Week 1 Problem Set

1. (Data exercise). From Yahoo Finance, download the last 5 years of data of daily SP500 index prices, first at the daily frequency and then at the weekly frequency.

   (a) Using the programme of your choice, construct the time series of weekly and daily returns as the percentage change in the stock price (or the difference in the logs)

   (b) Compute mean, skewness and kurtosis of both weekly and daily stock returns, and also plot the histograms. Compare these to the mean, skewness, kurtosis and histogram of a normal distribution. What do you find?

   (c) Compute the first order autocorrelation, $cov(y_t, y_{t-1})$ when y_t is either the stock price or the stock return, and the frequency is either daily or weekly.

2. (Data exercise). From FRED (https://fred.stlouisfed.org/), download data on 3 month (interbank) interest rates for the following currencies: USD (USD3MTD156N), EUR (EUR3MTD156N), JPY (JPY3MTD156N), GBP (GBP3MTD156N) and CHF (CHF3MTD156N).

   (a) Using the programme of your choice, compute the monthly average interest rates for the period January 1999 to December 2019. Plot the time series.

   (b) Compute mean, skewness and kurtosis of the interest rates and the change in the interest rates. How do these compare to the stock price data?

   (c) Compute the first order autocorrelation, $cov(y_t, y_{t-1})$ when y_t is either the interest rate or the change in the interest rate. How do these compare across different currencies?

   (d) Which two currencies have the highest crosscorrelation at time t both in levels and in changes? How about if you allow one currency to lead or lag another?

Week 2 Problem Set

1. Suppose that in the regression $y_t = \beta x_t + u_t$ you know that x_t is stationary and ergodic, with $E(x_t) = 1$ and x_t independent of u_t, however $E(u_t) = -1$ for $t < T/2$ and $E(x_t) = 1$ for $t < T/2$. Can you use standard inference in the regression?

2. Suppose in the regression in question 1 you instead find that $x_t u_t$ is stationary and ergodic with $E[x_t u_t] = .5$, and that it is autocorrelated, with $cov(u_t, u_{t-1}) = 1$ and $cov(u_t, u_{t-j}) = 0$ for $j > 1$. What are the asymptotic properties of $\hat{\beta}$?

3. Suppose in the regression in question 1 you have that $x_t u_t$ is stationary and ergodic with mean $E[x_t u_t] = 0$, and that it is autocorrelated, with $cov(u_t, u_{t-1}) = 1$ and $cov(u_t, u_{t-j}) = 0$ for $j > 1$. What are the asymptotic properties of $\hat{\beta}$?

4. Suppose $y_t \sim iidN(\mu, \sigma^2)$. Find $E(y_t)$, $Cov(y_t, y_{t-j})$ and the conditional distribution $f(y_t|\Omega_{t-1})$.

5. Suppose $y_t \sim iidN(\mu, \sigma^2)$ and let $z_t = \frac{y_t - \mu}{\sigma}$. Find $E(z_t)$, $Cov(z_t, z_{t-j})$ and the conditional distribution $f(z_t|\Omega_{t-1})$. Then find the log-likelihood, given by $log f(z_1, ..., z_T)$ (the joint density of the sample).

6. (Data exercise). From Yahoo Finance, download the last 5 years of data of the SP500 index and the price of a stock of your choice at the monthly frequency. Then compute the returns on your chosen stock. Then regress these stock returns on the SP500 monthly returns (make sure the two time series are aligned. i.e., the two returns are for the same month). Decide which standard errors to use to test the significance of the regression coefficient, using intuition, perhaps aided by analyzing the time series properties of the regressor and residuals (e.g., plotting the relevant series and computing the relevant autocorrelations)

Week 3 Problem Set

1. Consider the model
$$Y_t = \rho Y_{t-2} + \varepsilon_t, \varepsilon_t \sim \text{iid } N(0,1)$$
   (a) Under which conditions on ρ is the model covariance-stationary? Show how you reached your conclusion
   (b) Find the conditional mean $E(Y_t|\Omega_{t-1})$ when $\rho = .5$
   (c) Find the unconditional mean and variance $E(Y_t)$ and $Var(Y_t)$ when $\rho = .5$
   (d) Suppose that $\rho = 1$ and $(Y_0, Y_1) = (0, 0)$. Show that the variance of Y_t is not constant over time.
   (e) Suppose $\rho = 1$ in (1). Let $\Delta Y_t = Y_t - Y_{t-1}$. Find β so that $\Delta Y_t = \beta \Delta Y_{t-1} + \epsilon_t$. Is ΔY_t covariance stationary?

2. Consider the model
$$Y_t = \varepsilon_t + 2.4\varepsilon_{t-1} + 0.8\varepsilon_{t-2}$$
where ϵ_t is iid $N(0,1)$
   (a) Is this process covariance stationary?
   (b) Is it invertible?
   (c) Calculate the autocovariance function γ_j for $j = 1, 2, \ldots$

3. Consider the two AR processes $Y_t = .5Y_{t-1} + \epsilon_t$ and $Y_t = Y_{t-1} + \epsilon_t$. In both cases express Y_t in terms of Y_0 by recursive substitution. Comment on the effect of Y_0 on Y_t as $t \to \infty$ in the two cases.

4. Is the process $Y_t = .5Y_{t-1} + \epsilon_t$ invertible? If so, express it as an $MA(\infty)$.

5. Let $Y_t = Y_{t-1} + \epsilon_t$ and $X_t = \beta Y_t + \nu_t$ with ϵ_t, ν_t i.i.d.(0, 1), independent of each other. Show that X_t has a unit root (hint: express Y_t as a function of ϵ_t using lag polynomials and substitute into the expression for X_t).

6. Are the following processes covariance-stationary?
   (a) $Y_t = (1 + 2.4L + 0.8L^2)\epsilon_t$, ϵ_t white noise;
   (b) $(1 - 1.1L + 0.18L^2)Y_t = \epsilon_t$, ϵ_t white noise

7. Suppose $Y_t = 0.5Y_{t-1} + \epsilon_t$, ϵ_t iid(0,1) and that you take first differences $Z_t = \Delta Y_t = Y_t - Y_{t-1}$.
   (a) What is the process for Z_t?
   (b) Can this process be written as AR(∞)?
   (c) Is the OLS estimate of ϕ obtained by regressing Z_t on Z_{t-1} a consistent estimate of 0.5? Why or why not?

Weekly Problem Set 4

1. Consider the model $y_t = -1 * I(t < T/2) + I(t \geq T/2) + \varepsilon_t + \varepsilon_{t-1}$, where I() is the indicator variable taking value 1 if the inequality in parenthesis is true. What model is this? Is it stationary? If $\varepsilon_t \sim N(0, \sigma^2)$, what is the density of $y_t|\Omega_{t-1}$?

2. Assume that ϵ_t and η_t are mean zero, i.i.d. normal random variables, that are independent of each other and are of unit variance. Let x_t be an observable stationary time series satisfying:

$$x_t = \gamma x_{t-1} + \eta_t,$$

with $|\gamma| < 1$ amd y_t is an observable time series given by:

$$y_t = \beta x_t + \epsilon_t,$$

   (a) Show that y_t is weak stationary.

   (b) Suppose that you observe a sample of obeservations of (x_t, y_t) for $t = 1, \ldots, T$ and regress y_t on x_t. What is the asymptotic distribution of the least squares regression coefficient?

   For both questions you may assume that $x_0 = 0$ and can be taken as fixed.

3. Consider an ARMA(1,1): $y_t = \phi y_{t-1} + \varepsilon_t + \psi \varepsilon_{t-1}$, with $\varepsilon_t \sim Niid(0, 1)$ and $|\phi| < 1$.

   - What is the distribution of y_1?
   - Using the properties of a conditional normal compute $E(\varepsilon_1|y_1)$.
   - Using your answer to the above, compute $E(\varepsilon_2|y_1, y_2)$.

4. (Data excercise) Return to problem set 1, question 2 and the monthly interest rate data for the 5 currencies. Compute the ACF and PACF for the interest rates in levels and differences. Are the data stationary? What do the ACF and PACF suggest about potential time series models to fit the data?

Weekly Problem Set 5

1. A student fits an AR model to a time series and is concerned about structural breaks, particularly in the first half of the sample.

   - The student splits the sample in two and runs an Andrews 1993 break test on the first half sample with a 15 percent trim. The test rejects the null. The student repeats the exercise on the second half sample but the test does not reject the null. The student argues that there is only one structural break in the data in the first half of the sample. Do you agree or disagree? If so why?
   - The student then runs the Bai and Perron (1998) procedure on the first half sample which returns a result consistent with a single break. The student repeats her assertion. Do you agree or disagree? If so why?

2. Let the sequence $\{y_t\}_{t=1}^{T}$ be a stationary stochastic process such that $y_t = \alpha y_t - 1 + \epsilon_t$ and $\epsilon_t \sim N.i.i.d(0, \sigma^2)$. Show that the maximum likelihood estimator (conditional on y_1 – i.e. ignore $f(y_1)$) of α is equivalent to a LS regression of y_t on $y_t - 1$.

3. Consider a simple TAR model that is piecewise constant:

$$Y_t = \begin{cases} \phi_1 + \sigma_1 \epsilon_t & Y_{t-1} \leq r \\ \phi_2 + \sigma_2 \epsilon_t & Y_{t-1} > r \end{cases}.$$

   Where ϵ_t is a mean zero i.i.d. normal random variable with unit variance and r is a constant. Let $R_t = 1$ if $Y_{t-1} \leq r$ and 2 otherwise. Calculate the following probabilities: $Pr(R_t = 1|R_{t-1} = 1)$, $Pr(R_t = 2|R_{t-1} = 1)$, $Pr(R_t = 2|R_{t-1} = 2)$ and $Pr(R_t = 1|R_{t-1} = 2)$.

4. (Data exercise) Return to the dataset *FrozenJuice* used in the week 2 synchronous class. As in class, set up an ADL model regression of the real log difference in frozen juice prices regressed on freezing days and lagged changes in frozen juice prices. Assuming 18 monthly lags of freezing days are used, how many lags of the change in frozen juice prices have to be included in the regression for a Breusch-Godfrey test not to reject the null hypothesis of no autocorrelation in the estimated residuals.

Department of Economics Spring 2021
UCL

Time Series Econometrics (G0058)

1. Assume:

$$y_t = \mu + x_t + z_t \tag{3}$$
$$x_t = \epsilon_t + \theta\epsilon_{t-1} \tag{4}$$
$$z_t = \beta z_{t-1} + u_t \tag{5}$$

With $\epsilon_t \sim i.i.d.D(0, \sigma^2)$, $u_t \sim i.i.d.D(0, \phi^2)$, where D denotes a generic distribution with ϵ and η are independent of each other.

(a) What is the process for y_t?

(b) Given $|\theta| < 1$ and $|\beta| < 1$ (i.e. y_t stationary process). Find the forecast for the variable y_t at time t+s.

2. The goal of this exercise is to evaluate the median forecast of unemployment from the Survey of Professional Forecasters (SPF). The file unemp_forecast.xls contains the median one-quarter ahead forecast of unemployment. The file unemp_realizations.xls contains the realizations of unemployment for the corresponding quarter. Do the following:

(a) Plot the forecast and the realizations and comment (e.g., do we see consistent under- or over-predicting? Is there any pattern over time?)

(b) Conduct a forecast unbiasedness test using a squared error loss function. Are the forecasts unbiased?

(c) Test whether the one-quarter-ahead forecast errors are uncorrelated.

(d) Plot the autocorrelogram and partial autocorrelogram of the forecast errors. Do they suggest that the forecast errors are white noise?

(e) Construct sequences of one-step-ahead forecasts of unemployment using an AR(p) model estimated over the first $m = 50$ observations, selecting p by BIC over the first estimation window. Compare the out-of-sample performance of the AR(p) forecasts to that of the SPF forecasts over the remaining observations by comparing the Mean Square Forecast Error for the two forecasts over the out-of-sample portion.

(f) Briefly comment on the results.

1. Consider the model,

$$Y_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t z_t$$

Where $z_t \sim i.i.d.\ \ N(0,1)$.

(a) Suppose Y_t stationary and $\sigma_t^2 = w + \alpha\epsilon_{t-1}^2$, i.e. an ARCH(1) model. Show that the variance of Y_t is given by:

$$\sigma_y^2 = Var(Y_t) = \frac{w}{1-\alpha}$$

What are the restrictions for the variance to be well defined?

(b) Define $v_t \equiv \epsilon_t^2 - \sigma_t^2$. Show that v_t is a white noise process.

(c) Now suppose an ARCH(q) process. Show that if $\sum_{i=1}^{q} \alpha_i < 1$ then the process is stationary and with a well-defined second moment. Use v_t defined in b) to link an ARCH process with an AR process.

Consider the following ARCH(2) model for the stock returns r_t:

$$\begin{aligned} r_t &= \mu + \epsilon_t; \\ \epsilon_t &= \sigma_t z_t;\ z_t \sim iidN(0,1), \\ \sigma_t^2 &= \alpha_0 + \alpha_1^2\epsilon_{t-1}^2 + \alpha_2\epsilon_{t-2}^2; \end{aligned} \tag{1}$$

where r_t is the return on a stock. Assume that all necessary restrictions hold on the coefficients so that the process is stationary and σ_t^2 is always non-negative. Answer all the following questions.

1. Derive the conditional mean and variance of r_t, conditional on information up to time t-1.

2. Derive the unconditional mean and unconditional variance of r_t.

3. Show that ϵ_t^2 follows an AR(2) model, while ϵ_t is not autocorrelated. Which stylized fact in the data is consistent with this finding?

3. Assume now the following model:

$$\begin{aligned} Y_t &= \mu + \epsilon_t && (1) \\ \epsilon_t &= \sigma_t z_t && (2) \end{aligned}$$

$$\sigma_t^2 = w + \alpha_1 \mathbb{I}(\epsilon_{t-1} < 0)\epsilon_{t-1}^2 + \alpha_2 \mathbb{I}(\epsilon_{t-1} \geq 0)\epsilon_{t-1}^2 \quad (3)$$

where z_t is i.i.d. $(0, 1)$, and $\mathbb{I}(.)$ is the indicator function. That is, $\mathbb{I}(Y_{t-1} < 0) = 1$ if $Y_{t-1} < 0$ and zero otherwise.

(a) Show that if $\alpha_1 = \alpha_2$, the model collapses to the usual lineal ARCH. Use this to interpret the coefficients α_1 and α_2.

(b) Show that the volatility process can be express as:

$$\sigma_t^2 = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \gamma_2 \mathbb{I}(\epsilon_{t-1} \geq 0)\epsilon_{t-1}^2 \quad (4)$$

and get an expression for γ_0, γ_1 and γ_2 as a function of w, α_1 and α_2.

(c) Explain how you would formally test for asymmetric news impact using either model (3) or (4).

1. (Exam 2010/2011) Suppose you have $Y_t = (1 + \theta L)\epsilon_t$, with $\epsilon_t | I_{t-1} \sim N(0, \sigma_t^2)$ and $\sigma_t^2 = w + \alpha\epsilon_{t-1}^2$, ($I_{t-1}$ is the informational set at time t-1).
   (a) When is the model covariance-stationary?
   (b) Write down the sample likelihood of the model for a sample of size T.
   (c) Explain how you would test for the presence of conditional heteroskedasticity in the model[1].

2. Consider the following VAR model:

$$y_{1t} = 0.3y_{1t-1} + 0.8y_{2t-1} + \epsilon_{1t} \quad (1)$$
$$y_{2t} = 0.9y_{1t-1} + 0.4y_{2t-1} + \epsilon_{2t} \quad (2)$$

   With $E(\epsilon_{1t}\epsilon_{1\tau}) = 1$ for $t = \tau$ and 0 otherwise, $E(\epsilon_{2t}\epsilon_{2\tau}) = 2$ for $t = \tau$ and 0 otherwise and $E(\epsilon_{1t}\epsilon_{2\tau}) = 0\, \forall t, \tau$.
   (a) Is this system covariance-stationary?
   (b) Calculate $\Omega_s = \frac{\partial y_{t+s}}{\partial \epsilon_t}$, for s=0,1 and 2. What is the limit as $s \longrightarrow \infty$?

3. Consider the following bivariate VAR,

$$y_t = \rho_{yy}y_{t-1} + \rho_{ym}m_{t-1} + u_{yt} \quad (3)$$
$$m_t = \rho_{my}y_{t-1} + \rho_{mm}m_{t-1} + u_{mt} \quad (4)$$

   With $E(u_t u_t') = \begin{bmatrix} \sigma_y^2 & \gamma \\ \gamma & \sigma_m^2 \end{bmatrix}$.
   (a) Find a matrix H, which is lower triangular and ensures that if $Hu_t = \epsilon_t$, then $E(\epsilon_t\epsilon_t') = D$, where D is a diagonal matrix.
   (b) Given this matrix H calculate the structural representation of this VAR.
   (c) Calculate the VMA representation for the reduced form of this VAR.
   (d) Calculate the VMA representation of the structural form of the VAR.
   (e) Under what conditions will the reduced form and the structural form produce identical impulse response functions?

[1] There's an additional question but requires you to know about Forecasting, for latter this last question is: Find the density forecast for Y_{t+1}, i.e. the conditional density of Y_{t+1} given I_t. Do it while you are studying for the exam. Check past exams solutions

1. Let $Y_t = (IP_t, r_t, \pi_t)'$, where IP is industrial production growth, r interest rate and π inflation, measured at a quarterly frequency. Suppose you estimated the following model for Y_t :

$$\begin{aligned} Y_t &= \varepsilon_t + \begin{pmatrix} 1 & -.05 & -.5 \\ 0 & 1 & .4 \\ .1 & -.9 & 1 \end{pmatrix} \varepsilon_{t-1} + \begin{pmatrix} 1 & -.01 & -.1 \\ .2 & 1 & .7 \\ 0 & -.2 & 1 \end{pmatrix} \varepsilon_{t-2},\ \varepsilon_t\ iid(0,\Omega),\ \text{where} \\ \Omega &= \begin{pmatrix} 1 & 0 & .3 \\ 0 & 1 & .1 \\ .3 & .1 & 1 \end{pmatrix} \end{aligned}$$

   (a) What is the effect of a one-unit shock to interest rate on inflation 1-, 2- and 3-quarters ahead?

   (b) Does your answer to point a. represent how you would revise your future inflation expectations after learning new information about output and the interest rate today? Why or why not?

   (c) What is the effect of a one-unit shock to interest rate on industrial production growth one-quarter ahead?

2. Let x_t = log of output m_t = log of money supply u_{st} = iid supply shock u_{mt} = iid money shock. Consider the SVAR(1)

$$\begin{aligned} A_0 Y_t &= A_1 Y_{t-1} + u_t,\ E u_t u_t' = \Omega_u \\ Y_t &= \begin{pmatrix} \Delta x_t \\ \Delta m_t \end{pmatrix} \ u_t = \begin{pmatrix} u_{st} \\ u_{mt} \end{pmatrix} \end{aligned} \tag{1}$$

   and the corresponding reduced-form VAR(1)

$$Y_t = \Phi Y_{t-1} + \varepsilon_t,\ E\varepsilon_t\varepsilon_t' = \Omega_\varepsilon \tag{2}$$

   (a) Explain how you would test that money does not Granger-cause output in the model

   (b) In general, how many restrictions do you need to identify the structural parameters in (1) if you know the reduced-form parameters?

   (c) Suppose that Ω_u is diagonal and that the estimation of (2) yields reduced-form parameters $\Phi = \begin{pmatrix} .4 & -.4 \\ .1 & .7 \end{pmatrix}$, $\Omega_\varepsilon = \begin{pmatrix} 16 & 4 \\ 4 & 26 \end{pmatrix}$. Under the assumption that money does not contemporaneously affect output, derive the structural parameters in (1).

(d) Using the identifying assumption in point c), how would you compute the impulse response function (IRF) of both output and money to a supply shock? The IRF of output and money to a money shock?

3. The files hours.txt and realgdp.txt respectively contain quarterly data on average weekly hours worked and real GDP from 1964:2 to 2007:3. Compute the logarithmic growth rate of hours and of productivity (obtained as the GDP/Hours ratio).

   (a) Estimate a VAR(1) model for the productivity growth and the hours growth. Which model is selected by BIC? Which by AIC?[1] In each case, test whether the residuals of the model(s) selected by each information criterion are serially correlated.

   (b) For the model (or models) selected in question a., compute the impulse-response functions for all variables and all shocks, up to 10 quarters, with asymptotic confidence intervals (Eviews and Stata gives you the option). What is the effect of a shock to hours growth on productivity growth? How long does it take to die out?

   (c) For the model (or models) selected in question a., test whether the residuals are normally distributed. What are the implications for the use of an asymptotic confidence interval for the impulse-response function (IFR)?

   (d) For the model (or models) selected in question a., compute confidence intervals for the IRF that are based on re sling methods (in Eviews, you can do this by selecting the "Monte Carlo" standard errors option). Do your results change?

4. Download the series x1.txt, x2.txt, x3.txt. Do the following analysis in Eviews:

   (a) Test for a unit root in the three time series

   (b) Regress x1 on x2 and report the R^2 and significance of the regression coefficient. What do you conclude?

   (c) Plot the autocorrelogram of the residuals from the regression in point b. and do a unit root test on the residuals. What do you conclude?

   (d) Take first differences of x1 and x2 and repeat the analysis in point b. and c. for the first differences

   (e) Test whether x1 and x2 are cointegrated

   (f) Test whether x1, x2 and x3 are cointegrated find the number of cointegrating relationships and the cointegrating vectors.

   (g) Let $\beta = (\beta_1, \beta_2, \beta_3)$ be the cointegrating vector estimated in point f. above (if there are more than one, just choose one). Construct a new variable as $Y_t = \beta_1 x1 + \beta_2 x2 + \beta_3 x3$ and test whether it has a unit root

[1] In Stata, use the comand "varsoc". Eviews does it automatically for you by clicking on view-lag structure-lag length criteria.