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Lecture 2: Intr. Control Systems

ELG 3155 : Introduction to Control Systems

❑Objective:

- Defining and describing control systems.
- Mastering the terminologies of control systems.
- Understanding the difference between open and closed-loop control systems.

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Contents

Terminologies and Definitions

System

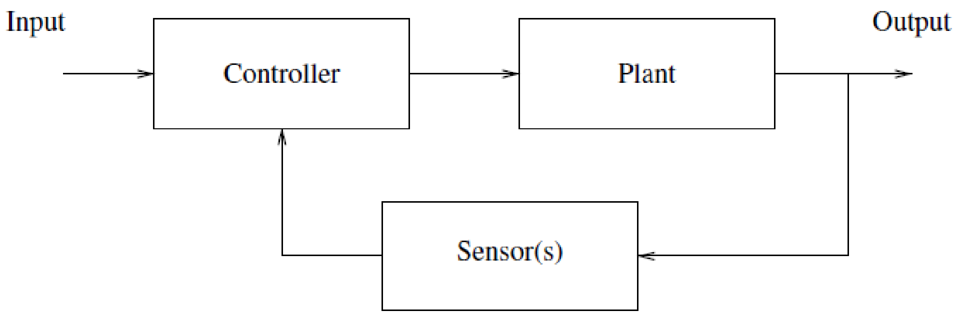
- A system can be defined as a set of elements collectively performing a given function. Each system (or each element it is composed of) establishes a dynamic relation between its input and output signals.

Definition (Automatic)

- A system is said to be automatic when it accomplishes a given task without human intervention. This automation has two main objectives :
  1. Substitute humans with machines to perform repetitive tasks. Examples : washing machine, assembly lines, mail processing.
  2. Perform tasks that are too complex or tedious for humans. Examples : guiding of missiles/satellites, automatic cruising, radar tracking.

Definition (Control System)

- In general, a control system is an automatic system that can be represented by the following block diagram:



- It consists of *subsystems* and *processes* (or *plants)* assembled for the purpose of obtaining a desired *output* with desired *performance*, given a specified *input*.
- Two major measures of performance are apparent:
  1. The transient response.
  2. The steady-state error.

Example (driving a car (manual cruise system))

- The driver sets a desired speed ; e.g., 100 km/h.
- The continuous comparison of this reference speed (input) with that (actual speed) displayed on the dashboard (output) generates a difference called error.
- Based on this error, the driver takes the decision of :
  - increasing the vehicle's speed, by stepping on the gas pedal, or
  - decreasing the speed, by taking his foot off the gas pedal.

- Because of the human intervention, the system in this example is not automatic, but it illustrates the operational principle of automatic systems.

In the example, the elements of the control system are :

Plant $\equiv$ Vehicle

Input signal $\equiv$ Reference speed

Output signal $\equiv$ Actual speed

Controller $\equiv$ Driver

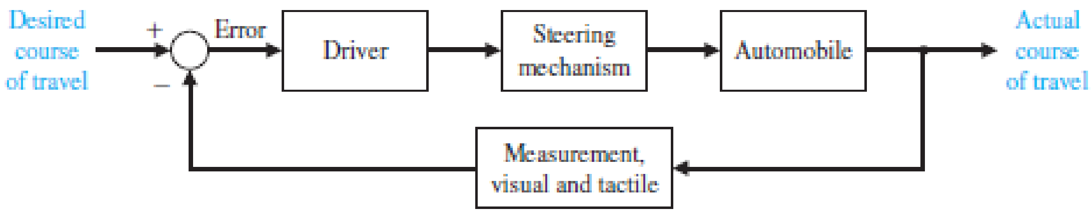
The aim of the controller is to :

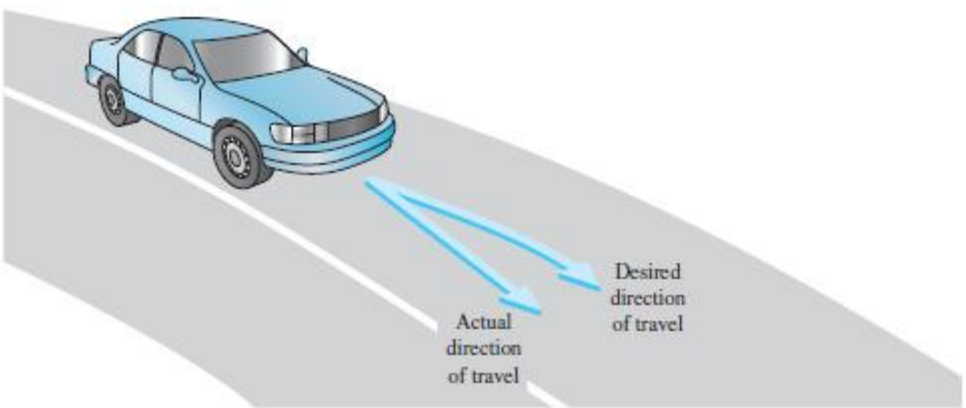
- first, determine the error between the reference input and actual output signals.
- then, act accordingly to minimize this error.

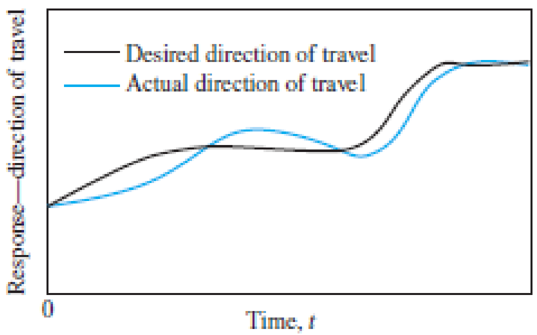
Exercise

Repeat the same analogy for a system controlling the direction of the vehicle (instead of its speed).

Solution:


Desired
course
of travel
+
−
Error
Driver
Steering
mechanism
Automobile
Actual
course
of travel
Measurement,
visual and tactile


Desired
direction
of travel
Actual
direction
of travel


Desired direction of travel
Actual direction of travel
Response—direction of travel
0
Time, t

More generally, a control system can be represented by the following block diagram :

- Disturbance is an external (extra input) signal, which is usually uncontrollable, acting directly on the plant.
- In the cruise system example, disturbances can come from wind, road condition, friction between the tires and the road, etc.

DEFINITION (LINEAR SYSTEM)

A system is said to be linear if and only if the mathematical relation between its "m" inputs $r_i, i = 1, 2, \ldots m$, and "n" outputs $y_i, i = 1, 2, \ldots n$, is linear, with out bias (i.e., zero output for zero input).

$$r_i(t) \longrightarrow \boxed{\text{System}} \longrightarrow y_i(t)$$

Superposition principle: Linear systems obey the superposition principle defined by the additivity and homogeneity properties.

Let y_1 and y_2 be the responses of a system for inputs u_1 and u_2, respectively.

- **Additivity**: The system is said to obey the additivity property if and only if for an input of $r = r_1 + r_2$ the system responds with an output $y = y_1 + y_2$.
- **Homogeneity:** The system is said to obey the homogeneity property if and only if for an input of αr_1 the system responds with an output αy_1.

Time invariance:

- **Definition: A system is time invariant for any arbitrary input signal $r(t)$ with corresponding output signal $y(t)$ and arbitrary time delay τ, the delayed input signal $r(t-\tau)$ produces output signal as $y(t-\tau)$**

- **The delay τ in the input signal must be carried out to the output.**

- **The system is called time varying if its not time invariant.**

- Linear systems whose parameters are invariant in time are called linear time-invariant (LTI) systems.
- The main scope of the course is to study LTI single-input single-output (SISO) systems.

Linear System (input-output)

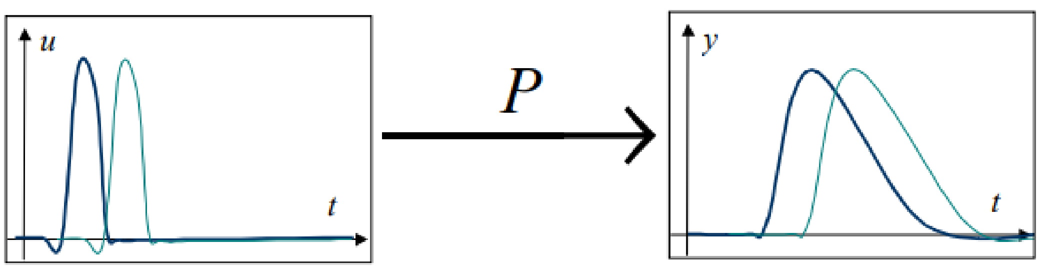
- Linearity

$$u_1(\cdot) \xrightarrow{P} y_1(\cdot) \qquad u_2(\cdot) \xrightarrow{P} y_2(\cdot)$$

$$au_1(\cdot) + bu_2(\cdot) \xrightarrow{P} ay_1(\cdot) + by_2(\cdot)$$

- Linear Time-Invariant systems - LTI

$$u(\cdot - T) \xrightarrow{P} y(\cdot - T)$$



Example

Is the following equation linear:

$$\frac{dy(t)}{dt} + y(t) = x(t)$$

To determine whether this system is linear, construct a new composite input:

$$x(t) = Ax_1(t) + Bx_2(t)$$

Now, create the expected composite output:

$$y(t) = Ay_1(t) + By_2(t)$$

Substituting the two into our original equation:

$$\frac{d[Ay_1(t) + By_2(t)]}{dt} + [Ay_1(t) + By_2(t)] = Ax_1(t) + Bx_2(t)$$

Factor out the derivative operator, as such:

$$\frac{d}{dt}[Ay_1(t) + By_2(t)] + [Ay_1(t) + By_2(t)] = Ax_1(t) + Bx_2(t)$$

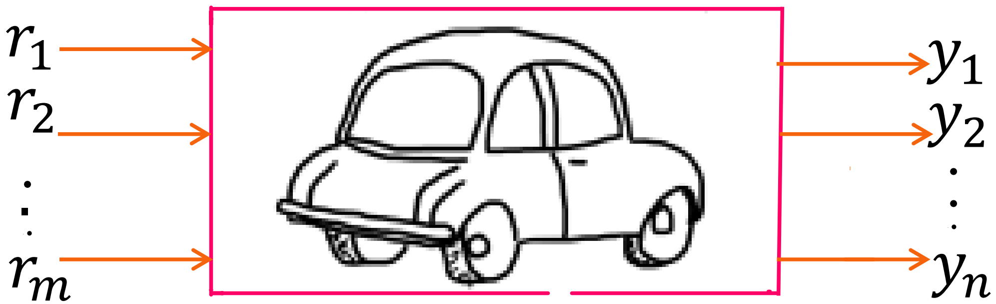
Finally, convert the various composite terms into the respective variables, to prove that this system is linear:

$$\frac{dy(t)}{dt} + y(t) = x(t)$$

For the record, derivatives and integrals are linear operators, and ordinary differential equations typically are linear equations.

Dynamic system:

Def.1 (Eng.): An interconnection of elements and devices for a desired purpose.



Def.2 (input/output paradigms): An object in which variables of different kinds interact via effects of external stimuli signals and produce an observable signals:

- **The external stimuli signals is the system input.**
- **The produced observable signals is the system output.**
- **The interaction is the process/system dynamics.**
- **The term plant is also used, especially for systems to be controlled.**

An alternative def. of dynamic system:

Is a system that is represented by a differential equation.

System Configurations

- we discuss two major configurations of control systems: open loop and closed loop.
- Control systems can be classified in two categories :
  1. open-loop systems
  2. closed-loop systems

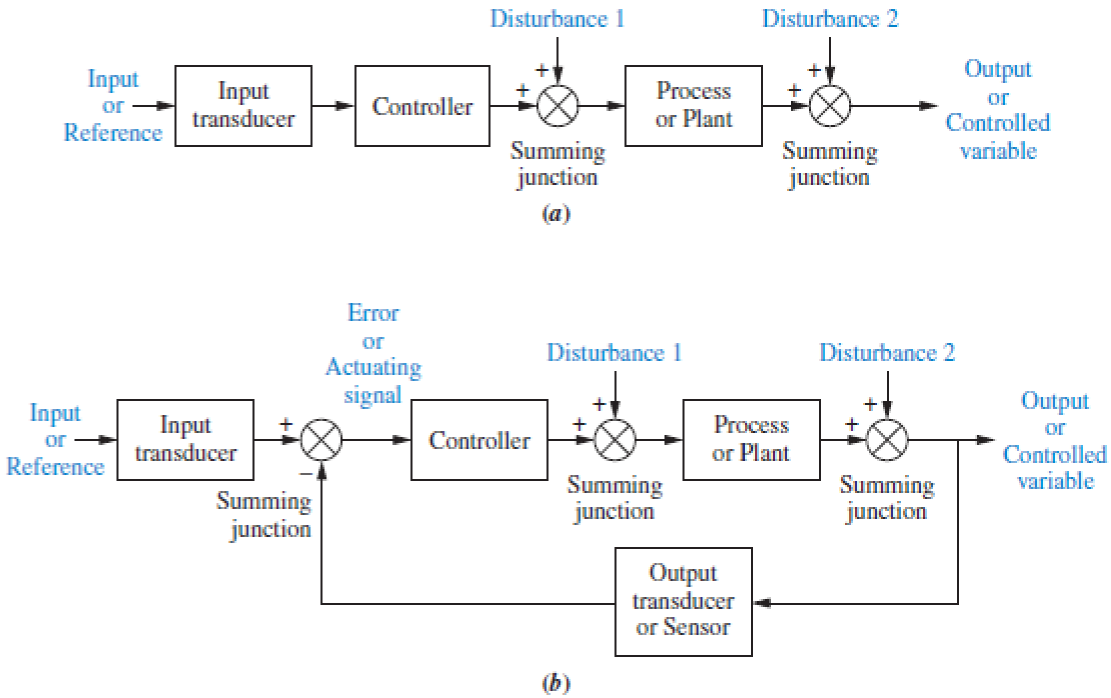


FIGURE 1.5 Block diagrams of control systems: a. open-loop system; b. closed-loop system

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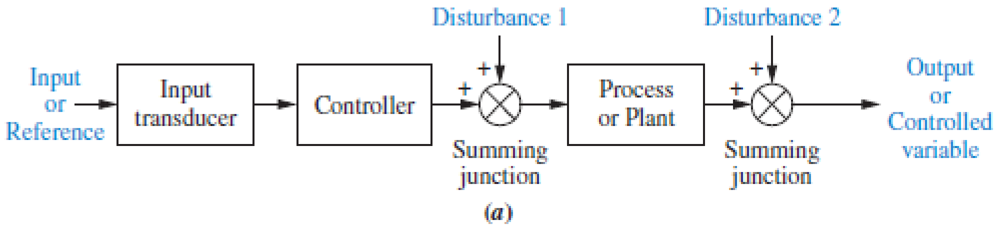
Open-loop system

- Definition : An open-loop control system is a system without the feedback loop containing the sensor(s)



- Advantage : Simple, inexpensive, fast (less blocks).
- Disadvantages :
  - Imprecise (blind) due to the absence of the error correction (feedback) loop.
  - Sensitive to disturbances ⇒ no correction is possible.

Example: Golf ball : Once shot, its trajectory cannot be rectified. Wind effect (disturbance), for instance, cannot be compensated for.

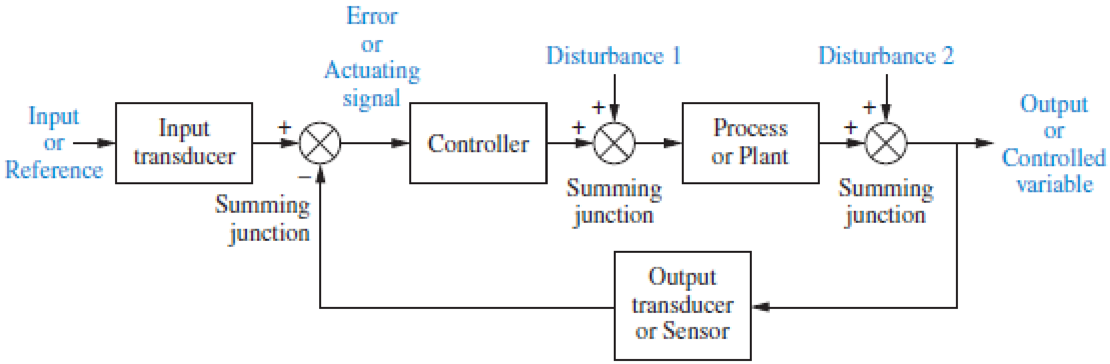
- Solution

Hence, the output signal must be measured and compared with the (desired) reference input signal to compute the error and annihilate it.

$\Rightarrow$ Add a feedback loop.

Closed-loop system

- Definition **:** A closed-loop system is a system with a feedback control loop to compute the system's error signal



(b)

- Advantage : Precise, with an error correction capability (to attenuate the effect of disturbance).
- Disadvantages :
  - More complex and slower than open-loop control systems,
  - more expensive (more blocks).

Example: Air conditioners- Laser-guided missiles automatic cruise systems

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Definition (Analysis and Design of a Control System

- Analysis is the process by which a system's performance is determined.
- Design is the process by which a system's performance is created or changed.

Objectives of control systems

- Rapidity : Control systems must respond rapidly to an excitation input.
- Precision : The output signal must be as close as possible to the reference signal.
- Stability : The output signal must converge to a finite value if the input signal is finite.

s-Plane partition

- Throughout this course, we will assume the following partition of the s-plane (Fig. 3) :

$$\text{Imaginary axis}: \{j\omega \mid \forall \omega \in \mathbb{R}\}$$
$$\text{Left-hand side (LHS)}: \{\sigma + j\omega \mid \forall \sigma < 0 \text{ and } \omega \in \mathbb{R}\}$$
$$\text{Right-hand side (RHS)}: \{\sigma + j\omega \mid \forall \sigma > 0 \text{ and } \omega \in \mathbb{R}\}$$

- Note that the imaginary axis does not belong to the LHS nor to the RHS of the s-plane.

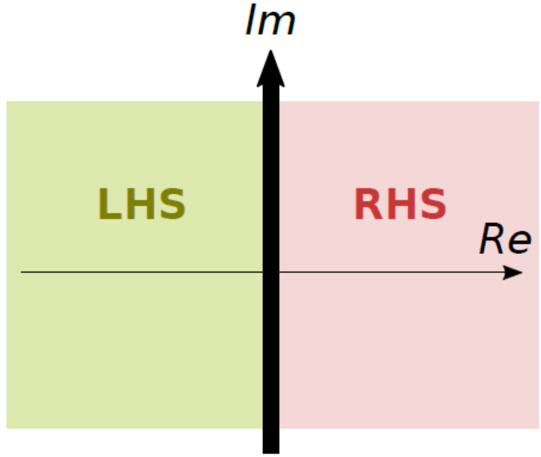


Figure 3: Partition of the s plane.

Design process of control systems

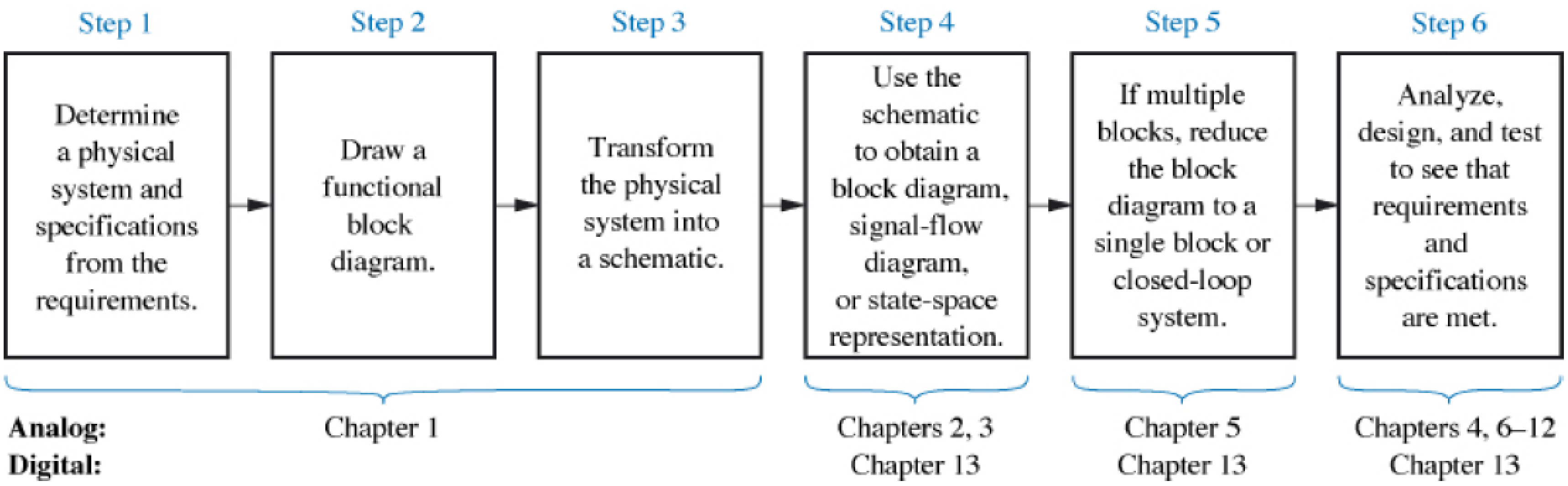
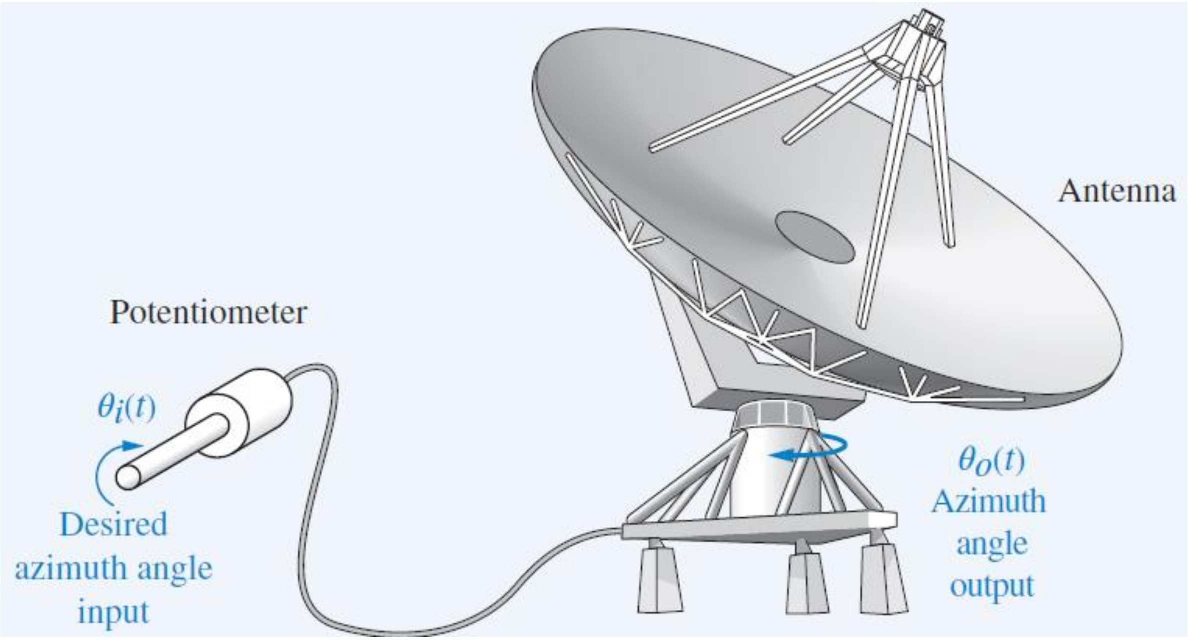


Figure 4: Design process of control systems.

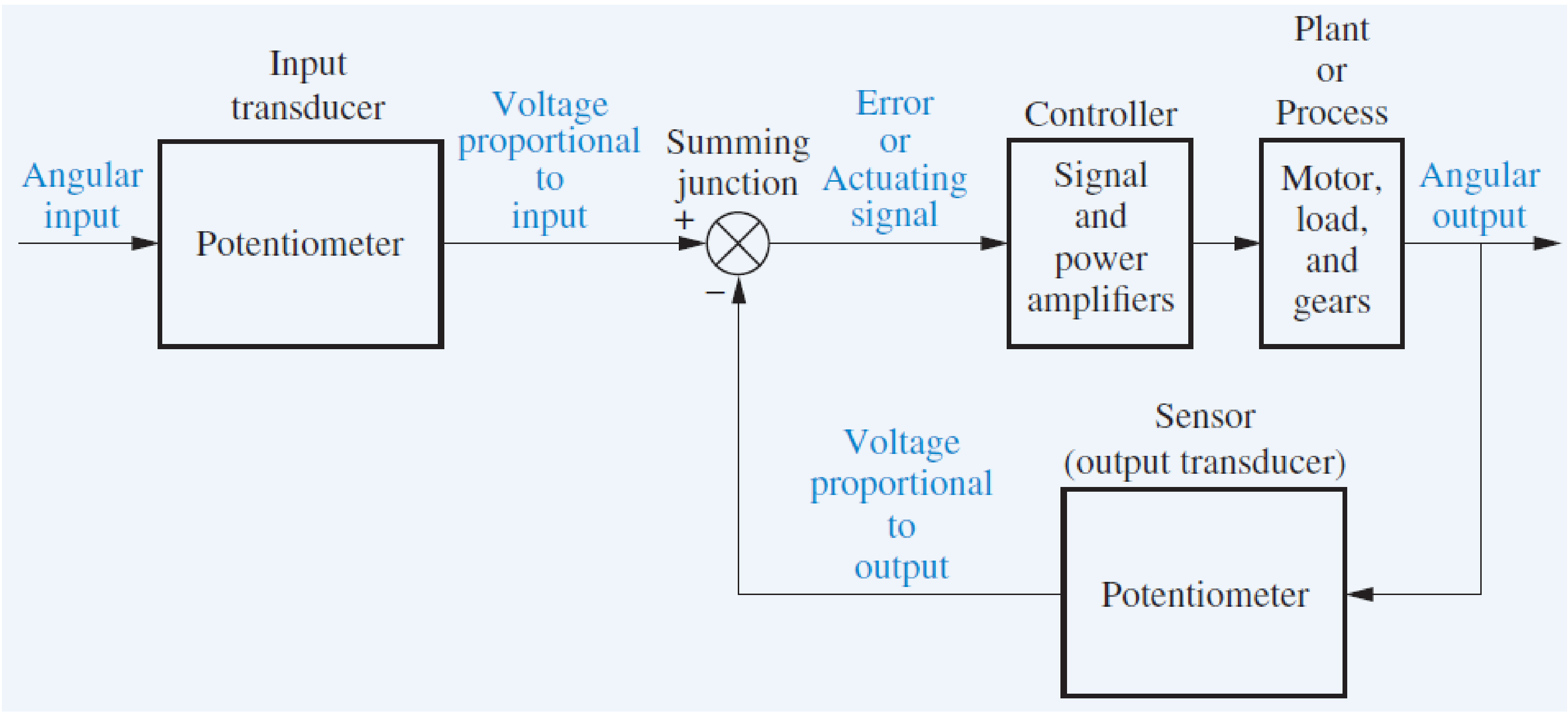
Case Study

Antenna Azimuth: An Introduction to Position Control Systems

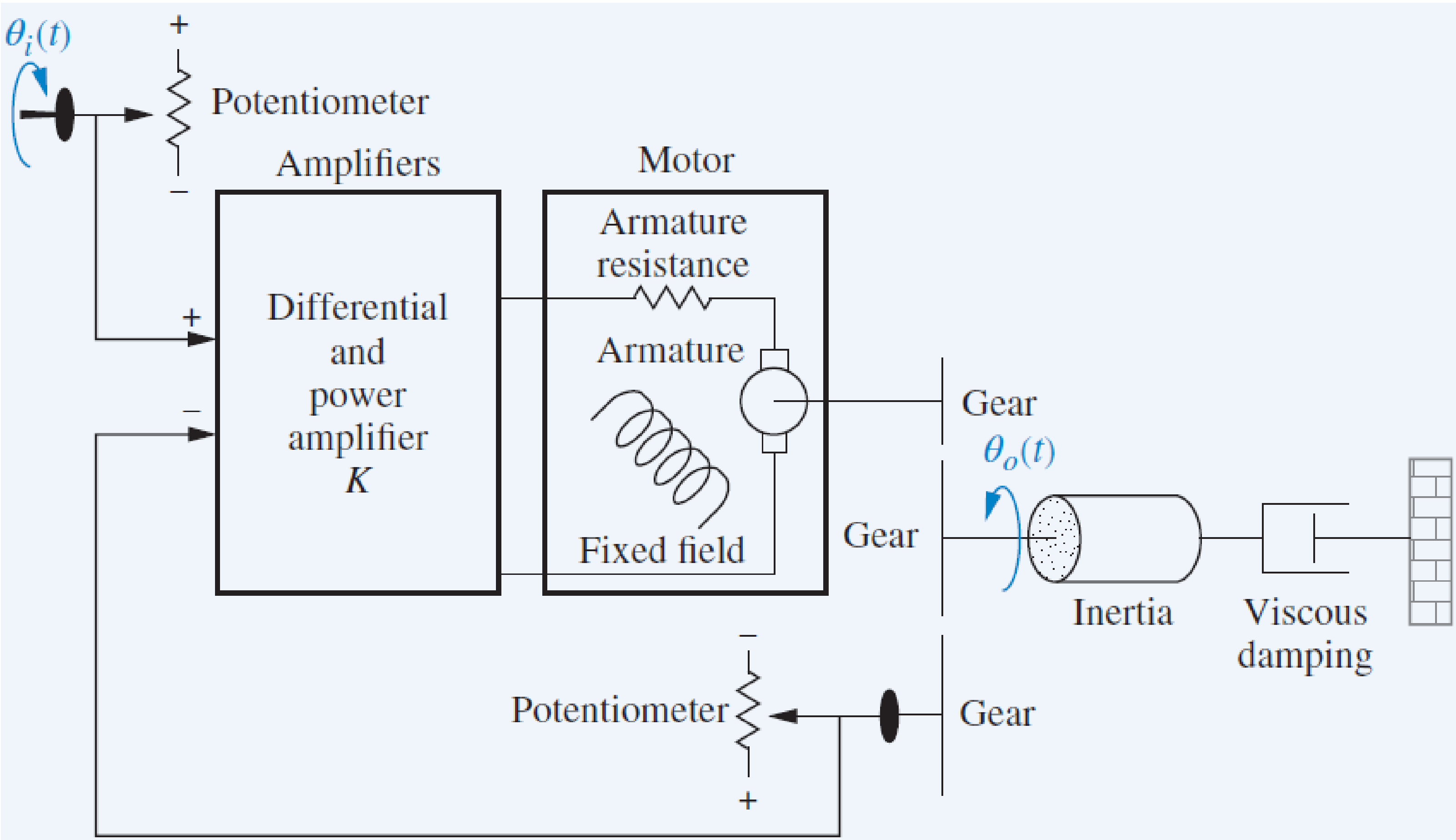
Step1: Transform Requirements into a Physical System



Step 2: Draw a Functional Block Diagram



Step 3: Create a Schematic

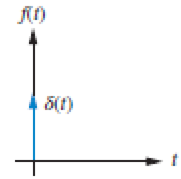
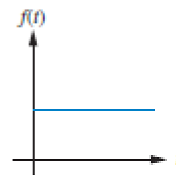
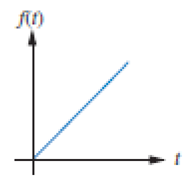
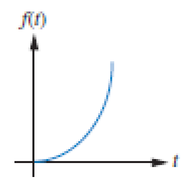
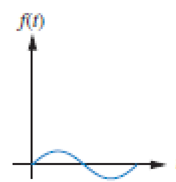


Test waveforms

- Control systems are often analyzed using well defined excitation signals called test waveforms (Table 1).

TABLE 1 – Test Waveform.

TABLE 1.1 Test waveforms used in control systems

Input	Function	Description	Sketch	Use
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t)dt = 1$		Transient response Modeling
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient response Steady-state error
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Sinusoid	$\sin \omega t$			Transient response Modeling Steady-state error

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Review Questions

PARTICIPATION ACTIVITY | 1.7.1: Control systems engineer.

1) Control system engineers work only with electrical and mechanical engineers.

- True
- False

Correct

Control system engineers may collaborate with a wide range of engineers and scientists depending on the project, from biologists, physicists, and chemists, to aerospace engineers, etc.

2) A top-down design approach starts from the components, a circuit is developed, and finally a product is assembled.

- True
- False

Correct

The approach described of starting from components, developing circuits, and assembling a prototype is a bottom-up approach. A top-down design can involve setting system requirements first, and then determining the system's hardware.

3) A controls engineer can be responsible for determining system performance specifications, subsystem functions, hardware and software design, and test plans and procedures.

- True
- False

Correct

A controls engineer can be engaged in several project aspects and can help make many important decisions.

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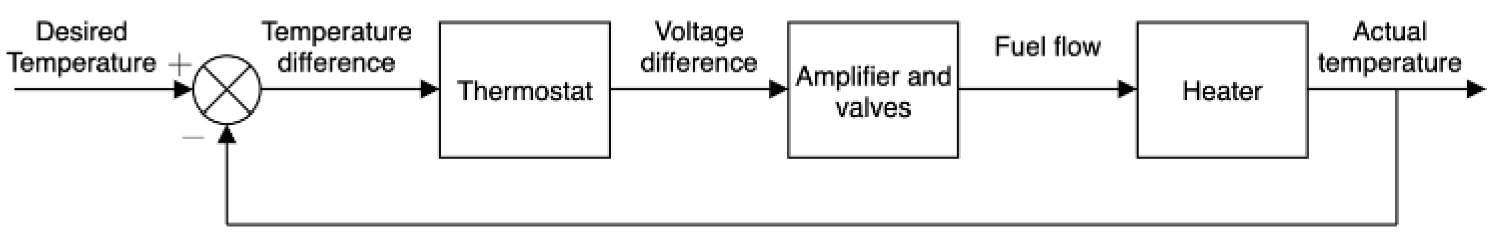
Exercise

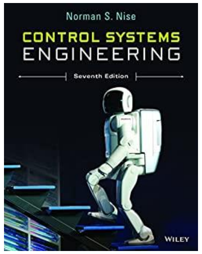
EXERCISE | (Chapter 1, Problem 2).

2. A temperature control system operates by sensing the difference between **the thermostat setting** and the **actual temperature** and then **opening a fuel valve** an amount proportional to this difference. Draw a functional closed loop block diagram similar to Figure 1.8(*d*) identifying the input and output transducers, the controller, and the plant. Further, identify the input and output signals of all subsystems previously described. [Section 1.4: Introduction to a Case Study]

Solution Visible to students



References



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