Chapter II-1

Plant Modeling and Identification in the Frequency Domain

II-1.1 Objectives

The objective of this lab is to find an open-loop model of the motor system, which will be used as the plant in the following lab experiments.

You will learn how to:

- Model sub-components of a motor system.
- Aggregate the models of these subsystems into the transfer function of the whole system.
- Approximate the real high-order model of a typical motor system by a first-order model estimate.

II-1.2 Modeling of a Motor System

A block diagram of the motor system is shown in Fig. II-1.1. The system is composed of a geared brushed DC motor and a fly wheel.

II-1.2.1 Motor Circuit

The motor circuit is schematically depicted on the left side of Fig. II-1.1. It comprises the windings around the armature (L_m) and their resistance (R_m). A back emf (v_{em}) is generated across the winding terminals when the rotor is in motion, such that

$$v_{em} = K_{em}\,\omega_{rm}$$

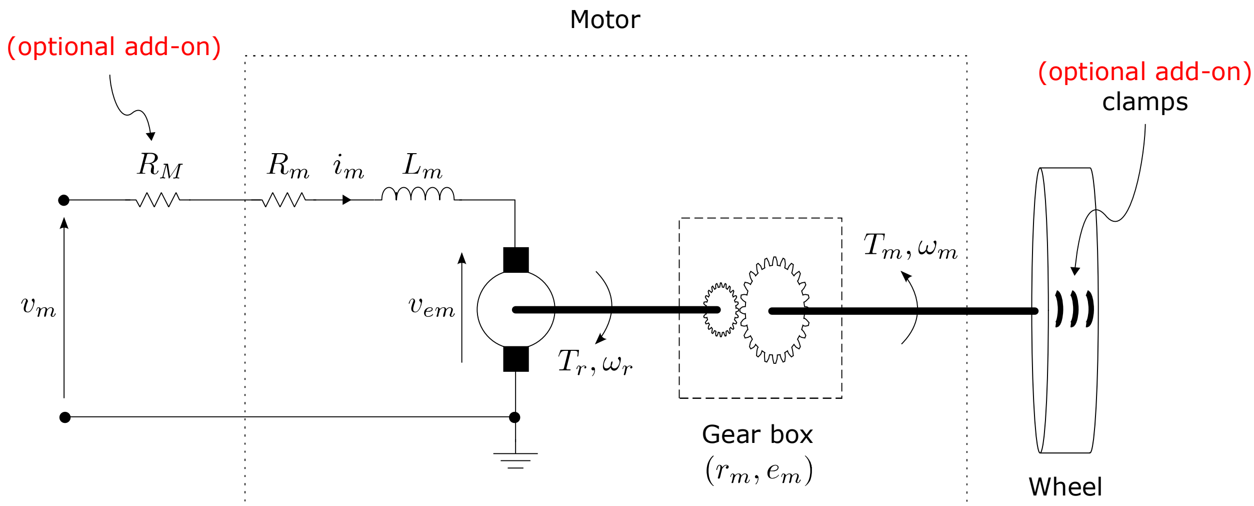

Figure II-1.1: Block diagram of the motor system (the plant)

where ω_{rm} is the motor rotor's angular speed and K_{em} is a motor-dependent back emf constant. The back emf acts on "weakening" the effective voltage running the motor. The motor's input, which is also the overall system's input in this case, is the DC voltage (v_m) applied across its terminals. The torque (T_{rm}) generated by the rotor is defined by

$$T_{rm} = K_{tm}\, i_m$$

where i_m is the current running through the motor windings and K_{tm} is a motor-dependent torque constant. The motor dynamics can be altered by adding an external resistor (R_M) in series with the motor circuit.

(5 pts) Pre-lab

P-II-1.1. In the Laplace domain, express v_m in terms of $\omega_{rm}(s)$, $T_{rm}(s)$, and motor constants/parameters (including R_M).

Hint: use the equation of the motor circuit. Note that signals like i_m, for example, are not constant.

II-1.2.2 Motor Gear

In general, a gear transfers mechanical energy from one side to the other. This is done by increasing the angular speed and decreasing the torque by a factor of r_m (known as the gear ratio), or vice versa, in such a way as to (ideally) conserve their product (the mechanical power). In other words, ideally, the motor's generated torque (T_m) and the motor shaft's angular speed (ω_m) are related by

$$\omega_m = \frac{-1}{r_m}\,\omega_{rm} \qquad\qquad T_m = -r_m\, T_{rm} \qquad\qquad \omega_m\, T_m = \omega_{rm}\, T_{rm}$$

The negative sign is due to the fact the gear switches the direction of rotation. In the following, and in order to be consistent with K-CSP's convention, we will assume that a positive rotation of the rotor is associated with a positive rotation of the motor shaft, and vice versa. Hence, the above equations become

$$\omega_m = \frac{1}{r_m}\,\omega_{rm} \qquad\qquad T_m = r_m\,T_{rm} \qquad\qquad \omega_m\,T_m = \omega_{rm}\,T_{rm}$$

In practice, however, gears are not ideal. They rather consume some of the energy they are supposed to transfer. Hence, for a gear of efficiency $e_m \in (0,1)$, the above relations are more precisely expressed as follows:

$$\omega_m = \frac{1}{r_m}\,\omega_{rm} \qquad\qquad T_m = e_m r_m\,T_{rm} \qquad\qquad \omega_m\,T_m = e_m\,\omega_{rm}\,T_{rm}$$

Pre-lab

P-II-1.2. Deduce the expression of $V_m(s)$ in terms of $\omega_m(s)$, $T_m(s)$, and motor constants defined so far. (5 pts)

II-1.2.3 Fly Wheel

A fly wheel is attached after the motor gear box to add some inertia to the system. Clamps may be added on or taken off the wheel to change the system's inertia and so its dynamics.

II-1.2.4 Overall System Model

In the following, it will be assumed that the system is subject to only a viscous-type friction torque $T_f = -b_f\omega_m$, where b_f is a positive friction coefficient (constant). Note that although conputationally convenient, considering that viscous friction is the only friction acting on the motor is not accurate. This friction model is often satisfactory at a relatively high motor speed but it may lead to a poor performance when the speed is around zero.

Let J be the total inertia of the system. Then, applying Newton's second law of motion (sum of the torques is equal to the inertia times the acceleration), we get

$$T_m + T_f + T_d = J\,\frac{\mathrm{d}\omega_m}{\mathrm{d}t} \tag{II-1.1}$$

where T_d is a lump of all other (disturbance) torques acting on the motor, which were not explicitly modeled.

Now that we defined the transfer functions of the interconnected subsystems, we are ready to derive the overall model of the entire system.

re-lab

P-II-1.3. Using (II-1.1), find the transfer function $[T_m(s) + T_d(s)]/\omega_m(s)$. (5 pts)

Pre-lab

(5 pts) P-II-1.4. Use the findings so far to fill-in the empty blocks in the motor's open-loop block diagram of Fig. II-1.2. Note that the feedback shown in the figure is inherently natural due to the electromagnetic nature of the motor. In other words, it can never be removed, which is why the model is still referred to as the "open-loop" model despite that feedback.

(5 pts) P-II-1.5. Assuming no external disturbances (i.e., $T_d = 0$), compute the expression of the system's transfer function $T_1(s) = \omega_m(s)/V_m(s)$. Express it as a ratio of two polynomials in s. What order is the system?

(5 pts) P-II-1.6. Now, disabling the main input (i.e., $V_m = 0$) and enabling the disturbance (i.e., $T_d \neq 0$), compute the expression of the transfer function $T_2(s) = \omega_m(s)/T_d(s)$. Express it as a ratio of two polynomials in s.

(5 pts) P-II-1.7. Deduce the expression of the motor's speed $\omega_m(s)$ when both inputs (V_m and T_d) are active.
Hint: think of the superposition principle.

(5 pts) P-II-1.8. Deduce the expression(s) of the open-loop motor system's pole(s).

(5 pts) P-II-1.9. In most cases, such a system can be approximated by a first-order system. This is because the dominant pole (the one closer the imaginary axis of the s-plane) is far less than 5 times in magnitude than the real parts of the other pole(s). This stems from the fact that, in practice, the total resistance in the armature circuit is by far much larger than its inductance. In other words, $\frac{L_m}{R_m+R_M} \approx 0$.

Assuming no external disturbances ($T_d = 0$), find the system's transfer function if it is approximated by a first-order model. Writing it in the form of $K/(\tau s+1)$, find the expressions of the system's DC gain (K) and its time constant (τ).

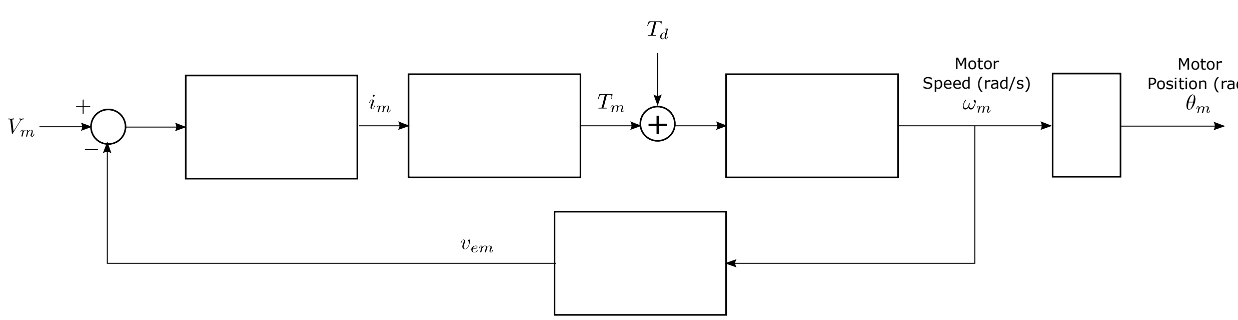

Figure II-1.2: Open-loop block diagram of the motor system (the plant)

II-1.3 Lab Procedure

II-1.3.1 Introduction

In the laboratory, you will run the motor system in an open-loop mode to experimentally compute its DC gain K and time constant τ while approximating the open-loop system by a first-order model;

$$\frac{\omega_m(s)}{V_m(s)} = \frac{K}{(\tau s + 1)} \tag{II-1.2}$$

Note that the response of such a family of systems to a step input of magnitude V_m in the time-domain is

$$\omega_m(t) = V_m\, K\, (1 - e^{-t/\tau}) \tag{II-1.3}$$

assuming zero initial conditions.

In practice, exciting the motor from rest using a step input with a zero initial condition may not lead to a first-order response. This is because the dynamics of the motor in the vicinity of zero speed is dominated by (unmodeled) nonlinear terms. Therefore, to make the system act more like a first-order system, it is better to have the motor running at an initial constant non-zero speed and from there excite it with a step input of amplitude $\Delta\, u$, as illustrated in Fig. II-1.3.

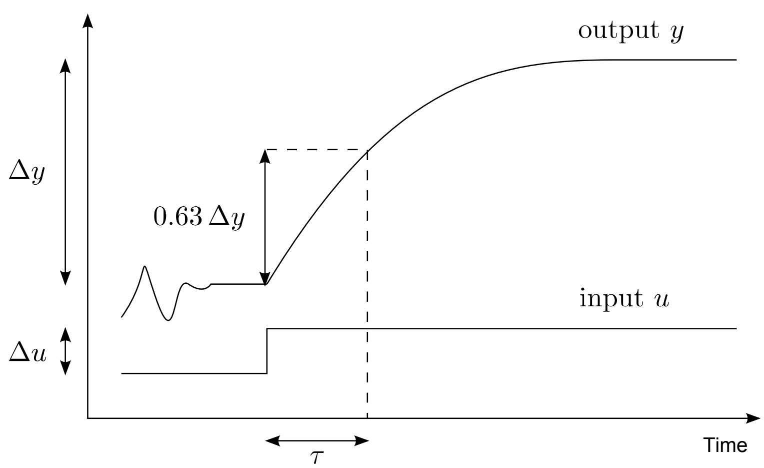

Figure II-1.3: Step response of a first-order system with non-zero initial condition

Let $\Delta\, y$ denote the change in the steady-state output (motor speed in this case). The DC gain (K) is the system's output response as a result of a **unit** step input. Thus,

$$K = \frac{\Delta\, y}{\Delta\, u}.$$

The time constant is the time needed for the system's response to reach 63% of the output **span** ($\Delta\, y$) (see Fig. II-1.3).

II-1.3.2 Implementation

To run the experiment, follow the following procedure.

L-II-1.1. Connect the K-MCK and the K-ECS boards and launch the K-CSP, as described in Chapter I-1.

L-II-1.2. Upload this lab's firmware (Hex file) to the K-ECS board.

L-II-1.3. Load this lab's configuration file (kcsp file) to the K-CSP.

L-II-1.4. Apply the settings shown in Fig. II-1.4 with the following exceptions:

a) Make sure you pick the right communication port.

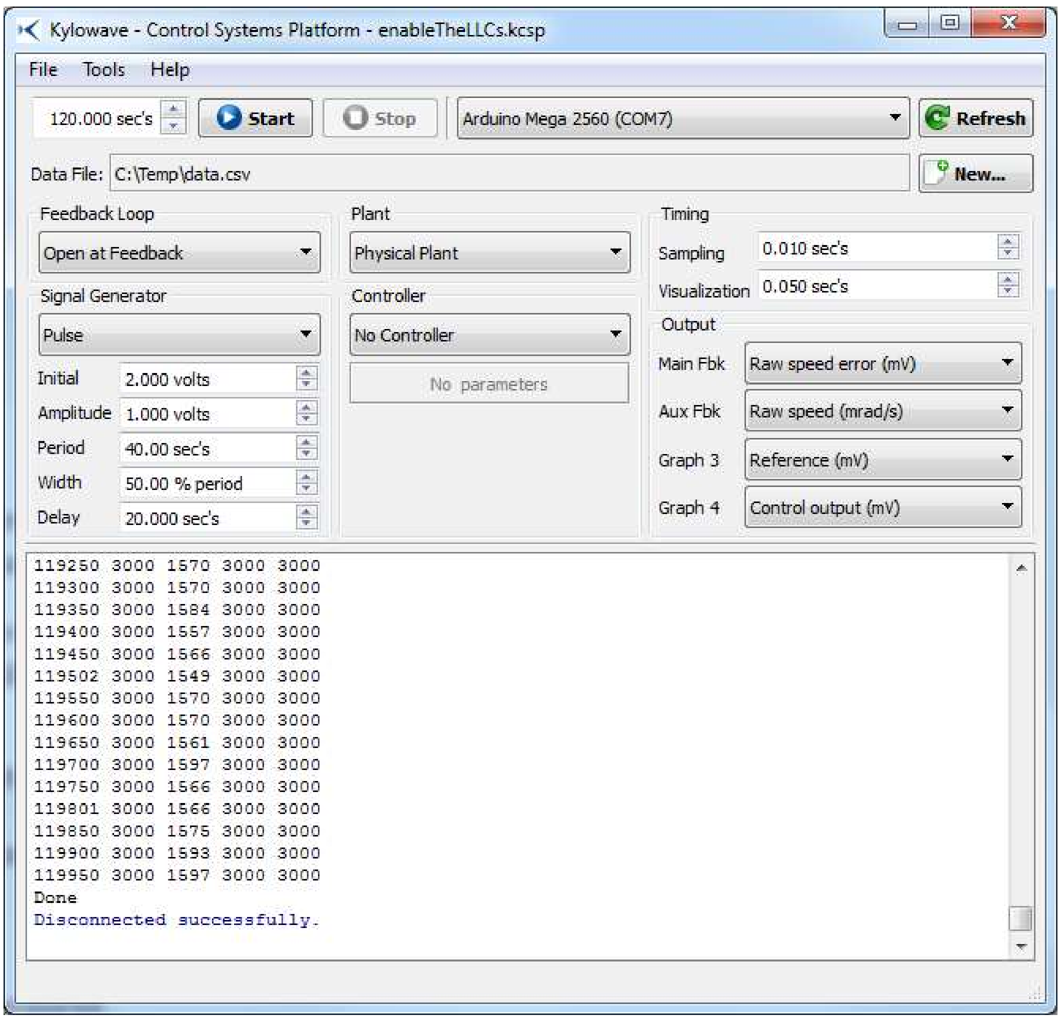

Figure II-1.4: K-CSP settings

L-II-1.5. Run the experiment with the reference signal's initial and amplitude values indicated in Table II-1.1. For information on the Pulse signal properties, see Fig. I-2.2.

Save your plots and copy the raw data (csv file) on your personal storage media, such as a USB key, for later use. You may want to give them meaningful names so that they do not get mixed up with the files of other experiments.

L-II-1.6. Using the raw speed data/plot, graphically compute the values of K and τ, as precisely as possible (use the zoom tool if necessary), and write them down in the proper cells of Table II-1.1. Avoid working on the first a few cycles of the plot. The numerical values will be computed later.

Demo

D-II-1.1. Make sure you demo to the TA at least one of your runs before leaving the lab. (10 pts)

						Speed			
						Graphically		Numerically	
Initial V_{ref} (V)	Amplitude (V)	ΔV_{Ref} (V)	Initial ω_m (rad/s)	Final ω_m (rad/s)	$\Delta \omega_m$ (rad/s)	K	τ (s)	K	τ (s)
2.0	1.0								
2.0	1.5								
2.0	2.0								
2.0	2.5								
2.0	3.0								
					Average:				

Table II-1.1: Model identification results

II-1.4 Laboratory Report

Include the following in your laboratory report:

Report

R-II-1.1. The plots of all the experiments corresponding to Table II-1.1. Make sure they are properly labeled. (4 pts)

R-II-1.2. A filled copy of Table II-1.1. (16 pts)

Furthermore, for each experiment, use the experimental raw data you saved in the lab, to plot the transient response of the system. One excitation cycle is enough for each experiment. Make sure to shift the x- and y-axis scales so that the graph starts from $(0, 0)$. The graphs must clearly show the time constant, settling time, and the output steady-state value.

Numerically process the speed data of each transient response graph to

Report

find the numerically computed values of the DC gain and the time constant.

To numerically compute the DC gain with a higher precision you may want to take the average of the output at steady state.

In Matlab, for instance, you may want to customize the following statement to find the index `idx` corresponding to the time constant:
`idx=max(find((yss-y)>=0.37*yss))`
where `yss` is the output's steady-state value and `y` is a vector of the output samples (i.e., speed).

Do not forget to calculate the average values of K and τ in Table II-1.1.

(10 pts) R-II-1.3. Model validation: For each row of Table II-1.1, generate two figures:

a) The first figure shows the response of the "ideal" system along with that measured experimentally. The latter is the same transient response plot you generated above. The "ideal" system response can be generated by applying the **average** of the numerically computed K and τ to the Matlab function `lsim` fed with the time and input vectors used to generate the first plot. For details on how to use `lsim` refer to http://www.mathworks.com/help/control/ref/lsim.html

b) In the second figure include the error between the ideal and the measured responses plotted in the first figure.

Analyze the validity of your model.

Include your answers to the following questions in your report.

Report

(5 pts) R-II-1.4. Does the DC gain or the time constant of a first-order system depend on the input? Justify your answer theoretically and back it up with your results in Table II-1.1.

(5 pts) R-II-1.5. What might be the origin of the discrepencies between the ideal and the experimental response of the motor system?

(30 pts) R-II-1.6. **Design of an LTI system:**
Design an electric circuit that has the same first-order transfer function as the motor system identified above (in its first-order approximation). It is preferred that the circuit's input and output be voltages (instead of current, for example).

Determine the following circuit characteristics:

1. The values of all the circuit's electric components (resistors, capacitors, etc.)

Report

2. The symbolic and numeric expressions of the circuit's transfer function.
3. The symbolic expressions and numeric values of the circuit DC gain and time constant.

The difference between symbolic and numeric expressions is that the symbols of the electric components are substituted with their numeric values in the latter.

Using Multisim, simulate the circuit with step functions of the amplitudes in Table II-1.1, and include the figures in your report. A guide on how to do that is at `http://www.ni.com/tutorial/12774/en/` You may want to search the Internet for other online tutorials on how to use Multisim, in general, if you do not know yet.

How do you compare the response of the circuit and that of the motor to the same inputs?

What do you think of the following statement? Explain.

> *Two systems with the same transfer function must have the same behavior (input-output wise) regardless of their type (electrical, mechanical, chemical, medical, economical, etc.)*

Submit your Multisim file as a separate document from your report so that the TA can run the simulations for verification. Do not submit your work in a zip archive. It is the student's responsibility to make sure that the Multisim document is compatible with the Multisim version installed in the computer labs of the Faculty of Engineering. That is where the TA will run the simulations. No debugging will be done. If the simulation does not work, for any reason, no points will be assigned.

Attention

When answering pre-lab or lab report questions, ALWAYS indicate the number of the question you are answering.