UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **ECONG021**

ASSESSMENT PATTERN : **ECONG021A**

MODULE NAME : **Microeconomics**

DATE : **13-May-15**

TIME : **10:00**

TIME ALLOWED : **3 Hours 0 Minutes**

TURN OVER

ECONG021 (Microeconomics), 2015

Examination time allowed: THREE hours

Answer all questions from Part A and TWO questions from Part B.

In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Part A: Answer all questions.

A.1. {*8 marks*} Give a definition of a weak sequential equilibrium.

A.2. {*8 marks*} Give a formulation of the Revenue Equivalence theorem.

A.3. {*9 marks*} Give a definition of an incentive-compatible direct mechanism. Write down an incentive compatibility condition for the model of single-object auction with agents who have independent private values.

A.4. {*9 marks*} Denote the Marshallian demand of a consumer by $\mathbf{f}(y, \mathbf{p})$. Derive the relationships between elasticities known as Engel aggregation and Cournot aggregation.

A.5. {*8 marks*} Consider an exchange economy. Which of the following statements are correct/incorrect? (You do not have to explain your answers.)

(a) Every allocation in the core is Pareto efficient.

(b) Fix the initial endowment vector $\boldsymbol{\omega}$. For every allocation $\mathbf{x}$ in the core, there is a Walrasian equilibrium such that $\mathbf{x}$ is the corresponding equilibrium allocation.

(c) If preferences are convex, there exists a unique Walrasian equilibrium.

(d) For convex preferences, the equilibrium allocation is in the core.

A.6. {*8 marks*} State the definition weak separability (a) in terms of preferences and (b) in terms of the utility representation.

Part B: Answer TWO questions. Each question is worth 25 marks.

B.1. Consider an economy with one commodity (call it money) and two individuals $i = a, b$. There are two states of the world $s = 1, 2$ that occur with equal probability. The initial endowments are

$$\omega_1^a = \omega_2^a = \omega_1^b = 1 \text{ and } \omega_2^b = 2.$$

Both individuals are expected utility maximizers with Bernoulli utility functions

$$u^s(x) = \ln x, \ s = a, b.$$

Ex-ante, i.e., before the state of the world is realized, the individuals can trade two assets X and Y: Asset X is a claim on one unit of money if state 1 is realized and nothing if state 2 is realized. Asset Y is a claim on one unit of money if state 2 is realized and nothing if state 1 is realized. Both assets can be bought and sold in continuous quantities and the price of asset X is normalized to $p_X = 1$. Denote the price of asset Y by p_Y.

(a) Compute the competitive equilibrium price p_Y^* without using the First Welfare Theorem.

(b) State the First Welfare Theorem and show that it is satisfied.

(c) Now suppose that asset X is no longer available. Instead, there is a new asset $\tilde{X}$ which is a claim on one unit of consumption in both states. Asset Y is as before. Again, normalize $p_{\tilde{X}} = 1$. Call the price of asset Y, $\tilde{p}_Y$.

(i) For each individual, formulate the initial endowment as a portfolio of assets.

(ii) Compute the new equilibrium price $\tilde{p}_Y^*$.

B.2. You observe the following partial data set about purchases of a consumer in two different years. There are only two goods and she always spends her entire budget.

	Year 1		Year 2	
	Quantity	Price	Quantity	Price
Good 1	100	100	120	100
Good 2	100	100	x_2^2	80

(a) For what range of observed quantities x_2^2 can you conclude (derive your answers formally):

(i) that the consumption bundle in year 1 is revealed preferred to the bundle in year two?

(ii) that the behavior violates WARP?

(b) If the observed behavior satisfies WARP, is it also consistent with maximization of a utility function? (Explain your answer)

In the following, assume that WARP is satisfied:

(c) Consider the price change from year one to year two.

(i) Does the Slutsky compensation increase or decrease the total budget? First, explain your answer without computing the compensation and then compute the exact amount.

(ii) Show that the observed quantities imply that good 2 is inferior (at some prices) if $x_2^2 \in (80, 100)$.

(c) For what levels of x_2^2 can you conclude that good 1 is inferior (at some prices)?

B.3. Consider a competitive labor market with a continuum of workers. Each worker is characterised by his productivity $\theta \in \{1, 2, 4\}$. A worker's productivity is his private information. The proportion of workers of type $\theta = 1$ is $\alpha \in [0, 1]$ and the proportion of workers of type $\theta = 4$ is $\beta \in [0, 1 - \alpha]$. Before applying for a job, each worker chooses the level of his education $e \geq 0$ (which is the amount of time spent in college). Education is costly and

has no effect on productivity. If a worker with productivity θ gets an education e and a wage w his payoff is

$$U(e, w, \theta) = w - \frac{e}{\theta}.$$

If a worker decides not to participate in the job market, he gets zero payoff.

(a) Assume that $\alpha = \beta = 1/3$. Solve for a pooling equilibrium in which agents obtain no education.

(b) Continue assuming that $\alpha = \beta = 1/3$. Are there equilibria in which agents with productivity $\theta = 2$ and $\theta = 4$ obtain the same level of education e, that is distinct from the amount of education obtained by agents with $\theta = 1$? If the there are such equilibria, find one in which the total cost of education across all agents is the lowest. Is it unique?

(c) Assume that $\beta = 0$ and $\alpha \in (0, 1)$. Consider a modification of the problem in which all colleges are owned by firms. All firms simultaneously offer programs to prospective students who can choose any program they like. A typical program offered by a firm consist of spending e amount of time in college and receiving a job offer conditional on graduation (terms of the program are a firm's choice). Formalise this problem as a game (adding reasonable assumptions where they are necessary to complete the model). Find an equilibrium in this game (whenever one exists) and provide conditions for its existence.

B.4. A government decides to tax the revenue of firms that produce rubber. There is a continuum of rubber-producing firms in the economy. A firm can produce an amount of rubber x at a cost cx^2, where $c > 0$ is a cost parameter (may be different for different firms). A firm can be of two types: high-tech or low-tech. High-tech firms have a cost parameter $c = c_g$ and low-tech firms have a cost parameter $c = c_b > c_g$. The type of a firm is its private information and is not observed by the government. However, the government

knows that the share of the high-tech firms in the economy is $\alpha \in (0, 1)$. Assume that the price of rubber is fixed and equal to 1. The government can observe a firm's revenue (recall the difference between the revenue and the profit of the firm) and taxes it using a non-linear tax $t(R)$, i.e. a firm that receives a revenue R has to pay $t(R) \leq R$ in taxes to the government.

(a) Assume first that the type of each firm is publicly observable. Find a tax scheme that maximizes government's tax revenue.

(b) Find a tax scheme that maximizes the government's tax revenue in the presence of private information.

(c) Compare the outputs and profits of the firms under tax schemes you found in B.4(a) and B.4(b). Provide an intuition for your findings.