Weekly Problem Set 4

1. Consider the model $y_t = -1 * I(t < T/2) + I(t \geq T/2) + \varepsilon_t + \varepsilon_{t-1}$, where I() is the indicator variable taking value 1 if the inequality in parenthesis is true. What model is this? Is it stationary? If $\varepsilon_t \sim N(0, \sigma^2)$, what is the density of $y_t|\Omega_{t-1}$?

2. Assume that ϵ_t and η_t are mean zero, i.i.d. normal random variables, that are independent of each other and are of unit variance. Let x_t be an observable stationary time series satisfying:

$$x_t = \gamma x_{t-1} + \eta_t,$$

with $|\gamma| < 1$ amd y_t is an observable time series given by:

$$y_t = \beta x_t + \epsilon_t,$$

   (a) Show that y_t is weak stationary.

   (b) Suppose that you observe a sample of obeservations of (x_t, y_t) for $t = 1, \ldots, T$ and regress y_t on x_t. What is the asymptotic distribution of the least squares regression coefficient?

   For both questions you may assume that $x_0 = 0$ and can be taken as fixed.

3. Consider an ARMA(1,1): $y_t = \phi y_{t-1} + \varepsilon_t + \psi \varepsilon_{t-1}$, with $\varepsilon_t \sim Niid(0, 1)$ and $|\phi| < 1$.

   - What is the distribution of y_1?
   - Using the properties of a conditional normal compute $E(\varepsilon_1|y_1)$.
   - Using your answer to the above, compute $E(\varepsilon_2|y_1, y_2)$.

4. (Data excercise) Return to problem set 1, question 2 and the monthly interest rate data for the 5 currencies. Compute the ACF and PACF for the interest rates in levels and differences. Are the data stationary? What do the ACF and PACF suggest about potential time series models to fit the data?