
重离子核物理

Heavy Ion Nuclear Physics

放射性束物理介绍

王惠仁 ONG Hooi Jin

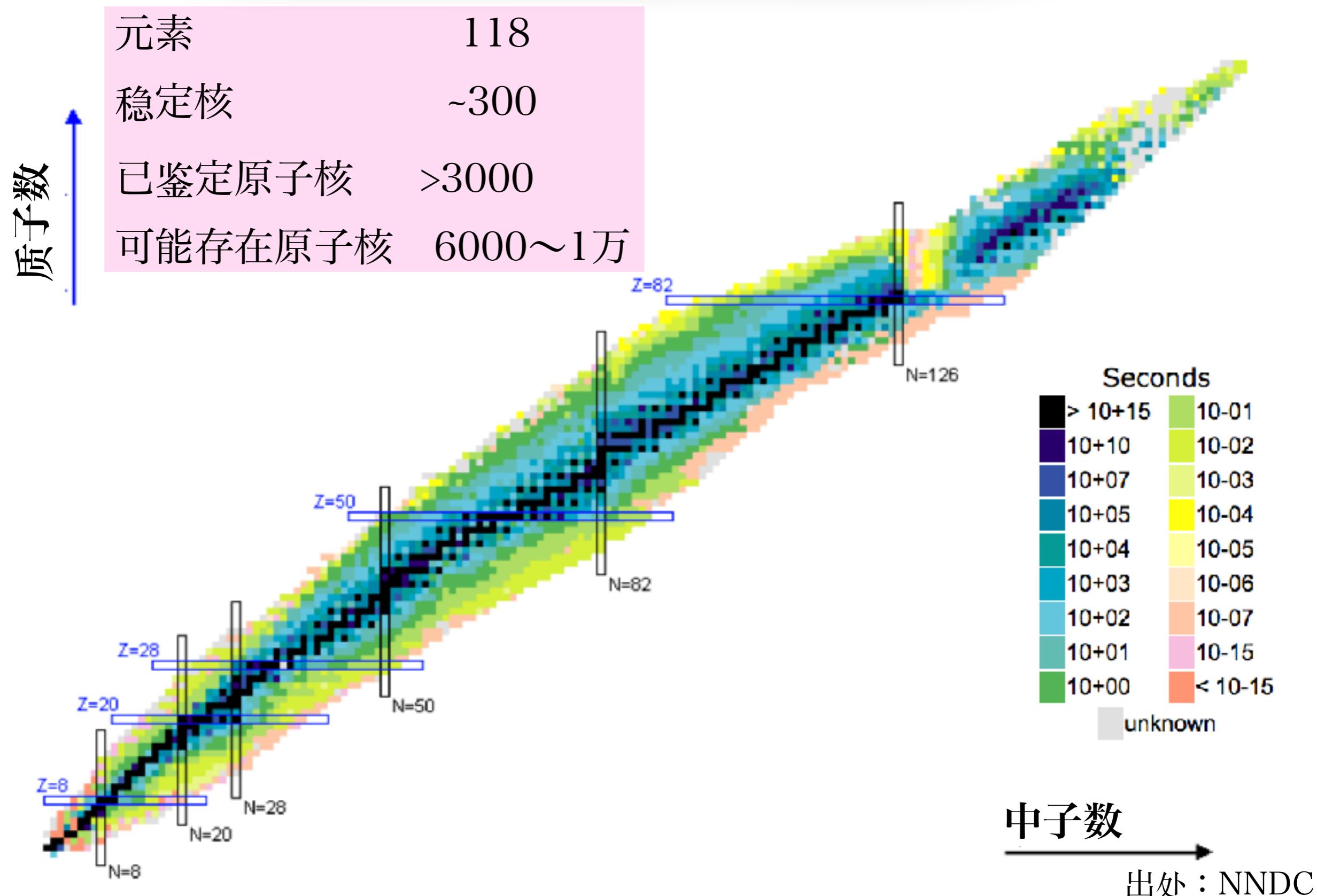
(近代物理研究所核物理中心放射性束物理室)

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提纲

- 1 原子核物理简介
- 2 核的稳定性
- 3 放射性束及实验装置
- 4 放射性束物理

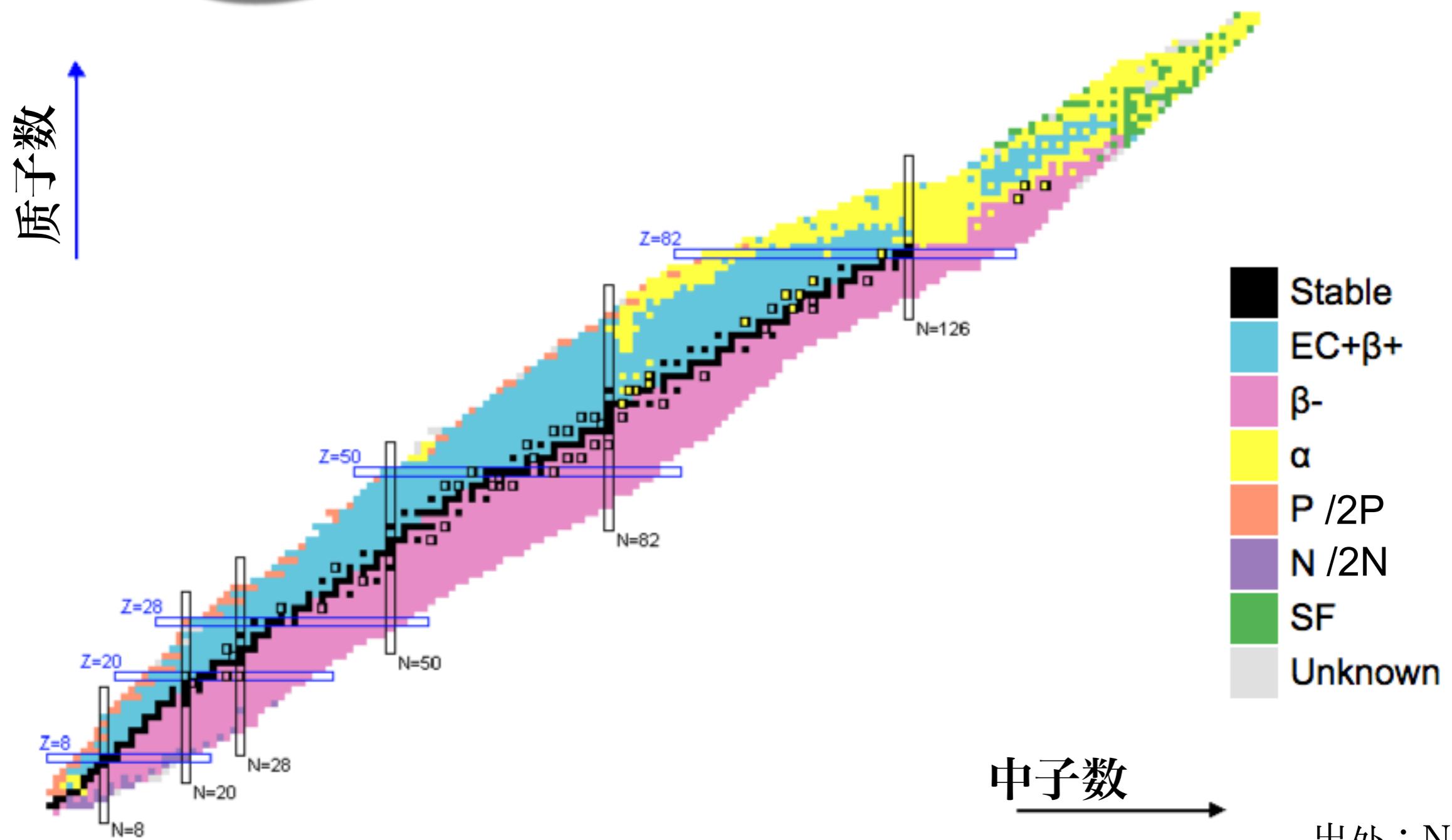
原子核 (Atomic Nuclei)



不稳定核

搜一搜

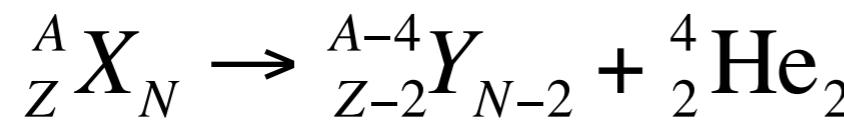
<https://www.nndc.bnl.gov/>



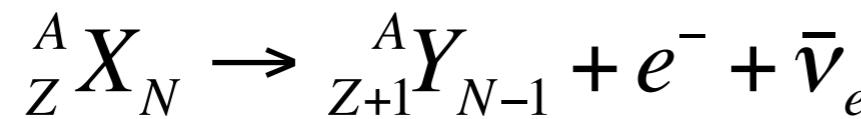
出处：NNDC

原子核的衰变

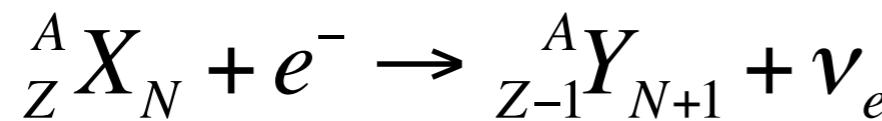
- α 衰变



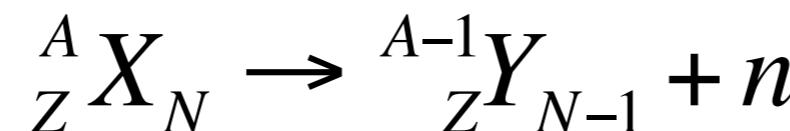
- $\beta-$ 衰变



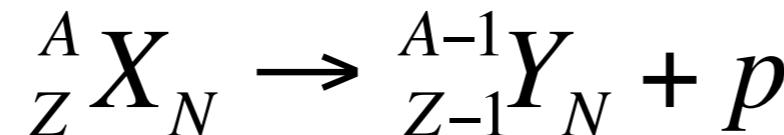
- $\beta+$ 衰变/电子俘获



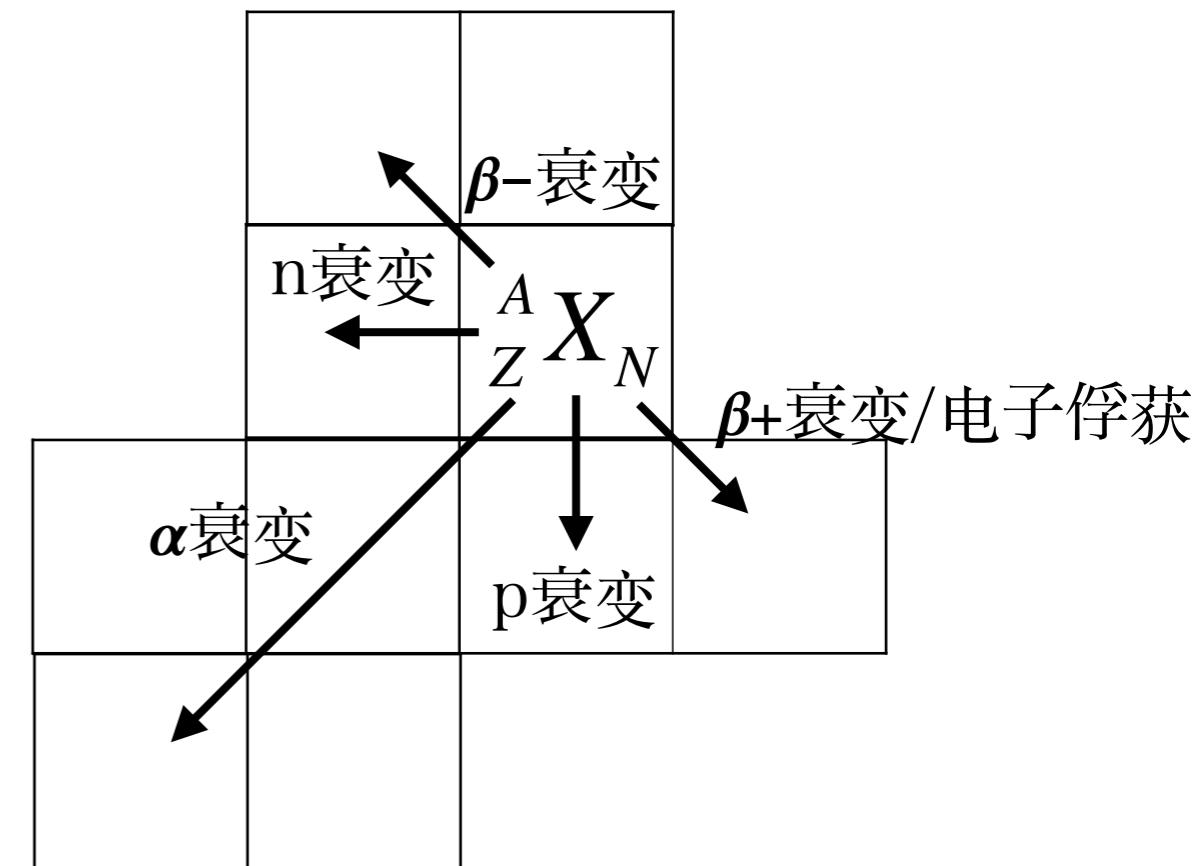
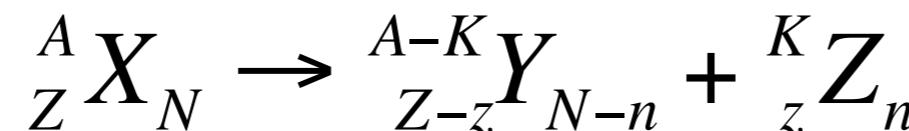
- 中子(n)衰变:



- 质子(p)衰变:



- 核裂变:



质量亏损、质量过剩、结合能

组成某一原子核的核子质量和与该原子核质量之差称为原子核的**质量亏损**：

$$\Delta m(Z, A) = Zm_p + (A - Z)m_n - m(Z, A)$$

由于电子的结合能很小可以忽略，质量亏损也可用原子质量表示：

$$\Delta M(Z, A) = ZM(^1H) + (A - Z)m_n - M(Z, A)$$

$(M - A)$ ，称为核素的**质量过剩**。 M 是核素的原子质量，以u为单位， A 是核素的核子数。 $(M - A)c^2$ ，是质量过剩所对应的能量，通常用符号 Δ 表示，即

$$\Delta(Z, A) = [M(Z, A) - A]c^2$$

这样列表，应用起来较为方便，用质量差计算能量变化时，就省去了单位之间的换算。

自由核子组成原子核所释放的能量称为**原子核的结合能**。

核素的结合能用 $B(Z, A)$ 表示。相据相对论质能关系，它与核素的质量亏损 $\Delta M(Z, A)$ 的关系是

$$B(Z, A) = \Delta M(Z, A)c^2$$

核子分离能

$$S_n(Z, N) \equiv B(Z, N) - B(Z, N-1)$$

$$S_{2n}(Z, N) \equiv B(Z, N) - B(Z, N-2)$$

$$S_p(Z, N) \equiv B(Z, N) - B(Z-1, N)$$

$$S_{2p}(Z, N) \equiv B(Z, N) - B(Z-2, N)$$

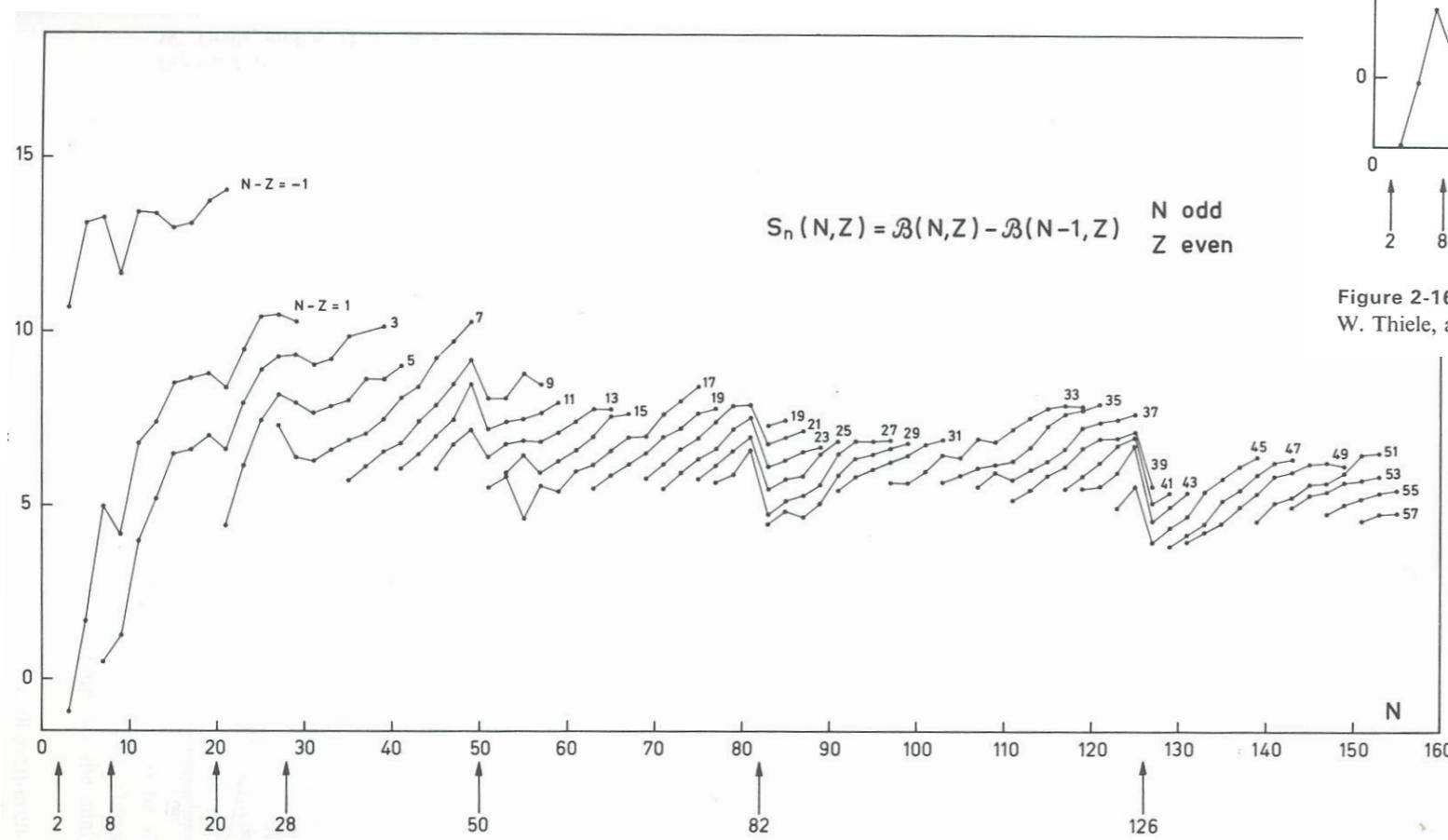


Figure 2-15 The neutron separation energies, S_n , are taken from the compilation by J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, *Nuclear Phys.* 67, 1 (1965).

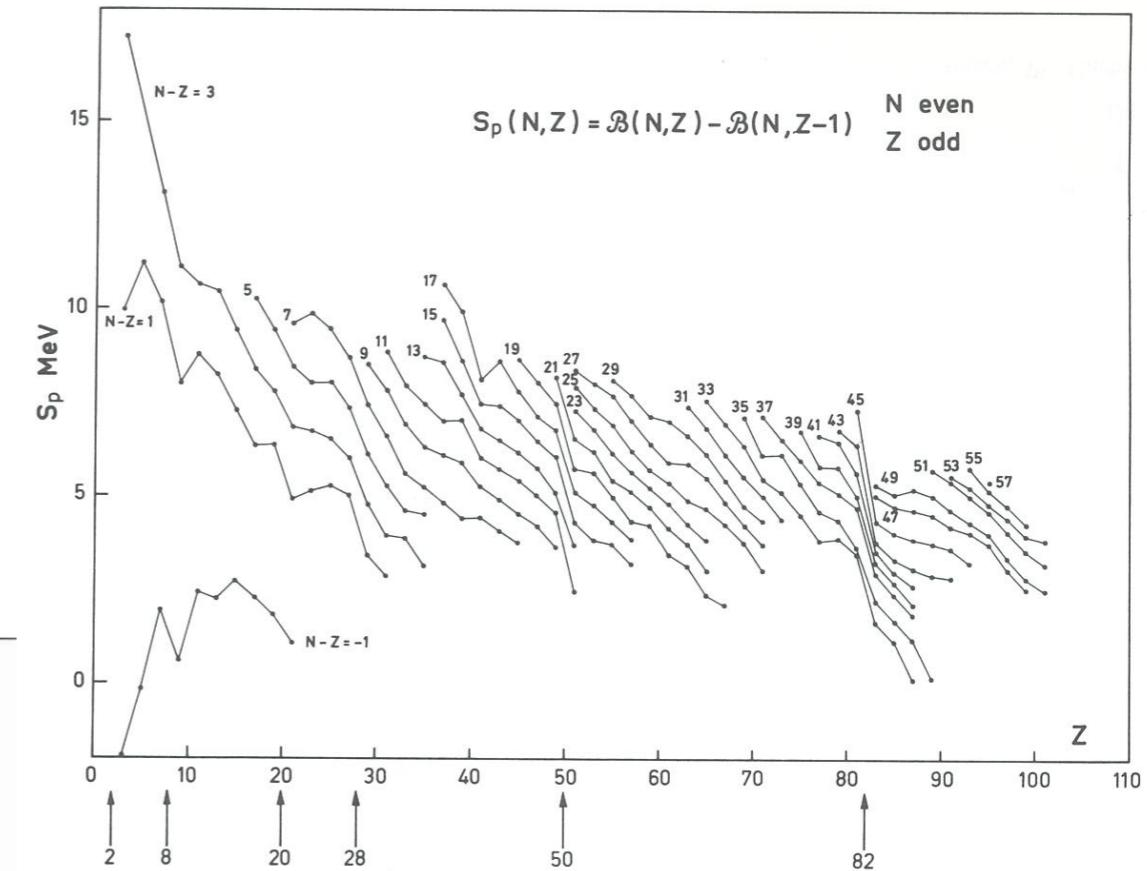
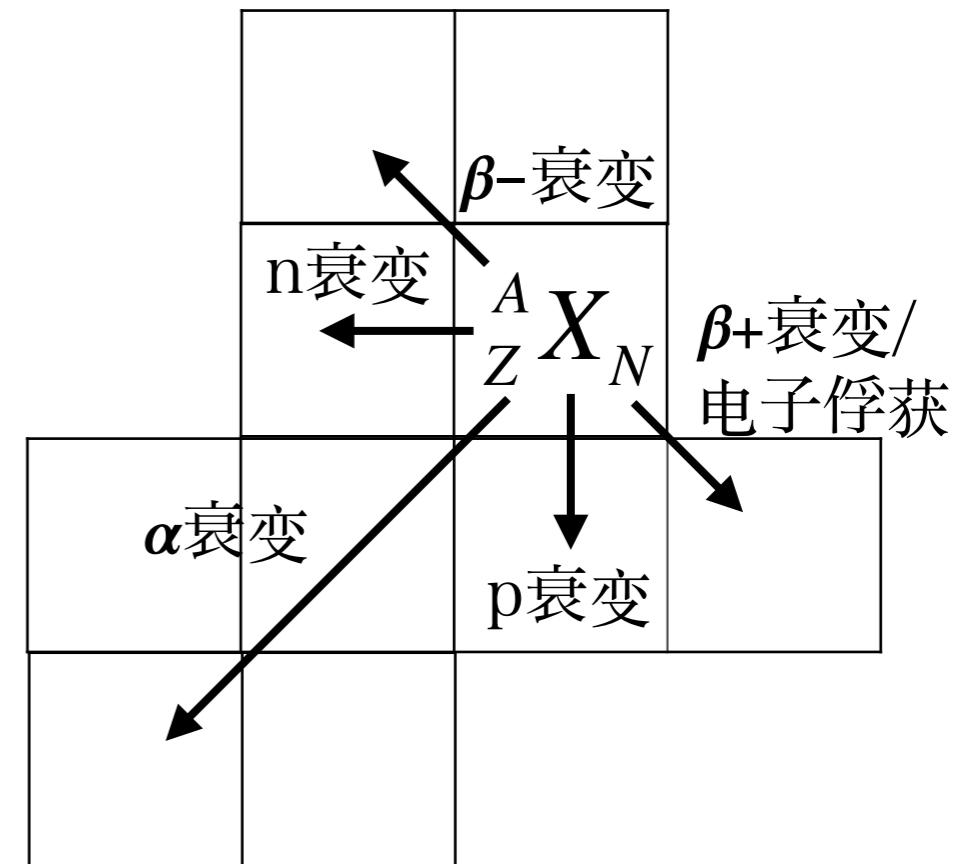


Figure 2-16 The proton separation energies, S_p , are taken from the compilation by J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, *Nuclear Phys.* 67, 1 (1965).

【习题】如何判断 ${}^7\text{Li}$ 、 ${}^7\text{Be}$ 是否稳定？

	mass excess (MeV)
${}^1\text{n}$	8.0713
${}^1\text{H}$	7.2890
${}^3\text{H}$	14.9498
${}^3\text{He}$	14.9312
${}^4\text{He}$	2.4249
${}^6\text{He}$	17.5928
${}^7\text{He}$	26.0731
${}^6\text{Li}$	14.0869
${}^7\text{Li}$	14.9071
${}^6\text{Be}$	18.3750
${}^7\text{Be}$	15.7690
${}^7\text{B}$	27.6765



结合能半经验公式

基于液滴模型，球形原子核的结合能半经验公式是

$$\begin{aligned}B(Z, A) &= B_V + B_S + B_C + B_A + B_P \\&= a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} - a_A \left(\frac{A}{2} - Z \right)^2 A^{-1} + \delta a_P A^{-1/2}\end{aligned}$$

其中的各个常量 a 是由此公式与很多原子核基态的结合能数据作最佳拟合定出的参量。虽然液滴模型给出的式中各项有明确的物理意义，但是系数 a 一般不能由理论本身导出，而依赖于实验数据。这样得到的公式是半经验公式，首先由魏茨泽克（Weizsäcker）在1935年提出。有一组参量是

$a_V = 15.835 \text{ MeV}$	$a'_V = 0.017000 \text{ u}$
$a_S = 18.33 \text{ MeV}$	$a'_S = 0.01968 \text{ u}$
$a_C = 0.714 \text{ MeV}$	$a'_C = 0.000767 \text{ u}$
$a_A = 92.80 \text{ MeV}$	$a'_A = 0.09962 \text{ u}$
$a_P = 11.2 \text{ MeV}$	$a'_P = 0.012 \text{ u}$

质量公式

由核素的结合能 $B(Z,A)$ 和质量 $m(Z,A)$ 的关系

$$m(Z, A) = Zm_p + (A - Z)m_n - B(Z, A) / c^2 ,$$

可以得到核素的原子质量

$$\begin{aligned} M(Z, A) &= ZM(^1H) + (A - Z)m_n - B(Z, A) / c^2 \\ &= ZM(^1H) + (A - Z)m_n - a'_v A + a'_s A^{2/3} \\ &\quad + a'_c Z^2 A^{-1/3} + a'_a \left(\frac{A}{2} - Z \right)^2 A^{-1} - \delta a'_p A^{-1/2} \end{aligned}$$

式中参量 $a' = a/c^2$ 。

β 稳定线

在Z-N平面上，连接具有 β 稳定性的核素的曲线，称为 **β 稳定线**。对于 $A<40$ 的原子核， β 稳定线近似为直线， $N=Z$ ，即中子数 N 与质子数 Z 之比（中质比）， $N/Z=1$ 。对于 $A>40$ 的原子核， β 稳定线的中质比 $N/Z>1$ 。对于 ^{208}Pb ，它的中质比 $N/Z=1.54$ 。

β 稳定线可用下列经验公式表示，

$$Z = \frac{A}{1.98 + 0.0155 A^{2/3}}$$

在 β 稳定线**丰中子**一侧的核素，具有 **β^- 放射性**。 β 稳定线**丰质子**一侧的核素具有**电子俘获EC**或 **β^+ 放射性**。 β 稳定线表示原子核中的核子有中子、质子对称相处的趋势，即中子数 N 和质子数 Z 相等的核素具有较大的稳定性。这种效应在轻核中很显著。对于重核，核内质子数增多，库仑排斥作用增大了，要构成稳定的原子核就需要含有更多的中子以抵消库仑排斥作用。随着质量数 A 的增大，具有 β 稳定性的核素，它的中质比也逐渐地增大。

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如何得到？

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放射性衰变的指数衰减规律

放射性核的衰变是自发的随机过程。即各个原子核的衰变是独立的，发生概率与历史无关。设 t 与 $t+\Delta t$ 时刻的核的数量为 $N(t)$ 与 $N(t+\Delta t)$ ， Δt 时间内的衰变数是

$$N(t) - N(t + \Delta t) \propto \Delta t \cdot N(t)$$

即

$$N(t) - N(t + \Delta t) = \lambda \cdot \Delta t \cdot N(t)$$

λ 是个别放射性核的衰变常数(radioactivity decay constant)，是在单位时间内每个原子核的衰变概率，它的量纲是时间的倒数。以上方程式写成微分方程，设 $t=0$ 时核的数量为 N_0 ，求积分得到

$$dN = -\lambda N dt, \quad N = N_0 e^{-\lambda t}$$

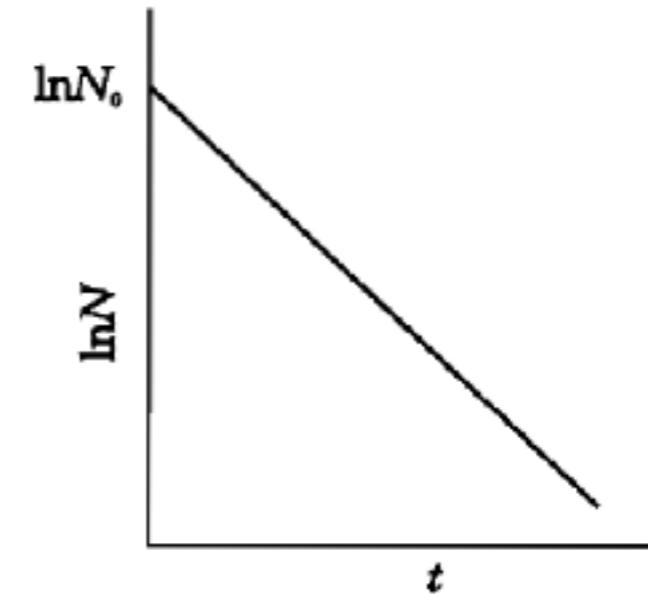
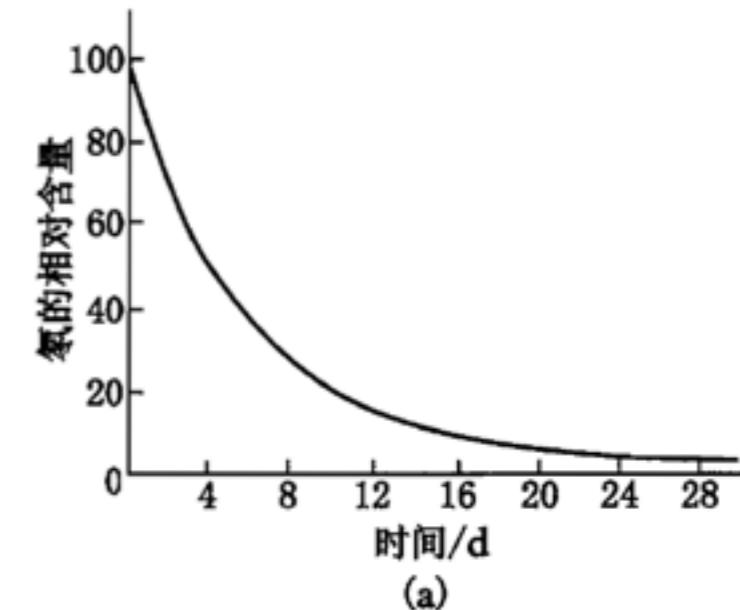


图 2-1 氚的衰变

放射性活度 (radioactivity)

$$A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$$

半衰期 $T_{1/2}$ 是放射性原子核数衰减到原来数目的一半所需的时间：

$$N = \frac{1}{2} N_0 = N_0 e^{-\lambda T_{1/2}}$$

所以

$$T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

平均寿命 τ 是指放射性原子核平均生存的时间：

$$\tau = \frac{1}{N_0} \int_0^\infty t \lambda N dt = \lambda \int_0^\infty t e^{-\lambda t} dt = \frac{1}{\lambda}$$

$$= \frac{T_{1/2}}{\ln 2} \approx \frac{T_{1/2}}{0.693}$$

平均寿命 τ 是指放射性原子核平均生存的时间：

$$\tau = \frac{\int_0^\infty tN / N_0 dt}{\int_0^\infty N / N_0 dt}$$

$$= \frac{\int_0^\infty te^{-\lambda t} dt}{\int_0^\infty e^{-\lambda t} dt}$$

$$= \lambda \int_0^\infty te^{-\lambda t} dt$$

$$= \frac{1}{\lambda}$$

$$= \frac{T_{1/2}}{\ln 2} \approx \frac{T_{1/2}}{0.693}$$

单个/少数放射性核的衰变几率

单个/少数放射性核的衰变几率

放射性核的衰变几率, $p = 1 - N / N_0 = 1 - e^{-\lambda t}$

不衰变的几率, $q = e^{-\lambda t}$

N 个($N \gg 1$)放射性核中 m 个在某个瞬间($\lambda t \ll 1$)发生衰变的几率:

$$P(m) = \frac{N!}{m!(N-m)!} (1 - e^{-\lambda t})^m (e^{-\lambda t})^{N-m}$$

由于 $N \gg 1$, $\lambda t \ll 1$, 有 $N - m \approx N$,

$$1 - e^{-\lambda t} \approx 1 - [1 - (-\lambda t) + \dots] = \lambda t$$

如果 $M \equiv N\lambda t$ 是有限值, 利用斯特灵(Stirling)公式

$$n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n,$$

$$\begin{aligned} P(m) &\approx \frac{(N/e)^N \sqrt{2\pi N}}{m! [(N-m)/e]^{N-m} \sqrt{2\pi(N-m)}} (1 - e^{-\lambda t})^m (e^{-\lambda t})^{N-m} \\ &\approx \frac{(N/e)^N}{m! [N/e]^{N-m}} (\lambda t)^m (e^{-\lambda t})^N = \frac{N^N e^N}{m! N^{N-m} e^{N-m} N^m} (N\lambda t)^m (e^{-\lambda t})^N \\ &\approx \frac{M^m}{m!} (e^{-M}) \end{aligned}$$

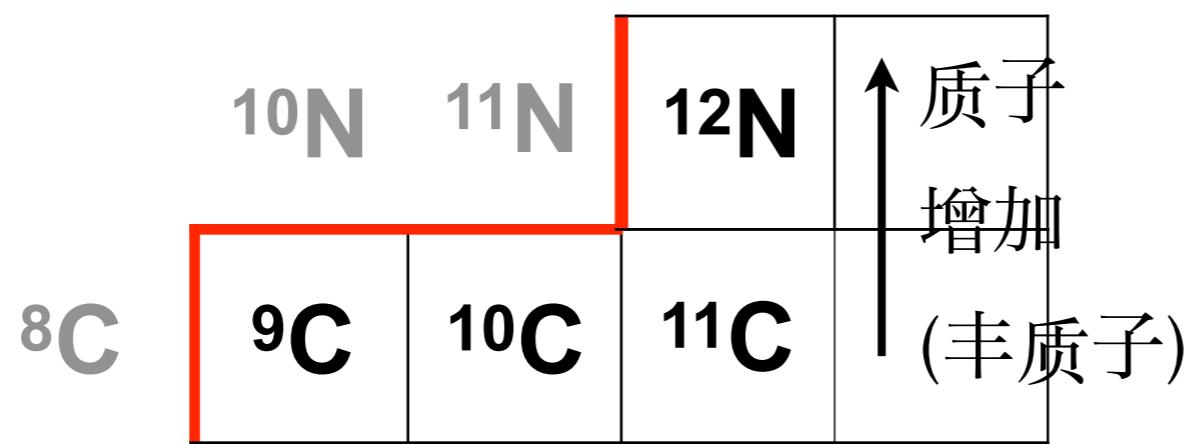
泊松分布(Poisson distribution)

不稳定核的存在极限

- 对于不稳定核，目前还没有严格的定义。这里我们把它定义为可以组成原子并进行化学反应—半衰期 10^{-6} s 以上的原子核。
- 中子滴线(dripline)

					21N	22N	23N
13C		→ 中子增加(丰中子)			20C	21C	22C

- 质子滴线



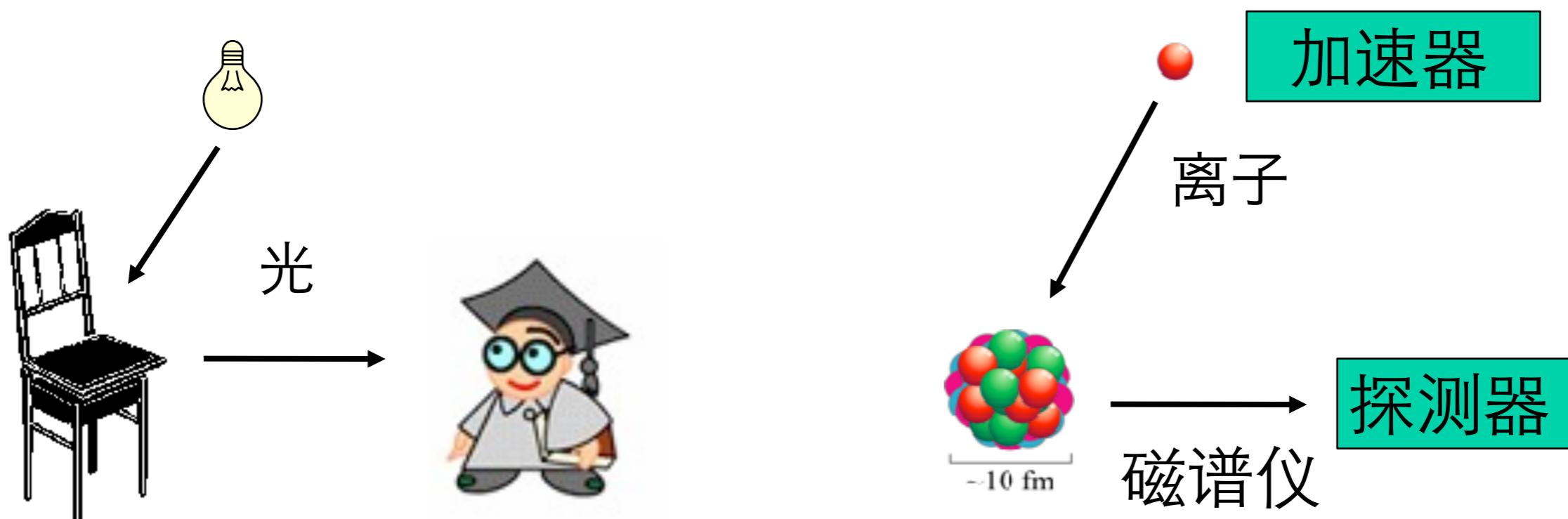
质子衰变的寿命并非都
 $<10^{-6}$ s。例： ^{45}Fe , ^{48}Ni 的
2质子衰变寿命 $\sim\text{ms}$ ；最近
十年的重大发现

- 超重元素：例) ^{113}Nh , ^{118}Og

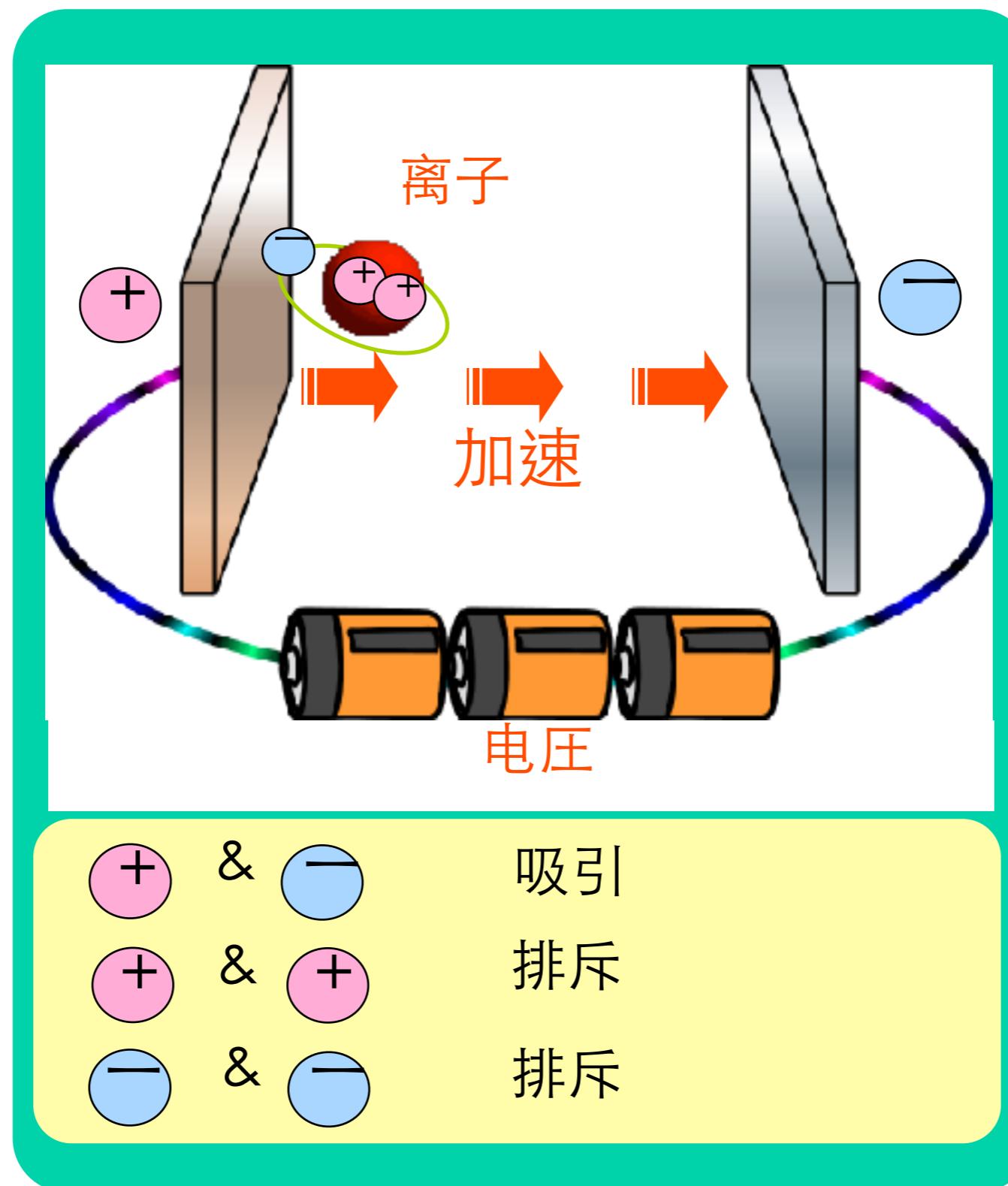
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核物理实验

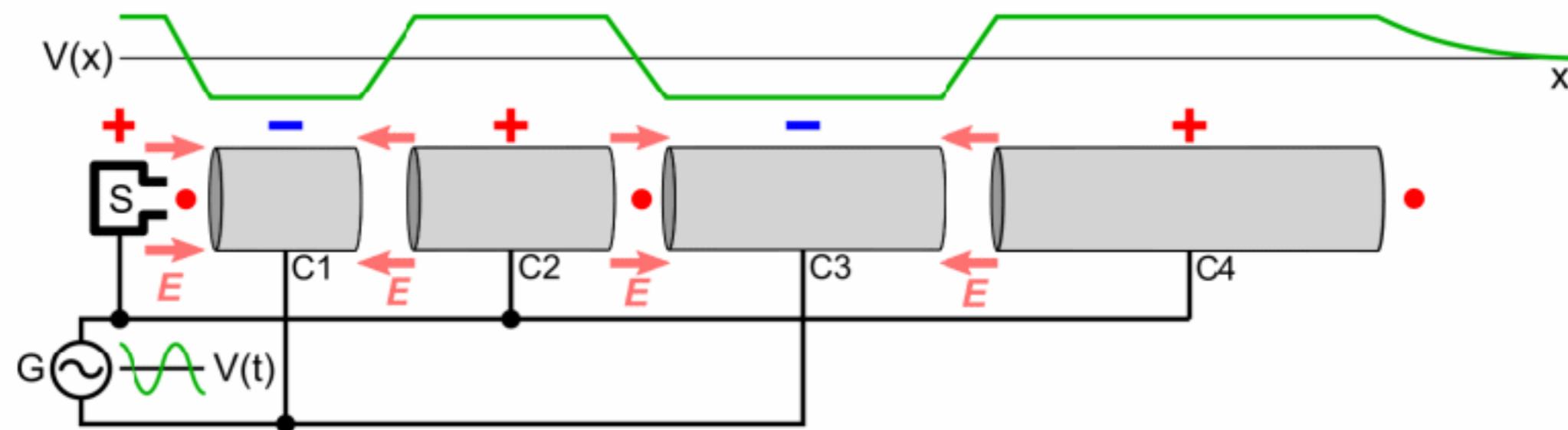


核物理实验：粒子加速器



直线加速器

通过一系列直线排列的射频 (RF) 腔体或波导结构，利用电磁场对粒子进行加速

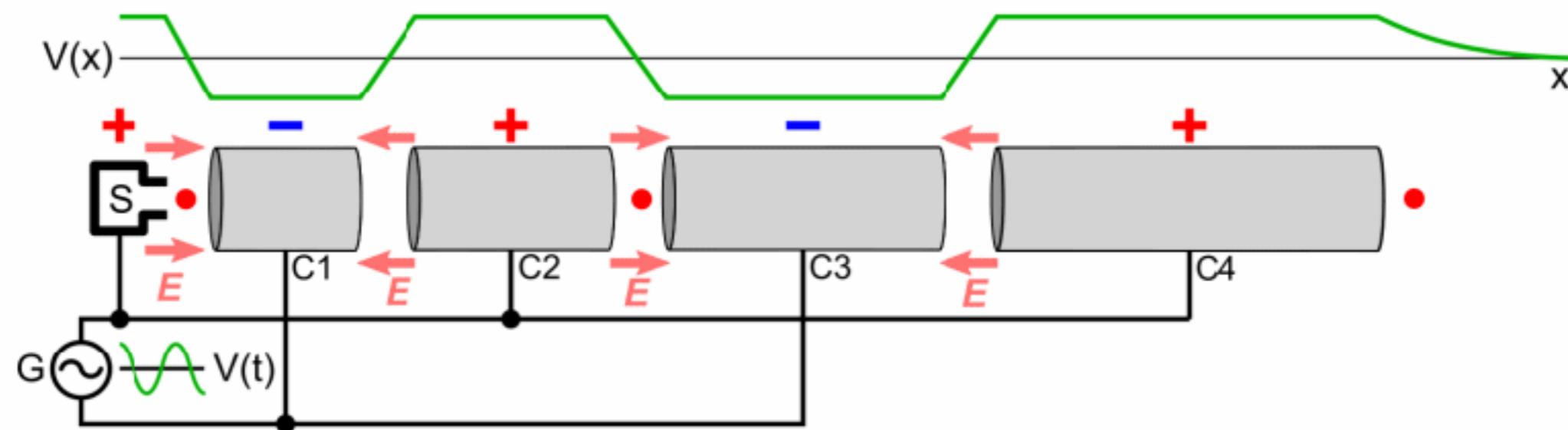


Courtesy of Chetvorno

<https://commons.wikimedia.org/w/index.php?curid=68477306>

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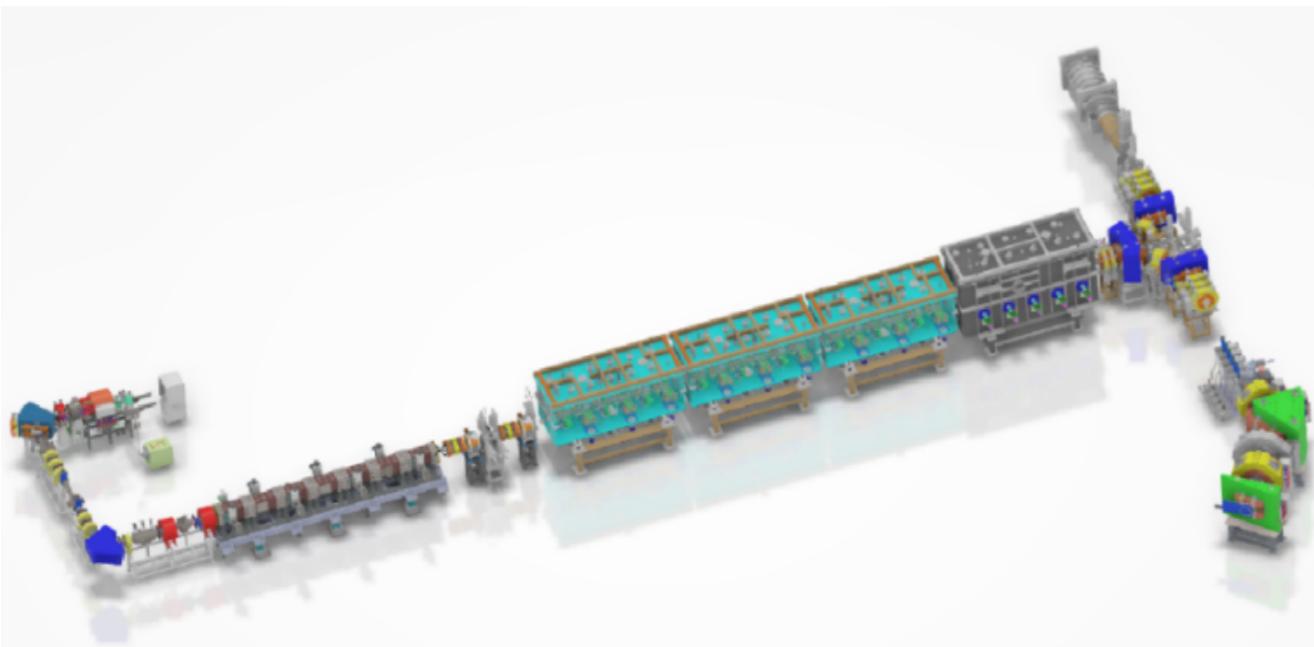


Courtesy of Chetvorno

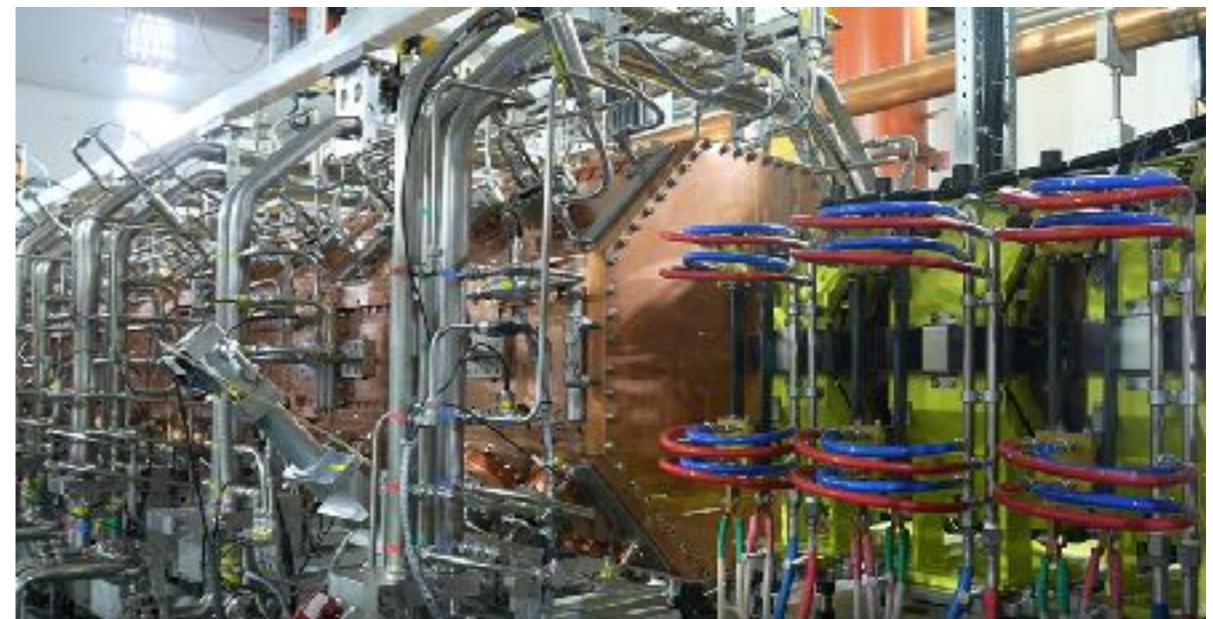
<https://commons.wikimedia.org/w/index.php?curid=68477306>

直线加速器举例

兰州重离子加速器国家实验室
强流超导直线加速器CAFé



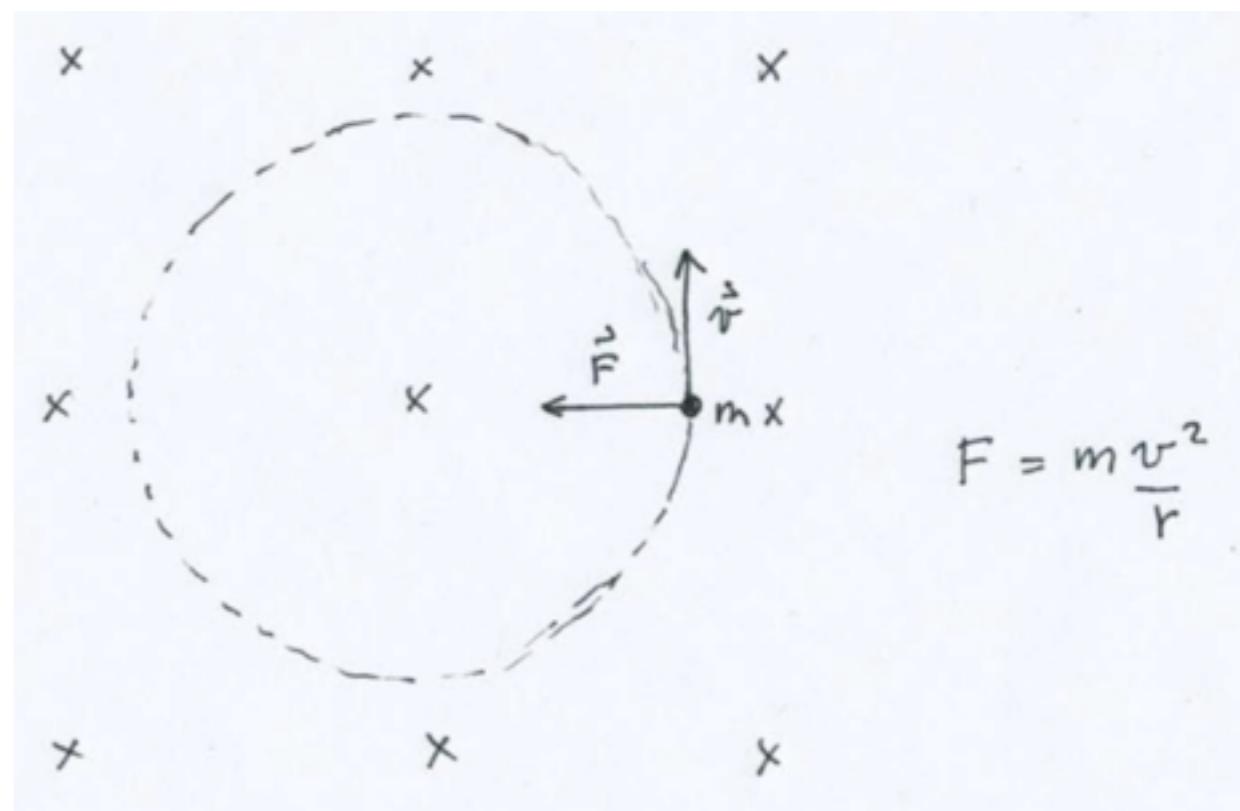
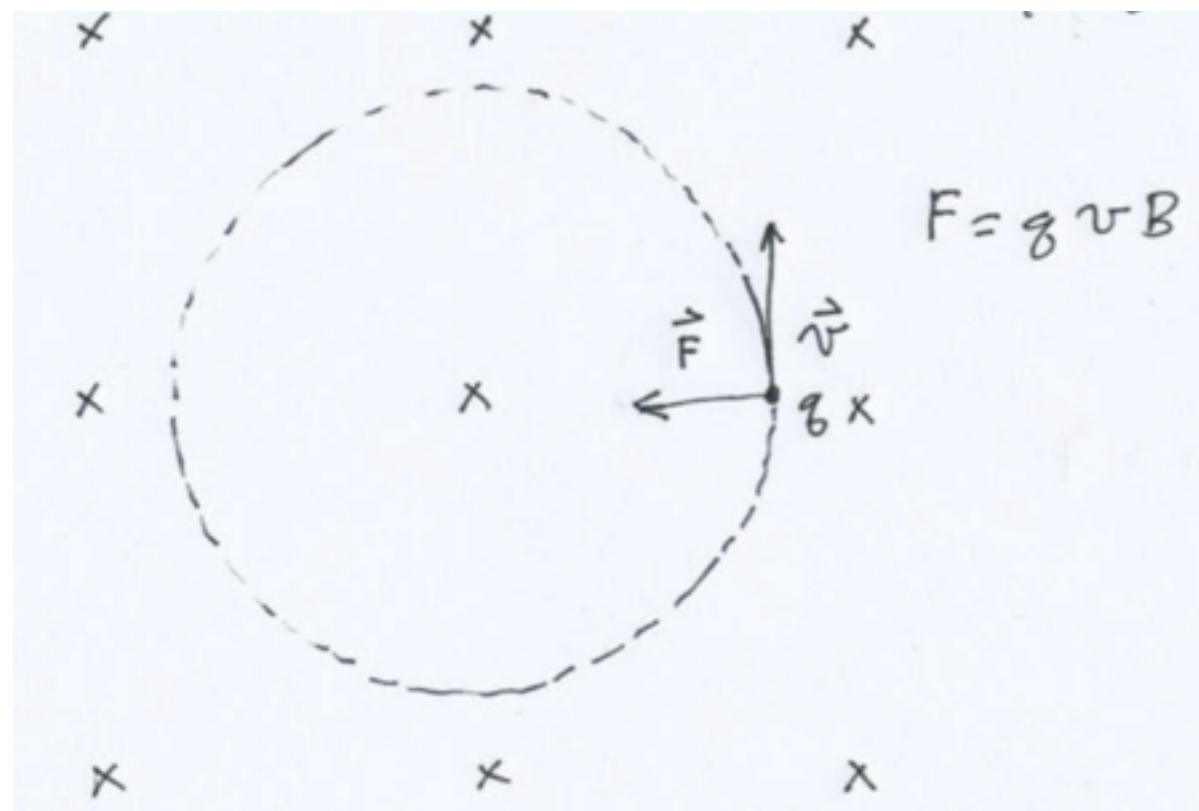
兰州重离子加速器国家实验室
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回旋加速器

带电粒子在垂直均匀磁场中的运动：

$$\frac{mv^2}{r} \hat{r} = q\vec{v} \times \vec{B}$$

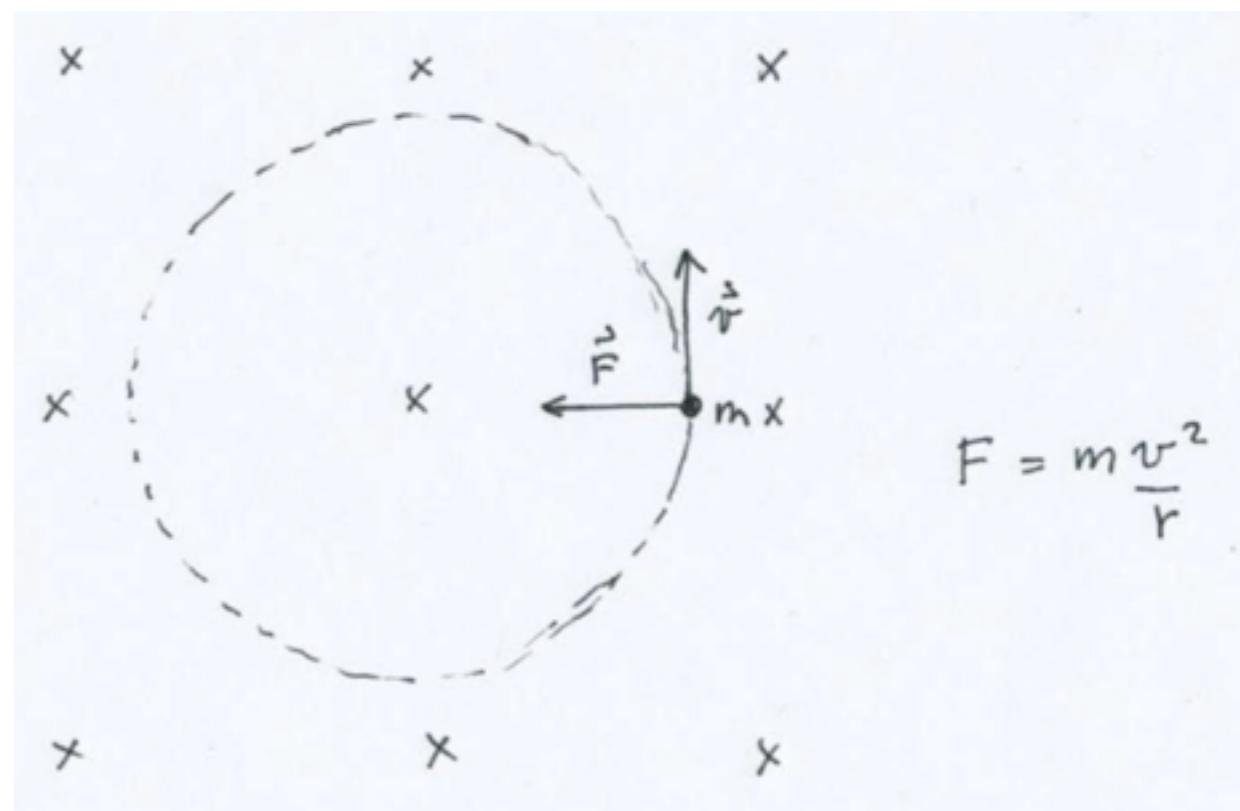
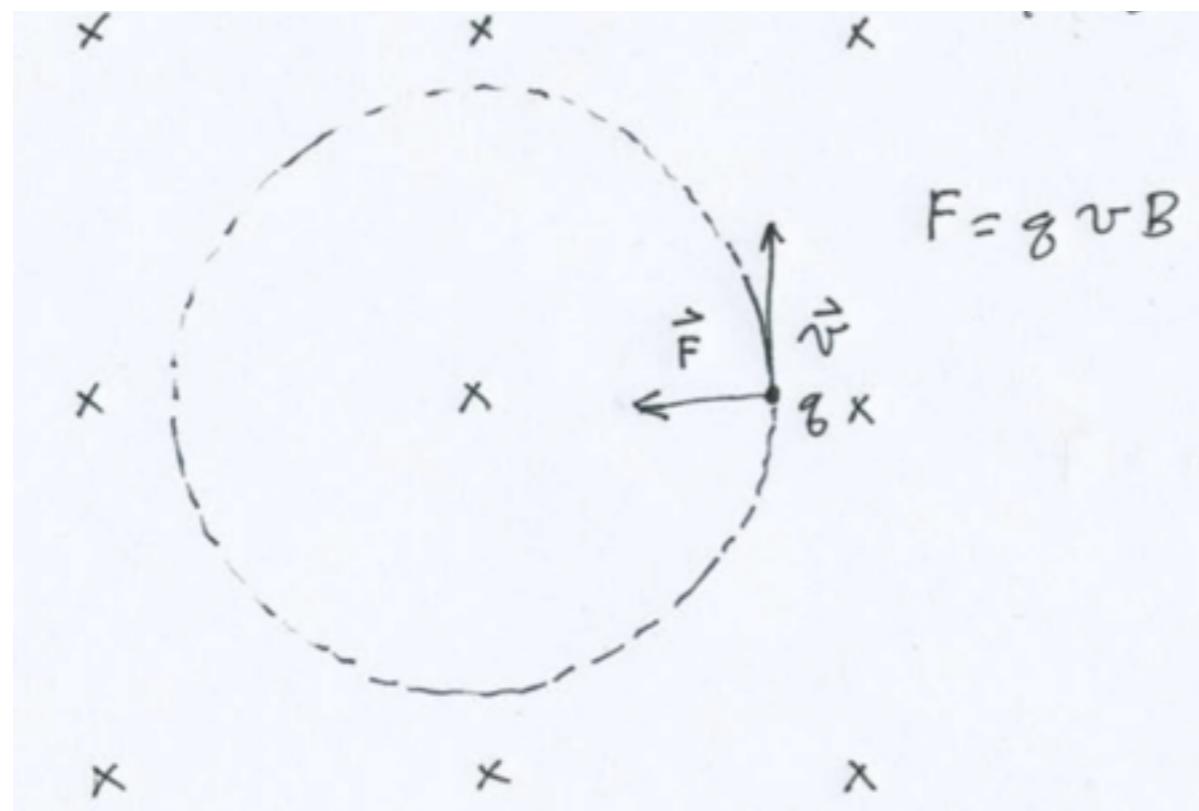


$$mv^2/r = qvB \longrightarrow \omega = v/r = qB/m$$

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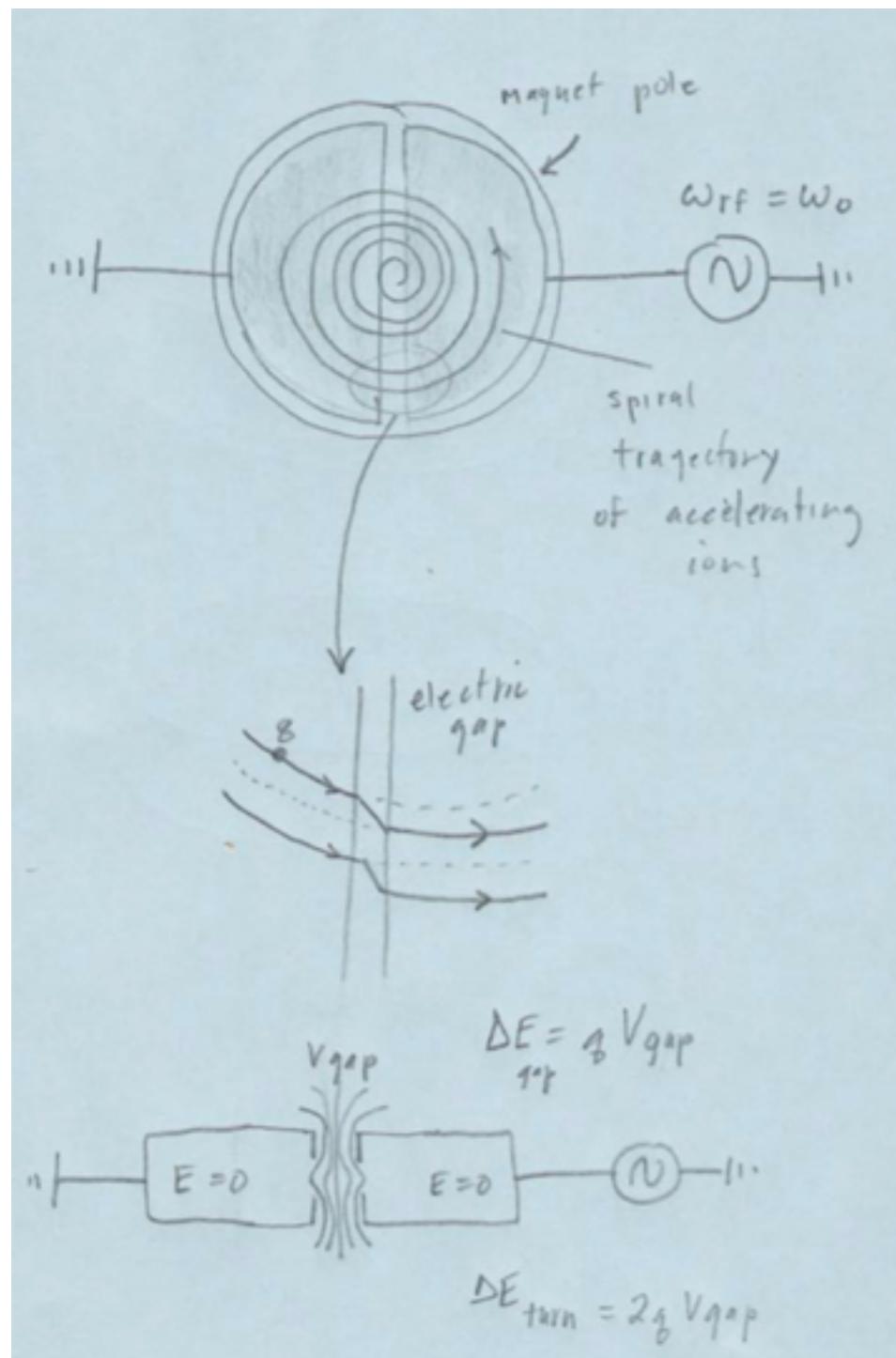
$$\frac{mv^2}{r} \hat{r} = q\vec{v} \times \vec{B}$$



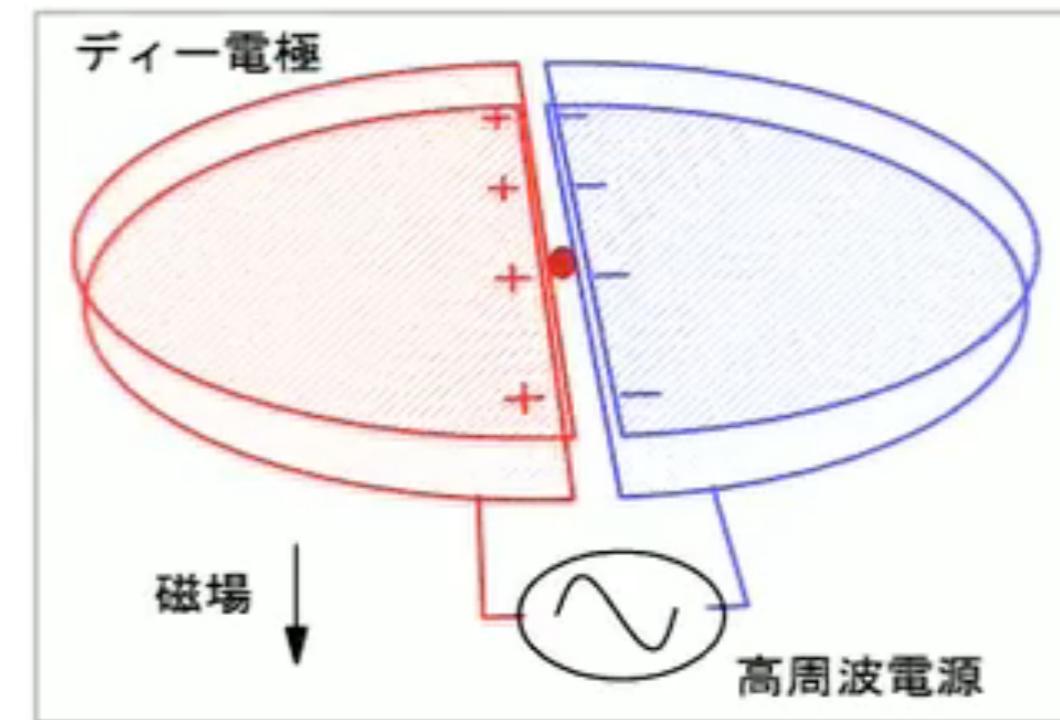
$$mv^2/r = qvB \longrightarrow \omega = v/r = qB/m$$

ω是固定值吗？

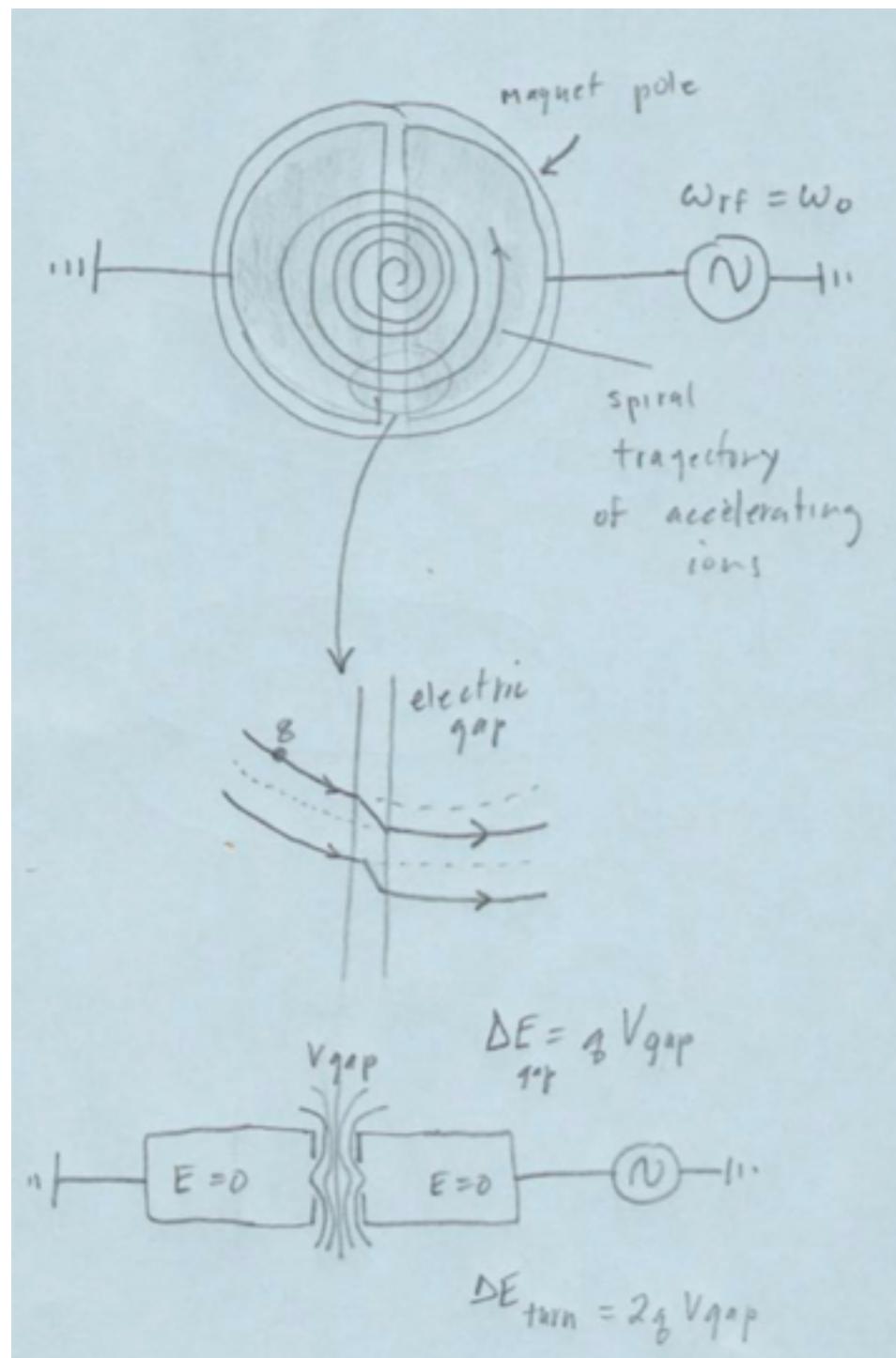
回旋加速器



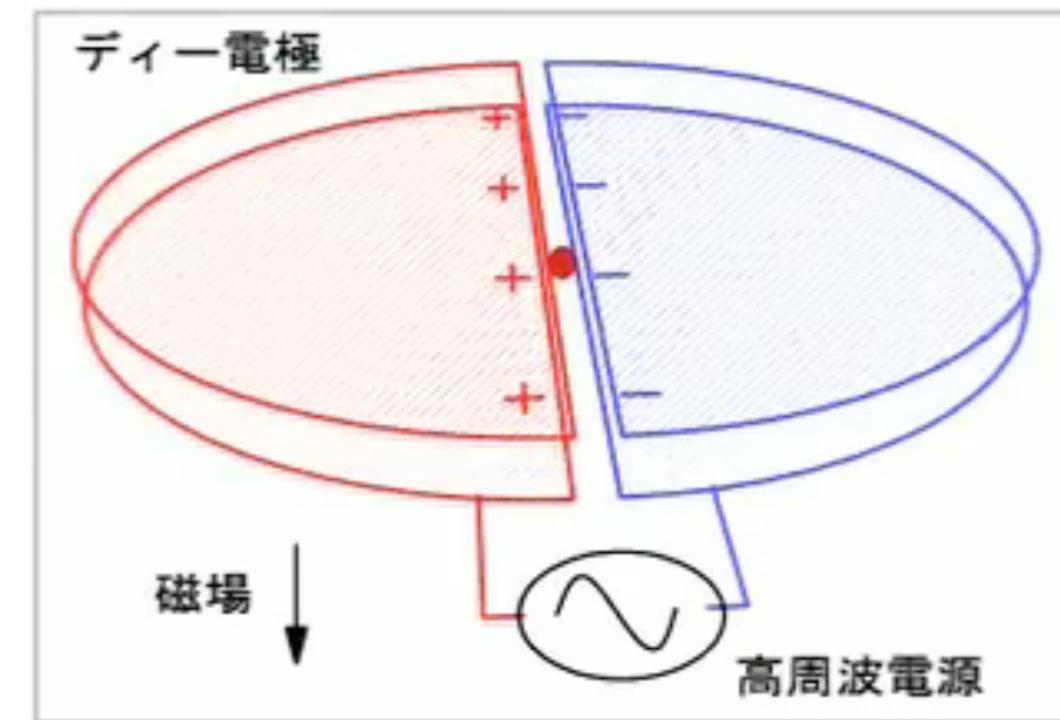
加速器内部有两个D形电极（称为“Dee”），当粒子通过D形电极之间的间隙时，会受到电场的作用而加速。高频电场的频率与粒子回旋运动的频率同步，确保粒子每次通过间隙时都能获得能量。



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回旋加速器

利用回旋加速器可以得到的每核子最大动能为

$$\frac{T}{A} = \frac{(eBr_{\max})^2}{2m_a} \left(\frac{Q}{A}\right)^2 = K \left(\frac{Q}{A}\right)^2$$

e : 元电荷

B : 磁场强度

r_{\max} : 回旋运动的最大半径

m_a : 原子质量单位

Q : 电荷

A : 质量数

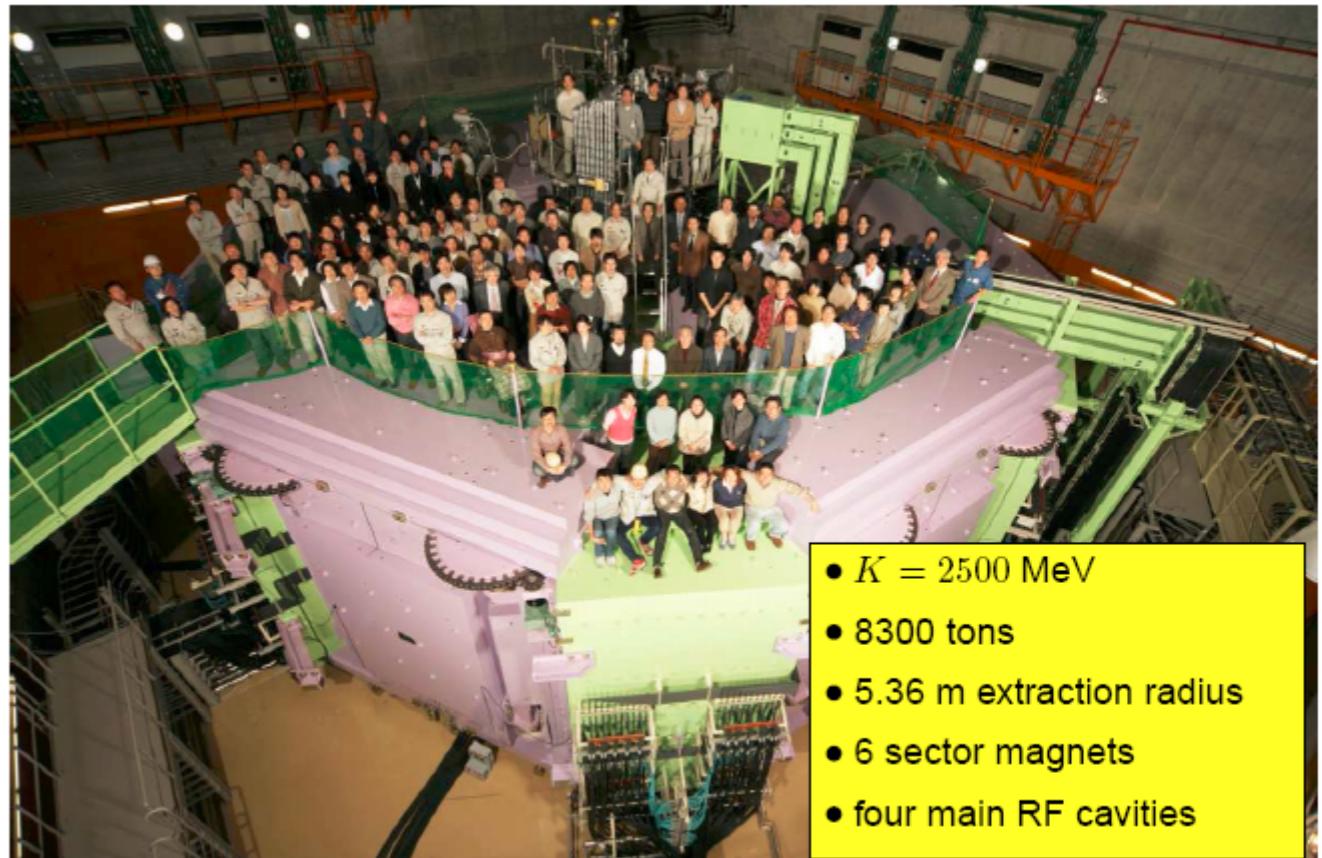
$$K = \frac{(eBr_{\max})^2}{2m_a} \rightarrow \boxed{\text{K值}}$$

回旋加速器举例

兰州重离子加速器国家实验室
分离扇形回旋加速器SSC



日本理化研究所
超导回旋加速器SRC

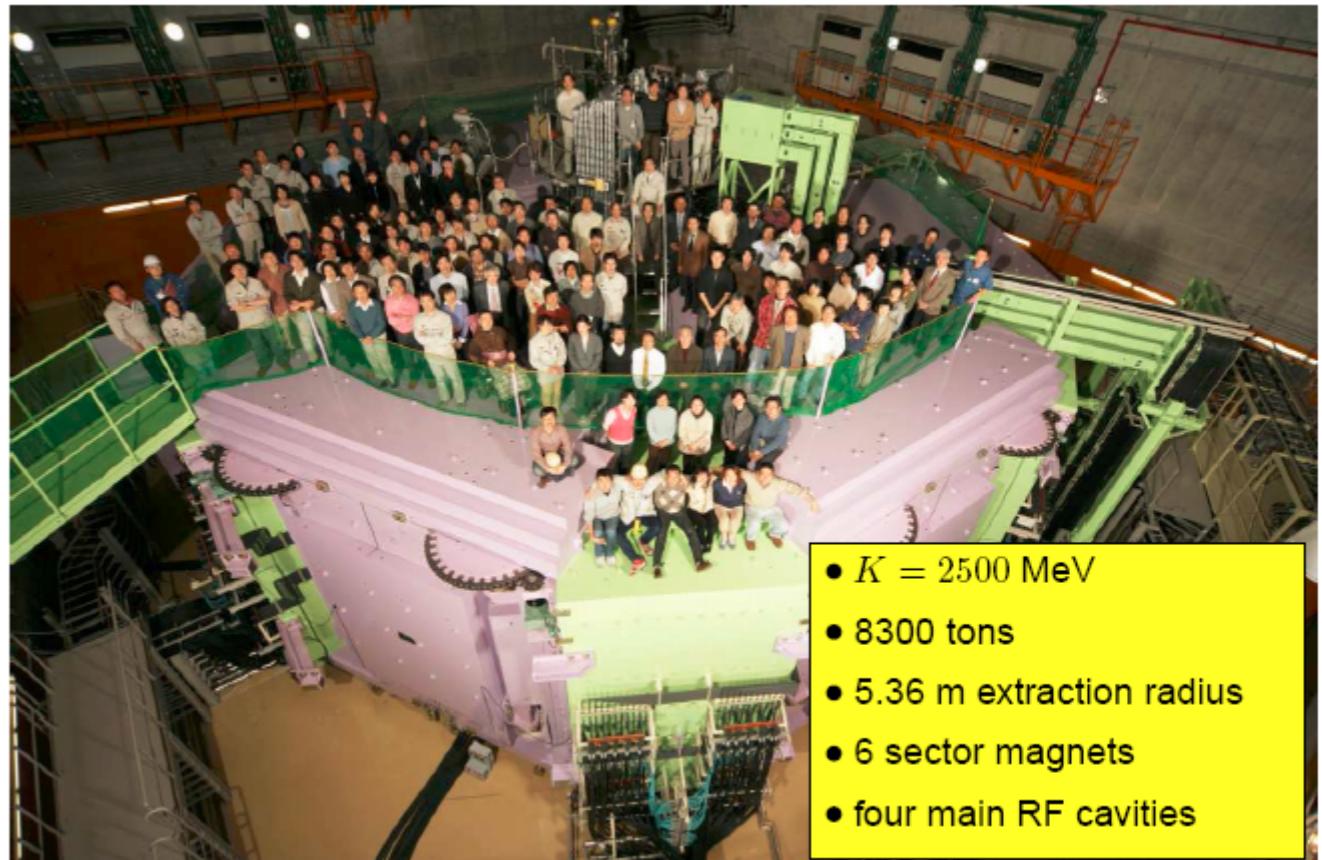


回旋加速器举例

兰州重离子加速器国家实验室
分离扇形回旋加速器SSC



日本理化研究所
超导回旋加速器SRC

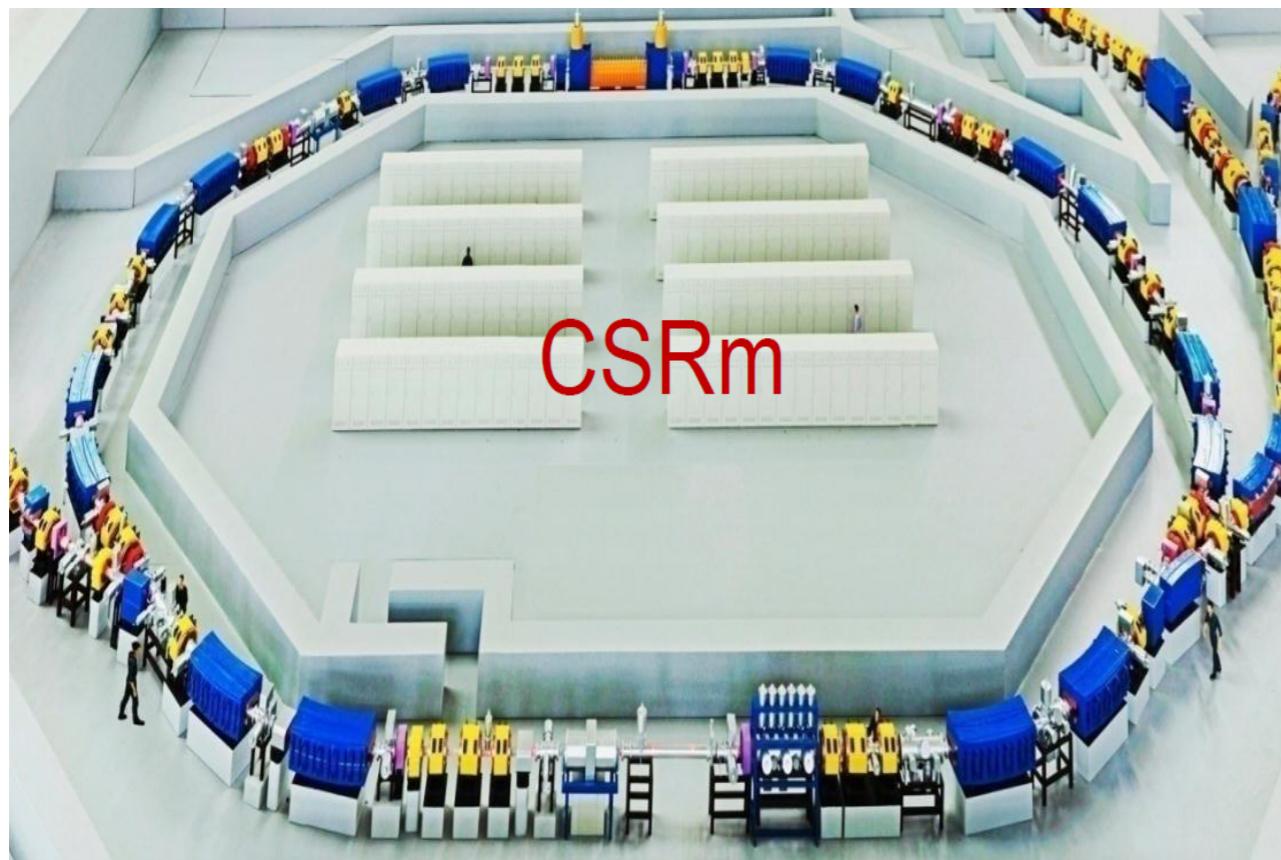


K值?

其他加速器举例

$$mv^2/r = qvB \longrightarrow \omega = v/r = qB/m$$

兰州重离子加速器国家实验室
同步加速器CSRm

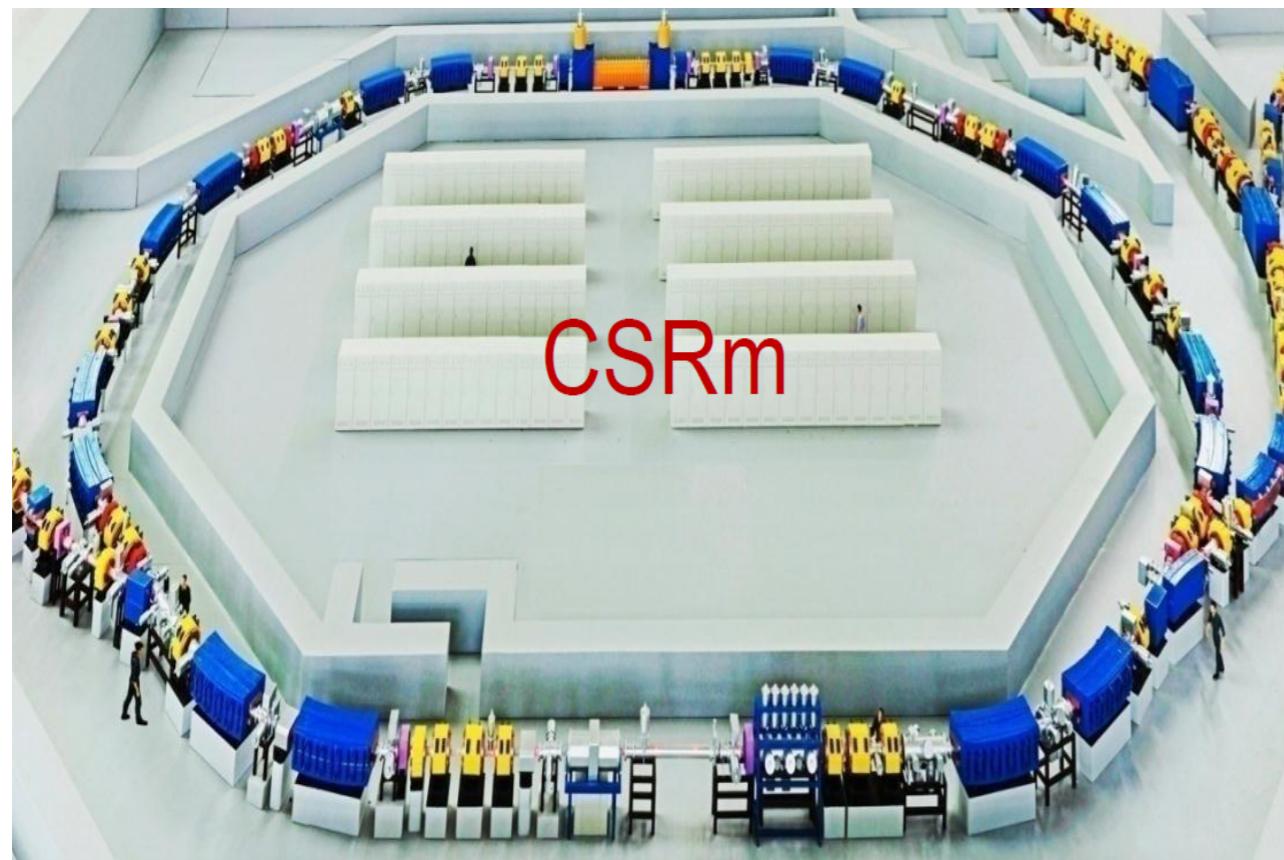


等时性加速器的原理?

其他加速器举例

$$mv^2/r = qvB \longrightarrow \omega = v/r = qB/m$$

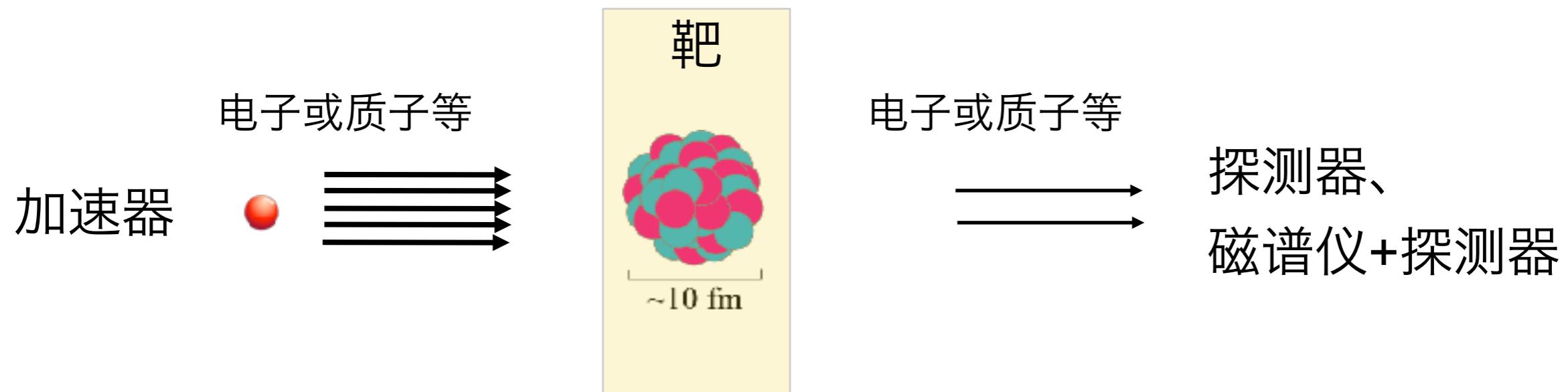
兰州重离子加速器国家实验室
同步加速器CSRm



原理？

等时性加速器的原理？

稳定核的物理实验



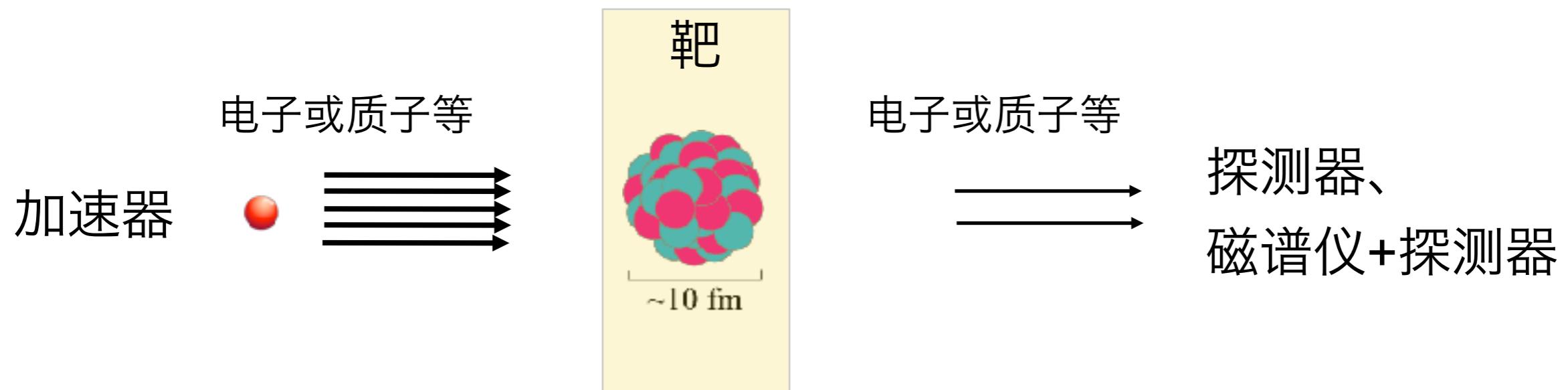
de Broglie 波长

$$\lambda = \frac{\hbar}{p} = \frac{197 \text{ MeV fm}}{pc}$$

e.g.) γ ray, 20 MeV : $\lambda \sim 10 \text{ fm}$
proton, 300 MeV: $\lambda \sim 0.2 \text{ fm}$

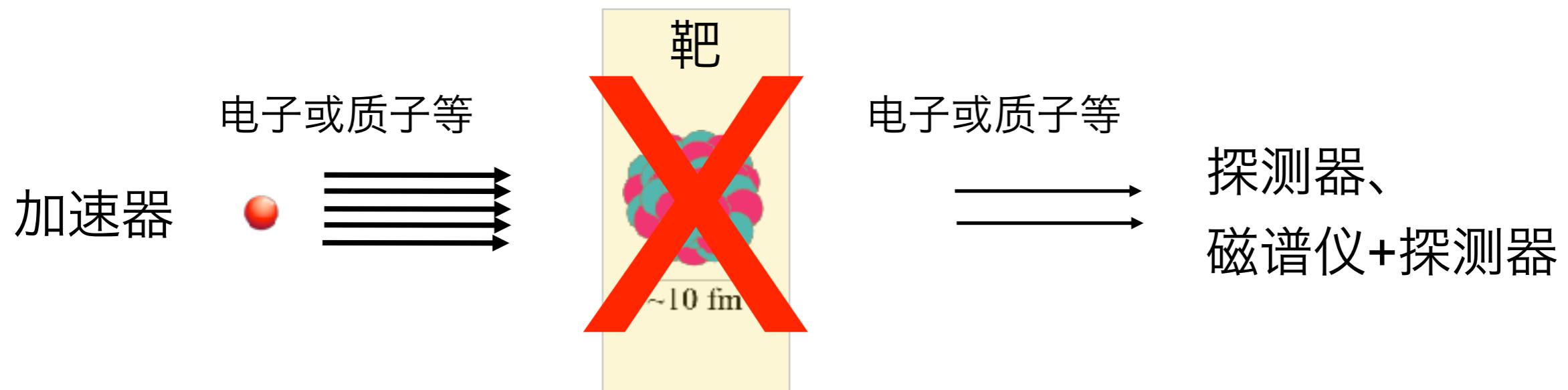
不稳定核的物理实验

放射性核的寿命都比较短: ms 量级



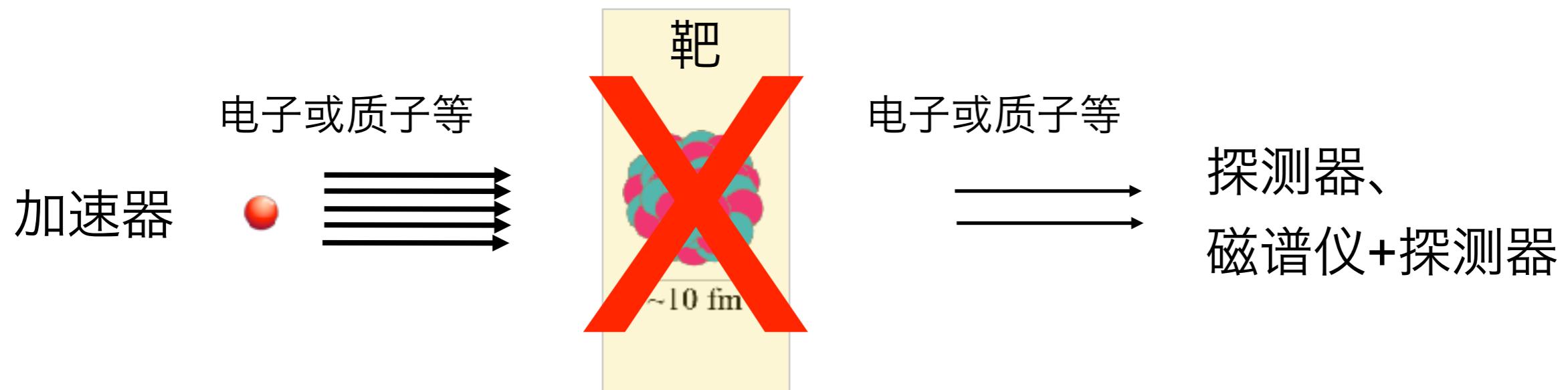
不稳定核的物理实验

放射性核的寿命都比较短: ms 量级



不稳定核的物理实验

放射性核的寿命都比较短: ms 量级



问：如何开展不稳定核的实验研究？

答：人工制造后立刻使用。

问：具体怎么做呢？

最早的放射性束物理实验

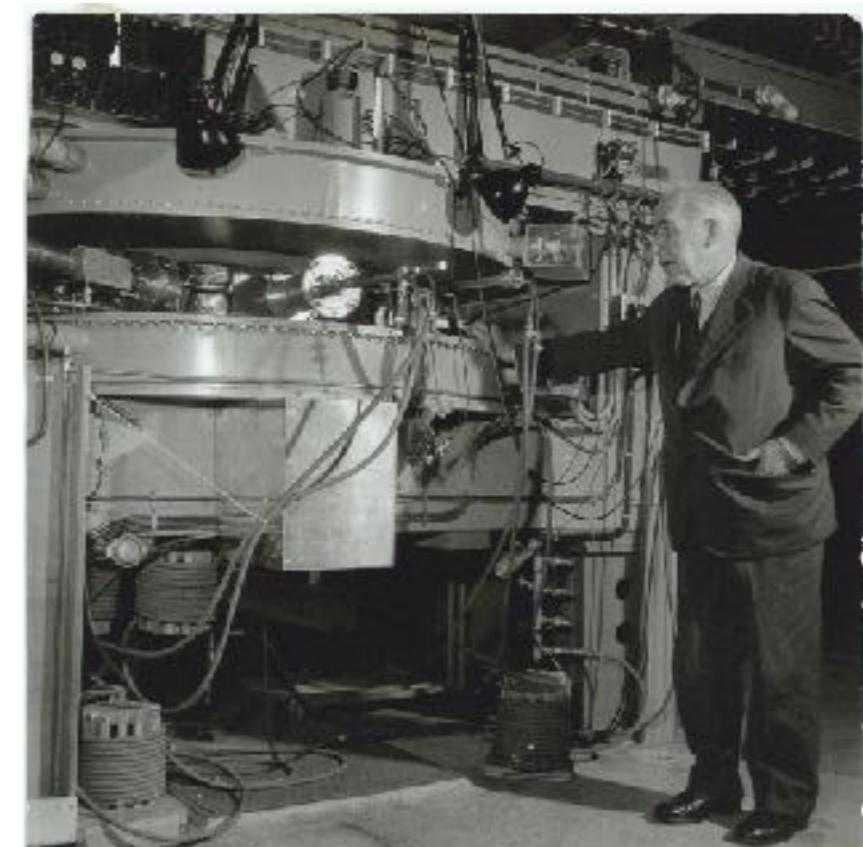
Short-Lived Krypton Isotopes and Their Daughter Substances

O. KOFOED-HANSEN AND K. O. NIELSEN

*Institute for Theoretical Physics, University of Copenhagen,
Copenhagen, Denmark*

(Received February 9, 1951)

THE isotopes Kr⁸⁹, Kr⁹⁰, Kr⁹¹, and their daughter substances have been investigated. Krypton formed in fission of uranium was pumped through a 10-m long tube directly from the cyclotron into the ion source of the isotope separator. The cyclotron and the isotope separator were operated simultaneously, and the counting could begin immediately after the interruption of the separation. The rubidium and strontium daughter substances were separated chemically; strontium was precipitated as carbonate. Half-lives were measured and an absorption analysis of the radiations was carried out. The results are given in Table I.



Physical Review 82 (1951) 96

早期的放射性束物理实验

在线同位素分离

Isotope Separation On-Line (ISOL) (1968年)

ISOTOPIC DISTRIBUTION OF SODIUM FRAGMENTS
EMITTED IN HIGH-ENERGY NUCLEAR REACTIONS.
IDENTIFICATION OF ^{27}Na AND POSSIBLE EXISTENCE OF HEAVIER Na ISOTOPES

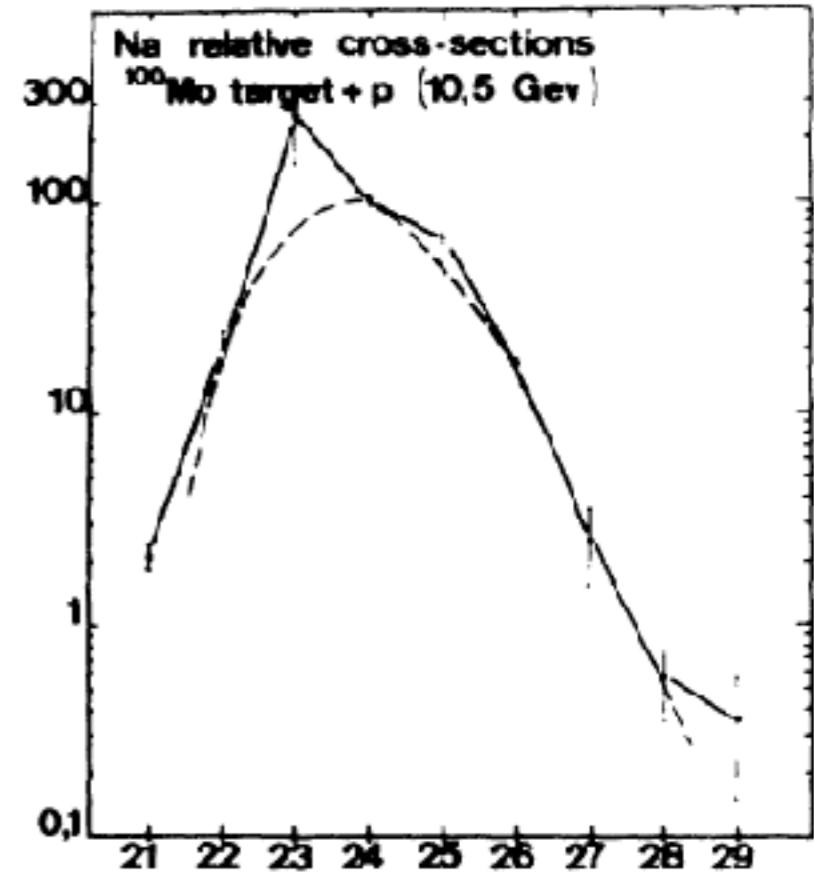
R. Klapisch, C. Philippe, J. Suchorzewska,* C. Detraz, and R. Bernas

Institut de Physique Nucléaire and Centre de Spectrométrie Nucléaire
et de Spectrométrie de Masse, Orsay, France

(Received 29 January 1968)

CERN 的 ISOLDE (Isotope Separation On-Line DEtector)

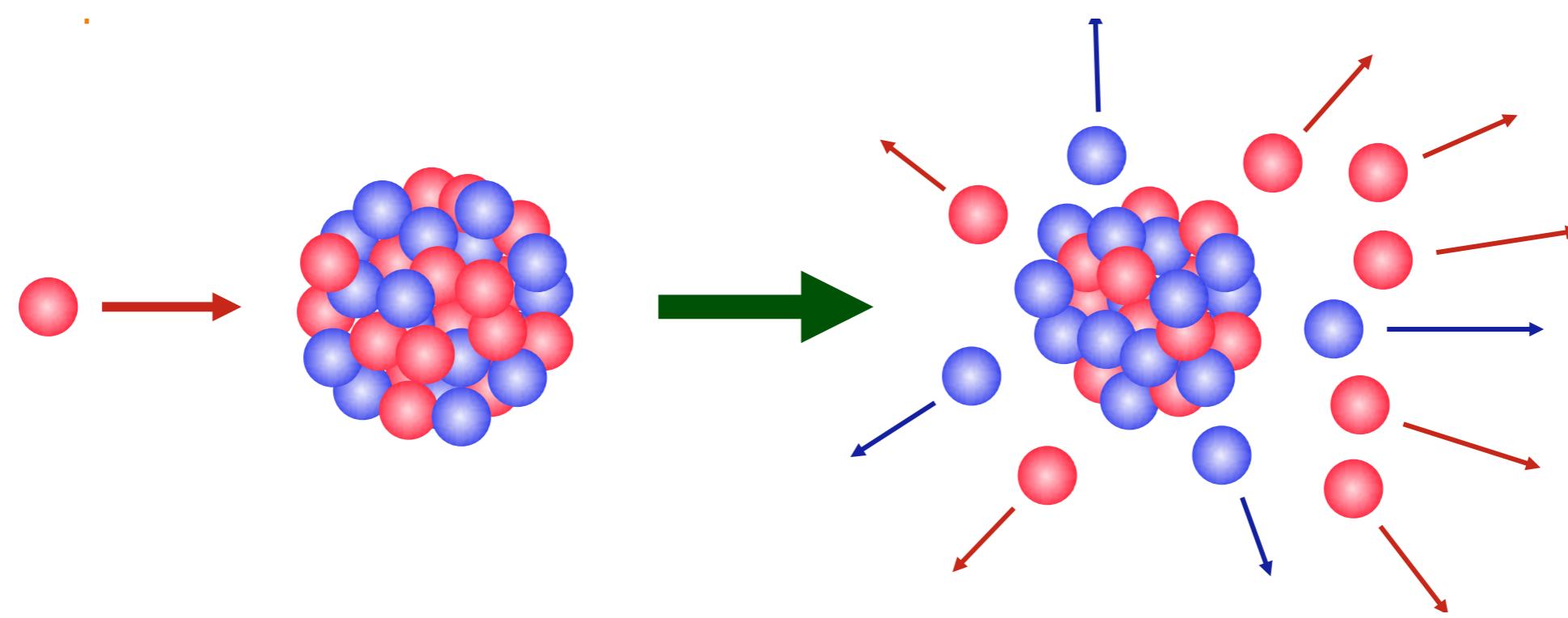
Phys. Rev. Lett. 20 (1968) 740



ISOL的方法

1. 利用高能轻粒子(如：1 GeV 的质子)打重靶

2. 轻粒子在重靶中引起散裂 (spallation)反应，导致重靶核丢失部分的质子和中子或引发核裂变

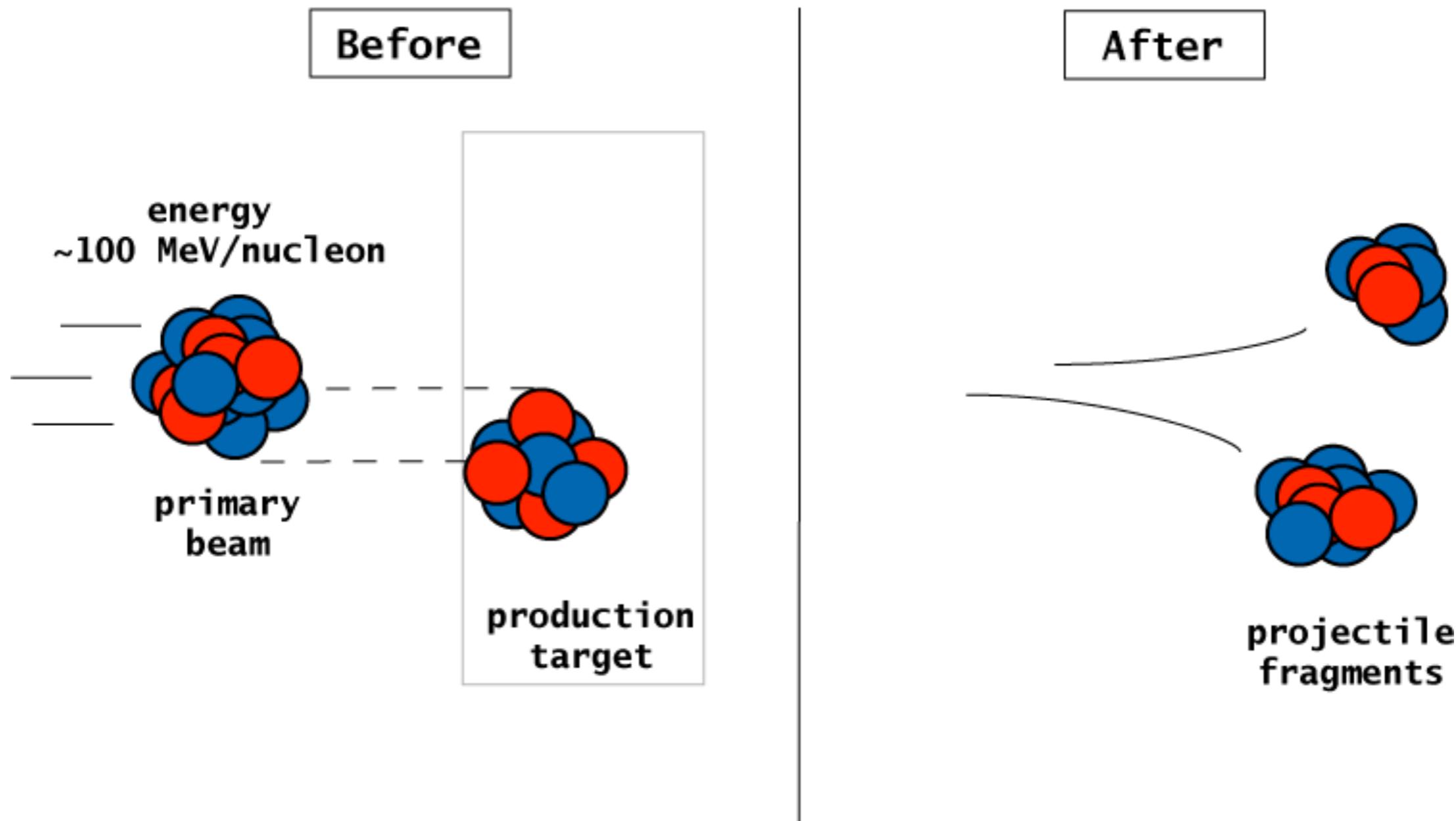


Courtesy of M. Thoennessen

3. 通过加热方式让核反应产物在靶介质中热化、扩散，把核反应产物从靶介质中吸附出来。随后立即输送到离子源内，被电离成离子后提取成束流，经同位素分离后再加速到所需能量。

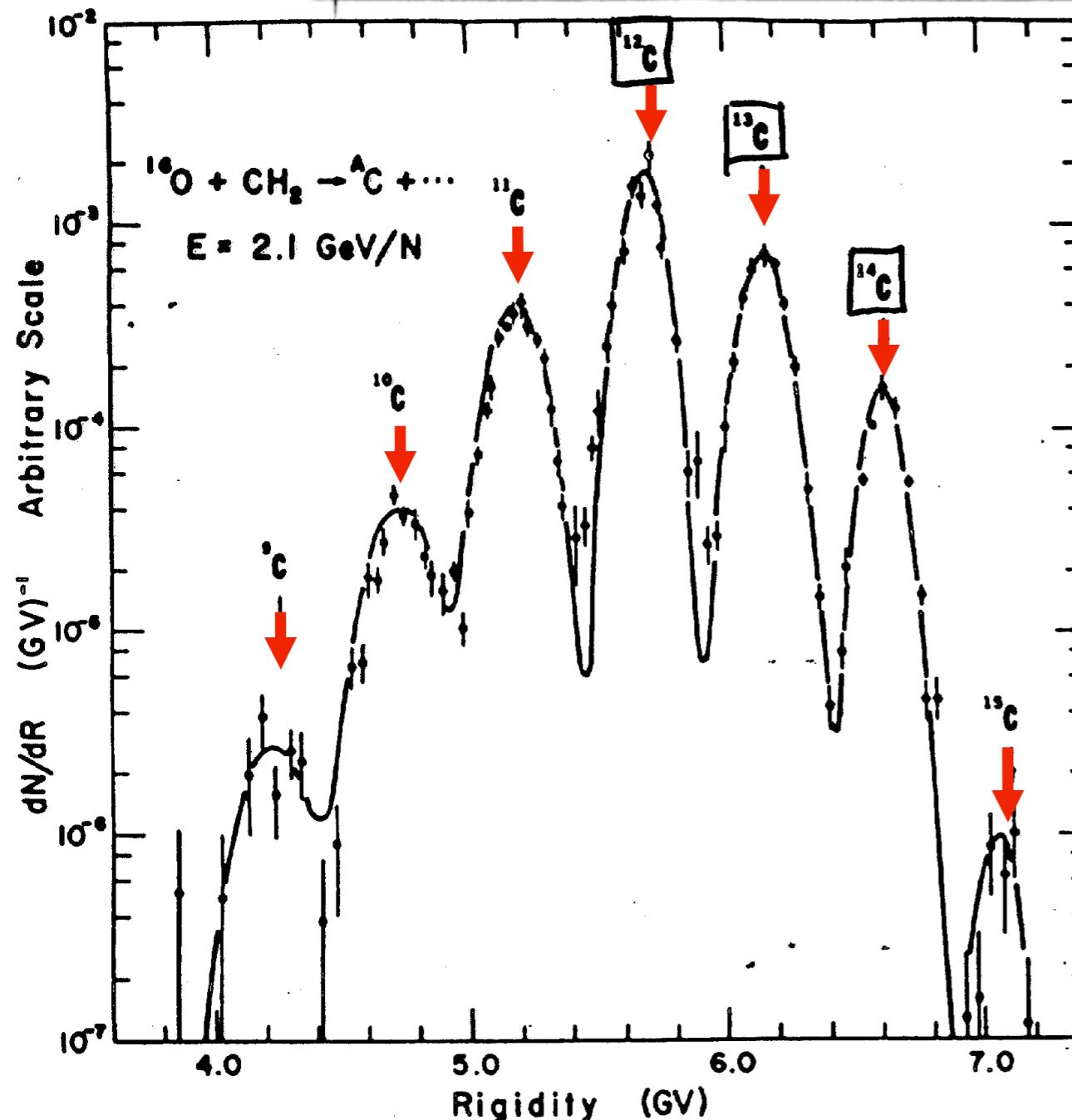
弹核碎裂反应

Production of Radioactive Nuclear Beam
via Projectile Fragmentation Reaction



原子核物理实验的革命

利用弹核碎裂反应产生放射性束流



(1970年代)
Heckman、Greiner 等
于美国劳伦斯伯克利国家实验室

弹核: ${}^{16}\text{O}^{8+}$
 $E_K = 2.1 \text{ GeV/nucleon}$
 $p \sim 2.9 \text{ GeV/c/nucleon}$

碎裂核的動量：
2.8 GeV/c/nucleon

$$p = P/A \sim \text{const.}$$

Figure 5 Rigidity spectrum of carbon isotopes produced by the fragmentation of ${}^{16}\text{O}$ projectiles at 2.1 A GeV . Arrows indicate the rigidities for each isotope evaluated at beam velocity (87).

碎裂核的動量分布

Goldhaber 模型 (1974)

(矢量) $P_A = \sum_{i=1}^A P_i = 0 \quad P_A^2 = 0$  K nucleons $\langle P_K^2 \rangle ?$

$$A\langle P^2 \rangle + \sum_{i \neq j} \langle P_i \cdot P_j \rangle = 0$$

$$\sum_{i \neq j} \langle P_i \cdot P_j \rangle = A(A-1) \langle \langle P_i \cdot P_j \rangle \rangle$$

$$\therefore \langle \langle P_i \cdot P_j \rangle \rangle = -\langle P^2 \rangle / (A-1)$$

i ≠ j 项的平均
值

$$\langle P_K^2 \rangle = \left\langle \left\langle \left(\sum_{i=1}^K P_i \right)^2 \right\rangle \right\rangle = \frac{K(A-K)}{(A-1)} \langle P^2 \rangle$$

3 dimensional Gaussian

$$\sigma_0^2 = \langle P^2 \rangle / 3$$

组成 K 核子的
所有可能组合
的平均值

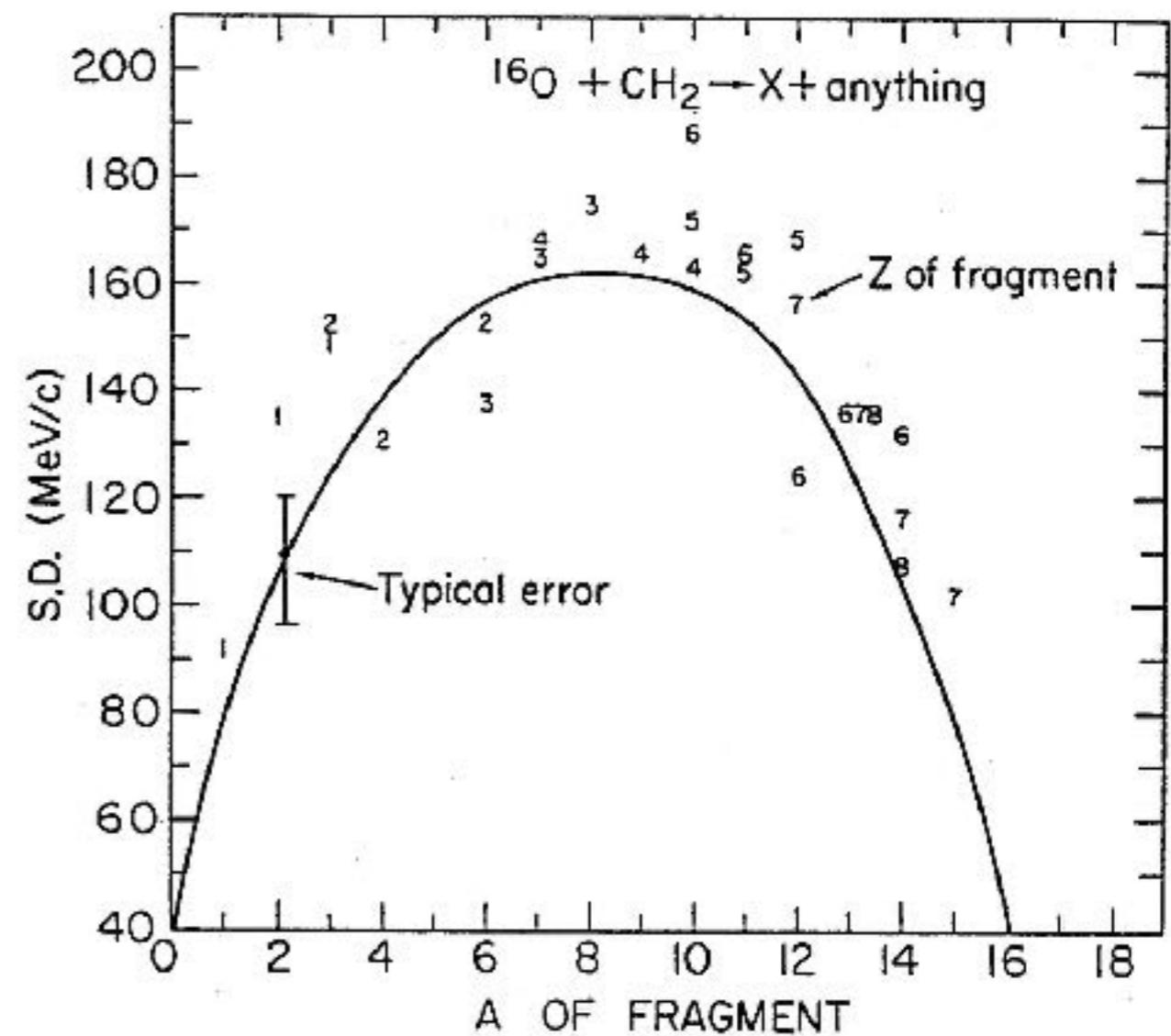
碎裂核的動量分布

Goldhaber 模型 (1974)

$$\sigma_{P//} = \sigma_0 \sqrt{\frac{K(A-K)}{A-1}}$$

K : mass number of fragment
 A : mass number of projectile

$$\sigma_0 = 90 \text{ MeV/c}$$



历史性重大发现

中子晕核 ^{11}Li 的发现

VOLUME 55, NUMBER 24

PHYSICAL REVIEW LETTERS

9 DECEMBER 1985

Measurements of Interaction Cross Sections and Nuclear Radii in the Light p -Shell Region

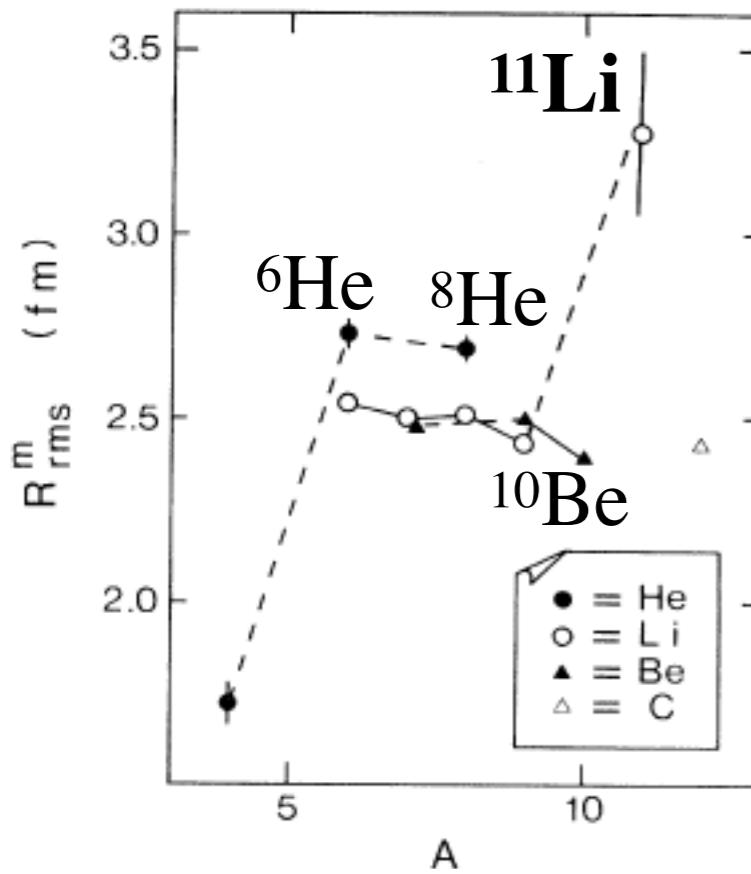
I. Tanihata,^(a) H. Hamagaki, O. Hashimoto, Y. Shida, and N. Yoshikawa
Institute for Nuclear Study, University of Tokyo, Tanashi, Tokyo 188, Japan

K. Sugimoto,^(b) O. Yamakawa, and T. Kobayashi

Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
and

N. Takahashi

College of General Education, Osaka University, Toyonaka, Osaka 560, Japan
(Received 11 July 1985; revised manuscript received 17 September 1985)

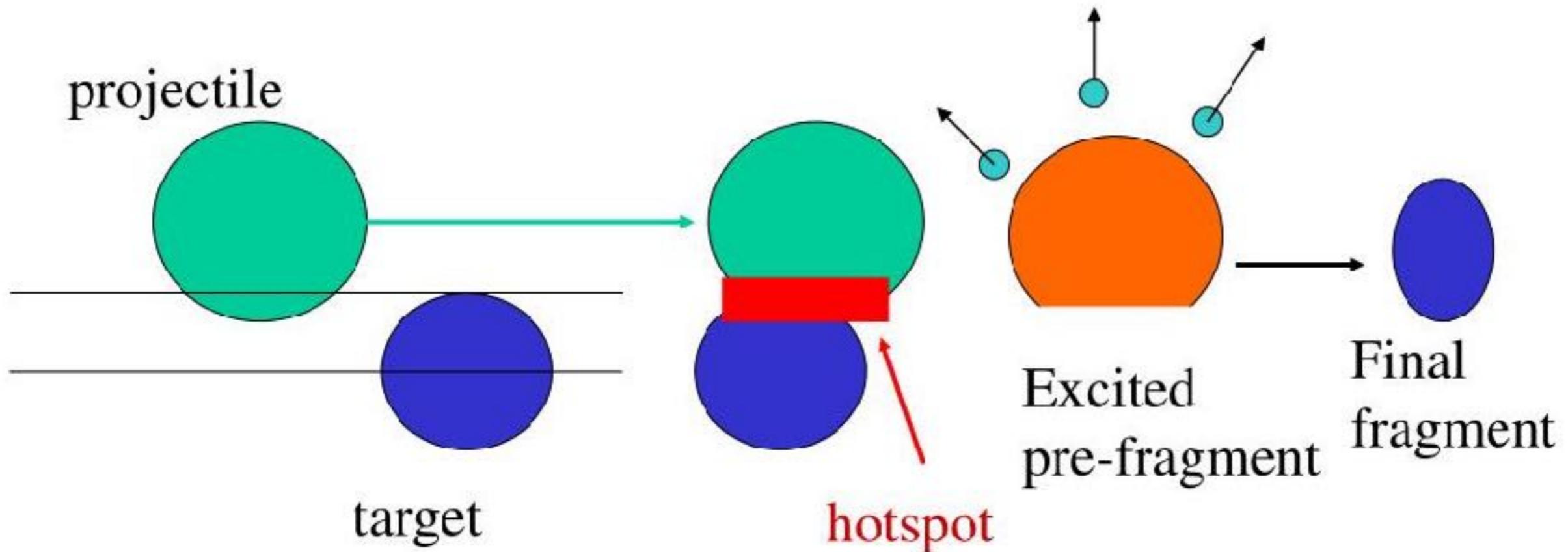


谷田勇夫教授：放射性束物理之父



弹核碎裂反应机制

Projectile Fragmentation Reactions



Energy (velocity) of beam > Fermi velocity inside nucleus ~30 MeV/u
Can ‘shear off’ different combinations of protons and neutrons.
Large variety of exotic nuclear species created, all at forward angles with ~beam velocity.

Courtesy: Paddy Regan

弹核碎裂反应截面

Empirical PArametrization of fragmentation CROSS section (EPAX)

$$\sigma(A, Z) = Y(A) \cdot n \cdot \exp(-R|Z_{\text{prob}} - Z|^{U_{n(p)}})$$

$$Y(A) = S \cdot P \cdot \exp[-R(A_p - A)]$$

$$S = S_2 (A_p^{1/3} + A_t^{1/3} + S_1)$$

$$\ln P = P_2 A_p + P_1$$

$Y(A)$: 质量 A 原子核的生成截面

Z_{prob} : 生成截面最大质子数

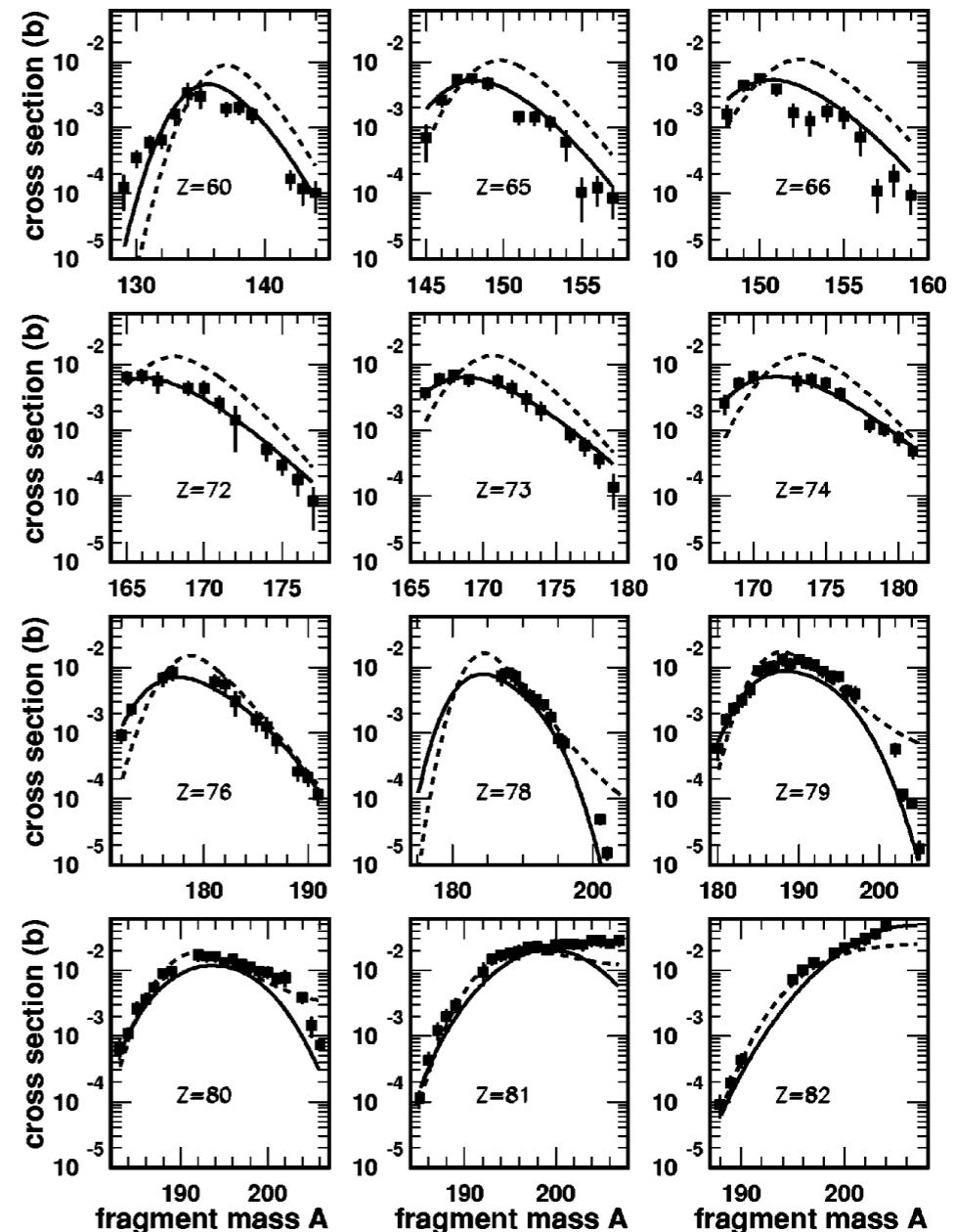
A_p : 入射弹核质量数

A_t : 靶核质量数

$n, R, U, S_1, S_2, P_1, P_2$: 常数

Physical Review C 42 (1990) 2546

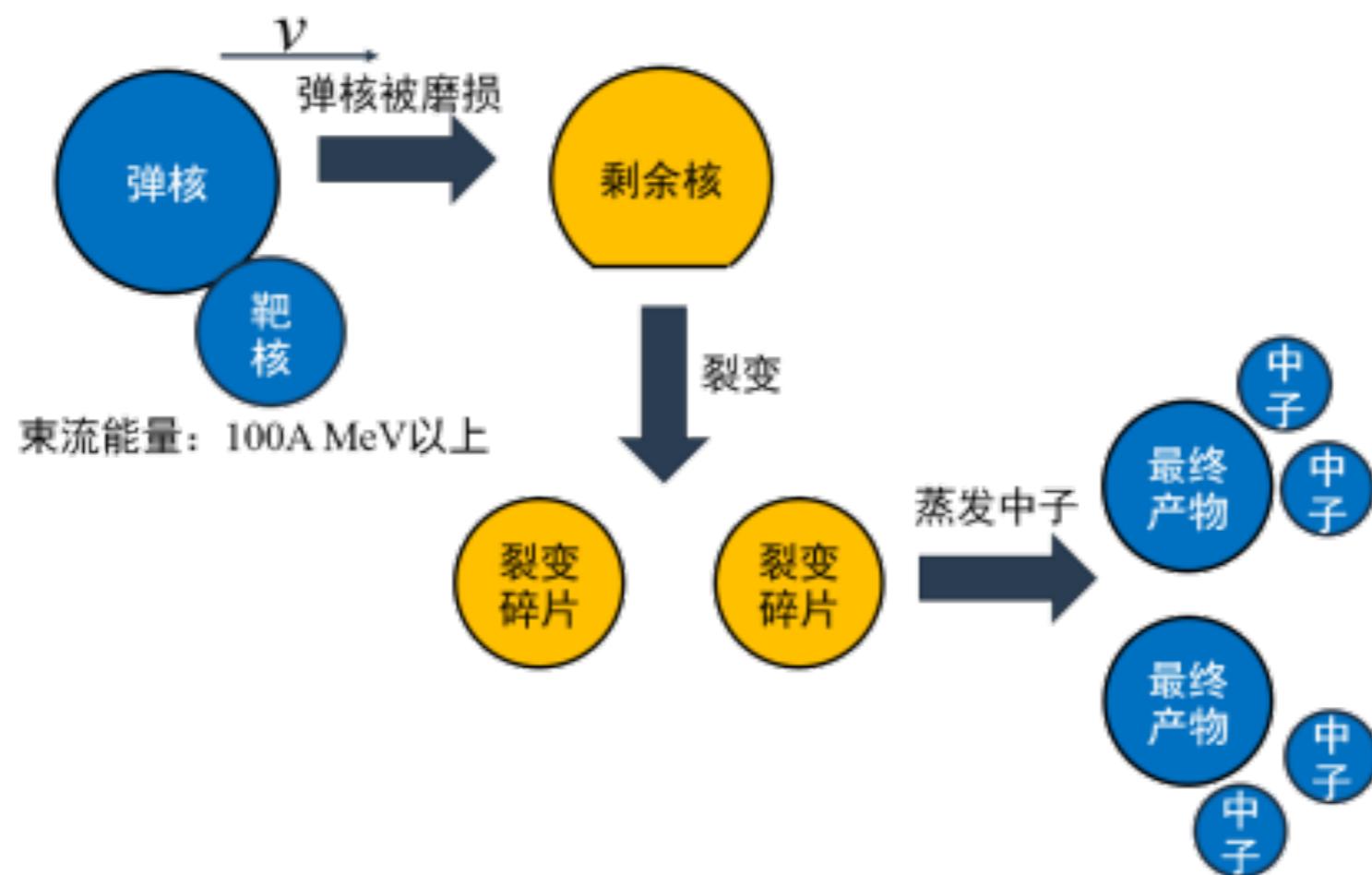
Physical Review C 61 (2000) 034607



飞行中裂变

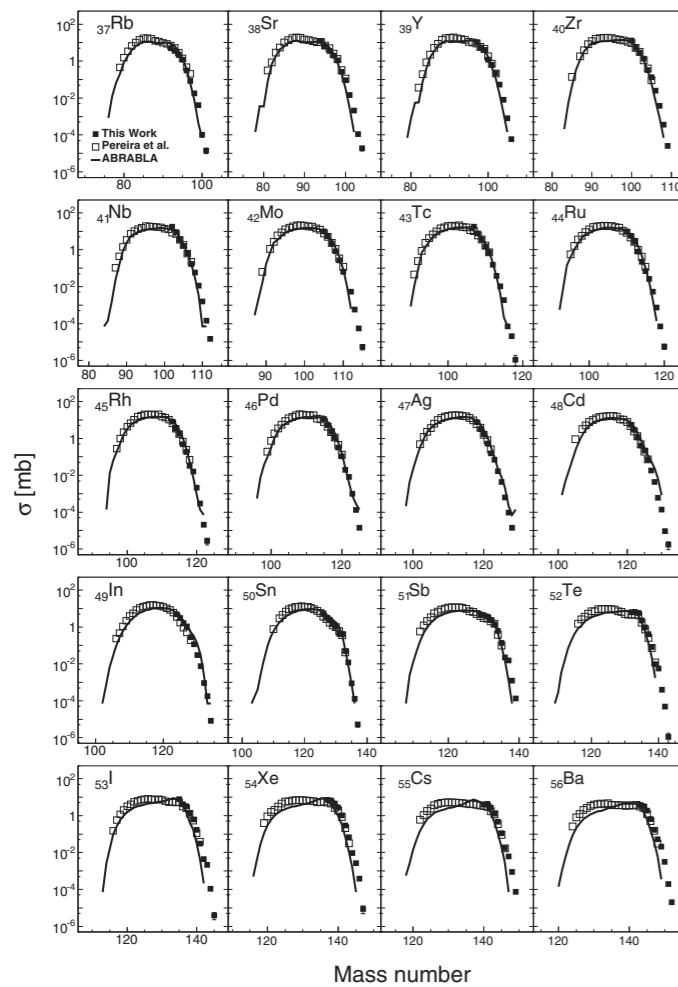
磨损-裂变模型 (Abrasion-Fission)

- 磨损 (Abrasion)
- 裂变 (Fission)
- 蒸发 (Ablation)

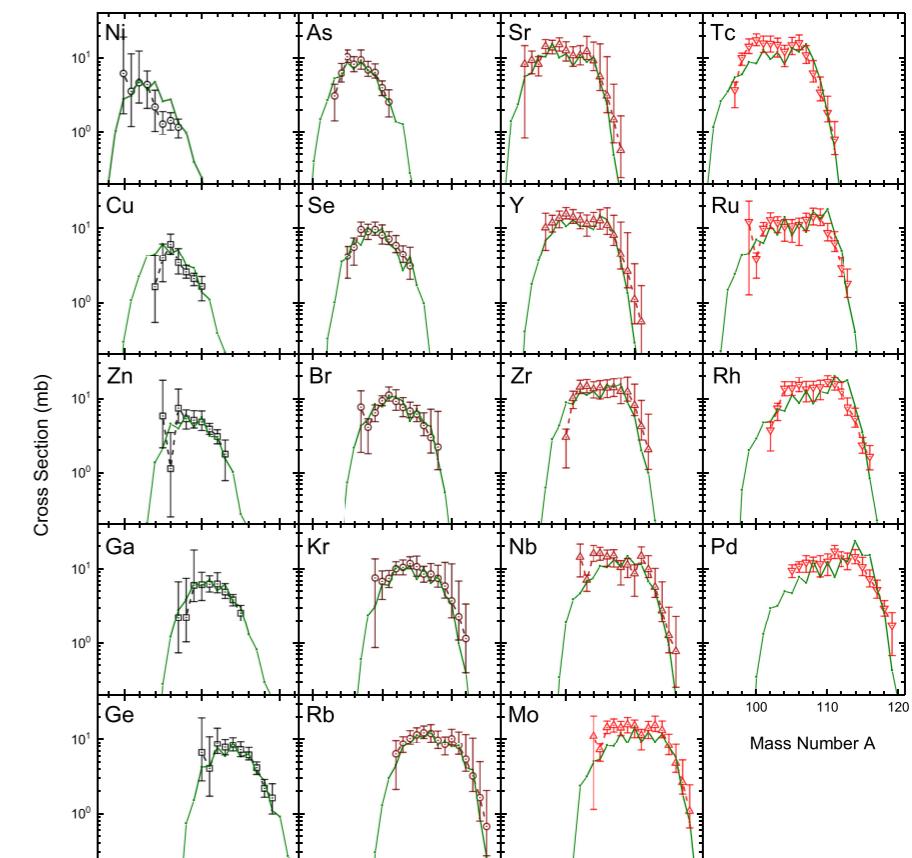
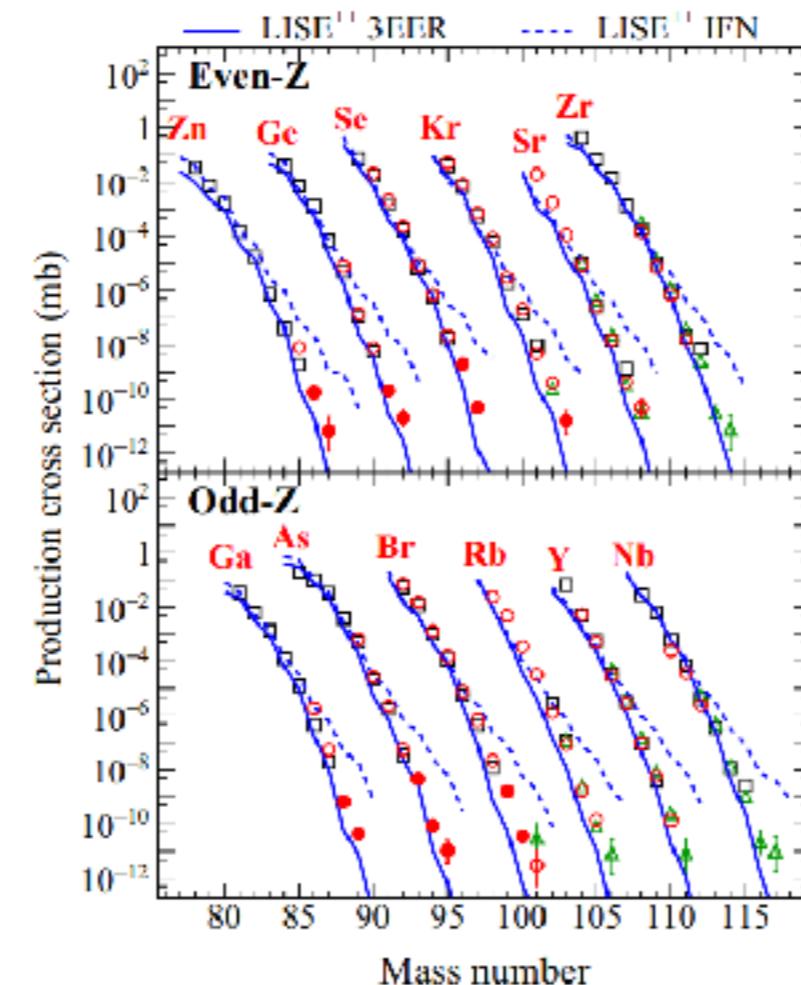


飞行中裂变

$^{238}\text{U}+\text{Be}$ @ 950A MeV



$^{238}\text{U}+\text{Be}$ @ 345 MeV/nucleon $^{238}\text{U}+\text{Be}$ @ 100 MeV/nucleon

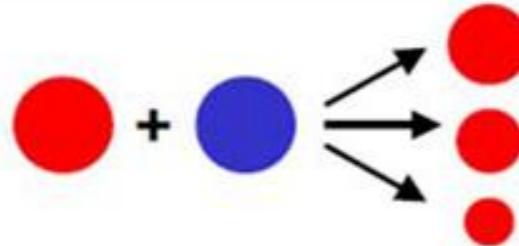
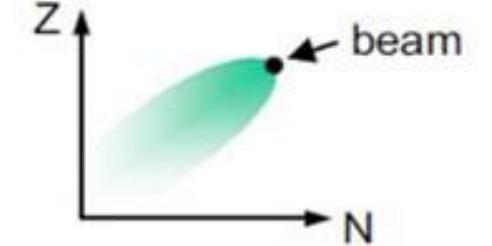
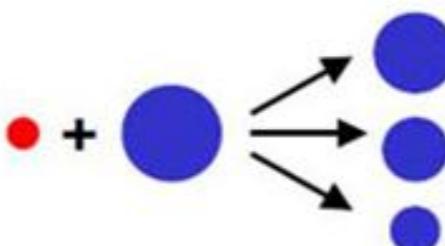
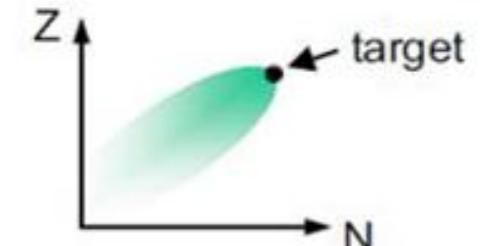
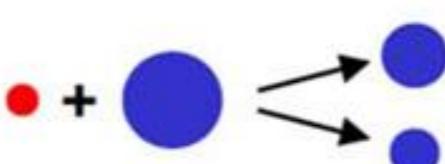
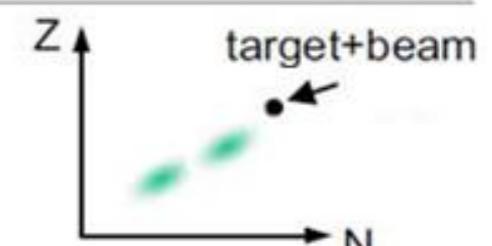
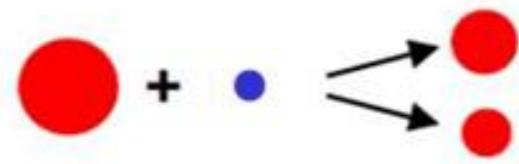
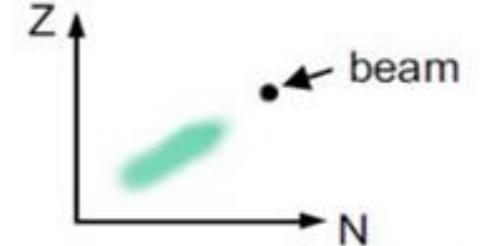
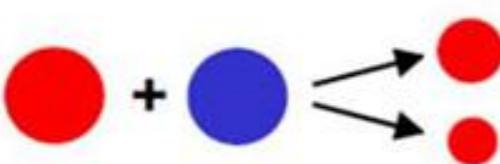
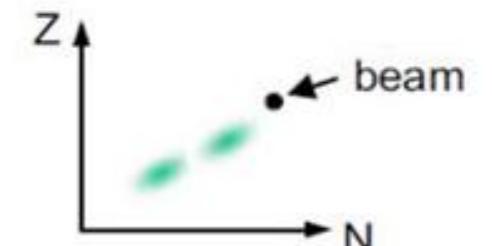
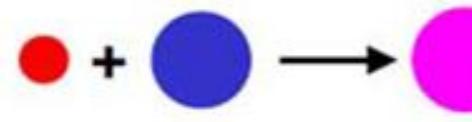
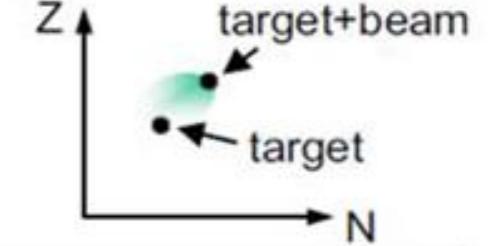


D. Pérez-Loureiro *et al.*, PRC
99, 054606 (2019)

Y. Shimizu *et al.*, PRC
109, 044313 (2024)

M. Bowry *et al.*, PRC 108,
034604 (2023)

其他反应机制

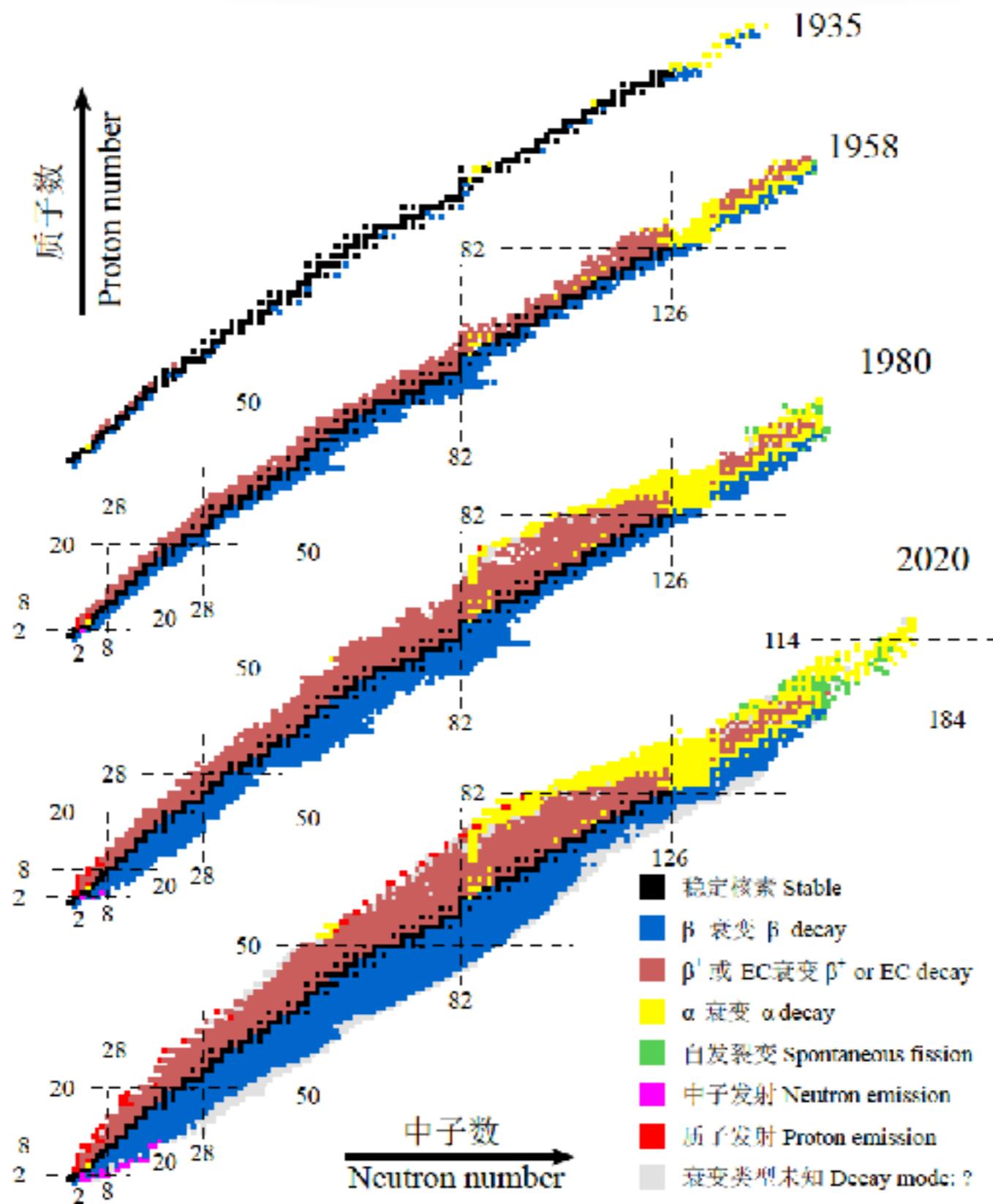
	projectile fragmentation	$v_{\text{product}} = v_{\text{beam}}$	up to 1000	
	spallation	few MeV/u	up to 1000	
	fusion- fission <i>new : inverse</i>	$\sim 1 \text{ MeV/u}$ $\sim 10-25 \text{ MeV/u}$	few 100	
	abrasion- fission	$v_{\text{product}} = v_{\text{beam}}$	few 100	
	Coulomb fission	$> 200 \text{ MeV/u}$ $v_{\text{product}} = v_{\text{beam}}$	few 100	
	fusion- evaporation	$E_R = \frac{m_p}{m_p + m_t} E_P$	few (≤ 20)	



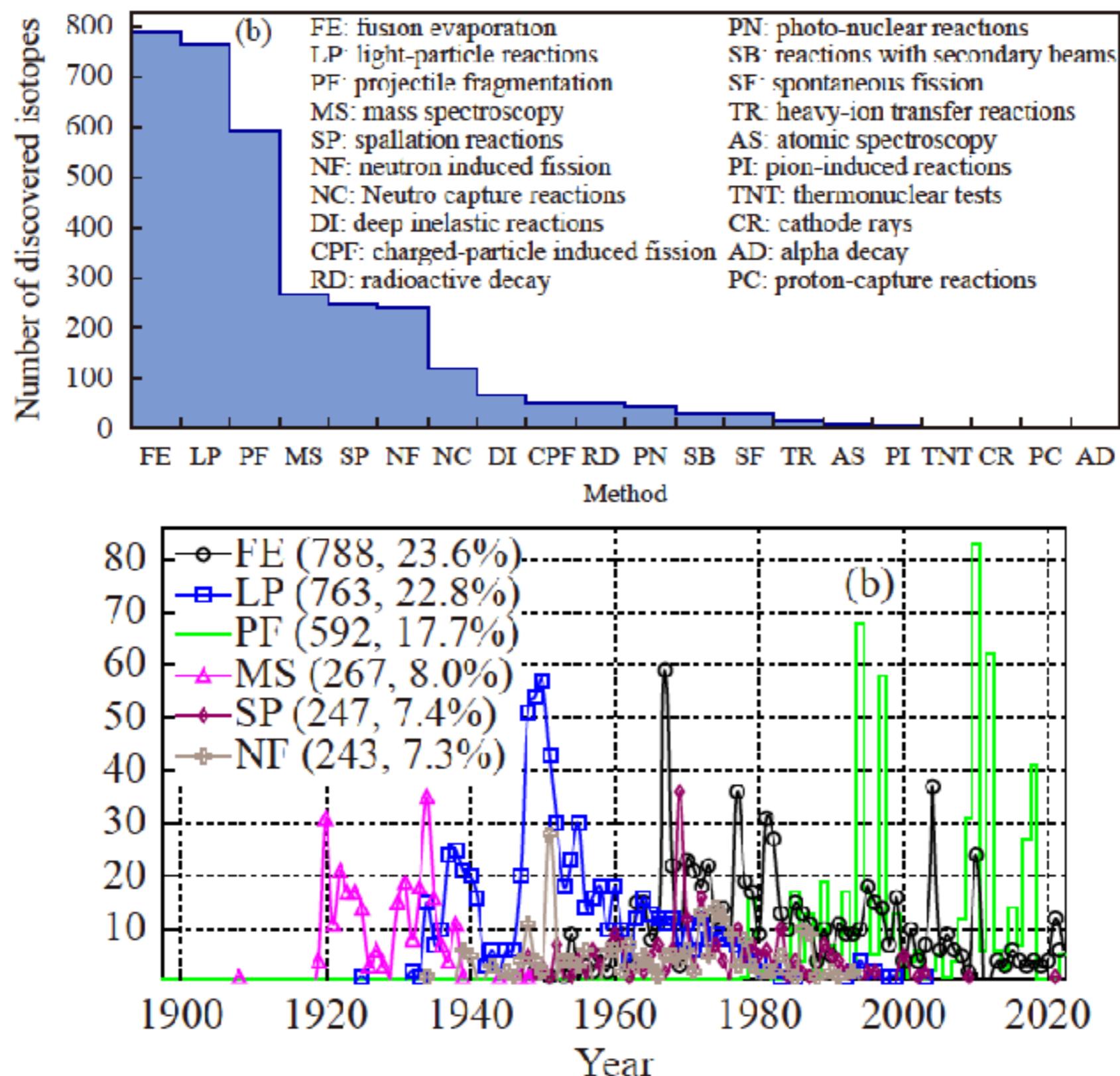
放射性束装置



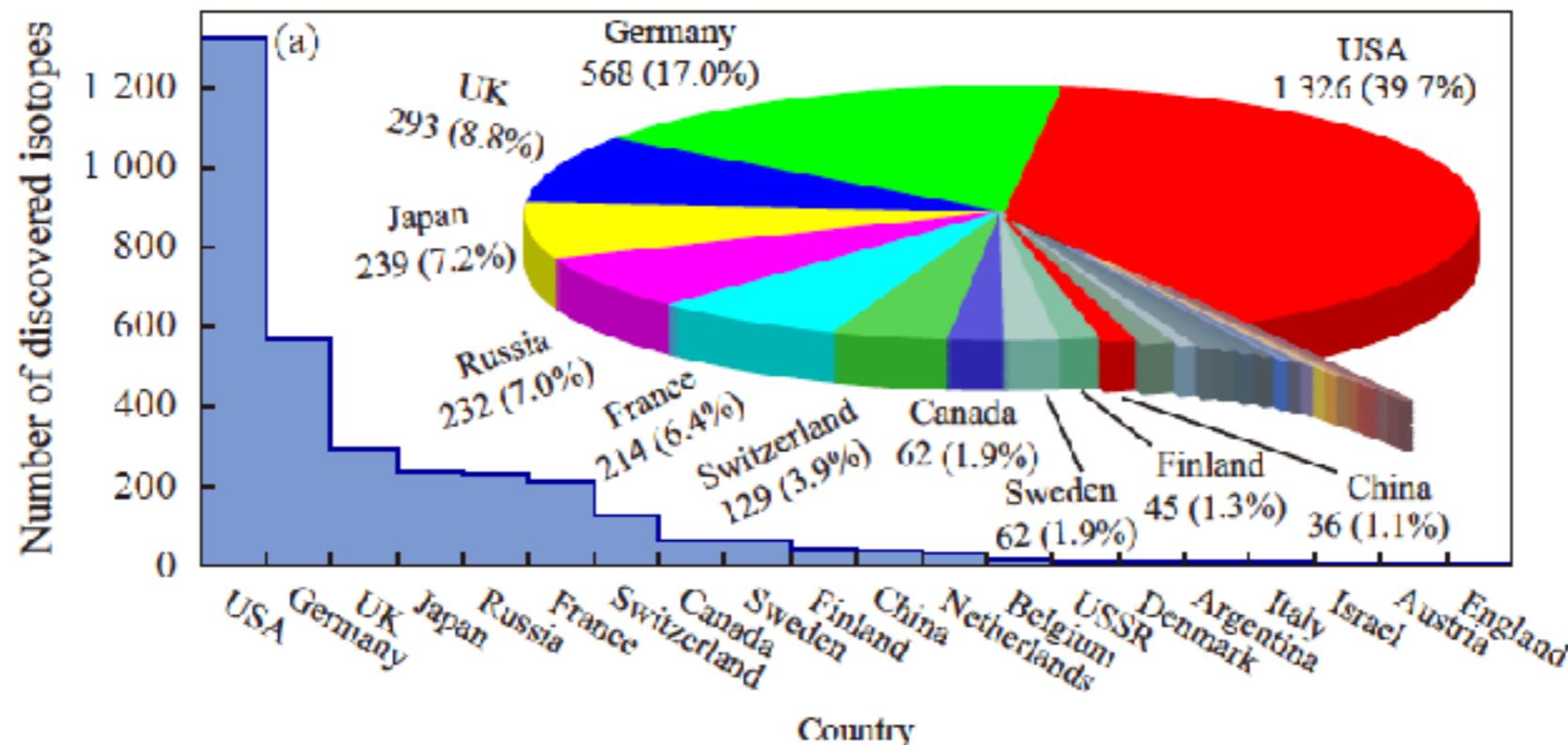
核素图的变迁



新核素产生手段

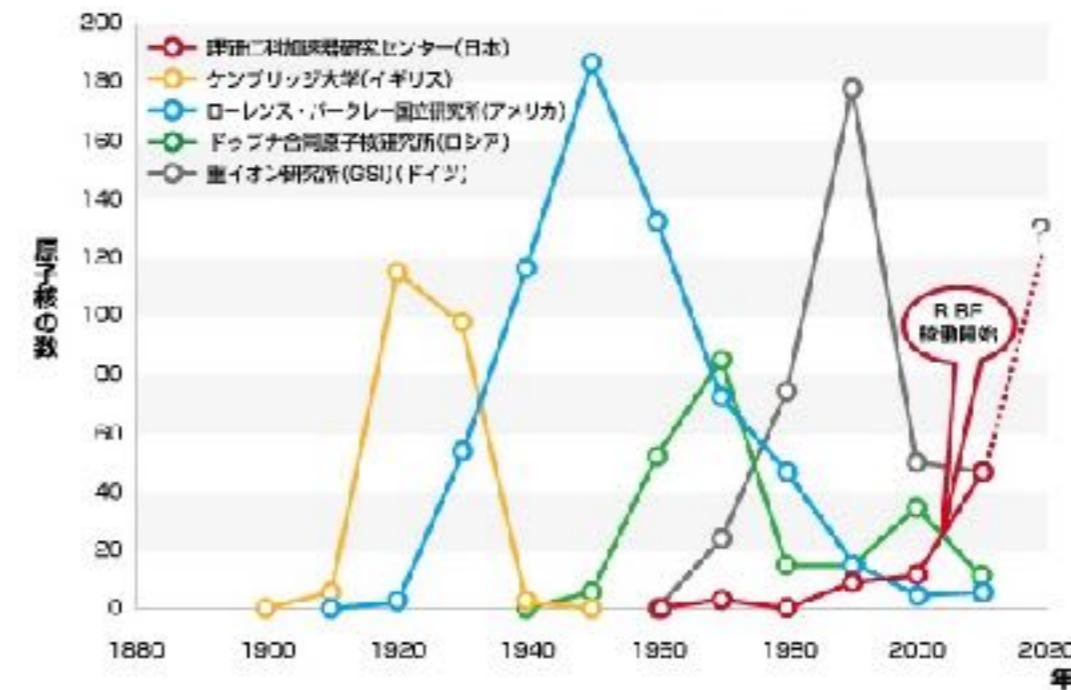


新核素发现国家/研究所



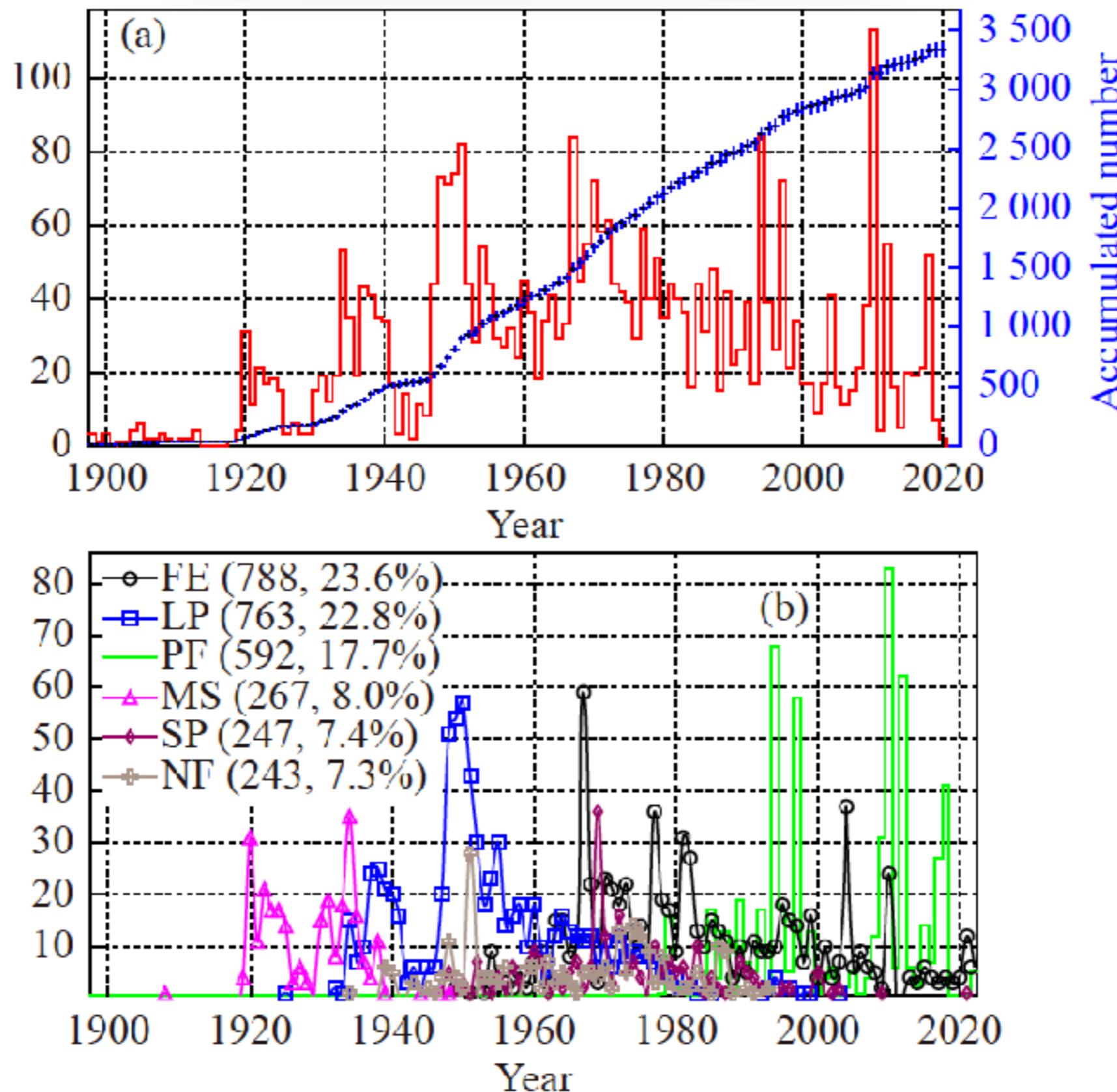
国旗 共41种（截至2024）

38在近物所发现

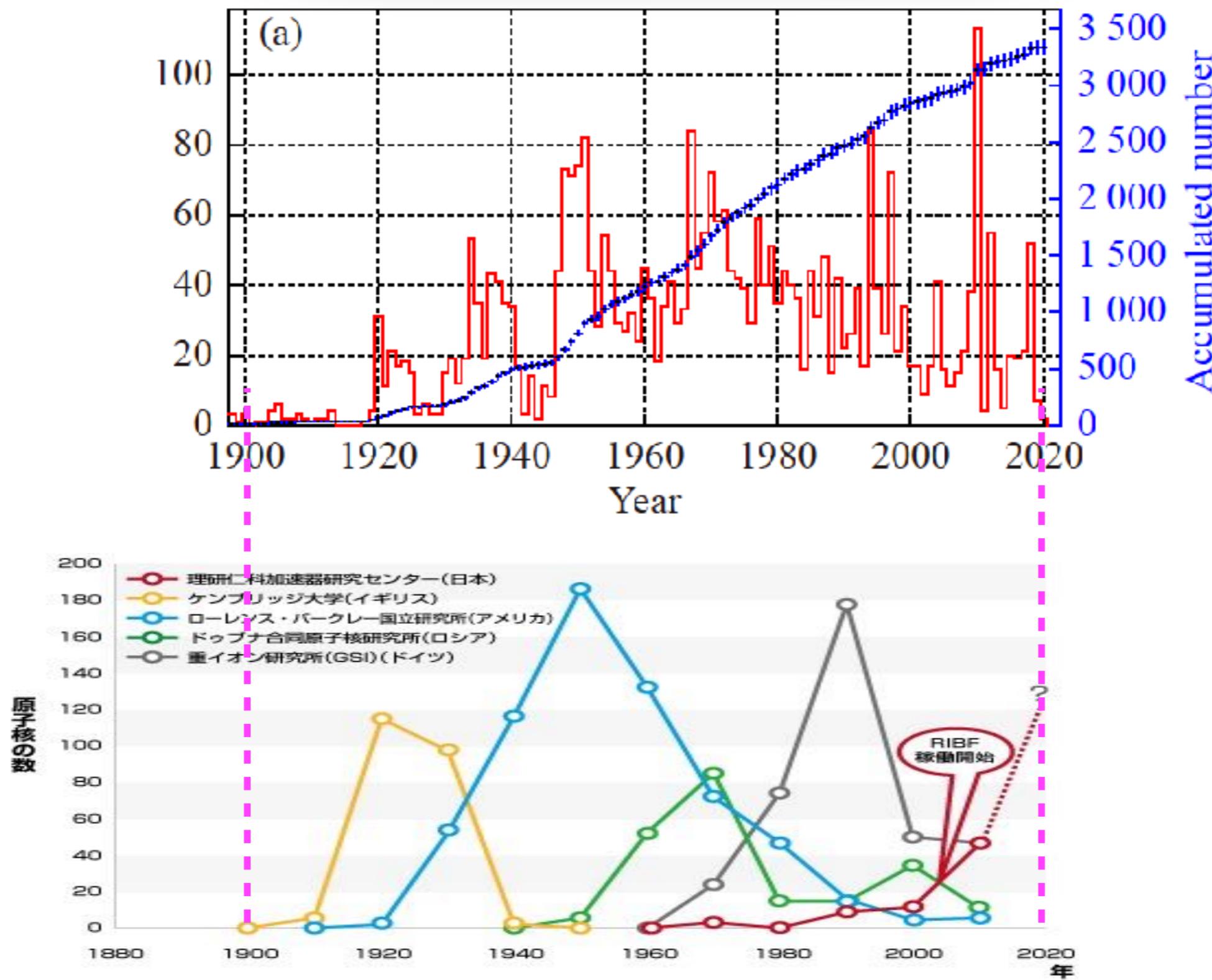


出处：日本理化研究所仁科加速器科学研究中心

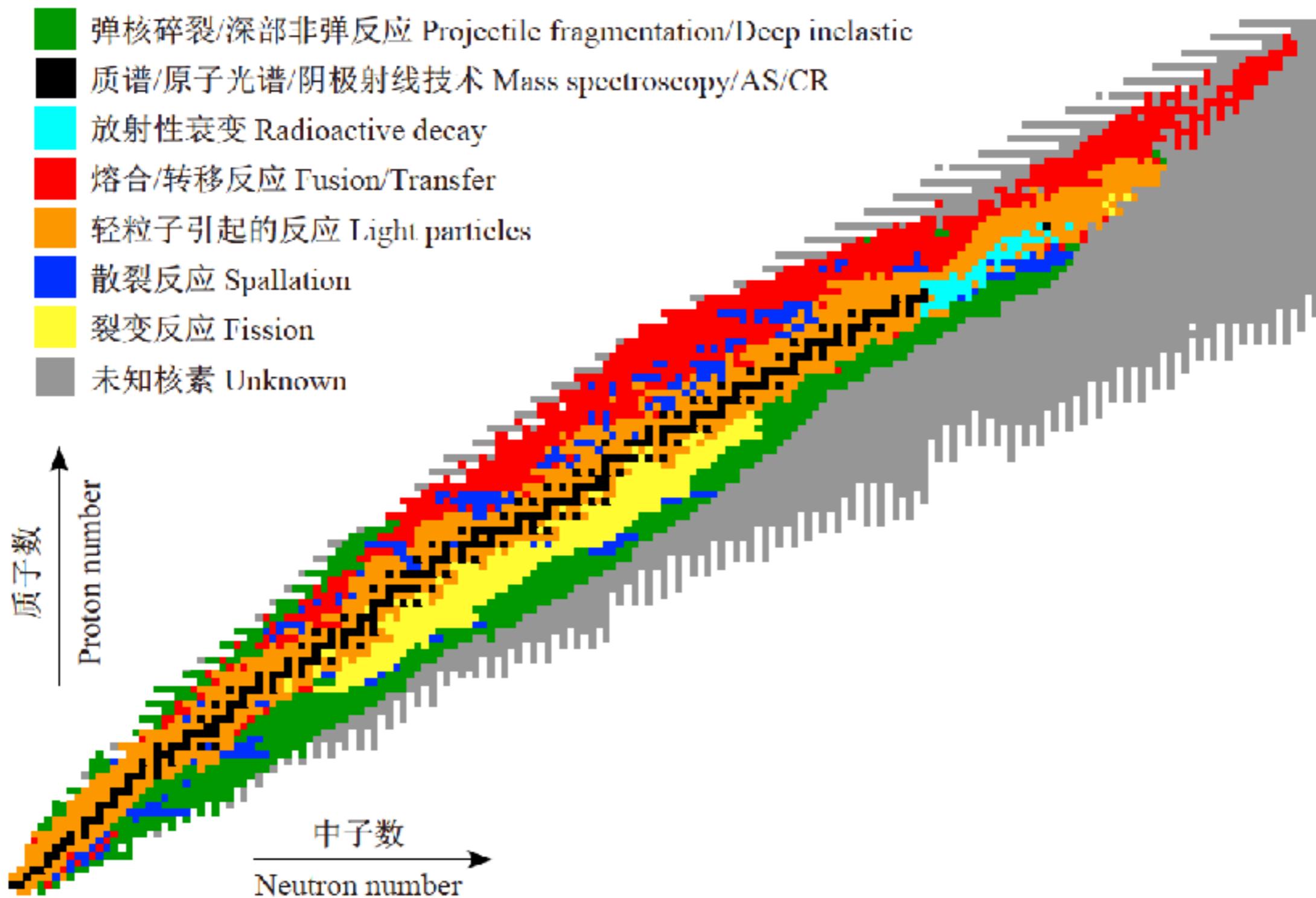
新核素发现年份



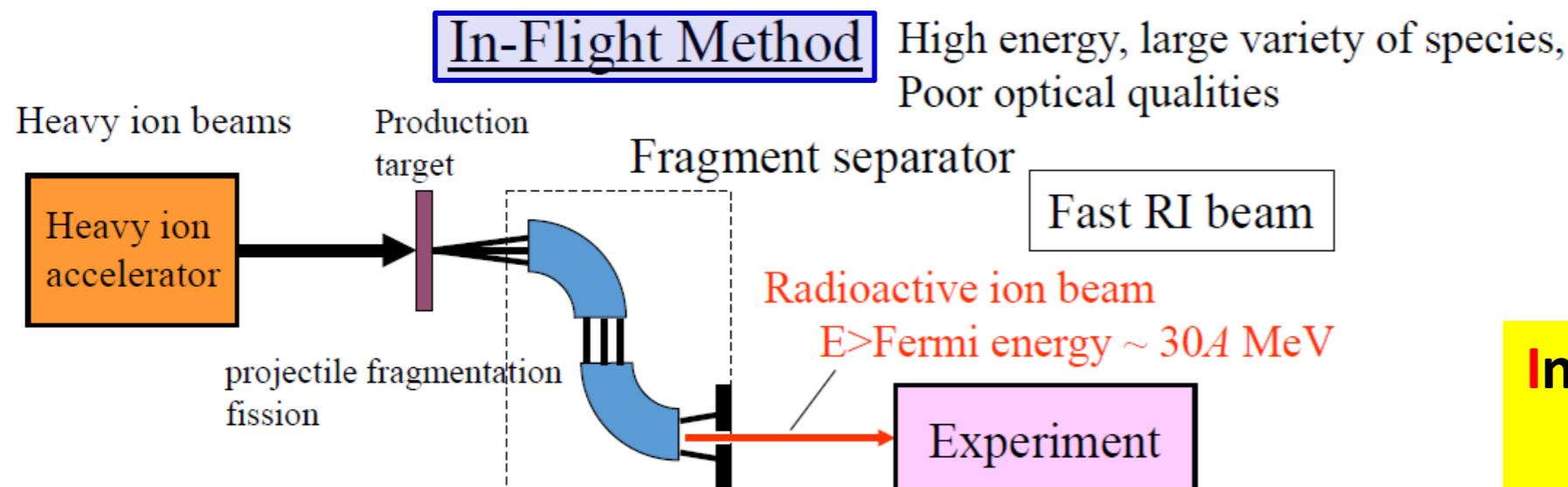
新核素发现年份



核素版图(2022)



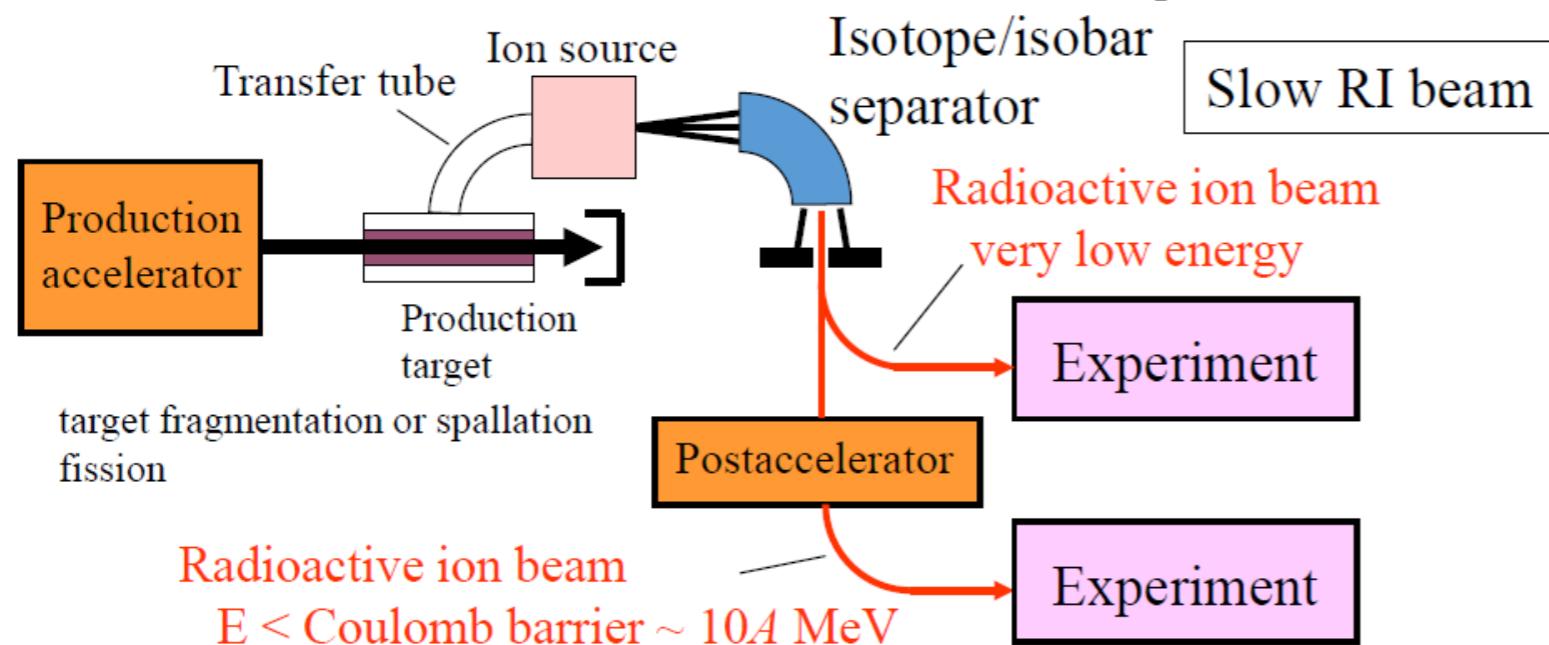
放射性束的生成与分离技术路线



中高能束流
 $\sim 30\text{MeV/u} \sim 1\text{GeV/u}$

In-Flight fragment separation
飞行中碎片分离

ISOL Method Low energy, chemistry is difficult, good
beam qualities



低能束流
 $\leq 10\text{MeV/u}$

Isotope Separation On-Line
在线同位素分离

ISOL vs IF

thought to be complementary

	ISOL	In-flight
Driver ion	light ions (p, d)	heavy ion
Origin of RI	target ion	projectile ion
Target	thick target	thin target
In-target yield	high	low
RI separation	sensitive to chemical properties	physical
Time scale	> 50 ms	> 1 μ s
Instrumentation	post accelerator	fragment separator
Beam quality	good	poor
Experiments	spectroscopy	reaction scheme

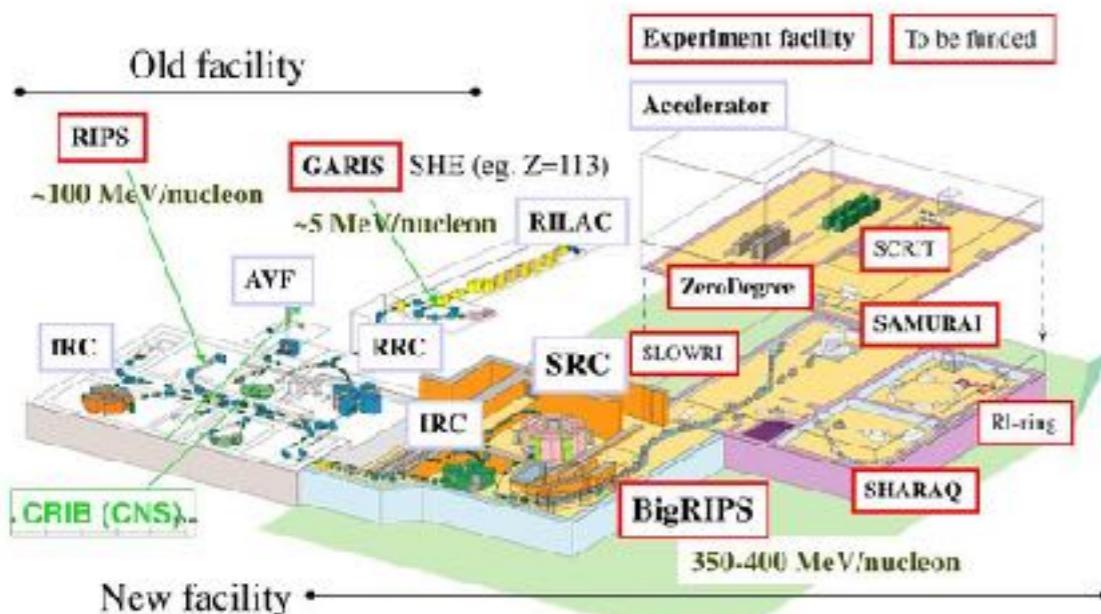
新一代In-Flight型放射性束装置



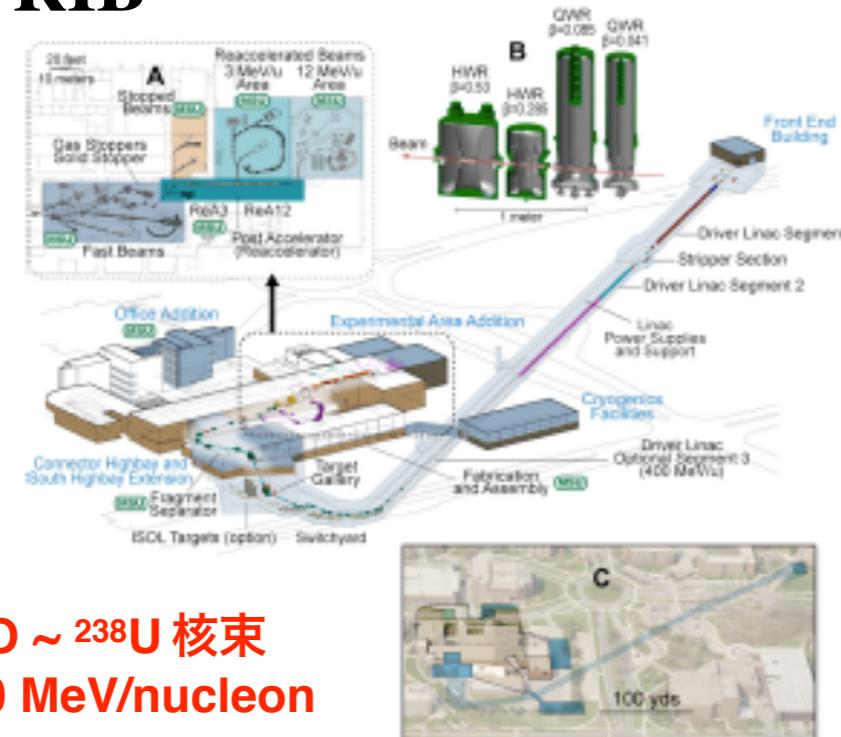
RIBF

$^{18}\text{O}^{8+} \sim ^{238}\text{U}^{86+}$ 核束
345 MeV/nucleon

RIKEN RI Beam Factory (RIBF)



FRIB

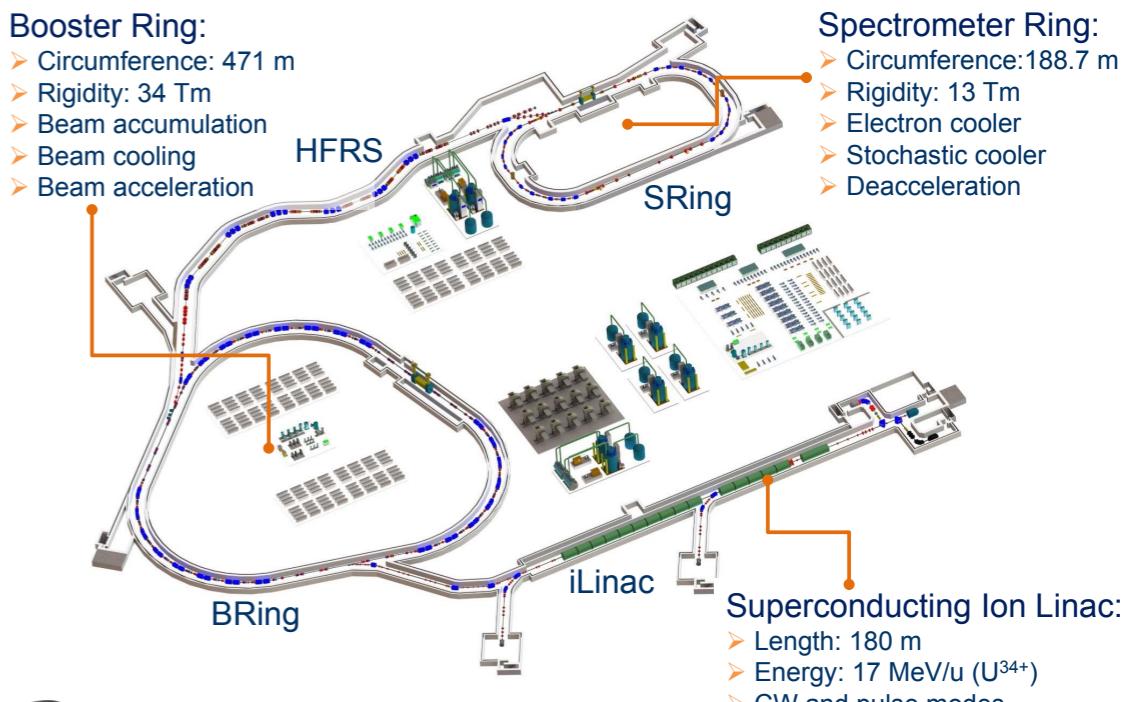


已运行 | 建设中

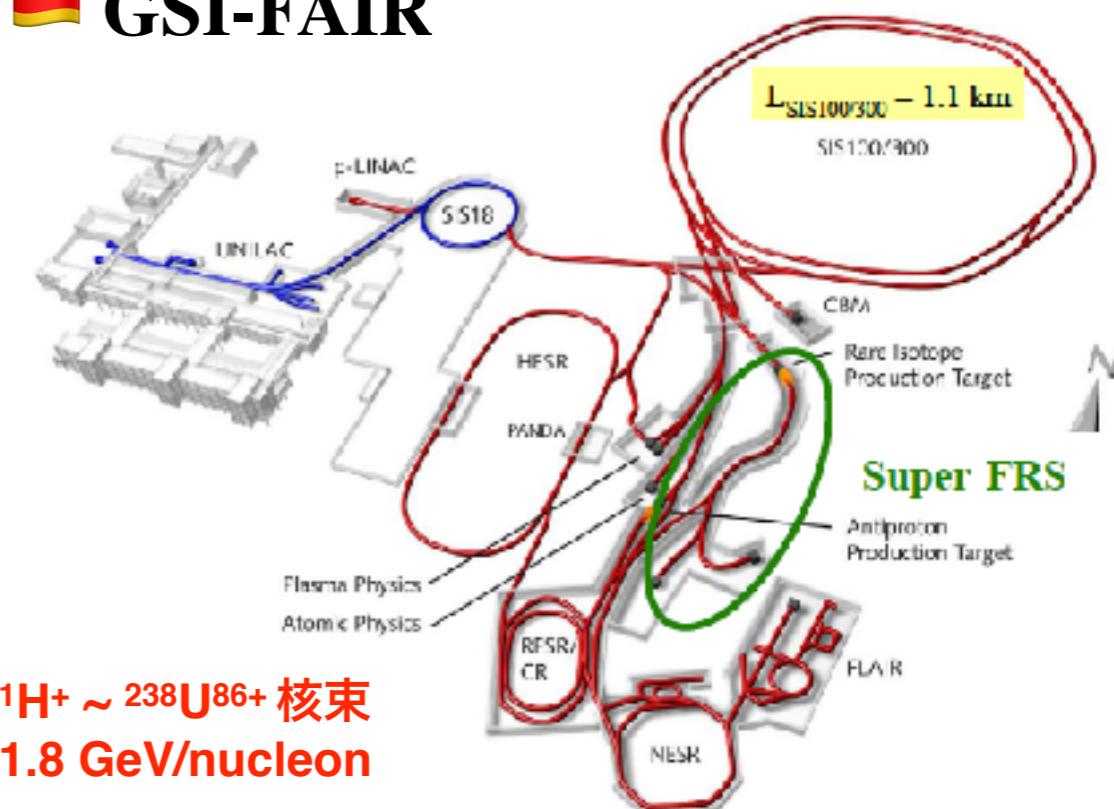


HIAF

$^1\text{H}^+ \sim ^{238}\text{U}^{86+}$ 核束
3 GeV/nucleon



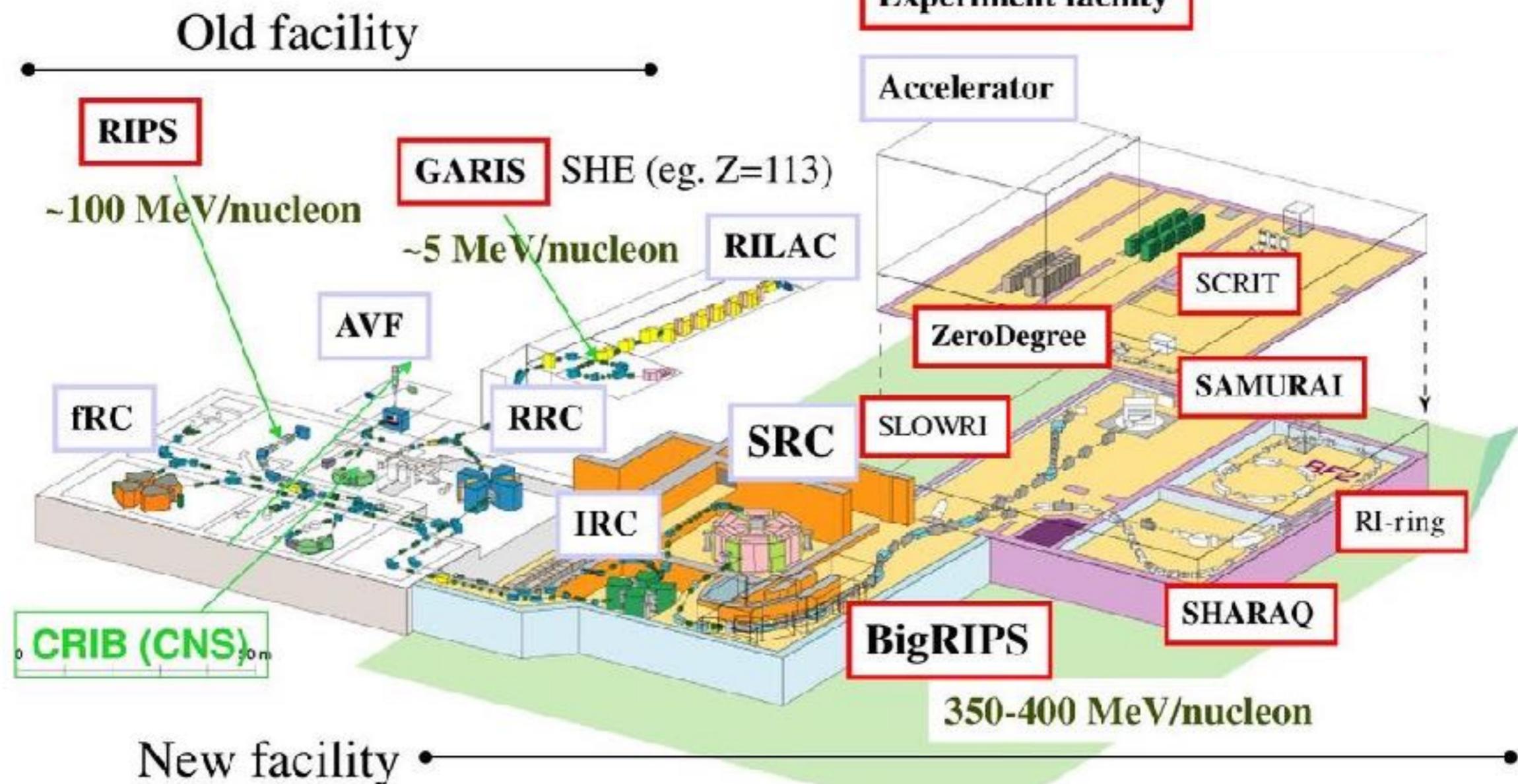
GSI-FAIR



RIBF

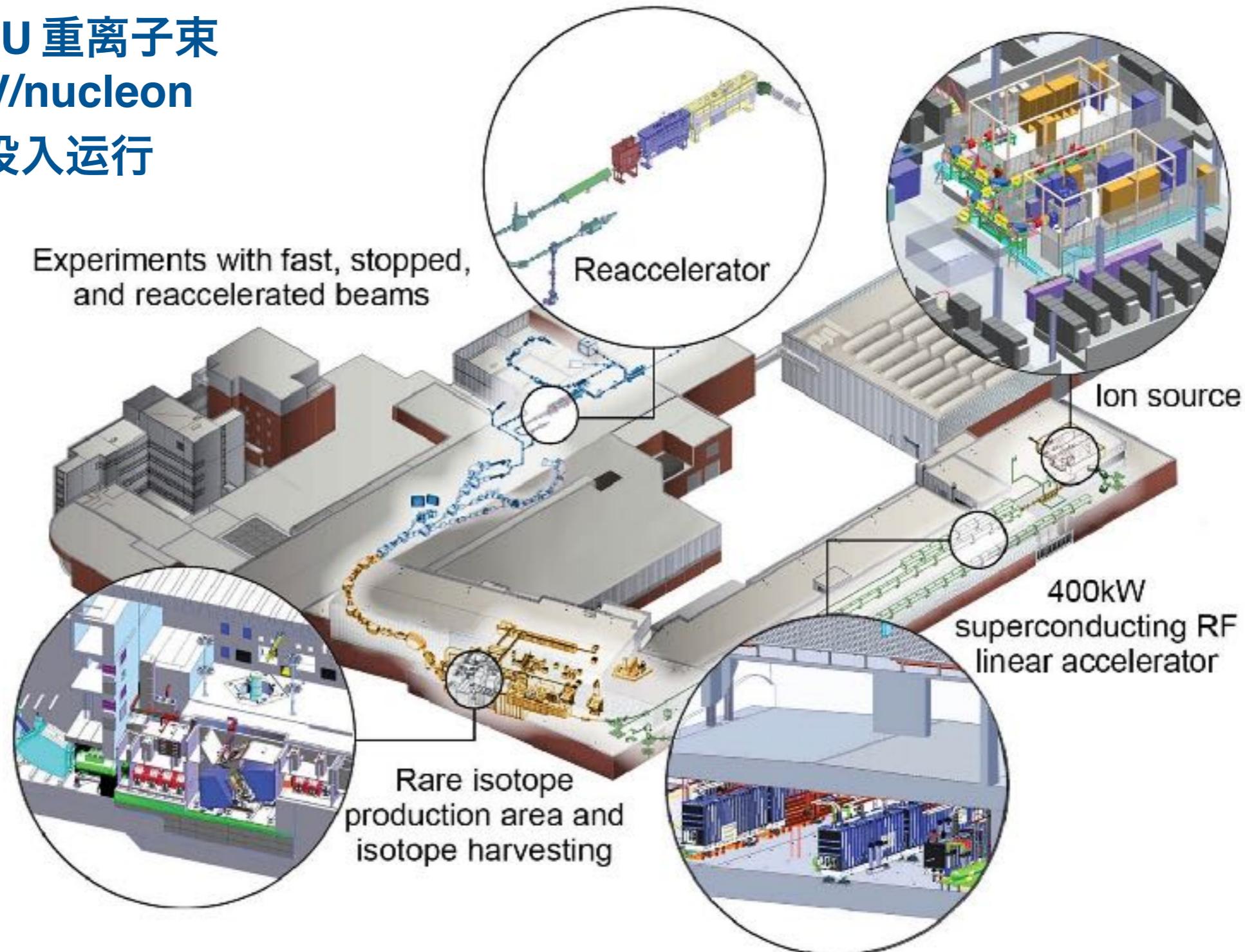
RIKEN RI Beam Factory

- $^{18}\text{O}^{8+} \sim ^{238}\text{U}^{86+}$ 重离子束
- 345 MeV/nucleon
- 2008年投入运行



Facility for Rare Isotope Beams

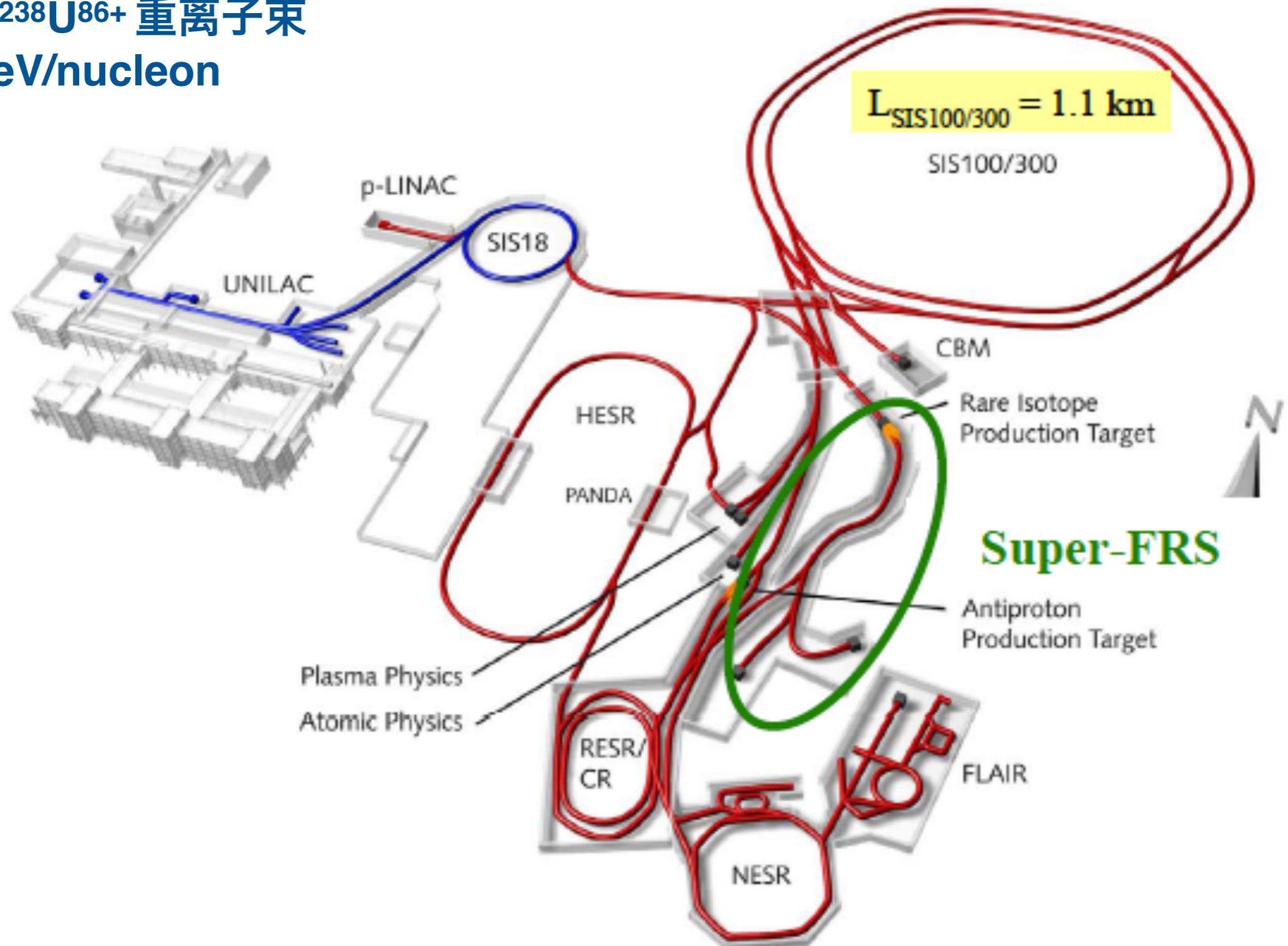
- $^{16}\text{O} \sim ^{238}\text{U}$ 重离子束
- 200 MeV/nucleon
- 2022年投入运行



FAIR

FAIR- Facility for Antiproton and Ion Research

- ${}^1\text{H}^+ \sim {}^{238}\text{U}^{86+}$ 重离子束
- 1.8 GeV/nucleon



新一代ISOL型放射性束装置

SC-IIinacs

- higher energy post-accelerator
PIAVE-ALPI, SPES
REX upgrade, HIE-ISOLDE
ISAC upgrade, ARIEL
- high intensity driver
200-kW driver (SPIRAL2)
5-MW driver (EURISOL)
100-kW electron driver (ARIEL/ISAC)

Target technology

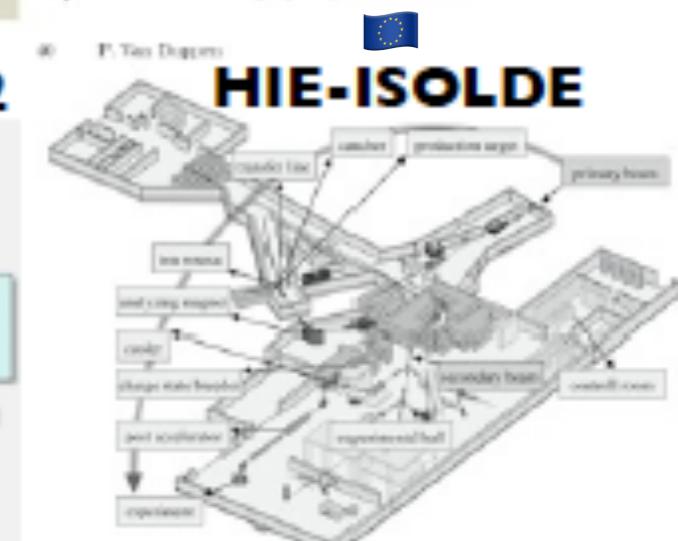
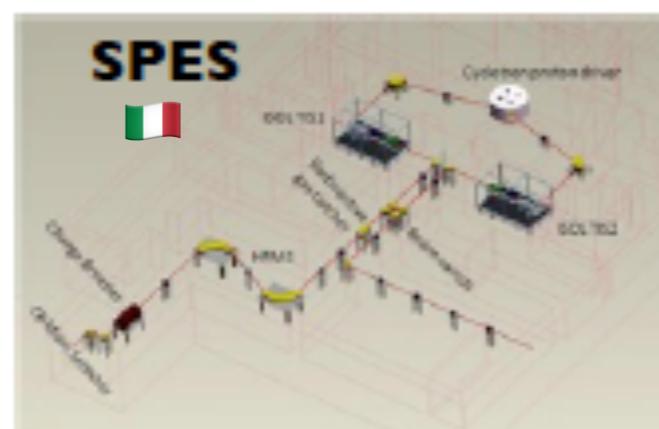
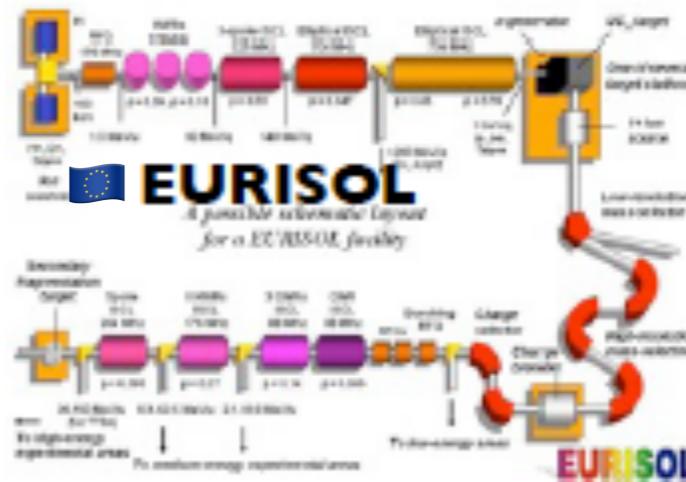
- neutron converter
carbon, SPIRAL2
Hg, EURISOL
- UCx target with a fast release time
SPES, SPIRAL2, EURISOL

Multi-user capability

- 5 experimental ports (GANIL/SPIRAL/SPIRAL2)
- Two new target stations (ARIEL)
- 6 UCx targets (EURISOL)

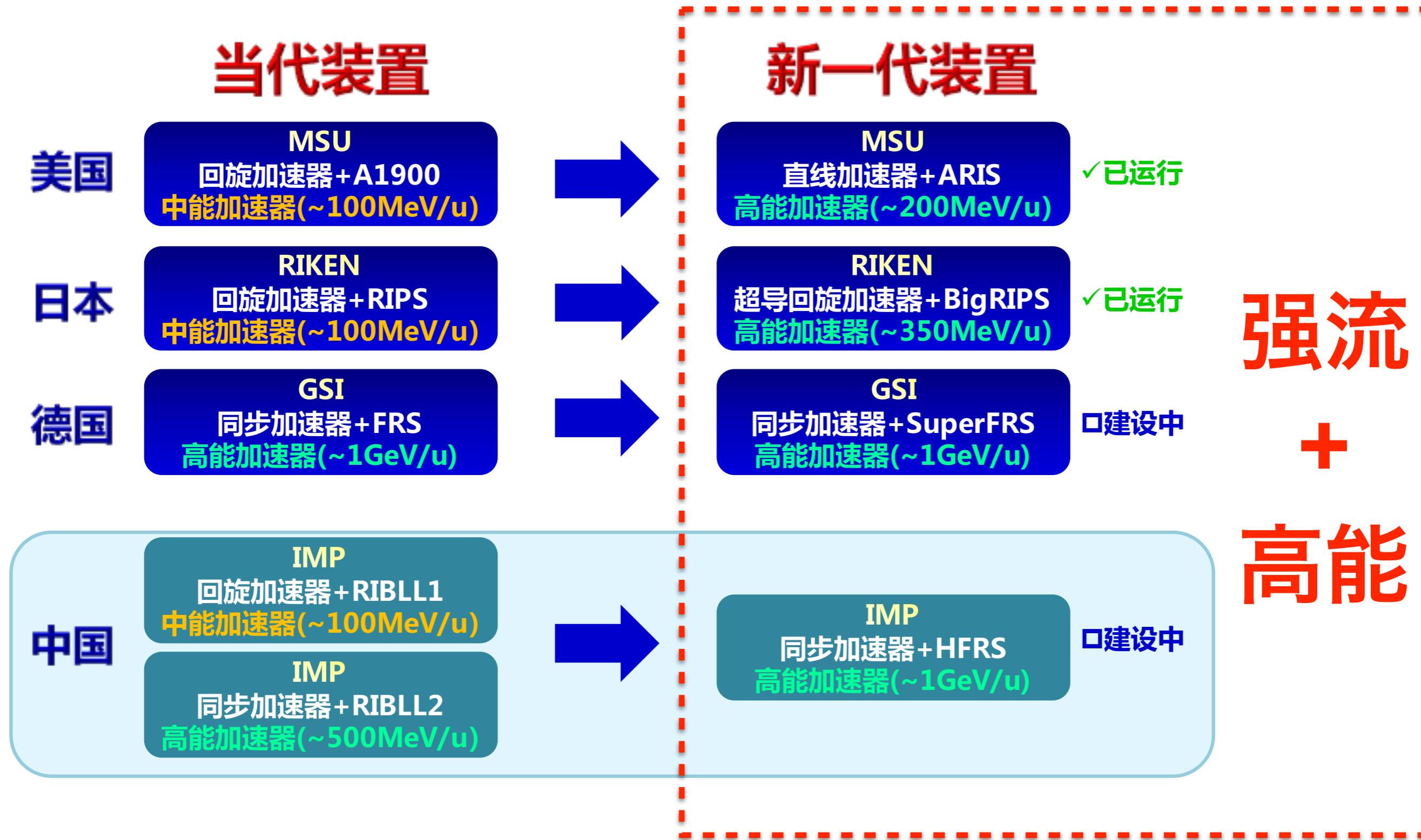
RI beam Intensity

- 10^9 pps (SPES, HIE-ISOLDE)
- 10^{10} pps (ARIEL, SPIRAL2)
- 10^{12} pps (EURISOL)



出处：日本理化研究所福西研究员会议报告

In-Flight型放射性束装置



高能放射性束装置的优势

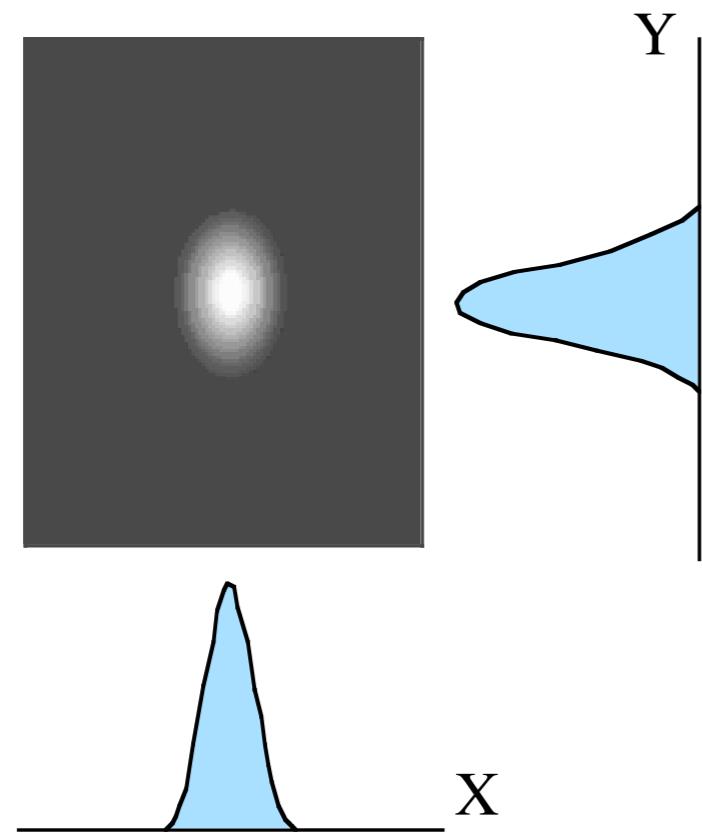
高能束流



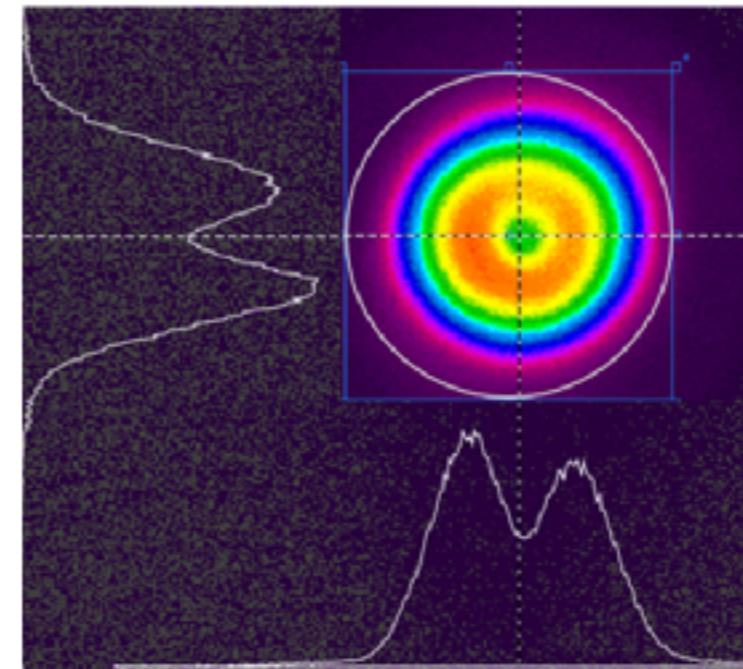
放射性束流线的束流光学

束流剖面

回旋加速器的束流

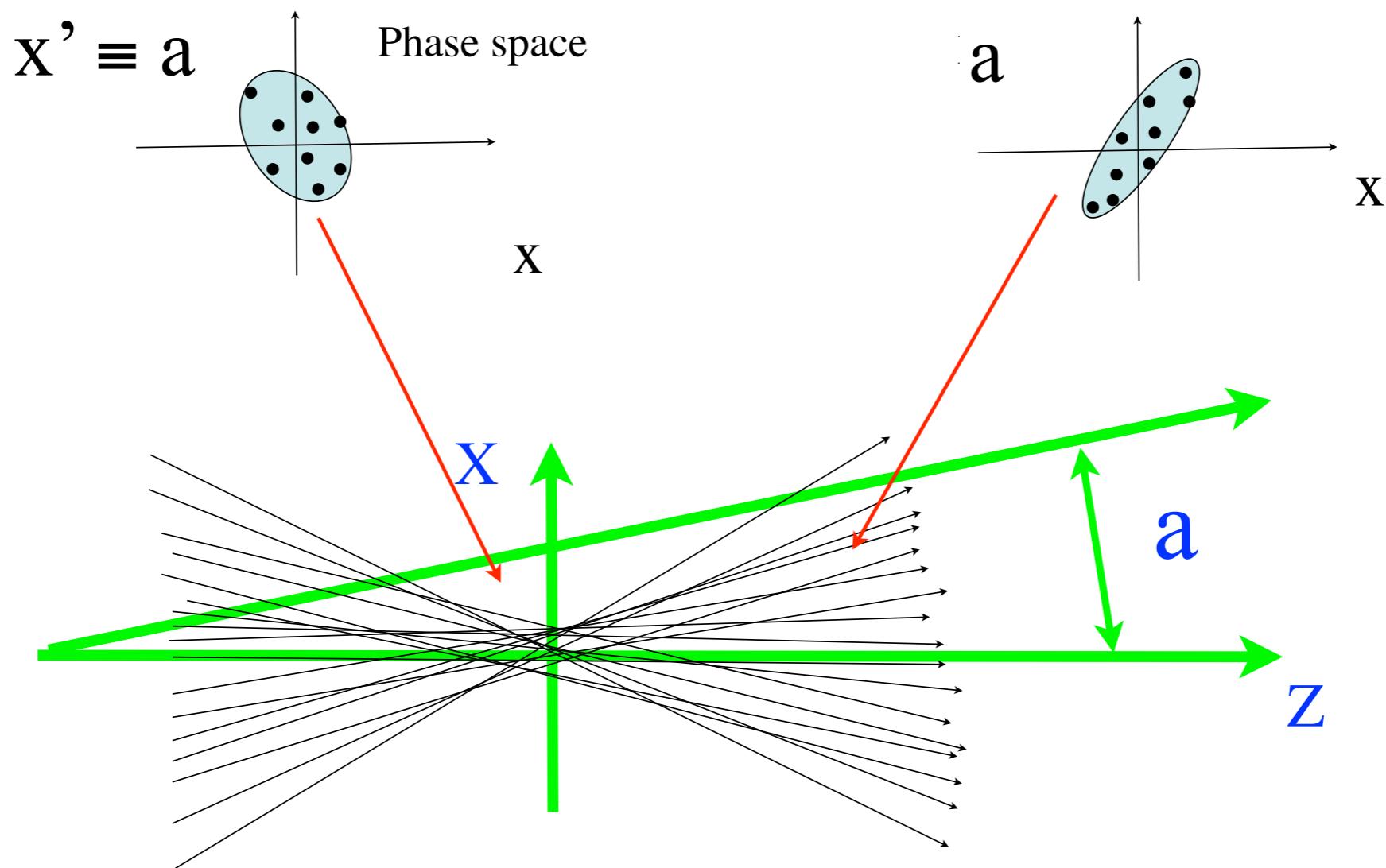


激光束

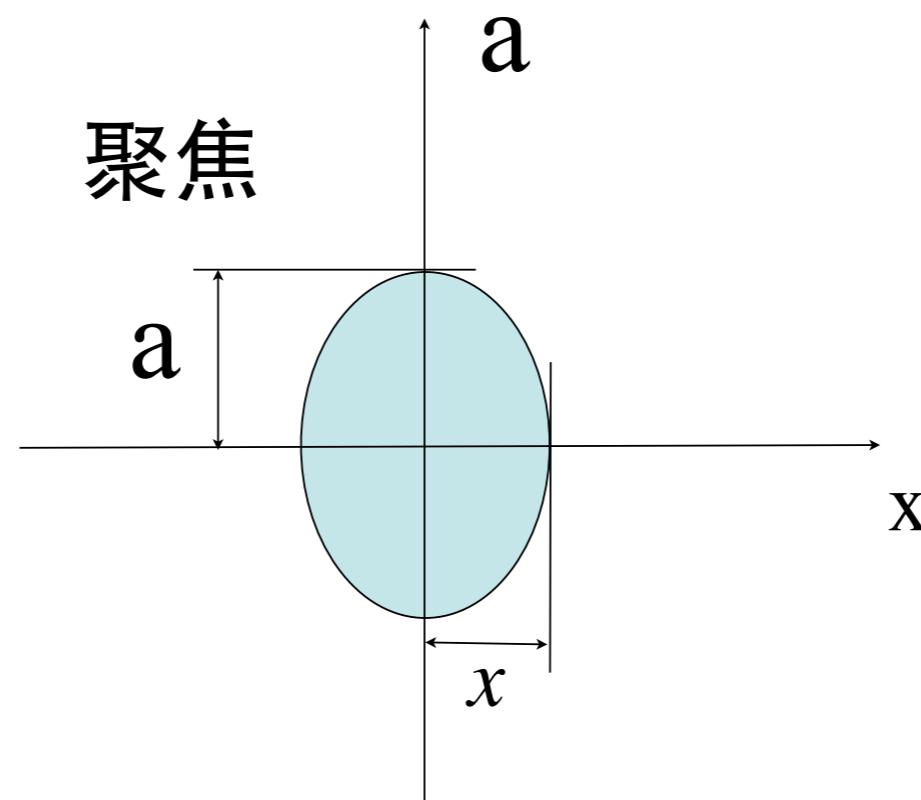


Proc. SPIE 9728, Fiber Lasers XIII, 972808 (2016)

束流相空间



聚焦面上的束流相空间



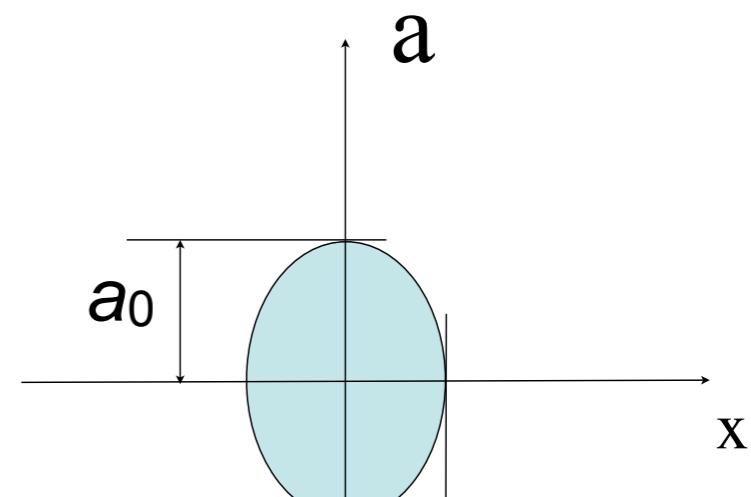
发射度

半径 x_0 的圆形相空间的“面积”：

$$x^2 + a^2 = x_0^2$$

$$a = \pm \sqrt{x_0^2 - x^2}$$

$$A_{\text{circle}} = \int_{-x_0}^{x_0} 2\sqrt{x_0^2 - x^2} dx = \pi x_0^2$$



椭圆形相空间的“面积” — 发射度(emittance)：

$$\varepsilon = A_{\text{ellipse}} = \int_{-x_0}^{x_0} 2a_0 \sqrt{1 - \frac{x^2}{x_0^2}} dx = \frac{a_0}{x_0} \int_{x_0}^{x_0} 2\sqrt{x_0^2 - x^2} dx = \frac{a_0}{x_0} A_{\text{circle}}$$

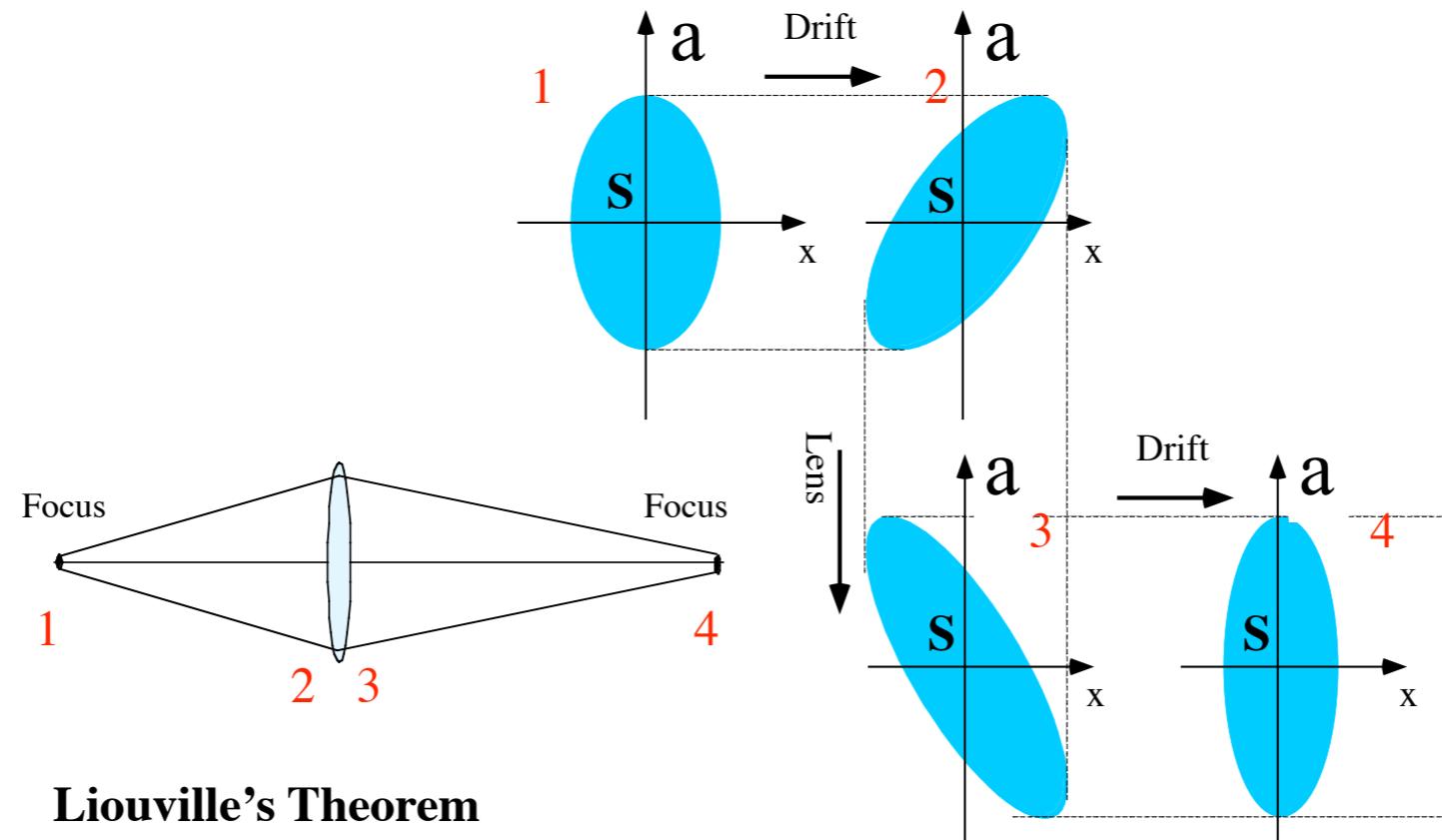
$$= \frac{a_0}{x_0} (\pi x_0^2)$$

$$= \pi a_0 x_0 \text{ [mm.mrad]}$$

$$\frac{x^2}{x_0^2} + \frac{a^2}{a_0^2} = 1$$

$$a = \pm a_0 \sqrt{1 - \frac{x^2}{x_0^2}}$$

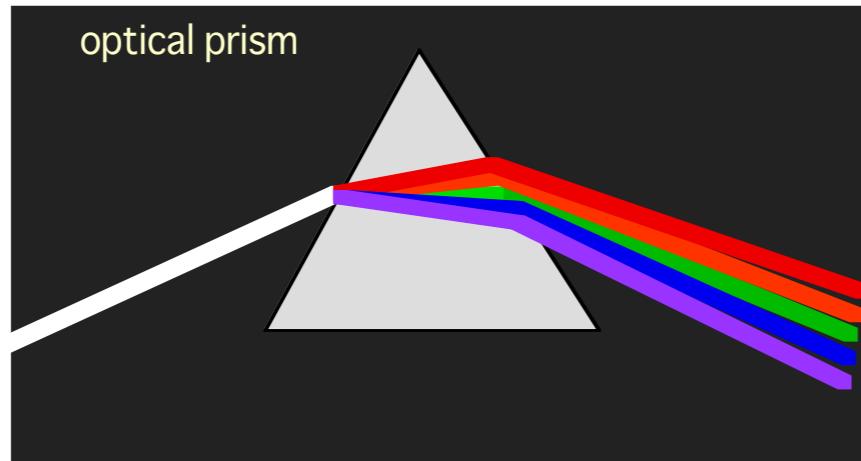
相空间守恒



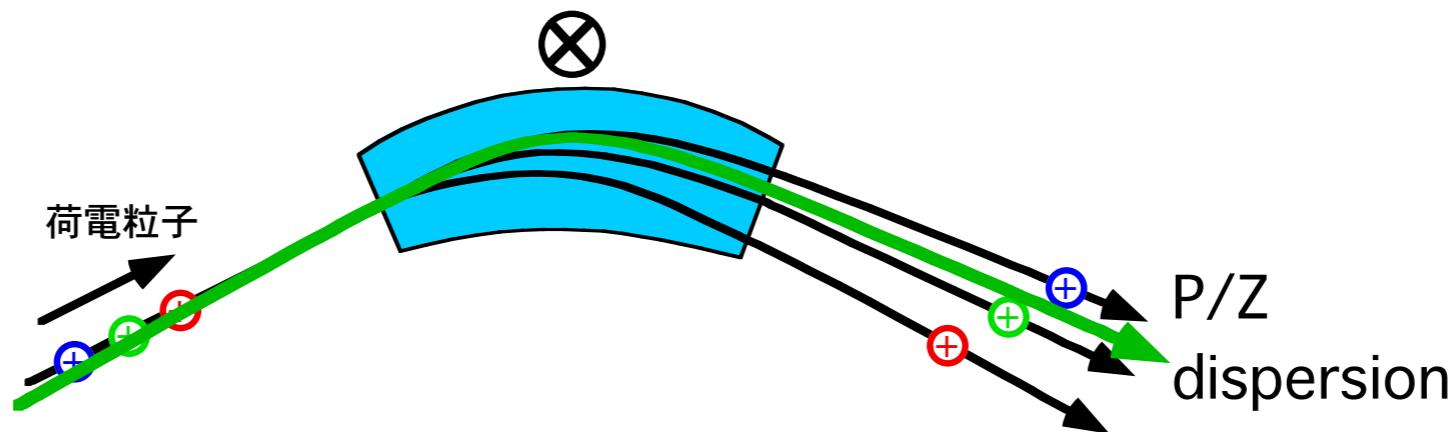
Note: ϵ shrinks (increases) with acceleration (deceleration);
Dissipative forces: ϵ increases in gases; electron, stochastic, laser cooling

Emittance is conserved under ONLY magnetic field influence

束流弯转：二极磁铁



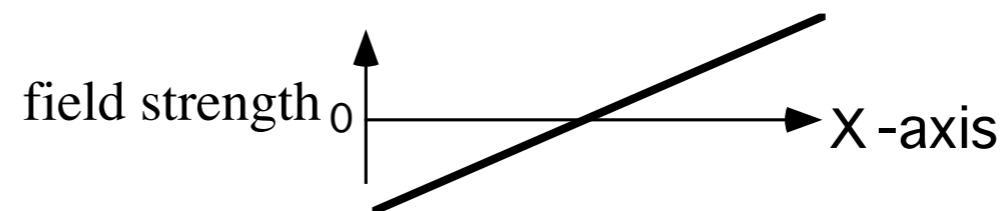
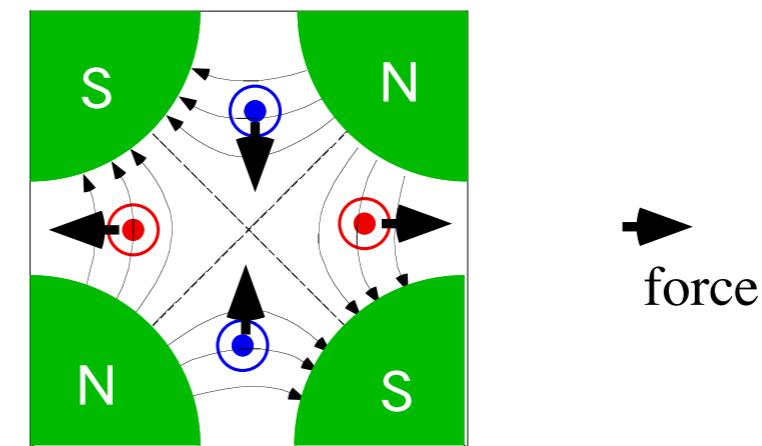
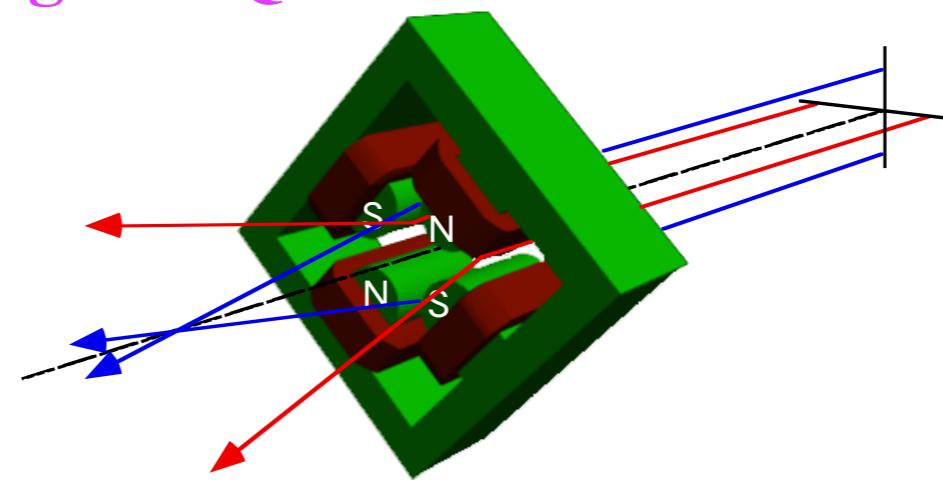
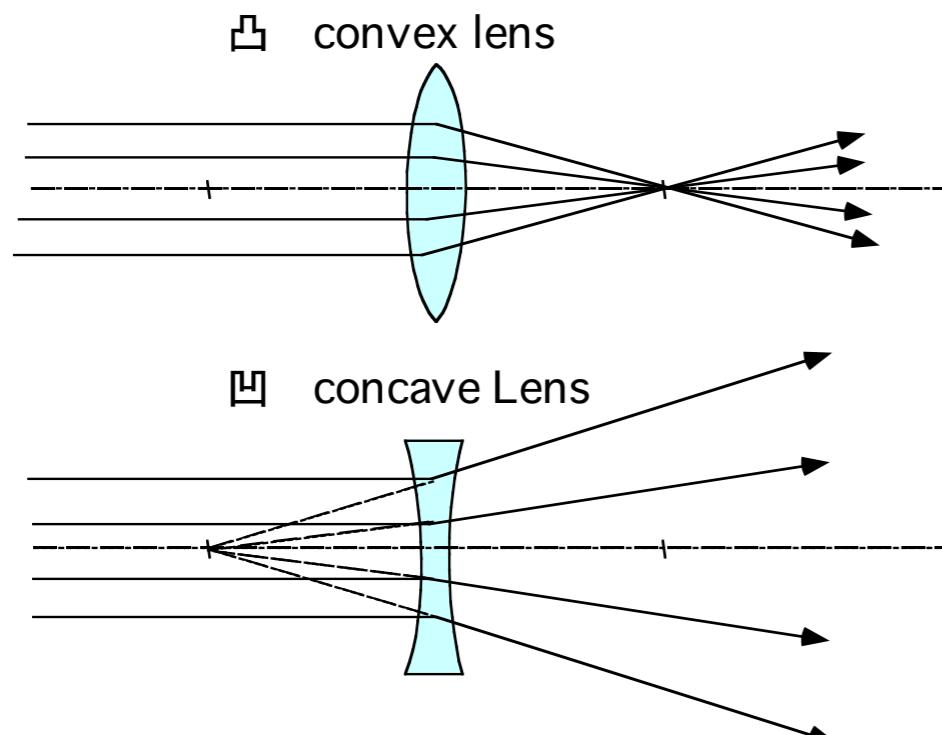
Dipole magnet 相等于“三棱镜”



Important component in particle/momentum spectrometer/analyizer

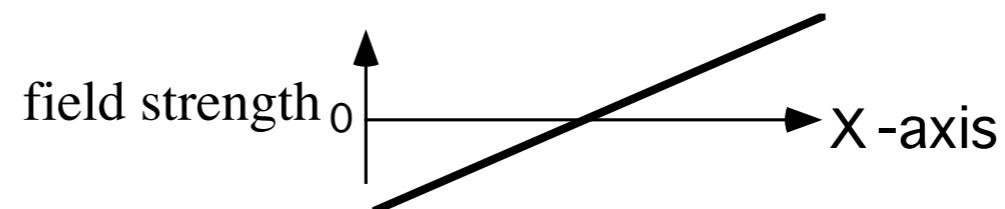
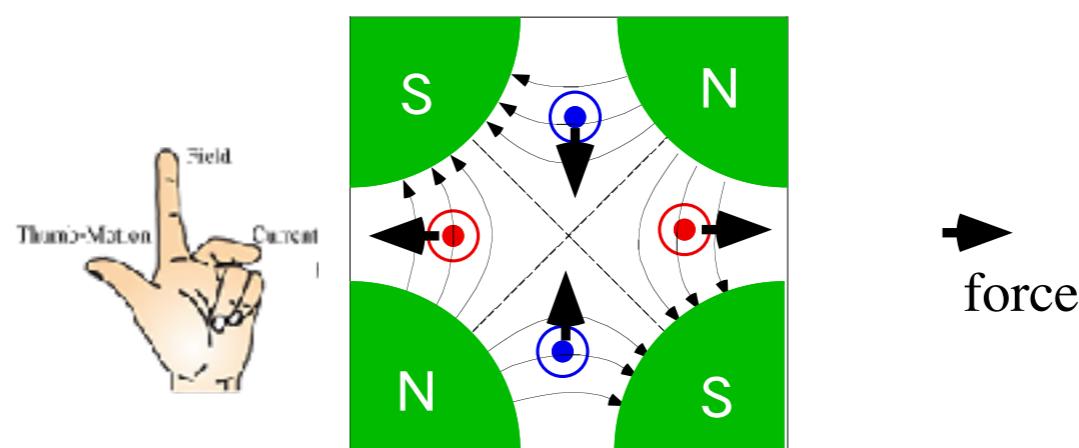
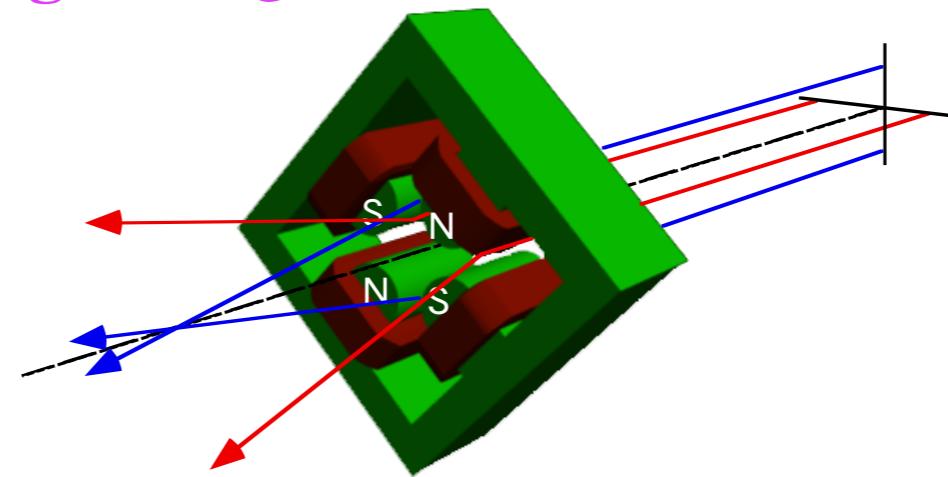
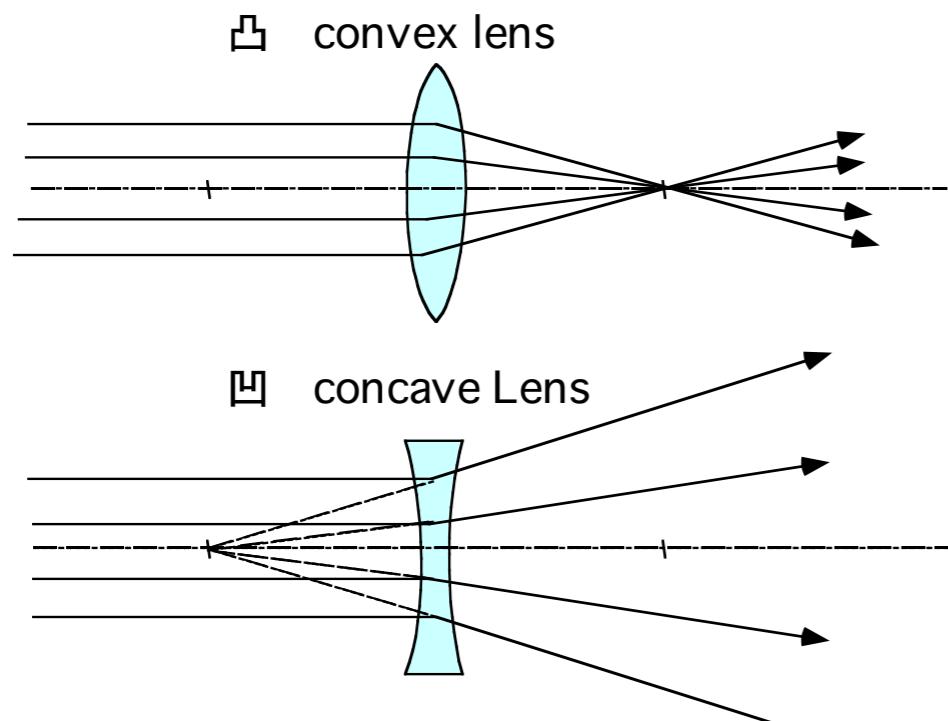
凹凸透镜：四极磁铁

quadrupole magnet : Q-lens



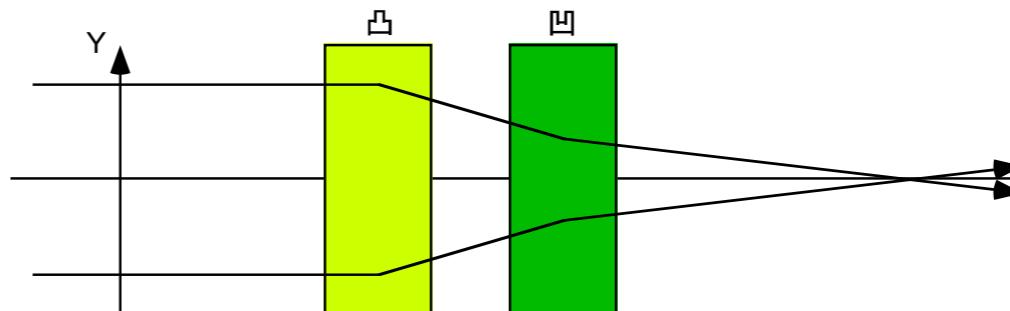
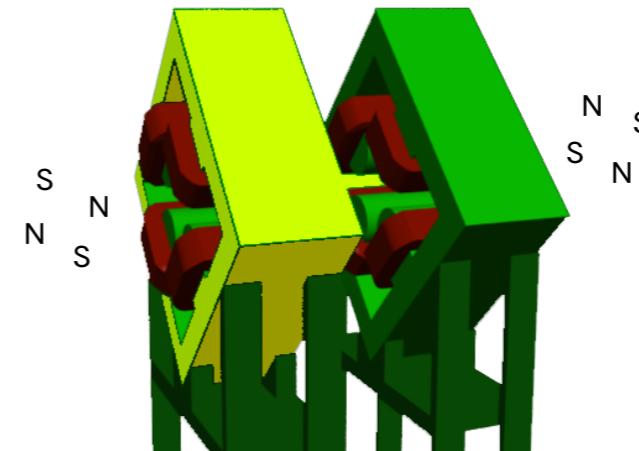
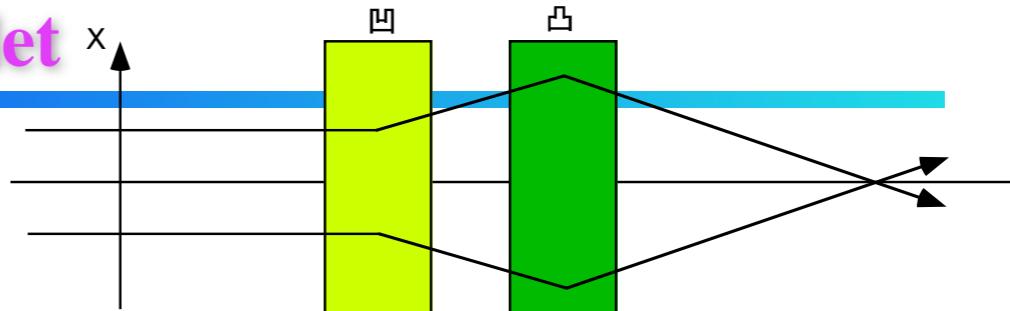
凹凸透镜：四极磁铁

quadrupole magnet : Q-lens



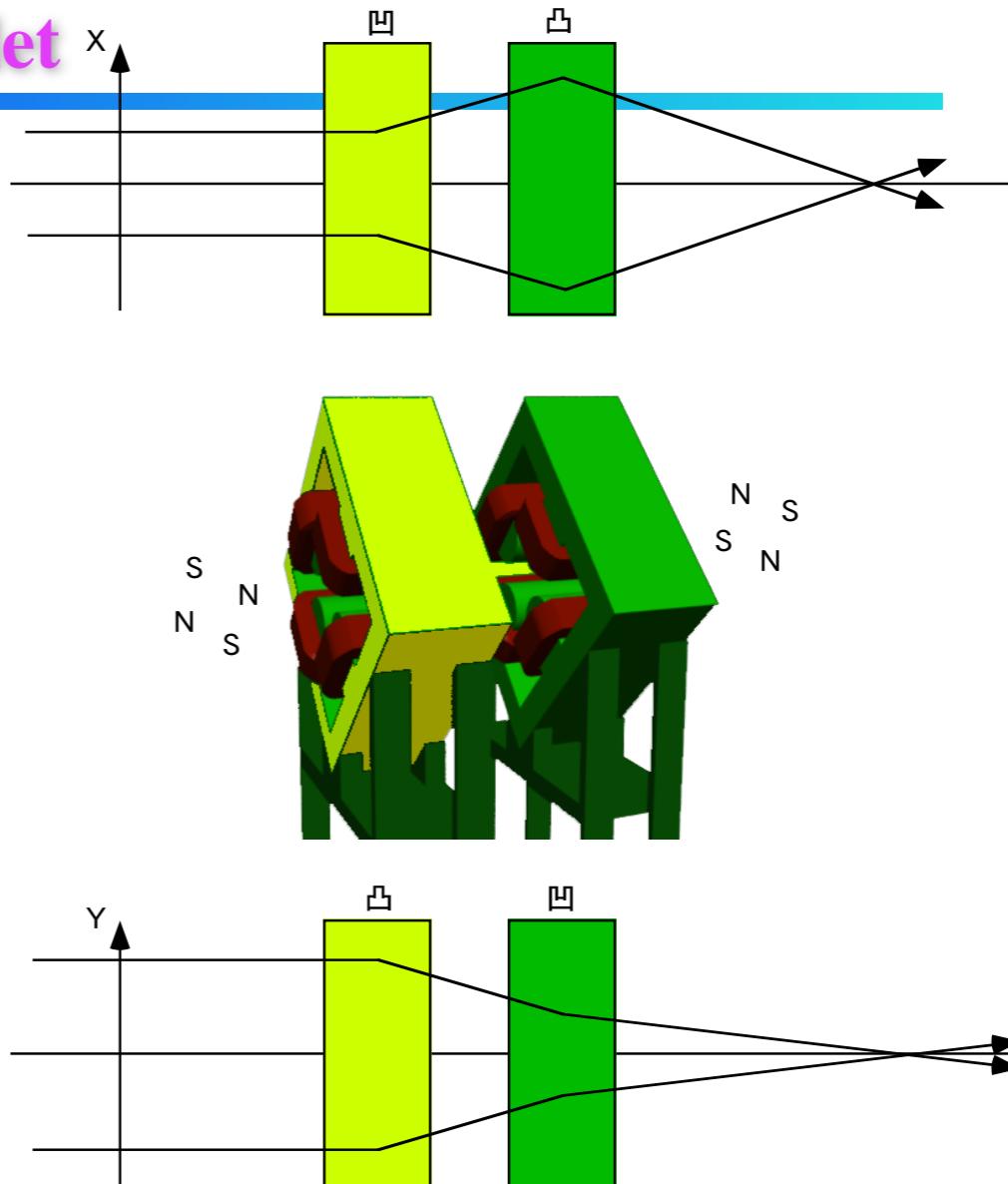
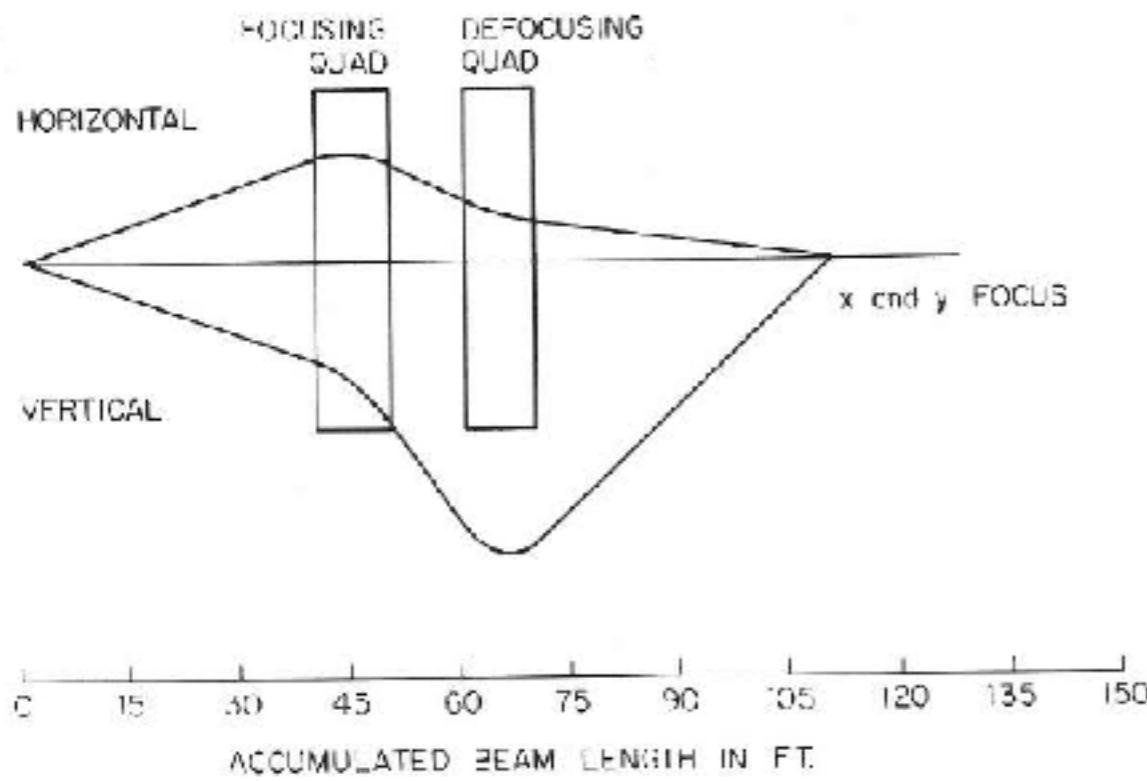
四极磁铁下的束流聚焦

Quadrupole Doublet



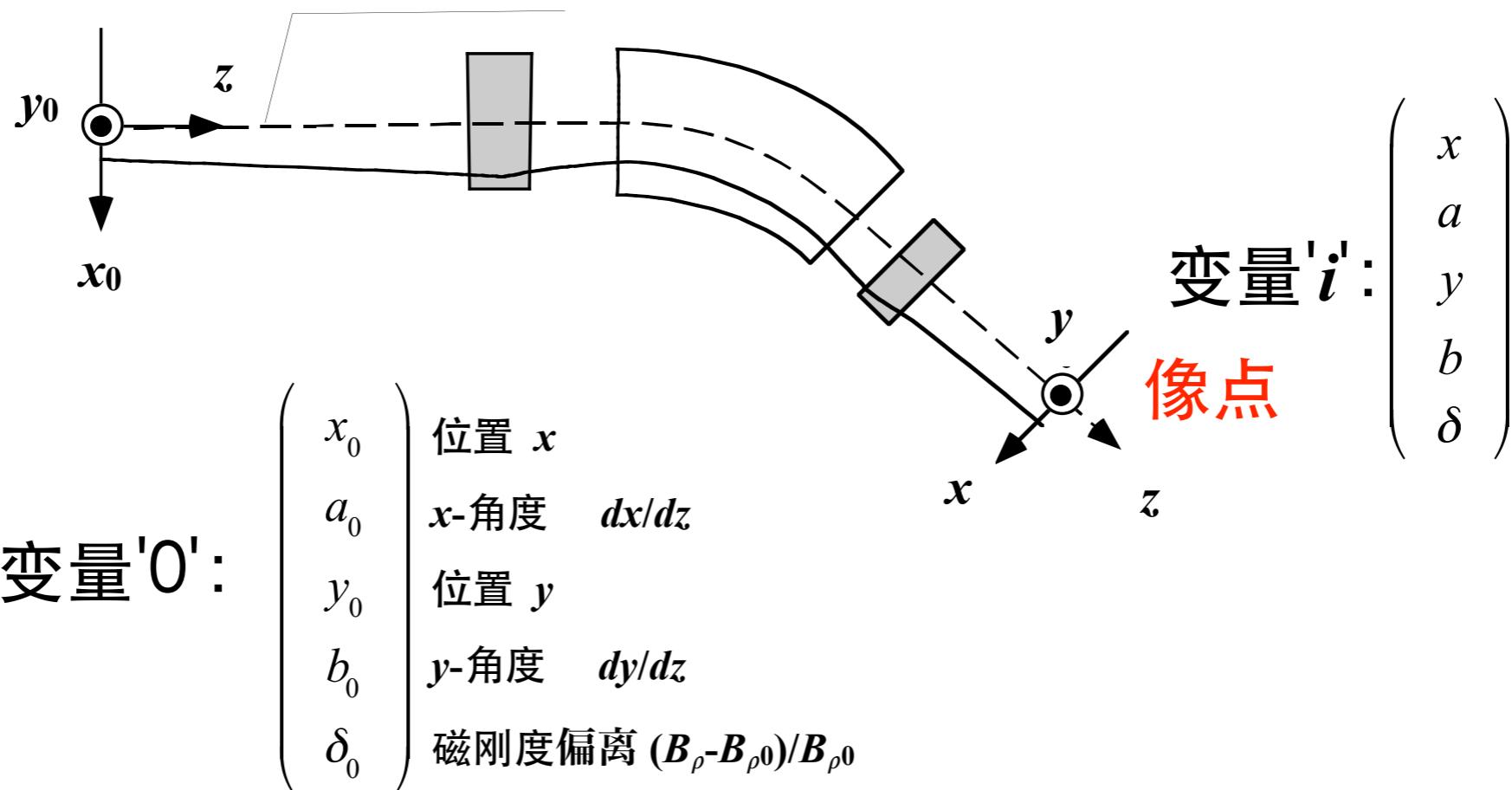
四极磁铁下的束流聚焦

Quadrupole Doublet



束流传输

物点



泰勒展开 (Taylor expansion):

$$\text{变量}'i': \xi_l = \sum_j \langle \xi_l | \xi_j \rangle \xi_{j0} + \sum_{j \geq k} \langle \xi_l | \xi_j \xi_k \rangle \xi_{j0} \xi_{k0} + \dots$$

$$\begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix} = M^1 \begin{pmatrix} x_0 \\ a_0 \\ y_0 \\ b_0 \\ \delta_0 \end{pmatrix} + M^2 \begin{pmatrix} x_0^2 \\ x_0 a_0 \\ x_0 y_0 \\ x_0 b_0 \\ x_0 \delta_0 \\ a_0^2 \\ \dots \\ \dots \end{pmatrix} + \dots$$

一阶 二阶
[传输矩阵]

$$\begin{aligned}
x &= \langle x | x \rangle x_0 + \langle x | a \rangle a_0 + \langle x | \delta \rangle \delta_0 + \langle x | x^2 \rangle x_0^2 + \langle x | x a \rangle x_0 a_0 + \langle x | a^2 \rangle a_0^2 \\
&\quad + \langle x | x \delta \rangle x_0 \delta_0 + \langle x | a \delta \rangle a_0 \delta_0 + \langle x | \delta^2 \rangle \delta_0^2 + \langle x | y^2 \rangle y_0^2 + \langle x | y b \rangle y_0 b_0 \\
&\quad + \langle x | b^2 \rangle b_0^2 + \dots \text{ 更高阶}
\end{aligned}$$

$a = \dots$

传输矩阵(transfer matrix) $M^i = M_n^i M_{n-1}^i \cdots M_1^i$

一阶传输矩阵

$$\begin{pmatrix} x \\ a \\ \delta \end{pmatrix} = \begin{pmatrix} < x | x > & < x | a > & < x | \delta > \\ < a | x > & < a | a > & < a | \delta > \\ < \delta | x > & < \delta | a > & < \delta | \delta > \end{pmatrix} \begin{pmatrix} x_0 \\ a_0 \\ \delta_0 \end{pmatrix}$$

双聚焦 : $\langle x | a \rangle = \langle y | b \rangle = 0$
double-angular focusing

双消色差 : $\langle x | \delta \rangle = \langle a | \delta \rangle = 0$
double-achromatic

$\langle x x \rangle$	放大倍数 magnification
$\langle x a \rangle$	聚焦 angular focusing
$\langle x \delta \rangle$	色散 chromatic (momentum) dispersion
$\langle a x \rangle$	焦距 focal length
$\langle a a \rangle$	角放大倍数 angular magnification
$\langle a \delta \rangle$	角色散 angular dispersion

漂移空间 drift space 薄透镜 thin lens
薄弯转磁铁
thin bending magnet

$$\begin{pmatrix} 1 & l & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -1/f & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & D \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ g(R, Z, A) & 0 & h(R, Z, A) \end{pmatrix}$$

薄物质 thin material

一阶传输矩阵

$$\begin{array}{c}
 \text{Magnification } M_x \\
 \text{Focusing fact} \\
 \text{Lateral Dispersion} \\
 \text{Angular Disp}
 \end{array}
 \begin{array}{c}
 \left[\begin{array}{c} x(t) \\ \theta(t) \\ y(t) \\ \varphi(t) \\ l(t) \\ \delta(t) \end{array} \right] = \left[\begin{array}{cccccc} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \cdot \left[\begin{array}{c} x_0 \\ \theta_0 \\ y_0 \\ \varphi_0 \\ l_0 \\ \delta_0 \end{array} \right]
 \end{array}$$

TRANSPORT-R-Matrix
 B for Beam Line
 & for Spektrometer

- 由于磁铁大都满足中线平面对称性，多数非对角矩阵元为0
- 对于设计完好的磁铁和发射度小的束流，可以采用线性近似(一阶矩阵)

四极铁的一阶传输矩阵

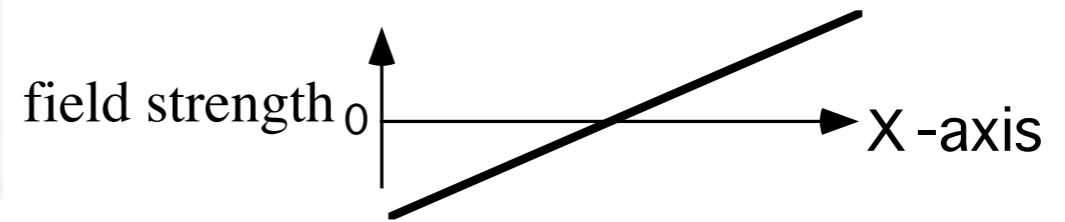
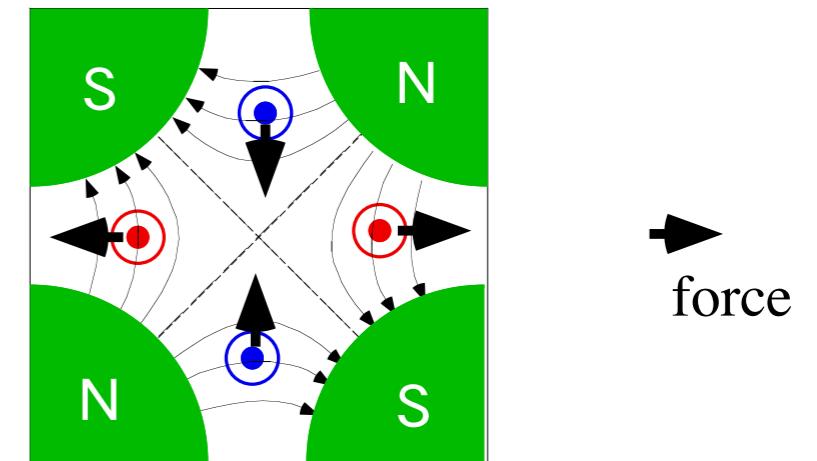
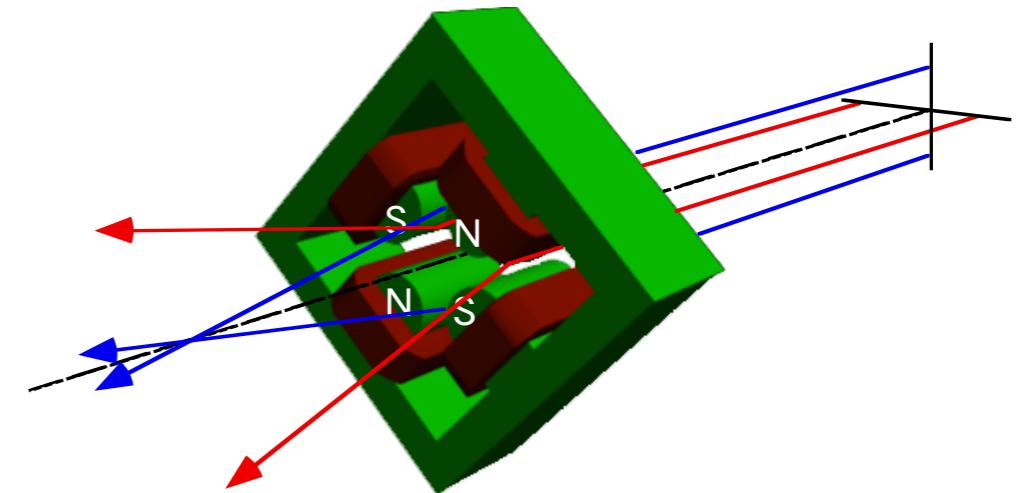
First-order quadrupole matrix $\frac{dB}{dx} \neq 0 \quad \frac{dB}{dy} \neq 0$

$\cos k_q L$	$\frac{1}{k_q} \sin k_q L$	0	0	0	0
$-k_q \sin k_q L$	$\cos k_q L$	0	0	0	0
0	0	$\cosh k_q L$	$\frac{1}{k_q} \sinh k_q L$	0	0
0	0	$k_q \sinh k_q L$	$\cosh k_q L$	0	0
0	0	0	0	1	0
0	0	0	0	0	1

These elements are for a quadrupole which focuses in the horizontal (x) plane (B positive). A vertically (y-plane) focusing quadrupole (B negative) has the first two diagonal submatrices interchanged.

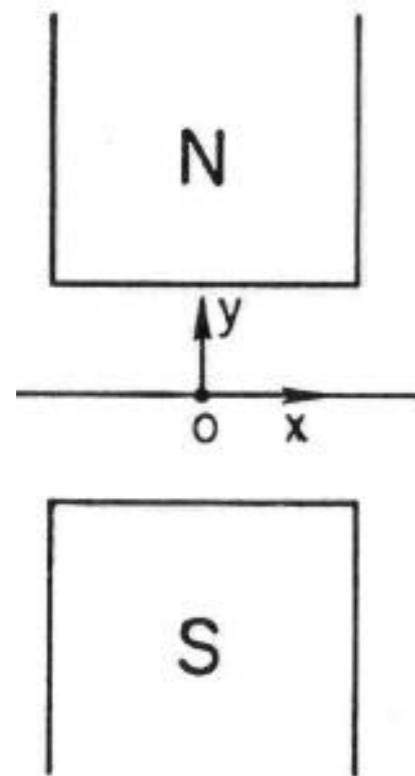
Definitions:

- L = the effective length of the quadrupole
- a = the radius of the aperture
- B_0 = the field at radius a
- $k_q^2 = (B_0/a)(1/B_{p0})$, where (B_{p0}) = the magnetic rigidity (momentum) of the central trajectory.

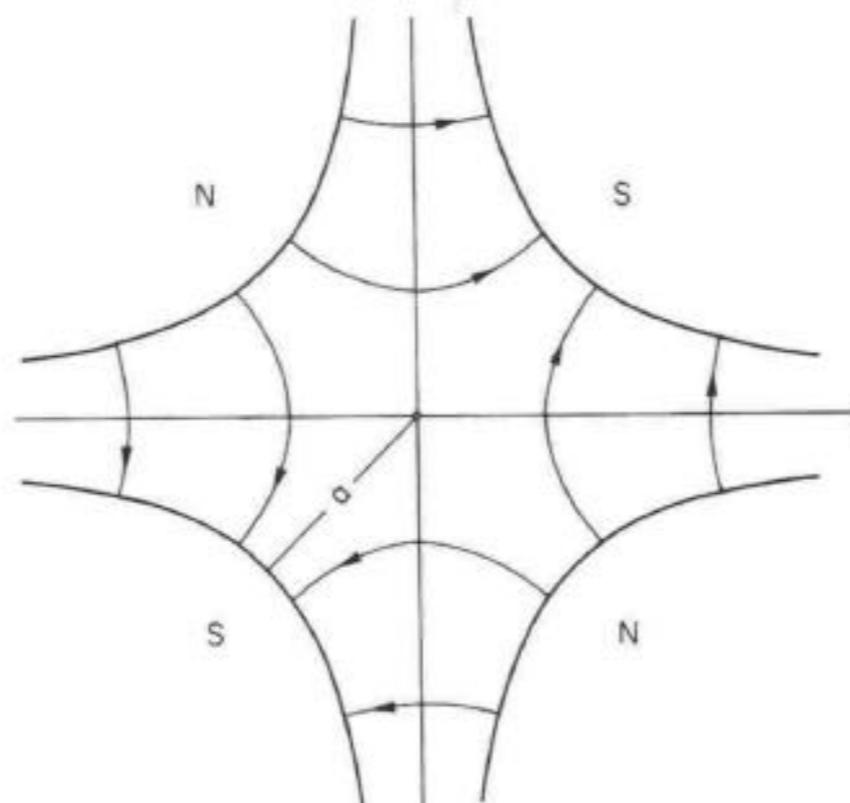


常用磁铁

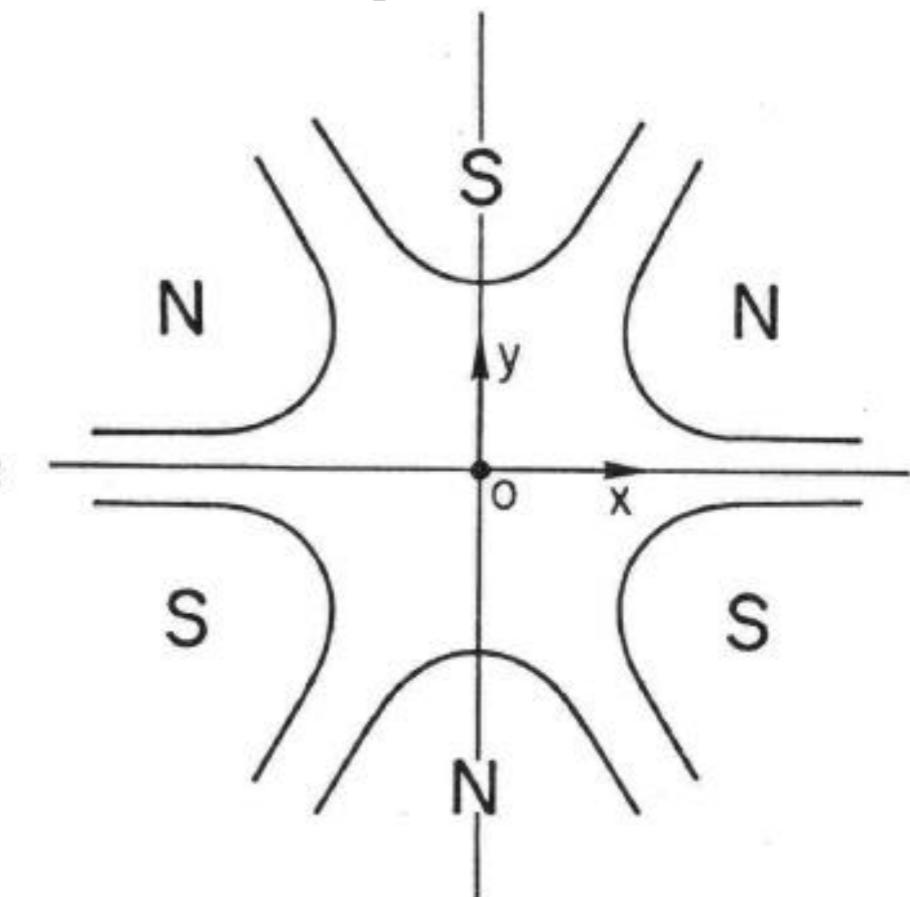
二极磁铁



四极磁铁



六极磁铁



DIPOLE

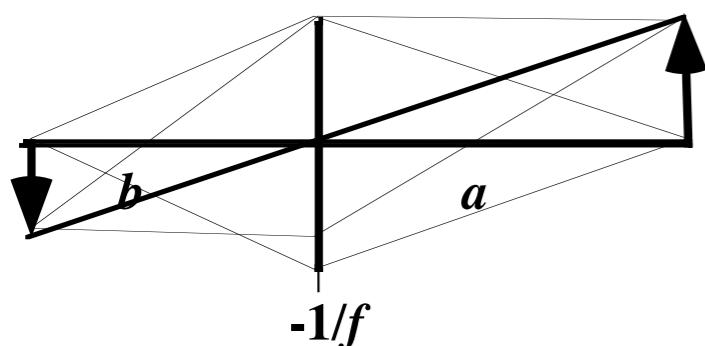
QUADRUPOLE

SEXTUPOLE

(束流)光学

漂移空间 drift space 薄透镜 thin lens

$$\begin{pmatrix} 1 & l & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ -1/f & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$M = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-b/f & a-ab/f+b \\ -1/f & 1-a/f \end{pmatrix}$$

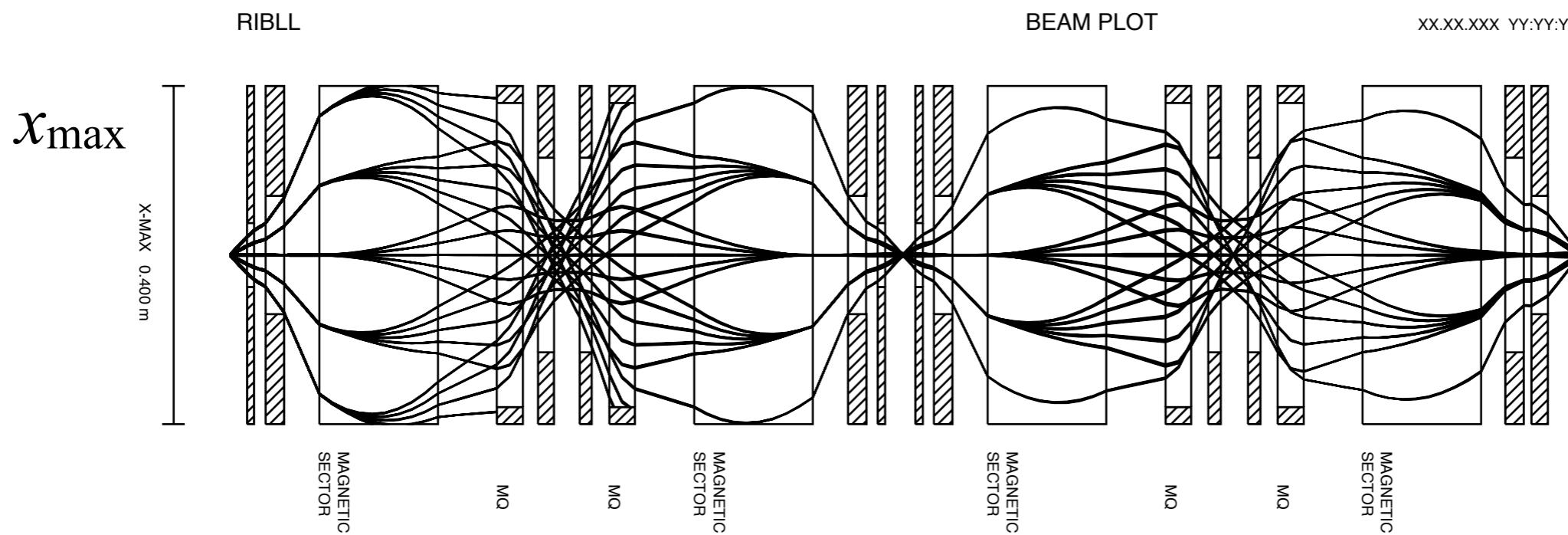
$$\text{聚焦: } a - ab/f + b = 0 \Rightarrow \frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$

$$M = \begin{pmatrix} -b/a & 0 \\ -1/f & -a/b \end{pmatrix}$$

束流包络线

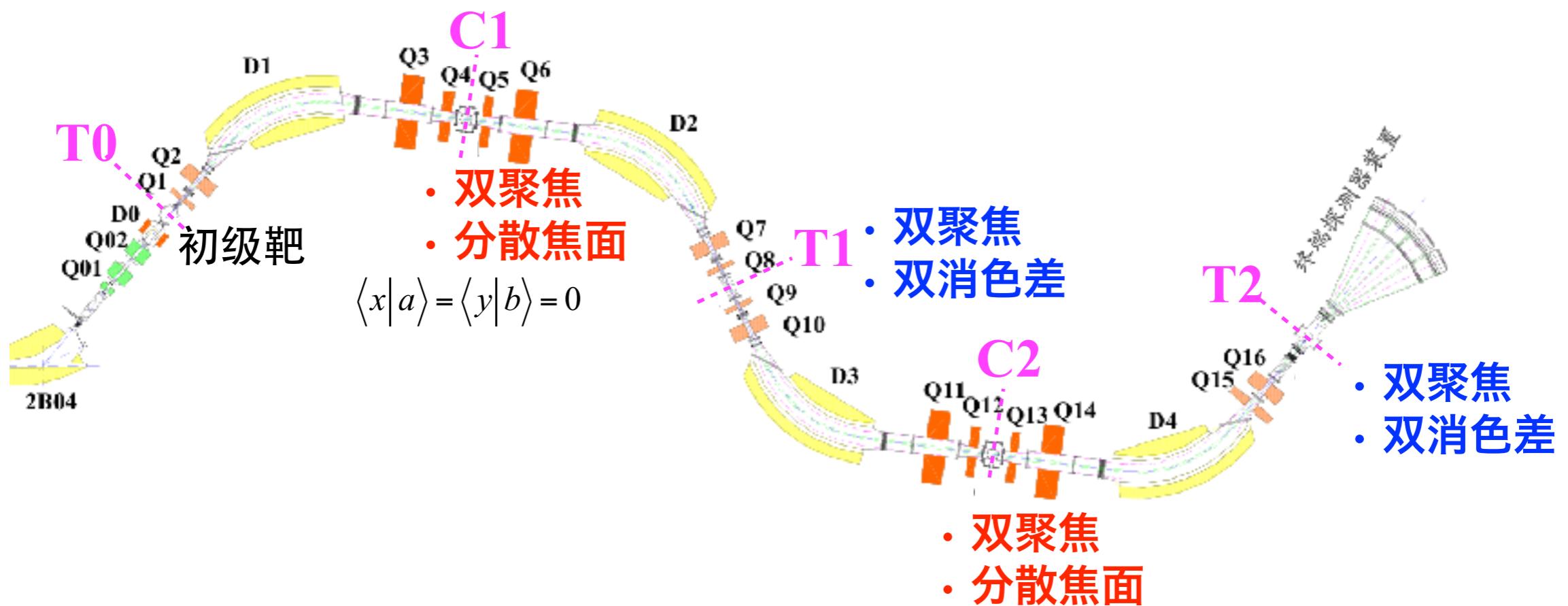
$$\begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix} = M \begin{pmatrix} x_0 \\ a_0 \\ y_0 \\ b_0 \\ \delta_0 \end{pmatrix}$$

- RMS of horizontal (x) and vertical (y) positions along the beam direction.
 $\langle x^2 \rangle^{1/2}$
 $\langle y^2 \rangle^{1/2}$ for centered beam
- maximum positions: x_{\max}, y_{\max}



检验束流传输

束流线(光学)设计



- 基于各焦平面的条件，分阶段($T_0 \rightarrow C_1$, $T_0 \rightarrow T_1$ (固定 $T_0 \rightarrow C_1$) $\rightarrow \dots$)求解四极磁铁的磁场强度

条件: $\langle x|a \rangle = \langle y|b \rangle = 0$ 、 $\langle x|\delta \rangle = \langle a|\delta \rangle = 0$ 、 $\langle x|x \rangle = M_x$ 、 $\langle y|y \rangle = M_y$ 等

RIBLL1的传输矩阵

T0 → C1

Block matrix						
1. X	-0.5428	0.0824	0	0	0	20.482
2. T	-0.0249	-1.8385	0	0	0	-0.0333
3. Y	0	0	-10.142	1.3268	0	0
4. P	0	0	-0.0169	-0.0964	0	0
5. L	-0.52889	-3.7652	0	0	1	7.0278
6. D	0	0	0	0	0	1
	/[mm]	/[mrad]	/[mm]	/[mrad]	/[mm]	/[%]

C1→T1

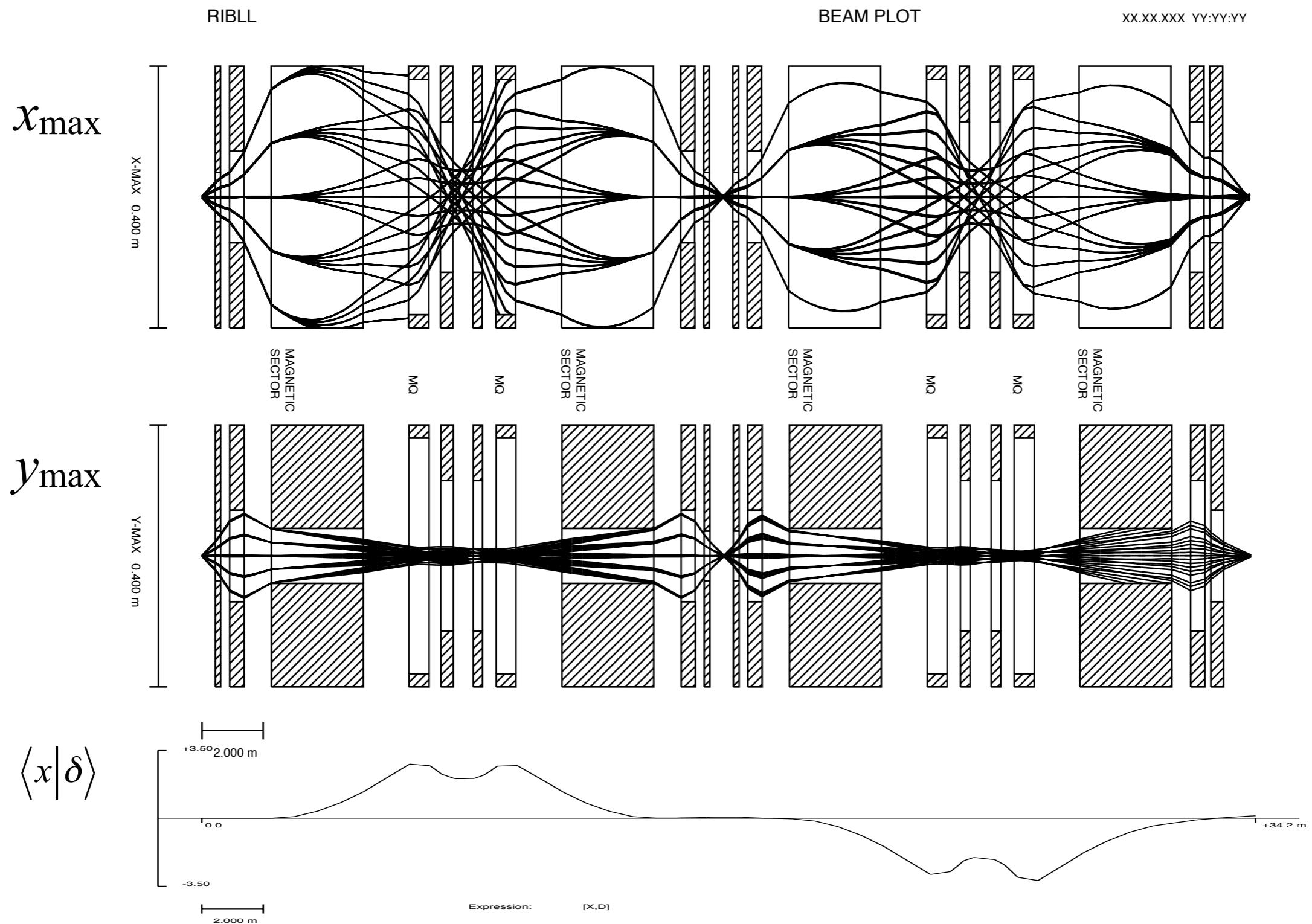
Block matrix						
1. X	-1.8464	-0.08232	0	0	0	37.788
2. T	-0.02503	-0.5427	0	0	0	0.5306
3. Y	0	0	-0.10036	-1.3253	0	0
4. P	0	0	-9.709e-3	-10.09	0	0
5. L	-0.81148	-2.0691	0	0	1	151.67
6. D	0	0	0	0	0	1
	/[mm]	/[mrad]	/[mm]	/[mrad]	/[mm]	/[%]

2-nd
order

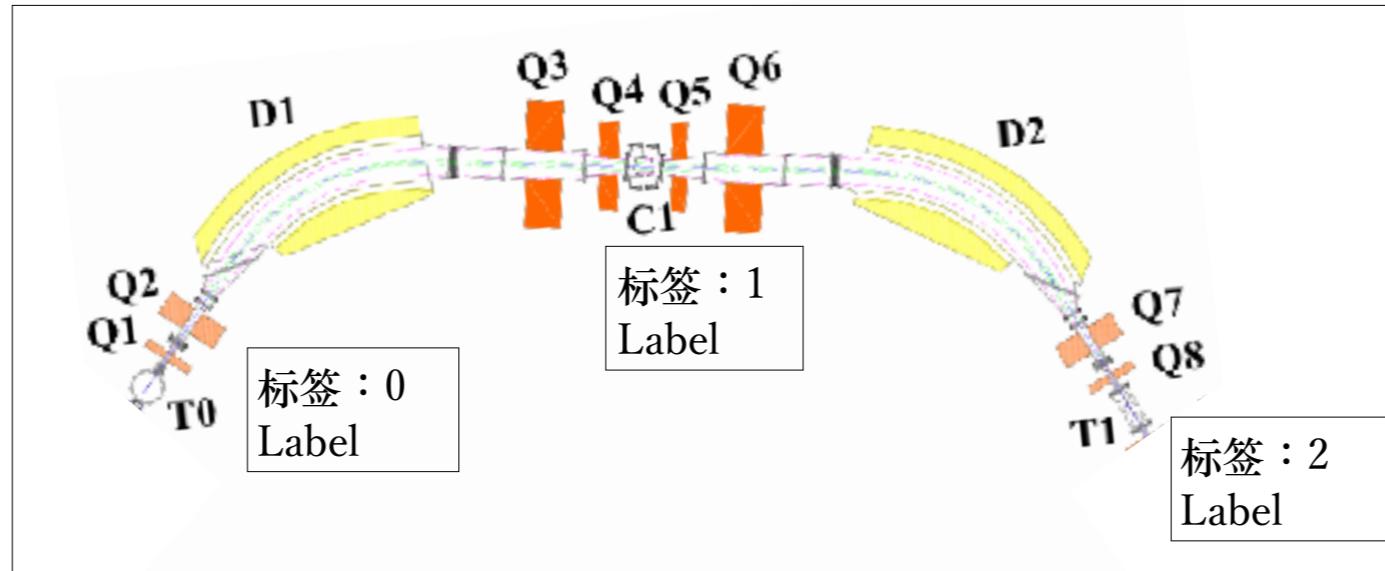
T0→T1

Global matrix						
1.00428	-7.98e-4	0	0	0	-0.02722	[mm]
0.0271	0.99569	0	0	0	0.03601	[mrad]
0	0	1.04025	-5.399e-3	0	0	[mm]
0	0	0.26899	0.95979	0	0	[mrad]
-0.0369	-0.02803	0	0	1	142.1459	[mm]
0	0	0	0	0	1	[%]
	/[mm]	/[mrad]	/[mm]	/[mrad]	/[mm]	/[%]

RIBLL1束流包络



降能器/物质下的束流光学



传输矩阵：

$$\begin{aligned}
 & \mathbf{T0} \rightarrow \mathbf{T1} \quad \mathbf{C1} \rightarrow \mathbf{T1} \quad \text{降能器} \quad \mathbf{T0} \rightarrow \mathbf{C1} \\
 \left(\begin{array}{ccc} \langle x|x \rangle_{0 \rightarrow 2} & \langle x|\theta \rangle_{0 \rightarrow 2} & \langle x|\delta \rangle_{0 \rightarrow 2} \\ \langle \theta|x \rangle_{0 \rightarrow 2} & \langle \theta|\theta \rangle_{0 \rightarrow 2} & \langle \theta|\delta \rangle_{0 \rightarrow 2} \\ \langle \delta|x \rangle_{0 \rightarrow 2} & \langle \delta|\theta \rangle_{0 \rightarrow 2} & \langle \delta|\delta \rangle_{0 \rightarrow 2} \end{array} \right) = \left(\begin{array}{ccc} \langle x|x \rangle_{1 \rightarrow 2} & \langle x|\theta \rangle_{1 \rightarrow 2} & \langle x|\delta \rangle_{1 \rightarrow 2} \\ \langle \theta|x \rangle_{1 \rightarrow 2} & \langle \theta|\theta \rangle_{1 \rightarrow 2} & \langle \theta|\delta \rangle_{1 \rightarrow 2} \\ 0 & 0 & 1 \end{array} \right) \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \langle \delta|x \rangle_d & \langle \delta|\theta \rangle_d & \langle \delta|\delta \rangle_d \end{array} \right) \left(\begin{array}{ccc} \langle x|x \rangle_{0 \rightarrow 1} & \langle x|\theta \rangle_{0 \rightarrow 1} & \langle x|\delta \rangle_{0 \rightarrow 1} \\ \langle \theta|x \rangle_{0 \rightarrow 1} & \langle \theta|\theta \rangle_{0 \rightarrow 1} & \langle \theta|\delta \rangle_{0 \rightarrow 1} \\ 0 & 0 & 1 \end{array} \right) \\
 & = \left(\begin{array}{ccc} \langle x|x \rangle_{1 \rightarrow 2} + \langle x|\delta \rangle_{1 \rightarrow 2} \langle \delta|x \rangle_d & \langle x|\theta \rangle_{1 \rightarrow 2} + \langle x|\delta \rangle_{1 \rightarrow 2} \langle \delta|\theta \rangle_d & \langle x|\delta \rangle_{1 \rightarrow 2} \langle \delta|\delta \rangle_d \\ \langle \theta|x \rangle_{1 \rightarrow 2} + \langle \theta|\delta \rangle_{1 \rightarrow 2} \langle \delta|x \rangle_d & \langle \theta|\theta \rangle_{1 \rightarrow 2} + \langle \theta|\delta \rangle_{1 \rightarrow 2} \langle \delta|\theta \rangle_d & \langle \theta|\delta \rangle_{1 \rightarrow 2} \langle \delta|\delta \rangle_d \\ \langle \delta|x \rangle_d & \langle \delta|\theta \rangle_d & \langle \delta|\delta \rangle_d \end{array} \right) \times \left(\begin{array}{ccc} \langle x|x \rangle_{0 \rightarrow 1} & \langle x|\theta \rangle_{0 \rightarrow 1} & \langle x|\delta \rangle_{0 \rightarrow 1} \\ \langle \theta|x \rangle_{0 \rightarrow 1} & \langle \theta|\theta \rangle_{0 \rightarrow 1} & \langle \theta|\delta \rangle_{0 \rightarrow 1} \\ 0 & 0 & 1 \end{array} \right)
 \end{aligned}$$

动能“ E_0 ”、质量数“ A ”、质子数“ Z ”的离子在降能器中的射程：

$$R = kA^{1-\gamma}E_0^\gamma/Z^2 \quad ; \quad k, \gamma : \text{参数}$$

经过厚度 “ d_0 ” 降能器后的离子能量： $E_d = E_0 \left(1 - \frac{d_0}{R}\right)^{1/\gamma}$

泰勒展开 $\frac{E_d(x, \theta, \delta, \dots) - E_d(0,0,0, \dots)}{E_d(0,0,0, \dots)} = \langle \delta | x \rangle_d x + \langle \delta | \theta \rangle_d \theta + \langle \delta | \delta \rangle_d \delta + \dots$

$$\text{降能器的矩阵元} \left\{ \begin{array}{l} \langle \delta | x \rangle_d = \left(\frac{1}{E_d} \frac{\partial E_d}{\partial x} \right)_{x=0} = \frac{-1}{\gamma R \left(1 - \frac{d_0}{R}\right)} \left(\frac{\partial d}{\partial x} \right)_{x=0} \\ \langle \delta | \theta \rangle_d = \left(\frac{1}{E_d} \frac{\partial E_d}{\partial \theta} \right)_{\theta=0} \\ \langle \delta | \delta \rangle_d = \left(\frac{1}{E_d} \frac{\partial E_d}{\partial \delta} \right)_{\delta=0} = \frac{1}{1 - d_0/R} \end{array} \right.$$

T1聚焦条件：

$$\begin{aligned}
 \langle x | \theta \rangle_{0 \rightarrow 2} &= \langle x | x \rangle_{1 \rightarrow 2} \langle x | \theta \rangle_{0 \rightarrow 1} + \langle x | \theta \rangle_{1 \rightarrow 2} \langle \theta | \theta \rangle_{0 \rightarrow 1} + \langle x | \delta \rangle_{1 \rightarrow 2} \langle \delta | x \rangle_d \langle x | \theta \rangle_{0 \rightarrow 1} + \langle x | \delta \rangle_{1 \rightarrow 2} \langle \delta | \theta \rangle_1 \langle \theta | \theta \rangle_{0 \rightarrow 1} \\
 &= \langle x | x \rangle_{1 \rightarrow 2} \langle x | \theta \rangle_{0 \rightarrow 1} + \langle x | \theta \rangle_{1 \rightarrow 2} \langle \theta | \theta \rangle_{0 \rightarrow 1} + \langle x | \delta \rangle_{1 \rightarrow 2} \{ \langle \delta | x \rangle_d \langle x | \theta \rangle_{0 \rightarrow 1} + \langle \delta | \theta \rangle_d \langle \theta | \theta \rangle_{0 \rightarrow 1} \} \\
 &= 0
 \end{aligned} \tag{1a}$$

T1消色差条件：

$$\begin{aligned}
 \langle x | \delta \rangle_{0 \rightarrow 2} &= \langle x | x \rangle_{1 \rightarrow 2} \langle x | \delta \rangle_{0 \rightarrow 1} + \langle x | \delta \rangle_{1 \rightarrow 2} \langle \delta | x \rangle_d \langle x | \delta \rangle_{0 \rightarrow 1} + \langle x | \theta \rangle_{1 \rightarrow 2} \langle \theta | \delta \rangle_{0 \rightarrow 1} + \langle x | \delta \rangle_{1 \rightarrow 2} \langle \delta | \theta \rangle_d \langle \theta | \delta \rangle_{0 \rightarrow 1} + \langle x | \delta \rangle_{1 \rightarrow 2} \langle \delta | \delta \rangle_d \\
 &= \langle x | x \rangle_{1 \rightarrow 2} \langle x | \delta \rangle_{0 \rightarrow 1} + \langle x | \theta \rangle_{1 \rightarrow 2} \langle \theta | \delta \rangle_{0 \rightarrow 1} + \langle x | \delta \rangle_{1 \rightarrow 2} \{ \langle \delta | x \rangle_d \langle x | \delta \rangle_{0 \rightarrow 1} + \langle \delta | \theta \rangle_d \langle \theta | \delta \rangle_{0 \rightarrow 1} + \langle \delta | \delta \rangle_d \} \\
 &= 0
 \end{aligned} \tag{1b}$$

平板降能器，或一般楔形降能器： $\langle \delta | \theta \rangle = 0$
 为减少降能器中多次散射的影响，要求 $\langle x | \theta \rangle_{1 \rightarrow 2} = 0$

代入方程(1a)、(1b)，T1聚焦条件： $\{ \langle x | x \rangle_{1 \rightarrow 2} + \langle x | \delta \rangle_{1 \rightarrow 2} \langle \delta | x \rangle_d \} \langle x | \theta \rangle_{0 \rightarrow 1} = 0 \Rightarrow \langle x | \theta \rangle_{0 \rightarrow 1} = 0$

$$\begin{aligned}
 \text{T1消色差条件} : \langle x | x \rangle_{1 \rightarrow 2} \langle x | \delta \rangle_{0 \rightarrow 1} + \langle x | \delta \rangle_{1 \rightarrow 2} \{ \langle \delta | x \rangle_d \langle x | \delta \rangle_{0 \rightarrow 1} + \langle \delta | \delta \rangle_d \} &= 0 \\
 D_{0 \rightarrow 1} M_{1 \rightarrow 2} + D_{1 \rightarrow 2} \{ D_{0 \rightarrow 1} \langle \delta | x \rangle_d + \langle \delta | \delta \rangle_d \} &= 0
 \end{aligned} \tag{2b}$$

放大倍数： $M_{1 \rightarrow 2} = \langle x | x \rangle_{1 \rightarrow 2}$ ；色散： $D_{0 \rightarrow 1} = \langle x | \delta \rangle_{0 \rightarrow 1}$ ， $D_{1 \rightarrow 2} = \langle x | \delta \rangle_{1 \rightarrow 2}$

● 无降能器下的束流光学

$$\langle \delta | x \rangle_d = 0, \langle \delta | \delta \rangle_d = 1$$

从方程(2b)： $D_{0 \rightarrow 1} M_{1 \rightarrow 2} + D_{1 \rightarrow 2} = 0$ (束流光学设计)

● 楔形降能器

为保证降能器不改变消色差条件，要求

$$D_{0 \rightarrow 1} \langle \delta | x \rangle_d + \langle \delta | \delta \rangle_d = 1 \quad \dots(3b)$$

$$D_{0 \rightarrow 1} \frac{-1}{\gamma R \left(1 - \frac{d_0}{R}\right)} \left(\frac{\partial d}{\partial x}\right)_{x=0} + \frac{1}{1 - d_0/R} = 1$$

$$D_{0 \rightarrow 1} \frac{-1}{\gamma R} \left(\frac{\partial d}{\partial x}\right)_{x=0} = -d_0/R$$

$$\left(\frac{\partial d}{\partial x}\right)_{x=0} = \gamma d_0 / D_{0 \rightarrow 1}$$

积分后得

$$d(x) = d_0 \left(1 + \frac{x\gamma}{D_{0 \rightarrow 1}}\right) \quad (\text{楔形降能器})$$

● 平板降能器

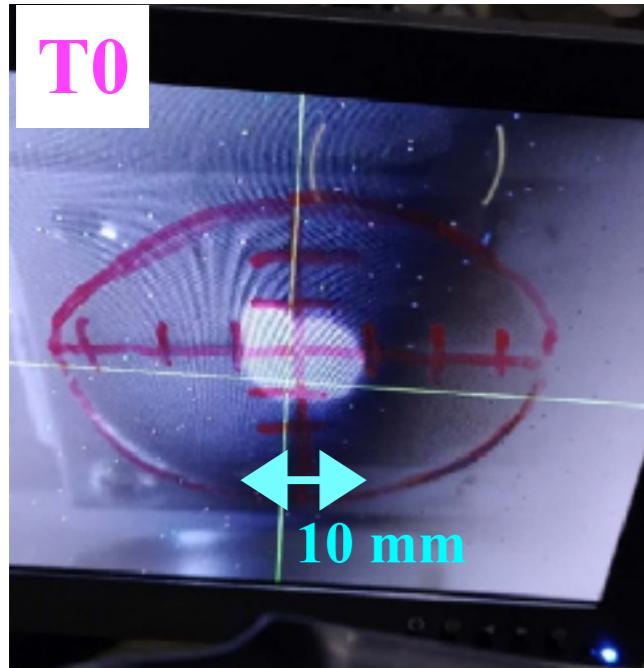
$$\langle \delta | x \rangle_d = 0$$

方程(2b)消色差条件： $D_{0 \rightarrow 1} M_{1 \rightarrow 2} + D_{1 \rightarrow 2} \left\{ D_{0 \rightarrow 1} \langle \delta | x \rangle_d + \langle \delta | \delta \rangle_d \right\} = D_{0 \rightarrow 1} M_{1 \rightarrow 2} + D_{1 \rightarrow 2} \langle \delta | \delta \rangle_d = 0$

$$D_{0 \rightarrow 1} M_{1 \rightarrow 2} + D_{1 \rightarrow 2} \langle \delta | \delta \rangle_d = D_{0 \rightarrow 1} M_{1 \rightarrow 2} + \frac{D_{1 \rightarrow 2}}{1 - d_0/R} = 0$$

对于不同厚度的降能器与动能(d_0/R)，需要采用**不同的束流光学设置**

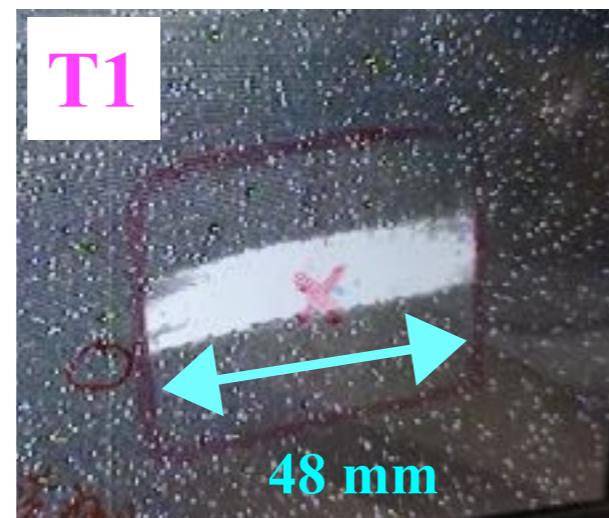
降能器/物质下的束流传输



• 无降能器下



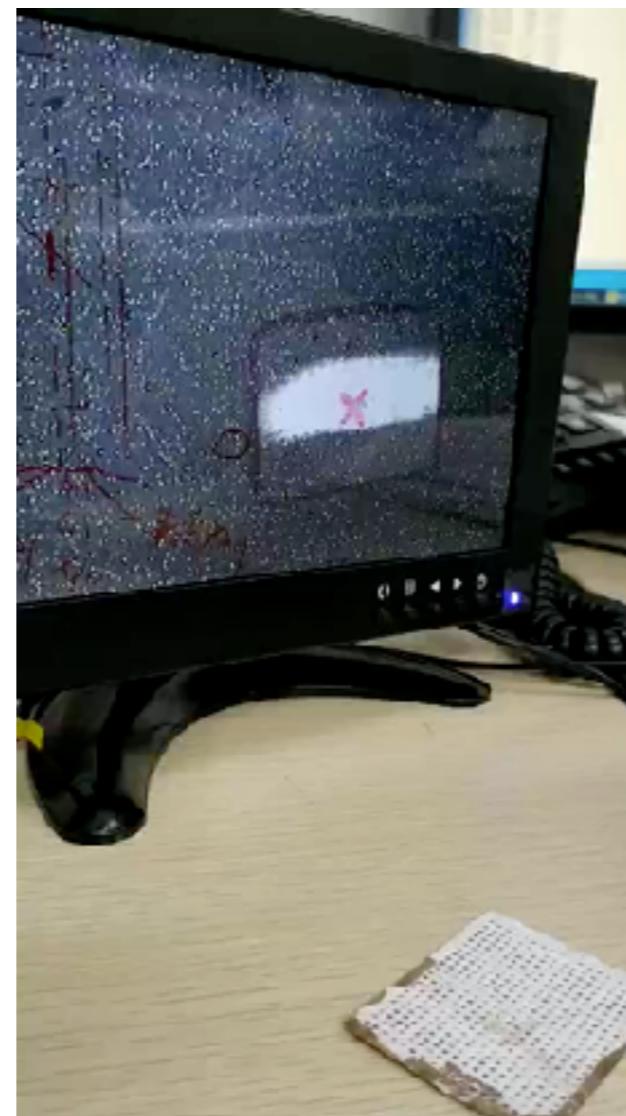
• 平板降能器($d_0/R \sim 0.4$)下



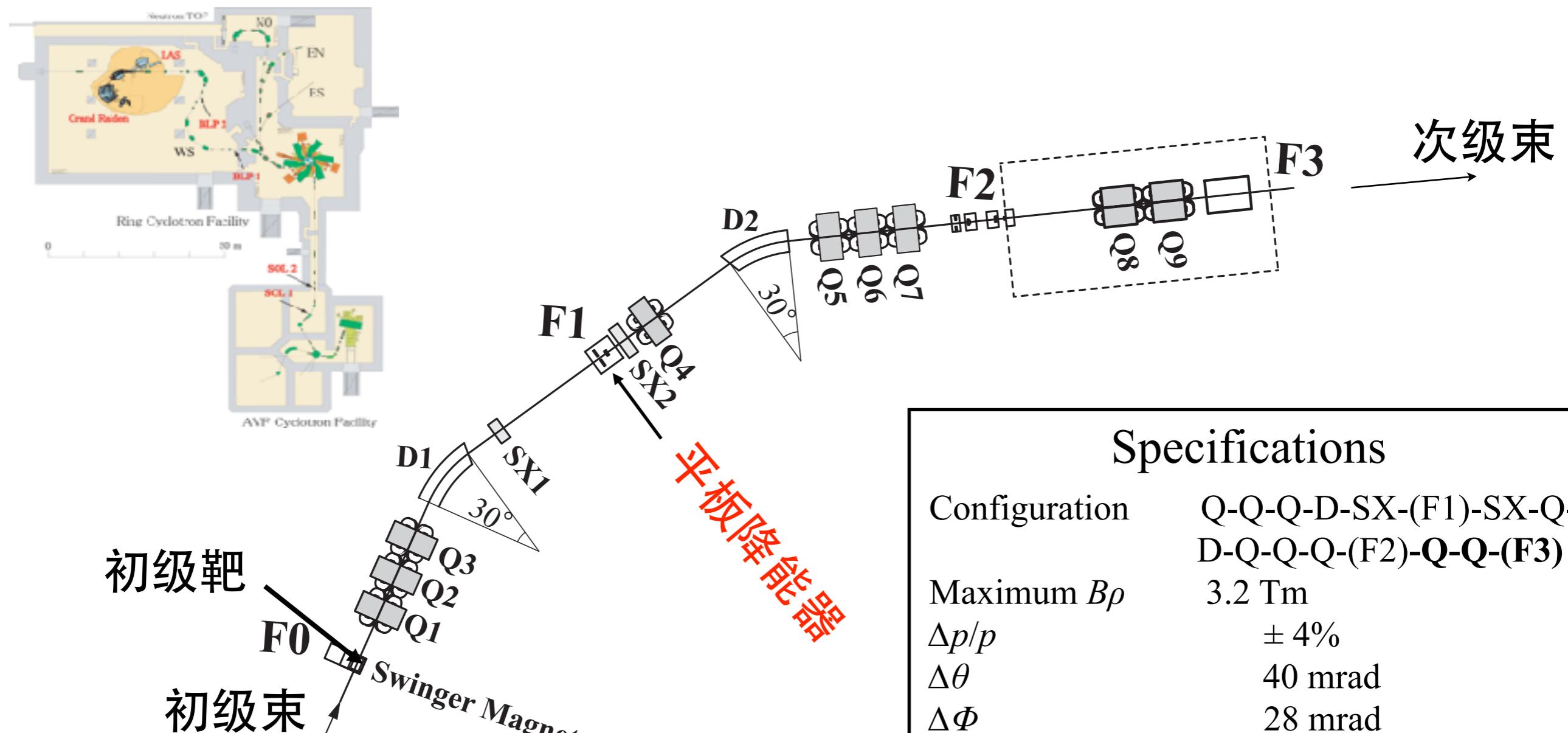
消色差
束流光学



降能器/物质下的束流传输



RCNP-EN：平板降能器



Specifications

Configuration	Q-Q-Q-D-SX-(F1)-SX-Q-D-Q-Q-Q-(F2)-Q-Q-(F3)
Maximum $B\rho$	3.2 Tm
$\Delta p/p$	$\pm 4\%$
$\Delta\theta$	40 mrad
$\Delta\Phi$	28 mrad
Momentum dispersion	17.32 mm/%

T. Shimoda, NIMB70 (1992) 320
S. Mitsuoka, NIMA372 (1996) 489
HJO, AIP Conf. Proc. 1588 (2014) 146

下载、安装

1. LISE++

<https://lise.frib.msu.edu/lise.html>

2. SRIM

<http://www.srim.org/SRIM/SRIMLEGGL.htm>