ECON0059: **Advanced Microeconomic Theory: Part 2**

Problem Set 2

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Question 1. A tenant (T) is deciding how much to invest in the apartment they are renting. The tenant can make a high or low investment $q \in \{L, H\}$ at cost c_q, where $c_H > c_L$. Investment determines the increase in value of the apartment to the tenant: v_q, where $v_H > v_L$. Then the owner (O) makes a take-it-or-leave-it offer on how much to increase the rent in the following year, x. The tenant either accepts ($a = 1$) or rejects ($a = 0$) the proposal, and the game ends. Payoffs are given by $u_T(q, x, a) = a(v_q - x) - c_q$ and $u_O(q, x, a) = ax$.[1] Solve for all subgame-perfect Nash equilibria. (While the game has infinitely many subgames, as x is continuous it can be solved by backward induction.)

Question 2. Let's revisit the alternating bargaining problem.

Two players, 1 and 2, bargain over the split of £$v > 0$.

There are up to T periods of bargaining, and players discount payoffs at a rate $\delta_1, \delta_2 \in (0, 1)$ per period.

Conditional on bargaining continuing up to period t, Player i gets to propose a split $b_t \in [0, v]$, which the opponent can accept or reject, where $i = 1$ if t is odd and $i = 2$ if otherwise.

If the opponent accepts, the game ends, with the proposer — Player i — getting $\delta_i^{t-1}(v - b_t)$, and the opponent $\delta_j^{t-1} b_t$, where $i, j = 1, 2$, $i \neq j$.

If the opponent rejects, the game moves on to the next period $t + 1$ if $t < T$, or it ends if $t = T$, in which case both players get zero.

(i) Let $T = 1$. What are the Nash equilibria of the game? And the subgame-perfect Nash equilibria?

(ii) Let $T = 2$. Solve for the subgame-perfect Nash equilibrium payoffs.

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[1] Where, implicitly, we assume that the owner does not value the investment the tenant made. This can very well be the case, as tenant-specific investment has to do with the tenant's idiosyncratic preferences, such as the color of the walls or some particular taste in furniture.

(iii) Let $\delta_1 = \delta_2 = \delta$. Show that patience pushes favors the last proposer and impatience favors the first.