

## Time Series Econometrics (G0058)

1. Assume:

$$y_t = \mu + x_t + z_t \quad (3)$$

$$x_t = \epsilon_t + \theta\epsilon_{t-1} \quad (4)$$

$$z_t = \beta z_{t-1} + u_t \quad (5)$$

With  $\epsilon_t \sim i.i.d.D(0, \sigma^2)$ ,  $u_t \sim i.i.d.D(0, \phi^2)$ , where D denotes a generic distribution with  $\epsilon$  and  $\eta$  are independent of each other.

(a) What is the process for  $y_t$ ?

(b) Given  $|\theta| < 1$  and  $|\beta| < 1$  (i.e.  $y_t$  stationary process). Find the forecast for the variable  $y_t$  at time  $t+s$ .

2. The goal of this exercise is to evaluate the median forecast of unemployment from the Survey of Professional Forecasters (SPF). The file `unemp_forecast.xls` contains the median one-quarter ahead forecast of unemployment. The file `unemp_realizations.xls` contains the realizations of unemployment for the corresponding quarter. Do the following:

(a) Plot the forecast and the realizations and comment (e.g., do we see consistent under- or over-predicting? Is there any pattern over time?)

(b) Conduct a forecast unbiasedness test using a squared error loss function. Are the forecasts unbiased?

(c) Test whether the one-quarter-ahead forecast errors are uncorrelated.

(d) Plot the autocorrelogram and partial autocorrelogram of the forecast errors. Do they suggest that the forecast errors are white noise?

(e) Construct sequences of one-step-ahead forecasts of unemployment using an AR(p) model estimated over the first  $m = 50$  observations, selecting p by BIC over the first estimation window. Compare the out-of-sample performance of the AR(p) forecasts to that of the SPF forecasts over the remaining observations by comparing the Mean Square Forecast Error for the two forecasts over the out-of-sample portion.

(f) Briefly comment on the results.