A novel LMS scheme for adaptive noise cancellation

Hitham Jleed

Department of Electrical and Computer Engineering
Elmergib University
ALkhoms, Libya
Email: hhjleed@elmergib.edu.ly

***Abstract*— A new scheme for adaptive noise cancellation (ANC) is presented in this paper. The method estimates signal corrupted by additive noise or interference. The primary input contains the corrupted signal, and the reference input contains noise that is correlated somehow with the primary noise. The algorithm minimizes the mean squared error of cost function, trying to converge in least mean square sense to the optimal Wiener-Hopf solution to filter the noise from the signal.**

Keywords—component, LMS, Adaptive Filter, Noise Cancellation, cost function, Wiener-Hopf, steepest descent.

I. Introduction

Adaptive noise cancellation is a process of canceling noise present in corrupted signals. The Adaptive Noise Canceller (ANC) has two inputs – primary and reference. The primary input receives a signal from the signal source, which is corrupted by the presence of noise. The reference input receives an interference noise [1]. The noise passes through a filter to produce an output that is a close estimate of noisy primary input. This noise estimates subtracted from the corrupted signal to produce an estimate of the pure signal, which is the aim of this system.

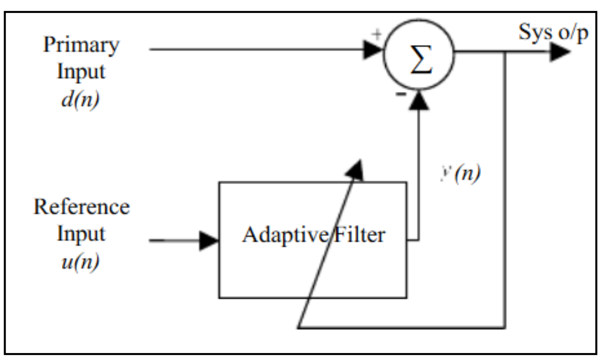

Fig. 1. Adaptive Noise Canceller

Figure.1 shows the schematic diagram of the adaptive noise canceller (ANC). Signal d(n) is the speech signal corrupted by noise and is made up of two components, s(n), the information-bearing signal, and u1(n) the noise component. The reference signal u(n) is the pure noise signal, which is correlated to u1(n) but is uncorrelated to s(n). u(t) is passed through the adaptive filter, the filter operates on this to generate an estimate of the interference, which is then subtracted from the primary to yield an estimate of the signal ŝ(t). Since the characteristics of the filter, h(t), is usually unknown, an adaptive filtering approach proved to be superior to any fixed-tap filter method. The filter parameters can be adjusted using the LMS algorithm by minimizing a cost function in the steepest descent sense, with the output being the estimate of the signal ŝ (t)[5].

This proposed paper discusses the concept of adaptive noise canceling that estimates a corrupted speech with additive noise. It describes the theoretical interview of steepest descent and the least mean square. It further shows the experimental result of adaptive noise cancellation on the speech signal, where the corrupted signal passes through a filter that tends to suppress the noise while leaving the signal relatively unchanged.

II. Steepest Descent Algorithm

The steepest descent is one of the oldest methods of optimization. It is based on finding the minimum value of the mean-squared error. J_{min} .The algorithm begins with an initial value ω(n) for the tap-weight vector. It is usually chosen to be a null vector unless some prior knowledge is available. The algorithm then computes the gradient vector ΔJ(n) and the tap-weight vector at time n+1 is computed by,

$$\omega(n+1) = \omega(n) + \frac{1}{2}\mu[-\nabla J(n)] \qquad (1)$$

Where
$$\nabla J(n) = -2P + 2R\omega(n) \qquad (2)$$

Therefore, the updated value of tap-weight vector ω(n+1) is computed using simple recursive relation

$$\omega(n+1) = \omega(n) + \mu[P - R\,\omega(n)] \qquad (3)$$

It can be seen that the step-size parameter μ controls the size of the incremental correction, and since the algorithm is recursive, it is possible to become unstable. It is observed that step-size parameter μ and the autocorrelation R control the feedback loop [3]. To determine the condition of stability, the correlation matrix R is represented in terms of its eigenvalues and associated eigenvectors at time n can be viewed as:

$$e(n) = \omega(n) - \omega_0 \qquad (4)$$

Where ω_0 is the optimum value defined by Weiner-Hopf equations. By eliminating the cross-correlation, the equation can be rewritten in terms of weight-error vector as

$$e(n+1) = (I - \mu R)e(n) \quad (5)$$

Using unity similarity transformation, the correlation matrix R can be written as

$$R = Q\Lambda Q^H \quad (6)$$

Where Q has its columns an orthogonal set of eigenvectors associated with eigenvalues of the matrix R. The matrix Λ is a diagonal matrix of eigenvalues.

$$e(n+1) = (1 - \mu Q\Lambda Q^H)e(n) \quad (7)$$

pre-multiplying both sides of this equation by Q^H, the equation can be rewritten as:

$$Q^H e(n+1) = (1 - \mu\Lambda)Q^H e(n) \quad (8)$$

A new set of coordinates can be set as,

$$v(n) = Q^H[\omega(n) - \omega_0] \quad (9)$$

$$v(n+1) = (1 - \mu\Lambda)v(n) \quad (10)$$

Then, $$v_k(n+1) = (1 - \mu\lambda_k)v_k(n)\,, \quad \forall k = 1,2,\dots,M \quad (11)$$

Where λ_k is the k^{th} eigenvalue of correlation matrix R. Since all eigenvalues of the correlation matrix R are positive and real, the magnitude of this geometric ratio must be less than 1 for all k.

$$-1 < 1 - \mu\lambda_k < 1 \qquad for\ all\ k \quad (12)$$

Therefore, for the convergence or stability of the steepest descent algorithm, it is required that μ satisfies the following condition:

$$0 < \mu < \frac{2}{\lambda_{max}} \quad (13)$$

Where λ_{max} is considered the largest eigenvalue of the correlation matrix-R.

III. The LMS Adaptation Algorithm

The least mean square algorithm, proposed by Windrow and Hoff's in 1960 [6], is the most popular adaptive noise canceller. It has good tracking properties and simplicity of implementation [1], and comparing to the Wiener filter, and it does not require correlation function or matrix inversion. The least mean square algorithm is an approximation of the steepest descent algorithm, which uses an instantaneous estimate of the gradient vector of a cost function. The LMS algorithm consists of two basic processes[4]:

- A Filtering process, which is computing the output of the transversal filter and generating an estimation error.
- An adaptive process automatically adjusts the tap weights of the filter according to estimated error.

The combining of these two processes generates a feedback loop in the LMS algorithm, which is responsible for performing the filter process. In addition, the adaptive weight-control mechanism performs the adaptive control process on the tap weight of the transversal. Figure.2 shows the detailed structure of the transversal filter component

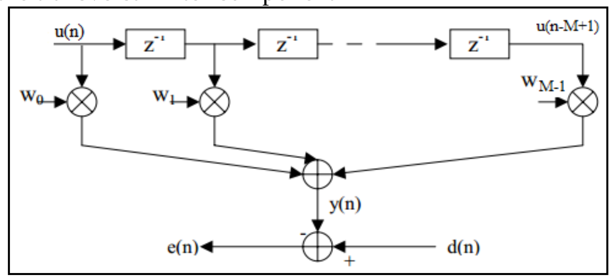

Fig. 2. The detailed structure of the transversal filter component

- The tap input *u(n), u(n-1), ..., u(n-M+1)* form the elements of *M-by1* tap-input vector *u(n),* where M-1 is the number of delay elements.
- Tap-weight vector
$$\overline{w} = [w_0\ w_1\ \dots\ w_{M-1}]^T \quad (14)$$
- signal input
$$\overline{u}(n) = [u(n)\, u(n-1) \dots u(n-M+1)]^T \quad (15)$$
- filter output
$$y(n) = \overline{w}^T\overline{u}(n) \quad (16)$$
- error signal
$$e(n) = d(n) - y(n) \quad (17)$$

The LMS algorithm uses the product $u(n-k)e^*(k)$ as an estimate of element k in the gradient vector $\nabla J(n)$ that characterizes the method of steepest descent. Therefore, the tap weight adaptation for the next cycle will be:

$$\hat{w}(n+1) = \hat{w}(n) + \mu\, u(n)e^*(n) \quad (18)$$

Where μ is the step size. The computation of error e(n) is based on the current estimate of the tap-weight vector $\hat{w}(n)$. The second term $\mu\, u(n)e^*(n)$ represents the correction applied to the current estimate of the tap vector $\hat{w}(n)$ [4]. The least mean square has some properties of :

- It can be used to solve the Wiener-Hopf equation without finding matrix inversion, and it is required information about the autocorrelation matrix of the input and the cross-correlation between the filter input and its desired signal.
- It is easy to implement and capable of delivering high performance during the adaptation process.
- Its iterative procedure involves computing the output of an FIR filter transversal filter coefficients, generation of an estimated error by comparing the output of the filter to the desired response and adjusting the tap weights of filter based on estimation error.
- It is stable and robust for a variety of signal conditions.

IV. Methodology

Noise Cancellation has many applications, but in generals, it uses a reference input derived from one or more sensors in the noise field where the signal is undetectable. This signal is somehow subtracted from primary input containing both signal and noise. As a result, the primary signal is eliminated by the cancellation. In this project, a speech signal is used as an input signal, which is random in nature and non-stationary. Natural speech signals are non-stationary; however, for mathematical tractability, a random sequence with piecewise stationary is used to approximate the speech signals. The adaptive noise cancellation setup is shown in figure.3. The algorithm utilizes two signals: one signal consists of speech and noise, whereas the second signal consists of only noise that is somehow related to the first signal. These signals are considered primary and reference signals, respectively. The noise reference x(n) passes through the adaptive filter, and output y(n) is produced as close to x1(n) as possible. The adaptive filter auto-adjusts to minimize the error between x1(n) and y(n). By substituting d(n)=s(n)+x1(n) in equation (3.4), the error becomes:

$$e(n) = s(n) + x1(n) - y(n) \tag{19}$$

The objective of work is accomplished by feeding the system back to the adaptive filter and adjusting the filter through an LMS algorithm to minimize e(n). To produce the noise cancellation, s, x1, and x are assumed to be statistically stationary and have zero means. By squaring the equation (4.1)

$$\begin{aligned} e(n)^2 = s(n)^2 + [x1(n) - y(n)]^2 \\ +2s(n)[x1(n) - y(n)] \end{aligned} \tag{20}$$

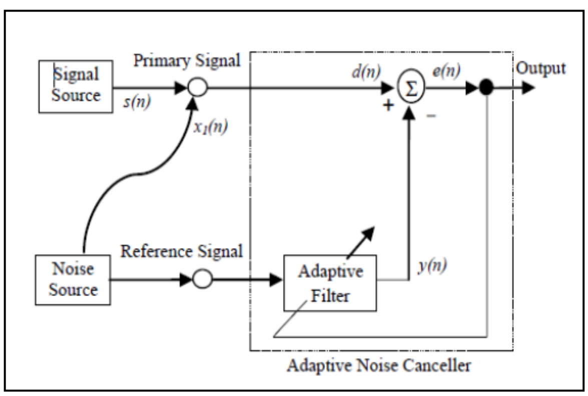

Fig.3 Adaptive Noise Cancellation System

Taking expectations of both sides of this equation and assuming the X1 is uncorrelated with S and Y.

$$\begin{aligned} E[e^2] = E[s^2] + E[(x1 - y)^2] + 2E[s(x1 - y)] \\ = \mathrm{E[s\^{}2\ ] + E[(x1 - y)\^{}2]} \end{aligned} \tag{21}$$

Considering that the source signal will not be affected by minimization,

$$minimize[e^2] = E[s^2] + minimize[(x1 - y)^2] \tag{22}$$

Therefore, when $[(x1 - y)^2]$ is minimized, $[(e - s)^2]$ is minimized as well. So

$$(e - s) \triangleq (x1 - y) \tag{23}$$

Thus, adjusting the filter to minimize the total power is giving the best least-square estimate of x1. The length of this adjustment depends on the number of tap weights of the adaptive filter. The output provides an estimate of the desired response. The algorithm converges to a set of tap weights, which, on average, are equal to the Wiener-Hopf solution.

$$w = w_0 = R^{-1}P \tag{24}$$

Where P is the cross-correlation vector M x 1 of the input signal and the desired signal, and R the Toeplitz autocorrelation matrix M x M of the input signal. If prior knowledge of the tap-weight vector w(n) is available, it will be used to select an appropriate value for w_0. Otherwise, the tap-weight of the filter is set as $w_0 = 0$ [4]. The LMS algorithm can be used to solve the Wiener-Hopf equation without having the availability of the autocorrelation matrix of the filter input and the cross-correlation between the filter input and its desired signal [7].

V. EXPERIMENTAL RESULT

In this section, the performance of the LNS algorithm is evaluated in noise cancellation based on the block diagram in figure.3. The database is retrieved from the free sound website [8]. For illustration, we have chosen one sample, as shown in figure.4. The input signal is a sound signal recorded over a 12 kHz sampling rate of 3 seconds lengths.

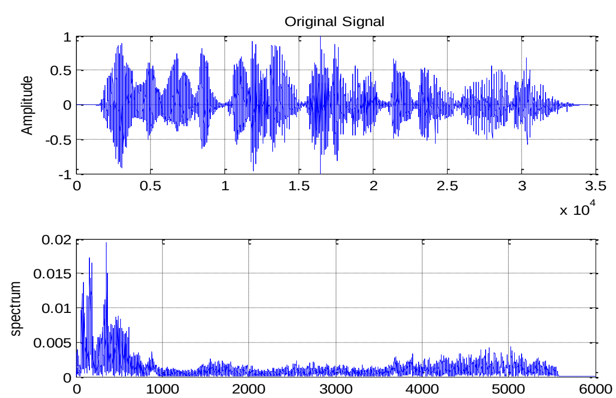

Fig.4 source signal of 3 seconds and sampled rate of 12 kHz

This signal was then corrupted by Gaussian noise x(n). Therefore the primary signal d(n) equals s(n)+x(n). Figure.5 shows the primary signal in the time domain and the frequency domain.

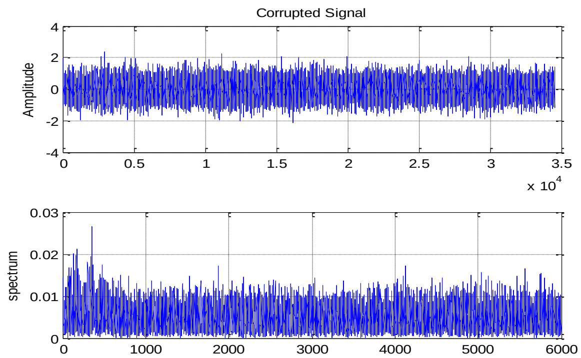

Fig.5 The primary signal "Corrupted signal."

The LMS adaptive filter uses reference noise signal $x_1(n)$, which is related to x(n) noise signal. In this project, the reference signal is generated using the following equation:

$$X_1(z) = \frac{X(z)}{1 - z^{-5}} \tag{25}$$

As the filter converges to the correct model, the filtered noise is subtracted, and the error signal should contain only the original signal. The order of the filter was set to M = 21. The parameter μ is varied. Different μ values have been tested, and as it can be seen in figure.6 the best value is around 0.002.

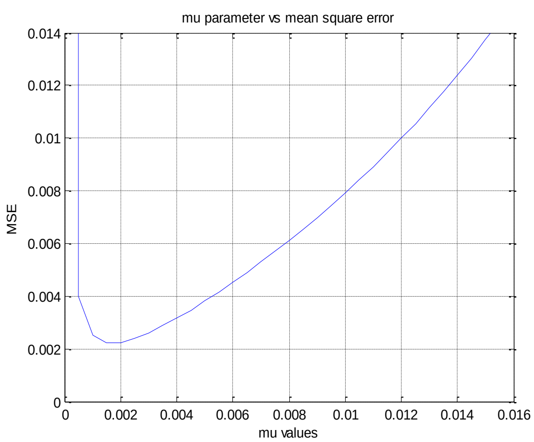

Fig.6 relationship between μ values and mean square error

By applying the parameters of M=21 and μ=0.002, the algorithm purifies the signal with a 2.23% error as shown in figure.7 below.

Overall, the output signal is clear and comparing to the original signal, and it sounds similar. It can be noticed that in 0.1 seconds, the signal has distortion because of convergence time, whereas the original signal was silent at this time.

VI. Conclusion

Adaptive Noise Cancelling has been presented in this work to remove the noise while leaving the original signal unchanged. The principal advantages of this algorithm are adaptation capability and lower output noise. In this paper, the LMS algorithm was applied on signal speech.

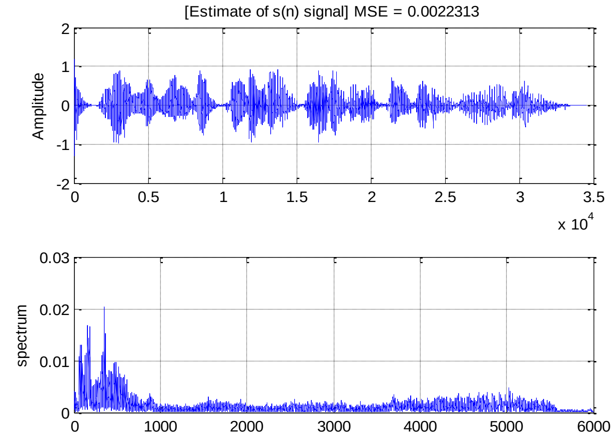

Fig.7 the output, the filtered signal

The original speech was corrupted by Gaussian noise to generate the primary signal, observed signal; whereas, the adaptive filter used reference noise, which correlated with signal noise in some sense, to estimate the noise and subtract it from the primary signal in each sample. The filter that was used is an FIR filter with a length of M=21. The algorithm has tried different values of step-size parameter μ and found μ=0.002 is the best value that gave a less mean square error of 2.23%.

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