SUMMER TERM 2022

ONLINE EXAMINATION

ECON0059: ADVANCED MICROECONOMIC THEORY

All work must be submitted anonymously. Please ensure that you add your **candidate number** and the module code to the template answer sheet provided. Note that the candidate number is a combination of four letters plus a number, e.g. ABCD9. You can find your candidate number in your PORTICO account, under "My Studies" then the "Examinations" container. Please, note that the candidate number is NOT the same as your student number (8 digits), which is printed on your UCL ID card. Submitting with your student number will delay marking and when your results might be available.

Word count: no more than 4,000
The above word count is provided as guidance only on the expected total length of your submitted answer sheets. You will not be penalised if you exceed the word limit.

*Answer **all three** questions. Each question carries 1/3 of the total mark.*

Allow enough time to submit your work. You are given 4 hours to solve your exam and an additional hour to submit your solutions on Moodle. This means that your work should be uploaded on Moodle **no later than 5 hours from the moment you accessed the examination paper for the first time**.

*By submitting this assessment, **you pledge your honour** that you have not violated UCL's Assessment Regulations which are detailed in* https://www.ucl.ac.uk/academic-manual/chapters/chapter-6-student-casework-framework/section-9-student-academic-misconduct-procedure, *which include (but are not limited to) plagiarism, self-plagiarism, unauthorised collaboration between students, sharing my assessment with another student or third party, access another student's assessment, falsification, contract cheating, and falsification of extenuating circumstances.*

Question 1.

Consider the following 2×2 game:

	Y	N
Y	c , c	c-1 , 0
N	0 , c-1	0 , 0

Suppose that c is some commonly known constant in (0,1).

1.1. Find the game's Nash equilibria (including both pure and mixed strategies).

1.2. What is players' max-min strategy (with randomization)? Is the game strictly competitive?

Now suppose that c is not common knowledge. It takes the values 0 or 1 with equal probability. Players are asymmetrically informed regarding the realization of c. Each player receives a conditionally independent binary signal of accuracy $q \in (½,1)$. That is, for every realization of c, each player receives the signal c with probability q and the signal 1-c with probability 1-q, independently of the other player's signal.

1.3. Formulate the interaction as a Bayesian game.

1.4. Does the game have symmetric pure-strategy Nash equilibria in which players choose a constant action, regardless of their signal? Characterize these equilibria.

1.5. Does the game have symmetric pure-strategy Nash equilibria in which players vary their action with their signal? Characterize these equilibria.

Question 2.

Two players, P1 and P2, are bargaining over how to split £100. They alternate in proposing a split. In each period $t = 1,2,\dots,T$, if no offer has yet been accepted, one of the players gets to propose a split (c_1, c_2), corresponding to the amount of money P1 and P2 get, respectively. The other player observes the offer and either accepts or rejects the offer.

- If the player accepts, the game ends and payoffs are $\delta^{t-1}c_1$ for P1 and $\delta^{t-1}c_2$ for P2, where $\delta \in (0,1)$ is a common discount factor.
- If the player rejects and $t < T$, the game moves on to period $t + 1$ and the other player gets to propose a split.
- If the player rejects and $t = T$, the game ends and both players get zero.

The game starts at period $t = 1$ with P1 proposing a split.

2.1. Assume that $T \in \mathbb{N}$ is odd. Describe the set of possible histories and the set of pure behavioral strategies for each player.

2.2. Assume that $T = 3$ and $\delta = 3/4$. Characterize all the equilibrium splits (c_1, c_2) that can arise as an outcome of a Nash equilibrium of the game. (You need to construct one NE for each split and rule out the existence of other possible equilibrium splits.)

2.3. Assume that $T = 2$ and $\delta = 3/4$. Characterize all subgame-perfect Nash equilibria.

Question 3.

Refer to the extensive-form game Q3 in Figure 1.

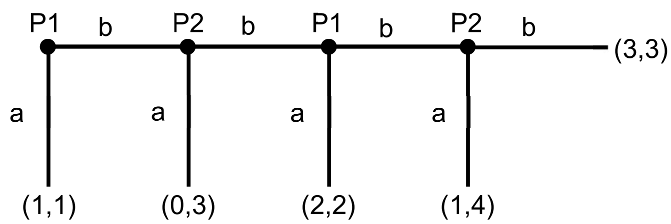

Figure 1: Game Q3

3.1. Characterize all subgame-perfect Nash equilibria.

3.2. Suppose that each player has a type $\theta_i \in \{G, N\}, i = 1,2$. Each player $i = 1,2$ is of type $\theta_i = G$(iving) with probability $p \in (0,1)$; and with probability $1 - p$ the player gets type $\theta_i = N$(eutral). Nature assigns types independently at random at the beginning of the game Q3.
If player i has type N, the player behaves in order to maximize their expected utility as usual; but if the player has type G, then the player always chooses action b.

(a) Characterize all pure-strategy weak perfect Bayesian Nash equilibria for $p = \frac{2}{5}$.

(b) Find all the pure-strategy sequential equilibria for $p = \frac{2}{5}$.

(c) Find a sequential equilibrium for $p = \frac{2}{5}$ in which there is a strictly positive probability of players choosing b in each of the first 3 periods.

3.3. Now refer back to the game Q3 in Figure 1 (i.e. no G types).

(a) What is the minimum payoff that each player can guarantee for themselves in the extensive-form game?

(b) Consider the repeated game in which game Q3 is a stage game that is infinitely repeated and payoffs of the stage game are discounted according to $\delta \in (0,1)$. Characterize a subgame-perfect equilibrium of the repeated game and discount factors under which the average discounted equilibrium payoffs are (3,3).