Université d'Ottawa Faculté de génie

École d'ingénierie et de technologie de l'information



University of Ottawa Faculty of Engineering

School of Information Technology and Engineering

L'Université canadienne Canada's university

ELG 3155 Introduction to Control Systems Winter 2023

**Final EXAMINATION (2.5 hours)** 

**Professor:** Hitham Jleed **Date:** 2023-07-13

This is a closed book exam

Answer all questions

The problems are not ordered by difficulty

C'est un examen à livres fermés

Répondre à toutes les questions

Les problèmes ne sont pas classés par difficulté

Question	Q1	Q2	Q3	Q4	Q5	Total
(weight)	(18)	(21)	(21)	(25)	(15)	(100)
Mark/Point						

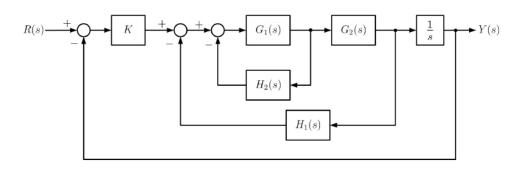
Helpful formula 
$$s_{1,2} = \sigma_d \pm j\omega_d = -\xi \omega_n \pm j\omega_n \sqrt{1 - \xi^2} \quad , \quad G(s) = \frac{\omega_n^2}{s^2 + 2\xi \omega_n + \omega_n^2}$$

$$f(\infty) = \lim_{s \to 0} sF(s) \quad , \quad e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}$$

$$\sigma_a = \frac{\sum p_i - \sum z_i}{\#p - \#z} \qquad \theta_a = \frac{(2k+1)\pi}{\#p - \#z} \qquad KG(s)H(s) = 1\angle(2k+1)180^0$$

$$Z_c(s) = \frac{1}{cs} \quad , \quad Z_L(s) = Ls \quad , \quad Z_R(s) = R$$

**Problem 1**: (18 points) Block-Diagram Reduction. Find the transfer function Y (s)/R(s).



**Problem 2**. (21 points) For the unity feedback system shown bellow

$$G(s) = \frac{G(s) + E(s)}{G(s) + E(s)}$$

$$G(s) = \frac{450(s + 7)(s - 8)(s + 15)}{s(s + 38)(s^2 + 2s + 28)}$$

find the steady-state errors for the following test inputs:

a. 15u(t);

b. 3tu(t);

c.  $4t^2u(t)$ 

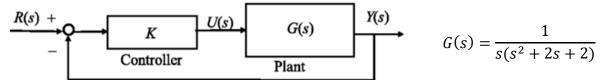
**Problem 3:** (21 points) In the given electrical network, the input is the dc-voltage  $(V_i)$  and the output voltage is  $V_o$ .

$$\begin{array}{c|cccc}
i(t) & R & 10\Omega & L & 0.2 \text{ H} \\
+ & & & + & \\
v_i(t) & & & v_o(t) & C \\
- & & & - & & -
\end{array}$$

- a. Find the transfer function  $V_o/V_i$
- **b.** Is the system underdamped or overdamped?
- **c.** Assuming  $x_1=i_L$  and  $x_2=v_c$ , write the state equation in the following form,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

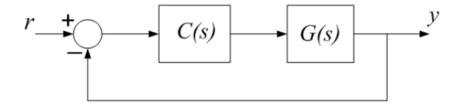
**Problem 4:** (25 points) A unity feedback control system is shown in this block diagram.



Make a rough sketch of root locus plot by determining the following.

- a. Centroid, number and angle of asymptotes.
- b. Angle of departure of root loci from the poles.
- c. Points of intersection with  $j\omega$

**Problem 5:** (15 points) Consider the following LTI system:



where 
$$C(s) = \frac{K}{s+4}$$
 and  $G(s) = \frac{3s+3}{s^2-2s-3}$ 

Determine for what values of K the system is stable and unstable.