Your **PRINTED FULL NAME**

Your **STUDENT ID NUMBER**

Number of additional sheets

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1. No computers, no tablets, no connected device (phone etc.)
2. Pocket calculator allowed
3. Closed book, closed notes, closed internet
4. Allowed: 1 page (double sided) Chi Chi
5. Additional sheets are available and may be submitted (e.g. for graphs).
6. Write your name below, and your SID on the top right corner of every page (including this one).
7. If you turn in additional sheets:
   - Write your name and/or SID on every sheet, and
   - Write the number of additional sheets you are turning in above where indicated
8. Do not write on the back of any page.

Part	1	2	3	4
Score				

1. Laplace Transforms

   (a) Use Laplace transform tables to derive the Laplace transform for the following time function

$$e^{-at}\sin(wt)\cos(wt)$$

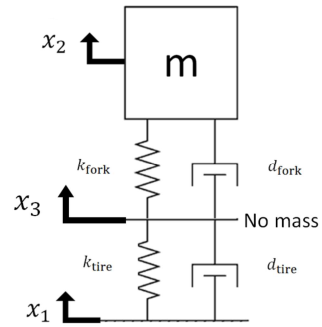
   (b) Use Laplace transforms to solve the following ODE. Assume all forcing functions are zero prior to $t = 0^{-}$.

$$\frac{d^2x}{dt^2} = 4e^{-t}\cos\left(\frac{t}{2}\right)\sin\left(\frac{t}{2}\right)$$

$$x(0) = 2,\ x'(0) = 3$$

More space for part (b)

2. Consider a simplified model of a front mountain bike suspension. The input is the position $x_1(t)$ of the rocky terrain and the output is the position $x_2(t)$ of the person with mass m. The spring and damping constants are defined as seen below in the figure. **Ignore the effect of gravity.**



(a) Derive the transfer function $T(s) = \frac{X_2(s)}{X_1(s)}$ in terms of $k_{\text{fork}}, k_{\text{tire}}, d_{\text{fork}}, d_{\text{tire}}$.
Note that $T(s)$ is the transfer function from position x_1 to position x_2, derive the transfer function accordingly.

(b) Make the approximation $k_{\text{tire}} = \infty$ and $d_{\text{tire}} = 0$ and derive the new transfer function $T(s) = \frac{X_2(s)}{X_1(s)}$.
Hint: this is equivalent to ignoring the dynamics of the tire.

(c) Using the second-order transfer function obtained from part (b) determine the settling time and percent overshoot for a step response given $k_{\text{fork}} = 200{\times}10^3\text{N/m}$, $d_{\text{fork}} = 10{\times}10^3\text{Ns/m}$, and $m = 10$ kg.

3. Consider the following system:

$$\dot{x} = Ax + Bu, \qquad y = Cx \quad \text{with} \quad A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \text{ and } x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (1)$$

(a) Compute the eigenvalues for A.

(b) Determine the associated eigenvectors for each eigenvalue you found.

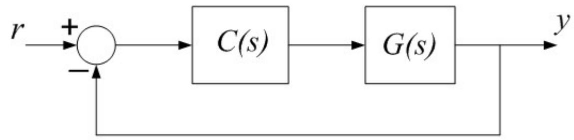
(c) Diagonalize the A matrix. Please write the P and P^{-1} matrix used in the diagonalization process.

(d) Calculate e^{At} explicitly.

(e) Compute the output $y(t)$ for a unit step $u(t)$. Use the results from part (d) to determine a soultion for $x(t)$, then find $y(t)$.

(f) Given the matrices A,B, and C above determine the transfer function $G(s)$ for the state-space system. How do the poles of the transfer function compare to the eigenvalues found in part (a)?

(g) Consider the following LTI system:



where $C(s) = \frac{K}{s+4}$ and $G(s) = \frac{3s+3}{s^2-2s-3}$

Determine for which values of K the system is stable and unstable.

(h) What is the steady-state error of the closed-loop system for a unit step input $u(t)$ as a function of K (for values of K that stabilize the system)?

4. Consider the following LTI system:

$$G(s) = \frac{(s+2)}{(s+.1)(s+1)(s+15)} \tag{2}$$

(a) Determine the number of branches and any asymptotes (center and angle) that exist.

(b) How many break-in/break away points are there? Find, but dont not solve, the polynomial which the break-in/away points must satisfy.

(c) Sketch the root locus. Make sure to label the asymptotes you found in part (a) and the direction of the root locus for increasing K. For ease of plotting assume the break aways occur at -6,-4,and $-.5$.

(d) **BONUS:** Plot the feasible region for the poles if the design requirment is to have a minimum damping ratio of 0.5 and a settling time less than 1.6 seconds. (You may use the second order approximation for settling time.) Given the region you drew, is it feasible to meet the design requirments? Explain.

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$\sin(\omega t)u(t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)u(t)$	$\frac{s}{s^2+\omega^2}$

Table 1: Laplace transforms of common functions

$\sin(2\theta)$	$2\cos(\theta)\sin(\theta)$
$\cos(2\theta)$	$\cos(\theta)^2 - \sin(\theta)^2$
$\tan(2\theta)$	$\dfrac{2\tan(\theta)}{1-\tan(\theta)^2}$

Table 2: Trigonometric functions