SUMMER TERM 2022

ONLINE EXAMINATION

ECON0061: PUBLIC MICROECONOMICS

Answer THREE questions.

Questions carry one third of the total mark each.

All work must be submitted anonymously. Please ensure that you add your **candidate number** and the module code to the template answer sheet provided. Note that the candidate number is a combination of four letters plus a number, e.g. ABCD9. You can find your candidate number in your PORTICO account, under "My Studies" then the "Examinations" container. Please, note that the candidate number is NOT the same as your student number (8 digits), which is printed on your UCL ID card. Submitting with your student number will delay marking and when your results might be available.

In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Allow enough time to submit your work. You are given 4 hours to solve your exam and an additional hour to submit your solutions on Moodle. This means that your work should be uploaded on Moodle **no later than 5 hours from the moment you accessed the examination paper for the first time**.

By submitting this assessment, **you pledge your honour** *that you have not violated UCL's Assessment Regulations which are detailed in*

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which include (but are not limited to) plagiarism, self-plagiarism, unauthorised collaboration between students, sharing my assessment with another student or third party, access another *student's assessment, falsification, contract cheating, and falsification of extenuating circumstances.*

1. A population consists of H identical individuals who each consume two private goods, bread q_1^h and cake q_2^h, $h = 1, \ldots, H$, and a public good, circuses Q. Preferences are captured in utility functions

$$u\left(q_1^h, q_2^h, Q\right) \;=\; \alpha \ln q_1^h + (1-\alpha)\ln\left(q_2^h + \sqrt{Q}\right).$$

Each individual has an endowment Y of grain which can be used to produce bread or cake or pooled with others to support circuses. The collective resource constraint requires

$$HY - \sum_{h=1}^{H}(q_1 + q_2) - PQ \;=\; 0$$

where P is a fixed marginal rate of transformation between bread or cakes and circuses.

(a) Suppose a central planner can decide each individual's consumption of bread and cake and collective consumption of circuses. What is the unique Pareto efficient level of circus provision?

Suppose instead that the government raises funds for circus provision by ad valorem taxes on bread and cake consumption. Pretax prices of the two goods are 1 and taxes are t_1 and t_2.

(b) Show that, given Q, individual demands are

$$q_1^h = \frac{\alpha}{(1+t_1)}\left[Y + (1+t_2)\sqrt{Q}\right]$$
$$q_2^h = \frac{1-\alpha}{(1+t_2)}\left[Y + (1+t_2)\sqrt{Q}\right] - \sqrt{Q}$$

and hence show that the government revenue constraint is

$$H\left(\left[\left(\frac{t_1}{1+t_1}\right)\alpha \;+\; \left(\frac{t_2}{1+t_2}\right)(1-\alpha)\right]\left[Y + (1+t_2)\sqrt{Q}\right] - t_2\sqrt{Q}\right) - PQ \;\geq\; 0.$$

(c) Suppose the government wants to maximise utility of a representative individual

$$\ln\left[Y + (1+t_2)\sqrt{Q}\right] - \alpha\ln(1+t_1) - (1-\alpha)\ln(1+t_2) + \alpha\ln\alpha + (1-\alpha)\ln(1-\alpha).$$

Given a Lagrangian for the problem

$$\ln\left[Y+(1+t_2)\sqrt{Q}\right]-\alpha\ln(1+t_1)-(1-\alpha)\ln(1+t_2)$$
$$+\lambda\left(\left[\left(\frac{t_1}{1+t_1}\right)\alpha\ +\ \left(\frac{t_2}{1+t_2}\right)(1-\alpha)\right]H\left[Y+(1+t_2)\sqrt{Q}\right]-Ht_2\sqrt{Q}-PQ\right)$$

with first order conditions

$$\begin{aligned}
0 &= -\frac{\alpha}{1+t_1}+\lambda\left(\frac{\alpha}{(1+t_1)^2}H\left[Y+(1+t_2)\sqrt{Q}\right]\right)\\
0 &= \frac{\sqrt{Q}}{Y+(1+t_2)\sqrt{Q}}\ -\ \frac{1-\alpha}{1+t_2}\\
&\quad +\ \lambda H\left(\frac{\alpha}{(1+t_2)^2}\left[Y+(1+t_2)\sqrt{Q}\right]+\left[\left(\frac{t_1}{1+t_1}\right)\alpha\ +\ \left(\frac{t_2}{1+t_2}\right)(1-\alpha)\ -\ 1\right]\sqrt{Q}\right)\\
0 &= \frac{1+t_2}{Y+(1+t_2)\sqrt{Q}}\\
&\quad +\ \lambda H\left(\left[\left(\frac{t_1}{1+t_1}\right)\alpha\ +\ \left(\frac{t_2}{1+t_2}\right)(1-\alpha)\right](1+t_2)\ -\ t_2\ -\ 2\frac{P}{H}\sqrt{Q}\right)
\end{aligned}$$

then show that it will tax bread and cake at the same rate. What will the provision of circuses be? Comment.

(d) Suppose now that the existence of a powerful cake lobby prevents taxation of cake so that $t_2=0$. All funds for circus provision must be raised by taxation of bread. What happens to the optimum provision of circuses? Comment.

2. A society has to decide social preference over three outcomes x, y and z. Individual opinions fall into three schools:

- A fraction p have preference: $x \succ y \succ z$
- A fraction q have preference: $y \succ z \succ x$
- A fraction $1 - p - q$ have preference: $z \succ x \succ y$

(a) For what values of p and q is social preference according to pairwise majority voting not transitive? Explain why this is a problem for social choice.

Consider the following method for determining social choice $\succ^*$ over the three options (known as instant runoff voting):

For any two outcomes a and b

- if a is the most preferred outcome of fewest individuals then $b \succsim^* a$
- if neither a nor b is the most preferred option of fewest individuals then $a \succsim^* b$ if and only if at least as many individuals prefer a to b as prefer b to a

In other words, the least socially preferred option is established by plurality voting then social preference between the other two is established by a pairwise majority vote.

(b) Are there any values of p and q for which social preference according to instant runoff voting is not transitive?

(c) Establish the values of p and q for which each of the three social outcomes is most socially preferred.

(d) What desirable properties does instant runoff voting fail to satisfy as a basis for determining social preference?

3. Original incomes y (which is to say, incomes before taxes and benefits) are distributed in a population according to distribution function F. Individuals pay taxes $T(y)$ and receive benefits $B(y)$, both positive at all incomes.

Define

$$\bar{y} \;=\; \int_0^1 y \,\mathrm{d}F(y) \qquad \bar{T} \;=\; \int_0^1 T(y)\,\mathrm{d}F(y) \qquad \bar{B} \;=\; \int_0^1 B(y)\,\mathrm{d}F(y)$$

$$L_y(p) \;=\; \frac{1}{\bar{y}}\int_0^p y \,\mathrm{d}F(y) \qquad L_T(p) \;=\; \frac{1}{\bar{T}}\int_0^p T(y)\,\mathrm{d}F(y) \qquad L_B(p) \;=\; \frac{1}{\bar{B}}\int_0^p B(y)\,\mathrm{d}F(y)$$

(a) $L_y(p)$ is the Lorenz curve for original incomes. Interpret $L_T(p)$ and $L_B(p)$.

Let $Y(y) = y - T(y) + B(y)$ denote final income. Assume $Y(y)$ is increasing everywhere in original income.

(b) Define the Lorenz curve for final incomes $L_Y(p)$ and show that

$$L_Y(p) - L_y(p) \;=\; \frac{\bar{T}}{\bar{y}-\bar{T}+\bar{B}}\left[L_y(p) - L_T(p)\right] \;+\; \frac{\bar{B}}{\bar{y}-\bar{T}+\bar{B}}\left[L_B(p) - L_y(p)\right].$$

Define original and final Gini coefficients

$$G_y \;= 1 - 2\int_0^p L_y(p)\,\mathrm{d}p \qquad G_Y \;= 1 - 2\int_0^p L_Y(p)\,\mathrm{d}p.$$

(c) Show that

$$G_y - G_Y \;=\; \frac{\bar{T}}{\bar{y}-\bar{T}+\bar{B}}\,D_T \;+\; \frac{\bar{B}}{\bar{y}-\bar{T}+\bar{B}}\,D_B$$

where

$$D_T \;=\; 2\int_0^p \left[L_y(p) - L_T(p)\right]\,\mathrm{d}p \qquad D_B \;= 2\int_0^p \left[L_B(p) - L_y(p)\right]\,\mathrm{d}p.$$

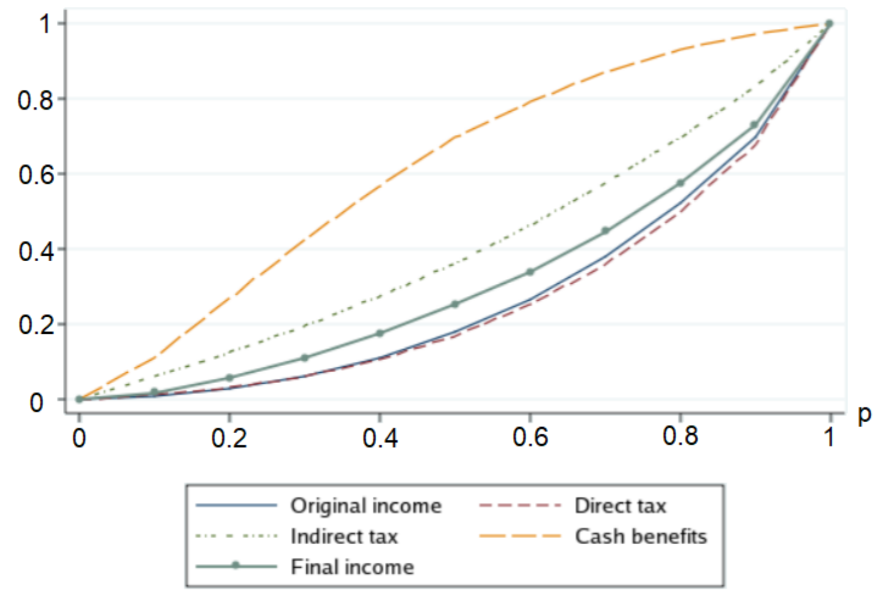
Interpret this equation.

Suppose that $T(y) = ty$ and $B(y) = (1-\phi)t\bar{y}$ where t is a constant marginal tax rate, ϕ is a parameter capturing administrative losses in the tax system and taxes and benefits are linked by government budget balance.

(d) Derive an expression for $G_y - G_Y$ in terms of t, ϕ and G_y.

(e) Figure 1 below shows cumulative shares of direct and indirect taxes, cash transfers, original and final incomes for the UK in 2007/8. Comment on the likely significance of the tax and benefit structure for the extent of redistribution.

Figure 1: Cumulative shares of taxes, transfers, original and final income



4. A population consists of individuals who supply labour L and consume c. Preferences are identical and described by utility functions

$$v(c, L) \;=\; c - \frac{1}{1-L}.$$

Wages w are distributed on the interval $[w_0, \infty)$ according to distribution function F where the lowest wage $w_0 > 1$. Earned income is taxed at a rate t and a uniform grant G paid from the revenue so that (assuming no other sources of revenue and no other forms of public expenditure)

$$G = t \int_{w_0}^{\infty} wL \, \mathrm{d}F.$$

Individuals have no other source of income so $c \;=\; w(1-t)L + G$.

(a) Establish an expression for chosen labour supply and show that all individuals work if $t < 1 - 1/w_0$.

(b) Assuming that this is true so that all individuals work, establish an expression for G and therefore show that utility for an individual with wage w is

$$U(w) \;=\; w(1-t) - 2\sqrt{w(1-t)} + \mu_1 t - \mu_{1/2} t / \sqrt{1-t}$$

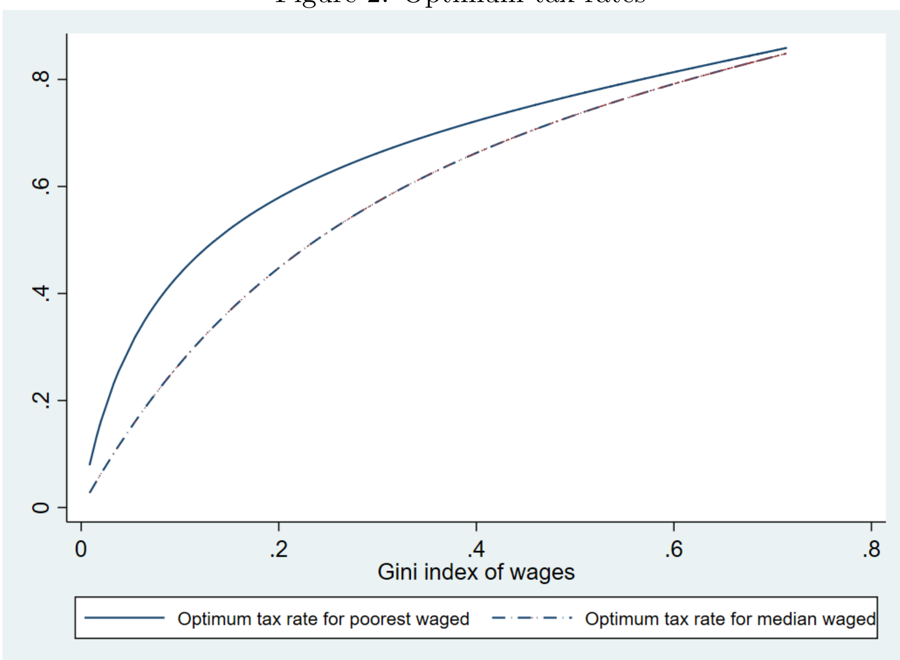
where $\mu_a \;=\; \int_{w_0}^{\infty} w^a \, \mathrm{d}F$.

(c) Suppose the government wants to set t so as to maximise mean utility $\int_{w_0}^{\infty} U(w) \, \mathrm{d}F$. What is the tax rate t^* which it sets? Comment.

(d) Suppose the government wants to set t to maximise utility of the worst off $U(w_0)$. Show that it sets a higher tax rate, $t_0 > t^*$.

(e) Figure 2 shows tax rates maximising utility of the lowest waged and the median waged individual as a function of the Gini index of wages under the assumption of Pareto-distributed wages with w_0 =10. (Note that at all values shown the condition for everyone to work is satisfied.) Comment.

Figure 2: Optimum tax rates



5. Individuals in a certain population supply labour and consume two goods, meat q_1 and rice q_2. They have wages w distributed according to distribution function F. Pretax prices of both meat and rice are 1. Government imposes ad valorem taxes t_1 and t_2, where $t_1,\ t_2\ > -1$, so that posttax prices are $p_1 = 1 + t_1$ and $p_2 = 1 + t_2$ and uses the revenue raised to fund a uniform lump sum grant G. There is no other government spending requirement and no direct tax on labour income. Assume throughout that we are only considering cases in which $q_1,\ q_2\ >\ 0$ for all individuals.

Preferences are identical and described by indirect utility function

$$V\left(G, w, p_1, p_2\right) \;=\; \frac{1}{2}\,\frac{w^2}{p_1^2} + \ln\left(\frac{p_1}{p_2}\right) + \frac{G}{p_1}.$$

Note that these preferences are characterised by weak separability between leisure and goods and linear Engel curves.

(a) Show that the grant paid equals

$$G \;=\; \tau_1\mu_2 \;+\; \frac{\tau_2 - \tau_1}{\left(1-\tau_1\right)^2}$$

where $\tau_i \;=\; t_i/\left(1+t_i\right),\ i = 1, 2$ and $\mu_\alpha = \int_0^\infty w^\alpha\,\mathrm{d}F$ for any α. and hence show that individual utility is

$$\begin{aligned} U \;=&\; \frac{1}{2}\left(1-\tau_1\right)^2 w^2 \;+\; \tau_1\left(1-\tau_1\right)\mu_2 \\ &-\; \ln\left(1-\tau_1\right) \;+\; \ln\left(1-\tau_2\right) \;+\; \frac{\tau_2-\tau_1}{1-\tau_1}. \end{aligned}$$

(b) Show that mean utility $\int U\ \mathrm{d}F$ is maximised by choosing $\tau_1 = \tau_2 = 0$.

(c) Suppose that the government chooses instead to maximise $\int \ln U\ \mathrm{d}F$. Show that it is still true that $\tau_1 = \tau_2$ at the optimum. Comment.

(d) Suppose that the government cannot administer a lump sum grant program and therefore only considers schemes such that $G = 0$. Note that, since quantities are positive, τ_1 and τ_2 therefore have opposite sign unless $\tau_1 = \tau_2 = 0$. Is it true now that $\tau_1 = \tau_2$ at the optimum?

6. Suppose the government is designing a tax schedule for taxable income Y. At some suitably high level of taxable income ζ tax liability is set at T_ζ and income above that point is taxed at a constant marginal rate τ.

   Suppose that

   - γ is the social marginal value of incomes of individuals with taxable incomes above ζ relative to public funds
   - η is the elasticity of taxable income of individuals with taxable incomes above ζ with respect to the retention rate $(1-\tau)$ holding after tax income at ζ fixed at $\zeta - T_\zeta$

   (a) Explain the argument for setting

   $$\frac{\tau}{1-\tau} = (1-\gamma)\frac{E(Y-\zeta \mid Y>\zeta)}{E(Y\eta \mid Y>\zeta)}$$

   (b) Suppose that $E(Y \mid Y>\zeta) \to \alpha\zeta$ as $\zeta \to \infty$. Show that

   $$\frac{\tau}{1-\tau} \to (1-1/\alpha)(1-\gamma)/\eta$$

   for high enough taxable incomes.

   (c) Assess the advantages and disadvantages of this sort of approach over the traditional Mirrleesian optimum income tax treatment. Include in your discussion some consideration of

   - the simplifying assumptions required
   - the formulation in terms of general taxable income rather than earnings .