Term 3 assessment cover sheet

ECON0059: Advanced Microeconomics

Assessment mode: Coursework

INSTRUCTIONS FOR PREPARING YOUR SUBMISSION

Please carefully read and follow all these instructions before your submission deadline so that we can ensure anonymity in marking and ensure compliance with UCL assessment policies.

1) **All answers must be uploaded via Turnitin on this Moodle course page** by the UCL advertised deadline for this module.

2) **All work must be submitted anonymously.**

   a) Do not put your name on any file name or inside any document.

   b) **Put your student candidate number (SCN) clearly at the top of the first page.**

      Note that the student candidate number (SCN) is a combination of four letters plus a number, e.g. ABCD9. You can find your SCN in your PORTICO account, under "My Studies" then the "Examinations" container.

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      ECON0059_student candidate number

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   a) Type your answers using word processing software such as Microsoft Word, LaTeX, or any alternative.

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By submitting this assessment, ***I pledge on my honour*** *that I have not violated UCL's Assessment Regulations which are detailed in https://www.ucl.ac.uk/academic-manual/chapters/chapter-6-student-casework-framework/section-9-student-academic-misconduct-procedure, which include (but are not limited to) plagiarism, self-plagiarism, unauthorised collaboration between students, access another student's assessment, falsification, contract cheating, and falsification of extenuating circumstances.*

Economics ECON0059: Advanced Microeconomics

2019-20

Answer all four questions.

Each question constitutes 25% of the final grade.

Please provide detailed derivations for all your answers.

CONTINUED

Question no. 1

Two profit-maximizing firms independently choose whether to offer a product of high or low quality. When a firm offers a high-quality product, it incurs a fixed cost (independently of whether it sells the product) of $c \in (0, \frac{1}{2})$. Offering a low-quality product entails no cost. Simultaneously with its quality choice, each firm also decides whether to advertise its product at a cost m.

A consumer is initially assigned to one of the two firms (with probability ½ each). If the other firm offers a higher-quality product and advertises it, the consumer switches to it. Otherwise, the consumer sticks to her initially assigned firm. The firm that is eventually selected by the consumer earns a gross revenue of 1, while the other firm earns zero gross revenues.

1. Describe the firms' interaction as a two-player strategic game.

2. Let $m=0$. Characterize the set of symmetric pure-strategy Nash equilibria in the game.

3. Now suppose $m \in (0, \frac{1}{2}-c)$.

    a. Show there exists no pure-strategy Nash equilibrium.

    b. Is there a symmetric mixed-strategy Nash equilibrium in which the equilibrium strategy has full support (i.e., every pure strategy is played with positive probability)? Explain.

    c. Find a symmetric mixed-strategy Nash equilibrium. Explain your calculations.

Question no. 2

An office party is about to be held, and two workers disagree over the important question of whether music will be played at the event. Worker 1 wants to have music, whereas worker 2 does not want any music. Specifically, if music is played, worker 1 will earn a gross utility v_1, whereas worker 2 will experience a gross disutility $-v_2$. The values v_1 and v_2 are independently and uniformly drawn from [0,1].

The two workers try to influence the plans for the party. Each worker chooses whether to remain silent or express her opinion. If the worker chooses the latter course of action, she incurs a cost of ¼. Music will be played if and only if agent 1 requests it and agent 2 remains silent.

1. Describe the interaction between the two workers as a Bayesian game.

2. Show that a pure-strategy Nash equilibrium must be in cutoff strategies: Each worker $i=1,2$ expresses her opinion if and only if v_i is above some threshold (which may be different for each player).

3. Find the game's pure-strategy Nash equilibrium. What is the probability that music will be played at the party? Explain your calculations.

Question no. 3

Consider the following infinitely repeated prisoner's dilemma, with discount factor $\delta \in (0,1)$ and payoff matrix:

	C	D
C	2,2	-1,3
D	3,-1	0,0

1. Describe all terminal histories of this game.

2. For each of the following strategies, characterize the set of δ (if any) for which there is a symmetric subgame-perfect equilibrium as described below:

   a. Play C in the first period. Then play C if in the last period the players both played C or both played D, and play D if last period one played C and the other played D.

   b. In odd-numbered periods, always play D. In even-numbered periods, play C if and only if both players played C is all previous even-numbered periods.

   c. Play C in the first period. After any subsequent history, play the action the opponent played in the previous period.

3. Are all of the strategies in parts (a)-(c) above sustained in a subgame-perfect equilibrium when δ is close to one? Provide an intuitive explanation why or why not.

Question no. 4

Consider the following signaling game.
Nature moves first and chooses player 1's type θ which is either A (with probability p) or B (with probability 1-p).
Player 1 observes Nature's move and chooses an action $a_1 \in \{U, D\}$.
Player 2 sees 1's move but not 1's type and chooses $a_2 \in \{L, R\}$.
The payoffs for each θ are given by the following matrices:

When θ =A

	L	R
U	3,3	0,0
D	0,0	2,2

When θ =B

	L	R
U	1,-1	-1,1
D	-1,1	1,-1

1. Draw the tree of this game. Make sure to indicate all information sets.

2. For what values of p does the game have a separating Perfect Bayesian Equilibrium (PBE)? Fully characterize at least one PBE for each of those values (if any).

3. For what values of p does the game have a pooling PBE? Fully characterize at least one PBE for each of those values (if any).

4. For what values of p does the game have a partially separating PBE in which type B plays U with probability 1 and type A assigns strictly positive probabilities to both actions?

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Please provide detailed derivations of all your answers.

Question no. 1

Two profit-maximizing firms independently choose whether to offer a product of high or low quality. When a firm offers a high-quality product, it incurs a fixed cost (independently of whether it sells the product) of $c \in (0,½)$. Offering a low-quality product entails no cost. Simultaneously with its quality choice, each firm also decides whether to advertise its product at a cost m.

A consumer is initially assigned to one of the two firms (with probability ½ each). If the other firm offers a higher-quality product and advertises it, the consumer switches to it. Otherwise, the consumer sticks to her initially assigned firm. The firm that is eventually selected by the consumer earns a gross revenue of 1, while the other firm earns zero gross revenues.

1. Describe the firms' interaction as a two-player strategic game.

2. Let m=0. Find a symmetric pure-strategy Nash equilibrium in the game.

Both firms choose high quality and advertise.

3. Now suppose $m \in (0, ½\text{-}c)$.

    a. Show there exists no pure-strategy Nash equilibrium.

If both firms offer high quality, they prefer not to advertise. But this gives a firm an incentive to deviate to low quality. If both firms offer low quality, a firm has an incentive to deviate to high quality and advertising. If one firm offers high quality and the other offers low quality, the high quality firm wants to advertise, but then the low quality firm prefers to deviate to high quality.

    b. Is there a symmetric mixed-strategy Nash equilibrium in which the equilibrium strategy has full support (i.e., every pure strategy is played with positive probability)? Explain.

CONTINUED

No. Given that the opponent offers a low-quality product with positive probability, offering a low-quality product and advertising it gives strictly lower payoff than offering low quality without advertising.

c. Find a symmetric mixed-strategy Nash equilibrium.

Each firm randomizes over three strategies: (h,0), (h,1) and (l,0). Indifference between (h,0) and (h,1) requires m=0.5p(l,0), because advertising enables a firm to steal consumers who are initially assigned to a low quality opponent. Indifference between (h,0) and (l,0) requires c=0.5p(h,1), because switching from low quality to high quality (without advertising) prevents consumers who are initially assigned to the firm from switching to a high quality opponent that advertises. The fourth strategy (l,1) is clearly inferior to (l,0). The above pair of equations (together with the condition that the probabilities add up to one) pins down the equilibrium strategy: p(1,0)=2m, p(h,1)=2c, p(h,0)=1-2c-2m.

Question no. 2

An office party is about to be held, and two workers disagree over the important question of whether music will be played at the event. Worker 1 wants to have music, whereas worker 2 does not want any music. Specifically, if music is played, worker 1 will earn a gross utility v_1, whereas worker 2 will experience a gross disutility $-v_2$. The values v_1 and v_2 are independently and uniformly drawn from [0,1].

The two workers try to influence the plans for the party. Each worker chooses whether to remain silent or express her opinion. If the worker chooses the latter course of action, she incurs a cost of ¼. Music will be played if and only if agent 1 requests it and agent 2 remains silent.

1. Describe the interaction between the two workers as a Bayesian game.

2. Define pure-strategy Nash equilibrium in this game.

3. Show that a pure-strategy Nash equilibrium must be in cutoff strategies: Each worker i=1,2 expresses her opinion if and only if v_i is above some threshold (which may be different for each player).

From a player's point of view, the opponent's strategy and distribution over types induce a probability that he remains silent, and that's all the player cares about. Given this probability, the player's expected utility is continuous and monotone in his own type, and therefore his best-reply is a cutoff strategy.

4. Find the game's pure-strategy Nash equilibrium. What is the probability that music will be played at the party?

Denote the cutoffs of players 1 and 2 by x and y. Then, the probabilities that players 1 and 2 are silent are x and y, respectively. Player 1's indifference at his cutoff x is $0 = xy - 0.25$. Player 2's indifference at his cutoff y is $-0.25 = -y(1-x)$. This implies $x=y=0.5$. The equilibrium probability that music will be played is 0.25.

Question no. 3

Consider the following infinitely repeated prisoner's dilemma, with discount factor $\delta \in (0,1)$ and payoff matrix:

	C	D
C	2,2	-1,3
D	3,-1	0,0

1. Describe all terminal histories of this game.

Terminal histories consist of infinite sequences $(\mathbf{a_1}, \mathbf{a_2}, \ldots)$ where each $\mathbf{a_t} \in \{(\mathbf{C}, \mathbf{C}), (\mathbf{C}, \mathbf{D}), (\mathbf{D}, \mathbf{C}), (\mathbf{D}, \mathbf{D})\}$.

2. For each of the following strategies, characterize the set of δ (if any) for which there is a symmetric subgame-perfect equilibrium described below:

a. Play C in the first period. Then play C if in the last period the players both played C or both played D, and play D if last period one played C and the other played D.

Without loss of generality consider the first agent and use the one-shot deviation principle. Moreover, consider only deviations in the first period because of the stationary nature of the game.

In any history where the prescribed play is (C,C), the total payoff in the first two periods is $\mathbf{2 + 2\delta}$. If she deviates to D, the play will be (D,C), (D,D), then (C,C) forever, so in the first periods she gets $\mathbf{3}$. Deviation is not profitable if $\mathbf{2 + 2\delta} \geq \mathbf{3}$, i.e. for $\boldsymbol{\delta} \geq \mathbf{\frac{1}{2}}$.

b. In odd-numbered periods, always play D. In even-numbered periods, play C if and only if both players played C is all previous even-numbered periods.

In odd periods there is no benefit from deviating since (D,D) is a Nash equilibrium of the stage game and there is no reward for deviating.

Consider even periods. In any history where the prescribed play if (C,C) (i.e. where nobody has deviated in even periods in the past), the total payoff is 2/(1-δ^2). A deviation induces (D,C), then (D,D) forever, with the total payoff of 3. Thus, she does not deviate when $\boldsymbol{\delta \geq 1/\sqrt{3}}$.

c. Play C in the first period. After any subsequent history, play the action the opponent played in the previous period.

In any history where the prescribed play is (C, C) forever, she can get the payoff stream (2, 2, …). By deviating to (D,C), (C,D), (D,C), …, she gets a payoff stream (3, -1, 3, -1, …). Thus, deviation is not profitable is $\boldsymbol{2+2\delta \geq 3-\delta}$, i.e. $\boldsymbol{\delta \geq \frac{1}{3}}$.

In any history where the prescribed play is (D,C) the expected continuation is (C,D), (D,C), …, and by deviating she can switch to (C,C) forever. This is exactly the opposite situation, and she would not want to deviate under the opposite condition $\boldsymbol{\delta \leq \frac{1}{3}}$.

In any history where the prescribed play is (C,D) the expected continuation is (D,C), (C,D), … (with a payoff stream -1, 3, -1, 3, …), while by deviating the first player can switch to (D,D) forever (with payoffs 0). Deviation is not profitable when $\boldsymbol{-1+3\delta \geq 0}$, i.e. $\boldsymbol{\delta \geq \frac{1}{3}}$ again.

But in any history where the prescribed play is (D,D) it is again the opposite, so we get $\boldsymbol{\delta \leq \frac{1}{3}}$. In sum, this SPE exists for $\boldsymbol{\delta = \frac{1}{3}}$ only.

3. Are all of the strategies in parts (a)-(c) above sustained in a subgame-perfect equilibrium when δ is close to one? Provide an intuitive explanation why or why not.

It is evident that strategies in parts (a) and (b) are sustained when δ is close to one while the strategy in part (c) isn't. The reason in (c) is that the punishment phase is not credible: a deviation from, for example, (D,C), takes the play back to the equilibrium path of cooperation, which is preferred by patient agents. This does not happen in parts (a) and (b) where punishment is always a pair of Nash strategies of the stage game (so deviating from it is not profitable in that period) and deviation from punishment does not shorten the punishment phase.

Question no. 4

Consider the following signaling game.

Nature moves first and chooses player 1's type θ which is either A (with probability p) or B (with probability 1-p).

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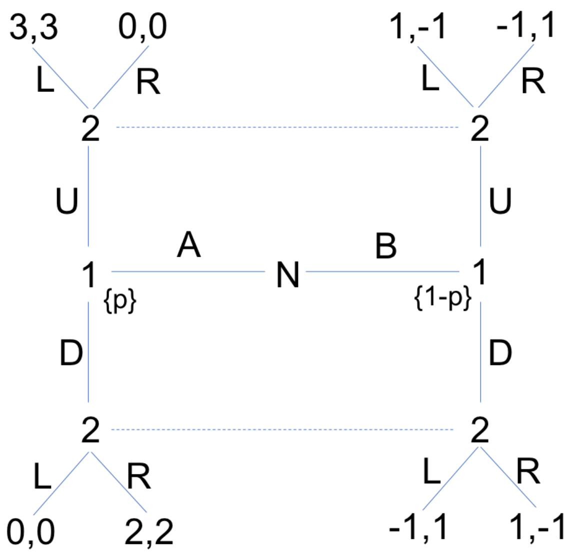
1. Draw the tree of this game. Make sure to indicate all information sets.



2. For what values of p does the game have a separating Perfect Bayesian Equilibrium (PBE)? Fully characterize at least one PBE for each of those values (if any).

We focus on the interesting case where p is strictly between 0 and 1 throughout. We show that there are no values of p for which there is a separating PBE.

END OF EXAM

Suppose player 1 plays U when observing θ=A and D when observing θ=B. Then player 2 (whose beliefs are correct) optimally responds by playing L when observing U and also L when observing D. But then type-B wants to deviate and play U.

Similarly suppose player 1 plays D when observing θ=A and U when observing θ=B. Then player 2 optimally responds by playing R after D and also R after U. Then type-B wants to deviate and play D.

3. For what values of p does the game have a pooling PBE? Fully characterize at least one PBE for each of those values (if any).

Answer: for all values of p.

Consider a pooling equilibrium where player 1 chooses U regardless of θ. Then in any PBE, $\mu(A \mid U) = p$. Player 2's expected payoff after observing U is 3p-(1-p)=4p-1 if she chooses L and 1-p if she chooses R. Thus L is the best response for $p \geq 0.4$ and R is the best response for $p \leq 0.4$ (with any mixture between them at p=0.4).

Is there belief $\mu(A \mid D)$ that supports this equilibrium play? Consider the case $p \geq 0.4$ first. Both types of player 1 get their maximum possible payoff in equilibrium and would never want to deviate. For example the following is an equilibrium: $\mu(A \mid U) = p$, $\mu(A \mid D) = 1$, player 1 chooses U regardless of the type, player 2 plays L after U and R after D.

Now consider $p < 0.4$. Here both types of player 1 get their minimum possible payoff in equilibrium (0 for type A, -1 for type B) and would not want to deviate only if they both get the same values by deviating. This indeed happens if player 2 responds to D by L. This would be the case for example when $\mu(A \mid D) = 0$. We arrive at an equilibrium: $\mu(A \mid U) = p$, $\mu(A \mid D) = 0$, player 1 chooses U regardless of the type, player 2 plays R after U and L after D.

END OF EXAM

4. For what values of p does the game have a hybrid PBE in which type B plays U with probability 1 and type A assigns strictly positive probabilities to both actions?

Answer: Never

By Bayes rule in such an equilibrium $\mu(A \mid D) = 1$. Thus player 2 best responds to D by R. This implies that type B by playing D can achieve her highest payoff of 1. Since type B instead plays U in the hypothesized equilibrium, she should also get 1 in equilibrium, which means that player 2 after observing U chooses L. But then type A will not mix, as playing U gives her a payoff of 3 compared to only 2 if she plays D.

Resit assessment cover sheet

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11) The assessment submission area has been set up to allow work to be submitted late. This is in place for those with permitted extensions due to SORAs or Extenuating Circumstances. **If you submit your work after the deadline** and do not have a permitted extension due to a SORA or Extenuating Circumstance your work will be subject to late penalties as set out in UCL's Academic Manual (Section 3.12 - https://www.ucl.ac.uk/academic-manual/chapters/chapter-4-assessment-framework-taught-programmes/section-3-module-assessment#3.9).

12) Once your submission has been accepted you will return to the 'My Submissions' tab where you will be able to see the details of your submission. If your submission is not confirmed for some reason, or you are experiencing problems uploading the document, get in touch with ISD (servicedesk@ucl.ac.uk) as soon as possible to figure out what the problem is. You should also alert Tina Fowler (tina.fowler@ucl.ac.uk). **Do not contact the module lecturer.**

By submitting this assessment, ***I pledge on my honour*** *that I have not violated UCL's Assessment Regulations which are detailed in* https://www.ucl.ac.uk/academic-manual/chapters/chapter-6-student-casework-framework/section-9-student-academic-misconduct-procedure*, which include (but are not limited to) plagiarism, self-plagiarism, unauthorised collaboration between students, access another student's assessment, falsification, contract cheating, and falsification of extenuating circumstances.*

Economics ECON0059: Advanced Microeconomics

2019-20, Resit exam

Answer all four questions.

Each question constitutes 25% of the final grade.

Please provide concise derivations of all your answers.

Question no. 1

Ten profit-maximizing firms play the following Cournot competition game. Each firm i simultaneously chooses a quantity x_i. Its payoff is $x_i(12\text{-}X) – C(x_i)$, where X is the sum of all ten firms' chosen quantities, and C is a fixed-cost function satisfying $C(0)=0$ and $C(x_i)=4$ for every $x_i>0$.

1. Show that there is no symmetric pure-strategy Nash equilibrium in the game.

2. Find an asymmetric pure-strategy Nash equilibrium in the game.

3. Find a symmetric mixed-strategy Nash equilibrium in the game, where each firm randomizes between only two quantity values.

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Question no. 2

Two agents with quasi-linear utility decide whether to submit a request for a single, indivisible object of common value v. Agent 1 has priority: If she requests the object, she gets it for sure. When agent 2 requests the object, he can get it only if agent 1 does not request it. For each agent, submitting a request entails a fixed cost $c \in (0,1)$, independently of whether the request is granted.

Assume v takes two possible values, 0 and 1, with equal prior probability. The agents are asymmetrically informed about v. Agent 2 receives no information. As to agent 1, with probability 1-q he receives no information. With probability q, he receives a signal with accuracy $p \in (½,1)$ - i.e., for every v, the signal is equal to v with probability p.

1. Describe the interaction between the two agents as a Bayesian game.

2. Define pure-strategy Nash equilibrium in this game.

3. Characterize the game's pure-strategy Nash equilibrium, as a function of the parameters c,q,p.

Question no. 3

Consider a three-period version of the Rubinstein bargaining model in which:

- Player A (she) makes a proposal (x,1-x) in period 1 (where throughout the problem the first number indicates the fraction of the pie that player A gets, before discounting).
- If player B (he) rejects (x,1-x), he makes a proposal (y,1-y) in period 2.
- If player A rejects (y,1-y), the players split the pie at (½, ½) in period 3.

Players discount time with a discount factor of $\delta \in (0,1)$ per period.

1. Fully characterize the subgame perfect equilibrium of this game.

2. For this part, suppose player A is "tough": In period 2, she never accepts an offer that gives her less than player B, even if this is ultimately to her detriment. Take this posture as given, and perform backward induction to derive the players' behavior in periods 1 and 2.

3. Now suppose player A is the tough type from part 2 with probability $p \in (0,1)$, while with probability 1-p she is "normal" as in part 1. Player A's type is observed by her but not by Player B. Does this game have a separating perfect Bayesian equilibrium in which the offer is always accepted in period 1? If so, fully characterize it; if not, explain why.

4. Consider the game from part 3 and assume p>½. Is there a perfect Bayesian equilibrium in which both types of player A make the same offer in period 1? If so, fully characterize it; if not, explain why.

Question 4

Consider the following simultaneous-move game:

	X	Y	Z	W
A	5, 5	0, 7	1, 4	0, -5
B	7, 0	4, 4	2, 1	0, -5
C	4, 1	1, 2	3, 3	0, -5
D	-5, 0	-5, 0	-5, 0	-5, -5

1. Find all pure-strategy Nash equilibria of this game.

For the rest of the question, suppose this stage game is repeated for T periods. Players in each period observe the actions taken in all previous periods. Payoffs in the repeated game are the sum of the payoffs received in each period; there is no discounting.

2. When T=2, show that there is a Nash equilibrium in which (A,X) is played in the first period. Fully characterize this equilibrium.

3. When T=2, is there a subgame perfect equilibrium in which (A,X) is played in the first period? If so, fully characterize it; if not, explain why.

4. When T=5, is there a subgame perfect equilibrium in which (A,X) is played in the first period? If so, fully characterize it; if not, explain why.

END OF EXAM

Economics ECON0059: Advanced Microeconomics
2019-20, Resit exam: Suggested solution

Answer all four questions.

Each question constitutes 25% of the final grade.

Please provide concise derivations of all your answers.

Question no. 1

Ten profit-maximizing firms play the following Cournot competition game. Each firm i simultaneously chooses a quantity x_i. Its payoff is $x_i(12\text{-}X) – C(x_i)$, where X is the sum of all ten firms' chosen quantities, and C is a fixed-cost function satisfying $C(0)=0$ and $C(x_i)=4$ for every $x_i>0$.

1. Show that there is no symmetric pure-strategy Nash equilibrium in the game.

   Suppose all firms choose x=0. Then, each firm earns zero profits. If a single firm deviates into, say, x=6, it will earn a net profit of 6*6-4 > 0, a contradiction. Now suppose all firms choose x>0. Then, x must maximize x(12-x-y), where y is the total quantity of all nine opponents of a single firm. Then, by first-order conditions and symmetry, x=(12-y)/2 and y=9x, such that the firm's profit is (12/11)^2 – 4 < 0, a contradiction.

2. Find an asymmetric pure-strategy Nash equilibrium in the game.

   Let us guess that five firms play x=0 whereas five firms play the same x>0. Then, by first-order conditions and symmetry, x=(12-y)/2 and y=4x, such that x=2. Thus, conditional on playing x>0, x=2 is optimal for each of the firms that play the positive quantity in equilibrium. Plugging this into the payoff function of a firm that plays x=2, we obtain 2*(12-2-4*2) – 4 = 0, hence the firm is indifferent between playing x=2 and x=0. In contrast, a firm that plays x=0 strictly prefers it to playing x>0, because that will imply strictly negative profits.

3. Find a symmetric mixed-strategy Nash equilibrium in the game, where each firm randomizes between only two quantity values.

   Suppose firms mix between x=0 and x=2, such that they assign probability 4/9 to x=2. Then, as far as an individual firm is concerned, the expected profit it faces as a function of its own choice of x is x*(12-x-9*(4/9)*2) – C(x), which

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reduces to x(4-x) – C(x), leading to the indifference between x=0 and x=2, and a strict preference to all other values of x. This completes the proof.

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Question no. 2

Two agents with quasi-linear utility decide whether to submit a request for a single, indivisible object of common value v. Agent 1 has priority: If she requests the object, she gets it for sure. When agent 2 requests the object, he can get it only if agent 1 does not request it. For each agent, submitting a request entails a fixed cost $c \in (0,1)$, independently of whether the request is granted.

Assume v takes two possible values, 0 and 1, with equal prior probability. The agents are asymmetrically informed about v. Agent 2 receives no information. As to agent 1, with probability 1-q he receives no information. With probability q, he receives a signal with accuracy $p \in (½,1)$ - i.e., for every v, the signal is equal to v with probability p.

1. Describe the interaction between the two agents as a Bayesian game.

2. Define pure-strategy Nash equilibrium in this game.

3. Characterize the game's pure-strategy Nash equilibrium, as a function of the parameters c,q,p.

   (Ignore the cases of ties, for notational simplicity.) When agent 1 receives no information, it requests the object when ½-c > 0. When it receives good information, it requests the object when p-c > 0, and when it receives bad information, it requests the object when 1-p-c > 0.
   Suppose c>p. Then, agent 1 never requests the object, and therefore agent 2's payoff from requesting it is 0.5-c < 0, hence no agent ever requests the object in equilibrium.
   Suppose c is between 0.5 and p. Then, agent 1 requests the object if and only if he receives good information. Then, agent 2's expected utility from requesting the object is 0.5*0 + 0.5*[q*(p*0+(1-p)*1)+(1-q)*1] – c = 0.5(1-pq) – c < 0, hence agent 2 does not request the object.
   Now suppose c is between 1-p and 0.5. Then, agent 1 requests the object if and only if he does not receive bad information. Then, agent 2's expected utility

from requesting the object is 0.5*0 + 0.5*[q*(p*0+(1-p)*1)+(1-q)*0] – c = 0.5q(1-p) – c, which is negative because c > 1-p. Therefore, agent 2 does not request the object.

Finally, suppose c is below 1-p. Then, agent 1 always requests the object. Then, agent 2 gets a payoff of –c if he requests the object, hence he does not request the object.

We see that agent 2 never requests the object in equilibrium, for all values of p,q,c.

Question no. 3

Consider a three-period version of the Rubinstein bargaining model in which:

- Player A (she) makes a proposal (x,1-x) in period 1 (where throughout the problem the first number indicates the fraction of the pie that player A gets, before discounting).
- If player B (he) rejects (x,1-x), he makes a proposal (y,1-y) in period 2.
- If player A rejects (y,1-y), the players split the pie at (½, ½) in period 3.

Players discount time with a discount factor of $\delta \in (0,1)$ per period.

1. Fully characterize the subgame perfect equilibrium of this game.

By backward induction:

- **In period 2, player A accepts any $y \geq \delta/2$ regardless of the history.**
- **Player B finds it optimal to propose $y=\delta/2$, again regardless of the history, resulting in payoffs $(\delta^2/2, \delta(1-\delta/2))$.**
- **In period 1, player B accepts any offer $1-x \geq \delta(1-\delta/2)$.**
- **Player A finds it optimal to offer $1-x=\delta(1-\delta/2)$ to B.**

2. For this part, suppose player A is "tough": In period 2, she never accepts an offer that gives her less than player B, even if this is ultimately to her detriment. Take this posture as given, and perform backward induction to derive the players' behavior in periods 1 and 2.

By backward induction again:

- **In period 2, player A accepts any $y \geq 1/2$ regardless of the history, because ½ is already better than rejecting and she cannot accept lower offers because of "toughness."**
- **Player B finds it optimal to propose $y=1/2$, again regardless of the history, resulting in payoffs $(\delta/2, \delta/2)$.**
- **In period 1, player B accepts any offer $1-x \geq \delta/2$.**
- **Player A finds it optimal to offer $1-x=\delta/2$ to B.**

3. Now suppose player A is the tough type from part 2 with probability $p \in (0,1)$, while with probability 1-p she is "normal" as in part 1. Player A's type is observed by her but not by Player B. Does this game have a separating perfect Bayesian equilibrium in which the offer is always accepted in period 1? If so, fully characterize it; if not, explain why.

No. In a separating equilibrium where the game ends in period 1 offers must be different. Therefore, the type that offers more in equilibrium will find it optimal to deviate to the lower offer of the other type.

4. Consider the game from part 3 and assume $p > ½$. Is there a perfect Bayesian equilibrium in which both types of player A make the same offer in period 1? If so, fully characterize it; if not, explain why.

Yes. As usual, we apply backward induction.

- **In period 2 player A behaves according to their type regardless of history: accept offers $y \geq \delta/2$ if she is a normal type and $y \geq 1/2$ if she is a tough type.**
- **Consider player B's offer in period 2 in the histories following the period 1 offer that is on the equilibrium path. Since both types make the same offer in period 1, B's belief about the type is the same as the prior. Out of all offers player B can make, two are reasonable: $y=\delta/2$ or $y=1/2$ (the others are immediately dominated). The offer $y=1/2$ is accepted by both types, resulting in a payoff $\delta/2$ for player A. The offer $y=\delta/2$ is only accepted by the normal type. Thus, with probability 1-p player A gets $\delta(1-\delta/2)$, and with probability p she gets $\delta^2/2$ (from half of the pie in the third period). The expected payoff is $(1-p)\delta(1-\delta/2)+p\delta^2/2=\delta(1-\delta/2)-p(\delta-\delta^2)$. It is straightforward to check that this is below $\delta/2$ whenever $p>1/2$, and so the offer of $y=1/2$ is optimal.**
- **In that case both types of player A optimally offer $1-x=\delta/2$ in period 1.**
- **It remains to characterize strategies and beliefs out of equilibrium path. A simple way is to suppose that the beliefs and strategies of player B are the same regardless of the period-1 action of player A.**

Question 4

Consider the following simultaneous-move game:

	X	Y	Z	W
A	5, 5	0, 7	1, 4	**0**, -5
B	**7**, 0	**4**, 4	2, 1	**0**, -5
C	4, 1	1, 2	**3**, 3	**0**, -5
D	-5, 0	-5, 0	-5, 0	-5, -5

1. Find all pure-strategy Nash equilibria of this game.

(B,Y) and (C,Z). Best responses for player one are highlighted in the table above, and they are symmetric for player two.

For the rest of the question, suppose this stage game is repeated for T periods. Players in each period observe the actions taken in all previous periods. Payoffs in the repeated game are the sum of the payoffs received in each period; there is no discounting.

2. When T=2, show that there is a Nash equilibrium in which (A,X) is played in the first period. Fully characterize this equilibrium.

Answer: yes. Play (A,X) in the first period. If nobody deviated, play (B,Y). If only player A deviated, play (A,W). If only player B deviated, play (D,X). If both deviated, play for example (C,Z).

Here the player who deviated in the first period is punished by the other player by minmaxing, which here means playing D or W. Deviating in the first period is not individually rational: by deviating to B in the first period player 1 only gets 7+0 instead of 5+4, regardless of her second-period play. And deviating in the second period only is not profitable because (B,Y) is a Nash equilibrium.

END OF EXAM

3. When T=2, is there a subgame perfect equilibrium in which (A,X) is played in the first period? If so, fully characterize it; if not, explain why.

Answer: no. In a hypothetical SPE a NE has to be played in the second period in all histories. Thus the maximum punishment is the payoff gap between (B,Y) and (C,Z), which is one. It is not enough to prevent deviations in the first period: for example, by player 1 from A to B.

4. When T=5, is there a subgame perfect equilibrium in which (A,X) is played in the first period? If so, fully characterize it; if not, explain why.

Answer: yes. Play (A,X) in the first period. Play (B,Y) if nobody deviated in the first period. Play (C,Z) otherwise (ignore deviations in periods other than the first one).

Deviating from A to B in the first period is not profitable: 7+3*4<5+4*4.

Economics ECONG008: Advanced Microeconomics
2014-15

Answer any three out of four questions.

Each question constitutes 33.3% of the final grade.

Please provide concise derivations of all your answers.

In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Exam duration: 2 hours

Question no. 1

Consider the following two-player simultaneous-move game:

	L	R
T	(2,6)	(5,0)
M	(5,0)	(2,6)
B	(3,4)	(3,4)

(a) Analyse the game using concept of mixed-strategy Nash equilibrium.

a. Formally define the concept.
b. Find the set of mixed-strategy Nash equilibria in this game.

Action B is strictly dominated. After deleting it, we have a game that is equivalent to matching pennies, and in Nash equilibrium both players mix uniformly between the two actions in the reduced game.

(b) Analyse the game using the concept of mix-minimization (with mixed strategies).

a. Formally define the concept.
b. Find the players' max-min strategies.

The game is strictly competitive (an affine transformation of the payoffs turns it into a zero-sum game). By the minimax theorem, Nash equilibrium is equivalent to max-minimization. Therefore, the max-min strategies are the same as the Nash equilibrium strategies

(c) Analyse the game using the concept of rationalizability.

a. Formally define the concept.
b. Find the set of rationalizable actions for each player.

The action B is strictly dominated. After it is eliminated, no action is strictly dominated. Therefore, all actions are rationalizable for player 2, and the actions T and M are rationalizable for player 1.

Question no. 2

Two politicians vote independently over some reform proposal. The reform is implemented only when both politicians support it – otherwise, the status quo prevails. The politicians are expected-utility maximizers whose vNM payoffs over outcomes are as follows:

	Status quo	Reform
Politician 1	1	0.5+v
Politician 2	0	1-2v

where v is a random variable that takes the values 0 or 1 with equal probability.

The two politicians are asymmetrically informed about the value of v. The information structure is as in the E-mail game. That is, politician 1 learns the value of v. If (and only if) v=1, an automatic message is sent from his computer to politician 2's computer. If this message arrives, an automatic confirmation message is sent back to politician 1's computer; if this confirmation message arrives, a re-confirmation message is sent to politician's computer; and so forth. Every message fails to arrive at its destination with independent probability ε>0. Thus, the process stops after finitely many transmission rounds. Each politician sees the total number of messages his computer sent; this number constitutes his information before he decides how to vote.

(a) Formally define this interaction as a Bayesian game (specify the state space, the prior distribution and the players' signal function).

(b) Define the concept of Nash equilibrium in a Bayesian game.

(c) Assume ε=1 – that is, politician 2's computer never receives any message. Find the set of Nash equilibria in this game.

Suppose player 2 votes for reform. Then, when player 1 learns v=0, he prefers the status quo and votes against the reform, and when he learns v=1 he prefers the status quo and votes against the reform. Therefore, it is not a best reply for player 2 to vote for reform. In equilibrium, he votes against reform, and player 1 is indifferent between the two actions.

(d) Now suppose that ε is strictly between 0 and 1. Is there a Nash equilibrium in which the reform is implemented with positive probability? If so, construct an example of such an equilibrium, and if not, prove it.

The question can be analysed by induction, as in the e-mail game. Alternatively, note that by an affine transformation of one of the player's utilities, we see that the players' payoffs add up to zero in every state. Then, the problem is isomorphic to a speculative trade problem, because voting against reform is equivalent to refusing to trade in the speculative trade model. By the no-trade theorem, reform is never implemented.

Question no. 3

Consider the game of the boy who cried wolf. A boy is playing in a field and his father is working nearby. With a probability 0.5 a wolf attacks the boy. In either case, if the wolf attacks or not, the boy has two actions: to cry for help or to not cry for help. The father then decides to either help his son or ignore him. The payoffs for each action are given in the following table, with the boy's payoff first and the father's second.

	Wolf attacks	Wolf does not attack
Boy cries wolf, Father helps	(1,1)	(2,-2)
Boy cries wolf, Father ignores	(-2,-2)	(-1,2)
Boy does not cry wolf, Father helps	(0,1)	(1,0)
Boy does not cry wolf, Father ignores	(-1,-1)	(1,1)

First, suppose that this is a game of complete information (i.e. the boy observes whether the wolf attacks or not and the father observes both whether the wolf attacks and the action of his son)

(a) Formally define this extensive form game and draw the game tree.

Answer:

Player: {nature,boy,father}

Terminal histories: {(attack,cry,help), (attack,cry,ignore), (attack,no cry,help), (attack,no cry, ignore), (no attack,cry,help), (no attack,cry,ignore), (no attack,no cry,help), (no attack,no cry, ignore)}

Player function: $P(\emptyset) = nature$, $P(attack) = boy$, $P(no\ attack) = boy$, $P(attack, cry) = father$, $P(attack, no\ cry) = father$, $P(no\ attack, cry) = father$, *and* $P(no\ attack, no\ cry) = father$

Preferences: u_1 *and* u_2 *functions from terminal nodes to real numbers – as given in question setup*

(b) Find all SPNE.

Answer: Boy: Cry,NoCry,

Father: (H,H,I,I)

(c) Draw the new game tree for this game.

(d) Is there a pooling equilibrium? If so, provide one. If not, explain why not. (Restrict yourself to pure strategies)

Answer: This is a signaling game.Let t_1 *to be type of boy that is attacked and* t_2 *be the type of boy that is not attacked.*

Let $p = Pr(t_1|Cry)$. $p = Pr(t_2|No\ Cry)$

There is a pooling equilibrium with

Boy: NC,NC

Father: I,I,H,H

$p<4/7$, $q=1/2$

(e) Is there a seperating equilibrium? If so, provide one. If not, explain why not. (Restrict yourself to pure strategies)

Answer: there are no seperating equilibrium

Question no. 4

Player 1 and player 2 are playing the following game. They first play a simultaneous game where each player chooses an action A or B. If (A,A) is played, player 1 moves, choosing between actions R and L with payoffs (3,2) and (6,2) respectively (where (x,y) represents the payoffs of player 1 and player 2 respectively). If (A,B) is played, player 2 moves, choosing between actions R and L with payoffs (4,3) and (2,2) respectively. If (B,A) is played, player 2 moves, choosing between actions R and L with payoffs (3,6) and (8,4) respectively. And, if (B,B) is played, player 1 moves, choosing between actions R and L with payoffs (3,2) and (2,8) respectively.

(a) Formally define this extensive form game.

Answer:

Player: {p1,p2}

Terminal histories: {(A,A,R),(A,A,L), (A,B,R), (A,B,L), (B,A,R), (B,A,L), (B,B,R), (B,B,L)}

Player function: $P(\emptyset) = \{1,2\}$, $P((A,A)) = 1$, $P((A,B)) = 2$, $P((B,A)) = 2$, $P((B,B) = 1$

Preferences: u_1 *and* u_2 *functions from terminal nodes to real numbers – as given in question setup.*

(b) Find all SPNE

Answer: P1: ALR and P2: BRR

Now suppose that the above (stage) game is infinitely repeated. Assume both players discount stage game payoffs with discount factor δ.

(c) Can you find a SPNE where player 1 achieves an average discounted payoff of 8? If so, find such a SPNE and the values of δ for which this is possible. If not, explain why.

Answer: Yes.

The following strategy will work:

In period 1: player 1 plays BLR and player 2 plays ARL

In period t: as long as player 1 received 8 for a stage game payoff in the previous round: continue playing as in period 1.

After any deviation, player 1 plays ALR forever and player 2 plays BRR forever.

Check that the above strategy is in fact a SPNE using one-shot deviation principle. $\delta \leq 2/3$ *in order to insure p2 not deviate.*

(d) Explain why there is no SPNE where player 2 achieves an average discounted payoff of 8?

Answer: If player 2 gets an average discounted payoff of 8, then p1 must get 2. But, this is below player 1's minmax payoff.

Economics ECONG008: Advanced Microeconomics

2015-16

Answer any three out of four questions.

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Please provide concise derivations of all your answers.

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Exam duration: 2 hour

Question no. 1

Two players play the following zero-sum game, where the payoff function of player 1 (the row player) is given by the following matrix:

	L	M	R
T	0	0.5	2
B	1	0.5	0

(a) Define the concept of mixed-strategy Nash equilibrium. What is the relation between this concept and max-minimization in a zero-sum game?

(b) Find the set of mixed-strategy Nash equilibria in this game.

Let us use the minimax theorem, and find the max-minimizing strategy for player 1. Let p denote the probability he plays T. The player's expected payoff from p is 1-p when player 2 plays L; 0.5 when player 2 plays M; and 2p when player 2 plays R. Taking the lower envelope of these three lines, we obtain the following. The max-min payoff is 0.5, and any p between 0.25 and 0.5 is a max-minimizer. Player 2's best reply to such p is M.

(c) Now suppose that the set of actions for player 2 is the unit interval [0,1]. Player 1's payoff function in the zero-sum game is given by $u_1(T,a_2)=2a_2$ and $u_1(B,a_2)=1-a_2$ for every action a_2 for player 2. Find a mixed-strategy Nash equilibrium in this game.

Once again, apply the minimax theorem. Every p for player 1 and a_2 for player 2 induce an expected payoff for player 1 of $p*2a_2+(1-p)*(1-a_2)$ = $(3a_2-1)p+(1-a)$. This function of p is a straight line. The maximum of the lower envelope must be when the slope of the line is zero, i.e. when $a_2=1/3$. We have thus pinned down player 2's strategy. To find a strategy for player 1, look for a value of p for which player 2 is indifferent among all

actions. Player 2's expected payoff can be written as –(3p-1)a-(1-p). If p=1/3, player 2's payoff is -2/3 for all values of a2.

Question no. 2

Annie (A) and Bob (B) are in a roller coaster competition against each other. The prize money they receive depends on how many roller coaster rides they take and on the number of rides the other takes. The prize money they receive is determined according to the following formulas: $P^A(r_A, r_B) = (30 - r_B)r_A$ and $P^B(r_B, r_A) = (30 - r_A)r_B$. Where $P^A(r_A, r_B)$ is the prize money Annie gets when she rides r_A rollar coasters and Bob rides r_B rollar coasters, and similarily for Bob. Both Annie and Bob like prize money but do not like riding roller coasters and pay a cost $\frac{1}{2}r_A^2$, and $\frac{1}{2}r_B^2$, respectively. Therefore, they have utilities over rides given by $U^A(r_A, r_B) = P^A(r_A, r_B) - \frac{1}{2}r_A^2$ and $U^B(r_B, r_A) = P^B(r_B, r_A) - \frac{1}{2}r_B^2$.

(a) Suppose Annie goes to the amusement park on Saturday and then Bob goes on Sunday. When Bob chooses his rides, he knows how many rides Annie has gone on. Find the SPNE.
Answer: $r_B^*(r_A) = 30 - r_A$ *and Answer:* $r_A^* = 0$

(b) How many roller coasters do Annie and Bob ride if they both simultaneously decide how many rides to go on? (i.e. find the NE)
Answer: $r_A^* = r_B^* = 15$

(c) Suppose Annie and Bob wanted to choose rides to maximize their joint weekly utility $U^A + U^B$. How many rides would they choose to go on each week? Assume $r_A = r_B$.
Answer: only option is to play NE in each stage-game

(d) Now, suppose Annie and Bob held a roller coaster competition every week for the rest of their lives (they never die). Suppose they each wanted to maximize their discounted sum of weekly utility (U_A and U_B, respectively). Could they sustain the joint utility maximizing outcome? If so, find a SPNE and a restriction on the discount factors that would support this.

Answer: yes they can sustain joint maximizing outcome. Play trigger strageies. Ride 10 in first period. If 10 has always been riden in every period before ride 10. Otherwise, rider 15 forever.
Will not want to deviate after history with no deviations:

$$\frac{150}{1-\delta} \geq 200 + \delta * \frac{112.5}{(1-\delta)}$$

$$150 \geq 200(1-\delta) + 112.5\delta$$

$$\delta \geq \frac{50}{87.5} \approx .57$$

Question no. 3

An incumbent firm in a market for some product chooses a production quantity. There are two, equally likely scenarios regarding the market structure. In one scenario, the firm is the sole producer. In the other scenario, a second producer (referred to as an "entrant") that makes a simultaneous production decision. The product's price P is determined according to the following inverse demand function: P = 1-Q, where Q is the total production quantity. Both firms maximize their revenues.

(a) Suppose that the incumbent is fully informed of the market structure prior to making its production decision.

1. What is the incumbent's choice when it is the sole producer?

   Standard monopoly with linear demand: choose q to maximize q(1-q), hence q=0.5

2. Formulate the situation when the incumbent learns that it faces an entrant as a strategic game, and find its pure-strategy Nash equilibrium.

   Standard Cournot competition with linear demand: the unique Nash equilibrium is (1/3,1/3).

(b) Now suppose that before the incumbent makes its production decision, it receives a binary signal regarding the market structure. The probability that the signal is accurate is 0.75.

1. Formulate the situation as a Bayesian game.

2. Find the game's pure-strategy Nash equilibrium.

Denote the entrant's action by e. Denote the incumbent's actions when it receives the monopoly and competition signals by m and c, respectively.
The entrant knows that the market structure is competitive. It believes that the incumbent received the competition signal with probability 0.75. Therefore, it chooses e to maximize

$$\mathbf{e*[1 - e - (0.75*c + 0.25*m)]}$$

Therefore, e = 0.5*[1-0.75c-0.25m].

The incumbent believes that the market structure matches its signal with probability 0.75 (thanks to the symmetric prior). Therefore, it chooses m to maximize

$$\mathbf{m*[1 - m - 0.25*e]}$$

Likewise, it chooses c to maximize

$$\mathbf{c*[1 - c - 0.75*e]}$$

Therefore, m = 0.5*[1-0.25e] and c = 0.5*[1-0.75e].

We now have three linear equations with three unknowns. The solution is e=0.21818, m=0.47273, c=0.41818.

Question no. 4

Father (F) and child (C) are going to the zoo. After seeing the elephants, bears, and lions, Nature decides with probability q that the child is tired and with probability 1-q that the child is not tired. After Nature moves, the child decides whether or not to ask for icecream. Father has to decide whether to give her ice cream or deny her ice cream. If the child does not ask for ice cream or if the Father decides to deny ice cream, they will go see the apes, which are Father's favourite.

The child gets a utility of 5 from eating ice cream when not tired and a utility of 4 from eating ice cream when she is. If she's not tired then her utility from seeing apes is 0 if she hasn't asked for ice cream and -1 if she has. If she's tired, her utility from seeing apes is -2 if she's been denied ice cream and 1 if she has not.

If his child is not tired, father gets a utility of 5 from seeing apes, unless he's had to deny ice cream, then it's 4. If his child is tired, his utility from seeing apes is 3, unless he's had to deny ice cream, then it's -2. His utility from ice cream is 1 if his child is tired and -1 if she is not.

(a) Draw the game tree.

Answer: Don't forget role of Nature – but all information sets are singleton.

(b) Find all SPNE.

Answer: 1 SPNE: Father gives icecream if tired and denies if not. Child asks for icecream if tired and doesn't ask if not.

Now suppose that the father cannot observe if his child is tired.

(c) For $q = \frac{1}{2}$, is there a PBNE where the child asks for ice cream only if she is tired? If so, find all such PBNE. If not, explain why not.

Answer: no such PBNE. The Child would also want to deviate and ask for icecream when she is not tired.

(d) For $q \in (0,1)$, is there a PBNE where the child asks for ice cream regardless of whether she is tired or not? If so, find all such PBNE. If not, explain why not.

Answer: Yes there is, as long as $q \geq \frac{5}{8}$. *Pr(T|ask)=q and Pr(NT|don't ask)=*α *for any* $\alpha \in [0,1]$. *Father gets icecream after ask. Child asks for icecream when tired and not tired.*

END OF EXAM

Economics ECONG008: Advanced Microeconomics
2016-17

Answer any three out of four questions.

Each question constitutes 33.3% of the final grade.

Please provide concise derivations of all your answers.

In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Exam duration: 2 hour

Question no. 1

Once upon a time, there was a pirate who hid a treasure in one of four horizontal locations:

A	B	C	D

The pirate is long dead. At the present time, a treasure hunter is looking for the treasure. She has a "treasure detector" and chooses to place it in one of the four locations. The detector can find the treasure if and only if the two are placed in the **same** location.

(a) Define the concept of max-minimization (allowing for mixed strategies).

(b) Find a mixed strategy that max-minimizes the treasure hunter's probability of finding the treasure.

The problem can be viewed as a zero-sum game between the hunter (player 1) and the pirate (player 2), where the hunter gets 1 if a_1=a_2 and -1 otherwise. Because of the symmetry across actions, it is easy to see there is a mixed-strategy Nash equilibrium in which both players mix uniformly over all four actions. By the minimax theorem, this is also a max-min strategy for the hunter.

(c) Now suppose that the detector can also detect the treasure if their locations are **adjacent**. Find a mixed strategy that max-minimizes the treasure hunter's probability of finding the treasure.

The payoff matrix In the zero-sum game is now changed. The hunter gets 1 if $|a_1$-$a_2|$<2, and -1 otherwise. We can guess and verify a mixed-strategy Nash equilibrium, where both players mix uniformly between the two extreme locations A and D. By the minimax theorem, this is also a max-min strategy for the hunter.

Question no. 2

Two countries make simultaneous armament decisions. Their payoffs are given by the following matrix:

	B	N
B	$-c_1$, $-c_2$	$1-c_1$, -2
N	-2 , $1-c_2$	0,0

where B denotes armament, N denotes refraining from armament, and c_i denotes the cost of armament for country i. This cost is country i's private information. Assume that c_i takes two possible values, 0.5 and 1.5. The probability that c_i=1.5 is p, independently across countries.

(a) Formulate the interaction as a game with incomplete information, and define the concept of Nash equilibrium in this game.

(b) For what values of p is there a Nash equilibrium in which countries always choose armament?

Suppose country 2 always plays B. Country 1's expected payoff from N is -2. Its expected payoff from B is $-c_1$, which is by definition always greater than -2. The calculation for country 2 is the same. Therefore, for all values of p there is an equilibrium in which both countries always play B.

(c) For what values of p is there a Nash equilibrium in which countries never choose armament?

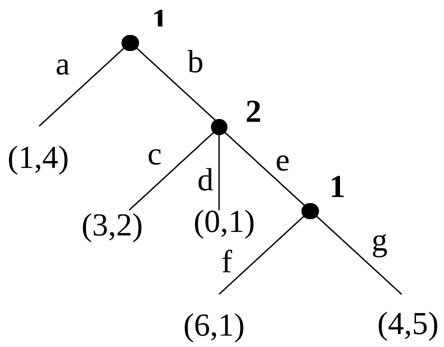
Suppose country 2 always plays N. Country 1's expected payoff from N is 0. Its expected payoff from B is $1-c_1$, and when c_1=0.5 it is greater than 0, hence B is a profitable deviation. Therefore, there is no value of p for which both countries always play N.

END OF EXAM

(d) For what values of p is there a Nash equilibrium in which each country chooses armament if and only if its armament cost is low?

Suppose country 2 plays B if and only if c_2=0.5. Country 1's payoff from N is (1-p)*(-2)+p*0 = 2p-2. Its payoff from B is p*1-c_1. We need the former expression to be higher than the latter if and only if c_1=1.5. That is, 2p-2 > p-1.5 and 2p-2<p-0.5. Therefore, we must have p>0.5.

Question no. 3



(a) Define subgame perfect equilibrium.

(b) Find all Nash equilibria and subgame perfect equilibria of the above game.

NE: ((b,f);c) ((a,f);d) ((a,g);d)

SPE: ((b,f);c)

(c) Let the above game be a stage game and suppose the stage game is repeated twice. Assume the discount factor is 1 for both players. Find all subgame perfect equilibria.

Play SPE in round 1. Play SPE in round 2 – regardless of history.

(d) Suppose the stage game is repeated infinitely. Assume both players have discount factors δ. Find the discount factors that support a subgame perfect equilibrium where player 2 earns 5 every round. If this is not possible, explain why not.

Such a subgame perfect equilibria exists. Supported by trigger strategies with SPE punishment:

Round 1: Play ((b,g);e)

Round t: If the outcome was (b,e,g) in all prior rounds then play ((b,g);e)

If the outcome was anything else in any prior round, then play ((b,f);c)

Question no. 4

A tutor offers to help a student prepare for an exam. If the student does not hire the tutor, then he will fail the class. If he hires the tutor, then the probability he passes the class depends on the quality of the tutor. If he hires a good tutor, he will pass with probability 3/4. If he hires a bad tutor, he will pass with probability ¼. The student believes that a good and bad tutor is equally likely. Assume that the student receives the equivalent of a monetary prize of V if he passes the exam and 0 if he fails. He seeks to maximizes his expected payoff net of any fees. So, if he pays x to the tutor and expects to pass the class with probability q, then his expected payoff is qV-x. The tutor seeks to maximize her expected earnings from tutoring. The tutor sets a price and the student can either hire the tutor at that price or not hire a tutor. You may assume that both players use pure strategies.

(a) Suppose the student knows she is playing against a good tutor. Find all subgame perfect equilibrium.

Tutor: Requests $x = \frac{3}{4}V$

Student: Hires if $x \leq \frac{3}{4}V$ *and doesn't hire otherwise*

(b) Is there a perfect Bayesian pooling equilibrium of the game in which both the good and bad tutors are hired? If so, find one, if not, explain why not.

There is a PBE: Good and bad tutors request same price x – and student believes good and bad tutors are equally likely, $\mu(G|x) = \frac{1}{2}$.

Student: Hires if $x \leq \frac{1}{2}V$ *and doesn't hire otherwise.*

Tutors: Requests $x = \frac{1}{2}V$.

Let $\mu\left(G \middle| x \neq \frac{1}{2}V\right) = 0$.

(c) Is there a perfect Bayesian separating equilibrium of the game in which both the good and bad tutors are hired? If so, find one, if not, explain why not.

No seperating. Suppose G tutor requests x_G *and bad tutor requests* $x_B \neq x_G$. *Thus,* $\mu(G|x_G) = 1$ *and* $\mu(G|x_B) = 0$. *There will always be an incentive for the bad tutor to deviate and offer* x_G.

(d) Now suppose the tutor sets a price and may also offer a money back guarantee in case the student fails. Again, the student can either accept the offer and hire the tutor or reject the offer and not hire the tutor. Is there a perfect Bayesian separating equilibrium of the game in which both the good and bad tutors are hired? If so, find one, if not, explain why.

Yes, there is a seperating PBE in this case. Suppose good tutor requests x_G *with money back guarantee (i.e.* (x_G, G)*) and bad tutor requests* x_B *and no guarantee (i.e.* (x_B, NG)*).*

Let, $\mu(G|x_G, G) = 1$ *and* $\mu(G|x, Z) = 0$ *for all* $(x, Z) \neq (x_G, G)$.

Student hires for (x,G) if $x \leq V$ *and hired for (x,NG) if* $x \leq \frac{1}{4}V$.

PBE when $x_G = V$ *and* $x_B = \frac{1}{4}V$.

END OF EXAM

SUMMER TERM 2018

ECONG008: ADVANCED MICROECONOMIC THEORY

TIME ALLOWANCE: 2 HOURS

Answer any three out of the four questions. Each question carries 33 1/3 marks.

In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 1 A group of *three* parents are organizing a party for their children. Each parent chooses independently whether to bring a cake. The party will be a success if and only if at least *two* parents bring a cake. All parents derive a gross payoff of 1 when the party is a success and 0 when it is not. A parent incurs a cost of $c \in (0,1)$ when he/she brings a cake.

(a) Formulate the interaction among the three parents as a strategic game.

(b) Define the concept of pure-strategy Nash equilibrium.

(c) Find all pure-strategy Nash equilibria in the game.

Any profile in which exactly two parent bring a cake is a pure-strategy Nash equilibrium. In addition, the profile in which no parent brings a cake is also a Nash equilibrium.

(d) Define the concept of symmetric Nash equilibrium, allowing for both pure and mixed strategies.

(e) Characterize the set of symmetric Nash equilibria in the game, as a function of the value of c.

For any c, the profile in which no parent brings a cake is a symmetric Nash equilibrium. In addition, there may exist a strictly mixed symmetric equilibrium. Let p denote the equilibrium probability that a parent brings a cake. The indifference condition is

$$\boldsymbol{v \cdot p^2 = v \cdot [p^2 + 2p(1-p)] - c}$$

The solution is $\boldsymbol{p = 0.5[1 + \sqrt{\square - 2c}]}$, for any $\boldsymbol{c \in (0, 0.5]}$. For c>0.5 there is no strictly mixed symmetric equilibrium.

Question 2 Consider the following two-player game with incomplete information. Each player $i=1,2$ chooses a real number a_i. Her payoff function is $u_i(a_1,a_2) = -(a_1-a_2)^2 - (a_i-\theta)^2$, where θ is an exogenous state of Nature which is distributed according to some probability distribution with mean μ. Assume that player 1 is fully informed of the realization of θ, whereas player 2 is entirely uninformed.

(a) Formulate the interaction as a game with incomplete information.

(b) Define the concept of Nash equilibrium in this game.

(c) Find the game's Nash equilibrium.

The informed player plays a=0.5(θ+μ). The uninformed player plays a=μ.

Question 3 Consider the following interaction between an expert (E) and a decision maker (DM). The state of the nature is $\theta \in \{0,1\}$, over which the common prior is given by $prob(\theta = 1) = 1/2$. The Expert privately observes the state of nature and then sends a report to the DM in which she can either truthfully report the state or lie. The decision maker observes the report and then chooses an action. The expert cares both about the action the DM takes and maintaining her reputation (i.e. it is costly for her to lie). The DM only cares about choosing the action that matches the state. The game has four stages:

Stage I Nature chooses θ according to the prior
Stage II Expert observes θ and reports $m \in \{0,1\}$
Stage III Decision maker observes m and chooses $x \in \{0,1\}$
Stage IV θ is observed and payoffs are realized

The payoff function of the DM is given by

$$u(x,\theta) = \left(\theta - \frac{1}{2}\right)x$$

The payoff function of the expert is given by

$$w(m,x,\theta) = \frac{1}{2}v(m,\theta) + \frac{1}{2}x$$

where v is the reputation function $v(m,\theta) = \begin{cases} 1, & m = \theta \\ 0, & m \neq \theta \end{cases}$.

a) Draw the game tree.

b) Is there a perfect Bayesian equilibrium in which the expert reports the state of nature truthfully? If your answer is yes, then specify such an equilibrium and verify that it is an equilibrium. If your answer is no, then provide a proof.
Beliefs for DM: $prob(\theta = 1|m = 1) = 1$, $prob(\theta = 1|m = 0) = 0$. *Strategy of DM: x(m=1)=1 and x(m=0)=0. Strategy of expert:* $m(\theta = 1) = 1$, $m(\theta = 1) = 0$.

c) Suppose now that in Stage II, the Expert does not observe θ but instead receives an informative but noisy signal $s \in \{0,1\}$. The precision of the signal is given by $prob(s = k|\theta = k) = 2/3$, for $k = 0,1$. Is there a perfect Bayesian equilibrium in which the expert reports his signal truthfully? If your answer is yes, then specify such an equilibrium and verify that it is an equilibrium. If your answer is no, then provide a proof.
Let $\mu_k = prob(\theta = 1|s = k)$. *Then by Baye's rule*

$$\mu_1 = 2/3$$
$$\mu_0 = 1/3$$

Suppose that the strategy of E is to truthfully report. Then the beliefs of DM are:

$$prob(\theta = 1|m = 1) = 2/3$$
$$prob(\theta = 1|m = 0) = 1/3$$

Consider a DM who recieves signal m=1.

$$EU_{DM}(1|m=1) = \frac{2}{3}\cdot\frac{1}{2} - \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6}$$
$$EU_{DM}(0|m=1) = 0$$

Thus, the DM plays x=1.

Consider a DM who recieves signal m=1.

$$EU_{DM}(1|m=0) = \frac{1}{3}\cdot\frac{1}{2} - \frac{2}{3}\cdot\frac{1}{2} = -\frac{1}{6}$$
$$EU_{DM}(0|m=0) = 0$$

Thus, the DM plays x=0.

Now, consider E who recieves signal s=1.

$$EU_{DM}(1|s=1) = \frac{2}{3}\cdot 1 + \frac{1}{3}\cdot\frac{1}{2}$$
$$EU_{DM}(0|s=1) = \frac{2}{3}\cdot 0 + \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6}$$

Thus, E plays m=1

Now, consider E who recieves signal s=0.

$$EU_{DM}(1|s=1) = \frac{1}{3}\cdot 1 + \frac{2}{3}\cdot\frac{1}{2}$$
$$EU_{DM}(0|s=1) = \frac{1}{3}\cdot 0 + \frac{2}{3}\cdot\frac{1}{2} = \frac{2}{6}$$

Thus, E plays m=1.

Therefore, there is no such equilibrium.

Question 4 Consider the following stage game payoff matrix:

		Player 2		
		Left	**Centre**	**Right**
Player 1	**Up**	6,6	0,0	1,8
	Middle	10,4	5,5	0,0
	Down	0,0	0,0	2,1

Suppose players play the infinitely repeated stage game. Assume players discount payoffs with the common discount factor $\delta \in (0,1)$.

a) Describe the set of individually rational payoffs for this game.
Individually rational for player 1 is any payoff >2 and for player 2 >1

b) Find the lowest δ that can sustain the outcome (Up, Left) in every round as part of a subgame perfect equilibrium.
1. play (U,L) in round 1
2. If (U,L) is played in all previous rounds play (U,L) otherwise play (D,R)
Minimum δ comes from P1 conditions. No optimal deviations for P1 requires

$$6 \geq 10(1-\delta) + 2\delta$$
$$\Rightarrow 8\delta \geq 4$$
$$\Rightarrow \delta \geq 1/2$$

Minimum $\delta = \frac{1}{2}$.

c) Suppose agents do not observe the actions played in the stage game perfectly. Specifically, assume that if the action profile played in the stage game is (s1, s2), then players observe (s1, s2) with probability γ and each of the other eight outcomes with equal probability $(1-\gamma)/8$. This means that players can only condition their round t behaviour on what was observed for the past t-1 rounds and not what was actually played. Payoffs depend on the action profiles played not on what was observed. Let $\delta = \gamma = 3/4$. Find a subgame perfect equilibrium where player 1 earns an average (expected) payoff greater than 6. HINT: Try a tit-for-tat strategy.
Strategy: Play (M,L) in round 1. Play (M,L) following any history where (M,L) or (D,R) was played in the previous round. Otherwise, play (D,R).
Player 2: Let V be the continuation value in histories following (M,L) or (D,R). Let V_p be the continuation value in all other histories.

$$V = (1-\delta)\cdot 4 + \delta(\gamma V + \frac{1}{8}(1-\gamma)V + \frac{7}{8}(1-\gamma)V_p)$$

$$V_p = (1-\delta) + \delta(\gamma V + \frac{1}{8}(1-\gamma)V + \frac{7}{8}(1-\gamma)V_p)$$
$$\Rightarrow V - V_p = 3(1-\delta)$$

Now, Check if SPE. There are no optimal one-shot deviations for player 1. Consider histories after (M,L) or (D,R). No optimal one-shot deviations for player 2 requires:

$$4(1-\delta) + \delta(\gamma V + \frac{1}{8}(1-\gamma)V + \frac{7}{8}(1-\gamma)V_p)$$
$$\geq 5(1-\delta) + \delta\left(\gamma V_p + \frac{6}{8}(1-\gamma)V_p + \frac{2}{8}(1-\gamma)V\right)$$

$$\Leftrightarrow \delta\gamma\left(V - V_p\right) \geq 1 - \delta$$

Therefore, the proposed strategies are a SPE.

Player 1 earns average payoff >6:

$$V = (1-\delta)\cdot 10 + \delta(\gamma V + \frac{1}{8}(1-\gamma)V + \frac{7}{8}(1-\gamma)V_p)$$
$$V_p = (1-\delta)\cdot 2 + \delta(\gamma V + \frac{1}{8}(1-\gamma)V + \frac{7}{8}(1-\gamma)V_p)$$
$$\Rightarrow V - V_p = 8(1-\delta)$$
$$V = 10 - 7\delta(1-\gamma)$$
$$V = 10 - 7\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{160}{16} - \frac{21}{16}$$
$$V = \frac{139}{16} \approx 8.6875$$