Tutorial 1 : Social Choice, Inequality and Poverty

(i) A population consists of H individuals distinguished by pretax income y_i and unobserved characteristics a_i, $i = 1, \ldots, H$. Mean income is $\bar{y} = \frac{1}{H}\sum_i y_i$. The government can redistribute income by imposing a proportional tax on income at rate $t \in [0, 1]$ and paying out a uniform grant G. However administrative costs mean that resources are lost in the process so that the value of the grant affordable is only $\phi t\bar{y}$ where $\phi < 1$. After-tax incomes are therefore

$$x_i \;=\; y_i(1-t) + \phi t\bar{y} \qquad i = 1, \ldots, H.$$

Individual preferences over tax rates are summarised in utility functions

$$U_i(t) = V(x_i, a_i)$$

where nothing is known about the form of dependence on a_i and it is known only that V depends positively on x_i for all values of a_i.

Government wishes to base decisions about which tax rate to implement on a social preference ordering which is based on individual preferences as captured in these utilities $U_i(t)$, $i = 1, \ldots, H$.

(a) i. Explain what it means for preferences to be *single-peaked* and show that individual preferences are single-peaked over tax rates $t \in [0, 1]$.

ii. Majority voting regards one tax rate t^A as weakly socially preferred to another t^B if and only if no fewer individuals prefer t^A to t^B than prefer t^B to t^A. Given that individual preferences are single peaked, explain why majority voting gives a social preference ordering that is complete, transitive, nondictatorial and satisfies both the Pareto principle and independence of irrelevant alternatives.

iii. Explain what a *Condorcet winner* is and why there is a Condorcet winning tax rate in this economy. What is the Condorcet winning tax rate?

(b) Suppose instead that it is known that individual utilities do not depend on characteristics a_i so that $U_i(t) = V(x_i)$, but that it is still true that nothing is known about dependence on x_i except that V is

increasing. Explain why it is now possible, unlike before, to base social preference on the utility of the worst-off person, $W_\infty(t) = (1-t)y_{min} + \phi t\bar{y}$. Show that the optimal tax rate according to such a social preference ordering depends critically on a comparison of ϕ and $y_{min}/\bar{y}$.

(c) i. Suppose instead that individual utilities are known to have the form

$$U_i(t) = \lambda V(x_i) + F(a_i)$$

where λ is an unknown positive constant which is the same for all individuals, F is some unknown function of individual characteristics but V is a *known* increasing, concave function of x_i. Explain why it is now feasible to base social judgment on the Benthamite social welfare function

$$W(t) \;=\; \sum_i U_i(t).$$

ii. Explain what it means for a social welfare function to be *Schur-concave* in after-tax incomes and why this might be desirable. Show that $W(t)$ is Schur-concave.

iii. Suppose that $V(x_i) = \ln x_i$. Show that $W'(0)$ depends on

$$\mathcal{I} \;=\; 1 - \frac{\hat{y}}{\bar{y}}$$

where

$$\hat{y} = \frac{H}{\sum_i (1/y_i)}$$

is the harmonic mean income. Interpret $\mathcal{I}$ and discuss why it might be relevant to tax setting.

(d) Discuss alternative assumptions on cardinality and comparability of utilities and their relationship to possible social welfare functions.

(e) Suppose that the government's objective is to minimise the proportion of the population with posttax income falling below a poverty line z. What now is the optimum tax rate? How sensible an objective is this if the government is concerned about poverty?