Tutorial 4 : Public Goods

(i) Individuals in a population of N individuals, $i \in I$, have endowments of a private good, bread, ω_i. They have preferences over consumption of bread q_i and of a public good, circuses, Q, as represented by Cobb-Douglas utility functions

$$u_i = \alpha_i \ln q_i + (1 - \alpha_i) \ln Q$$

where α_i are individual-specific preference parameters. There is a constant-returns-to-scale technology linking the two goods with one unit of bread needed to produce p units of circuses.

(a) Suppose production of circuses is by private contribution so that $Q = p \sum_i g_i$ where $q_i = \omega_i - g_i$. Consider the Nash equilibrium in which each individual contribution $\hat{g}_i$ is chosen optimally given the chosen contribution of every other individual. Let $C = \{i \mid \hat{g}_i > 0\}$ be the set of contributors in equilibrium and $Y^C = p \sum_{i \in C} \omega_i$ be the value of their collective endowment. Establish expressions for each individual's consumption of bread and the collective circus provision. Show that equilibrium circus provision is independent of the distribution of endowments among contributors. What must be true of the distribution of endowments for everyone to contribute?

Ans: Individual choices solve

$$\max_{g_i} \alpha_i \ln [\omega_i - g_i] + (1 - \alpha_i) \ln \left(p \sum_{j \in C} g_j \right)$$

which is the same as

$$\max_{q_i} \alpha_i \ln q_i + (1 - \alpha_i) \ln \left(Y^C - p \sum_{j \in C} q_j \right)$$

First order conditions require

$$pq_i = \frac{\alpha_i}{1 - \alpha_i} \left(Y^C - p \sum_{j \in C} q_j \right)$$

Summing over the set of M contributors gives

$$p\sum_{j\in C} q_j = \frac{A_C}{1+A_C}Y^C$$

where $A_C = \sum_{j\in C}\frac{\alpha_i}{1-\alpha_i}$ so that

$$\begin{aligned} Q &= \frac{1}{1+A_C}Y^C \\ pq_i &= \frac{\alpha_i}{(1-\alpha_i)(1+A_C)}Y^C && i\in C \\ pq_i &= p\omega_i && i\notin C \end{aligned}$$

Notice that Q and $q_i,\ i\in C$ depend only upon Y^C so that the distribution of endowments among contributors has no bearing on the equilibrium quantities so long as the set of contributors stays the same.

Individuals contribute if the value of their endowment exceeds the value of their equilibrium private consumption if a contributor. Hence all contribute if

$$p\omega_i > \frac{\alpha_i}{(1+\alpha_i)(1+A_I)}Y$$

where Y is the economy-wide endowment $Y = p\sum_{i\in I}\omega_i$, which will be true if the smallest value of $\omega_i(1-\alpha_i)/\alpha_i$ exceeds $Y/p(1+A_I)$

(b) Suppose now that the government decides to provide the circuses with funds raised through taxation. Say first that it divides the bread cost of provision Q/p equally across all voters and that provision is decided by exhaustive pairwise majority voting. Explain why there is a Condorcet winning provision and what it is.

Ans: Each individual's preferred level solves

$$\max_Q \alpha_i \ln(\omega_i - Q/pN) + (1-\alpha_i)\ln Q$$

which has an optimum $Q = (1-\alpha_i)p\omega_i N$. The Condorcet winning level is the choice of the voter with median value of $(1-\alpha_i)\omega_i$.

(c) Say instead that the government is elected with a mandate to implement a social consensus on maximising a particular social welfare function

$$W = \sum_{i\in I}\beta_i\left[\alpha_i\ln(\omega_i - T_i) + (1-\alpha_i)\ln Q\right]$$

by choice of Q and individual tax payments T_i with the constraint that

$$p\sum_{i\in I}T_i \geq Q.$$

The welfare weights β_i sum to unity, $\sum_{i \in I} \beta_i = 1$. Find the government's optimal circus provision and the tax payments to fund it.

Ans: The government's problem is

$$\max_{T_1,\ldots,T_N} \sum_{i \in I} \beta_i \alpha_i \ln(\omega_i - T_i) + (1-\alpha_i) \ln\left(p \sum_{i \in I} T_i\right)$$

This is a straightforward Cobb-Douglas maximisation with solutions

$$\begin{aligned} Q &= \sum_i \beta_i (1-\alpha_i)\, Y \\ pq_i &= \beta_i \alpha_i\, Y \end{aligned}$$

(d) What are the Lindahl prices at this social optimum? Show that the government's optimum coincides with a notional competitive equilibrium reached by imposing an appropriate set of lump sum transfers and individual-specific public good prices set at these Lindahl prices. Discuss.

Ans: Lindahl prices, π_i, equal the private good price times the marginal rates of substitution

$$\begin{aligned} \frac{\pi_i}{p} &= \frac{\partial u_i/\partial Q}{\partial u_i/\partial q_i} \\ &= \frac{(1-\alpha_i)/\sum_j \beta_j(1-\alpha_j)Y}{\alpha_i p/\beta_i \alpha_i Y} = \frac{\beta_i(1-\alpha_i)}{p\sum_j \beta_j(1-\alpha_j)} \end{aligned}$$

In the Lindahl equilibrium, endowments need to be such that individuals demand the socially optimal quantities at the Lindahl prices. It is therefore necessary to impose lump sum taxes δ_i such that

$$\begin{aligned} (1-\alpha_i)p(\omega_i - \delta_i)/\pi_i &= (1-\alpha_i)(\omega_i - \delta_i)/p \sum_j \beta_j(1-\alpha_j)/\beta_i(1-\alpha_i) \\ &= \sum_j \beta_j (1-\alpha_j)\, Y \\ \alpha_i p(\omega_i - \delta_i) &= \beta_i \alpha_i\, Y \end{aligned}$$

which are simultaneously satisfied by $\delta_i = \omega_i - \beta_i Y/p$. As a competitive equilibrium, the Lindahl equilibrium is Pareto efficient and public provision satisfies the Samuelson condition since $\sum_{i \in I} \pi_i = 1$.

(e) Discuss issues raised by the problems of identifying variation across individuals in the preference parameters α_i.

Ans: Implementation of Lindahl equilibrium would require knowledge of the Lindahl prices which in turn would require knowledge of preference parameters α_i. But individuals have no incentive to reveal these if they expect to be charged higher for declaring a stronger preference for the public good. Mechanisms such as Clarke-Groves schemes are designed to incentivise truthful revelation. Individuals' declared preferences *affect the quantity of the public good provided but contributions depend on the excess of cost of provision over other individuals' declared willingness to pay.*