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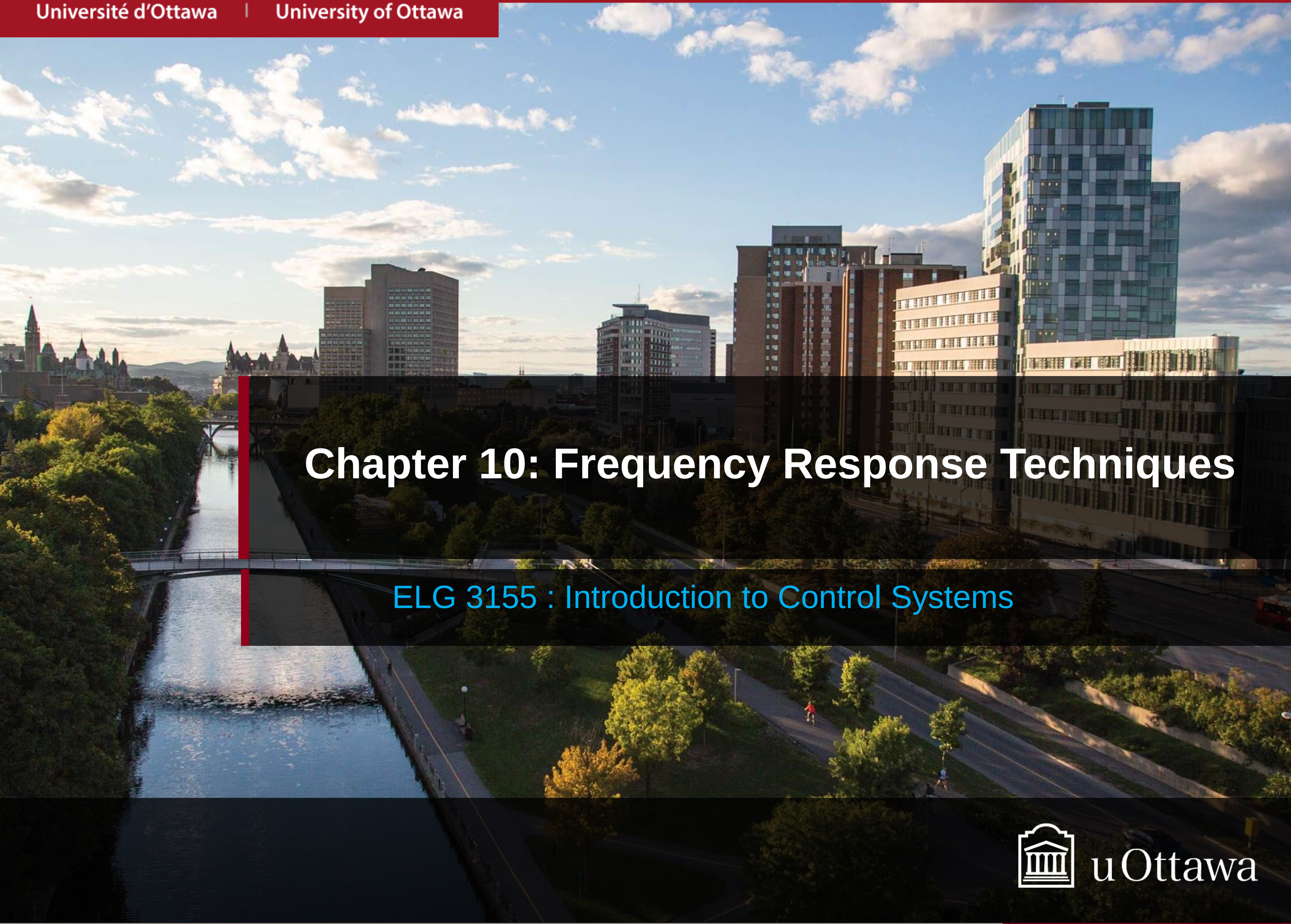

Chapter 10: Frequency Response Techniques

ELG 3155 : Introduction to Control Systems

uOttawa

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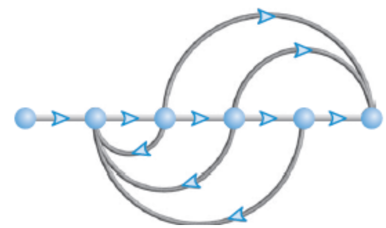
Outline

Frequency Response Techniques 10



- Advantages of FR techniques over RL
- Define FR
- Define Bode & Nyquist plots
- Relation between poles & zeros to Bode plots (slope, etc.)
- Features of 1st- & 2nd-order system Bode plots
- Define Nyquist criterion
- Method of dealing with OL poles & zeros on imaginary axis
- Simple method of dealing with OL stable & unstable systems
- Determining gain & phase margins from Bode & Nyquist plots
- Define static error constants
- Determining static error constants from Bode & Nyquist plots
- Determining TF from experimental FR data

The concept of FR,

- At steady-state, sinusoidal inputs to a linear system generate sinusoidal responses of the same frequency with different amplitudes and phase angle from the input, each of which are a function of frequency.
- *Phasor* – complex representation of a sinusoid
  - $||G(\omega)||$ – amplitude
  - $\angle G(\omega)$ – phase angle
  - $M\cos(\omega t + \phi)$ ... $M\angle\phi$

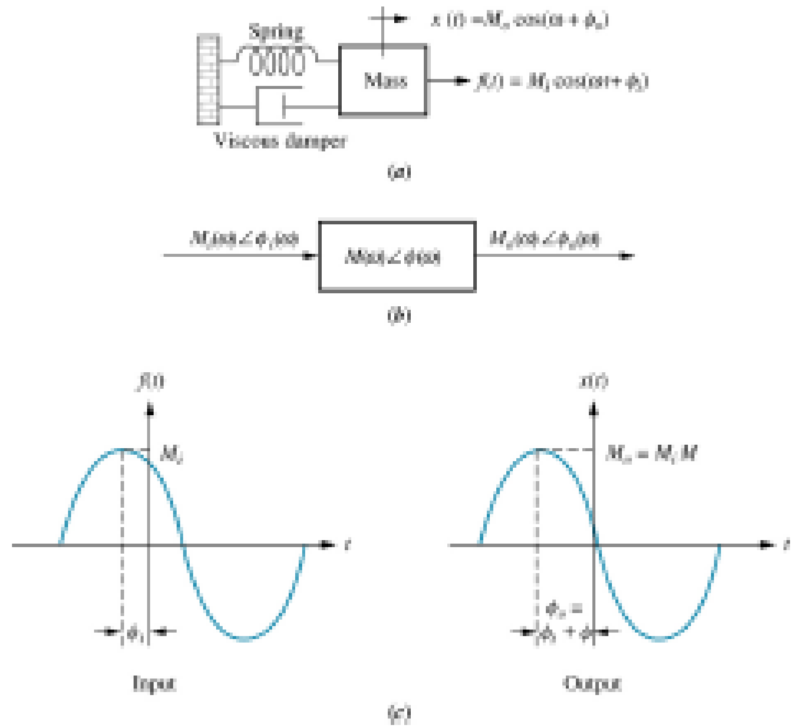


Figure: Sinusoidal FR: a. system; b. TF; c. IO waveforms

Analytical Expressions for Frequency Response

Now that we have defined frequency response, let us obtain the analytical expression for it (*Nilsson, 1990*). Later in the chapter, we will use this analytical expression to determine stability, transient response, and steady-state error. Figure 10.3 shows a system, $G(s)$, with the Laplace transform of a general sinusoid, $r(t) = A\cos\omega t + B\sin\omega t = \sqrt{A^2+B^2}\cos[\omega t - \tan^{-1}(B/A)]$ as the input. We can represent the input as a phasor in three ways: (1) in polar form, $M_i\angle\phi_i$, where $M_i = \sqrt{A^2+B^2}$ and $\phi_i = -\tan^{-1}(B/A)$; (2) in rectangular form, $A - jB$; and (3) using Euler's formula, $M_i e^{j\phi_i}$.

We now solve for the forced response portion of $C(s)$, from which we evaluate the frequency response. From Figure 10.3,

FIGURE 10.3 System with sinusoidal input

- General input sinusoid

$$r(t) = A\cos(\omega t) + B\sin(\omega t)$$
$$= \sqrt{A^2+B^2}\cos\left(\omega t - \tan^{-1}\left(\tfrac{B}{A}\right)\right)$$

- Input phasor forms
  - Polar, $M_i\angle\phi_i$

$$M_i = \sqrt{A^2+B^2}$$
$$\phi_i = -\tan^{-1}\left(\tfrac{B}{A}\right)$$

  - Rectangular, $A - jB$
  - Euler's, $M_i e^{j\phi_i}$

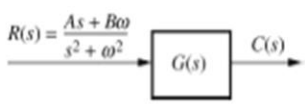


Figure: System with sinusoidal input

- Forced response

$$C(s) = \frac{As + B\omega}{s^2+\omega^2}G(s)$$

- Steady-state forced response after partial fraction expansion

$$C_{ss}(s) = \frac{\frac{M_iM_G}{2}e^{-j(\phi_i-\phi_G)}}{s+j\omega} + \frac{\frac{M_iM_G}{2}e^{j(\phi_i-\phi_G)}}{s-j\omega}$$

where $M_G = ||G(j\omega)||$ and $\phi_G = \angle G(j\omega)$

- Time-domain response

$$c(t) = M_iM_G\cos(\omega t + \phi_i + \phi_G)$$

- Time-domain response in phasor form

$$M_o\angle\phi_o = (M_i\angle\phi_i)(M_G\angle\phi_G)$$

- FR of system

$$G(j\omega) = G(s)|_{s\to j\omega}$$

Bode plots approximations,

- TF

$$G(s) = s + a$$

- Low frequencies

$$G(j\omega) \approx a\angle 0^\circ$$

- High frequencies

$$G(j\omega) \approx \omega\angle 90^\circ$$

- *Asymptotes* – straight-line approximations
  - *Low-frequency*
  - *Break frequency*
  - *High-frequency*

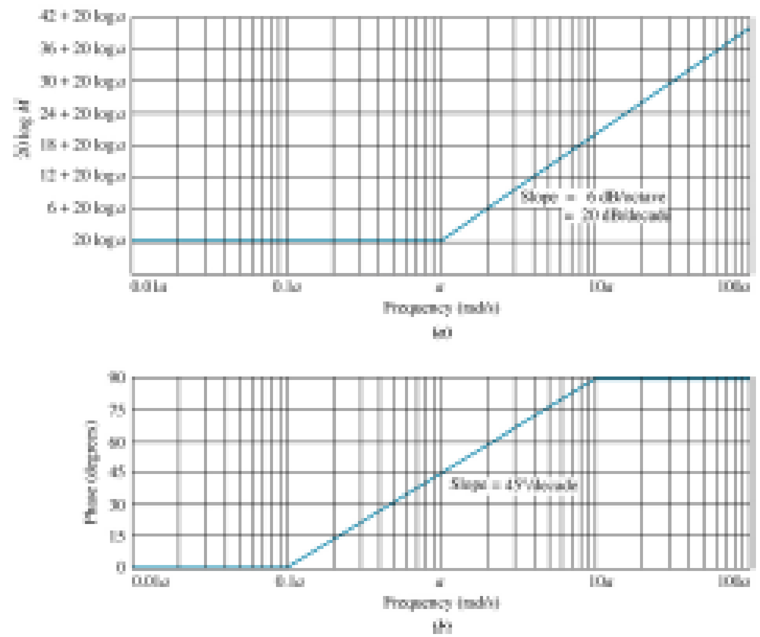


Figure: Bode plots of $s + a$: a. magnitude plot; b. phase plot

References

Nise, N. S. (2015).
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Wiley, 7 edition.