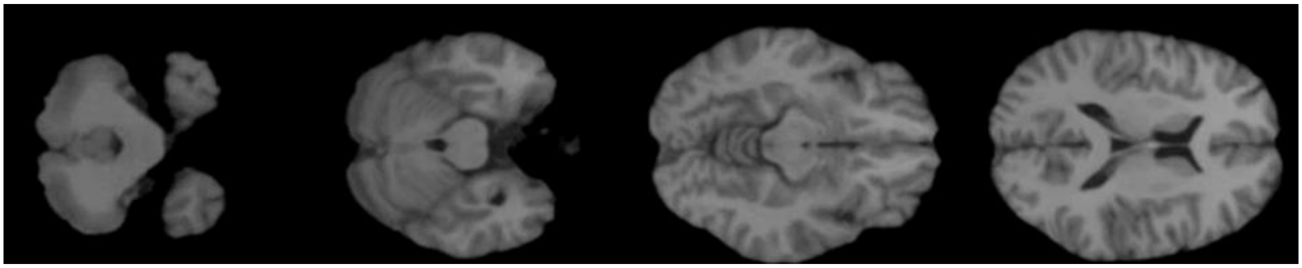


Fig 2g: Original (Top view) - (*observed under ImageJ*)

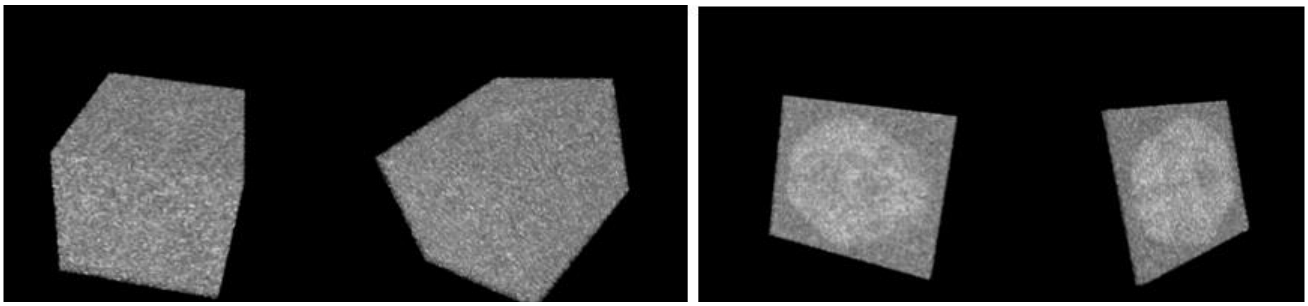


Fig 2h: Noisy (Round view of Full and selected slices) - (*observed under ImageJ*)

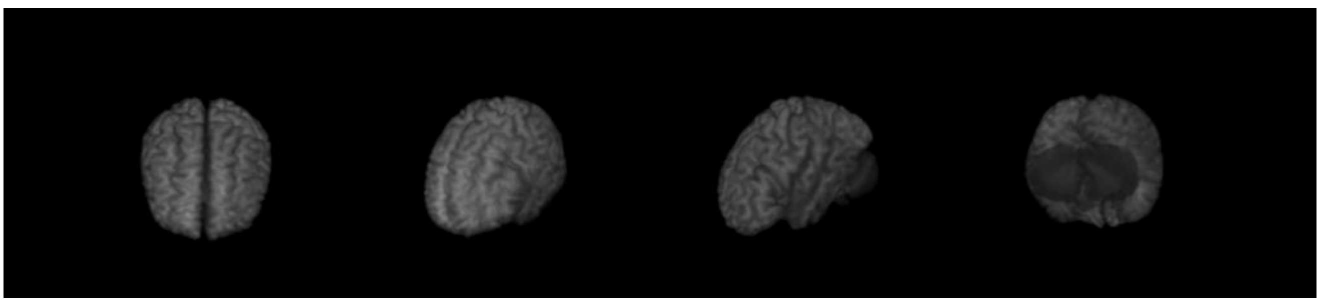


Fig 2i: Denoised (Round view) - (*observed under ImageJ*)

Fig 2j: Denoised (Top view) - (*observed under ImageJ*)

From the above illustrations, it is clearly demonstrated that the denoising has very successfully on the 3D bioimedical image.

The 2^{nd} part of the experiment concerns with the case of being given only 1 slice of 2D image. The principle of applying 3D wavlet transform and denoising is by translating the image representations in terms of the gray or bit level as the third dimension. For instance, for a grayscale image of size 128x128, the image may be mapped onto a 3D image of size 128x128x8 with each 'slice' containing intensity information along the slice-wise axis. 8 slices for this case, as each pixel is denoted with 8 bits gray level quantization.

The following steps may be summarized as

a) Translating the 2D image in 3D representation
b) Conducting 3D wavelet transform
c) Computation of optimal coefficients
d) Thresholding the image using the polynomial thresholding algorithm on the optimal coefficients
e) Image recovery

The following depicts the results.

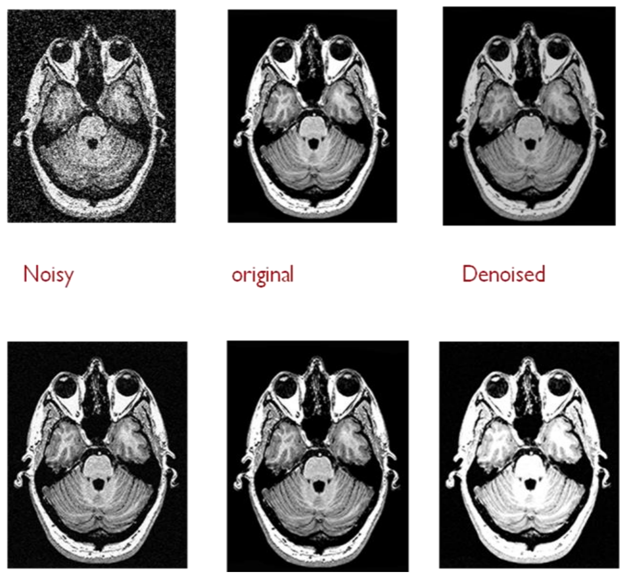


Fig 2k: Denoising in slice-wise gray level 3D images

3. Fractals

Fractal is a natural way of thinking in geometrical sense in describing the highly convoluted surface, and shapes. This can be application especially relevant to quantify by geometrical means in the images of structures of nature, such as river branchings, snowflakes , blood vessels, and intricate textures of brains or the moon. It can also be used to delineate mechanics such as turbulent flow or Brownian motion.

Fractals are uniquely identified by its phenomenon of self similarity. Akin to a random noise generator, the fractal is seeded parametrically with a starting point, a base form, and a motif. These parametric seed is iteratively divided prescribed to a specified algorithm which implicates an affine transformation resulting in rotation, translation, shearing, scaling, reflection, and possible forms of orthogonal projection.

Colored fractals such as fractal trees are drawn by programs for observation of such phenomenon.

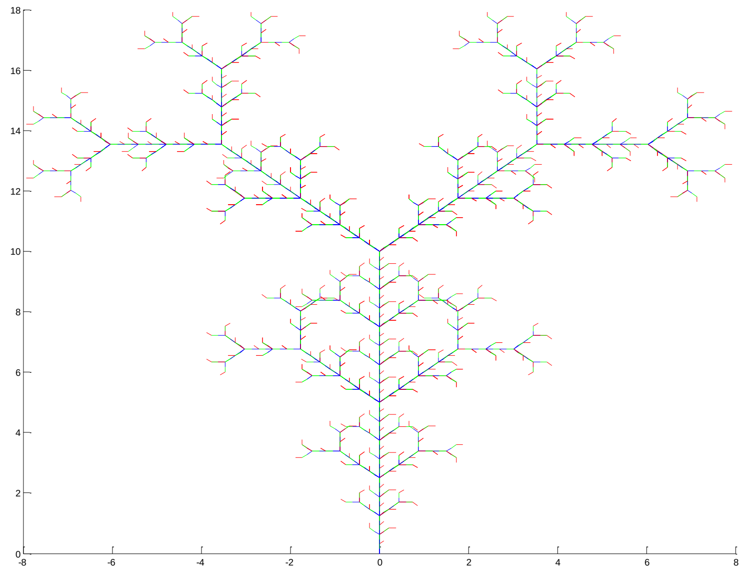


Fig 3a: Deterministically Colored Fractal Tree

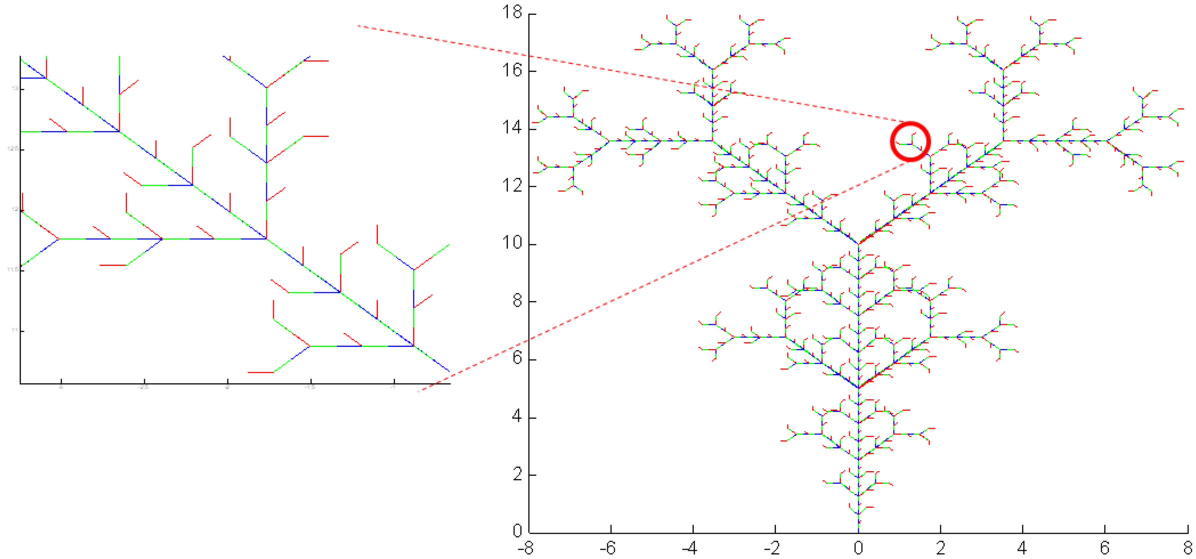


Fig 3b: Randomly Colored Fractal Tree at Level 5

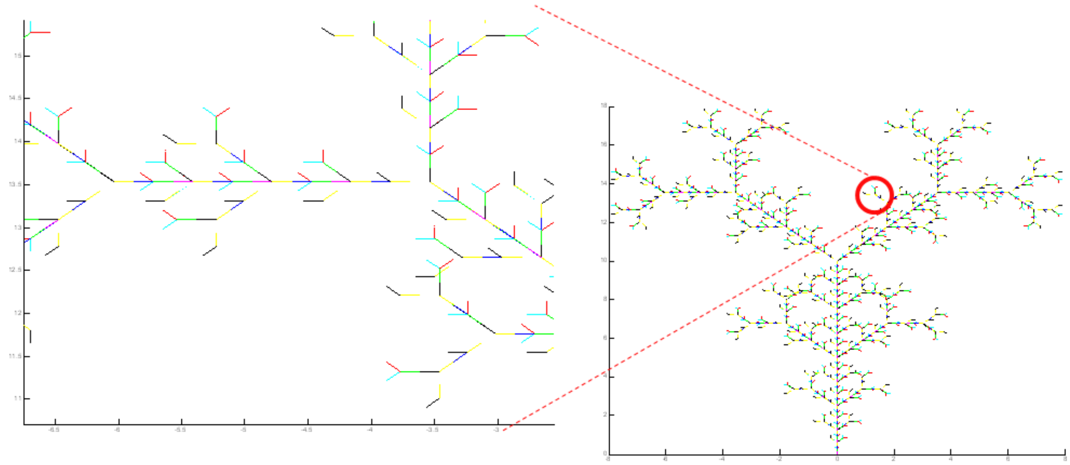


Fig 3c: Randomly Colored Fractal Tree at Level 8

It can be seen that as fractal gets intense at higher level, the inner details is not obvious to the human eye. Zooming in provides an insight to the details of self-similarity with the respective level.

4. Box counting

The concept of fractals provides a way to see and measure things. One of such ideas is the calculation of fractal dimension. The measurement of dimensionality measure is log(N)/log(r) where N is the number of pieces the seed is split into, and r the ratio of the size of the new pieces to the size of the original. In this project, the box counting dimension methods is chosen to measure the distribution of pieces as a gauge to how close the denoised image is to the original image.

The following shows the result of box counting on the Brain 3D images.

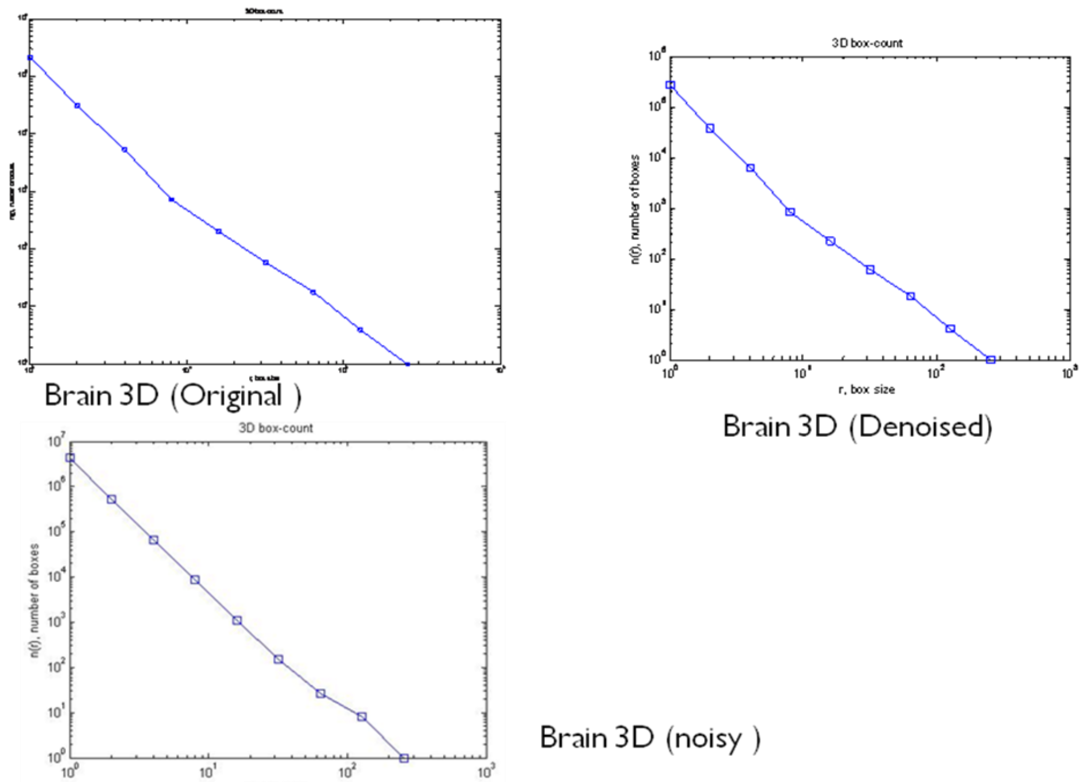


Fig 4a: Box Counting on the Brain 3D Images (Original, Noisy, and Denoised)

From the computation, estimation is made on the box distribution of the respective images. The same thing is done for fractal images.

Image 3D	**Box Distribution**
Brain 3D (Original)	215067 30632 5321 714 201 58 18 4 1 *log(n)./log(2) =* *(18.3663 15.4919 12.9069 9.9069 7.9069 5.9069 4.1699 2.0000 0)*
Brain 3D (Noisy)	4300571 537600 67200 8640 1080 150 27 8 1 *log(n)./log(2) =* *(17.7144 14.9028 12.3775 9.4798 7.6511 5.8580 4.1699 2.0000 0)*
Brain 3D (Denoised)	269024 36805 6301 828 220 60 18 4 1 *log(n)./log(2) =* *(18.0374 15.1676 12.6214 9.6935 7.7814 5.9069 4.1699 2.0000 0)*
DFT-Real Fractal (Original)	129160 16384 2048 256 64 16 4 1
DFT-Real Fractal (Noisy)	589361 102394 13312 1792 256 32 4 1
DFT-Real Fractal (Denoised)	815169 102400 13312 1792 256 32 4 1

Apart from the Brain 3D, denoising is done on the fractals 3D. The following displays the mentioned.

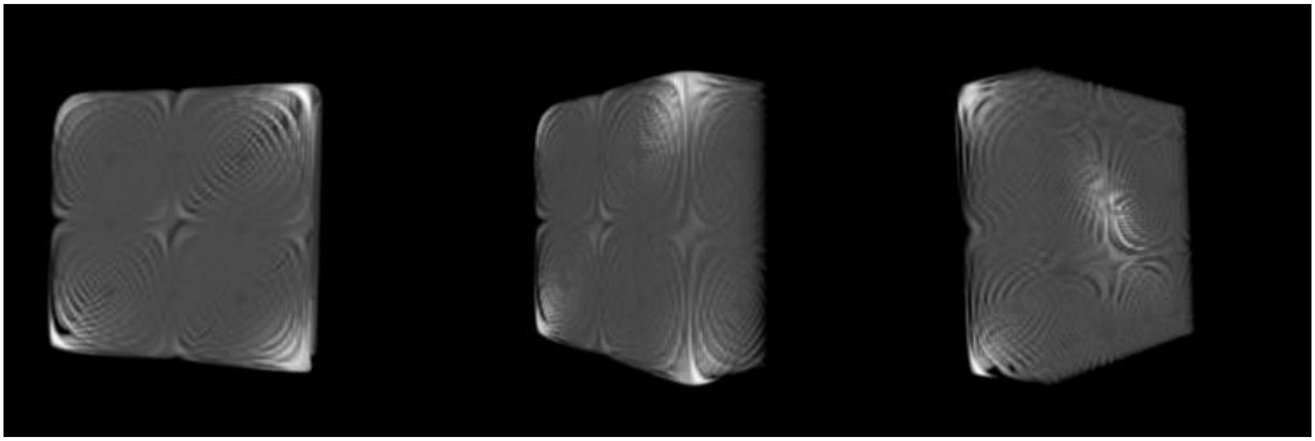


Fig 4b: Fractals Created By DFT

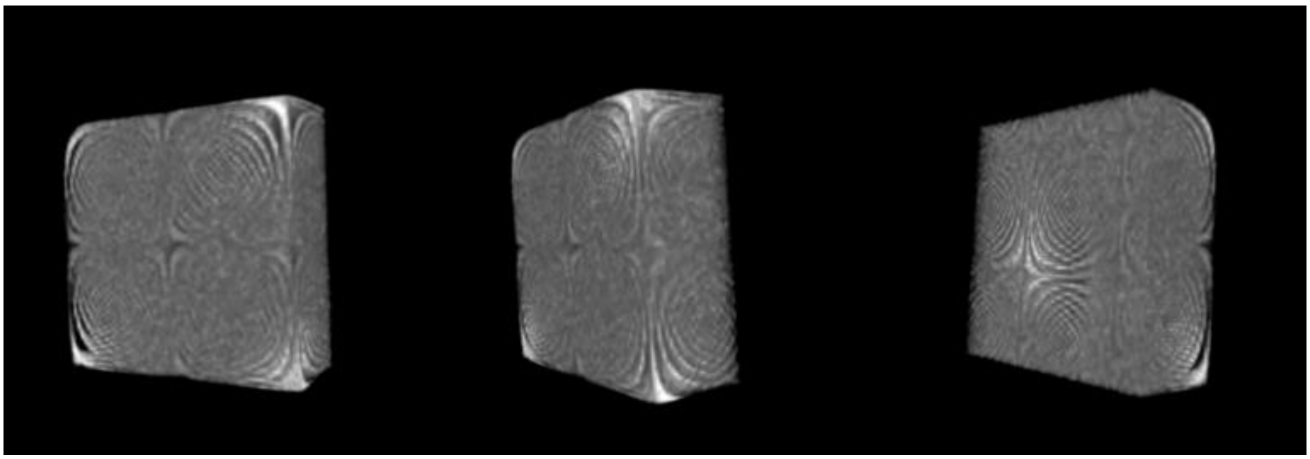


Fig 4c: Fractals Under DFT With Noise

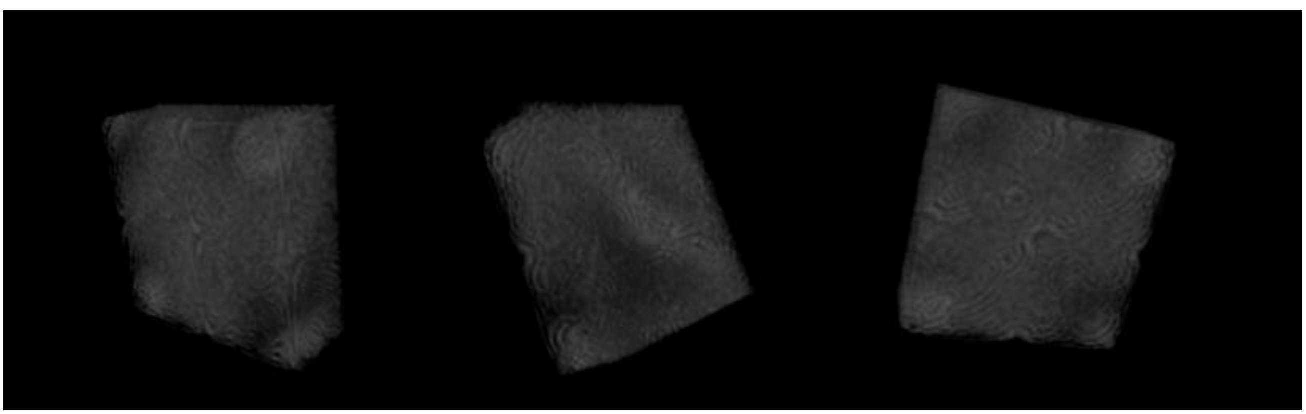


Fig 4d: Fractals Under DFT With Denoised

Clearly, as can be seen, the level of detail in fractals is too great to allow for thresholding the same as a 3d image, such as the biomedical images as we have shown.2D images such as the CATSCAN image we have used are also capable of being denoised with a 3D transform such as the 3D wavelet transform.

The efficacy of such denoising methods is dependent on what is chosen to regard as the 3rd dimension. Sets of gray levels, or bit planes are considered, and it is found that, generally, the gray level choice works better even at higher variances of noise.

In all, box counting may be used as a metric for denoising, and as clearly seen, it provides a reliable indicator as to how close the denoised image is to the original image.

5. Conclusion

The denoising using polynomial thresholding works significantly in bioimedical images such as the Brain 3D in the mitigation of Gaussian noise with different variances. In general, at high variances, gray level slicing is a better bet for denoising than bit level slicing. Gray level slicing tends to produce some unwanted pixels of noise after thersholding ,which are best removed by filtering by low pass filters. Last but not least, Soft thresholding yields a better result than hard thresholding.

In comparison of the efficacy of different denoising algorithms on noisy 3D biomedical images and fractals using 3D wavelet transform, it is seen that fractal images have very high level of detail which makes thresholding more difficult. This is due to the detail being almost inevitably lost during the thresholding process.

The performance metric of box counting dimension gives an idea to determine how close the denoised image was to the original image.

References:

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