1. (Exam 2010/2011) Suppose you have $Y_t = (1+\theta L)\epsilon_t$, with $\epsilon_t | I_{t-1} \sim N(0, \sigma_t^2)$ and $\sigma_t^2 = w + \alpha\epsilon_{t-1}^2$, ($I_{t-1}$ is the informational set at time t-1).
   (a) When is the model covariance-stationary?
   (b) Write down the sample likelihood of the model for a sample of size T.
   (c) Explain how you would test for the presence of conditional heteroskedasticity in the model[1].

2. Consider the following VAR model:

$$y_{1t} = 0.3y_{1t-1} + 0.8y_{2t-1} + \epsilon_{1t} \tag{1}$$
$$y_{2t} = 0.9y_{1t-1} + 0.4y_{2t-1} + \epsilon_{2t} \tag{2}$$

   With $E(\epsilon_{1t}\epsilon_{1\tau}) = 1$ for $t = \tau$ and 0 otherwise, $E(\epsilon_{2t}\epsilon_{2\tau}) = 2$ for $t = \tau$ and 0 otherwise and $E(\epsilon_{1t}\epsilon_{2\tau}) = 0\, \forall t, \tau$.
   (a) Is this system covariance-stationary?
   (b) Calculate $\Omega_s = \frac{\partial y_{t+s}}{\partial \epsilon_t}$, for s=0,1 and 2. What is the limit as $s \longrightarrow \infty$?

3. Consider the following bivariate VAR,

$$y_t = \rho_{yy}y_{t-1} + \rho_{ym}m_{t-1} + u_{yt} \tag{3}$$
$$m_t = \rho_{my}y_{t-1} + \rho_{mm}m_{t-1} + u_{mt} \tag{4}$$

   With $E(u_t u_t') = \begin{bmatrix} \sigma_y^2 & \gamma \\ \gamma & \sigma_m^2 \end{bmatrix}$.
   (a) Find a matrix H, which is lower triangular and ensures that if $Hu_t = \epsilon_t$, then $E(\epsilon_t\epsilon_t') = D$, where D is a diagonal matrix.
   (b) Given this matrix H calculate the structural representation of this VAR.
   (c) Calculate the VMA representation for the reduced form of this VAR.
   (d) Calculate the VMA representation of the structural form of the VAR.
   (e) Under what conditions will the reduced form and the structural form produce identical impulse response functions?

[1] There's an additional question but requires you to know about Forecasting, for latter this last question is: Find the density forecast for Y_{t+1}, i.e. the conditional density of Y_{t+1} given I_t. Do it while you are studying for the exam. Check past exams solutions