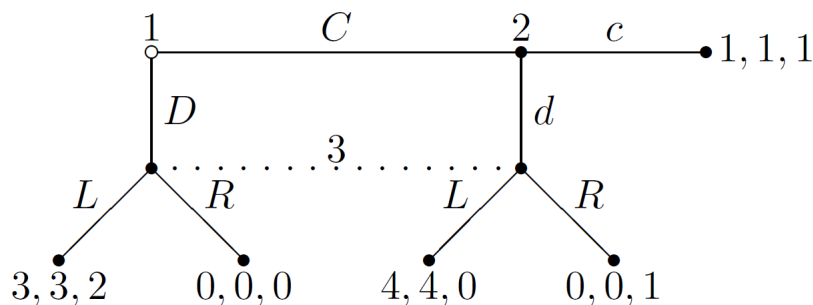


MIDTERM 3

Question 1 (Selten's horse). Find all sequential equilibria in the game below and defend your answer:



Let $((r, 1 - r), (q, 1 - q), (p, 1 - p))$ be a strategy profile. Let $(\mu, 1 - \mu)$ denote the beliefs of player 3 at his information set.

- 1 Player 2 chooses c if $p \in [0, 1/4]$ (he's indifferent at $p=1/4$).
- 2 Player 3 chooses L if $\mu \in (1/3, 1]$.
- 3 Player 1 chooses D if $u(D) = 3p$ is greater than $u(C) = q4p + (1 - q)$.

Assume $p = 0$. Then, by (1), $q = 0$. By (3), P1's utility from C is 1 while his utility from D is 0, so he's happy to choose C . It remains to find a belief of Player 3 that makes it rational for him to choose $p = 0$. Any $\mu \in (0, 1/3]$ works, and we can come up with a sequence of fully mixed strategies converging to (q, r) s.t. the implied belief converges to μ .

Therefore, $((0, 1), (0, 1), (0, 1))$ together with any $\mu \in (0, 1/3]$ is a sequential equilibrium.

Assume $p \in (0, 1/4]$. Then P3 is playing a mixed strategy, so his utility from the two actions is equal. It follows that $\mu = 1/3$. By (3), P1's utility from D is $3/4$ and his utility from C is 1. He therefore chooses C with probability $(r = 1)$. The only way to justify $\mu = 1/3$ is if P2 chooses c with probability 1, which indeed is a best response.

Therefore, $((0, 1), (0, 1), (p, 1 - p))$ with $\mu = 1/3$, $p \in (0, 1/4)$ is a sequential equilibrium.

Finally, assume $p > 1/4$. Then Player 2 chooses d ($q = 1$). Player 1's utility from D is $3p$ while his utility from C is $4p$. Therefore, Player 1 chooses C . Since Player 3's beliefs must be consistent, he'll know he's at the second node and choose R . But this means $p = 0$ a contradiction.

Question 2 (Repeated Bertrand). Consider the Bertrand duopoly model described as follows. The firms name prices simultaneously; demand for firm i 's product is $a - p_i$ if $p_i < p_j$, is 0 if $p_i > p_j$ and is $(a - p_i)/2$ if $p_i = p_j$; marginal costs are $c < a$.

(A) Find the unique Nash equilibrium and defend your answer.

Claim: The only NE is the one where both firms produce at marginal cost.

Proof: First, consider a situation where one of the firms sets $p_i < c$. Then its profits are negative, a contradiction.

Now, assume one firm produces at cost and the other firm charges a strictly higher price. Then both firms get 0 profit. The first firm could instead increase its price a little bit and get $(a - p_i)(p_i - c)$, which is positive. A contradiction.

Finally, assume both firms charge prices above cost. If the prices are the same, then one firm could lower its price a little bit and get higher profit. If the prices are different, the firm with the lower price could lower its price a little bit and get higher profit. Either way, we have a contradiction.

The only remaining case is that both firms produce at marginal cost.

(B) Consider the infinitely repeated game based on this stage game. Show that the firms can use trigger strategies (that switch forever to the stage-game Nash equilibrium after any deviation) to sustain the monopoly price level in a subgame perfect Nash equilibrium if and only if $\delta \geq 1/2$.

If both firms produce at cost, the profit is zero. The profit in a monopoly can be obtained by solving

$$\max p_i(a - p_i) - c(a - p_i)$$

Let's denote the monopoly profit by Π_M . The first order conditions for sustaining

collusion are:

$$\Pi_M/2 \geq (1 - \delta)\Pi_M,$$

which rearranging gives $\delta \geq 1/2$.

Question 3. (a) Explain why always defect is the only subgame perfect Nash outcome in a finitely repeated prisoner's dilemma.

The argument is backward induction. In the very last period, each player's dominant strategy is to defect. Then in the previous next to last you know that whatever you do will not affect the next period. So it's a dominant strategy to defect again, etc.

(B) Consider a prisoner's dilemma played in continuous time. Let $[0, 1]$ denote the time interval and assume, for example, that players make choices at every point in the interval $[0, 1)$. Does the argument you used to argue for always defect apply here? Why or why not?

It doesn't work here because the "period next to last" is not defined.