

## Weekly Problem Set 5

1. A student fits an AR model to a time series and is concerned about structural breaks, particularly in the first half of the sample.
  - The student splits the sample in two and runs an Andrews 1993 break test on the first half sample with a 15 percent trim. The test rejects the null. The student repeats the exercise on the second half sample but the test does not reject the null. The student argues that there is only one structural break in the data in the first half of the sample. Do you agree or disagree? If so why?
  - The student then runs the Bai and Perron (1998) procedure on the first half sample which returns a result consistent with a single break. The student repeats her assertion. Do you agree or disagree? If so why?
2. Let the sequence  $\{y_t\}_{t=1}^T$  be a stationary stochastic process such that  $y_t = \alpha y_{t-1} + \epsilon_t$  and  $\epsilon_t \sim N.i.i.d(0, \sigma^2)$ . Show that the maximum likelihood estimator (conditional on  $y_1$  – i.e. ignore  $f(y_1)$ ) of  $\alpha$  is equivalent to a LS regression of  $y_t$  on  $y_{t-1}$ .
3. Consider a simple TAR model that is piecewise constant:

$$Y_t = \begin{cases} \phi_1 + \sigma_1 \epsilon_t & Y_{t-1} \leq r \\ \phi_2 + \sigma_2 \epsilon_t & Y_{t-1} > r \end{cases}.$$

Where  $\epsilon_t$  is a mean zero i.i.d. normal random variable with unit variance and  $r$  is a constant. Let  $R_t = 1$  if  $Y_{t-1} \leq r$  and 2 otherwise. Calculate the following probabilities:  $Pr(R_t = 1 | R_{t-1} = 1)$ ,  $Pr(R_t = 2 | R_{t-1} = 1)$ ,  $Pr(R_t = 2 | R_{t-1} = 2)$  and  $Pr(R_t = 1 | R_{t-1} = 2)$ .

4. (Data exercise) Return to the dataset *FrozenJuice* used in the week 2 synchronous class. As in class, set up an ADL model regression of the real log difference in frozen juice prices regressed on freezing days and lagged changes in frozen juice prices. Assuming 18 monthly lags of freezing days are used, how many lags of the change in frozen juice prices have to be included in the regression for a Breusch-Godfrey test not to reject the null hypothesis of no autocorrelation in the estimated residuals.