

## **Advanced Microeconomic Theory**

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### **Problem Set no. 4**

#### **Problem no. 1 (common-value auction)**

Find a symmetric Nash equilibrium in the first-price version of the common-value auction studied in class.

#### **Problem no. 2 (Cournot competition with uncertainty about demand)**

Two profit-maximizing firms compete in a market for a homogeneous good, according to the following model. Each firm independently chooses its production quantity. The market price  $P$  is determined according to the equation  $P = A - Q$ , where  $Q$  is the sum of the firms' production quantities, and  $A$  is a parameter that gets three possible values,  $L$ ,  $M$  and  $H$ , with probabilities  $1/2$ ,  $1/3$  and  $1/6$ , respectively. Firm 1 is informed of whether or not  $A=L$ , while firm 2 is informed of whether  $A=H$  or not. Formulate this interaction as a Bayesian game and characterize a Nash equilibrium as a function of the underlying parameters. (For simplicity, assume that negative quantities are permissible.)

#### **Problem no. 3 (market entry game)**

Two profit-maximizing firms, denoted 1 and 2, simultaneously decide whether to enter a market. When a firm chooses not to enter, it earns zero profits. When a firm enters while its opponent stays out, the firm earns  $1-c$ , where  $c$  is an exogenous parameter in  $(\frac{1}{2}, 1)$ . Finally, when both firms enter the market, firm 1 earns  $x-2c$  while firm 2 earns  $1-x-2c$ , where  $x$  is an exogenous state of Nature that takes the values 0 and 1 with equal prior probability. Firm 1 knows the value of  $x$ . When  $x=1$ , firm 2 receives an alert with probability  $\frac{3}{4}$ . When  $x=0$ , firm 2 receives no alert.

Formulate the interaction as a Bayesian game and characterize its Nash equilibria as a function of  $c$ .