BOUNDED RATIONALITY and INDUSTRIAL ORGANIZATION

RAN SPIEGLER

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OXFORD
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Published by Oxford University Press, Inc.
198 Madison Avenue, New York, New York 10016
www.oup.com

Library of Congress Cataloging-in-Publication Data
Spiegler, Ran.
Bounded rationality and industrial organization / Ran Spiegler.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-19-539871-7 (cloth : alk. paper)
1. Consumer behavior. 2. Decision making. 3. Consumption
(Economics)–Psychological aspects. 4. Industrial organization
(Economic theory) I. Title.
HF5415.32.S67 2011
658.8′342–dc22 2010015046

1 3 5 7 9 8 6 4 2

Printed in the United States of America
on acid-free paper

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PREFACE

This book summarizes and synthesizes recent developments in the theory of Industrial Organization that incorporate departures from the standard model of consumer behavior in the direction of a "richer psychology." It cannot be denied that the challenge of enriching the psychology of decision makers in economic models has been at the very frontier of theoretical research in the last decade, given an enormous boost by the behavioral economics movement.

However, while the subject has given rise to a large and varied literature that includes numerous research articles as well as a number of surveys and anthologies, one can think of very few proper *textbooks* that could serve a graduate-level course in economic theory. Indeed, the most recent theory-oriented textbook of any relevance that I am aware of is Ariel Rubinstein's 1998 *Modeling Bounded Rationality*, which preceded many of the developments that caused such a stir in our profession. This book aims to narrow this gap, albeit in the very specific domain of industrial organization.

The book is meant to serve as a textbook for graduate courses in microeconomic theory, as well as theory-oriented courses in industrial organization or behavioral economics. It is partly based on lecture notes from courses given at Tel Aviv University, University College London and the Helsinki Center of Economic Research. In the course of writing this book, I have greatly benefitted from financial support by the European Research Council and the ESRC (UK). Parts of the research it is based on were supported by BSF and ISF grants.

I wish to thank several colleagues who gave comments on earlier drafts of book chapters: Eddie Dekel, Kfir Eliaz, Susana Esteban, Ignacio Esponda, Erik Eyster, Yves Guéron, Michael Grubb, Paul Heidhues, Philippe Jehiel, Barton Lipman, Marco Mariotti, Erik Mohlin, Michele Piccione, Jidong Zhou, and especially Ayala Arad for her consistently superb feedback. I am hugely indebted to Ariel Rubinstein, who not only contributed concrete suggestions that helped me improve the exposition and proofs of several results, but also provided valuable encouragement throughout this project. Ariel got me "hooked" on the subject of bounded rationality fifteen years ago, and I have continued to benefit from his work and his thoughts ever since. Ayala Arad and Yves Guéron have provided truly spectacular research assistance. In particular, Yves helped typesetting the manuscript and wrote the solution manual. I am forever grateful to both. My coauthors, Kfir Eliaz and Michele Piccione, deserve some of the credit for the material in the book, as our joint work has formed the basis of several chapters. The Oxford University Press editors, Terry Vaughn and Joe Jackson, provided warm and reliable support. Finally, I wish to thank the composer Steve Reich, whose music provided an indispensable soundtrack for the writing process. If the book feels monotonous or repetitive at times, blame it on him.

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1 Introduction

Consider the following market situations you may have encountered as a consumer:

- You receive an invitation to accept a new credit card. You are reluctant to accept it because you are concerned that, armed with this new credit card, you will be tempted to spend more than you wish.
- You consider acquiring a new service at what seems like an attractive price. You suspect, however, that you may be unaware of certain future contingencies that will make it considerably more expensive than you can estimate at present.
- You try to choose a mutual fund from a vast array of market options, and you find the task of evaluating and comparing the numerous alternatives daunting. It is not that you lack information. On the contrary, information is superabundant, and it overwhelms you.
- You have spent the last hour reading the latest telephone bill, and you are still trying to figure out how the bottom line was reached.
- You listen to a sales pitch / watch a TV commercial / stare at the package of a product, and although these have no informational content, you feel more willing to consider buying the product.

These situations should be familiar to most members of an advanced economy. However, they present the modern economic theorist with a dilemma. Does conventional economic theory offer adequate tools for making sense of these situations and their welfare implications, or do we need new modeling tools? This book springs from the belief that the latter is true. In each of the above situations, there is an aspect of consumer psychology that takes the situation outside the scope of standard economic theory. My objective in this book is to present and synthesize recent theoretical research that has tried to formalize some of these aspects of decision making and examine their implications for market interactions—specifically, for the subfield of microeconomic theory that studies individual markets, namely Industrial Organization (henceforth I.O.).

The theory of I.O. has been developed in several major waves. The first wave involved the formulation of basic models of market structure (particularly perfect competition and monopoly). The second and third waves came more or less simultaneously in the 1970s. One wave saw the introduction of Game Theory into I.O., enabling thorough exploration of static and dynamic models of oligopoly. The other incorporated asymmetric information into I.O. models. Economists have come to realize that informational asymmetries constitute a major source of market friction, and that certain market and non-market institutions may be explained as responses to this market failure.

Throughout these developments, a maintained assumption has been that all market agents are rational. They are assumed to hold well-defined preferences. Although the rational choice paradigm allows preferences to be defined over very general domains, in virtually all I.O. applications rationality is narrowly practiced: preferences are defined over "simple" consequences, fully specified by the amount of money the consumer pays and the quantity or quality of the product he consumes. In addition, agents in standard models have full understanding of the market model. When they are imperfectly informed, they have perfect ability to draw Bayesian inferences in accordance with correct knowledge of the market model and market equilibrium.

Bounded rationality is another potential source of market friction. When some agents have limited understanding of their market environment (including their own behavior in certain circumstances), limited ability to process information, and preferences that are highly unstable, context-dependent, and malleable, market outcomes may differ in interesting and economically significant ways from the rational-consumer benchmark. Moreover, introducing boundedly rational agents into our market models may challenge conventional wisdom regarding the welfare properties of market interactions.

Here are a few of the theoretical questions I address:

- Can we view certain aspects of firms' pricing, marketing, and product differentiation strategies as responses to consumers' bounded rationality?
- To what extent are boundedly rational consumers vulnerable to exploitation by profit-maximizing firms? Does market competition protect consumers from being exploited?
- Does interaction between firms and boundedly rational consumers give rise to inefficiencies, and how are these affected by competition?
- What is the impact of various regulatory interventions in markets with boundedly rational consumers?
- Do market forces impel firms to "educate" or "de-bias" boundedly rational consumers?
- Does greater consumer rationality imply more competitive market outcomes?
- What sort of methods do firms use to discriminate between consumers according to the type and magnitude of their deviation from perfect rationality?

As can be gleaned from these questions, the book maintains an important distinction: firms will always be rational profit maximizers with a correct understanding of the market model, as in standard theory. On the other hand, consumers will depart from the standard model. The primary justification for this simplistic dichotomy is that a firm is more likely to conform to the standard model, in the sense that it focuses its attention, intelligence, and internal organization on a small set of markets. In contrast, consumers devote a fraction of their attention and intelligence to any individual market. Firms interact repeatedly with the market and therefore have many opportunities to learn its regularities. In contrast, consumers often have limited opportunities to learn the market model and the market equilibrium. Firms deliberately apply systematic reasoning, relying on experts and statistical data, whereas consumers often rely on intuition. These are essentially asymmetries in *rationality*.

This is not to say that firms do not have their own types of non-rational behavior. For example, firms are perhaps more vulnerable than individual consumers to the follies of groupthink. Communication failures and frequent staff changes within the firm can lead to weak organizational memory. However, this book completely abstracts from these issues. The distinction between rational, profit-maximizing firms and boundedly rational consumers is maintained throughout.

WHY BOUNDED RATIONALITY?

The motivation for market models that involve boundedly rational consumers is threefold. First, there are the casual, everyday observations of the type that traditionally inspired economic thought, with which I opened this book. Second, there is a growing sentiment among economists that certain phenomena (advertising, consumption of addictive goods, complicated financial products) are not captured in a satisfactory manner by standard rational-choice models. Finally, experimental psychologists have made a powerful case for the claim that decision makers systematically deviate from the model of rational choice as it is typically practiced by economists.

The term "bounded rationality" is notoriously vague. Many readers (not to mention authors of some of the works I shall discuss) would justly view some of the decision models as entirely compatible with rationality—even if not in the narrow sense described above and practiced in traditional I.O. models. Rubinstein (1998) classifies decision models in the bounded-rationality category when they explicitly incorporate procedural elements of decision making that are absent from the standard model of rational choice. Gilboa & Schmeidler (2001) use bounded rationality as a way to judge decision-making quality: a decision is not rational if it embarrasses the decision maker once the situation is explained to him. Other economists view decision models with unstable and context-dependent preferences as manifestations of bounded rationality, even if the modeling tools they end up using are more or less standard.

Therefore, the book's title is inescapably a misnomer. It might just as well have been called "I.O. Applications of Non-standard Models of Consumer Behavior." My main excuse for using the B-word is that I think it will create the right expectations among its potential readership. My other excuse is that it is no less a misnomer than "rational expectations."

WHY I.O.?

Bounded rationality manifests itself in many environments. Why did I choose to focus entirely on applications to I.O.? First, this is an area of fundamental importance in economics. The consumer is the decision unit that receives the greatest attention in our undergraduate textbooks, and the analysis of how rational consumers interact with profit-maximizing firms in single markets is the bedrock of undergraduate microeconomics. For this reason, there is something very basic about investigating the implications of introducing boundedly rational consumers into individual markets.

Second, there is a methodological advantage in focusing on I.O. models. Throughout this book, consumers will be *non-strategic* agents, just as in the basic market models familiar from undergraduate microeconomics. The only strategic agents in these models will be the firms. Since firms will be assumed to be rational profit maximizers, it will be legitimate to analyze their behavior with standard tools (constrained maximization, Nash equilibrium). This greatly simplifies analysis and allows us to gain more mileage than if we also tried to construct and analyze models in which boundedly rational agents are also strategic players.

Third, narrowing the scope of investigation to a certain class of I.O. models enables me to achieve, within these restrictions, greater coherence and generality than is often the case with treatments of bounded rationality. It will allow me to trace recurring questions and themes and suggest generalizations.

However, none of these arguments would be relevant if this were not an area that has seen significant progress in the last decade. The literature on I.O. models with boundedly rational consumers is not huge. At this stage of its development, it does not justify a "bible" akin to Jean Tirole's 1988 *Theory of Industrial Organization*. However, in my opinion, it is ripe for a concise, pedagogical synthesis that distills the literature's major recurring questions as well as its main theoretical insights. This is my task in this book.

■ PLAN OF THE BOOK

The book presents a few strands in the development of I.O. models with boundedly rational consumers. Each of its first three parts addresses a different major aspect of consumer psychology and examines its implications for pricing, marketing, or product differentiation strategies employed by firms, as well as for welfare analysis.

Part I: Anticipating Future Preferences

This part presents a collection of closely related dynamic models, in which consumers are confronted with price schemes that pertain to future consumption decisions. It mostly deals with *dynamically inconsistent preferences*, covering a large class of situations in which consumer tastes change over time as a result of various psychological forces, such as temptations that appeal to visceral urges, addiction, or changing reference points. A key feature of the models analyzed in this part is that some consumers are "naive," in the sense that they fail to fully appreciate the likelihood or magnitude of future taste changes. Variants include an extended model that allows consumers to exercise self-control in response to temptations, and a model with biased beliefs about future preferences without dynamic inconsistency.

Part II: Responding to Market Complexity

This part examines situations in which market alternatives are complicated (either inherently or as a result of firms' deliberate obfuscation), and the ways in which boundedly rational consumers cope with this complexity. First, I introduce a decision model in which consumers evaluate market alternatives by sampling small parts and extrapolating naively. Second, I study a model in which consumers reason in terms of "coarse representations" of market alternatives.

Part III: Reference Dependence

This part examines decision models in which consumer choice is sensitive to reference points. I begin by tackling situations in which preferences exhibit loss aversion, and then turn to models in which consumers' default options exert a pull on their decision process, resulting in consumer inertia.

While each part is based on a different aspect of the psychology of decision making, the treatment is unified by several factors. First, the non-standard decision models are invariably embedded in I.O. models that are among the most basic in economics, such that if consumers were rational, everything would collapse to an undergraduate textbook model.

Second, there is considerable overlap among the I.O. questions analyzed in the three parts. For the most part, I explore the implications of non-standard decision models for the firms' pricing behavior (including price discrimination, the structure of price plans, and marketing effects such as add-on pricing or the use of irrelevant alternatives), product differentiation strategies, and welfare analysis (including the effect of regulatory interventions).

Third, the three topics studied in Parts I–III are interrelated. The formation of biased beliefs about future preferences is closely related to the problem of evaluating a complex contract when the number of possible future contingencies is so large that the consumer may be unaware of some of them. Likewise, the complexity of evaluating market alternatives may strengthen consumer inertia. When preferences are sensitive to a reference point, they may display dynamic inconsistency because the reference point may shift over time. These are only a few of the examples I could give for such links.

Although the selection of topics for Parts I–III covers a significant portion of the theoretical literature on I.O. with boundedly rational consumers, it is not intended to be exhaustive. For example, limited consumer memory is almost entirely absent from the book. I selected those topics I thought I knew how to turn into compelling lecture notes. And of course, there is the natural bias in favor of research I have been involved with myself.

Part IV concludes the book and takes a more global view of the material, by revisiting themes—substantive economic themes as well as methodological ones—that recur throughout the book.

HOW TO USE THE BOOK

The book is primarily meant to be used as a set of lecture notes for graduate courses in microeconomic theory, industrial organization, or behavioral economics. I hope that certain features of the presentation—simple pedagogical models, a well-defined domain of economic applications, complete proofs for most of the results, and close attention to choice-theoretic considerations—will make the book appealing to researchers in various fields.

The book is modular. Its three main parts can be studied independently of each other. The themes that recur in Parts I–III are discussed from a bird's-eye view in Part IV. An instructor who wishes to give graduate students a taste of developments in economic theory that involve elements of bounded rationality can use any of the first three parts, coupled with a glance at the final part.

In terms of necessary background, the book presupposes that the reader knows basic microeconomic theory, preferably in the form of a core graduate course. It does not require any additional mathematical knowledge. Part I is perhaps the one most suited for advanced undergraduates.

STYLE OF THE BOOK

The pedagogical orientation of this book dictates a style of presentation that seeks the simplest framework that could express the ideas I wish to convey. Thus, instead of presenting a catalogue of the literature, in each chapter I study thoroughly a single model (or a family of closely related models) and use it to synthesize and adapt ideas from a number of papers, which themselves often follow diverse modeling approaches. This has the unavoidable consequence that my mode of presentation and emphases often depart from the original works. In this sense, this book occasionally fails to do full justice to the works it is based on. This aspect of the book's style also forces me to relegate virtually all references to the bibliographic notes that appear at the end of almost every chapter.

Two decades ago, the quest for psychologically richer models of decision making was pursued by a small group of economic theorists. Since then, the success and wide exposure of behavioral economics have changed the field. As a result, many intellectual traditions have been brought into contact with issues that concern the psychology of decision making. This has often led to intense disagreements about the "right" way of doing research in this area.

Against this background, this book is eclectic, in the sense that its themes have been explored by economists from diverse traditions. Nevertheless, I maintain a uniform, distinctive style. Although the study of I.O. models with boundedly rational consumers may provide an intellectual background for consumer protection policies (analogous to the role that conventional I.O. theory plays in competition policy), this book has a clear theoretical orientation, and discussions of applications or policy implications are highly stylized. Models are constructed for their pedagogical and "story-telling" value. Although the models are deliberately simple, they are pitched at a level of abstraction that immediately suggests generalizations and encourages links with the decision-theoretic literature. The I.O. models in which I embed the non-standard decision models are themselves extremely basic, and I do not complicate them in any way other than in the bounded rationality dimension. Although I invoke relevant psychological research to motivate certain models, there is almost no discussion of experimental psychology, and readers who are interested in this material are referred to excellent available sources.

PREVIEW OF THE MAIN THEMES

The book's repeated use of the most basic I.O. models as a template and the recurrence of certain general questions allow us to draw a few economic "lessons." It may be useful to list them at this stage:

- The presence of boundedly rational consumers may impel firms to adopt price schemes that are more complex than if consumers were rational (e.g., three-part tariffs, add-on pricing, excessive price variation over time and across circumstances).
- Boundedly rational consumers are often vulnerable to exploitative contracts. Competitive forces do not necessarily mitigate the exploitation, and may sometimes exacerbate it. Ordinary competition and consumer protection policies (increasing the number of competitors, introducing simple alternatives or simplifying existing ones) may be ineffective, and even counterproductive when consumers are boundedly rational.
- Consumers' bounded rationality is often a force that generates greater product differentiation, which is "spurious" in the sense that it does not enhance consumer welfare.
- It is not necessarily the case that the more rational the consumers, the more competitive the market outcome. A related point is that rational consumers do not always exert a positive externality on boundedly rational consumers.

The economic message that emerges from these "lessons" is critical of conventional accounts of the market mechanism. In particular, aspects of firms' behavior that are conventionally viewed as "healthy" responses to consumers' preferences and constraints are here viewed as irrelevant for consumer welfare at best and harmful at worst. At the same time, I exhibit traditional caution and skepticism regarding the power of regulatory intervention to curb the frictions that result from consumers' bounded rationality. To put it in coarse terms, consumers' bounded rationality can generate market failure, but it is far from clear whether it is a failure that can be "fixed."

1.1 BIBLIOGRAPHIC NOTES

Thanks to the flourishing of behavioral economics, there is a multitude of excellent sources on the psychology of decision making and its relevance for economics. Kahneman, Slovic & Tversky (1982), Kahneman & Tversky (2000) and Thaler

(1994) are authoritative. Recent popular books include Gilbert (2006), Iyengar (2010), and Thaler & Sunstein (2008).

There are fewer texts on economic models of bounded rationality. Simon (1982) is a collection of Herbert Simon's early writings on the subject. Rubinstein (1998) offers a collection of bounded rationality models in various economic contexts. Ellison (2006) and Armstrong (2008) are surveys of I.O. models that incorporate bounded rationality, the latter being more policy-oriented than the former. Rabin (1998) is an early survey of behavioral economics. DellaVigna (2009) is a survey of empirical work in behavioral economics.

PART ONE
Anticipating Future Preferences

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2 Dynamically Inconsistent Preferences I: Unconstrained Contracting

Choice behavior is rational if it is consistent with maximization of a preference relation $\succsim$ over the set of all potentially feasible alternatives X. If the decision maker's choices sometimes reveal that he strictly prefers x to y and sometimes reveal that he strictly prefers y to x, then he lacks stable preferences and in this sense, his choice behavior is not rational. In many economic situations of interest, such displays of choice inconsistency tend to have some temporal regularity. Here are a few salient examples.

Example 2.1 (Present bias)**.** The following is a classic experiment on intertemporal saving and consumption choices. A dated prize is a pair (M, T), where M is the dollar amount of money and T is the calendar day in which the amount is obtained. In the experiment, people are faced with the problem of choosing between such pairs (M, T). Let $T > 0$. On day 0, experimental subjects tend to prefer $(11, T+1)$ to $(10, T)$. On day T itself, however, they tend to prefer $(10, T)$ to $(11, T+1)$. That is, people are willing to delay income in return for interest when both options are in the future, yet when the choice is between *immediate* and delayed rewards, their willingness to wait is diminished.

Example 2.2 (Temptations)**.** Consider a two-stage decision problem. In the first stage, you choose a restaurant. In the second stage, while at the restaurant, you decide which dish to order. You are on a diet, so from the point of view of the first stage, eating steak is inferior to eating salad. If you could commit yourself to a particular dish ex ante, you would commit to eating salad rather than steak. However, once at the restaurant, facing a menu that contains both salad and steak, you are tempted by the latter and go for it.

Example 2.3 (Dynamic implications of reference-point effects)**.** Imagine that you are considering signing up for a service. Before you sign up, you are willing to pay at most \$20. Suppose that at the time you are offered the service contract, the stated price is \$20. After you sign up for it, the firm raises the price unexpectedly to \$25. You are still able to cancel the deal at no cost. From your point of view before you signed up, you would prefer to cancel. However, having signed up, your point of view has changed; you weigh the pros and cons differently, and decide that you do not want to cancel.

Note that in the first example, the change in preferences is due to the mere passage of time. In the second and third examples, preferences change as a result of a particular contingency that may arise in the course of a dynamic decision problem. In the second example, it is the experience of actually sitting in a restaurant in front of a menu, which makes temptations more tangible. In the third example, it is the experience of signing up for the service, which changes the consumer's perspective.

These examples suggest a large variety of market situations in which changing tastes could be relevant: consumption of credit services, lifestyle activities that are demanding in the short run but rewarding in the long run, addiction, and so on. The prevalence of these situations presents a challenge for I.O. models. On one hand, inconsistent preferences mean that we are unable to account for consumer behavior with the model of maximization according to a preference relation. However, the fact that the dynamic inconsistencies are systematic means that we may be able to find alternative modeling tools to account for them.

In this part of the book, I introduce such a modeling tool and apply it to market situations that fit the following two-period scenario. In period 1, consumers choose whether to sign up for a service provided by a supplier (in monopolistic or competitive environments). In period 2, having signed up for the service, they make their consumption decision. Our objective is to understand the implications of consumers' dynamic inconsistency (and the extent to which they are aware of this predicament) for the firms' pricing decisions and for welfare.

Dynamic inconsistency is the most developed subject in the literature on bounded rationality and I.O.—partly because the methods that economists employ to analyze this phenomenon are quite standard. Many of the central themes of this book will make their first appearance in this part of the book, which is divided into four chapters. In this chapter, we will assume that firms have unlimited ability to formulate and enforce contracts. In Chapter 3, we will relax this assumption. Consumers' ability to anticipate their future preferences will be a major issue in both chapters. This aspect is taken up again and analyzed more thoroughly in Chapters 4 and 5.

2.1 THE MULTI-SELVES MODEL

The most widely practiced modeling tool that economists use to analyze decision making under changing tastes is known as the "multi-selves model." Given an extensive-form decision problem, we model the decision maker as a collection of different players ("selves") having idiosyncratic preferences. We assign a self to a decision node, in the same way that we would assign a player to a node in an extensive-form game. Thus, instead of modeling the decision maker's choices as the outcome of a maximization problem, we model it as the outcome of some game-theoretic solution concept, in a game played among the different selves.

In this chapter, we apply this tool to two-period decision problems of the type captured by Example 2.2. In period 1, the decision maker chooses a menu A—i.e., a non-empty subset of the set of possible consumption decisions Z—from a set of menus that are available in the market. In period 2, he chooses an element from the chosen menu A. This is modeled as a two-stage game, such that self $j \in \{1, 2\}$ moves in period j and has a preference relation $\succ_j$ over Z. The "standard" solution concept for such games is subgame perfect equilibrium. This solution concept means in particular that the decision maker perfectly anticipates the future change in his tastes. Relaxing this perfect-foresight assumption will be central to our discussion later in this chapter.[1]

This model generates a taste for commitment that people often display in situations such as Example 2.2. For instance, let $Z = \{steak, lettuce\}$, $lettuce \succ_1 steak$ and $steak \succ_2 lettuce$. Then, in subgame perfect equilibrium, self 1 will choose the menu $\{lettuce\}$ in period 1, and consequently, self 2 will be forced to eat lettuce in period 2. In equilibrium, self 1 strictly prefers $\{lettuce\}$ to any other menu—in particular, to the grand set Z.

This strict taste for commitment is a general real-life phenomenon. For instance, knowing that you will be tempted to eat an entire packet of nuts, you choose to constrain your future options by buying a small packet. Similarly, knowing that you will be tempted to squander your entire disposable income on fun and games, you lock a substantial portion of your income in an illiquid savings account. The precise structure of the preferences for commitment induced by the multi-selves model is discussed in the decision-theoretic appendix to Part I.

2.1.1 Naivete

When we apply subgame perfect equilibrium as a solution concept for the multi-selves model, we assume that the consumer is fully aware of the future change in his preferences. In other words, he is "sophisticated." In many situations, it seems reasonable to assume that consumers underestimate the likelihood that, or the extent to which, their tastes will change over time or as a result of a change in circumstances.

I refer to such consumers as being "naive." For example, a consumer may overestimate the degree to which his second-period preferences will resemble his current preferences—what is known in the psychology literature as a "projection bias." Alternatively, he may be overconfident of his ability to resist temptations. In the two-period model described above, full naivete means that in period 1, self 1 believes that self 2 has identical preferences. Note that the first-period behavior of a naive consumer is indistinguishable from a standard rational decision maker, because he believes that his choice from the menu will be consistent with his current preferences. We will discuss notions of *partial naivete* in Chapter 4.

Comment: Naive consumers as novices

It should be emphasized that while beliefs over future tastes can be subjected to systematic biases, long-run experience may help consumers correct these biases and reach correct beliefs. Thus, to some extent the distinction between biased and unbiased consumers is a distinction between inexperienced and

[1] When the set of $\succsim_2$-maximal elements in A is not a singleton, most treatments of the multi-selves model assume that the consumer selects the $\succsim_1$-maximal element among them. In contrast, we will assume that such second-period ties are broken in favor of the firm that interacts with the consumer.

experienced consumers. Economists sometimes invoke the latter interpretation as a cause for dismissing the importance of the distinction. I find this attitude misplaced. First, experimental research by psychologists suggests that people are not very good at learning their future preferences, even as they gain experience. Second, even if we assume that people successfully learn from experience, consumers often sign long-term contracts, and by the time these contracts expire, market conditions may have changed considerably such that consumers need to form beliefs afresh. Third, learning often involves constant switching between suppliers, which carries physical and psychological costs. Finally, in industries such as insurance, products are by definition relevant in rare events, such that opportunities to learn are inherently scarce. Therefore, in many market situations of interest, the fraction of "novices" in the consumer population is likely to be non-negligible.

■ 2.2 MONOPOLY PRICING

Let us now turn to our first application of the multi-selves model. Consider a monopolistic firm that interacts with a population of consumers with changing tastes. Let X be a set of actions that a consumer can take in period 2, conditional on having accepted the firm's price scheme in period 1. A price scheme is a function $t : X \to \mathbb{R}$ that specifies a transfer (possibly negative) from the consumer to the firm for every second-period action. If the consumer accepts a price scheme, he is committed to it in period 2—no renegotiation is possible. (We will relax this assumption in Chapter 3.)

The consumer has quasi-linear preferences. However, these preferences change over time. In period 1, the consumer's willingness to pay for any action is given by the function $u : X \to \mathbb{R}$, whereas in period 2, his willingness to pay is given by the function $v : X \to \mathbb{R}$. Thus, when confronted with the price scheme t, in period 1 the consumer evaluates the action x by $u(x) - t(x)$, whereas in period 2 he evaluates the action x by $v(x) - t(x)$. In both periods, the consumer evaluates the outside option of not signing any price scheme with the firm at zero. This option is not available in period 2 if the consumer accepted the firm's price scheme in period 1.[2]

Let $c(x)$ denote the cost that the firm incurs when the consumer takes the action x. Assume that each of the functions $u - c$, $v - c$, and $u - v$ attains a non-negative maximum at a unique point. Note that in this model, all consumers are identical in their first-period and second-period preferences. However, we will allow consumers to vary in their level of sophistication.

Note that the model is silent regarding the timing of the consumer's payment to the firm. One interpretation is that all payments are made at the end of period 2, after the consumer's second-period consumption decision is observed. However, if t involves a lump-sum payment, it is natural to assume that this payment is made already in period 1, at the time the consumer accepts the price scheme. At any rate,

[2]The assumption that the second-period self evaluates the outside option at zero is behaviorally meaningless. It is only relevant for welfare analysis.

consumers in this model do not care about how the payment is divided between periods. In particular, they do not discount delayed payments.

2.2.1 Optimal Price Schemes for Sophisticated Consumers

When designing a price scheme t for a consumer who is known to be sophisticated, we can assume without loss of generality that all the firm needs to fix is a pair (x^*, T^*), where x^* is the action the consumer chooses in period 2 and $T^* = t(x^*)$ is the payment he makes. The reason is that both parties agree that the consumer's second-period preferences will be given by v. Therefore, we can set $t(x)$ to be arbitrarily large for any $x \neq x^*$, without loss of generality. This is essentially a commitment device that forces the consumer to choose x^* in period 2, conditional on having accepted the price scheme in period 1. The feasibility of such a payment scheme follows from a maintained assumption in this chapter, namely that the firm can perfectly monitor the consumer's second-period action.

The firm's problem is then reduced to choosing x and T to maximize $T - c(x)$ subject to $u(x) - T \geq 0$. The solution is:

$$x^* = \arg\max(u - c)$$

$$T^* = u(x^*)$$

This is the same outcome that would be induced by an optimal price scheme if the consumer's preferences were given by u in both periods. The difference is that in our model, the design of $t(x)$ for $x \neq x^*$ needs to ensure that $v(x) - t(x) \leq v(x^*) - T^*$. In contrast, when the consumer's willingness to pay is u in both periods, the design of $t(x)$ for $x \neq x^*$ needs to ensure that $u(x) - t(x) \leq u(x^*) - T^* = 0$. This difference was masked by our convenient assumption that $t(x) = +\infty$ for all $x \neq x^*$.

2.2.2 Optimal Price Schemes for Naive Consumers

When the firm interacts with a naive consumer, the relevant features of the price scheme can be summarized by a 4-tuple, (x^u, T^u, x^v, T^v), where (x^u, T^u) is the action-payment pair in the event that the consumer's willingness to pay is u, and (x^v, T^v) is the action-payment pair in the event that the consumer's willingness to pay is v. The consumer believes in the former event, while the firm believes in the latter event. Our interpretation is that the latter event is the "real" one, while the former is "imaginary" due to the consumer's naivete. Note that in this model, the naive consumer does not infer anything from the price scheme offered to him by the monopolist. We will return to this interpretational issue later in this chapter.

A price scheme offered to a naive consumer is essentially a *bet* over the consumer's second-period consumption decision. The motivation for the bet is that the firm and the consumer hold conflicting prior beliefs regarding the consumer's second-period preferences.

The firm's maximization problem is thus reduced to:

$$\max_{x^u, T^u, x^v, T^v} \quad T^v - c(x^v)$$

subject to

$$v(x^v) - T^v \geq v(x^u) - T^u \qquad \text{(IC}_2\text{V)}$$
$$u(x^u) - T^u \geq u(x^v) - T^v \qquad \text{(IC}_2\text{U)}$$
$$u(x^u) - T^u \geq 0 \qquad \text{(IR)}$$

The first constraint ensures that the consumer indeed chooses x^v when his preferences are given by v, as the firm expects. The second constraint ensures that he chooses x^u when his preferences are given by u, as the consumer expects. The final constraint is a standard participation constraint, determined by the consumer's first-period preferences and what he expects will happen in period 2. To derive the optimal (x^u, T^u, x^v, T^v), we can ignore all other actions, because the firm can set $t(x) = +\infty$ for all $x \neq x^u, x^v$.

Let us solve this maximization problem. First, observe that the constraint (IR) must be binding in optimum—otherwise, the firm can raise both T^u and T^v by the same arbitrarily small $\varepsilon > 0$, and this would raise its profit while satisfying all constraints. Similarly, the constraint (IC_2V) must be binding in optimum—otherwise, the firm can raise T^v by an arbitrarily small $\varepsilon > 0$, and this would raise its profit while satisfying all constraints. The interpretation of a binding (IC_2V) is that the consumer's second-period willingness to pay for the "real" action x^v relative to the "imaginary" action x^u is fully extracted.

We can rewrite the condition that (IR) and (IC_2V) are binding as follows:

$$T^u = u(x^u)$$
$$T^v = v(x^v) + u(x^u) - v(x^u)$$

This reduces the firm's maximization problem into:

$$\max_{x^u, x^v} [v(x^v) - c(x^v)] + [u(x^u) - v(x^u)] \qquad (2.1)$$

subject to the constraint (IC_2U). The solution to the unconstrained problem is

$$x^v = \arg\max(v - c)$$
$$x^u = \arg\max(u - v)$$

When we plug the expressions for T^v and T^u into IC_2U, we see that the constraint holds because $x^u = \arg\max(u - v)$.

Note that if we forced the firm to set $x^v = x^u$, the (IC_2V) and (IC_2U) constraints would hold trivially and the problem would be reduced to the case of a sophisticated consumer. When $\arg\max(u - v) \neq \arg\max(v - c)$, $\max(v - c) + \max(u - v) > \max(u - c)$, hence optimal price schemes for naive consumers are different (and more profitable) from optimal price schemes for sophisticated consumers.

Optimal price schemes for naive consumers have a number of noteworthy features. First, they induce the same action as optimal pricing with a dynamically consistent consumer whose willingness-to-pay function is v. This is in contrast to optimal price schemes for sophisticated consumers, which induce the same action as optimal pricing with a dynamically consistent consumer whose willingness-to-pay function is u. Second, the price scheme offered to the naive consumer is essentially a bet on his second-period preferences, where the bet is motivated by the two parties' different prior beliefs. If the two parties could directly condition on the consumer's second-period preferences, it would be beneficial to both to agree on infinite bets. However, since they can only condition on the consumer's action, and since $u - v$ is bounded by assumption, the stakes of the bet are bounded as well.

Note that in general, there are multiple optimal price schemes, because of the indeterminacy of $t(x)$ for $x \neq x^u, x^v$. However, every optimal price scheme induces the same (x^u, T^u, x^v, T^v).

The following exercise demonstrates that if the firm assigns some probability p to the possibility that the naive consumer is actually right about his second-period preferences, the optimal price scheme changes. In particular, if p is sufficiently high, the optimal price scheme coincides with the scheme aimed at sophisticated consumers.

Exercise 2.1. *Assume that $u - v$ is a one-to-one function. Characterize the monopolist's optimal price scheme for a naive consumer, under the assumption that the monopolist's belief is that the consumer's second-period willingness to pay is u with probability p and v with probability $1 - p$.*

Comment: First-period consumption

Our framework fails to capture situations in which the consumer's preferences change as a result of previous consumption. A salient example is addiction: the consumer's second-period preferences over second-period consumption depend on his consumption quantity in the first period. However, it is easy to extend the model in this direction. Let x_1 and x_2 denote first-period and second-period consumption. Let $t(x_1, x_2)$ denote the consumer's payment when he takes the action x_1 in period 1 and then the action x_2 in period 2. Assume that the consumer's first-period preferences over consumption paths is given by $u(x_1, x_2) - t(x_1, x_2)$. His second-period preference over consumption paths is $v(x_1, x_2) - t(x_1, x_2)$. (Note, however, that by the time the consumer acts in period 2, his first-period consumption x_1 is sunk.) The characterization of optimal price schemes for sophisticated and naive consumers in this extended model is essentially the same as in the basic model.

2.2.3 Screening the Consumer's Type

We will now show that when the population of consumers consists of both naifs and sophisticates and the consumer's "cognitive" type is his private information, the firm can nevertheless screen the consumer's type at no cost.

Consider the menu consisting of the first-best price schemes characterized in the preceding two sub-sections. All consumer types agree that an optimal price scheme designed for a sophisticated consumer provides a perfect commitment device, and all consumer types evaluate it at 0. However, the two types differ in their first-period evaluation of the optimal price schemes designed for a naive consumer. The latter evaluates it at 0, because his participation constraint is binding. In contrast, a sophisticated consumer will avoid it because he believes that it is an exploitative price scheme that yields a negative payoff. The reason is that, as we saw, this price scheme generates a higher profit for the firm than any price scheme that a sophisticated consumer would accept.

Thus, contrary to standard models of monopolistic second-degree price discrimination, in this model the monopolist is perfectly able to screen the consumer's type. This is an example of the new kind of price discrimination effects that arise when consumer heterogeneity lies in cognitive characteristics rather than preference characteristics.

To summarize, any optimal menu will consist of two types of price schemes: (i) perfect commitment devices aimed at sophisticated consumers, inducing an action-payment pair (x^*, T^*) satisfying $x^* = \arg\max(u - c)$ and $T^* = u(x^*)$; and (ii) "betting" price schemes aimed at naive consumers, inducing $x^v = \arg\max(v - c)$ and $x^u = \arg\max(u - v)$, $T^u = u(x^u)$ and $T^v = v(x^v) + u(x^u) - v(x^u)$.

2.3 COMPETITIVE PRICING

Let us turn from monopolistic interaction with dynamically inconsistent consumers to competitive interaction in a market with such consumers. Assume that two identical firms now face the same population of consumers as in the previous section—that is, all consumers have the same first-period and second-period preferences, but they may differ in their ability to predict future preferences. Assume that firms are unable to identify the consumer's type. Therefore, in period 1 the firms play a simultaneous-move game in which each firm offers a menu of price schemes. Subsequently, each consumer chooses a price scheme from the union of the firms' menus; he is committed to this price scheme in period 2.

Given a price scheme t_s that a sophisticated consumer chooses in period 1, let (x_s^*, T_s^*) be the action-payment pair that is realized in period 2. Given a price scheme t_n that a naive consumer chooses in period 1, let (x_n^v, T_n^v) be the action-payment pair that is realized in period 2 in the ("real") event that the consumer's willingness-to-pay function is v; similarly, let (x_n^u, T_n^u) be the action-payment pair that is realized in period 2 in the ("imaginary") event that the consumer's willingness-to-pay function is u. To simplify our exposition, assume that when the consumer is indifferent among several actions in either period, we (as analysts) are free to break the tie at our will.

The following is an example of a symmetric Nash equilibrium in this game.

Proposition 2.1. *There is a symmetric Nash equilibrium in which firms offer the menu of price schemes* $\{t_s, t_n\}$*, such that:*

(i) $x_s^* = \arg\max(u - c)$, $T_s^* = c(x_s^*)$.
(ii) $x_n^v = \arg\max(v - c)$, $T_n^v = c(x_n^v)$, $x_n^u = \arg\max(u - v)$, $T_n^u = c(x_n^v) + v(x_n^u) - v(x_n^v)$.
(iii) $t_s(x) = \infty$ *for every* $x \neq x_s^*$*, and* $t_n(x) = \infty$ *for every* $x \neq x_n^v, x_n^u$
(iv) Sophisticated consumers choose t_s*, naive consumers choose* t_n*, and firms earn zero profits.*

Proof**.** Suppose that both firms offer the menu $\{t_s, t_n\}$. If a consumer chooses t_s, he takes the action x_s^* in period 2, because t_s is a perfect commitment device. If a consumer chooses t_n, he takes the action x_n^v in period 2, because x_n^v maximizes $v(x) - t_n(x)$. Since $t_s(x_s^*) = T_s^* = c(x_s^*)$ and $t_n^v(x_n^v) = T_n^v = c(x_n^v)$, both price schemes generate zero profits for firms.

The pair (x_s^*, T_s^*) maximizes $u(x) - T$ subject to the constraint $T - c(x) \geq 0$. This has two implications. First, sophisticated consumers necessarily prefer t_s to t_n in period 1. Second, no firm can deviate to a price scheme t_s' that sophisticated consumers will prefer to t_s and earn strictly positive profits.

The tuple $(x_n^u, T_n^u, x_n^v, T_n^v)$ maximizes $u(x^u) - T^u$ subject to the constraints (IC_2V), (IC_2U) and $T^v - c(x^v) \geq 0$. This means that the price scheme t_n maximizes the perceived first-period payoff of naive consumers, subject to the constraint that the price scheme generates non-negative profits for firms. Again, this has two implications. First, naive consumers necessarily prefer t_n to t_s in period 1. Second, no firm can deviate to a price scheme t_n' that naive consumers will prefer to t_n and earn strictly positive profits.

It follows that no firm has a profitable deviation, hence $\{t_s, t_n\}$ is a symmetric Nash equilibrium strategy. □

It can be shown that in every Nash equilibrium, firms earn zero profits, every price scheme that sophisticated consumers choose induces the same action-payment pair (x_s^*, T_s^*) as in Proposition 2.1, and every price scheme that naive consumers choose induces the same tuple $(x_n^u, T_n^u, x_n^v, T_n^v)$ as in Proposition 2.1. The reason is that in any equilibrium, price schemes aimed at sophisticated (naive) consumers maximize their real (perceived) first-period payoff subject to the constraint that firms earn non-negative profits.

The perfect screening result continues to hold in the competitive case. That is, the fact that consumers' type is their private information does not change the price schemes they are offered in equilibrium. The reason is that while consumers differ in their first-period evaluation of price schemes, they all make the same consumption decision conditional on accepting a given price scheme. Therefore, when a firm lowers its prices in order to attract consumers of a certain type away from its competitor, it does not mind if another consumer type accepts the price scheme as well.

2.4 WELFARE ANALYSIS

Once we perceive the consumer as a collection of selves with different preferences over the set of consequences, and then proceed to analyze consumption behavior as the outcome of a game played between selves, welfare evaluation becomes problematic. Should we view the different selves as genuinely separate entities, and conduct welfare analysis in the same way that we do in standard, interpersonal games? If so, what would constitute a proper "social" welfare function?

In some cases, even the primitives of such a quasi-social-choice analysis are ill-defined. For instance, consider the extended model in which consumers make a consumption decision in period 1 as well as in period 2 . If we wish to aggregate the two selves' preferences, we need to define their preferences over the same domain. However, while the first-period self can in principle commit to both first- and second-period actions, the second-period self cannot by definition choose first-period consumption. According to the doctrine of revealed preferences, the second-period self's preferences over consumption paths are meaningless.

Economists who apply the multi-selves model often adopt one self's preference relation as the welfare criterion. For instance, when the model describes a consumer's attempt to maintain a diet, then first-period considerations appear to have a higher normative status than second-period considerations, because the former reflect long-run planning, whereas the latter reflect visceral urges. In other cases, none of the selves' preferences are used as welfare criteria, because both reflect visceral urges to some extent. In these cases, a third preference relation that disregards these urges is employed for welfare analysis.

Such practices have been controversial, because they introduce an element of paternalism that economists traditionally try to avoid. There is also an element of subjective judgment in these practices. In my opinion, there is no escape from such judgments when changing tastes seem to be an intrinsic aspect of the economic situation. Furthermore, in many cases our welfare judgments are aided by strong intuitions. For example, in the case of consumption of addictive substances, the pre-addiction preference is intuitively superior to the post-addiction preference as a normative criterion. In other cases—for example, in the case of consumption of art—welfare judgments are more elusive, and we may even privilege the second-period self's attitude as being more instinctive and free of inhibitions. At any rate, I will usually carry out a double welfare analysis, one according to the first-period self's preferences and another according to the second-period self's preferences. However, at times I will make do with the controversial practice of adopting the perspective of the first-period self.

After this long methodological digression, let us turn to the actual welfare analysis. In the monopoly case, under the price scheme chosen by the sophisticated consumer type, the outcome is efficient according to the consumer's *first-period* preferences, because it induces an action that maximizes $u - c$. The price scheme fully extracts the consumer's surplus according to u. As to the price scheme chosen by naive consumer types, the outcome is efficient according to the consumer's *second-period* preferences, because it induces an action that maximizes $v - c$. The amount t^v that the consumer ends up paying is strictly higher than his

u-willingness to pay for x^v. It is also higher than his v-willingness to pay for x^v (strictly as long as $\max(u - v) > 0$). Thus, the price scheme aimed at naive consumers is unambiguously exploitative ex-post, given the action they end up taking. Of course, the reason a naif does not find the price scheme exploitative ex-ante is that he believes he will take the action x^u rather than x^v.

Does competition eliminate the element of exploitation inherent in the price schemes aimed at naive consumer types? Given that $x^v = \arg\max(v - c)$ and $t^v = c(x^v)$, the equilibrium price scheme for naifs is clearly not exploitative according to v. However, when $u(x^v) < c(x^v)$, this price scheme is exploitative according to u. Thus, when second-period willingness to pay is significantly higher than first-period willingness to pay, it is possible that competition will not eliminate exploitation. In contrast, when second-period preferences reflect a lower willingness to pay than first-period preferences, competition eliminates the exploitation of naive consumer types.

2.5 EDUCATING NAIVE CONSUMERS

Recall one of the questions highlighted in Chapter 1: Do market forces impel firms to "educate," or "de-bias," consumers with systematic decision or belief errors? In the present context, suppose that each individual firm could explain the situation (at no cost) to consumers, thereby turning all naive consumers into sophisticates. Would firms have an incentive to do so?

The answer depends on the market structure and the composition of the consumer population. In the case of monopoly, recall that the optimal price scheme aimed at naifs generates a higher profit than the optimal price scheme aimed at sophisticates. Therefore, it is clear that the monopolist would not want to correct consumers' naivete.

When the market is competitive, the answer is subtler. If the consumer population consists entirely of naifs, equilibrium price schemes satisfy $x_n^v = \arg\max(v - c)$ and $T_n^v = c(x_n^v)$. Suppose that $\arg\max(v - c) \neq \arg\max(u - c)$. (Otherwise, the outcome in the "real" state v is the same as it would be if consumers were sophisticated.) Then, a firm could deviate by turning all consumers into sophisticates and simultaneously offering them a perfect commitment device that implements the action $x^* = \arg\max(u - c)$ for the payment $T^* = c(x^*) + \varepsilon$. By definition, $u(x^*) - c(x^*) > u(x_n^v) - c(x_n^v)$. Therefore, if $\varepsilon > 0$ is sufficiently small, all naifs-turned-sophisticates would strictly prefer the new price scheme, which generates a strictly positive profit for the deviating firm. Thus, the firms' ability to de-bias naive consumers destabilizes the competitive market outcome.

However, if there are already sophisticated consumers in the population, we saw that in the competitive model, the equilibrium menu contains a price scheme aimed at sophisticates, which generates—like the price scheme aimed at naifs—zero profits. In this case, no firm has an incentive to educate naive consumers, because it cannot attract them with a price scheme that generates strictly positive profits.

2.6 THE INTERPRETATION OF NAIVETE

How should we interpret naivete in the context of our model? One interpretation is that the consumer is aware of the possibility that his preferences will change, as well as of the fact that the firm has different beliefs, yet he confidently holds his own belief and "agrees to disagree" with the firm. For instance, the consumer may believe that while the firm's belief is correct as far as the average consumer is concerned, his ability to predict his own preferences is "better than average." According to this interpretation, the consumer does not believe that the monopolist is better informed about his own second-period preferences. The two parties play a game with non-common priors, and in period 1 they are equally uninformed of the consumer's second-period preferences.

Under this agreeing-to-disagree interpretation, the two parties would ideally sign an explicit bet on the consumer's second-period preferences. Furthermore, because both parties' payoff functions are linear in money, there is no upper bound on the stakes of the bet they would want to sign. The only thing that prevents them from signing such an explicit bet is the fact that the consumer's preferences are unverifiable, and therefore cannot be contracted upon. The two parties can only condition on the consumer's second-period actions, and therefore the price scheme is only implicitly a bet on the consumer's second-period preferences. The stakes of the bet are constrained by (IC_2V) and (IC_2U).

Another interpretation of naivete is that the consumer is simply unaware of the possibility that his tastes will change. However, the structure of optimal price schemes aimed at naive consumers forces us to question this interpretation. Suppose, for instance, that $X = \mathbb{R}_+$ and that v lies above u at all $x > 0$. That is, second-period willingness to pay is uniformly higher than first-period willingness to pay. Then, $u - v$ attains a maximum at $x = 0$, which means that when the naive consumer accepts the price scheme, he is certain that his second-period action-payment pair will be $(0, 0)$. Even if the consumer is unaware of the possibility that his tastes will change, he should ask himself why the firm would go through the trouble of offering him a price scheme that induces zero consumption. And one conclusion may indeed be that the firm believes that his tastes will change. Therefore, even if the consumer begins the interaction in a state of unawareness, the structure of the optimal price scheme may trigger a chain of reasoning that will bring the possibility of changing tastes to his awareness. In this sense, the interpretation of naivete as unawareness is not robust.

This interpretation becomes even more problematic when firms offer a menu of price schemes in order to screen the consumer's type. Consider the following example. Let $X = \{L, H\}$ and suppose that u, v, and c are given by Figure 2.1, where $c(H) \in (1, 2)$. This example captures a situation in which the consumer initially looks for a low-intensity service and derives no pleasure from higher quality, whereas in period 2, his tastes change and he is willing to pay for enhancing service intensity more than it costs the firm.

Given our characterization of Nash equilibrium under competitive pricing, the equilibrium menu consists of two price schemes, denoted t_s and t_n. The former is aimed at sophisticated consumers, $t_s(L) = 0$ and $t_s(H) > v(H) - v(L) = 2$.

	L	H
u	1	1
v	0	2
c	0	$c(H)$

Figure 2.1. A two–action example

The latter is aimed at naive consumers, $t_n(L) = c(H) + v(L) - v(H) = c(H) - 2$ and $t_n(H) = c(H)$.

This menu has an intriguing property: the payment associated with each outcome is always higher under the price scheme aimed at the sophisticated consumer. In other words, the naive consumer will find that the other price scheme in the menu is *dominated* by the price scheme he chooses. Moreover, he may observe that other consumers actually choose the dominated price scheme, and then he might ask himself: Why would all these other consumers choose a dominated price scheme? Perhaps they hold a different model of the market situation? And could this suggest that the firm shares their model? Again, such a chain of reasoning can ultimately lead the consumer to question his prior beliefs.

Thus, we see that although the naivete assumption per se may be interpreted in a variety of ways, the structure of the price schemes on offer in the market may render some of these interpretations implausible. And this may also be a consideration that firms take into account when devising a menu of price schemes. Although the firm has an incentive to exploit naive consumers, it may try to avoid making the exploitation too transparent, in order to prevent naive consumers from reconsidering their beliefs. Capturing this consideration in a formal model is a difficult challenge.

Exercise 2.2. *Let* $X = \{L, H\}$.

(i) Show that in an optimal monopoly menu of the form $\{t_s, t_n\}$*, no price scheme strictly dominates another. Show, however, that it is possible that* t_n *weakly dominates* t_s*; that is,* $t_s(x) \geq t_n(x)$ *for all* $x \in X$*, with at least one strict inequality.*

(ii) Find conditions on u and v that will imply the existence of a competitive menu in which one price scheme strictly dominates another.

2.7 TWO APPLICATIONS

Given the pervasiveness of dynamically inconsistent preferences in real life, it may be useful to investigate the relevance of the theoretical considerations highlighted in this chapter. Although this book is primarily theoretical and does not delve deeply into applications, it will be instrumental to examine several real-life industries in which we should expect dynamically inconsistent preferences to have a first-order effect, and see if our conclusions regarding firms' response to this feature of consumer psychology are consistent with what we observe in reality.

Credit card "teaser" rates

Credit card companies often try to attract consumers with low "teaser rates" on small-size loans and other "welcome benefits," and then switch to post-introductory interest rates far above marginal cost. Ausubel (1991, 1999) presents evidence suggesting the presence of consumers who respond to the teaser rates in a way that appears harmful in retrospect. Comparing two loans with the same post-introductory interest rate, Ausubel (1999) shows that the acceptance rate of an offer of a low introductory rate for six months was higher than that of a higher introductory rate for nine months. However, the data on actual account usage suggested that most of the consumers who chose the former price scheme would have been better off with the latter. This apparently systematic error suggests that these consumers may have had biased beliefs about their future preferences at the time they chose between price schemes.

This interpretation of the data can be demonstrated with the two-action example given by Figure 2.1. Suppose that the action L (H) stands for a low (high) level of borrowing after the introductory phase. In period 1, when the consumer contemplates acquiring a credit card, he would like to utilize it at a low level, whereas in period 2, after the credit card has been issued, the consumer is tempted to use the credit card for additional borrowing. The payment $t(L)$ is interpreted as the introductory rate, while the post-introductory rate broadly corresponds to $t(H) - t(L)$, as this is the consumer's payment to the credit card company if he chooses to request extended credit. As we saw, this pattern of dynamic inconsistency gives rise to a menu of price schemes that (apparently) dominate one another, even in a competitive market environment. The credit card aimed at sophisticated consumers will be characterized by a moderate introductory rate and an exceedingly high post-introductory rate. The latter serves as a deterrent against excessive borrowing. By comparison, the credit card aimed at naifs will be characterized by a low "teaser" introductory rate and a post-introductory rate that is high, but not so high as to deter the consumer from extra borrowing. To an outside observer, the latter offer appears to dominate the former. Naive consumers choose the seemingly dominant price scheme, expecting to borrow little, yet they end up borrowing excessively and would be better off under the price scheme with the moderate introductory rate.

Negative option offers

A prevalent marketing device is to offer a product, typically accompanied by some immediate benefit such as a free trial period or a gift voucher, and require the consumer to *explicitly* reject it later on in order to avoid additional charges. This practice is called a "negative option offer."

Negative option offers can be accounted for by our framework, by assuming that the consumer's second-period action set contains a specific action interpreted as cancellation of the price scheme. This means in particular a zero level of consumption. Consumers' attitudes to the cancellation option may change over time. For instance, the consumer may exhibit a "reference point effect" that causes his evaluation of a price scheme to depend on whether he has already accepted it. Alternatively, cancellation may be time consuming, and the consumer's present and future selves may disagree on the cost of this time spent.

In these environments, a negative option offer is a means of exploiting naive consumers. The consumer accepts the price scheme because of its short-term benefits, thinking he will cancel it in period 2. However, when the time comes to exercise the cancellation option, he may find it too time-consuming to cancel, or he may attach greater weight to the reasons against cancellation. On the other hand, sophisticated consumers will reject the price scheme because they anticipate that they will not want to cancel it in period 2.

Comment: Rationalizations

In some of the examples above, one could think of a conventional explanation for the observed phenomenon that does not appeal to dynamic inconsistency and naivete. For example, benefits such as teaser rates can be viewed as an attempt to poach customers in a competitive environment. However, it is harder to construct a standard model that will replicate the totality of the effects that our model generates. The question of whether predictions of models based on consumers' bounded rationality can be replicated by standard models with rational consumers will accompany us throughout this book. In fact, it will be the subject of the final chapter.

2.8 OTHER TOPICS

We conclude this chapter with a brief discussion of two extensions of the models presented here.

2.8.1 The (β, δ) Model

The presentation in this chapter has generally refrained from making substantive assumptions about the utility functions u and v. Economists often prefer to work with particular parametric forms of utility functions, as this facilitates analysis and makes comparative statics exercises more straightforward. A functional form known as the (β, δ) model, which aims to capture the phenomenon of present bias discussed at the beginning of this chapter, has been tremendously popular in the last decade, especially in long-horizon models of consumption and saving (most famously, in Laibson (1997)). This model modifies conventional discounting by changing the discount factor in period 0 from β to $\beta\delta$.

The following is one way of fitting the (β, δ) model into our (u, v) framework. Suppose that while consumers choose a pricing scheme in period 1 and make their consumption decision in period 2, the consequences of their decisions are only realized in a third (and final) period. Each consumption decision $x \in X$ carries a cost $e(x)$ for the consumer as well as a benefit $b(x)$. However, the cost and the benefit are realized at different times.

For instance, in situations like choosing a diet or a health club, the cost is realized in period 2 and the benefit is realized in period 3. In these cases, we can write $u(x) = -\beta\delta e(x) + \beta\delta^2 b(x)$ and $v(x) = -e(x) + \beta\delta b(x)$. The parameter $\delta \in [0, 1]$ is a standard discount factor, whereas $\beta \in [0, 1]$ is a novel discounting parameter that captures the notion of "present bias." The first-period self discounts the future cost and benefit uniformly, according to a constant discount factor. In contrast, the second-period self discounts the third-period benefit more heavily, because the second-period cost is incurred in the present, from this self's point of view. Similarly, in situations in which the cost associated with the consumer's consumption decision is incurred in period 3 and the benefit is incurred in period 2 , the translation into our (u, v) model is $u(x) = \beta\delta b(x) - \beta\delta^2 e(x)$ and $v(x) = b(x) - \beta\delta e(x)$.

However, some applications of the (β, δ) model lie outside the scope of the (u, v) framework. The latter assumes that the timing of payments is irrelevant. In some contexts, one could argue that what changes over time is the consumer's evaluation of monetary payoffs. This means that our quasi-linearity assumption fails, and when firms design their pricing schemes, the timing of payments will be a non-trivial aspect of the design.

One advantage of the (β, δ) functional form is that it offers a very convenient tool for analyzing situations that involve a long time horizon. The following exercise abandons our two-period framework in favor of a long-horizon setting, and shows that when consumers have (β, δ) preferences and exhibit naivete, subsidizing consumer search can be very effective.

Exercise 2.3. *A consumer is offered a product labeled g or b (with equal probabilities) in every period $t = 1, \ldots, T$. The process is terminated as soon as the consumer accepts an offer. (He is forced to accept any offer made in the final period T.) The consumer has (β, δ) preferences with $\delta = 1$ and $\beta < 1$. Specifically, from his period t point of view, the payoff from accepting $x \in \{g, b\}$ in period $t' > t$ is βx, whereas the payoff from immediate acceptance is x. The consumer is naive—that is, in every period he believes that in future periods he will have $\beta = 1$. Assume $g > \beta(g + 1) > b > \beta g$. Suppose that a social planner can commit in advance to a subsidy scheme that pays 1 to the consumer if and only if he searches for at least $T^* < T$ periods. Show that as T tends to infinity, the social planner can choose T^* such that the probability that the consumer will choose g approaches one, while the probability that the subsidy will actually be paid approaches zero.*

2.8.2 Preference Heterogeneity

Throughout this chapter, we assumed that the population of consumers is homogeneous in their first- and second-period preferences. The scope for price discrimination derived entirely from the consumers' diverse degree of sophistication. In this sub-section, we show that when first- and second-period preferences are positively correlated, a monopolist may be able to fully discriminate among perfectly sophisticated consumers, using a menu of price schemes that contains a perfect commitment device (namely, a contract that implements a particular action regardless of the consumer's second-period preferences) as well as a flexible price scheme (namely, a contract induces consumption decisions that may vary with the consumer's second-period preferences). Although this is qualitatively the same feature that characterized the menu of price schemes designed to screen consumers' degree of naivete, the interpretation of such a menu will be quite different when all consumers are sophisticated and their heterogeneity lies in their first- and second-period preferences.

Consider the following example. In period 2, the consumer chooses between two actions, denoted L and H. There are two consumer types, denoted 1 and 2, both sophisticated. Their first- and second-period preferences are given by Figure 2.2, where $2 > u_H > u_L > 1$ and $v_H - v_L > 2$. That is, type 2 has a higher first-period willingness to pay for both actions than type 1. At the same time, he finds H more tempting relative to L in the second period than type 1. The interpretation is that consumers with a bigger appetite for the product also tend to be more tempted by the high-intensity action. The monopolist has zero costs.

	L	H
u	1	1
v	0	2

(a) Type 1 preferences

	L	H
u	u_L	u_H
v	v_L	v_H

(b) Type 2 preferences

Figure 2.2. A two actions–two types example

Let us now construct a menu of price schemes that fully extracts consumer surplus even when the monopolist cannot identify the consumer's type. One price scheme is a perfect commitment device t_2, with $t_2(H) = u_H$ and $t_2(L) = +\infty$. This price scheme is aimed at type 2. The other price scheme t_1 is flexible, with $t_1(L) = 1$ and $t_1(H) \in (2, v_H - v_L + 1)$.

Type 1 prefers the flexible price scheme t_1. The reason is that, although t_1 is not a perfect commitment device, it effectively commits him to L, as his temptation to switch from L to H in the second period is not too large. The price scheme extracts his entire first-period willingness to pay for L, while the perfect commitment device t_2 charges more than type 1 is willing to pay for the prescribed action H. On his part, type 2 prefers the perfect commitment device t_2. The flexible price scheme t_1 does not serve as an effective commitment device for type 2, because $t(H)$ is too small to prevent him from switching to H in the second period. And by our assumptions regarding type 2's first- and second-period preferences, he anticipates that he will pay more for H than his first-period willingness to pay for this action.

Note that in this example, type 2's first-period preferences have the kind of structure that creates screening problems in standard models of second-degree price discrimination. However, the positive correlation between first- and second-period preferences allows the monopolist to extract consumer's entire first-period surplus. This is achieved with a menu of prices schemes that effectively introduces an "irrelevant alternative" (the action H in the flexible price scheme) that acts as a deterring temptation for the high type while posing no such problem for the low type. Thus, although the monopolist could offer perfect commitment devices to both consumers if he knew their types, he prefers to offer a flexible price scheme when he does not know the consumer's type, in order to screen the consumer's intensity of second-period temptation.

We can see that menus that contain commitment devices as well as flexible price schemes do not have a single explanation. In this chapter we proposed two explanations: an attempt to screen a consumer's degree of naivete and an attempt to screen his intensity of second-period temptations. In order to distinguish between such conflicting interpretations, one needs to impose more structure on the consumers' preferences, or simply observe their second-period behavior after accepting a price scheme.

2.9 SUMMARY

In this chapter we examined optimal non-linear pricing when consumers have dynamically inconsistent preferences. We made a key distinction between sophisticated and naive consumers—that is, those who can and those who cannot anticipate the future change in their preferences. The main lessons can be summarized as follows:

- Price schemes aimed at sophisticated consumers act as commitment devices, whereas price schemes aimed at naifs are flexible contracts that essentially act as bets over the consumer's consumption decision. Real-life pricing strategies such as credit card "teaser" rates and negative option offers can be viewed as examples of such flexible schemes.
- The menu of price schemes is independent of whether the consumer's "cognitive" type (sophisticated or naive) is known to the firms.
- The optimal flexible price scheme aimed at naive consumers is unambiguously exploitative under monopoly. Competition need not eliminate exploitation of naifs (however, this does depend on the welfare criterion).
- The menu of price schemes may contain one price scheme that appears to dominate another scheme. The former is, however, chosen by sophisticates, whereas the latter is chosen by naifs.

2.10 BIBLIOGRAPHIC NOTES

For surveys on intertemporal choice patterns and their interpretation, see Frederick, Loewenstein & O'Donoghue (2002) and Mariotti & Mazini (2009) . The multi-selves approach was originally introduced by Strotz (1956) and developed further by Peleg & Yaari (1973). Strotz (1956) also introduced the distinction between sophistication and naivete. O'Donoghue & Rabin (1999) pioneered the study of incentive design for dynamically inconsistent agents.

DellaVigna & Malmendier (2004) were the first to analyze price setting in the presence of dynamically inconsistent preferences. The market model presented in this chapter is based on Eliaz & Spiegler (2006), who introduced unconstrained contracting and the notion of a mechanism that discriminates among consumers according to their degree of sophistication. Ausubel (1991, 1999) studies relevant empirical aspects of the credit card industry. For a theoretical study of credit card markets with dynamically inconsistent preferences, see Heidhues & Kőszegi (2010). Bar-Gill (2003) contributes a legal perspective to this application. Kőszegi (2005) discusses some features of market competition in the presence of dynamically inconsistent consumers. Ashraf, Karlan & Yin (2006) provide field-experimental evidence for consumer demand for commitment devices in the context of saving accounts.

The (β, δ) model was originally introduced by Phelps & Pollak (1968) in the context of intergenerational consumption-savings models, and in the last decade it was extensively applied to situations in which decision makers have dynamically inconsistent preferences that involve a present bias. DellaVigna & Malmendier (2004) and Heidhues & Kőszegi (2010), for instance, rely on this parametrization. For a critical assessment of the (β, δ) model, see Rubinstein (2003). Exercise 2.3 is based on Rubinstein (2006). The discussion of correlation between consumers' first- and second-period preferences borrows from Esteban & Miyagawa (2006*a*).

3 Dynamically Inconsistent Preferences II: Constrained Contracting

In the previous chapter, we introduced the multi-selves model of dynamically inconsistent consumers, and used it to study the pricing behavior of firms who face such consumers. We assumed "unconstrained contracting," in the sense that firms faced no restrictions on the manner in which they condition payments on consumers' second-period actions, or on their ability to enforce these payments. For sophisticated consumers, unlimited contracting is important because it ensures the feasibility of perfect commitment devices. For naive consumers, unconstrained contracting is important because it allows the firm to devise ornate "betting" price schemes.

In this chapter we relax the unconstrained contracting assumption in various ways. First, we restrict attention to two-part tariffs, a classic example of a "simple" price scheme. Next, we will see what happens when firms are unable to commit not to renegotiate the terms of the contract or prevent consumers from dealing with other suppliers in the second period. Finally, we will explore the role of consumers' own self-control in situations where perfect commitment devices are infeasible. For this purpose, we will need to extend the multi-selves model, which itself is unable to capture self-control. In all three cases, we will see that the distinction between constrained and unconstrained contracting is important in the context of interactions between profit-maximizing firms and consumers with changing tastes, and that it leads to new insights.

3.1 TWO-PART TARIFFS

In this section we assume that $X = [0, 1]$ and restrict attention to pricing schemes that take the form of two-part tariffs: $t(x) = 0$ if $x = 0$ and $t(x) = A + px$ if $x > 0$. The restriction to two-part tariffs is made for several reasons. First, they are commonly observed. Second, two-part tariffs are known to maximize monopoly profits when consumers have dynamically consistent, quasi-linear preferences that satisfy diminishing marginal willingness to pay. Therefore, it is interesting to see if this property carries over to the case of dynamically inconsistent preferences. Finally, while optimal two-part tariffs in standard models have the property that the per-unit price p is equal to marginal cost, we will see that when the consumer is dynamically inconsistent, the model can account for real-life departures from marginal-cost pricing.

Throughout this section, we assume that $u(0) = v(0) = 0$, and that both u and v are continuously twice differentiable, with $u', v' > 0$ and $u'', v'' < 0$. We also assume that the firm faces a constant marginal cost κ, where $\kappa > u'(1), v'(1)$ and $\kappa < u'(0), v'(0)$. We will examine two cases: (i) $u'(x) < v'(x)$ for all x; and (ii) $u'(x) > v'(x)$ for all x. The first case fits situations in which it is tempting to consume more in period 2 (e.g., when x denotes the amount of food consumption and the consumer is on a diet). The second case fits situations in which consuming the product is rewarding in the long run but taxing in the short run (e.g., when x denotes the intensity of physical exercise).

3.1.1 Departure from Marginal-Cost Pricing

Let us begin our analysis of two-part tariffs with the finding that in the presence of dynamically inconsistent consumers, two-part tariffs depart from marginal-cost pricing. For brevity, I restrict attention to the case of monopoly pricing.

Proposition 3.1. *When the consumer is known to be fully sophisticated, a monopolist restricted to two-part tariffs will choose* (A^*, p^*) *such that in case (i),* $p^* > \kappa$ *and in case (ii),* $p^* < \kappa$.

Proof. Suppose that the consumer is known to be sophisticated. Then, he knows that given a two-part tariff (A, p), he will choose in period 2 the level of consumption x^v defined by $v'(x^v) = p$. The maximal A that he is therefore willing to accept in period 1 satisfies $A = u(x^v) - px^v$. Thus, the firm effectively chooses p to maximize $\pi_s(p) = px^v(p) + [u(x^v(p)) - px^v(p)] - \kappa x^v(p) = u(x^v(p)) - \kappa x^v(p)$, yielding $u'(x^v) = \kappa$. Since $v'(x^v) = p$, it follows that in case (i), where $v' > u'$, we obtain $p^* > \kappa$, and in case (ii), where $v' < u'$, we obtain $p^* < \kappa$. □

Thus, optimal two-part tariffs aimed at sophisticates depart from marginal-cost pricing. When the consumer's marginal willingness to pay increases in period 2, the price per unit is above marginal cost. And when the consumer's marginal willingness to pay diminishes in period 2, the price per unit is below marginal cost. Note that in case (i), the optimal value of A may be *negative*. This may capture immediate benefits that consumers receive when they sign up for the service. In case (ii), the optimal value of A is necessarily positive.

Let us turn to the monopolist's optimal two-part tariff when the consumer is naive. In the previous chapter, we observed that the monopolist's maximal profit is higher when the consumer is naive than when he is sophisticated. This conclusion extends to the restricted domain of two-part tariffs. To see why, consider any two-part tariff (A, p) that leaves a sophisticated consumer indifferent between accepting and rejecting. Then, $A = u(x^v) - px^v$. If a naive consumer is offered the same two-part tariff (A, p), he will evaluate it at

$$u(x^u) - A - px^u = u(x^u) + p \cdot [x^v(p) - x^u(p)] - u(x^v) \tag{3.1}$$

Recall that $u'(x^u) = v'(x^v) = p$. The R.H.S of equation (3.1) is strictly positive, due to an elementary "envelope" argument that makes use of the strict concavity of u. It follows that the naive consumer strictly prefers to accept (A, p). If the firm raised A by an arbitrarily small amount, he would still accept the contract and generate a strictly higher profit.

It turns out that our qualitative conclusions regarding the departure from marginal-cost pricing hold also when the consumer is known to be fully naive.

Proposition 3.2. *When the consumer is known to be fully naive, a monopolist restricted to two-part tariffs will choose* (A^*, p^*) *such that in case (i),* $p^* > \kappa$ *and in case (ii),* $p^* < \kappa$.

Proof**.** Suppose that the consumer is known to be fully naive—that is, he expects his second-period preferences to be given by u. The firm expects the consumer to choose $x^v(p) = \arg\max[v(x) - px]$. In period 1, the consumer expects that he will choose $x^u(p) = \arg\max[u(x) - px]$. The maximal A that he is therefore willing to accept in period 1 satisfies $A = u(x^u(p)) - px^u(p)$. It follows that the firm effectively chooses p to maximize $u(x^u(p)) + p \cdot [x^v(p) - x^u(p)] - \kappa x^v(p)$. For expositional simplicity, we will analyze the problem as if $x^u(p)$ and $x^v(p)$ are always strictly between 0 and 1. We can rewrite the firm's maximization problem as follows:

$$\max_p \{[u(x^u(p)) - px^u(p)] + (p - \kappa)x^v(p)\}$$

subject to

$$x^u(p) = \arg\max[u(x) - px]$$
$$x^v(p) = \arg\max[v(x) - px]$$

The derivative of the objective function with respect to p can be written as follows:

$$x^v(p) - x^u(p) + \frac{dx^v(p)}{dp} \cdot (p - \kappa) + \frac{dx^u(p)}{dp} \cdot \left[\frac{du}{dx}(x^u(p)) - p\right]$$

Let us evaluate this derivative in case (i). Since $v' > u'$, $x^v(p) - x^u(p) > 0$. By the concavity of v, $dx^v(p)/dp$ is negative. Finally, since $x^u(p)$ is defined by the implicit function $u'(x) = p$, the term inside the square brackets is zero. It follows that the overall derivative of the objective function with respect to p is positive for every $p \leq \kappa$. Hence, it is optimal for the firm to set $p^* > \kappa$. In case (ii), a similar calculation implies that it is optimal for the firm to set $p^* < \kappa$. □

Therefore, both when the consumer is sophisticated and when he is naive, optimal two-part tariffs induce the same qualitative departure from marginal-cost pricing. When the consumer is sophisticated, this is meant as a commitment device, whereas when he is naive, it is a device for exploiting his naivety.

An application: Health clubs

An empirical study by DellaVigna & Malmendier (2006) examined consumer behavior in the context of health clubs. Working out regularly at a health club is an activity that is costly in the short run and rewarding in the long run. Therefore, consumers with a taste for immediate gratification will display dynamically inconsistent preferences. From an ex-ante point of view, they would like to commit to go to the gym on a regular basis. However, when the time comes to carry out this commitment, consumers may turn lazy and procrastinate. Health clubs generally offer two types of payment schemes: membership and non-membership. Roughly speaking, this is a menu consisting of two kinds of two-part tariffs. A member makes a large up-front payment for a long period, and subsequently faces a small price per visit. A non-member simply pays a relatively high price per visit.

DellaVigna & Malmendier (2006) gathered data on consumers' contract choice and subsequent attendance. They showed that many of the consumers who chose the membership option attended the gym so irregularly that they would have been better off if they had stuck to the non-membership option. That is, the effective price per visit implicit in their total payment and actual attendance is higher than the explicit price per visit under the non-membership plan. A natural interpretation of this effect is that some of the consumers who opt for the membership plan are naive. They falsely believe that their strong initial willingness to exercise will persist in the future, and therefore find the membership plan more attractive than they would if they correctly anticipated their future preferences.

3.1.2 Welfare Analysis

Is the restriction to two-part tariffs an effective constraint for the monopolist? In the proof of Proposition 3.1, we saw that when consumers are sophisticated, the optimal two-part tariff induces an action x^v that maximizes $u - c$ and extracts a total revenue equal to $u(x^v)$. Therefore, the optimal two-part tariff mimics the optimal unconstrained price scheme. (Of course, this conclusion is relevant only for the environments we restricted attention to, in terms of assumptions on u and v, in which two-part tariffs are of interest in the first place.)

In contrast, the restriction to two-part tariffs carries a loss of generality in case of a naive consumer. Recall that the optimal unrestricted price scheme induces an action x^v that is efficient according to v. Given our assumptions on v, this means $v'(x^v) = \kappa$. But the optimal two-part tariff induces an action x^v that maximizes $v(x) - p^*x$, and since $p^* \neq \kappa$, the outcome is inefficient according to v. Hence, when facing a naive consumer, the monopolist could do better by offering a price schedule that is more complex than a two-part tariff.

The intuition for this difference is as follows. When dealing with a naive consumer, the monopolist is interested in two actions: the action x^v that it expects the consumer to take and the action x^u that the consumer expects to take. The restriction to two-part tariffs implies that x^v and x^u are chosen such that $v'(x^v) = u'(x^u)$. In contrast, the optimal unrestricted price scheme induces $x^v = \arg\max[v(x) - \kappa x]$ and $x^u = \arg\max[u(x) - v(x)]$. There is no particular reason for these solutions to coincide. In contrast, when dealing with a sophisticated consumer, the monopolist is interested in a single outcome, x^v. This is also the outcome the consumer expects. Therefore, by setting $p = v'[\arg\max(u - c)]$, the monopolist can implement the optimal commitment device.

Let us now turn to the implications of the restriction to two-part tariffs for the welfare of naive consumers, focusing on the *first-period self*. Under the optimal two-part tariff (A, p), the first-period self's net payoff is $u(x^v_{tpt}) - px^v_{tpt} - [u(x^v_{tpt}) - px^u_{tpt}]$, where x^v_{tpt} and x^u_{tpt} are given by $v'(x^v_{tpt}) = u'(x^u_{tpt}) = p$. As we saw in our discussion of equation (3.1), this net payoff is negative. It follows that the restriction to two-part tariffs does not eliminate the exploitation of naive consumers (from the point of view of their first-period preferences).

Does the restriction to two-part tariffs at least curb exploitation, relative to the optimal unrestricted price scheme? Let us examine case (i), in which $v' > u'$. (In this case, $\max(u - v) = 0$.) Under the optimal unrestricted price schemes derived in Chapter 2, the first-period self's net payoff is $u(x^v_o) - v(x^v_o)$, where x^v_o is the "real" action induced by the optimal price scheme, given by $v'(x) = \kappa$. Recall that under case (i), $p > \kappa$. Therefore, $x^v_o > x^v_{tpt}$. By our assumptions on u and v, $u(x^v_o) - v(x^v_o) < u(x^v_{tpt}) - v(x^v_{tpt}) < 0$. Since x^u_{tpt} and x^v_{tpt} are given by $u'(x^u_{tpt}) = v'(x^v_{tpt}) = p$, we have $u(x^u_{tpt}) + p \cdot (x^v_{tpt} - x^u_{tpt}) < v(x^v_{tpt})$. Combining these inequalities, we obtain that the first-period self's net payoff is strictly lower under the optimal unrestricted price scheme relative to the optimal two-part tariff. In other words, the restriction to two-part tariffs mitigates the exploitation of naive consumers, even if it does not eliminate it altogether.

Exercise 3.1. *Show that the same welfare analysis holds under case* (*ii*)*, i.e.,* $v' < u'$.

This finding is a first illustration in this book of the following principle: optimal pricing schemes for boundedly rational consumers often have a complexity that would not be called for if the consumer were rational. Therefore, forcing the firm to forgo this complexity in favor of a simple pricing scheme may mitigate the exploitation of boundedly rational consumers.

■ 3.2 DESTABILIZATION OF COMMITMENT DEVICES: RENEGOTIATION AND SPOT MARKET COMPETITION

In this sub-section I introduce the possibility that consumers who accepted a price scheme in period 1 will face new alternatives in period 2. This may take several forms. First, the same firm that offered the first-period price scheme may try to renegotiate its terms. Second, other firms, which did not sign contracts with the consumers in period 1, may enter the market and offer "spot" contracts that compete with the price schemes the consumers accepted in period 1. As these are both forms of second-period competition, we will assume that the first-period market itself is competitive as well, as analyzed in Section 2.3 of Chapter 2.

Assume that there are no constraints on the price scheme that firms can offer in period 1. Let us first consider the case of sophisticated consumers.

Renegotiation

Second-period renegotiation can destabilize the commitment device chosen by sophisticated consumers in period 1. Recall that a competitive price scheme for sophisticates, denoted t_s, induces an action x_s that maximizes $u - c$ and a payment $t_s(x_s) = c(x_s)$, and thus generates zero profits for the firms. Typically, x_s does not maximize $v - c$, and so a firm can propose replacing the original price scheme with a new price scheme t_r, which induces an action x_r^v that maximizes $v - c$ and a payment $t_r(x_r^v)$ such that $t_r(x_r^v) > c(x_r^v)$ and $v(x_r^v) - t_r(x_r^v) > v(x_s) - t_s(x_s)$. Both the firm and the sophisticated consumer will prefer the new price scheme in period 2.

The only renegotiation-proof commitment devices that survive first-period competition are those that induce the action $x^* = \arg\max(v - c)$ and the payment $T^* = c(x^*)$. Such commitment devices enforce an outcome that is efficient according to the second-period self's preferences, but inefficient according to the first-period self's preferences. Thus, the possibility of renegotiation distorts the interaction with sophisticated consumers.

The possibility of renegotiating commitment devices is interesting from a legal point of view. According to a powerful tradition in legal theory, when two parties agree to renegotiate an existing price scheme, the court should not void the newly signed contract. However, the rationale behind this libertarian stance is typically that the renegotiation was a result of new information, whereas in our case the renegotiation is a result of *predictably* changing tastes. The ability to enforce commitment contracts thus calls for a legal doctrine that acknowledges the distinction between these different motivations for renegotiation.

Spot markets

Second-period competition from "spot" contracts is another force that can destabilize commitment contracts signed in period 1. Suppose that the commitment device sophisticates choose in period 1 induces a low level of consumption, and that second-period preferences exhibit *increased* willingness to pay for additional units. Then, the consumer can obtain the extra units from another supplier without breaking the first-period contract. (For instance, think of a consumer who signs in period 1 a loan contract with his bank, in the hope that the stringent terms of the contract will deter him from over-borrowing in period 2. Spot market competition in period 2 can come from other lenders, such as credit card companies and loan sharks.) The only first-period equilibrium commitment devices that are immune to such spot market competition are the renegotiation-proof price schemes described above.

Renegotiation and spot market competition with naive consumers

Renegotiation and spot market competition are irrelevant for the price schemes aimed at naive consumers, because these price schemes maximize $v - c$ in the first place—that is, they are efficient according to the second-period self's preferences. As the first-period market is assumed to be competitive, second-period competition will have no effect on naifs.

■ 3.3 SELF-CONTROL

When consumers do not have a perfect commitment device at their disposal, an alternative channel for overcoming temptations is the exertion of self-control. A consumer with self-control acknowledges the temptation yet manages to incorporate his first-period preferences into his second-period consumption decision. In this section I present an extension of the multi-selves model that incorporates self-control in a particular manner, and analyze some of its implications for firms' pricing behavior.

Let Z be a set of consumption alternatives. These will typically involve quantities consumed and payments, but for the moment we will not commit to a particular specification of what a consumption alternative is. As before, we will analyze consumption problems in which the consumer chooses a *menu*, namely a non-empty subset $A \subseteq Z$ in period 1, and selects an alternative z from A in period 2. The cornerstone of the model of self-control is the idea that consumer preferences are defined not over Z, but over the extended set of *decision paths* $\{(A, z) \mid \varnothing \subset A \subseteq Z, z \in A\}$.

Preferences over decision paths are an appealing way to model self-control. When a consumer who exercises self-control evaluates his consumption decision, he takes into account not only the action he eventually took, but also feasible actions he was tempted to take yet chose not to by exerting self-control. The fact that he has to exert self-control in order to avoid tempting actions means that he is not indifferent to their inclusion in the menu.

In the model we present, it is costly to exert self-control in period 2, at the time the consumer selects an alternative from a menu he chose in period 1. The greater the temptations in the menu, the higher the cost of self-control. This cost may be purely mental, or it can be the cost of a physical activity that is not explicitly modeled as an element in Z (e.g., participation in a support group). The following class of utility functions that represent preferences over decision paths captures this consideration:

$$U(A, z) = u^*(z) - s(A, z)$$

where $u^*(z)$ is the "commitment utility" attached to z (namely, the payoff that z yields when it is the only available element) and $s(A, z)$ is the *cost* of the self-control needed to choose z from A.

These preferences over decision paths represented by U are assumed to be *dynamically consistent*: they are stable and do not change over time. This assumption eliminates the possibility of *naivete*, because a consumer who chooses according to a stable preference relation over decision paths behaves as if he knows in period 1 that his second-period choice will maximize $U(A, \cdot)$.

We will employ a particular specification of the cost of self-control function:

$$s(A, z) = \max_{y \in A}(v^*(y)) - v^*(z)$$

where $v^*(z)$ is the "temptation utility" attached to z. That is, the self-control cost of choosing z from A is simply the difference in temptation utility between z and the maximally tempting element in A. This leads to an alternative way of writing the utility function over decision paths:

$$U(A, z) = [u^*(z) + v^*(z)] - \max_{y \in A} v^*(y)$$

Note that since the last term is fixed given a menu A, the consumer's second-period choices are consistent with maximizing the utility function $u^* + v^*$ defined over Z. Thus, second-period choices reflect a rational compromise between "commitment preferences" and "temptation preferences."

Comment: Rationalization via consequence space expansion

If we only observed how the consumer chooses from menus in period 2 and how he ranks singleton menus in period 1, we would find this observed behavior consistent with a multi-selves model in which the first-period self has preferences over Z represented by u^* and the second-period self has preferences over Z represented by $u^* + v^*$. Our model of self-control preferences eliminates this dynamic inconsistency because the preferences are defined over the extended domain of decision paths. The first-period ranking of singleton menus is viewed as a preference over decision paths of the form $(\{x\}, x)$, while choices from a menu A in period 2 are interpreted as preferences over decision paths of the form (A, x). The inconsistency between these choices is "rationalized away" by treating them as outcomes of two different choice problems. This is a general methodological principle: we can always "rationalize" choice inconsistencies by redefining the set of consequences, such that the description of a consequence includes not only the consumption decision but also the menu from which it was selected.

Let us compare self-control preferences to the case of a *sophisticated* multi-selves consumer. Consider the first-period preferences over menus that are induced by the two models. The self-control model allows the consumer to prefer commitment (i.e., a menu with fewer options) even if his second-period choice behavior does not change as a result of the commitment. For example, he may prefer the menu $\{z\}$ to the menu $\{z, y\}$ even if he chooses z from the larger menu. This will happen if $v^*(y) > v^*(z)$ and $u^*(z) + v^*(z) > u^*(y) + v^*(y)$. The reason the smaller menu is preferred is that this saves the cost of self-control, which is not required at the singleton menu but is required at the larger menu. In contrast, a multi-selves consumer prefers a smaller menu only if this affects his second-period choice.

To illustrate this comparison, let $Z = \{b, i\}$—where b and i stand for broccoli and ice cream. Suppose that $u^*(b) > u^*(i)$ and $v^*(i) > v^*(b)$. This captures a situation in which the consumer is on a diet, and he finds broccoli better than ice cream on dietary grounds, yet he finds ice cream more tempting than broccoli. It is easy to check that if $u^*(b) - u^*(i) > v^*(i) - v^*(b)$, then

$$U(\{b\}, b) > U(\{b, i\}, b) > U(\{b, i\}, i) = U(\{i\}, i)$$

such that the consumer chooses b from the menu $\{b, i\}$. The interpretation of these rankings is as follows. When $u^*(b) - u^*(i) > v^*(i) - v^*(b)$, the temptation of ice cream is insufficiently strong to override the diet consideration, and therefore the consumer chooses broccoli. However, resisting the temptation to eat ice cream requires costly self-control, and for this reason the consumer would like to commit to broccoli rather than having a free choice that demands self-control. That is, self-control is a costly substitute for a perfect commitment device.

Conversely, if $u^*(b) - u^*(i) < v^*(i) - v^*(b)$, then:

$$U(\{b\}, b) > U(\{b, i\}, i) = U(\{i\}, i) > U(\{b, i\}, b)$$

such that the consumer chooses i from the menu $\{b, i\}$. The interpretation of these rankings is as follows. When $u^*(b) - u^*(i) < v^*(i) - v^*(b)$, ice cream is too tempting for the diet to be sustainable. Note that the consumer is indifferent between breaking his diet (the decision path $(\{b, i\}, i)$) and not being on a diet in the first place (the decision path $(\{i\}, i)$). That is, if the consumer totally succumbs to the temptation, he does not incur any self-control cost. There are no other motives such as regret that make him worse off when he attempts a diet and then breaks it, relative to not attempting a diet at all.

In general, if $\left|v^*(z) - v^*(y)\right| > \left|u^*(z) - u^*(y)\right|$ for all distinct $z, y \in Z$, then in period 2 the consumer chooses according to v^*. That is, temptation is always overwhelming. Consumer behavior in this case is indistinguishable from the multi-selves model in which u^* and v^* represent the first- and second-period selves' preferences. Thus, the multi-selves model is a special case of our model of self-control preferences.

Exercise 3.2. *Suggest interesting specifications of the function s, for which the consumer's second-period choice function cannot be rationalized by a utility function over Z.*

In the decision-theoretic appendix to Part I, I conduct an elaborate comparison between the model of self-control preferences and the multi-selves model.

3.3.1 Implications for Monopoly Pricing

To see the implications of self-control for pricing behavior, relative to the multi-selves model, let us consider a special setting, in which a monopolist offers at no cost one unit of a product. The monopolist is restricted to a simple pricing scheme: it can only charge an entry fee from the consumer in period 1. An alternative is a pair (q, T), where $q \in \{0, 1\}$ is the consumer's consumption decision and T is the payment he ends up making. Assume that both u^* and v^* are quasi-linear. Thus, there exist u and v such that $u^*(q, T) = u(q) - T$ and $v^*(q, T) = v(q) - T$. Assume $u(0) = v(0) = 0$ and $u(1) > 0 > v(1)$.

This example fits the health club example discussed in Section 3.1.1, where $q = 1$ (0) represents doing (not doing) physical exercise. Ex ante, the consumer wants to do physical exercise and he would be willing to pay a positive amount for a mechanism that would force him to work out. However, in period 2 he is tempted not to do any physical exercise. Correspondingly, his willingness to pay for physical exercise goes down, below zero ("you would have to pay me to lift those weights . . . ").

If the firm could force the consumer to choose $q = 1$, it could charge $T = u(1)$ for such a commitment device. Now suppose that the monopolist cannot monitor whether the consumer chooses $q = 0$ or $q = 1$ after paying the entry fee T. The fee is sunk in period 2. Having paid the fee, the consumer will choose $q = 1$ in period 2 as long as $u(1) + v(1) \geq u(0) + v(0) = 0$. However, when this condition holds, the consumer agrees to pay the entry fee in period 1 only if

$$
\begin{aligned}
&u(1) - T + v(1) - T - \max\{v(1) - T, v(0) - T\} \\
&= u(1) + v(1) - \max[v(1), v(0)] - T \\
&= u(1) + v(1) - T \geq 0
\end{aligned}
$$

Since $v(1) < 0$, the consumer's first-period willingness to pay for entry is strictly below $u(1)$.

To sum up, when the monopolist cannot monitor the consumer's second-period action, it is possible that the consumer will enter and consume the product for a fee that is lower than the price of a perfect commitment device. This effect is impossible under the multi-selves model. If $q = 0$ is a more tempting alternative in period 2 than $q = 1$, then in the absence of a commitment device, a sophisticated multi-selves consumer will simply refuse to pay any entry fee, anticipating that he will choose $q = 0$ in period 2 (while a naive consumer's willingness to pay for entry is independent of whether a commitment device is available).

Exercise 3.3. *Following the quasi-linear specification of this section, let $X = [0, 1]$ and assume that u is continuously twice differentiable, with $u(0) = 0$, $u' > 0$ and $u'' < 0$. Define $v \equiv \delta u$, where $\delta > 1$, and $c(x) = \kappa x$, where $\kappa > 0$. Show that any optimal two-part tariff for a monopolist sets the price per unit above k.*

3.3.2 Do Self-Control Costs Hamper Competition?

I conclude this section with an exercise that applies the self-control model to a simple competitive market setting, in order to demonstrate that the presence of temptations can obstruct competition. In general, when firms face a population of consumers with diverse tastes, they can act competitively by offering a large variety at attractive prices. However, consumers with self-control problems dislike variety, and this attitude weakens competitive forces. As we shall see, the analysis will not depend on whether consumers follow the multi-selves model or the more general self-control model. Nevertheless, the exercise will be presented in terms of the latter because it is a generalization of the former.

Consider the following situation. Two food stores can provide two products, broccoli and ice cream (denoted b and i) at zero cost. The stores compete for a measure one of consumers, by choosing simultaneously which products to offer and at what price. Consumers move after the firms make their decisions. They go through a two-stage decision process. First, given the menu of product-price pairs offered by each store, they choose which store to enter. (They also have the option of entering none of the stores.) Having entered a store, they choose which food item to buy at the store. (They also have the option of not buying anything at the store.)

Consumers are divided into two groups of equal size and different self-control preferences. Let us first specify the two groups' commitment utilities. Denote the prices of broccoli and ice cream by p_b and p_i. Commitment utilities are given by Figure 3.1. As to the consumers' temptation utility v^*, consider first a benchmark situation in which v^* is a constant function: $v^*(b, p_b) - v^*(i, p_i) = 0$ for all consumers. In other words, there are no self-control costs, hence consumer behavior collapses to a standard model of rational choice with heterogeneous preferences. For all consumers, the u^* and v^* values associated with not entering any store are 0.

	Group I	Group II
$u^*\left(b, p_b\right)$	$1 - p_b$	$-p_b$
$u^*\left(i, p_i\right)$	$-p_i$	$1 - p_i$

Figure 3.1. Group I and II commitment utilities

In this environment, there is a unique Nash equilibrium, in which both stores offer both products at zero price. This is a perfectly competitive outcome. The logic behind this result is standard: as long as one store offers any product at a strictly positive price (or fails to offer it altogether), the other firm has an incentive to offer the same product at a slightly lower price and win the entire group of consumers who prefer this product.

Now consider an alternative scenario, which differs from the benchmark scenario only in the v^*-values of the first group of consumers, so that $v^*(i, p_i) - v^*(b, p_b) = \delta > 1$. The interpretation is that consumers whose commitment utility ranks broccoli above ice cream are simply on a healthy diet and they find ice cream more tempting than broccoli. Note that temptation utility ignores product prices in this example.

We will now show that in this alternative environment, there is a Nash equilibrium in which store 1 offers only b at a price $p_b = 1$, while store 2 offers only i at a price $p_i = 1$. First, note that under this strategy profile, group I is indifferent between shopping at store 1 and not shopping at all, while group II is indifferent between shopping at store 2 and not shopping at all. Since the two consumer groups are equal in size, no store has an incentive to *change* the product they offer. Therefore, the only potentially profitable deviation is to *add* a product to the store's menu. To see why such a deviation is unprofitable, suppose that store 1 adds i to its menu, at a price p_i. The store may also change the price it charges for broccoli p_b. In order for the deviation to be profitable, p_b cannot be negative. In order to attract consumers away from store 2, store 1 must set $p_i < 1$. However, this makes store 1 unattractive for group-I consumers: if they enter the store and buy broccoli, their payoff is $1 - p_b - \delta < 0$; and if they enter the store and buy ice cream, their payoff is $-p_i < 0$. Thus, group-I consumers prefer not to enter store 1 as a result of this deviation. It follows that the deviation cannot be profitable, because store 1's payoff is $\frac{1}{2}p_i$, whereas prior to the deviation it was $\frac{1}{2}$. The argument for store 2 is similar and therefore omitted.

Thus, in this Nash equilibrium, stores specialize in different products and offer them at monopolistic prices. The reason for this equilibrium differentiation is that in order to engage in profitable price competition, firms must offer both products. However, such variety is anathema to consumers on a diet, and therefore competitive forces are weakened. If each store could costlessly set up a second branch, so that each branch would specialize in a different product, then the ordinary competitive force would be restored because firms would be able to offer broccoli to dieting consumers without requiring them to exert any self-control cost. (Alternatively, if there are at least four firms, there is an equilibrium in which at least two firms offer b alone, at least two other firms offer i alone, and each firm prices its product competitively.)

As mentioned above, the equilibrium specialization result does not depend on whether $\left|u^*(i, p_i) - u^*(b, p_b)\right| \geq \left|v^*(i, p_i) - v^*(b, p_b)\right|$—that is, on whether consumers obey the model of self-control preferences or the more special multi-selves model (in which temptations are always overwhelming). Finding general conditions for the two models to yield identical equilibrium predictions in market models is an interesting open problem.

3.4 SUMMARY

In this chapter we continued our investigation of market models in which consumer preferences over consumption alternatives change over time. We examined considerations that arise when firms' ability to formulate and enforce contracts is limited. Constrained contracting limits both the commitment devices that firms can offer to sophisticates and the exploitative "betting" price schemes that they can offer to naifs.

- When firms are restricted to two-part tariffs, we should expect to observe deviations from marginal-cost pricing. The direction of the deviation depends on whether second-period preferences display greater or lower willingness to pay than first-period preferences, but not on whether consumers are naive or sophisticated.
- Restriction to two-part tariffs compromises profit maximization when dynamically inconsistent consumers are naive, but not when they are sophisticated. Moreover, such a restriction makes naive consumers better off.
- Renegotiation between the firm and the consumer in period 2, or market competition with other suppliers in period 2, may restrict the commitment devices that firms can offer to sophisticated consumers. However, they do not upset the “betting” price schemes signed with naifs.
- When firms have limited ability to offer perfect commitment devices, self-control can serve as a substitute mechanism. However, when self-control is costly for consumers, this has implications for the firms’ pricing decisions:
    - Monopolistic firms restricted to flat rates (“admission fees”) may be impelled to lower their prices.
    - Competitive firms that offer menus of products and are able to price each product differently may be impelled to specialize in different products and raise their prices.

3.5 BIBLIOGRAPHIC NOTES

The characterization of optimal two-part tariffs in the presence of dynamically inconsistent consumers is based on DellaVigna & Malmendier (2004). The discussion of renegotiation and spot market competition builds on Kőszegi (2005). The self-control model is due to Gul & Pesendorfer (2001). The two market applications of the self-control model are free variations on Esteban & Miyagawa (2006*a*,*b*).

Gottlieb (2008) analyzes a competitive model that combines spot-market competition with a restriction to two-part tariffs. This combination creates an additional instability. When the equilibrium per-unit price in the model without spot-market competition is above the firms’ marginal cost, this gives a spot-market provider an incentive to undercut this per-unit price in period 2, thus destabilizing the equilibrium price scheme even when consumers are naive.

For a legal-studies perspective into the problem of renegotiating contracts that are specifically designed to serve as commitment devices, see Jolls (1997) and Bar-Gill & Ben-Shahar (2004).

4 Dynamically Inconsistent Preferences III: Partial Naivete

So far, our analysis of the two-period multi-selves model assumed that consumers are either fully sophisticated or fully naive. Interesting new effects emerge when we introduce intermediate, "partially naive" types whose level of sophistication lies between these two extremes. Throughout this chapter, we are back in the unconstrained-contracting world of Chapter 2: firms face no restriction on the price schemes they offer in period 1; and in period 2, consumers are bound by the price scheme they accepted in period 1. Renegotiation, spot market transactions, or self-control are ruled out. The only departure from the multi-selves model as defined in Chapter 2 is that consumers are allowed to hold richer first-period beliefs regarding their second-period preferences.

In general, a partially naive consumer holds a belief that is somehow "in between" full sophistication and full naivete. Since the set of beliefs over second-period preferences that can be viewed as "partially naive" is huge and rather intractable, I examine two different restrictions on the set of "partially naive" beliefs, one at a time. Each approach captures a different aspect of partial naivete, and the two approaches complement each other in terms of the insights they generate.

4.1 MAGNITUDE NAIVETE

According to our first definition, a consumer is partially naive if he believes that his second-period preferences will be given—*with probability one*—by a payoff function $w = \alpha u + (1 - \alpha)v$, where $\alpha \in (0, 1)$. Thus, α is the consumer's type; a higher α represents greater naivete. This notion of partial naivete reflects the idea that while consumers may realize that their future preferences will change, they underestimate the magnitude of this change. A partially naive consumer is "almost fully sophisticated" if his value of α is close to zero, such that w is close to v. However, note that even such a consumer assigns zero probability to the true state v.

4.1.1 Monopoly Pricing

What is the optimal price scheme for a monopolist against a partially naive consumer? As in the case of full naivete, without loss of generality we can reduce the price scheme to a 4-tuple, (x^w, T^w, x^v, T^v), where (x^w, T^w) is the action-payment pair in the ("imaginary") event that the consumer's second-period willingness-to-pay payoff function is w, and (x^v, T^v) is the action-payment pair in the ("real") event that the consumer's second-period willingness-to-pay function is v. The firm's maximization problem is:

$$\max_{x^w, T^w, x^v, T^v} T^v - c(x^v)$$

subject to

$$v(x^v) - T^v \geq v(x^w) - T^w \tag{IC$_2$V}$$

$$w(x^w) - T^w \geq w(x^v) - T^v \tag{IC$_2$W}$$

$$u(x^w) - T^w \geq 0 \tag{IR}$$

As in Chapter 2, assume that $u - c$, $v - c$ and $u - v$ attain non-negative maxima and have unique maximizers.

The solution of this maximization problem turns out to be *exactly the same as for the fully naive consumer*! The reason is as follows. The objective function and the constraints (IC$_2$V) and (IR) are the same as in the full-naivete case. Therefore, we saw that the two constraints are binding in optimum. The problem is thus simplified into maximizing

$$[v(x^v) - c(x^v)] + [u(x^w) - v(x^w)]$$

subject to the constraint

$$w(x^w) - v(x^w) \geq w(x^v) - v(x^v)$$

The unconstrained solution to the simplified problem is $x^v = \arg\max(v - c)$, $x^w = \arg\max(u - v)$. Thanks to the definition of w as a *convex combination* of u and v, $\arg\max(w - v) = \arg\max(u - v)$, and therefore the unconstrained solution satisfies the constraint.

It follows that the "real" and "imaginary" actions are the same as in the full-naivete case, and therefore the payments associated with these actions are the same, too. Thus, there is a discontinuity at $\alpha = 0$. Optimal price schemes pool together all consumers with $\alpha > 0$. Even if a consumer is "almost fully" sophisticated according to our notion of partial magnitude naivete, he is treated by the monopolist as if he were fully naive. Since all consumer types evaluate the optimal price scheme for a fully sophisticated consumer at 0, it follows that the perfect screening result we obtained in Chapter 2 persists for an arbitrary population consisting of diversely (magnitude) naive consumers.

Note that although all the types $\alpha > 0$ choose the same price scheme and end up taking the same action, fully and partially naive consumers perceive this price scheme differently. Let us revisit the two-action example of Figure 2.1 introduced in Chapter 2, and reproduced in Figure 4.1. Assume $c(H) \in (1, 2)$.

	L	H
u	1	1
v	0	2
c	0	$c(H)$

Figure 4.1. A two-action example

The monopolist's optimal price scheme for $\alpha > 0$, denoted t_n, is given by $t_n(L) = 1$ and $t_n(H) = 1 + v(H) - v(L) = 3$.

In this example, the action L maximizes $u - c$ and the action H maximizes $v - c$. The fully naive consumer believes that he has no self-control problem and therefore does not even seek a commitment device. On the other hand, partially naive consumers know that they have a self-control problem, and would therefore demand a commitment device that deters them from choosing H. However, because these consumers underestimate the extent to which their second-period preferences depart from their first-period preferences, they falsely regard t_n as such a commitment device. Their choice can be described as a "futile attempt at self-control."

The following exercise extends our analysis to more complicated "partially naive" beliefs, namely mixtures over payoff functions w as defined in this section.

Exercise 4.1. *Let $X = [0, 1]$. Assume that $u - v$ is concave, and suppose that the consumer's first-period belief assigns positive probability to an arbitrary (finite) number of utility functions that are all convex combinations of u and v. Show that the characterization of optimal price schemes is the same as in the full-naivete case.*

4.1.2 Are More Sophisticated Consumers Always Better Off?

The notion of magnitude naivete enables us to highlight in a simple manner a non-monotonicity in the susceptibility of partially naive consumers to market exploitation. Specifically, given a *fixed* menu of available price schemes, making a consumer more (but not fully) sophisticated does not necessarily make him better off in terms of his first-period self's preferences.

To illustrate this effect, consider the two-action example of Figure 4.1. Suppose that consumers can choose between two price schemes, t_1 and t_2, presented in Figure 4.2.

	L	H
t_1	0	0.9
t_2	0.1	1.2

Figure 4.2. Two price schemes

A fully naive consumer believes that his second-period willingness to pay will be $u(L) = u(H) = 1$, hence he believes that he will choose L in period 2 under both price schemes. Since $t_1(L) < t_2(L)$, the consumer will select the price scheme t_1.

Now consider a partially naive consumer with $\alpha = \frac{1}{2}$. This consumer believes that in period 2, his willingness to pay will be given by $w = \frac{1}{2}u + \frac{1}{2}v$, such that $w(L) = 0.5$ and $w(H) = 1.5$. Therefore, he believes that if he selects t_1, he will choose H in period 2, while if he selects t_2, he will choose L in period 2. In other words, he believes that only t_2 is an effective commitment device that deters him from choosing H. This consumer will prefer t_2 to t_1, because given the way he expects to behave in period 2, the first-period self's payoff from t_1 is $u(H) - t_1(H) = 1 - 0.9 = 0.1$, while the first-period self's payoff from t_2 is $u(L) - t_2(L) = 1 - 0.1 = 0.9$.

In reality, however, neither price scheme is an effective commitment device, because the consumer's actual second-period willingness to pay in period 2 is given by v. He will choose H under both price schemes. The fully naive consumer's first-period payoff is $u(H) - t_1(H) = 1 - 0.9 = 0.1$, while the partially naive consumer's first-period payoff is $u(H) - t_2(H) = 1 - 1.2 = -0.2$. Thus, the partially naive consumer is worse off than the fully naive consumer, in terms of his first-period self's preferences.

The intuition is that while the fully naive consumer does not believe he has any self-control problems, the partially naive consumer acknowledges the problem but underestimates its magnitude. Therefore, he prefers the price scheme t_2 even though it charges more for L than t_1, because he perceives that only t_2 provides a sufficiently strong incentive—in the form of a surcharge for H—to choose L. However, since he plays down the extent of his self-control problem, the partially naive consumer ends up paying both the higher price for L and the surcharge for switching to H. This is a very simple illustration of an idea we will revisit later in the book: *consumers' market performance is not always monotonic with respect to their degree of rationality or market sophistication.*

4.2 FREQUENCY NAIVETE

Let us now turn to an alternative definition of a partially naive consumer as one who assigns probability $\theta \in (0, 1)$ to u and probability $1 - \theta$ to v. In particular, an "almost sophisticated" consumer is one with θ close to zero. Unlike the previous definition, here a partially naive consumer does not underestimate the extent to which his preferences change in period 2. Instead, he underestimates the likelihood of v, possibly because he does not appreciate the full set of circumstances in which his preferences change. For instance, think of a person on a diet who is unable to foresee all the cues and temptations that can cause him to lose his willpower and break his diet.

As we shall see, this notion of partial naivete eliminates the discontinuity that characterized magnitude naivete. As a result, it also attenuates the perfect screening result, and gives rise to an interesting observation regarding the effect of competition on the exploitation of naive consumers.

4.2.1 First-Best Monopoly Pricing

Let us first analyze monopoly pricing when the firm knows it faces a partially naive consumer of type θ. The monopolist's maximization problem is:

$$\max_{x^u, T^u, x^v, T^v} T^v - c(x^v)$$

subject to the by-now-familiar second-period incentive compatibility constraints

$$v(x^v) - T^v \geq v(x^u) - T^u \tag{4.1}$$

$$u(x^u) - T^u \geq u(x^v) - T^v \tag{4.2}$$

and the following modified participation constraint:

$$\theta[u(x^u) - T^u] + (1 - \theta)[u(x^v) - T^v] \geq 0 \tag{4.3}$$

The solution to this maximization problem is:

$$\begin{aligned} x^v &\in \arg\max(\theta v + (1 - \theta)u - c) \\ x^u &= \arg\max(u - v) \\ T^v &= \theta v(x^v) + (1 - \theta)u(x^v) + \theta[u(x^u) - v(x^u)] \\ T^u &= \theta u(x^u) + (1 - \theta)v(x^u) + (1 - \theta)[u(x^v) - v(x^v)] \end{aligned} \tag{4.4}$$

This solution displays several differences from the case of magnitude naivete. The consumer ends up taking an action that maximizes a *weighted* surplus. The more sophisticated (naive) the consumer, the closer the "real" action is to maximizing $u - c$ ($v - c$). As θ approaches zero, the action-payment pair (x^v, T^v) converges to the action-payment pair induced by the sophisticated consumer's price scheme. Thus, the solution is continuous with respect to the consumer's degree of naivete, unlike the case of magnitude naivete. (However, the structure of the price scheme differs from the perfect commitment device aimed at fully sophisticated consumers, because it retains the distinction between the consumer's "real" and "imaginary" actions, regardless of how close θ is to zero. In this sense, there is a discontinuity at $\theta = 0$.)

There are two commonalities with the case of magnitude naivete. First, the constraint (4.1) is binding in optimum. That is, the consumer's second-period surplus in state v is fully extracted, relative to the "imaginary" action x^u. Second, the imaginary action is the same as in the case of magnitude naivete.

The next exercise shows that the monopolist's profit is monotonically increasing with the consumer's degree of naivete.

Exercise 4.2. *Show that the monopolist's maximal profit is increasing with θ.*

4.2.2 Second-Best Monopoly Pricing

Let us turn to the case in which the degree of the consumer's frequency naivete is his private information. The monopolist's problem is to find a menu of price schemes that maximizes its expected profit, subject to the constraint that each consumer type chooses in period 1 what he perceives to be the best price scheme in the menu (including the outside option of not accepting any price scheme), and given that the consumer's second-period consumption decision satisfies the second-period incentive compatibility constraints with respect to the price scheme he chooses in period 1.

Unlike the case of magnitude naivete, perfect screening breaks down when consumers display partial frequency naivete. To see why, consider two partially naive consumer types θ, ϕ, where $\phi > \theta$. Suppose that type θ selects a price scheme t that the firm would find optimal if it knew his type. In particular, the participation constraint for this consumer type is binding. That is:

$$\theta[u(x^u) - T^u] + (1 - \theta)[u(x^v) - T^v] = 0$$

From the solution to the monopolist's first-best problem, given by the system (4.4), it is easy to see that $u(x^u) - T^u > 0$ and $u(x^v) - T^v < 0$. But this means that a consumer of type $\phi > \theta$ will evaluate the price scheme strictly above zero. Thus, since the firm cannot directly observe the consumer's type, it cannot fully extract both types' surplus. This is a standard "single crossing" argument.

Complete analysis of second-best monopolistic pricing is beyond the scope of this book. We will make do with demonstrating an important feature of this model (as well as its competitive counterpart), using the two-action example given by Figure 4.1, which runs through this chapter. Suppose that the monopolist believes that the consumer's unobserved type θ is distributed uniformly over [0, 1].

Recall that if the monopolist knew that it faces a fully sophisticated consumer (type $\theta = 0$), it would be optimal for the monopolist to offer the consumer a perfect commitment device t_s, satisfying $t_s(L) = u(L) = 1$ and $t_s(H) = +\infty$. This price scheme commits the consumer to the action L and fully extracts his first-period willingness to pay for this action. Therefore, all consumer types evaluate it at zero. In other words, the fully sophisticated consumer does not exert an informational externality on more naive consumers, and so it is optimal for the monopolist to include t_s in its menu. This price scheme generates a profit of 1 to the firm.

The more difficult problem is to characterize the remaining price schemes in an optimal menu. It turns out that in the two-action example, there is an optimal menu that contains at most *one* price scheme, denoted t_n, apart from t_s. This property, which I will not prove here, is due to the assumption that X consists of only two actions. At any rate, it reduces the monopolist's problem into the following:

$$\max_{x_n^u, T_n^u, x_n^v, T_n^v} \mu \cdot 1 + (1 - \mu) \cdot [T_n^v - c(x_n^v)]$$

subject to the constraint that $(x_n^u, T_n^u, x_n^v, T_n^v)$ satisfies (4.1) and (4.2), as well as the constraint that μ is the fraction of consumer types who prefer t_s to t_n. We are interested in characterizing this fraction, and how it changes when competition is introduced.

Proposition 4.1. *There is an optimal solution to the monopolist's second-best maximization problem, in which the fraction of consumers who choose t_s is*

$$\mu = \frac{1}{2} + \frac{1}{4}c(H)$$

***Proof*.** Assume that the firm offers the menu $\{t_s, t_n\}$, where t_s is the perfect commitment device described above. Our objective is to show that this menu generates a strictly higher expected profit than the singleton $\{t_s\}$. We prove the result in a series of steps.

Step 1: $x_n^u = L$, $x_n^v = H$.

Proof. If $x_n^u = x_n^v$, then we have a price scheme that effectively commits the consumer to a particular action. But we have already seen that t_s is the monopolist's optimal commitment device. Therefore, it must be the case that $x_n^u \neq x_n^v$. By the constraint (4.2), $u(x_n^u) - T_n^u \geq u(x_n^v) - T_n^v$. But since $u(L) = u(H) = 1$, this means that $T_n^v \geq T_n^u$. Constraint (4.1) then implies that $x_n^u = L$ and $x_n^v = H$.

Step 2: There is a type θ^* such that all types $\theta < \theta^*$ ($\theta > \theta^*$) choose t_s (t_n). If $\theta^* \in (0, 1)$, then type θ^* is indifferent between the two price schemes.

Proof. If a consumer of type θ chooses t_n over t_s, then

$$\theta[u(x_n^u) - T_n^u] + (1-\theta)[u(x_n^v) - T_n^v] \geq 0 \tag{4.5}$$

Since $u(L) = u(H) = 1$, it follows that $\theta T_n^u + (1-\theta)T_n^v \leq 1$. By assumption, t_n generates a higher profit than t_s. Therefore, $T_n^v - c(H) > 1$. It follows that $T_n^v > 1$ and $T_n^u < 1$. If type θ satisfies inequality (4.5), then every type $\phi > \theta$ satisfies this inequality strictly. This immediately implies the existence of a cutoff type θ^*. Because of the continuity of the L.H.S of (4.5), it is satisfied with equality if θ^* is interior. In the sequel, I will take it for granted that θ^* is indeed interior, for the sake of brevity.

Recall that one feature of the first-best price scheme aimed at type θ is that the constraint (4.1) is binding. Our task now is to show that this property carries over to the second-best problem.

Step 3: The price scheme t_n satisfies the constraint (4.1) *with equality.*

Proof. Assume that a solution to the monopolist's problem has the property that

$$v(H) - t_n(H) > v(L) - t_n(L)$$

Suppose that the monopolist deviates from t_n into the price scheme t'_n defined as follows:

$$t'_n(H) = t_n(H) + \varepsilon$$
$$t'_n(L) = t_n(L) - \frac{1-\theta^*}{\theta^*} \cdot \varepsilon$$

where $\varepsilon > 0$ is arbitrarily small. By the previous step, type θ^* evaluates t_n at zero and every type $\theta > \theta^*$ evaluates t_n strictly above zero. Therefore, by construction, type θ^* evaluates t'_n at zero and every type $\theta > \theta^*$ evaluates t'_n strictly above zero. This means that all types $\theta > \theta^*$ choose t'_n over t_s. Moreover, if some types $\theta < \theta^*$ switched from t_s to t'_n, the deviation would still be profitable because t'_n generates a higher profit than t_n, which in turn (as we saw above) is more profitable than t_s.

Step 4: $t_n(H) = 1 + 2\theta^*$.

Proof. Because t_n satisfies the constraint (4.1), and because (4.3) is satisfied with equality for type θ^*, $t_n(L)$ and $t_n(H)$ are given by two equations, which yield the above solution.

By the assumption that θ is uniformly distributed over [0, 1], the fraction of consumers who choose t_s is $\mu = \theta^*$. Having characterized $(x_n^u, T_n^u, x_n^v, T_n^v)$ as a function of the cutoff type θ^*, we can reduce the monopolist's problem into:

$$\max_{\mu} \; \mu \cdot 1 + (1-\mu) \cdot [1 + 2\mu - c(H)]$$

which leads to the solution

$$\mu = \frac{1}{2} + \frac{1}{4} c(H)$$

And since by assumption $c(H)$ is strictly between 1 and 2, we can see that $\mu \in \left(\frac{3}{4}, 1\right)$. □

Thus, a large fraction of partially naive consumers end up selecting the perfect commitment device which is optimal in the first-best sense only for fully sophisticated consumers. This pooling of partially naive consumers with the fully sophisticated consumers has a close analogue in conventional models of second-degree price discrimination, where incentive compatibility considerations impel the monopolist to exclude consumers with low willingness to pay. Likewise, in the present model, inability to identify the consumer's exact degree of frequency naivete impels the monopolist to refrain from offering mildly naive consumers the kind of exploitative "betting" price schemes it would ideally want to offer them. Instead, the monopolist prefers to pool them with the fully sophisticated consumers.

Exercise 4.3. *Assume that X, u, v, c are given by Figure 4.1. Suppose that there are only two consumer types in the population, θ_L and θ_H, where $\theta_L < \theta_H$. The fraction of each type in the population is $\frac{1}{2}$. Find a menu of price schemes that maximizes the firm's profits, under the constraint that the menu contains at most one price scheme which is not a perfect commitment device.*

4.2.3 Does Competition Curb Exploitation?

Let us now turn to the competitive counterpart of the model of second-best monopoly pricing analyzed in the previous section. In period 1, two profit-maximizing firms play a game in which each firm simultaneously offers a menu of price schemes. Subsequently, each consumer chooses a price scheme from the union of these two menus that he perceives to be the best. In period 2, the consumer makes his consumption decision, and he is bound by the terms of the price scheme he accepted in period 1. The specification of X, u, v, c is as in the two-action example given by Figure 4.1.

Proposition 4.2. *There is a symmetric Nash equilibrium in the game, in which firms offer the menu $\{t_s, t_n\}$ satisfying:*

(i) $t_s(L) = 0$, $t_s(H) = +\infty$; *consumers who choose t_s take the action L in period 2.*

(ii) $t_n(L) = c(H) - 2$, $t_n(H) = c(H)$; *consumers who choose t_n take the action L in state u and the action H in state v.*

Both firms earn zero profits in this equilibrium.

I leave the proof as an exercise for the reader. The crucial property of the menu $\{t_s, t_n\}$ is that any other price scheme that some consumer type will rank over *both* t_s and t_n generates negative profits. Note that under the price scheme t_n, the first-period self's payoff given the action taken in state v is $u(H) - t_s(H) = 1 - c(H) < 0$. In other words, the "betting" price scheme that survives competition is exploitative in terms of the first-period self's welfare.

The price scheme t_s is a commitment device preferred by relatively sophisticated consumers, while the price scheme t_n is a "betting" price scheme preferred by relatively naive consumers. The cutoff type θ^{**} who is indifferent between the two price schemes is given by the following equation:

$$u(L) - t_s(L) = \theta^{**} \cdot [u(L) - t_n(L)] + (1 - \theta^{**}) \cdot [u(H) - t_n(H)]$$

yielding the solution

$$\theta^{**} = \frac{1}{2}c(H)$$

Let us now compare the value of this cutoff to the analogous cutoff θ^* in the monopoly case. By the assumption that $c(H) \in (1, 2)$, $\theta^{**} < \theta^*$. That is, a greater fraction of the consumer population chooses a "betting" price scheme over the commitment device when the market is competitive.

This finding resonates with the familiar insight that competition mitigates the exclusion of low types in standard models of second-degree price discrimination. However, this effect has a novel interpretation in the present context. In the competitive model, the types above θ^{**} select a price scheme that is exploitative from their first-period self's point of view. The magnitude of exploitation is of course lower than in the monopoly case, because competitive forces drive prices down. However, the *ubiquity* of exploitation is actually higher in the case of competition, as a larger fraction of the population choose an exploitative price scheme.

To conclude, under the notion of frequency naivete, mildly naive consumer types are excluded from exploitation. Since competition weakens the incentive to exclude, it makes exploitation more prevalent, even as it reduces its magnitude. This has an interesting implication for discussions of market interventions. If a central planner's objective is to maximize consumer welfare from the first-period self's point of view, then opening a monopolistic market to competition has an ambiguous effect.

4.3 SUMMARY

The implications of introducing consumer types with intermediate degrees of naivete enriches our analysis of markets with dynamically inconsistent consumers. In particular:

- The concept of magnitude naivete captures situations in which consumers underestimate the extent to which their preferences will change. Using this concept, we illustrated the idea that greater sophistication does not necessarily make the consumer better off.
- The concept of frequency naivete captures situations in which consumers disregard some of the circumstances that cause their preferences to change. Using this concept, we illustrated the idea that greater market competition can make exploitative price schemes more ubiquitous.

4.4 BIBLIOGRAPHIC NOTES

The notion of magnitude naivete is due to Loewenstein, O'Donoghue & Rabin (2003), who regard it as a model of projection bias. Our analysis of the implications of magnitude naivete is based on Heidhues & Kőszegi (2009, 2010). The notion of frequency naivete was introduced and applied to monopoly pricing by Eliaz & Spiegler (2006).

5 Biased Beliefs without Dynamic Inconsistency

In Chapters 2–4, we devoted considerable attention to what we referred to as "naivete"—namely, biased beliefs that consumers hold regarding their future preferences. However, naivete was always discussed in the context of dynamically inconsistent preferences. Consumers had definite first-period preferences over their second-period consumption options; these preferences changed in the second period, and naivete meant underestimating the magnitude or likelihood of this change.

In this chapter, we discuss situations in which consumers hold biased beliefs regarding their future preferences, without any element of dynamic inconsistency. In the two-period models we shall examine, the consumer's second-period preferences over consumption alternatives are "state-dependent." In the first period he holds a belief over the possible states. Firms possess different prior beliefs, and we as modelers judge those as unbiased.

So far this is just like the two-period models in Chapters 2–4. The difference is that in the present chapter, the consumer will not have a distinct first-period preference relation over second-period market outcomes. In the first period, he will choose a price scheme as a standard expected utility maximizer, whose beliefs happen to be judged by the modeler as biased. In this respect, this chapter—unlike most others in this book—stays within the boundaries of the standard model of consumer choice. The primary reason for including it is its linkage with the market models inhabited by naive, dynamically inconsistent consumers.

We will consider three types of systematic belief biases:

1. *Over-optimism.* One future state may be "better" than another, in the sense that if consumers could choose between them, they would choose the former. Beliefs are over-optimistic if they systematically assign excessive probability to the good states. For instance, a consumer who is about to sign up for a cable TV package may be over-optimistic about the amount of leisure that he will have, and therefore exaggerate the value of having more channels on the package. Likewise, a consumer who considers buying car insurance may falsely believe that he has below-average accident risk.
2. *Over-confidence.* The consumer may over-estimate the precision with which he estimates his second-period valuation of various consumption decisions. For instance, a consumer who is about to sign up for mobile phone services considers evidence from past consumption (his own and others') and reaches an estimate that he will demand 1,000 minutes of airtime, with little uncertainty around this estimate. In reality, his demand for airtime will vary more than he anticipates.

3 *Unforeseen contingencies*. The consumer may assign zero probability to certain states because he is simply unaware of them at the time of purchase. For instance, a consumer who is about to acquire a vacation package may fail to think about various expenses he will need to incur during the vacation. Alternatively, a consumer who considers buying a home entertainment product may fail to think about various additional components (add-ons) he will perceive as necessary or valuable after the purchase.

My objective in this chapter is to expose some of the implications of such belief biases for firms' pricing decisions (in both monopolistic and competitive settings). In particular, we will see how biased consumer beliefs can account for effects such as three-part tariffs and add-on pricing. We will also be interested in a comparison with the effects discussed in previous chapters, where biased beliefs and dynamic inconsistency were coupled.

5.1 MONOPOLY PRICING WITH OVER-OPTIMISTIC CONSUMERS

Let us begin with a close variant on the model of monopoly pricing with partially (frequency) naive consumers, which was studied in Chapter 4. There are two time periods, $t = 1, 2$. A monopolistic firm offers a service to a single consumer. Let $X = [0, 1]$ be the set of consumption decisions that the consumer can take in period 2, conditional on having accepted the firm's price scheme in period 1. I typically interpret x as a consumption quantity. The firm's cost of providing the service is given by a continuously increasing function $c : [0, 1] \to \mathbb{R}$. A price scheme is a function $t : X \to \mathbb{R}$ that specifies a transfer (possibly negative) from the consumer to the firm for every second-period action. If the consumer accepts the firm's price scheme, he is obliged by it in period 2. If he rejects it, he is left with an outside option, the value of which is zero.

The consumer has quasi-linear, state-dependent preferences over second-period outcomes. The state space is $\{u, v\}$. In state u, the consumer's willingness to pay for any action is given by a continuous function $u : X \to \mathbb{R}$, whereas in state v, it is given by a continuous function $v : X \to \mathbb{R}$. Assume that $u(0) = v(0) = 0$ and $u(x) \geq v(x)$ for all $x > 0$, with a strict inequality for some x. Thus, u and v can be viewed as "good" and "bad" states, because in the state u the consumer derives more pleasure from the firm's service, and therefore his willingness to pay is higher in that state. Assume that each of the functions $u - c$, $v - c$, and $u - v$ attains a non-negative maximum at a unique point.

As in previous chapters, I assume that neither the firm nor the consumer have any additional information about the consumer's second-period preferences. To facilitate comparison with the model of Chapter 4, assume for the moment that the monopolist's prior belief assigns probability one to the state v. The consumer's prior on the state u is denoted θ. I interpret the monopolist's prior belief as correct and unbiased. If $\theta > 0$, we say that the consumer's belief is *over-optimistic*, because it assigns excessive weight to the "good" state u.

The monopolist's first-period maximization problem is thus to find a 4-tuple (x^u, T^u, x^v, T^v)—where (x^u, T^u) and (x^v, T^v) are the action-payment pair in the states u and v, respectively—that maximize

$$T^v - c(x^v)$$

subject to the second-period incentive compatibility constraints

$$u(x^u) - T^u \geq u(x^v) - T^v \tag{5.1}$$

$$v(x^v) - T^v \geq v(x^u) - T^u \tag{5.2}$$

as well as a participation constraint

$$\theta \cdot [u(x^u) - T^u] + (1-\theta) \cdot [v(x^v) - T^v] \geq 0 \tag{5.3}$$

This maximization problem is very similar to the one we studied in Section 4.2.1. The difference lies in the consumer's participation constraint. Since we now assume that the consumer is *dynamically consistent*, his first-period net evaluation of the action x in state $\omega \in \{u, v\}$ is $\omega(x) - t(x)$. In contrast, in the model of Section 4.2.1, the consumer's first-period evaluation of x was $u(x) - t(x)$ in *both* states, because he had a distinct first-period willingness-to-pay function u over his second-period consumption decisions.

When $\theta = 0$, the model is reduced to a standard problem of monopoly pricing, as both parties agree that the consumer's preferences over second-period consumption are given by v. Therefore, a price scheme that commits the consumer to $x^* = \arg\max(v - c)$ in return for a payment equal to $v(x^*)$ is optimal. The following result characterizes optimal price schemes when the consumer is over-optimistic.

Proposition 5.1. *Let $\theta > 0$. Under any optimal price scheme, the consumer's second-period consumption decisions are*

$$x^v = \arg\max(v - c)$$

$$x^u = \arg\max(u - v)$$

and the payments he makes are

$$T^v = v(x^v) + \theta \cdot [u(x^u) - v(x^u)]$$

$$T^u = \theta \cdot u(x^u) + (1-\theta) \cdot v(x^u)$$

The proof is straightforward and therefore omitted. The key to the result is the observation that in optimum, the participation constraint and the second-period incentive compatibility constraint in state v (namely, (5.2)) are binding. The other models of monopoly pricing examined in Chapters 2 and 4 shared this feature.

Note that when $\theta > 0$, $v(x^v) - T^v < 0$ and $u(x^u) - T^u > 0$. That is, the payment the consumer ends up making in state v (u) is higher (lower) than his willingness to pay for the action he takes in that state. This reflects the speculative aspect of the consumer-firm interaction: optimal price schemes are essentially bets over the consumer's second-period consumption. A consumer who shares the firm's (unbiased) prior belief would not accept these price schemes. In this sense, they are *exploitative*.

The overall shape of optimal price schemes depends on finer details of u and v. For example, when $u'(x) > v'(x)$ for all x, optimal price schemes will induce $x^u = 1 \geq x^v$ (the inequality is strict if $\arg\max(v - c) < 1$). To an outside observer, the price scheme will appear to grant an exceedingly generous quantity discount. In this case, over-optimistic consumers expect to enjoy these discounts, yet they end up consuming little at a relatively high average price. In contrast, consider the following configuration: for every $\omega \in \{u, v\}$, there exists a satiation point s^ω, such that $\omega(x) = \min(\frac{x}{s^\omega}, 1)$. Assume further that $s^u < s^v$. The interpretation is that the consumer has an "aspiration utility" of 1, and he is uncertain about how quickly it can be attained. The state u (v) represents an optimistic (pessimistic) scenario, in which the consumer reaches satiation with a low (high) quantity of consumption. In this case, an optimal price scheme will induce $x^u < x^v$. To an outside observer, the price scheme will appear to charge a low payment for low consumption quantities, and a large surcharge for high quantities.

Full-support unbiased beliefs

How robust is Proposition 5.1 to the extreme assumption that the monopolist assigns probability one to v? Modify the model by assuming that the monopolist's (unbiased) prior belief assigns positive probability q_ω to every state $\omega \in \{u, v\}$. Assume that $u'(x) > v'(x)$ for all $x \geq 0$. It turns out that under any optimal price scheme, the consumer's second-period consumption decision in each state ω is

$$x^\omega = \arg\max_x \{q_\omega \cdot [\omega(x) - c(x)] + \max(\theta_\omega - q_\omega, 0) \cdot [\omega(x) - \omega'(x)]\} \tag{5.4}$$

where $\omega' \neq \omega$ and θ_ω denotes the consumer's prior on ω. Thus, as in our basic model, the consumer's second-period action continues to maximize social surplus in the state that he deems less likely relative to the firm. In contrast, the action the consumer takes in the state he deems more likely relative to the firm maximizes a linear combination of the social surplus in that state and the consumer's excess willingness to pay, relative to the other state. Proving this is left as an exercise.

Exercise 5.1. *Show that when the monopolist assigns positive probability q_ω to every state $\omega \in \{u, v\}$, the consumer's second-period consumption decision in each state ω under any optimal price scheme is given by* (5.4).

Note that the more general model allows for situations in which the consumer assigns *lower* probability to u than the monopolist—that is, the consumer is over-pessimistic. Expression (5.4) covers this possibility. However, optimal price schemes in this case are *not* exploitative. That is, consumers with unbiased beliefs would be willing to accept them. The following exercise goes further: any price scheme that satisfies the participation and second-period incentive compatibility constraints does not exploit over-pessimists.

Exercise 5.2. *Show that if* (x^u, x^v, T^u, T^v) *satisfies the second-period incentive compatibility constraints* (5.1)–(5.2) *and the participation constraint* (5.3) *of a consumer with* $\theta_u < q_u$, *it also satisfies the participation constraint of a consumer with* $\theta_u = q_u$.

5.1.1 Comparison with Related Models

Let us turn to a description of how our analysis of optimal monopoly pricing with overoptimistic consumers relates to the analogous analysis of monopoly pricing with partially (frequency) naive, dynamically inconsistent consumers in Section 4.2.1. One motivation behind this comparison is that in applications of market models with naive consumers, it is not always obvious whether the proper assumption is that consumers have changing tastes.

For instance, recall the health club example in Section 3.1.1. We assumed that when the consumer contracts with the health club, he has a clear perception of the amount of physical exercise he would like to implement in period 2, but he has an unrealistic expectation regarding his ability to summon the self-discipline required to implement these plans. However, it is equally plausible to assume that at the time the consumer contracts with the health club, he does not have such a clear perception of the amount of physical exercise he wants to engage in. His naivete lies in an underestimation of the amount of pain he will derive from exercising, or an overestimation of the amount of time he will be able to spare for physical exercise; both translate into an unrealistic expectation of his future willingness to pay for exercise. Since both assumptions are reasonable a priori, it is interesting to see if they have different implications for consumer behavior, and how these, in turn, differ from those of a conventional model in which consumers have unbiased subjective beliefs about their future demand at the time they contract with the health club.

In the following comparison between the model of this current section and the model of Section 4.2.1, I return to the original assumption that the monopolist's prior belief assigns probability one to the state v, unless explicitly stated otherwise.

Consumption decisions

In the present model, the actual second-period action x^v maximizes $v - c$, independently of the consumer's belief. In contrast, when the consumer has changing tastes, his actual second-period action depends on his own belief, and only gravitates toward $\arg\max(v - c)$ when his belief approaches perfect naivete ($\theta = 1$). However, the "imaginary" action that the consumer believes he will take in state u is the same under both models. This action maximizes the difference in willingness to pay between the two states. The intuition is that the optimal price scheme has a speculative component due to the difference between the monopolist's and the consumer's prior beliefs. By setting $x^u = \arg\max(u - v)$, the monopolist maximizes the speculative gain, and this consideration is common to both models.

Recall that although the two models' predictions of x^v coincide in the case $\theta = 1$, the welfare evaluation of this outcome varies across models. In the present chapter, we unequivocally use v to evaluate the outcome. In contrast, in the model of monopoly pricing with a fully naive dynamically inconsistent consumer, we used both u and v to evaluate market outcomes. If $\arg\max(v - c) \neq \arg\max(u - c)$, then x^v is ex-post efficient from the point of view of the second-period self, but inefficient from the first-period self's perspective.

Contractual forms

When the consumer holds biased beliefs of his future preferences, the qualitative structure of the optimal price scheme is the same in both models. It induces two relevant second-period actions, x^u and x^v. The firm believes that the consumer will take the latter, while the consumer assigns positive probability to the former. The consumer's net surplus is positive (negative) in state u (v). This reflects the element of speculative trade in optimal price schemes.

When the consumer's belief over second-period preferences is unbiased, the two models imply different contractual forms. This comparison is sharper if we consider the more general case in which the firm assigns positive probability to both states. As can be seen from expression (5.4), optimal monopoly pricing when the consumer is dynamically consistent with $\theta = q_u$ induces a state-dependent action that maximizes the social surplus function $\omega - c$ in each state ω. In contrast, a fully sophisticated, dynamically inconsistent consumer seeks a commitment device that will implement his first-period preferences. In this case, an optimal price scheme would commit him to the action $x^* = \arg\max(u - c)$ in both states, independently of the two parties' prior beliefs.

The next two comments concern differences between the two models that arise when the consumer's degree of naivete, captured by his value of θ, is his private information. In this case, the monopolist offers a menu of price schemes, and each consumer selects a price scheme from the menu to maximize expected utility, according to his own subjective prior belief.

The possibility of dominated contracts

One of the effects emphasized in Chapter 2 was that when consumers are dynamically inconsistent, it is possible to devise a menu of price schemes that appear to dominate one another (in the sense of prescribing lower payments for all second-period actions), such that naive consumers will prefer one price scheme while sophisticates will opt for another. Moreover, such menus are possible outcomes of market competition.

This effect is impossible when consumers have dynamically consistent preferences. Consider two price schemes, t and t', such that $t(x) < t'(x)$ for all $x \in X$. Then, any expected utility maximizer will prefer t to t', regardless of his belief over second-period preferences. The reason that a sophisticated dynamically inconsistent consumer may choose an apparently dominated contract is that he does not conform to expected utility maximization—rather, he is playing a game with his future self, and choosing an apparently dominated contract is a strategic decision designed to tie the future self's hands.

Screening the consumer's type

In Chapter 2, we saw that when the consumer population is divided into perfectly sophisticated and perfectly naive types, a monopolist is perfectly able to screen the consumer's type. A menu consisting of an optimal contract for naifs and an optimal contract for sophisticates continues to be optimal when the firm cannot identify the consumer's type. The reason is that sophisticated consumers exert no informational externality on naifs, because optimal contracts for sophisticates are perfect commitment devices that are evaluated identically by all consumers, regardless of their degree of naivete.

This property is no longer true when consumers are dynamically consistent. Let us return to the simple case in which the firm assigns probability one to v. As we saw, there is an optimal price scheme for unbiased consumers that commits them to the action $x^* = \arg\max(v - c)$ and charges a payment equal to $v(x^*)$, such that their participation constraint is binding. An over-optimistic consumer assigning probability $\theta > 0$ to the state u will evaluate this contract at $\theta \cdot [u(x^*) - v(x^*)]$, which may be strictly positive. In this case, the biased consumer will strictly prefer this price scheme to the contract aimed at him, given by Proposition 5.1. In other words, the over-optimistic consumer enjoys an informational rent due to the firm's inability to distinguish him from unbiased consumers.

Comparison with the case of an unbiased, dynamically consistent consumer

The case of $\theta = q_u \in (0, 1)$ reflects a situation in which the consumer is uncertain of his future demand at the time he contracts with the firm, but his beliefs are unbiased. This is a natural rational-consumer benchmark that is interesting to compare with our twin models of monopoly pricing with naive consumers.

As can be seen from expression (5.4), the consumer's second-period action is ex-post efficient in both states when $\theta = q_u$. In contrast, when the consumer's prior belief regarding his future preferences diverges from the firm's prior, optimal monopoly pricing generates an ex-post inefficient outcome in one state (recall, however, the subtlety of efficiency judgments when the consumer is dynamically inconsistent).

Another major difference concerns the possibility of implementing optimal price schemes with two-part tariffs, as the following exercise demonstrates.

Exercise 5.3. *Return to Exercise 5.1. Assume that both u and v are twice continuously differentiable, with $u' > v' > 0$ and $u'', v'' < 0$, and that $c(x) = \kappa x$, where $\kappa > 0$ is the firm's constant marginal cost. Show that when $\theta = q_u$, optimal price schemes can be implemented by a two-part tariff with a price per unit equal to κ. Show that when $\theta \neq q_u$, optimal price schemes cannot be implemented by a two-part tariff.*

Thus, when dynamically consistent consumers have unbiased beliefs, there exists a two-part tariff that maximizes the monopolist's profits and conforms to marginal-cost pricing. In contrast, when consumers have biased beliefs, the optimum cannot be implemented by a two-part tariff. This distinction mirrors our observation in Chapter 3 that when a dynamically inconsistent consumer is sophisticated (naive), two-part tariffs can (cannot) implement the optimum.

5.2 OVERCONFIDENCE: THREE-PART TARIFFS

Our model of monopoly pricing restricted attention to two future preference states. While this simplifying assumption is pedagogically useful, it rules out interesting belief biases. Moreover, because of the two-state restriction, all that matters in a price scheme is the two action-payment pairs that it induces. As a result, we were only able to characterize the structure of optimal price schemes in broad strokes. This section is devoted to an example of a model of monopoly pricing with biased consumers, in which the number of preference states is large. This will enable us to capture a new psychological phenomenon that is relevant for consumer behavior, and at the same time to provide an account of a particular price scheme often observed in reality.

The bias referred to above as over-optimism captured the tendency to overestimate the expected future benefit from the firm's product. Another bias, referred to as *over-confidence*, is defined as underestimation of the *variance* of the product's future benefit. This can arise from failure to conceive of future shocks to the consumer's willingness to pay. Alternatively, it may be a result of naive extrapolation from a limited set of past observations. The consumer may have had casual experience with the product, either through direct consumption or via word of mouth, and he may infer that this experience will persist in the future, failing to account for noise. A similar kind of rash inference will be the subject of our next chapter.

I turn to a model of monopoly pricing in a many-state environment with overconfident consumers. Let $X = [0, 1]$ be the set of second-period actions that the consumer is able to take conditional on accepting a price scheme $t : X \to \mathbb{R}$ offered by the monopolist in period 1. Let $S = [0, 1]$ be a set of states. The state is realized in period 2. An element $s \in S$ represents a second-period preference relation over X. In state s, the consumer's second-period willingness to pay is given by $u_s(x) = \min(x, s)$. The interpretation is that the consumer has a random satiation point, yet as long as his consumption quantity falls below the satiation point, his marginal willingness to pay is equal to 1 in all states. Assume that the monopolist's prior belief is uniform over S. The monopolist incurs zero costs. The consumer's first-period outside option has a certain value of 0.

When the consumer's belief is unbiased—that is, his prior over S is uniform—the price scheme $t(x) = x$ maximizes the firm's profit. The consumer's consumption quantity is equal to his satiation point in every state, hence it is ex-post efficient. Showing that this is the case is left as an exercise. Now assume that the consumer's belief displays an extreme form of overconfidence. Specifically, it assigns probability one to some $s^* \in S$. Thus, the consumer completely ignores the noisiness of his satiation point, and regards his point estimate as a sure prediction. Under these assumptions, the firm's objective is to find a price scheme t and a collection of consumption decisions $(x_s)_{s \in S}$, that maximize expected profits

$$\int_0^1 t(x_s)ds$$

subject to the consumer's participation constraint

$$u_{s^*}(x_{s^*}) - t(x_{s^*}) \geq 0$$

as well as second-period incentive compatibility constraints:

$$x_s \in \arg\max_x [u_s(x) - t(x)]$$

for every $s \in [0, 1]$.

It turns out that in this simple example, there is an optimal price scheme with the structure of a *three-part tariff*. Such a tariff consists of a lump-sum payment, zero marginal price for quantities below some critical level, and a strictly positive, constant marginal price for quantities above it. Three-part tariffs are commonly observed in a variety of industries, mainly telecommunication.

Proposition 5.2. *There is a solution* $(t^*, (x_s^*)_{s \in [0,1]})$ *to the monopolist's problem, such that:*

(i) $x_s^* = s$ for all $s \in [0, 1]$.
(ii) $t^*(x) = \max(x, s^*)$.

Proof. It is easy to see that $(t^*, (x_s^*)_{s \in [0,1]})$ satisfies the consumer's first-period participation constraint bindingly. As to the second-period incentive compatibility constraints, we need to show that for every s, $u_s(x_s^*) - t^*(x_s^*) \geq u_s(x) - t^*(x)$ for all x. This inequality can be rewritten as follows:

$$s - \max(s, s^*) \geq \min(x, s) - \max(x, s^*)$$

It is easy to verify that this inequality indeed holds for all triples x, s, s^*.

In order to prove the optimality of t^*, consider some $(t, (x_s)_{s \in [0,1]})$ that satisfies the participation and second-period incentive compatibility constraint. We will show that $t(x_s) \leq \max(s, s^*)$ for all s. By second-period incentive compatibility and the assumption that $u_s(x) \leq x$ for all s, x, the following inequality holds for every s:

$$s - t(x_s) \geq u_s(x_{s^*}) - t(x_{s^*}) \tag{5.5}$$

Consider $s \geq s^*$. Then, $u_s(x) \geq u_{s^*}(x)$ for all x . Therefore, $u_s(x_{s^*}) - t(x_{s^*}) \geq u_{s^*}(x_{s^*}) - t(x_{s^*})$. The R.H.S. of the latter inequality is non-negative, by the participation constraint. It follows that $t(x_s) \leq s$ for all $s \geq s^*$. Now consider $s < s^*$. Let us distinguish between two cases.

Case (i): $x_{s^*} < s$.

Since x_{s^*} is lower than both s and s^*, $u_s(x_{s^*}) = u_{s^*}(x_{s^*}) = x_{s^*}$. It follows that the R.H.S. of (5.5) is equal to $u_{s^*}(x_{s^*}) - t(x_{s^*})$, which itself is non-negative by the participation constraint. Therefore, $t(x_s) \leq s$.

Case (ii): $x_{s^*} \geq s$.

Since x_{s^*} is above s, $u_s(x_{s^*}) = s$, hence the R.H.S. of (5.5) is equal to $s - t(x_{s^*})$. Therefore, $s - t(x_{s^*}) \leq s - t(x_s)$. Since we have already established that $t(x_{s^*}) \leq s^*$, it follows that $t(x_s) \leq s^*$.

Combining the inequalities we have derived, we obtain $t(x_s) \leq \max(s, s^*)$ for all s. This completes the proof. □

The intuition behind the optimality of a three-part tariff is as follows. Since the consumer underestimates the probability of consuming below s^*, he does not mind paying the lump sum because the average price is not prohibitively high under the plan to consume s^*. The average price *is* prohibitively high if the consumer chooses $x < s^*$, but he fails to take this scenario into account. On the other hand, when the consumer learns that he wants to consume more than s^*, the lump sum is a sunk cost, and he is willing to pay the added payment for increased consumption. The consumer regrets having signed the contract whenever $s < s^*$ because he ends up paying more than his willingness to pay in these states. However, when $s \geq s^*$, the consumer's net ex-post payoff is zero. Thus, the three-part tariff exploits consumers only to the extent that their over-confident belief reflects over-optimism. This resonates with the message of Exercise 5.2.

5.3 UNFORESEEN CONTINGENCIES: ADD-ON PRICING

Another common example of a belief bias is unawareness of future contingencies that affect willingness to pay for a product. In this section I analyze a two-period model of a competitive market, in which firms offer a basic product in period 1. In period 2, consumers who bought the product may purchase an additional component that complements the basic product, referred to as an "add-on." For example, consider mini-bars and telecommunication services in hotel rooms. These can be viewed as add-ons with respect to the basic product, namely the hotel room itself. Similarly, many electric appliances are sold "bare" with a number of missing features, which are offered as add-ons only after the sale. Sophisticated consumers are initially aware of the possibility of buying an add-on from the same firm, whereas naive consumers are unaware of it at the time they purchase the basic product. They become aware of it only in period 2.

Intuitively, when some consumers in these situations fail to conceive of future contingencies in which they will demand the add-on, firms will compete fiercely over the basic product; by the time the consumer learns that he needs the add-on, he is captured by the firm that sold him the basic product, and therefore subjected to its monopoly power. As a result, the market price of the basic product (add-on) will tend to be low (high), an effect called "add-on pricing." In this section we formalize this intuition and explore some of its implications. In particular, we will pursue two questions: How does the market outcome depend on the fraction of unaware consumers? Do firms in a competitive environment have an incentive to "educate" consumers, in the sense of heightening their awareness and alerting them to the future contingency that calls for an add-on?

Let us turn to the two-period model. There are two firms and a continuum of consumers. The firms offer a basic product and an add-on. For simplicity, set the costs of both components to zero. In period 1, each firm i simultaneously chooses a strategy (p_i^1, p_i^2), where p_i^1 is the price of firm i's basic product and p_i^2 is the price of firm i's add-on. Consumers choose in period 1 whether to buy the basic product, and from which firm. Conditional on buying the basic product from firm i in period 1, the consumer decides in period 2 whether to buy the firm's add-on at the price it charges. At that point the consumer cannot buy the add-on from firm j, because each firm's add-on is compatible only with the firm's own basic product. Note that each firm *i commits* to p_i^2 in period 1.

All consumers are willing to pay 1 for the basic product, independently of the firm from which it is bought. In contrast, there is heterogeneity among consumers in their evaluation of the add-on. Conditional on having purchased the basic product, the consumer's willingness to pay for the add-on is distributed over [0, 1] according to a strictly increasing and differentiable *cdf* F. When the consumer does not acquire the basic product, his payoff is 0. Denote $R(p) = p \cdot [1 - F(p)]$. This function represents a firm's second-period revenues from selling add-ons at a price p to consumers who bought its basic product in period 1. Assume that R is hump-shaped and denote $p^m = \arg\max_p R(p)$. That is, p^m is the monopoly price for the add-on.

Our analysis will contrast two extreme cases in terms of the consumers' level of market sophistication, as well as a third case with a mixed consumer population.

Case 1: Sophisticated consumers

This is a rational-consumer benchmark, where consumers are fully aware of the future demand for the add-on, and in particular they know their own valuation. Therefore, consumers choose according to the prices that firms charge for both the basic product and the add-on. Consider a consumer whose valuation of the add-on is v. His payoff from choosing to buy the basic product from firm i in period 1 is

$$(1 - p_i^1) + \max(v - p_i^2, 0)$$

because in period 2 the consumer buys the add-on if and only if $p_i^2 \leq v$. The consumer will choose the market alternative that maximizes this payoff. Ties are resolved symmetrically.

Proposition 5.3. *When consumers are sophisticated, firms charge $p^2 \leq 0$ and $p^1 = -p^2$ in symmetric Nash equilibrium.*

Proof. Let us first establish that firms earn zero profits in symmetric Nash equilibrium. Consider an equilibrium strategy $\left(p^1, p^2\right)$. By the equilibrium symmetry, each firm attracts half the consumer population, thus earning a profit of $\frac{1}{2} \cdot [p^1 + R(p^2)]$. Suppose that this profit is strictly positive. Consider a deviation to the strategy $(p^1 - \varepsilon, p^2 - \varepsilon)$, where $\varepsilon > 0$. Because the deviating firm offers a strictly lower price for both components, it will attract all consumers. If ε is sufficiently small, the deviation is profitable.

Consider a putative equilibrium strategy (p^1, p^2) such that $p^2 > 0$. By the zero-profit result above, $p^1 + R(p^2) = 0$. Since $p^2 > 0$, $R(p^2) < p^2$, hence $p^1 + p^2 > 0$. Suppose that firm 1, say, deviates to $(p^1 + \varepsilon, p^2 - 2\varepsilon)$, where $\varepsilon > 0$ is arbitrarily small. Then, the firm will attract all consumers for whom $v \geq p^2 - 2\varepsilon$ and will generate a strictly positive profit from them. At the same time, the firm will fail to attract all consumers for whom $v < p^2 - 2\varepsilon$. Therefore, the deviation is profitable.

Thus, we are led to examine putative equilibrium strategies (p^1, p^2) satisfying $p^2 \leq 0$. Since $p^2 \leq 0$, $F(p^2) = 0$, hence $R(p^2) = p^2$. It follows that $p^1 + p^2 = p^1 + R(p^2) = 0$. To see that any strategy (p^1, p^2) satisfying $p^2 = -p^1 \leq 0$ is an equilibrium strategy, note that any deviation either leads to negative profit or to zero clientele. □

Thus, marginal cost pricing for both components is consistent with Nash equilibrium when consumers are sophisticated. However, charging an add-on price below marginal cost is also consistent with equilibrium.

Case 2: Naive consumers

In this case, consumers are unaware of the possibility that they will demand an add-on. Therefore, they choose in period 1 entirely according to the basic product's price—that is, they choose to buy the basic product from firm i if $p_i^1 < p_j^1$ and $p_i^1 \leq 1$. Ties between firms are resolved symmetrically. In period 2, consumers become aware of the add-on, learn their valuation, and buy the add-on if and only if $p_i^2 \leq v$.

Proposition 5.4. *When consumers are naive, firms charge $p^2 = p^m$ and $p^1 = -R(p^m)$ in symmetric Nash equilibrium.*

***Proof*.** Firms earn zero profits in symmetric Nash equilibrium. The proof for the case of sophisticated consumers applies here, too. Therefore, any symmetric equilibrium strategy (p^1, p^2) satisfies $p^1 = -R(p^2)$. Assume that $p^2 \neq p^m$. For brevity, let us ignore the case of $p^1 > 1$, which is easy to rule out. Suppose that firm 1, say, deviates to the strategy $(p^1 - \varepsilon, p^m)$. Since consumers are naive and choose only according to the price of the basic product, the firm will attract all consumers as a result of the deviation. By assumption, $p^2 \neq p^m$, and therefore, $R(p^m) > R(p^2)$. If $\varepsilon > 0$ is sufficiently small, $p^1 - \varepsilon + R(p^m) > 0$, hence the deviation is profitable. □

Thus, when the consumer population consists entirely of naifs, firms set the price of the add-on at the monopolistic level p^m, and the basic product's price is negative, such that the loss from the basic product is exactly offset by the profit from the add-on. This simple result provides an account of ultra-competitive pricing of "bare" products coupled with non-competitive pricing of their add-ons. The add-on cross-subsidizes the basic product.

Case 3: A mixed consumer population

Now suppose that the consumer population consists of both naifs and sophisticates, and that the fraction of the former is $\lambda \in (0, 1)$. This case turns out to be non-trivial to analyze, as it forces us to abandon pure-strategy equilibrium analysis, as the following exercise demonstrates.

Exercise 5.4. *Show that there exists no symmetric pure-strategy Nash equilibrium when the consumer population consists of both naifs and sophisticates.*

The following variant turns out to be much simpler to analyze. In many situations, awareness of future contingencies that generate a demand for add-ons also allows the consumer to obtain a cheaper substitute for the firm's add-on. For instance, a consumer anticipating a need for a cold drink after checking into a hotel will get a beverage in advance, thus mitigating "mini-bar exploitation."

To take an extreme case, assume that sophisticated consumers have costless access to a perfect substitute for the add-on. In this case, sophisticated consumers buy the basic product only (as long as its price does not exceed 1). If firms charge a negative price for the basic product, this means that firms make a loss in their transactions with sophisticates. This loss is offset by the gains that firms generate from naifs. Since firms are not interested in attracting sophisticates, they can afford to set p^2 at the monopoly level, thus extracting full monopoly profits from naifs. They price the basic product below marginal cost as part of a competitive strategy designed to attract naifs, but they are mindful of the way in which sophisticates exploit this pricing strategy. The following exercise formalizes this intuition.

Exercise 5.5. *Assume that sophisticates have access to a costless, perfect substitute for the add-on. Let the fraction of naifs in the consumer population be $\lambda \in (0, 1)$. Show that firms charge $p^2 = p^m$ and $p^1 = -\lambda R(p^m)$ in symmetric Nash equilibrium.*

This exercise demonstrates an important point: *rational consumers can exert a negative externality over boundedly rational ones*. In the present context, the reason is that firms are unable to discriminate explicitly between the two types of consumers. As a result, any attempt to exploit naifs is met with counter-exploitation by the sophisticates, who buy the basic product. As the fraction of sophisticates in the consumer population increases, the equilibrium price of the basic product goes up toward zero, because the total monopoly profit from selling the add-on to naifs, which is meant to cover the loss from the basic product, goes down. This means that the total price that naifs end up paying rises.

Educating unaware consumers

When consumers are unaware of a future contingency, a firm can fix this by explicitly alerting the consumer's attention to the contingency, or by advertising the price it charges in that contingency. In this way, the consumer immediately becomes aware of the price that the rival firm charges for the add-on as well. Will firms have an incentive to do it? To address this question, let us modify the model by enabling each firm to turn all naive consumers into sophisticates, unilaterally and at no cost. The firm's decision whether to educate consumers in this fashion is simultaneous with its pricing decision.

Consider the original model in which sophisticates do not have a substitute for the add-ons. In Nash equilibrium, at least one firm will educate consumers, such that all consumers will act as sophisticates. Consequently, the firms' equilibrium pricing decisions are the same as in the case in which consumers are sophisticated to begin with. The reasoning behind this result is as follows. When consumers are naive, they do not take the add-on price into account when choosing between the two firms. By the time they become aware of the add-on, they are captured by whichever firm they chose for the basic product. The firm thus enjoys monopoly power in the provision of the add-on, and this leads to an inefficient outcome. If any firm could costlessly make consumers aware of the add-on, it could at the same time raise p^1 by ε and lower p^2 by 2ε, where $\varepsilon > 0$ is arbitrarily small. This is precisely the profitable deviation that we used to knock out putative equilibria that sustain $p^2 > 0$ when consumers were assumed to be sophisticated.

Let us now turn to the extended model in which $\lambda \in (0, 1)$ and sophisticates have access to a costless perfect substitute for the add-ons. In this case, even when firms are able to educate consumers, the equilibrium characterized in Exercise 5.5 survives, where firms choose not to educate naifs. To see why, note that if a firm deviates by turning all naifs into sophisticates, this has an unwarranted by-product that these consumers will not buy the add-on unless it is negatively priced. In other words, educating naifs about add-ons only leads them to take advantage of the basic product's subsidization.

Unawareness versus zero probability

Is there a meaningful behavioral distinction between unawareness of a future contingency and a belief that assigns zero probability to that contingency?

The answer is negative in the basic model in which "educating consumers" is infeasible. The distinction in this case between unawareness and zero-probability belief is purely a matter of interpretation. It becomes behaviorally meaningful in the extended model, in which firms are able to "educate" consumers, because an unaware consumer can update his beliefs in a way that a Bayesian rational agent never would. First, no Bayesian who assigns zero probability to an event would ever change this belief. Second, a Bayesian would never update his beliefs when confronted with statements that convey no information. In contrast, a consumer who is initially unaware of an event E may change his beliefs when encountering the statement "E is either true or false" because this statement, while tautological and therefore completely uninformative in the usual sense, may heighten his awareness of E.

Can the model be rationalized?

Our basic model of competitive add-on pricing when the consumer population consists entirely of naifs can be reformulated as a model with conventionally rational consumers. Assume that consumers are fully aware of all contingencies. However, instead of assuming that firms commit to (p^1, p^2) at the outset, suppose that firms can only commit to p^1 in period 1, and they determine the add-on price p^2 only in period 2. Thus, the firms' monopoly power in the provision of add-on does not arise from consumer naivete, but from their inability to commit to the add-on price as part of their competitive strategy in period 1. This is a classic "hold-up" problem. In sub-game perfect equilibrium of the ensuing extensive-form game, firms will set $p^2 = p^m$ in period 2, and competitive pressures will drive profits to zero, such that $p^1 = -R(p^m)$.

This reformulation does not survive some of the extensions of the basic model. The analysis of the mixed population case, initiated in Exercise 5.5, is nonsensical if firms cannot commit to p^2 in period 1. In addition, our discussion of educating consumers—that is, using "uninformative information" to bring add-ons to their awareness—becomes irrelevant under this reformulation.

This discussion illustrates an important methodological principle. Even if the most basic version of a market model with boundedly rational consumers can be reinterpreted as a conventional rational-choice model, the distinction between the two interpretations is important because the bounded-rationality interpretation suggests extensions that are strange or meaningless in the context of the "rationalizing" model. We will have more opportunities to see this principle in action later in the book.

■ 5.4 A SUMMARY EXERCISE: INSURANCE MARKETS WITH BIASED CONSUMERS

Insurance markets provide an important environment in which belief biases should be expected. The events consumers aim to insure themselves against are rare, and assessing their likelihood is a difficult task for most people. (Is the chance of catching a certain disease on the order of 1:100 or 1:10,000?) Even if a consumer could access the statistical tables that guide the insurance companies, he would find it hard to tell how his own private information should change his assessment relative to the insurance company's unconditional probabilities. These difficulties naturally magnify some of the systematic belief biases that we have encountered in this chapter.

In this section I briefly discuss a variation on the textbook Rothschild-Stiglitz model of competitive insurance markets. I will present the model in terms of a car-insurance scenario. In this case, it is realistic to assume that some consumers overestimate their own driving skills, and as a result underestimate the probability of a car accident. Therefore, consumers who objectively belong to the high-risk pool may perceive themselves as low-risk agents.

Although the model will retouch the main themes of this chapter, it also departs from the models examined so far in several dimensions. First, the consumers' reservation value is sensitive to their subjective risk assessment. Second, consumers are risk averse, hence their utility is not linear in money. Third, I assume that consumers are not required to take any action after learning the true state of Nature. Thus, all second-period incentive issues are eliminated. Fourth, the model assumes heterogeneity both in consumers' objective risk levels and in their subjective beliefs. Finally, the model restricts firms to proper insurance contracts, thereby ruling out "betting" contracts that magnify the consumers' risk instead of shrinking it.

Consider a market that consists of many insurance firms and a continuum of consumers who are interested in insuring themselves against idiosyncratic wealth shocks. Each consumer has the same initial wealth level W. With some probability, he experiences an accident that causes a negative wealth shock of size d, such that his final wealth is $W - d$. Consumers are divided into low-risk and high-risk groups, for which the objective accident probabilities are $q^L > 0$ and $q^H > q^L$, respectively. A fraction of the high-risk consumers mistakenly think that they belong to the low-risk group. These consumers are referred to as *over-optimistic*. All other consumers have unbiased beliefs. The fractions of the three consumer types (low-risk, unbiased high-risk, and over-optimistic) in the population are λ_L, λ_H and λ_O.

An insurance contract is a pair $\alpha = (\alpha_1, \alpha_2)$, where $\alpha_1, \alpha_2 \geq 0$. The amount α_1 is the insurance premium, whereas $\alpha_1 + \alpha_2$ is the insurance coverage. The implicit price of the insurance contract α is $\alpha_1/(\alpha_1 + \alpha_2)$. When a consumer acquires the contract α, his net wealth is $W - \alpha_1$ when the accident does not occur, and $W - d + \alpha_2$ when the accident occurs. The option of acquiring no insurance corresponds to $\alpha = (0, 0)$. All consumers maximize expected utility over final wealth levels, and they all share the same vNM function u. Assume that u is twice differentiable, with $u' > 0$, $u'' < 0$, that is consumers are risk averse. A consumer whose subjective probability of an accident is $r \in \{q^L, q^H\}$ evaluates a contract α at

$$U(\alpha, r) = r \cdot u(W - d + \alpha_2) + (1 - r) \cdot u(W - \alpha_1)$$

Given that a consumer with an objective accident probability $q \in \{q^L, q^H\}$ chooses an insurance contract α, the profit that this contract generates for the insurance company is

$$\Pi(\alpha, q) = \alpha_1 - q \cdot (\alpha_1 + \alpha_2)$$

Let us turn to the notion of market equilibrium. Elsewhere in the book, we formalize models of market competition using game-theoretic tools. In this section, we depart from this modeling practice and instead adopt the Rothschild-Stiglitz definition of a competitive equilibrium in the insurance market model, because it is a textbook definition that should be familiar to most readers. We will say that a set of contracts A is a *competitive equilibrium* if it satisfies the following conditions: (i) every contract $\alpha \in A$ generates non-negative profits, given that each consumer chooses a contract from A to maximize his own subjective expected utility; (ii) every contract $\alpha \notin A$ generates non-positive profits, given that each consumer chooses a contract from $A \cup \{\alpha\}$ to maximize his own subjective expected utility.

5.4.1 Equilibrium Analysis When Subjective Beliefs Are Observable

As a benchmark for our equilibrium analysis, it will be useful to characterize competitive equilibria in a related model, in which firms perfectly identify the consumers' subjective assessment of an accident. Assume that the firms cannot distinguish between types L and O, since these types make the same choices between insurance contracts. Effectively, we analyze competitive equilibria for two separate markets: one for the group of (unbiased) high-risk consumers, denoted H, and another for the pooled group of low-risk and over-optimistic consumers, denoted LO. From the firms' perspective, the objective risk of group LO is

$$q^{LO} = \frac{\lambda_L q^L + \lambda_O q^H}{\lambda_L + \lambda_O}$$

which is strictly between q^L and q^H.

Competitive equilibrium contracts maximize consumers' expected utility subject to the zero-profit constraint. That is, for each group $i \in \{H, LO\}$, the equilibrium contract α^i maximizes $U(\alpha, r^i)$ subject to $\Pi(\alpha, q^i) = 0$, where $r^H = q^H$ and $r^{LO} = q^L$. The zero-profit condition itself implies $\alpha_1^i = q^i(\alpha_1^i + \alpha_2^i)$ for all consumer types i—that is, the insurance premium is equal to the expected payout. Assuming that first-orders condition characterize equilibrium contracts, the solution is

$$\frac{u'(W - \alpha_1^i)}{u'(W - d + \alpha_2^i)} = \frac{r^i}{1 - r^i} \cdot \frac{1 - q^i}{q^i}$$

for every group i, where the L.H.S. of this equation represents the slope of the indifference curve that runs through the state-contingent outcome $(W - \alpha_1^i, W - d + \alpha_2^i)$.

Since $q^H = r^H$, α^H offers full insurance, that is, $\alpha_1^H + \alpha_2^H = d$. In contrast, objective and subjective risks diverge for the pooled group LO: $r^{LO} = q^L < q^{LO}$. As a result, the equilibrium contract α^{LO} does not provide full insurance. In fact, if q^{LO} is sufficiently high relative to q^L, the first-order condition may fail to hold under the restriction $\alpha_1, \alpha_2 \geq 0$, such that the equilibrium contract is (0, 0). If we relaxed this restriction, the first-order condition would always characterize the equilibrium contract.[1]

In general, the betting motive due to the conflicting priors between firms and *LO* consumers is tempered by the insurance motive due to risk aversion. However, if the difference in beliefs is sufficiently large, the former force will outweigh the latter, and in equilibrium *LO* types will not be insured. It can be shown that this effect lowers the welfare of genuinely low-risk consumers.

5.4.2 Equilibrium Analysis When Subjective Beliefs Are Private Information

When consumers' subjective beliefs are their private information, firms face the familiar incentive problem that high-risk consumers may want to pretend to be in the low-risk group, in order to enjoy a lower insurance premium. Firms will design their menu of insurance contracts to screen the consumer's risk type. The equilibrium analysis shares many features with the conventional Rothschild-Stiglitz model. Therefore, I will only skim arguments that merely extend the basic model.

Result 1: If a competitive equilibrium exists, it must separate LO and H.

The reasoning is essentially the same as in the basic model without belief biases. Consider an equilibrium in which all consumer types choose the same contract. From the firms' point of view, the objective risk of the pooled population is $\bar{q} = \lambda_L q^L + (\lambda_O + \lambda_H) q^H$. Because of competitive forces, the equilibrium contract generates zero profits. Since $q^{LO} < \bar{q}$, a firm can offer a new contract that slightly lowers insurance coverage and premium at the same time, such that *LO* consumers alone will choose this contract, thus rendering it profitable.

Result 2: If a competitive equilibrium exists, the equilibrium set of contracts consists of two contracts, α^H and α^{LO}, such that α^H coincides with the complete-information benchmark and is selected by high-risk consumers only, whereas α^{LO} is given by

$$\alpha^{LO} \in \arg\max_{\alpha} U(\alpha, q^L)$$

[1] Note that ruling out "betting" contracts with $\alpha_1, \alpha_2 < 0$ is justified on moral-hazard grounds. Whether an accident has actually occurred is the consumer's private information, and the insurance contract needs to provide an incentive to report the accident.

subject to

$$\Pi(\alpha, q^{LO}) = 0$$
$$U(\alpha^H, q^H) \geq U(\alpha, q^H)$$

and is selected by low-risk and over-optimistic consumers only.

Again, the reasoning behind this result is conventional. As mentioned above, firms face an incentive problem, namely that high-risk consumers may want to pretend to belong to the low-risk group in order to get better insurance terms. Therefore, optimal screening of consumer types requires that high-risk consumers be fully insured, whereas *LO* consumers (whom firms treat as a single group) get the best insurance contract that generates non-negative profits and does not give high-risk consumers an incentive to deviate from their designated contract. As in the conventional Rothschild-Stiglitz model, competitive equilibrium may fail to exist. The reason is that, given the set of contracts $\{\alpha^{LO}, \alpha^H\}$ characterized by Result 2, a firm may be able to deviate to a contract that will attract all consumers and yet generate a strictly positive profit.

The following result describes a crucial difference between the present model and the conventional Rothschild-Stiglitz model.

Result 3: The incentive compatibility constraint of type H need not be binding in competitive equilibrium.

This is an effect that has surfaced repeatedly in our models of price discrimination according to consumers' prior beliefs. For a raw intuition in the present context, suppose that the set of contracts available in the market is the one characterized in the complete-information benchmark. In the conventional model, all consumers are fully insured, and high-risk consumers pay a higher premium, which gives them a stark incentive to pretend to be in the low-risk group. In contrast, in the present model, *LO* consumers are not fully insured in the complete-information benchmark. Therefore, it is not obvious that high-risk consumers have an incentive to pretend to be in the *LO* group.

For a particularly stark demonstration of the irrelevance of incentive compatibility—and the departures from the conventional Rothschild-Stiglitz model it implies—suppose that $q^{LO} \gg q^L$, such that in the complete-information benchmark model, $\alpha^{LO} = (0, 0)$; that is, low-risk and over-optimistic consumers are left totally uninsured. The high-risk consumers' incentive compatibility constraint is redundant, as it coincides with their participation constraint, and therefore high-risk consumers are offered the same contract as in the complete-information benchmark. In general, the equilibrium menu under private information mimics the complete-information benchmark when the fraction of over-optimists is sufficiently large.

Exercise 5.6. *Characterize competitive equilibrium when $\lambda_L = 0$; that is, all consumers have the same objective risk and they differ only in their subjective beliefs.*

Competitive equilibria in the present model have other features that depart from the conventional Rothschild-Stiglitz model. I list them briefly.

- Compulsory partial insurance can be Pareto improving in the conventional model, because it relaxes the high-risk consumers' incentive constraint and thus enables the market to offer greater coverage to low-risk types without violating the separation between types. This effect is irrelevant in the present model when type *H*'s incentive constraint is slack to begin with. In this case, this form of government intervention will not be Pareto-improving, because it lowers low-risk consumers' welfare.
- The structure of equilibrium contracts in the conventional model is entirely determined by the preferences of different consumer types, and therefore invariant to the composition of the consumer population. In contrast, in the present model, we saw that competitive equilibrium can replicate the complete-information outcome, where the contract offered to *LO* consumers is sensitive to the fraction of over-optimists.
- The relation between consumers' equilibrium choice of insurance coverage and their actual accident frequency is stark in the conventional model, where high-risk consumers choose full insurance coverage and low-risk consumers purchase partial insurance. As a result, an outside observer would see that consumers who bought full insurance tend to have a higher accident rate. (In reality, this correlation could also be due to moral hazard considerations that we have assumed away.) Biased beliefs blur this correlation. In particular, when $\lambda_L = 0$, all consumers have the same accident frequency, yet their different beliefs impel them to select different levels of coverage.

Comment: Risk attitudes vs. belief biases

Fix the set of insurance contracts that are available to consumers. If we observe that a consumer chooses low coverage, this can have several explanations. First, his risk aversion may be mild. Second, he may be over-optimistic. However, once we turn from individual behavior to analysis of market equilibrium, diminished risk aversion and greater over-overoptimism turn out to have different implications. In the conventional model with unbiased consumers, introducing high-risk consumers with lower risk aversion makes the incentive-compatibility constraint more stringent because these consumers have a stronger incentive to deviate to the lower-coverage contract aimed at low-risk consumers. This ultimately lowers the amount of insurance coverage that low-risk consumers get in equilibrium. In contrast, increasing the fraction of over-optimists in the present model raises q^{LO}. By the zero-profit condition, $\alpha_1^{LO} = q^{LO} \cdot (\alpha_1^{LO} + \alpha_2^{LO})$, hence the insurance price implicit in the contract aimed at *LO* consumers goes up. This relaxes the incentive-compatibility constraint because it lowers the high-risk consumers' incentive to deviate to the low-coverage contract. As a result, *LO* consumers get greater coverage in equilibrium. This is an interesting illustration of the way equilibrium analysis can expose qualitative distinctions that are absent from the analysis of individual behavior.

5.5 SUMMARY

Let us summarize the lessons we have gained from the collection of models studied in this chapter.

- When consumers have over-optimistic prior beliefs regarding their future preferences, a monopolist wants to offer an exploitative contract—that is, a contract that a consumer with unbiased beliefs would not accept. The quantity induced by the monopolist's optimal exploitative contract is efficient in the state the consumer deems less likely relative to the firm, but may be inefficient in the other state.
- Unlike the model with dynamically inconsistent consumers, unbiased consumers may exert an informational externality on consumers with biased beliefs.
- Pricing effects such as add-on pricing and three-part tariffs can be interpreted as consequences of plausible belief biases.
- In competitive insurance markets with over-optimistic consumers, incentive compatibility is not necessarily binding. In particular, consumers who believe that they belong to a low-risk group may remain uninsured in competitive equilibrium.

5.6 BIBLIOGRAPHIC NOTES

The model of monopolistic pricing with over-optimistic consumers is due to Eliaz & Spiegler (2008), where the unbiased-consumer benchmark builds on Armstrong (1996) and Courty & Li (2000). The discussion of three-part tariffs and over-confident beliefs is based on Grubb (2009), who also provides empirical evidence for this belief bias in the market for mobile phone services. Michael Grubb also provided useful suggestions for the proof of Proposition 5.2. The discussion of add-on pricing and consumer education relies on Gabaix & Laibson (2006). Uthemann (2005) studies competitive screening of consumers with non-common priors. The discussion of insurance markets with biased consumers is based on Sandroni & Squintani (2007). Spinnewijn (2009*a*) analyzes a related model that also introduces moral hazard concerns. Miravete (2003) evaluates the empirical relevance of biased consumer beliefs in telecommunication markets. The literature on contracting with agents who hold biased prior beliefs is not exclusively limited to industrial organization applications—see, for example, Landier & Thesmar (2009) on financial contracting and Spinnewijn (2009*b*) on unemployment insurance.

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PART TWO

Responding to Market Complexity

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6 Sampling-Based Reasoning I: Price Competition and Product Differentiation

In many market situations, consumers need to carry out a comparative evaluation of probability distributions. The prices of products offered by stores tend to fluctuate (special offers, sales, etc.). Experts extend services of varying quality, for exogenous reasons (the outcome of the service depends on a state of nature) as well as endogenous ones (the expert varies the effort he exerts). Evaluating insurance policies and financial products requires an assessment of the likelihood of many future contingencies. Being able to evaluate these stochastic variables requires probabilistic sophistication. In particular, the consumer needs to understand how to draw correct inferences from information regarding these variables.

In this chapter as well as the next one we examine the implications of market competition when consumers have limited ability to evaluate stochastic variables. We will focus on a particular way in which consumers cope with this limitation: examining carefully a small part of the environment and extrapolating naively from the sampled part. This mode of behavior is multiply determined psychologically. People tend to reason anecdotally, rather than probabilistically, about random variables. They often find anecdotes more persuasive and informative than they should be according to probabilistic thinking, because anecdotes are concrete stories filled with vivid details that register more powerfully in the decision maker's memory than abstract and general information. Another source of sampling-based reasoning is the phenomenon known as "the law of small numbers," namely people's tendency to exaggerate the informational content of a small sample.

As in Part I, we will first introduce a simple modeling tool that enables us to capture the aspect of the psychology of decision making in question—namely, sampling-based reasoning. Next, we will apply it to a sequence of market models that vary in the extent to which the randomness consumers face is endogenous. In each case, we will obtain an insight into the way in which consumers' anecdotal reasoning affects industrial organization.

6.1 A SAMPLING-BASED CHOICE PROCEDURE

Let Z be a set of outcomes. A consumer has a preference relation ▫ over Z. The consumer faces a profile of probability distributions over Z, $(F_1, \dots, F_n)$. He chooses a probability distribution according to the following sampling procedure. He draws a *single* sample point from the joint distribution $(F_1, \dots, F_n)$ and thus obtains a sample $(z_1, \dots, z_n)$. He chooses the alternative i^* for which z_{i^*}

is ◻-maximal in the sample (with a symmetric tie-breaking rule). The outcome of the consumer's choice is a new, independent draw from F_{i^*}.

The role of sampling in this model is fundamentally different from conventional I.O. models with consumer search. In both cases, the consumer chooses i^*, the best alternative in his sample. However, in a search model the consumer ends up getting z_{i^*}, whereas in the present model, the actual outcome is drawn anew from F_{i^*}.

The element of bounded rationality captured by the sampling-based procedure is the evaluation of stochastic variables on the basis of naive extrapolation from a small sample. As mentioned before, the psychological forces behind this type of reasoning are a tendency to be influenced by vivid anecdotal evidence more than by dry statistical data, and an exaggerated belief in the informativeness of small samples. One could argue that the reliance on small samples is not necessarily an aspect of intrinsic bounded rationality, but a feature of the consumers' objective constraints. In particular, they may lack market experience, and therefore their understanding of the market alternatives is limited to a collection of anecdotes. According to this interpretation, we are dealing less with inherent limited rationality and more with poor understanding of a given market due to lack of experience in that particular market. I find both interpretations equally legitimate.

A natural generalization of the sampling-based procedure—which we shall not analyze here—is to assume that each consumer draws K sample points from $(F_1, \ldots, F_n)$, and chooses the alternative with the best average performance in the sample. This generalization has the advantage that it quantifies the magnitude of the departure of consumer behavior from standard rationality. In particular, when $K \to \infty$, consumer behavior converges to the rational choice benchmark.

6.2 PRICE COMPETITION AND TECHNOLOGY ADOPTION

We begin with the simplest model of market competition over consumers who evaluate probability distributions on the basis of anecdotes. Imagine a market that consists of n identical firms and a continuum of identical consumers who enter the market with some need. The value of satisfying the need is 1 for all consumers. Each firm sells at zero cost a product that satisfies this need with independent probability $\alpha \in (0, 1)$. In addition to the n products traded in the market, consumers have the option of "not doing anything," labeled $i = 0$. This option, occasionally referred to as the consumers' outside option, satisfies their need with probability $\alpha_0 \in [0, 1)$. The existence of an outside option will be relevant for some of our welfare analyses. We do not impose any relation between α_0 and α. Firms are standard profit maximizers. They compete by choosing prices simultaneously. Let $p_i \in [0, 1]$ denote the price chosen by firm i. Assume that $p_0 = 0$; that is, the price of "doing nothing" is zero.

The case of $\alpha = \alpha_0$ will have special importance in this section. In this case, firms have no skills relative to the outside option, and therefore it is apt to refer to them as "quacks." There are several real-life situations that fit this special case. Actively managed mutual funds are a case in point. According to the *Efficient Market Hypothesis*, prices in financial markets fully reveal private information. Consequently, an actively managed mutual fund cannot generate (risk-adjusted) returns in excess of the market portfolio. Thus, under the Efficient Market Hypothesis, the market for actively managed mutual funds is a "market for quacks." And of course, as the term "quacks" indicates, practitioners of non-scientific medicine often fall into this category.[1]

Suppose that consumers choose rationally given a correct understanding of the model. If $\alpha_0 \geq \alpha$, consumers recognize that firms have no skills relative to the outside option, and so the market is inactive. When $\alpha_0 < \alpha$, the model is reduced to standard Bertrand competition, because the firms are selling a homogeneous product. Consumers pay a zero price in Nash equilibrium and firms earn zero profits.

Instead, let us assume in this section that consumers choose according to the sampling-based procedure, reflecting an "anecdotal" perception of their stochastic market environment. That is, each consumer independently samples each of the $n+1$ market alternatives (including the outside option) and selects the alternative i that maximizes $x_i - p_i$ in the sample, where $x_i = 1$ (0) if alternative i satisfies (fails to satisfy) the consumer's need in the sample. Ties are resolved symmetrically between firms and in favor of the outside option.

The consumers' choice procedure thus induces a complete-information, simultaneous-move game played by the firms. As in the competitive models of Part I, firms are standard, profit-maximizing agents, and therefore Nash equilibrium is a reasonable solution concept to apply to this game. To illustrate the firms' payoff function in this game, suppose that all market alternatives have different prices, and order them as follows: $p_n > p_{n-1} > \cdots > p_1 > p_0 = 0$. Then, firm k's profit is

$$p_k \cdot \alpha \cdot (1-\alpha_0) \cdot (1-\alpha)^{k-1}$$

The reason is that firm k's clientele consists of all the consumers who heard a good anecdote about it and a bad anecdote about all cheaper alternatives. Note that this means that firms are able to secure a strictly positive profit. The max-min payoff is $\alpha \cdot (1-\alpha)^{n-1}(1-\alpha_0)$.

Comment: The interpretation of n

The most straightforward interpretation of n is the number of firms that are physically present in the market. However, one could view n as the number of market alternatives the consumer happens to consider, through an unmodeled process of gathering anecdotes. According to this interpretation, the number of firms in the market is arbitrarily large, and a higher n means that it is easier for the consumer to generate anecdotes about market alternatives (e.g., because of a richer social network that facilitates word-of-mouth learning).

[1] However, it is sometimes argued that alternative medicine is better than conventional medicine at generating a Placebo Effect, precisely because of its faith-based, non-scientific style.

Comment: The interpretation of mixed strategies

We assume that firms have access to mixed pricing strategies. Mixed-strategy Nash equilibrium has received multiple interpretations in the context of price competition models. The conventional interpretation is based on the view of the set of n firms as fixed, such that a mixed strategy is a description of how the firm varies its price over time. Under this interpretation, mixed-strategy equilibrium describes firms' use of sales and temporary price changes.

An alternative interpretation that pertains to *symmetric* mixed-strategy equilibrium is based on the alternative view of n described in the previous paragraph: the n firms are randomly drawn from a large population following an unmodeled search process; each firm plays a deterministic pricing strategy. A symmetric mixed-strategy Nash equilibrium represents an equilibrium distribution of market prices. When a firm makes its pricing decision, it knows that the consumer will compare it to $n-1$ other firms drawn from the same market distribution, as well as the outside option. This interpretation turns symmetric mixed-strategy equilibrium into a model of *price dispersion*. It is difficult to extend this interpretation to asymmetric equilibria.

Under both interpretations, once a price p_i has been realized, firm i is committed to it as far as consumers are concerned. They know the exact prices; the only source of variation in their sample is the imperfect success rates of alternatives, which are exogenously given. A mixed strategy inflicts uncertainty on the firm's opponents, not on the consumers. Thus, the randomness faced by consumers is purely exogenous.

6.2.1 Nash Equilibrium

Let us now characterize symmetric Nash equilibrium in the game.

Proposition 6.1. *In symmetric Nash equilibrium, firms play the mixed strategy given by the cdf:*

$$G(p) = \frac{1}{\alpha} - \frac{1-\alpha}{\alpha} \cdot p^{-1/(n-1)} \tag{6.1}$$

defined over the support $[(1-\alpha)^{n-1}, 1]$.

***Proof*.** The firms' equilibrium strategy induces a *cdf* G defined over an interval $[p^L, p^H]$. Because the max-min payoff is strictly positive, the Nash equilibrium payoff must be strictly positive as well. In mixed-strategy Nash equilibrium, each action in the support of a firm's equilibrium strategy is a best-reply. By continuity arguments, the same holds for every action in the closure of the support. This implies that the infimum of the support of G must be strictly positive. If G has an atom at some price $p \in (0, 1]$, then it is profitable for any firm to deviate by shifting this atom to a slightly lower price. Thus, G is continuous. Finally, if G is flat over some interval (p, p')—where p belongs to the support of G—then p is not a best-reply, because any price $p'' \in (p, p')$ would generate the same clientele. This means in particular that $p^H = 1$. It follows that G is a strictly increasing, continuous function over the interval $[p^L, 1]$, where $p^L > 0$.

It remains to determine p^L and the exact expression for G. Since $p^H = 1$, the firms' equilibrium payoff is $\alpha(1-\alpha)^{n-1}(1-\alpha_0)$. Therefore, this is also the payoff generated from every price in the support of the equilibrium strategy. It follows that for every $p \in [p^L, 1]$:

$$\alpha(1-\alpha)^{n-1}(1-\alpha_0) = p \cdot \alpha \cdot [1 - \alpha G(p)]^{n-1} \cdot (1-\alpha_0)$$

since the market share of a firm that charges p is equal to the probability that a consumer gathers a good anecdote about the firm and a bad anecdote about all cheaper alternatives. The value of p^L follows from the identity $G(p^L) = 0$, and the expression for G is immediately given by this equation. □

Given the formula (6.1) for $G(p)$, it is easy to calculate expected equilibrium price:

$$E(p) = \begin{cases} -\frac{1-\alpha}{\alpha}\ln(1-\alpha) & \text{for } n = 2 \\ \frac{1-\alpha}{\alpha(n-2)}[1-(1-\alpha)^{n-2}] & \text{for} n > 2 \end{cases}$$

It follows that the firms' expected equilibrium price is strictly decreasing with α. In particular, $E(p) \to 0$ as $\alpha \to 1$ and $E(p) \to 1$ as $\alpha \to 0$. Note that the characterization of the firms' equilibrium pricing strategy is independent of the value of α_0.

When $\alpha_0 \geq \alpha$, we have an active "market for quacks." Firms charge a positive price for a product that has no value relative to the outside option. The reason they are able to earn a positive profit is that consumers' anecdotal reasoning causes them to attribute to skill a good outcome that is due to sheer luck. Expected price increases as α decreases—that is, as the success rate that characterizes the industry goes down. The intuition for the comparative statics is simple. When α is lower, a consumer's sample is less likely to contain multiple successes. This weakens competitive pressures and causes prices to go up. In the limit $\alpha \to 0$, the market tends to a state of *monopolistic competition*: the price level is close to the monopoly price $p = 1$, and industry profits are close to zero.

Comment: Reinterpreting the model in terms of rational choice

A recurring theme in this book is the confrontation between market models based on consumers' bounded rationality and more conventional, rationality-based models. In particular, a common question is whether one could replicate the predictions of a bounded-rationality model with a standard model. In the case of the market model of this chapter, the answer is affirmative. In fact, the model can be entirely reformulated conventionally. Under this alternative formulation, consumers are rational with quasi-linear utility $u(z) - p_z$, where $z \in \{0, 1, \ldots, n\}$, and $u(z)$ gets the values 1 or 0, with probabilities α and $1 - \alpha$, independently across alternatives and consumers. This formulation is behaviorally equivalent to our sampling-based model.

It follows that consumers who evaluate alternatives according to the sampling-based procedure behave as if they are rational with private, independent values. Of course, for welfare analysis the two formulations are radically different. Under the sampling-based model, when $\alpha_0 \geq \alpha$, whenever the consumer chooses a firm that charges a positive price over the outside option, he is making a choice error. In contrast, in the reformulated model, he simply exhibits an idiosyncratic preference for the firm's product.

What should we make of this rationalization? Despite economists' traditional preference for rationality-based explanations of human behavior, assessing the plausibility of this explanation in the present case depends on the context. For example, if products are treatments designed to solve a medical problem, or investment strategies proposed by money managers, then the independent, private values assumption seems highly implausible: in these contexts consumers obviously care mostly about product quality, and they share essentially the same definition of quality. It is more convincing to assume that the appearance of private, independent values is due to consumers' inference errors rather than a reflection of truly idiosyncratic tastes.

We will revisit this discussion in Chapter 13. At any rate, the observation that sampling-based reasoning leads consumers to behave as if they have heterogeneous tastes will enable us to generate novel insights in the sequel.

6.2.2 Welfare Analysis

As in previous chapters that dealt with consumers' misperception of their market environment, welfare has to be carefully defined because consumers' choice behavior does not reveal their preferences over actual outcomes. Let us first examine the case of a "market for quacks"–that is, $\alpha = \alpha_0$. In this case, since all market alternatives have the same success rate, it makes sense to view industry profits, equal to $n\alpha(1 - \alpha)^n$, as a measure of the welfare loss that the industry inflicts on consumers. Thus, the very existence of an active market for quacks inflicts a welfare loss on consumers. The welfare loss is hump-shaped with respect to n. Thus, increasing the number of competitors does not monotonically curb the consumers' welfare loss.

The intuition behind this observation is straightforward. On one hand, a greater number of firms increases the incentive to cut prices. This is a standard "competitive" effect. On the other hand, a greater number of market alternatives increases demand for the industry as a whole, because there is a higher chance of hearing a good anecdote about some product. This is an "exploitative" effect. Fixing α, the competitive effect outweighs the exploitative effect when n is sufficiently large (but the critical value of n increases as α decreases), and the welfare loss vanishes completely as $n \to \infty$.

In fact, the competitive and exploitative effects can be neatly separated. Recall that a firm's max-min payoff is $\alpha(1 - \alpha)^{n-1}(1 - \alpha_0)$. Yet this is also the payoff that each firm earns in Nash equilibrium. Thus, the exploitative effect finds expression in the strictly positive max-min payoff, while the competitive effect prevents equilibrium payoffs from rising above the max-min level.

Let us turn to the case of $\alpha_0 < \alpha$. When consumers are rational, their expected utility in Nash equilibrium is α. By comparison, "our" consumers' equilibrium expected utility is given by the following expression:

$$\alpha \cdot A + \alpha_0 \cdot (1 - A) - n\alpha(1 - \alpha)^{n-1}(1 - \alpha_0)$$

where

$$A = (1 - \alpha_0) \cdot [1 - (1 - \alpha)^n]$$

is the probability that a consumer ends up choosing a firm. Note that if consumers did not enter the market, their expected utility would be α_0. It is easy to see that if both α_0 and α are sufficiently close to zero, consumer welfare can fall below α_0—that is, consumers experience market exploitation.

We already saw that the most conventional competition policy—namely, increasing the number of market alternatives n available to the consumer—may have an adverse impact on consumer welfare, because it may raise aggregate demand for an industry consisting of "quacks."

Another competition policy is to introduce a high-quality competitor into the market. If consumers were rational and understood the model, they would all switch to the high-quality firm, hence the equilibrium outcome would be efficient. However, as the following exercise shows, when consumers follow the sampling-based procedure, merely adding a single high-quality competitor does not eliminate the problem of "active quacks."

Exercise 6.1. *Let $n = 2$. Modify the model by letting the success rate of firm 2 be $\alpha_2 > \alpha = \alpha_1 = \alpha_0$.*

- *(i) Derive the firms' Nash equilibrium profits. Show that firm 1's profit is independent of α_2. (Hint: When the two firms have different success rates, their equilibrium pricing strategies have the same support, but firm 2's cdf has an atom on $p = 1$.)*
- *(ii) Suppose that in a stage prior to the price competition game, firms choose their success rates simultaneously and at no cost. Which profiles of success rates are consistent with sub-game perfect equilibrium in this two-stage game?*

Thus, market interventions that are viewed as proper competition policies in a world with rational consumers are ineffective when consumers behave according to the sampling-based procedure. Without lifting consumers' reasoning above the anecdotal level, it is hard to correct the market failure that their bounded rationality creates.

6.3 SPURIOUS PRODUCT DIFFERENTIATION

Economists traditionally view product differentiation as an industry's response to preference heterogeneity among consumers. However, when consumers systematically misperceive the value of the products they face in the market, we may observe *spurious product differentiation*, as firms have an incentive to create an *impression* of a differentiated product in order to weaken competitive pressures. The sampling-based procedure provides a good opportunity to examine this intuition because, as we already saw, consumers who follow this procedure behave as if they have diverse private values.

Let A be a finite set of *actions* that firms may recommend to consumers. Denote $|A| = m$ and assume $m > 1$. Each action $a \in A$ satisfies the consumer's need with probability $\frac{1}{m}$. The actions' successes are mutually exclusive: conditional on a satisfying the consumer's need, the probability that any $a' \neq a$ satisfies it is zero. The consumer's payoff when his need is (is not) satisfied is 1 (0).

This description fits situations in which the firm's recommended action is a *prediction* of some future outcome (e.g., the identity of the winner in a contest). In this case, A is the set of possible future outcomes, and the consumer's need is satisfied if and only if the prediction he adopts is correct. Assume that when a consumer acquires the services of a firm, he automatically adopts its recommended action. Firms have no information beyond this description.

As in the previous section, the market consists of n identical firms and a continuum of identical consumers. A pure strategy for firm $i \in \{1, \ldots, n\}$ is a pair (p_i, a_i), where $p_i \in [0, 1]$ is the price the firm charges and a_i is the action it recommends. Thus, each firm has a success rate of $\frac{1}{m}$, regardless of its choice of action. For simplicity, assume that consumers have an outside option of known value 0.[2]

Consumers choose a firm according to the sampling-based procedure. In the present context, the following is a consistent story behind this procedure. Each consumer recalls a random past episode in which firms made predictions, and isolates those firms whose predictions proved correct in that episode (if none of the firms was successful, the consumer sticks to the outside option). Among those firms, the consumer selects the firm that charges the lowest price.

Formally, each consumer independently draws an action $a \in A$ from the uniform distribution (this action is interpreted as the recommendation that turned out to be correct in the randomly recalled past episode), and chooses the alternative $i \in \{0, 1, \ldots, n\}$ that maximizes $x_i - p_i$ in the sample, where $x_0 = p_0 = 0$, and for each firm $i = 1, \ldots, n$, $x_i = 1$ (0) if the firm recommends (does not recommend) the action a. Assume a symmetric tie-breaking rule. Thus, when all firms charge distinct prices, each firm's market share is $\frac{1}{m}$ multiplied by the probability that every opponent that shares the firm's recommendation charges a higher price.

[2] The analysis below is easily extendible to the case in which the outside option is (p_0, a_0), where $p_0 = 0$ and a_0 is drawn from the uniform distribution over A. According to the forecasting interpretation, the outside option corresponds to a "lay prediction."

6.3.1 Nash Equilibrium

Product differentiation in this model manifests itself in a firm making a distinct recommendation from its opponents. Such differentiation is spurious because the firms' recommendations do not matter for consumer welfare—only the prices that they charge do. There is a clear differentiation motive in this model. If all firms recommend the same action, the most expensive firm among them is never chosen, and would therefore benefit from making a different recommendation, thus securing a probability $\frac{1}{m}$ of being chosen. The structure of Nash equilibria in this model reflects this motive.

When $n \leq m$, the model has asymmetric, pure-strategy Nash equilibria in which each firm recommends a distinct action and charges the monopoly price $p = 1$. To see why this is an equilibrium configuration, observe that each firm has a market share of $\frac{1}{m}$. But the market share of an individual firm can never exceed $\frac{1}{m}$, because this is the probability that the action it recommends is successful in the consumer's sample. Since each firm charges the monopoly price $p = 1$, it is clear that no deviation could be profitable. Industry profits in these equilibria are equal to $\frac{n}{m}$.

When $n > m$, these full-differentiation pure-strategy equilibria do not exist, because it is impossible for each firm to make a distinct recommendation. In fact, when $2m > n > m$, no pure-strategy equilibrium exists. When $n \geq 2m$, there are pure-strategy Nash equilibria in which for each action a there are at least two firms that charge $p = 0$ and recommend a. All firms earn zero profits in these equilibria. No other pure-strategy equilibria exist.

Turning to mixed-strategy equilibria, the following symmetric equilibrium holds for every $n \geq 2$. Each firm recommends each action with probability $\frac{1}{m}$, and randomizes independently over prices according to the *cdf* given by expression (6.1), where α is substituted by $\frac{1}{m}$. There is product differentiation ex post—the probability that all firms make the same recommendation is $(\frac{1}{m})^{n-1}$—yet there is no differentiation ex ante because all firms play the same mixed strategy. Industry profits in this equilibrium are given by the expression

$$n \cdot \frac{1}{m} \cdot (1 - \frac{1}{m})^{n-1}$$

To see why this is an equilibrium, note that when an individual firm considers its recommendation, it realizes that its opponents mix over A uniformly and independently of their price. Therefore, the firm is indifferent among all actions. Mixing uniformly over A is as good as any other recommendation strategy. As to the firms' pricing decisions, recall that in the basic model of the previous section, we assumed that each firm has an independent success rate of α. In comparison, in the present context the firms' success probabilities are independent because they randomize uniformly and independently over A. Thus, the equilibrium pricing strategy is exactly the same as in the basic model, except that $\frac{1}{m}$ replaces α. When m increases, expected equilibrium price in the symmetric equilibrium approaches the monopoly level $p = 1$, whereas industry profits behave non-monotonically in m. In terms of the forecasting metaphor invoked earlier, the interpretation is that as the future becomes harder to predict, professional forecasters charge a higher price and industry profits may be higher.

The following exercise illustrates that there may exist asymmetric mixed-strategy equilibria as well.

Exercise 6.2. *Construct an asymmetric mixed-strategy Nash equilibrium when $n > m$ and both n and m are even.*

The next exercise illustrates that when we relax the assumption that all actions have the same success rate, some firms make inferior recommendations as a differentiation strategy (an effect reminiscent of the "favorite long-shot bias" observed in gambling markets—namely, the market's tendency to over-bet on low-probability outcomes).

Exercise 6.3. *Let $A = \{a_1, a_2\}$. Assume that with probability α $(1 - \alpha)$ the action a_1 (a_2) alone satisfies the consumer's need. Assume $\alpha > \frac{1}{2}$. Consider a symmetric Nash equilibrium in which firms randomize over prices and recommendations independently. What is the equilibrium probability that the sub-optimal action a_2 is recommended? What happens to this probability as n tends to infinity?*

6.3.2 Product Complexity as a Differentiation Device

In the product differentiation model we analyzed so far, firms make unambiguous recommendations. In reality, experts who offer forecasts, medical treatments, self-help advice, or money management services tend to condition their recommendation on the specifics of the situation that the consumer confronts. Furthermore, in many situations, these recommendations tend to vary across experts, and there seems to be very little consensus among experts regarding the best course of action in a given case. In this section I offer a way to interpret these observations. As the reader may have guessed, case-specific recommendations can be viewed as a means of spurious product differentiation when consumers follow the sampling-based procedure.

As in the model that opened this section, assume that A is a set of actions that firms can recommend. However, let us enrich the model by introducing a set of *cases* C. Let $t : C \to A$ be a case-specific recommendation (CSR henceforth). Each firm simultaneously chooses a pair (p_i, t_i), where p_i is the firms' price and t_i is its CSR. We will say that firm i makes an exclusive recommendation in case c if $t_j(c) \neq t_i(c)$ for all $j \neq i$.

A *state* is specified by a case and the (unique) action that satisfies the consumers' need in that case. The state (c, a) is drawn according to uniform distribution over $C \times A$. As before, each consumer evaluates a satisfied need by 1. A firm's CSR is successful in state (c, a) if and only if $t(c) = a$. Thus, when a consumer chooses firm i, his payoff in state (c, a) is $1 - p_i$ if $t_i(c) = a$ and $-p_i$ if $t_i(c) \neq a$. Consumers choose a firm according to the sampling-based procedure. That is, each consumer independently samples a state and chooses the best-performing alternative in that state. Let us retain the assumption that consumers have an outside option of known value 0. Since all actions are equally likely to satisfy the consumers' need in each case, case-specific recommendations involve a complexity, which is redundant in terms of consumer welfare.

To illustrate the firms' payoff function in this game, let C be finite, and suppose that the strategy profile $(p_i, t_i)_{i=1,\dots,n}$ has the property that all firms charge different prices. Order the firms as follows: $p_n > p_{n-1} > \cdots > p_1$. Define $z_k(c) = 1$ if $t_j(c) \neq t_k(c)$ for all firms $j < k$, and $z_k(c) = 0$ otherwise. Then, firm k's payoff is:

$$\frac{p_k}{m \cdot |C|} \sum_{c \in C} z_k(c)$$

The reason is that firm k's clientele consists of all the consumers who sampled a state in which the firm's recommended action is successful and different from the recommended actions of all cheaper firms.

Suppose that all firms play degenerate CSRs. That is, for each firm i there is an action a_i such that $t_i(c) = a_i$ for all c. Then, the present model is immediately reduced to the simpler model of the previous sub-section. Therefore, all pure-strategy equilibria identified in the previous sub-section survive the current extension.

When $m < n < 2m$, the extended model with case-specific recommendations gives rise to a new class of Nash equilibria, in which each firm's strategy consists of a pure, non-degenerate CSR and a mixed pricing strategy. I refer to such equilibria as *hybrid*. For instance, let $C = [0, 1]$. We can construct hybrid equilibria with the following structure:

- For each firm, the fraction of cases in which it makes an exclusive recommendation is

$$\mu = \frac{2m - n}{n}$$

- In each case, $2m - n$ actions are recommended by one firm each, and the remaining $n - m$ actions are recommended by two firms each.[3]
- All firms play the pricing strategy given by the *cdf*

$$G(p) = \frac{p - \mu}{p - p\mu} \tag{6.2}$$

  over the support $[\mu, 1]$.

To see why this configuration is consistent with equilibrium, note first that no firm has any incentive to change its recommendation in any case, for the

[3] We can construct such a profile of CSRs as follows. Partition C into $\binom{n}{2m-n}$ equal intervals. Associate a distinct subset of $2m - n$ firms with each interval, and assume that these firms make exclusive recommendations in all cases that belong to the interval.

following reason. Prior to the deviation, the firm shares a recommendation with at most one other firm. After the deviation, it shares a recommendation with one or two other firms. Since all actions have the same success rate and since all firms play the same pricing strategy, the deviation is unprofitable.

As to the firms' pricing strategies, observe that for every price $p \in [\mu, 1]$, a firm's profit is

$$p \cdot \frac{1}{m} \cdot [\mu + (1-\mu)(1-G(p))]$$

because with probability μ, the firm makes an exclusive recommendation in the case the consumer samples, while with probability $1-\mu$ it shares its recommendation with exactly one firm (playing the pricing strategy G) in the case the consumer samples. Since $G(1) = 1$, the firm's equilibrium payoff is $\frac{\mu}{m}$, and this immediately yields formula (6.2).

Industry profits in this class of hybrid equilibria are equal to

$$\frac{2m-n}{m}$$

When n is relatively close to m, industry profits exceed the level attained in the symmetric mixed-strategy equilibrium discussed above. Thus, the ability to make case-specific recommendations allows firms to attain a degree of spurious product differentiation that they would be unable to achieve otherwise. The intuition is that when n is close to m, the multiple cases function as "sunspots" that allow firms to coordinate their recommendations in a way that increases the probability that each firm makes an exclusive recommendation, relative to the mixed-strategy equilibrium in which firms mix over actions independently of each other and of the case. As far as an outside observer is concerned, firms' recommendations in each case are "maximally differentiated," in the sense that they are as diffuse and unconcentrated as possible. Moreover, the recommendations are overly complex, in the sense that they are extremely case-sensitive without contributing to consumer welfare. This matches the observations made at the beginning of this sub-section.

It turns out that the maximal differentiation of recommendations displayed by the above equilibria is a general feature of hybrid equilibria when $m < n < 2m$. For simplicity, I prove the result under certain restrictions on the firms' equilibrium pricing strategies.

Proposition 6.2. *Let $m < n < 2m$ and suppose that C is finite. Consider a hybrid Nash equilibrium in which the firms' pricing strategies are continuous and the intersection of their supports is of positive measure. Then, in each case, exactly $2m-n$ actions are recommended by one firm each, and each of the remaining $n-m$ actions is recommended by two firms.*[4]

[4] An analogous result can be obtained when C is a continuum, except that the claim should be restated to hold for all cases except possibly in a measure-zero set.

***Proof*.** Consider a hybrid Nash equilibrium, in which the profile of CSRs is $\mathbf{t} = (t_i)_{i=1,\dots,n}$. For each firm i, let p_i^H denote the supremum of its equilibrium *cdf* over prices. Let $e(c, a)$ denote the number of firms that recommend action a in case c under the profile $\mathbf{t}$. Since $m < n < 2m$, for every $c \in C$ there exist $a, a' \in A$ such that $e(c, a) < 2$ and $e(c, a') > 1$. Suppose that $e(c, a) = 0$ for some state (c, a). Consider an action a' for which $e(c, a') > 1$. A firm i that recommends a' in c under $\mathbf{t}$ can profitably deviate to the pure strategy (p_i^H, t_i'), where $t_i'(c) = a$ and $t_i'(c') = t_i(c')$ for every $c' \neq c$. Thus, $e(c, a) > 0$ for every (c, a). This immediately proves the result for $n = 3$. From now on, assume $n > 3$. Let B_i denote the set of cases c for which firm i makes an exclusive recommendation—that is, $t_i(c) \neq t_j(c)$ for all $j \neq i$. The proof proceeds in two steps.

Step 1: $B_i \neq \varnothing$ for all i.

Proof. Because $n < 2m$, for every case c there is a firm i that makes an exclusive recommendation in c. Therefore, firm i necessarily makes positive profits. Suppose that some firm $j \neq i$ earns zero profits. Then, j can deviate to (p, t_i), where $p > 0$ belongs to the support of firm i's *cdf* over prices and yet $p < p_i^H$. Because this strategy mimics firm i's CSR and at the same time ensures that j is cheaper than i with positive probability, it generates a positive profit. It follows that all firms earn positive profits in equilibrium.

Suppose that $B_i = \varnothing$ for some firm i. Order the firms as follows: $p_1^H \geq \cdots \geq p_n^H$. (In case of ties, the ordering is arbitrary.) Note that $B_1 \neq \varnothing$; otherwise, firm 1 earns zero profits, a contradiction. Define firm k as follows: $B_k = \varnothing$ and $B_i \neq \varnothing$ for all $i < k$. Since firm k earns positive profits, it must be the case that $p_k^H < p_1^H$. Let $c \in B_1$. By the definition of k, there exists $j \neq k$ such that $t_j(c) = t_k(c)$. Consider a deviation by firm $i^* = \min(j, k)$ to a pure strategy consisting of the price $p_{i^*}^H$ and a CSR t_{i^*}' satisfying $t_{i^*}'(c) = t_1(c)$ and $t_{i^*}'(c') = t_{i^*}(c')$ for all $c' \neq c$. Compare the strategy $(p_{i^*}^H, t_{i^*}')$ with the pure strategy $(p_{i^*}^H, t_{i^*})$, which belongs to the support of player i's equilibrium strategy and is therefore a best-reply. The two strategies yield the same expected payoff whenever consumers sample a case $c' \neq c$. However, when a consumer samples the case c, the probability that he ends up choosing firm i^* is strictly higher under $(p_{i^*}^H, t_{i^*}')$. It follows that the deviation is profitable.

Step 2: $e(c, a) \in \{1, 2\}$ for all (c, a).

Proof. We have already shown that $e(c, a) > 0$ for all c, a. For every c, let $b(c)$ denote the number of actions a for which $e(c, a) = 1$. By definition, $b(c) \geq 2m - n$ for all c. If $b(c) = 2m - n$ for all c, the proof is complete. If $b(c) > 2m - n$, then there must be an action a such that $e(c, a) > 2$. Define $b^* = \max_{c \in C} b(c)$. Let c^* be a case satisfying $b(c^*) = b^*$, and let a be an action such that $e(c^*, a) > 2$. Let i, j, k be three distinct firms such that $t_i(c^*) = t_j(c^*) = t_k(c^*) = a$. Let l be a firm that recommends a distinct action from all other firms in c^*. By assumption, there exists a price p that belongs to the supports of all firms' equilibrium pricing strategies. If $G_l(p) \leq G_i(p)$, then j can profitably deviate to a pure strategy (p, t_j') satisfying $t_j'(c^*) = t_l(c^*)$ and $t_j'(c) = t_j(c)$ for every $c \neq c^*$. It follows that $G_l(p) >$

$G_i(p)$. By Step 1, $B_i \neq \varnothing$. Consider a case $c^{**} \in B_i$. Suppose that $c^{**} \notin B_l$—that is, there exists a firm g such that $t_g(c^{**}) = t_l(c^{**})$. We have shown that $G_l(p) > G_i(p)$. Firm g can profitably deviate to a pure strategy (p, t'_g) satisfying $t'_g(c^{**}) = t_i(c^{**})$ and $t'_g(c) = t_g(c)$ for every $c \neq c^{**}$. Therefore, $c^{**} \in B_l$. We have established that $B_i \subset B_l$. But this holds for *any* firm l among the b^* firms that make exclusive recommendations in c^*. Therefore, $b(c^{**}) > b^*$, contradicting the definition of b^*.

Step 2 immediately implies that in each case, exactly $2m - n$ actions are recommended by one firm each, and each of the remaining $n - m$ actions is recommended by two firms. This completes the proof. □

The lesson from this result is that when firms in "soft expertise" industries give highly differentiated and case-specific recommendations, this may be interpreted as a consequence of their response to consumers' sampling-based reasoning.

6.4 CAN THE MARKET EDUCATE CONSUMERS?

Consumers who follow sampling-based reasoning make a systematic inference error: they draw exaggerated conclusions from anecdotal evidence. As in some of the models examined in Part I of this book, this raises the question of whether market forces alone can provide firms with an incentive to "de-bias" the consumers and correct their error. In this section, we revisit the basic model of Section 6.2, and allow firms to disclose their success rates to consumers. In addition, relax the assumption that all firms have the same success rate, and allow α_i to take any value in (0, 1) for each market alternative $i = 0, 1, \ldots, n$. Assume that a firm's disclosure of its success rate is credible.

How shall we adapt our model of consumer behavior to this extended environment? Since disclosure is credible, assume that when firm i discloses its success rate α_i and charges the price p_i, all consumers evaluate this market alternative at $\alpha_i - p_i$. The crucial question is how consumers evaluate a firm when it does *not* disclose its success rate. We will make a strong assumption: in this scenario, consumers infer nothing from the lack of disclosure, and rely on the sampling-based procedure to evaluate the firm.

Formally, the n firms in this extended model play the following simultaneous-move, complete-information game. A strategy for firm i is a pair (p_i, r_i), where $r_i = Y$ (N) means that the firm discloses (does not disclose) its success rate. As in the basic model, x_i denotes an individual consumer's evaluation of the quality of firm i's product. When $r_i = Y$, $x_i = \alpha_i$ with probability one. When $r_i = N$, $x_i = 1$ with probability α_i and $x_i = 0$ with probability $1 - \alpha_i$. As before, the consumer chooses the alternative that maximizes $x_i - p_i$ in his sample.

Before this section, the sampling-based procedure captured a particular aspect of bounded rationality, namely the tendency to reason anecdotally about stochastic variables and draw strong inferences from anecdotal evidence. However, in the extended model with disclosure, the adapted procedure attains an additional meaning. The assumption that consumers continue to rely on the procedure when there is no disclosure implies that they are *strategically*, as well as statistically, naive. If they were strategically sophisticated, they would realize that a firm's decision not to disclose its success rate is a bad signal. Indeed, this would be a correct inference in sequential equilibrium of an appropriately defined game with imperfectly informed consumers. The equilibrium outcome in our model is different.

Proposition 6.3. *For every p, the strategy (p, Y) for firm i is weakly dominated by some other strategy (p', N).*

***Proof*.** Suppose that firm i plays the strategy (p, Y). Denote $\alpha_i = \alpha$, for notational convenience. I consider two cases.

First, suppose that $p \geq \alpha$. In this case, $x_i - p \leq 0$ with probability one in the consumer's sample. As a result, the consumer will never pick alternative i over the outside option, hence firm i earns zero profits. If the firm deviates to (p', N), where $p' \in (0, 1)$, then $x_i - p' > 0$ with probability $\alpha > 0$. Therefore, the firm earns strictly positive profits for some strategy profiles of its opponents. Of course, it never earns negative profits. It follows that (p', N) weakly dominates (p, Y).

Second, suppose that $p < \alpha$. In this case, firm i's payoff from the strategy (p, Y) is bounded from above by:

$$p \cdot \Pi_{j \neq i} \Pr(x_j - p_j \leq \alpha - p)$$

In contrast, when the firm adopts the strategy (p', N), its payoff is bounded from below by:

$$p' \cdot \alpha \cdot \Pi_{j \neq i} \Pr(x_j - p_j < 1 - p')$$

(The reason that these two expressions provide bounds on firm i's profits is the possibility of ties with other firms.) Let $p' = p/\alpha$. We now show that (p, Y) is weakly dominated by (p', N). Note that $p' \in (p, 1)$ and $\alpha - p < 1 - p'$. Therefore:

$$\Pi_{j \neq i} \Pr(x_j - p_j < 1 - p') \geq \Pi_{j \neq i} \Pr(x_j - p_j \leq \alpha - p)$$

This inequality is strict if $p_j \in (p', 1 - \alpha + p)$ for some $j \neq i$. It follows that (p', N) weakly dominates (p, Y). □

While this result demonstrates that "educating consumers" is a weakly dominated strategy, it can be shown that disclosure is impossible in Nash equilibrium, too. The lesson from this result is that "market education" will not take place when consumers use the sampling-based procedure to evaluate market alternative, and when in addition they do not infer anything from lack of disclosure. Compare this result to the classic treatment of market disclosure in Milgrom & Roberts (1986). When consumers are strategically naive yet probabilistically sophisticated (in the sense of drawing Bayesian inferences from whatever data they have), Milgrom and Roberts show that full disclosure emerges in equilibrium, thanks to competitive forces. Similarly, as we already noted, the same result holds when consumers are probabilistically naive, yet strategically sophisticated. However, we cannot rely on the market to "educate," or "de-bias," boundedly rational consumers when they are naive both probabilistically and strategically.

Comment: The rationalization revisited

The present extension would be entirely meaningless under the reinterpretation that consumers have independent private valuations of each market alternative. This is an example of how apparently equivalent rational-choice and bounded-rationality models can lead to very different extensions. This demonstrates the value of pursuing market models with boundedly rational consumers, even if it appears at certain points that similar results could be obtained from conventional models with rational consumers.

6.5 SUMMARY

The main lessons of this chapter can be summarized as follows:

- Consumers' tendency to over-infer from anecdotal evidence about firms' quality may result in a thriving market for a product of little intrinsic value.
- In a price competition model, equilibrium prices rise as the success rate that characterizes the industry falls. In the limit, as the success rate tends to zero, the market outcome approaches a state of monopolistic competition. Ordinary competition policies (e.g., extending the supply of market alternatives) may diminish consumer welfare.
- Consumers who apply sampling-based reasoning behave as if they have independent private valuations of market alternatives. Therefore, firms have an incentive to differentiate their product in the sense of giving contradictory recommendations. This lowers the correlation between the firms' success rates and thus weakens competitive forces. This type of product differentiation is spurious: it is entirely due to consumers' inference errors and has nothing to do with the products' objective, payoff-relevant features. When firms can make case-specific recommendations, they may end up giving superfluously circumscribed, maximally diffuse recommendations, as yet another means of product differentiation.
- When consumers are strategically naive as well, firms will not disclose their quality in market equilibrium. In other words, the market will not correct the inference errors made by consumers who follow sampling-based reasoning.

6.6 BIBLIOGRAPHIC NOTES

This chapter is based on Spiegler (2006*b*). For precursors of models of technology adoption by consumers with limited market experience, see Smallwood & Conlisk (1979) and Ellison & Fudenberg (1995). For traditional models of spatial competition and product differentiation, see Tirole (1988, pp. 279–287).

The sampling-based procedure is borrowed from Osborne & Rubinstein (1998), who analyzed two-player games in which all players choose according to a sampling-based procedure, and defined an equilibrium concept that is suitable for such situations. In Osborne and Rubinstein's model, a player evaluates each action by testing it against an independent random draw from the opponent's mixed strategy. This is somewhat different from the procedure employed in this chapter, in which the consumer uses a single draw from the entire profile of probability distributions in order to evaluate each alternative. This difference has some important implications—for example, the procedure that Osborne and Rubinstein assume allows dominated actions to be taken with positive probability under their notion of equilibrium, provided that players have at least three actions. In the context of the model of this chapter, the distinction is irrelevant whenever the distributions that the consumer faces are statistically independent, as in the case of the basic model, or as in the symmetric mixed-strategy equilibrium in the extended model with product differentiation. Osborne & Rubinstein (2003) apply a variant on the sampling procedure in the context of a voting game.

The law of small numbers was introduced by Kahneman & Tversky (1971, 1974). Kareev, Arnon & Horwitz-Zeliger (2002) study individual perception of the variability of random variables. For evidence on the impact of vivid versus dry information, see Borgida & Nisbett (1977). Stewart, Chater & Brown (2006) present a different model of "decision by sampling," in which individuals assign values to alternatives according to their rank in a sample.

Szech (2009) extends the basic model of this chapter by endogenizing the firms' success rates as the result of a prior stage in the game, in which the firms choose their own success rates. Part (ii) in Exercise 6.1 is based on this extension. Radner & Sundararajan (2005) analyze dynamic monopoly pricing of a network good when consumers form beliefs regarding future prices by extrapolating naively and myopically from current prices. This gives the monopolist an incentive to employ a low price at the beginning in order to facilitate technology adoption throughout the network, and to subsequently raise the price.

7 Sampling-Based Reasoning II: Obfuscation

In this chapter we continue our investigation of the implications of sampling-based reasoning for market outcomes. In the preceding chapter we assumed that firms are restricted in the amount of randomness they can inflict on consumers. Let us now take the opposite extreme and assume that the uncertainty consumers face is entirely endogenous. We will refer to such endogenous randomness as a deliberate attempt to confuse consumers, or to *obfuscate*. There are various ways in which firms can introduce randomness into consumers' task of evaluating market alternatives. Firms are able to change price or quality over time. They can discriminate among consumers on an arbitrary basis (e.g., insurance companies have some discretion when they determines payouts). And as we shall see, complicated multi-dimensional pricing schemes can also be viewed as a way of inflicting randomness on consumers.

When consumers rely on a sampling-based procedure to evaluate market alternatives, they are prone to making inference errors in the face of such obfuscation strategies. I will analyze the following questions. Does sampling-based reasoning give firms an incentive to obfuscate? How does this motive respond to competitive pressures? What are the welfare implications of obfuscation?

7.1 A MODEL OF COMPETITIVE OBFUSCATION

Our market consists of a continuum of identical consumers and a set $\{1, \ldots, n\}$ of identical profit-maximizing firms that offer a homogeneous product. Each consumer has a unit demand for the product, with a willingness to pay normalized to 1. Consumers have no outside option. Firms have a constant marginal cost of $c \in [0, 1)$. The firms play a simultaneous-move game in which a strategy for firm i is a cumulative distribution function (*cdf*) G_i over the product's price.

After firms make their strategic decisions, each consumer chooses a firm according to the sampling-based procedure described in the previous section. That is, he independently draws a single sample point from each of the n *cdf* s, and chooses the alternative that gives him the highest payoff in the sample. (Assume a symmetric tie-breaking rule.) The outcome of the consumer's decision is a *new, independent draw* from the *cdf* associated with the chosen alternative. A firm incurs its cost only when it sells the product.

For now, allow the prices that firms charge to get any value in $(-\infty, 1]$. The upper bound means that consumers cannot be forced to pay more than their willingness to pay. In other words, if the realization of a consumer's chosen *cdf* turns out to be a price higher than his willingness to pay, he is not forced to buy the product. (However, at that stage the consumer does not have access to other market alternatives.) The lack of a lower bound on prices means in particular that firms are allowed to offer their product at a price below marginal cost. We will discuss the role of the bounds on prices later in this chapter.

We may interpret the firm's strategy as a genuinely random strategy. Firms often change their product prices over time, by means of sales and special offers. They also discriminate on the basis of criteria that are not always transparent to the consumer. In both cases, the price that an inexperienced consumer who makes a one-off consumption decision is asked to pay is random to all intents and purposes. An alternative interpretation will be proposed in the sequel.

I now construct firm i's payoff function. Let $T_{G_i} \subseteq (-\infty, 1]$ denote the support of G_i. Let μ_{G_i} denote the expected price according to G_i. Fixing a profile $G_{-i} = (G_j)_{j \neq i}$ of rival firms' strategies, define $H_i(p \mid G_{-i})$ as the probability that a consumer chooses firm i, conditional on the realization of G_i in the sample being p. We may interpret H_i as a *residual demand function* that firm i faces—for every price p, $H_i(p \mid G_{-i})$ is the fraction of consumers who end up choosing firm i. The ex ante expected fraction of consumers who choose firm i under the strategy profile $(G_1, \ldots, G_n)$ is thus $E_{G_i}[H_i(p \mid G_{-i})]$. When each firm i plays a continuous *cdf* with a well-defined density g_i, the definitions of H_i and $E_{G_i}[H_i(p \mid G_{-i})]$ take a simple form:

$$H_i(p \mid G_{-i}) = \prod_{j \neq i} [1 - G_j(p)]$$

$$E_{G_i}[H_i(p \mid G_{-i})] = \int_{-\infty}^{1} \prod_{j \neq i} [1 - G_j(p)] g_i(p) dp$$

Firm i's payoff function can be written as follows:

$$\pi_i(G_1, \ldots, G_n) = [\mu_{G_i} - c] \cdot E_{G_i}[H_i(p \mid G_{-i})]$$

In other words, the firm's payoff is equal to its expected markup multiplied by the size of its clientele.

The firms' strategies in this model are probability distributions. However, they are *not* "mixed strategies." Technically, this is because π_i is quadratic, rather than linear, in g_i. In particular, we should not expect firm i to be indifferent between G_i and the individual elements in T_i. Indeed, the following example illustrates that firms may have a strict incentive to randomize. Let $n = 3$ and $c = 0$. Suppose that firms 1 and 2 both play the *cdf* $G \equiv U[0, 1]$. If firm 3 assigns probability one to some price $\beta \in (0, 1)$, then its payoff is $\beta \cdot (1 - \beta)^2$. If the firm switches to a distribution that assigns probability β to $p = 1$ and probability $1 - \beta$ to $p = 0$, then its payoff is $[\beta \cdot 1 + (1 - \beta) \cdot 0] \cdot (1 - \beta)$, which is a strict improvement.

We interpret this type of randomization as an "obfuscation device." The price is "competitive" in some situations and "monopolistic" in others. The former situations generate a clientele by creating an impression that the firm is cheap, whereas the latter situations generate revenue by exploiting this impression. The obfuscation is extreme, because the consumer ends up choosing firm 3 if and only if $p_3 = 0$ in his sample, yet the actual price that he pays is the highest in the market with probability β. Moreover, when $\beta > \frac{1}{2}$, firm 3 is the worst choice in terms of expected price.

Comment: Sampling versus search

In Chapter 6, we noted the difference between the sampling-based procedure and conventional search models. It is worthwhile to revisit this discussion, because it has interesting implications for the model of competitive obfuscation. First, note that the sampling procedure itself is costless for all parties. It is a model of the consumer's decision process, rather than a model of a physical search process.

In both types of models, consumers rely on a sample $(p_1, \ldots, p_n)$ to choose from among n firms. If, say, $p_1 < p_j$ for all $j \neq 1$, the consumer will choose firm 1 in both cases. However, in a search model, he will actually pay p_1 to firm 1, whereas in the sampling model, the price he will pay to firm 1 is a new, independent realization of the same *cdf* G_1 from which p_1 was drawn.

To demonstrate the difference between the two models, consider the way you may choose between gas stations in a particular neighborhood. The prices that individual stations charge may differ, and they may also change over time. A search model implicitly assumes that each time you want to fill your gas tank, you survey a sample of the available stations and choose the cheapest among them. In contrast, the sampling model fits a scenario in which you make price comparisons only sporadically. After you make such a comparison, you temporarily stick to the cheapest gas station you found, and it may take a while before you conduct a new round of price comparisons. But in the meantime, the stations' price fluctuations may have caused the identity of the cheapest station to change.

7.1.1 Nash Equilibrium

We now turn to an analysis of symmetric Nash equilibrium in the game played by the firms. For illustration, let $n = 2$ and $c = 0$. Let us show that in this case, the uniform distribution over the interval [0, 1] is a symmetric equilibrium strategy. Given that firm 1 plays this strategy, $H_2(p) = 1 - p$ for every $p \in [0, 1]$. As to prices outside this range, recall that by assumption, assigning weight to prices above $p = 1$ is infeasible, whereas assigning prices below $p = 0$ would be suboptimal because it would lower the firm's expected revenue conditional on being chosen by the consumer, without changing the firm's market share. Thus, firm 2 only considers price distributions whose support is contained in [0, 1]. Firm 2's profit function is:

$$\mu_{G_2} \cdot E_{G_2}[H(p \mid G_1)] = \mu_{G_2} \cdot E_{G_2}(1 - p) = \mu_{G_2}(1 - \mu_{G_2})$$

Thus, firm 2's profit is purely a function of its expected price. Since $\mu_{G_2} = \frac{1}{2}$ maximizes this expression, it follows that every *cdf* G for firm 2 that has an expected price of $\frac{1}{2}$ is a best-reply to firm 1's strategy. Since the uniform distribution over [0, 1] satisfies this property, it follows that it is a symmetric equilibrium strategy.

Proposition 7.1. *For every $n \geq 2$, the game has a unique symmetric Nash equilibrium. Each firm plays the cdf*

$$G(p) = 1 - \sqrt[n-1]{\frac{2(1-p)}{n(1-c)}} \tag{7.1}$$

over the support $[1 - \frac{n}{2}(1-c), 1]$.

Proof. The proof proceeds stepwise. It will be notationally convenient to rewrite the firm's object as choice as a *cdf* over the payoff that a consumer receives when choosing the firm. Let $x = 1 - p$ denote the consumer's payoff induced by the price p, and define $F(x) = 1 - G(p)$. Let us use S to denote the support of F. We need to redefine the demand function accordingly. Let $J_i(x \mid F_{-i})$ be the probability that a consumer whose realized payoff from F_i is x ends up choosing firm i, given that the rival firms play the strategy profile F_{-i}. Thus, when all firms play a continuous *cdf* with a well-defined density, we can write:

$$J_i(x \mid F_{-i}) = \prod_{j \neq i} F_j(x)$$

$$E_{F_i}[J_i(x \mid F_{-i})] = \int_0^{\infty} \prod_{j \neq i} F_j(x) f_i(x) dx$$

I will use the abbreviated notations $Ex_i = E_{F_i}(x_i)$ and $EJ_i = E_{F_i}[J_i(x \mid F_{-i})]$ when there is no risk of confusion.

Step 1: The support of the equilibrium strategy

Let us first establish that F is a continuously increasing function over an interval whose lower bound is 0. The continuity argument is standard: if F places an atom on some $x \geq 0$, then it is profitable for any firm to deviate by shifting the weight from x slightly upward. The argument that the support of F is an interval is somewhat less standard. Let us first show that $\inf S = 0$. Assume the contrary. Denote $\inf S = x^*$. Suppose that firm i deviates by shifting to $x = 0$ all the weight it assigns to $(x^*, x^* + \varepsilon)$. Then, it reduces Ex_i by more than $x^* \cdot F(x^* + \varepsilon)$. At the same time, it reduces EJ_i by less than $[F(x^* + \varepsilon)]^2$. If $\varepsilon > 0$ is sufficiently small, the reduction in EJ_i is negligible in comparison to the reduction in Ex_i, such that the deviation is profitable. A similar argument establishes that the support of F contains no holes.

Step 2: Linearity of the residual demand function

We now turn to the key step in the proof, which establishes that in symmetric equilibrium, $J(x) = x/\bar{x}$. That is, the residual demand function faced by each firm in symmetric equilibrium is linear. To prove this, consider three outcomes

x_1, x_2, x_3 in the support of $F(x, n)$, $x_1 < x_2 < x_3$, and suppose that the point $(x_2, J(x_2)) \in \mathbb{R}^2$ lies *above* the line connecting the points $(x_1, J(x_1))$ and $(x_3, J(x_3))$. Then, any firm can deviate to a strategy that shifts weight from the neighborhoods of x_1 and x_3 to x_2, in a way that preserves Ex. The deviation increases EJ, hence it is profitable. Conversely, if $(x_2, J(x_2)) \in \mathbb{R}^2$ lies *below* the line connecting $(x_1, J(x_1))$ and $(x_3, J(x_3))$, then any firm can profitably deviate by shifting weight from the neighborhood of x_2 to x_1 and x_3 in a way that preserves Ex and increases EJ. It follows that J is linear over the support of $F(x, n)$. In particular, this implies that the support of $F(x, n)$ must have a finite upper bound $\bar{x}$.

Step 3: Deriving the equilibrium formula

Recall that the firms' objective function is $(1 - c - Ex) \cdot EJ$. By Step 2, $EJ = Ex/\bar{x}$ in symmetric equilibrium. Thus, each firm i chooses F_i to maximize $(1 - c - Ex) \cdot Ex$. It follows that in symmetric equilibrium, $Ex = \frac{1-c}{2}$ under F. By symmetry of equilibrium, $EJ = \frac{1}{n}$ for each firm. Therefore, $\bar{x} = \frac{n}{2}(1 - c)$. By the formula for $J(x)$ obtained in Step 2, $J(x) = \frac{2x}{n(1-c)}$ for every $x \in [0, \frac{n}{2}(1 - c)]$. Since $J(x) = [F(x)]^{n-1}$, we obtain the following expression for F:

$$F(x) = \sqrt[n-1]{\frac{2x}{n(1 - c)}}$$

which completes the proof. □

This result has an important corollary: *the expected price according to $G(p)$ is $\frac{1+c}{2}$, independently of the number of firms n.* The basic idea behind this result is the following. In equilibrium, the residual demand that each firm faces is linear—otherwise, the firm could modify its randomization in a mean-preserving way so as to increase its clientele. Moreover, the slope of this linear residual demand function is independent of n, which immediately implies the invariance of the equilibrium price to n.

The corollary is in fact stronger: for all $n \geq 2$, $G(p; n + 1)$ is a *mean preserving spread* of $G(p; n)$. The reasoning behind this corollary is as follows. A simple calculation shows that for every $p \in (-\infty, 1)$, $\int_p^1 G(w; n)dw$ is increasing with n. By an existing basic result (see Mas-Colell, Whinston & Green (1995, p. 198)), this is equivalent to saying that $G(p; n + 1)$ is a mean-preserving spread of $G(p; n)$. Thus, *an increase in n results in an increase in the variance of the equilibrium cdf, without affecting expected price.*

The broad intuition behind this effect is that firms have two strategic considerations: the usual competitive motive, which induces firms to offer an attractive price distribution, and the incentive to confuse the consumer by introducing greater variance into their price distribution. A firm may raise the variance of its distribution in order to increase the probability that the consumer will choose it over another distribution, which itself may be more attractive in terms of expected price. It turns out that in equilibrium, firms respond to greater competition by cultivating the "obfuscatory effect" only.

Is the firms' equilibrium preference for obfuscation strict? Note that for all n, firms are indifferent between the equilibrium strategy $G(p; n)$ and the degenerate *cdf* that assigns probability one to $p = \frac{1+c}{2}$. They strictly prefer $G(p; n)$ to all other deterministic strategies. In this sense, the incentive to obfuscate is weak in equilibrium.

Another effect that characterizes the equilibrium strategy when $n > 2$ is that it assigns positive probability to $p < c$. Thus, the model of competitive obfuscation generates an effect similar to loss-leader pricing. The probability that prices fall below marginal cost is

$$\Pr\left(p < c \mid G(p)\right) = 1 - \sqrt[n-1]{\frac{2}{n}}$$

Interestingly, this expression does not behave monotonically with n. It attains a maximum of approximately 20% when $n = 4$, and then decreases monotonically. As n tends to infinity, the probability of below-marginal-cost prices vanishes. (However, note that the lower bound of the price distribution diverges.)

The following pair of exercises extend the model by examining what happens when consumers take multiple sample points from firms' price distributions.

Exercise 7.1. *Let $n = 2$ and $c = 0$. Modify the model by allowing prices to get values in $\{0, 1\}$ only. In addition, assume that for every firm $k = 1, 2$, the consumer observes $m_k = k$ independent sample points from the firm's price distribution and chooses the firm with the lowest average price in the sample. In case of a tie, he chooses firm 2. If the average price of both firms in the consumer's sample is 1, he opts out and chooses none of the firms. Formulate the interaction between the firms as a strategic game and find its Nash equilibrium.*

Exercise 7.2. *Let $n = 2$ and $c = 0$. Modify the model by assuming that consumers draw a sequence of two sample points (p_i^1, p_i^2) from each firm i's cdf G_i. If $p_i^k < p_j^k$ for both $k = 1, 2$ (i.e., if firm i dominates firm j in the consumer's sample) the consumer chooses firm i. If no firm dominates another in the sample, the consumer chooses each firm with probability $\frac{1}{2}$. Formulate the interaction as a strategic game and show that there is a Nash equilibrium in which $G_1 = G_2 \equiv U[0, 1]$.*

Comment: The role of the bounds on prices

The main role of the assumption that prices have no lower bound is to simplify the proof and obtain a simple formula for the symmetric equilibrium strategy. The result that in symmetric Nash equilibrium $Ep = \frac{1+c}{2}$ and $G(p; n+1)$ is a mean-preserving spread of $G(p; n)$ for all $n \geq 2$ would continue to hold if we assumed that p is bounded between c and 1. In this case, as $n \to \infty$, the equilibrium strategy converges to a simple distribution that assigns probability $\frac{1}{2}$ to each of the extreme values $p = c$ and $p = 1$. This limit distribution has the maximal possible variance subject to the constraint that $p \in [c, 1]$.

In contrast, the assumption that prices have an upper bound is crucial. If we allowed p to get arbitrarily large values, firms would be able to earn unboundedly large profits, by assigning probability $\alpha \in (0, 1)$ to some p with $H_i(p) > 0$ and probability $1 - \alpha$ to an arbitrarily large p^1.

7.1.2 Welfare Analysis

For the purpose of welfare analysis, we need to define consumers' "true" preferences over probability distributions. If we rank distributions according to their expected value, then the conclusion from our equilibrium analysis is that raising the number of competitors in the market does not make consumers better or worse off. If, however, consumers' true preferences over probability distributions display risk aversion, then our equilibrium analysis implies that raising the number of competitors makes consumers strictly worse off, because $G(p; n + 1)$ is a mean-preserving spread of $G(p; n)$.

Note, however, that any definition of consumer welfare that incorporates an explicit assumption regarding the consumers' risk attitudes is problematic in this model. Our consumer is *unaware* of the actual structure of the *cdf*s he is facing. At the time he makes a decision, there is no reason to assume that the risk he perceives has anything to do with the actual risk. This is another demonstration that welfare analysis with boundedly rational consumers is subtle and should not be carried out mechanically.

Consider the variant of the model discussed at the end of the previous section, where firms are constrained to price distributions the support of which is contained in $[c, 1]$. Notice that under this constraint, the max-min payoff of an individual firm is $\frac{1-c}{4n}$. The reason is that for any strategy the firm employs, its worst-case scenario is when all other firms charge $p = c$ with probability one. The best-reply to this scenario is to offer $p = c$ and $p = 1$ with probability $\frac{1}{2}$ each, yielding a payoff of $\frac{1-c}{4n}$. This is half the profit that firms earn in symmetric Nash equilibrium.

The significance of this finding is that when firms can obfuscate, competitive forces perform an imperfect role in limiting the extent to which firms take advantage of consumers' sampling-based reasoning. As in the model of Chapter 6, the max-min profit in the present model is strictly positive, due to the decision errors brought about by the sampling-based procedure. However, unlike the model of Chapter 6, equilibrium behavior has an implicit collusive component: obfuscatory pricing strategies prevent competitive forces from pushing the firms' profits down to the max-min level.

7.2 PRODUCTION INEFFICIENCIES

Let us assume now that firms choose not only the price of their product but also its quality. This will enable us to address the impact of consumers' sampling-based

[1] One justification for the upper-bound assumption is that consumers can cancel the transaction ex post if they are charged a price above their willingness to pay.

reasoning on production efficiency. As before, let p denote the product's price. Let $q \in [0, \infty)$ denote product *quality*, measured in production cost units. Let $v(q)$ be the consumers' willingness to pay for quality level q. Thus, when a consumer chooses a firm that offers the realization (q, p), his payoff is $v(q) - p$, whereas the firm's payoff is $p - q$. Assume that v is unboundedly increasing and strictly concave, with $v(0) = 0$, and that total surplus $v(q) - q$ attains maximum at a unique $q^* > 0$. Normalize $v(q^*) - q^* = 1$.

Unlike the previous section, here we impose a lower bound on prices. Specifically, we restrict p to lie in the interval $[0, v(q)]$. This has implications for the Pareto frontier. Pareto-efficient pairs (q, p) fall into two categories: (i) $q = q^*$ and $p \in [0, v(q^*)]$, or (ii) $q > q^*$ and $p = 0$. Thus, as long as we wish to sustain a consumer payoff below $v(q^*)$, it is Pareto-efficient to produce the quality level q^* and use prices to induce the desired payoff. However, if we wish to sustain a consumer payoff $\bar{v} > v(q^*)$, it is Pareto-efficient to produce $q > q^*$ such that $v(q) = \bar{v}$ and set the price to zero.

A strategy for a firm is a probability distribution over pairs (q, p). At first glance, the added quality dimension seems to complicate the model relative to the basic model of the previous section. However, a simplification is immediately made possible, thanks to the observation that it is always sub-optimal for firms to assign positive probability to quality-price pairs below the Pareto frontier. Thus, a strategy for firm i can be reduced without loss of generality to a *cdf* G_i over the scalar variable $\pi = p - q$, namely the firm's profit conditional on being chosen.

Correspondingly, $H_i(\pi \mid G_{-i})$ is the probability that the consumer chooses firm i, given that the firm chooses a quality-price pair that induces a profit level π and that the rival firms play the strategy profile G_{-i}. Because the firm assigns positive probability only to quality-price pairs on the Pareto frontier, H_i decreases with π. When all firms play continuous *cdf*s, we can write:

$$H_i(\pi \mid G_{-i}) = \prod_{j \neq i} [1 - G_j(\pi)]$$

As in the basic model, firm i's payoff function is $E_{G_i}(\pi_i) \cdot E_{G_i}[H_i(\pi \mid G_{-i})]$, where $E_{G_i}(\pi_i)$ is the firm's expected profit conditional on being chosen, and $E_{G_i}[H_i(\pi \mid G_{-i})]$ is the size of the firm's clientele.

Having reduced the game played among the firms to that of the previous section, it follows that the characterization of symmetric Nash equilibrium is exactly the same as in the previous section: each firm plays the *cdf*

$$G(\pi) = 1 - \sqrt[n-1]{\frac{2(1 - \pi)}{n}}$$

over the support $[1 - \frac{n}{2}, 1]$. (In comparing formulas, recall that we have normalized the maximal surplus to 1.) This has several implications for welfare analysis.

Proposition 7.2. *The symmetric equilibrium has the following properties:*

(i) $E\pi = \frac{1}{2}$ *for every* $n \geq 2$.
(ii) For every $n > 2v(q^*)$, $q > q^*$ *with positive probability.*
(iii) The expectation of $v(q) - q$ *is strictly decreasing with* n *in the range* $n > 2v(q^*)$.

The number of competitors does not affect equilibrium industry profits: those are equal to half the maximal surplus, regardless of n. However, when the number of firms is sufficiently high, expected total surplus falls below the maximal level 1. The efficiency loss increases with n, and is borne entirely by the consumers. In this regard, greater competition has a strictly *negative* effect on both consumer welfare and social welfare.[2]

What drives the adverse welfare effects of competition? As in the basic model, when there are sufficiently many competitors, the equilibrium strategy assigns positive probability to negative profits. But in the present model, the lowest profit that a firm can generate under surplus-maximizing production is $-q^*$. When $n > 2v(q^*)$, the strength of competition pushes firms to assign weight to profit levels below $-q^*$. The only way to attain such levels is to produce $q > q^*$ (i.e., a quality above the efficient level).

Why does expected total surplus diminish with n? Since $G(\cdot; n+1)$ is a mean preserving spread of $G(\cdot; n)$, the expected value of any concave function of π is lower under $G(\cdot; n+1)$ than under $G(\cdot; n)$. (See Mas-Colell, Whinston & Green (1995, p.198).) Thus, all we need to show is that total surplus is a concave function of π. Recall that the surplus function $v(q) - q$ is strictly concave. In addition, along the Pareto frontier, every $\pi \in [-q^*, 1]$ corresponds to the surplus-maximizing quality level q^*, while every $\pi < -q^*$ corresponds to a quality level above q^*. Therefore, total surplus as a function of π is strictly increasing and strictly concave in the range below $-q^*$ and constant in the range $[-q^*, 1]$. And since the firms' equilibrium strategy assigns positive probability to $q > q^*$ whenever $n > 2v(q^*)$, expected total surplus decreases with n.

Thus, the forces that drive the inefficiency result are: (i) the mean-preserving-spread property of G, which reflects the firms' increased effort to obfuscate in response to greater competition, and (ii) diminishing marginal utility from quality.

Comment: The role of the lower bound on prices

The assumption that firms cannot charge negative prices is crucial for the production inefficiency result. If we allowed firms to charge arbitrarily negative prices, we would have $q = q^*$ throughout the Pareto frontier. And since the firms' equilibrium strategy assigns positive probability to Pareto-efficient quality-price pairs only, expected total surplus would be maximal for all n.

[2]There is no contradiction between this inefficiency result and the observation made above that firms assign positive probability only to Pareto-efficient quality-price pairs, since the welfare analysis in the proposition pertains to *expected social surplus* induced by probability distributions over quality-price pairs.

7.3 MULTI-DIMENSIONAL PRICES

The primary interpretation of G_i we adopted in this chapter is that it is a genuinely random strategy, where the randomization can be over time or across circumstances. An alternative interpretation is that G_i is a reduced form of a multidimensional pricing strategy. According to this interpretation, firms offer a service with a large number of attributes, or covering a large number of contingencies. Marginal cost is c for all dimensions. A pure strategy for a firm specifies a price for every dimension, such that $G_i(p)$ is the fraction of dimensions for which the price is at most p. Evaluating the firm's multidimensional pricing strategy is a difficult task for the consumer, because it requires him to calculate trade-offs across an enormous number of attributes or contingencies. The consumer simplifies his decision problem by examining one dimension *at random* and choosing the firm that offers the best terms along this particular dimension. The outcome of his choice is determined by the terms that the firm offers in an independent random subset of dimensions.

This interpretation is attractive in terms of the scope of applications that it enables (e.g., consider the fee structure at commercial banks, or health insurance policies), and we will rely on it in the next section. The firms' incentive to randomize can be interpreted as an incentive not to inform consumers of the "bottom line"–that is, the true price of their product (e.g., the APR on a credit card). We saw that in symmetric equilibrium, firms have a weak incentive not to educate consumers.

As the number of dimensions tends to infinity, this multidimensional model converges to our basic model. In the infinite-dimensionality limit, there exists a symmetric *mixed-strategy* equilibrium in which the price in each dimension is an independent realization of the *cdf* $G(p)$ given by expression (7.1). When the number of dimensions is $M < \infty$, this link is imprecise, because with probability $\frac{1}{M}$ the consumer gets exactly what he observed in the sampling stage, and this contradicts the assumption in the basic model that the sampled and realized outcomes are statistically independent.

Comment: Predictability of the sampled dimension

Another crucial aspect of this interpretation is that firms are unable to predict the dimension that consumers consider in the sampling stage. If firms knew with certainty that consumers examine dimension d^*, they would compete fiercely along that dimension, while behaving monopolistically in all other dimensions. When prices in each dimension are restricted to the interval $[c, 1]$, all firms would charge $p = c$ in dimension d^* and $p = 1$ in all other dimensions. When prices are not bounded from below, the equilibrium price in dimension d^* will be determined such that firms earn zero profits, hence $p_{d^*} + 1 \cdot (M - 1) = M \cdot c$.

This is essentially the same argument we used in the model of add-on pricing in Section 5.3. The existence of a known dimension that consumers use for comparing firms enables firms to focus their competitive efforts on this dimension, and this gives rise to the zero profit outcome. When firms do not know which dimension consumers sample, such focusing is infeasible; therefore competitive forces are weaker and equilibrium profits are higher.

Comment: The interpretation of obfuscation

Under the multiple-dimensions interpretation of the model, we have used the term "obfuscation" as if it is synonymous with large variation across dimensions. Inferring the product's expected value from one sampled dimension entails an error. Obfuscation enhances this inference error; the larger the price variation across dimensions, the greater the obfuscation.

However, this may be different from what we often have in mind when we say that a contract is complicated or confusing. For instance, a contract may be complicated in the sense that its exact terms are hard to understand. Alternatively, a contract may state broad terms prominently and leave qualifications to the small print. These notions of obfuscation are not captured by our model.

Furthermore, whether or not a multidimensional pricing strategy is complicated or confusing depends on the language we use to describe it. For example, a price distribution that assigns probability $\frac{1}{2}$ to $p = c$ and $p = 1$ each maximizes variance among all distributions whose support is restricted to $[c, 1]$, but it is certainly simple to describe, as long as we accept such a probabilistic description. However, if a multidimensional pricing strategy can only be described by a detailed specification of the contractual terms that pertain to every individual dimension, then all multidimensional pricing schemes are equally difficult to write or to understand. In this case, sampling-based reasoning is a natural simplifying heuristic for the consumer, and our notion of obfuscation as cross-section variation that enhances inference errors seems appropriate.

7.4 A MARKET INTERVENTION: INTRODUCING "SIMPLE" OPTIONS

We already saw that increasing the number of competitors need not make consumers better off, and may even lower their welfare. Another market intervention that a priori seems to benefit consumers is to endow them with an attractive additional option, which is also *simple to understand*. In the context of a market for financial services, say, this "plain vanilla" option can be provided by the government (or by a government-regulated provider). Alternatively, a regulator can force firms to offer a simple option in addition to their "regular," possibly complex product.

Let us see how to capture these regulatory interventions in a stylized manner with our model of competitive obfuscation. For the purposes of our discussion, assume that the support of firms' *cdf*s is restricted to the interval $[c, 1]$. This will simplify our analysis considerably without changing the qualitative results. Consider the first intervention mentioned above: introducing an additional alternative, denoted 0. As in the basic model, the consumer applies the sampling-based procedure, except that now he faces a profile of $n + 1$ alternatives $(G_0, G_1, \ldots, G_n)$, where G_0 is the *cdf* associated with the outside option.

To capture the notion of a simple, "plain vanilla" outside option, assume that G_0 is a degenerate *cdf* that assigns probability one to some price p_0. Taking the multiple-dimensions interpretation of the model, such a *cdf* is simple in the sense that the price is flat and does not vary at all with the many attributes or contingencies that characterize the product.

This market intervention has an important implication for the demand function that each firm faces. For every $i = 1, \ldots, n$, $H_i(p) = 0$ for all $p > p_0$. Consider the case in which the "simple" outside option is priced at marginal cost (i.e., $p_0 = c$). Although the exact value of $H_i(c)$ depends on $(G_j)_{j \neq i}$, firm i's best-reply turns out not to depend on it. Because $H_i(p) = 0$ for all $p > c$, firm i has no incentive to assign any weight to prices in $(c, 1)$. Thus, firm i's best-reply assigns some probability α to $p = c$ and probability $1 - \alpha$ to $p = 1$, yielding an expected payoff of:

$$\alpha H_i(c) \cdot [\alpha \cdot 0 + (1 - \alpha) \cdot (1 - c)].$$

The value of α that maximizes this expression is $\frac{1}{2}$, independently of $H_i(c)$. Hence, firm i has a strictly dominant strategy, which assigns probability $\frac{1}{2}$ to each of the extreme prices $p = c$ and $p = 1$.

The next result extends this observation. I omit the proof.

Proposition 7.3. *Let $n > 2$. If $p_0 \in [c, \frac{1+c}{2})$, then in symmetric Nash equilibrium, firms play a distribution that assigns probability $\frac{1}{2}$ to each of the extreme prices $p = c$ and $p = 1$.*

Thus, adding a simple and attractively priced option to the consumers' set of available market options causes firms to react by maximizing the variance of their *cdf*. It does not, however, affect the expected equilibrium price. In other words, firms respond to the simple option by cultivating the obfuscatory motive without trying to behave more competitively in expectation. The intuition is that firms do not compete at all in the range $p > p_0$. They prefer to shift weight in this range to the extreme point $p = 1$. This generates a large revenue for the firm conditional on being chosen, and thus increases the incentive to behave competitively in the range $p \leq p_0$.

According to the proposition, when $p_0 \in (c, \frac{1+c}{2})$, the probability that the consumer ends up choosing the simple outside option in symmetric equilibrium is $(\frac{1}{2})^n$. This expression decreases with n, and converges to zero as $n \to \infty$. Conditional on abandoning the simple option in favor of a market alternative, the consumer experiences an expected welfare loss of $\frac{1}{2}(1 + c) - p_0$, relative to the value of the simple outside option. The consumer's expected welfare loss in symmetric equilibrium is thus

$$\left[1 - \left(\frac{1}{2}\right)^n\right] \cdot \left[\frac{1+c}{2} - p_0\right]$$

This expression increases with n.

Consumers are "exploited" in equilibrium, in the sense that they choose alternatives that are worse in expectation than the simple, externally provided option. The reason for this exploitative effect is that consumers place their full understanding of the simple option and their more anecdotal understanding of the other market alternatives on equal footing. This involves two mistakes. First, the consumer treats a single sample point drawn from a firm's *cdf* as if it has the same informational content as full knowledge of the simple option. Second, the consumer disregards the fact that a firm's *cdf* is the result of a strategic choice, while the simple option is exogenous. For failing to draw these distinctions, the consumer suffers a welfare loss.

I find it interesting that introducing an attractive simple option has the same qualitative effect as raising n in the basic model. Both interventions are effective competition policies in standard Industrial Organization models. They continue to have a similar effect in the present model, albeit in the orthogonal direction of enhancing obfuscation.

The next exercise illustrates how a firm's dilemma whether to obfuscate or act competitively depends on the relation between its marginal cost and the price of the simple outside option.

Exercise 7.3. *Suppose that there is only one firm in the market. The firm chooses a cdf over* $[c, 1]$. *In addition to this firm, the consumer has access to a simple option at a fixed price* p_0. *For simplicity, assume that the consumer breaks ties in favor of the firm. Show how the firm's optimal cdf varies with the firm's marginal cost* c.

Let us turn to the second market intervention. Rather than offering a simple additional option, the regulator forces each firm to offer a simple alternative, modeled as a degenerate *cdf*, in addition to its regular, possibly complex product. This extends our model in the following direction. A strategy for firm i is a pair (p_i, G_i), where $p_i \in [c, 1]$ is the price of the firm's simple product and G_i is a *cdf* over $[c, 1]$ which represents the firm's complex product. Assume that when the realization x_i of the consumer's draw from G_i is weakly below p_i, the consumer chooses the firm's complex product over its simple product. (If we allowed firms to offer *cdf*s that assign weight to prices below c, this tie-breaking rule would not be necessary.)

The consequences of this intervention for symmetric Nash equilibrium are very similar to those of the first intervention. Because consumers never err in evaluating the simple products, competitive forces push their prices to marginal cost. Thus, when firms design the pricing strategy for their complex product, they are in the same strategic situation as in the first regulatory intervention. The residual demand for the firm's complex product satisfies $H_i(p) = 0$ for all $p > c$. Thus, the best-replying *cdf* for each firm i assigns probability $\frac{1}{2}$ to each of the extreme prices, $p = c$ and $p = 1$.

As in the case of the first intervention, the fraction of consumers who end up choosing complex products grows with n, and converges to one as n tends to infinity. These consumers choose an inferior product, because its expected price is $\frac{1}{2}(1 + c)$, whereas the simple products, provided by the same firms, are priced at marginal cost.

Comment: Preference for simple alternatives

The market interventions considered in this chapter lead to rather pessimistic conclusions. While they improve consumer welfare when we take the firms' pricing strategies as fixed, they become ineffective once the firms' response to the interventions is taken into account. But is this conclusion too pessimistic?

Try to put yourself in the consumer's shoes. You have a limited understanding of the market environment and little information about the value of market alternatives. Therefore, you rely on the sampling-based procedure to simplify your decision problem. At the same time, you have a good understanding of the available simple options. With some thought, you may come to realize that firms know something about your limitations and try to take advantage of them when designing their complex products. Will this thought make you more cautious when applying the sampling procedure in such a market environment? Will it make you develop a preference for the simple products?

We encountered a similar problem in Chapter 2. In an individual decision-making setting, naivete was interpreted simply as an incorrect belief regarding future preferences. However, in an interactive context, the consumer faced choice from a menu that seemed to contain a dominated alternative, and perhaps he could even observe other consumers selecting the supposedly dominated alternative. This may have led the consumer to rethink his beliefs. Likewise, in the present model with additional simple options, our consumer may lack a perfect understanding of the market and therefore resort to a simplifying procedure. Yet, a minimal understanding of the strategic situation could lead the consumer to rethink his procedure. I expect considerable heterogeneity among consumers in this regard.

This is a major difficulty in modeling boundedly rational consumers in market environments. We have deviated from the standard rational-choice model because we wished to relax, with good reasons, the assumption that all consumers have a perfect understanding of their market environment. However, when we substitute perfect rationality with some alternative decision model, the amount of "market intelligence" (or lack thereof) implied by the model is not invariant to the market situation. In some situations, a consumer who follows the alternative decision model displays far too little market intelligence than we may wish to attribute to him. Resolving this difficulty remains an open problem.

■ 7.5 SUMMARY

The sampling-based procedure studied in the last two chapters captures one way in which people respond to a complex environment: they carefully examine a small part, and their evaluation of the "big picture" is based on simple extrapolation. The question we addressed was how profit-maximizing firms respond to this type of decision making, both when market complexity is exogenous and when it is the making of the firms themselves.

Let us summarize the lessons from this chapter.

- When consumers use sampling-based reasoning to evaluate market alternatives and firms can generate essentially any probability distribution over prices, market competition gives firms a strict incentive to "obfuscate"—namely, to generate noisy price distributions.
- This incentive responds strongly to the intensity of competition, at the expense of "proper" competitive behavior. In particular, increasing the number of competitors induces a mean-preserving spread of the distribution over prices. Regardless of the number of competitors, consumers always get half the surplus in equilibrium.
- When firms determine both price and quality, and the production of quality has diminishing marginal returns, the welfare conclusions become even more drastic: increasing the number of competitors may strictly reduce total expected surplus and consumer welfare.
- A regulatory intervention that introduces "simple" options into the market (either by offering them through a government agency or by forcing firms to offer them) may be ineffective, because firms respond to these policies with obfuscation rather than with "proper" competitive behavior.

7.6 BIBLIOGRAPHIC NOTES

This chapter closely follows Spiegler (2006*a*). In Chapter 10 we will analyze a model of price competition that offers a different perspective into the phenomenon of price obfuscation.

Rubinstein & Spiegler (2008) analyze a dynamic model of price obfuscation when agents rely on the sampling-based procedure. The Rubinstein-Spiegler model is presented in the context of a stylized asset market. However, it can be reformulated as a model of dynamic monopoly pricing, in which consumers who enter the market can choose between a fixed-price, long-term service contract and a short-term contract, such that if they choose to renew the latter in future periods they will pay a new price. Consumers rely on the sampling procedure to guess the new price and figure out whether they should opt for one long-term or a sequence of short-term contracts. The monopolist finds it optimal to let prices fluctuate over time, in order to take advantage of consumers' inference errors. However, the excess profits it is able to generate in this way are lower than in a corresponding static model.

The literature contains more conventional modeling approaches to price obfuscation. Ellison & Wolitzky (2009) and Wilson (2009) adapt standard models of sequential search, and formalize obfuscation as any action that increases consumers' search costs. Carlin (2009) adapts a conventional model of price competition in which some consumers choose randomly, and formalizes obfuscation as any action that increases the fraction of these consumers in the population. These papers either rely on conventional frictions or treat consumers' bounded rationality as a black box, in contrast to the approach taken here—namely, explicit modeling of elements of bounded rationality in consumers' choice procedures.

To illustrate the difference between the two modeling approaches, it may be useful to present one of these works in more detail. The model due to Ellison & Wolitzky (2009) is an extension of a sequential search model due to Stahl (1989). In this model, consumers examine firms in random order. When they visit a firm, they learn the firm's price and decide whether to buy from the firm or to continue searching. In Stahl's model, consumers have a constant cost per one search (which is zero for some consumers and positive for the others). In the Ellison-Wolitzky model, each firm determines the amount of time that the consumer would have to spend in order to learn its price. Increasing this time above a certain minimum is interpreted as obfuscation. In addition, Ellison and Wolitzky assume that the consumer's overall search costs are convex in the total amount of time that he shops around. Consumers are rational and have a correct knowledge of the distribution of prices and obfuscation decisions.

In the Ellison–Wolitzky model, obfuscation has nothing to do with the consumer's perception of the firm's price. The reason that firms may want to obfuscate is that by doing so, they raise the consumer's cost of further search and therefore raise the threshold price below which he is willing to stop searching and buy from the firm. The search and bounded-rationality approaches to modeling obfuscation thus appear to complement each other, as they illuminate different ways in which firms try to obstruct consumer shopping.

For field-empirical investigations of obfuscatory pricing practices in e-commerce environments, see Ellison & Fisher Ellison (2009) and Brown, Hossain & Morgan (2010). These studies are of interest because the Internet seemingly reduces traditional search frictions to a minimum. Obfuscation on the Internet can thus be viewed as the market response to the low-friction environment.

8 Coarse Reasoning

The sampling-based procedure studied in Chapters 6 and 7 captured the idea that when consumers face a complex market object, they use a simplifying heuristic: examine a small part of the object in detail and extrapolate naively in order to assess the object as a whole. In this chapter we continue our exploration of consumers with bounded ability to cope with complex market environments, but from a different, complementary angle. In the models examined in this chapter, the consumer will process the entire available data on the market objects he faces (typically complex pricing strategies). However, to simplify his decision problem, the consumer will create a "coarse representation" of the data and reason entirely in terms of this representation. The decision error that may result from coarse reasoning is that the consumer will fail to perceive patterns that underlie the data, a failure which may be exploited by firms.

8.1 A MODELING FRAMEWORK

The models of consumer behavior that we shall examine fall into the following general framework, which I describe loosely. A pricing strategy for a firm is a mapping f from a set of *situations* Ω (states of Nature, histories in a dynamic model, etc.) into prices. A fully rational consumer would make a consumption decision that maximizes his (expected) utility given a perfect perception of f. In contrast, the boundedly rational consumer in this chapter will maximize his expected utility with respect to a *coarse representation* of f. Specifically, we endow the consumer with a *perceptual partition* $\Pi = \{\Pi_1, \ldots, \Pi_K\}$ of Ω. Let $\Pi(\omega)$ denote the partition cell that contains the situation ω. The consumer's coarse representation of f is a mixed strategy σ^Π. The probability that σ^Π assigns to a given price p conditional on a situation ω is equal to the fraction of situations $\omega' \in \Pi(\omega)$ for which $f(\omega') = p$. In other words, for every $\omega \in \Omega$, the coarse representation σ^Π replaces $f(\omega)$ with a mixed strategy that mimics the price distribution in the entire cell $\Pi(\omega)$. A fully rational consumer is a special case that corresponds to a maximally fine perceptual partition: $\Pi = \{\{\omega\}\}_{\omega\in\Omega}$.

It is important to emphasize that consumers have a perfect understanding of the set Ω. Indeed, if the consumer receives some private information about ω, he uses it to update his beliefs as in the standard model. This information does not affect his perceptual partition. The only limitation on the consumer's rationality is in his ability to attribute complex strategies to the firm.

Thus, from a formal point of view, coarse reasoning in the models of this chapter consists of a very simple procedure: replace the firm's actual strategy f with its coarse representation σ^Π, and adhere to the Bayesian rationality paradigm in every other respect. A boundedly rational consumer in this chapter behaves just like a rational consumer, except that his perception of the firm's strategy is mediated by his coarse perceptual partition of Ω. It is important to distinguish between coarse perception and imperfect information, despite the modeling similarities. In standard models, agents may have a coarse knowledge of the state of the world. This is captured by their information partition. The coarser the information partition, the more limited the agent's information. However, standard models always assume that *in equilibrium*, agents have a perfect knowledge of the *strategies* played by their opponents. This is where the model of coarse reasoning departs from the standard model. It is the equilibrium knowledge of the strategy played by the opponent (here the firm) that is coarsened by the perceptual partition Π, and that is in principle independent of the information partition.

An illustrative example

To see how consumer behavior can be affected by coarse reasoning, consider the following simple example. A consumer is interested in purchasing a single unit of some product from a monopolist. The consumer's willingness to pay takes two values, h and l, with equal probability, where $h > l > 0$. The monopolist incurs no costs. The consumer makes his consumption decision *before* observing the price.

Suppose that both parties observe the consumer's willingness to pay. In particular, the monopolist can condition the product price on it. Assume that the monopolist always charges a price equal to the consumer's willingness to pay. If the consumer were rational, he would always buy the product. However, suppose that the consumer's perceptual partition is "fully coarse." That is, he believes that the firm charges $p = l$ or $p = h$ with equal probability, independently of his willingness to pay. One interpretation for this coarse belief is that when the consumer enters the market, his access to historical data is restricted to the monopolist's past prices. Using the simplest theory to explain the monopolist's past behavior, the consumer hypothesizes that the monopolist randomizes uniformly between the two price levels, h and l. An alternative interpretation is that the consumer is unaware that the monopolist knows his willingness to pay.

Our consumer is risk-neutral and chooses to buy the product if and only if the expected price according to his belief does not exceed his willingness to pay. Since the expected price is $\frac{1}{2}(l + h)$, the consumer will purchase the product in the high state only. Thus, a pricing strategy that extracts the whole surplus from a rational consumer leads to inefficient trade.

The origin of the perceptual partition

The perceptual partition is a primitive of the model. However, it is natural to ask where it comes from, and what distinguishes a consumer with a fine partition from his coarse fellow consumer. One interpretation is that consumers *learn* the firm's strategy through repeated observations of past cases. The feedback that the consumers receive during this learning process may itself be coarse. For instance, it may neglect certain aspects of observed past cases. Alternatively, even when consumers have "unlimited access" to the "database" of past consumption experiences, their ability to process large amounts of historical data may be limited. To use the "database" metaphor, they are only able to submit coarse queries.

Under both scenarios, consumers can only use statistical information about the firms' behavior within any class of past cases that their feedback pools together. When consumers form a belief of the firm's strategy, they implicitly apply the simplifying assumption (in the manner of Occam's razor) that the firm's strategy is measurable with respect to the partition their feedback generates. That is, they do not allow their theory of the firm's behavior to distinguish between cases that are indistinguishable according to their perceptual partition.

The "database" image can help us perceive the connection between the models of coarse and sampling-based reasoning, despite their superficial differences. In both cases, the database contains plenty of information that a fully rational consumer can analyze to produce a useful prediction of the value of market alternatives. Both models assume that the consumer has limited ability to perform calculations over the database. The two models capture different aspects of this limitation. The sampling-based procedure assumes that he fully surveys a limited number of entries, while the model of coarse reasoning assumes that he breaks the database into cells and correctly calculates averages within each cell.

In the rest of this chapter, I present two market applications of the coarse reasoning framework. First, I present a model in which a monopolistic firm creates a complex price pattern to discriminate between consumers with diverse degrees of coarseness, interpreted as heterogeneity in consumers' ability to perceive patterns. Second, I discuss the implications of coarse reasoning for the consumers' ability to follow the logic of adverse selection in markets with asymmetric information regarding product quality.

8.2 COMPLEX PRICE PATTERNS AS A DISCRIMINATION DEVICE

Consider the following model of temporal price setting. Time is discrete, $t = 1, 2, \ldots$. There is a long-lived monopolistic firm that offers a product in each period. The firm can serve any number of consumers at no cost. The firm's strategy is an infinite price sequence $(p^1, p^2, \ldots)$, to which it commits at the outset. Its objective is to maximize its long-run average per-period profit. In every period t, a large population of short-lived consumers choose whether or not to enter the market. The decision is taken before they get to observe p^t. If a consumer enters the market in period t, he incurs a fixed entry cost $\varepsilon > 0$ and learns p^t upon entry. If he subsequently purchases the product, he pays p^t, consumes the product, and exits the market. In the next period, a new generation of consumers faces a similar dilemma. The consumer population is divided into two groups of equal size. A consumer of type θ is willing to pay $\theta \in \{h, l\}$, where $h > l > \varepsilon$. Thus, if the consumer enters and subsequently buys the product upon learning its price, his payoff is $\theta - p - \varepsilon$. If he enters and does not buy the product, his payoff is $-\varepsilon$, and if he does not enter his payoff is 0.

Let us turn to modeling the consumers' coarse reasoning. Each consumer type θ is also characterized by an integer T_θ. Given an infinite price sequence

$(p^1, p^2, \dots)$, consumer type θ can form a prediction of the price in period $t > T_\theta$ only on the basis of the T_θ *most recent prices*, $(p^{t-T_\theta}, \dots, p^{t-1})$—occasionally referred to as a "T_θ-truncated price history at period t." Specifically, given that the sequence of T_θ most recent prices is $(p^{t-T_\theta}, \dots, p^{t-1})$, the probability that the consumer assigns to $p^t = p$ is equal to the *long-run frequency* with which p follows the same sequence of T_θ most recent prices.[1] The consumer chooses to enter in period t if and only if the expected price, given his prediction, does not exceed $\theta - \varepsilon$.

Thus, although all consumers have access to the same actual price history, they differ in their ability to process the data. To use the "database" metaphor introduced above, when a consumer mines a long price history in order to make a prediction, he looks for patterns, and T_θ is an indicator of the maximal complexity of price patterns his data mining is able to elicit. An alternative interpretation is that the consumer himself is not necessarily bounded in his ability to mine the data, and T_θ is a measure of the maximal complexity of price patterns that the consumer is willing to attribute to the firm.[2]

Define $T^* = \max(T_l, T_h)$. For each consumer type $\theta \in \{l, h\}$ and for every $t > T^*$, there is a partition Π_θ of the set of finite price histories $(p^1, \dots, p^{t-1})$, such that two histories belong to the same partition cell if and only if they share the same sequence of T_θ most recent prices. This can be viewed as the consumer's *perceptual partition* of the set of finite price histories. As T_θ increases, the perceptual partition becomes finer. The correlation between the consumer's "preference type" θ and his "cognitive type," as captured by T_θ, will be important for the analysis.

If all consumers in the population had the same preference type θ, the monopolist would charge a price of $\theta - \varepsilon$ in each period. This is obvious in the case of fully rational consumers. To see why it extends to the case of "coarse" consumers, note that no matter how coarse a consumer's perception of the firm's strategy, by assumption he knows the long-run average price conditional on every T_θ-truncated price history. Since the consumer is risk-neutral, he will enter the market after a T_θ-truncated price history if and only if the long-run average price conditional on that truncated history does not exceed $\theta - \varepsilon$. Thus, when all consumers have the same willingness to pay, the firm does not have any incentive to obfuscate—that is, to create a complex temporal price pattern.

In the remainder of this section, I assume that the fraction of each of the two preference types l and h in the population of consumers who enter the market is $\frac{1}{2}$ in every period. A consumer's preference type is his private information. If all consumers were fully rational, the monopolist would be unable to discriminate among consumers because of their limited action space. The monopolist would choose between two constant price sequences: $p^t = h - \varepsilon$ for all t, or $p^t = l - \varepsilon$ for all t. It would opt for the former strategy whenever $2(l - \varepsilon) < h - \varepsilon$. We will now

[1] I set aside the question of whether this long-run frequency is a mathematically well-defined limit. Throughout this section, we will focus on cyclical price sequences, for which the limit *is* well-defined.

[2] Yet another, quite obvious interpretation is that T_θ represents a limitation on the consumer's *memory*. I will not pursue this interpretation here.

see that when consumers are "diversely coarse," the firm may have an incentive to create complex price patterns as a discrimination device.

8.2.1 "DeBruijn" Price Sequences

The consumers' coarse reasoning means that they will be unable to make perfect price predictions when the temporal price sequence obeys a rule that conditions on price realizations in historically distant periods. From now on, assume (without loss of generality) that prices can take only two values, L and H. Assume that the firm's strategy induces a price sequence that follows the cycle (L, L, H, H). If $T_\theta = 2$, the consumer can perfectly predict the price at each period: $p^t = H$ whenever $(p^{t-2}, p^{t-1}) \in \{(L, L), (L, H)\}$ and $p^t = L$ whenever $(p^{t-2}, p^{t-1}) \in \{(H, H), (H, L)\}$. However, for a consumer with $T_\theta = 1$, historical prices have no informational value: for every finite history $(p^1, \dots, p^{t-1})$ in which $p^{t-1} = p$, p^t is equally likely to be L or H, independently of whether $p = L$ or $p = H$.

Similarly, suppose that the firm's strategy induces a price sequence that follows the cycle (L, L, L, H, L, H, H, H). Then, a consumer with $T_\theta = 3$ can perfectly predict the price at each period, because it is uniquely determined by the three most recent price realizations. However, if $T_\theta = 2$, the consumer will be unable to predict the price, because the two most recent price realizations are entirely uninformative: given any (p^{t-2}, p^{t-1}), p^t is equally likely to be L or H.

In both examples, we constructed a cyclic price sequence with the following properties: (1) a consumer with $T_\theta = d$ can perfectly perceive the pattern and make correct predictions; (2) a consumer with $T_\theta < d$ can perceive no pattern, and in fact cannot even make a statistically valuable prediction that will make use of his access to historical price data. Such magical sequences are called *DeBruijn* sequences (after the mathematician Nicolaas Govert de Bruijn) and they will play an important role in our analysis.

Definition 8.1. *An infinite price sequence $(p^1, p^2, \dots)$ is a DeBruijn sequence of order T if for every period $t > T$:*

(i) The T-truncated history $(p^{t-T}, \dots, p^{t-1})$ uniquely determines p^t.
(ii) For every $p \in \{L, H\}$, the long-run frequency of $p^t = p$ conditional on any possible $(T-1)$-truncated history $(p^{t-T+1}, \dots, p^{t-1})$ is $\frac{1}{2}$.

Note that condition (*i*) in this definition implies that the price sequence follows a cycle, and therefore the long-run frequencies referred to in condition (*ii*) are well-defined limits.

Proposition 8.1. *For every $T = 1, 2, \dots,$ there exists a DeBruijn sequence of order T.*

Proof. For $T = 1$, a DeBruijn sequence is simply $(L, H, L, H, \dots)$. We have already demonstrated the existence of a DeBruijn sequence for $T = 2, 3$. Let $T > 3$. We will make use of the following auxiliary structure. A directed graph is a pair

(V, E), where V is a finite set of *nodes* and $E \subseteq V \times V$ is a set of *directed links*, where $(x, y) \in E$ means that there is a link from the node x into the node y. Define $V = \{L, H\}^{T-1}$. For every node $x = (a_1, \ldots, a_{T-1})$ and any $a \in \{L, H\}$, assume that there is a link from x into $(a_2, \ldots, a_{T-1}, a)$. This graph is connected. It is also "regular"—for every node x, there are two links that go into x and two links that go out of x. A basic result in Graph Theory states that connectedness and regularity are necessary and sufficient for the existence of an "Euler cycle," that is, a path that passes through every link exactly once. The length of this path given our graph is 2^T.

We can now define an infinite, cyclic price sequence (p^t) entirely in terms of the links in the Euler cycle, where the t^{th} link in the Euler cycle is from $(a_1, \ldots, a_{T-1})$ into $(a_2, \ldots, a_{T-1}, p^t)$. The constructed sequence satisfies the two conditions of a DeBruijn sequence. First, since the length of the Euler cycle is 2^T, this is also the length of the cycle in the price sequence. Second, by the definition of an Euler cycle, it passes through every link in the graph exactly once. Therefore, knowledge of the $T - 1$ most recent price realizations conveys no information about the next price, because it is equivalent to knowing only that the next link in the cycle will originate from a particular node. □

The existence of DeBruijn sequences of an arbitrary order has a strong implication in our context. Suppose that the two consumer types differ in their degree of coarseness: $T_h \neq T_l$. The monopolist can adopt a pricing strategy that induces a DeBruijn sequence of order $T = \max(T_h, T_l)$. The consumer type θ with the higher T_θ will be perfectly able to predict prices in each period, while the other consumer type will be completely unable to make valuable predictions because he will regard both price levels to be equally likely after every history. In other words, the complex price pattern will confuse only the coarser consumer, in a way that may enable the monopolist to implement profitable price discrimination.

Exercise 8.1. *Construct a DeBruijn sequence of order 4.*

8.2.2 Conditions for Profitability of Complex Price Patterns

Let us now check whether the monopolist does in fact have an incentive to obfuscate in such a manner. It turns out that the correlation between the consumer's preference and cognitive types is crucial for this analysis. First, let us show that when the consumer type with the highest willingness to pay is also the more sophisticated consumer type, obfuscation is unprofitable.

Proposition 8.2. *If $T_h \geq T_l$, then a constant pricing strategy (i.e., $f(\omega) = p$ for all $\omega \in \Omega$) maximizes the monopolist's long-run profit.*

Proof. Suppose that $T_h \geq T_l$. Then, if type h is unable to distinguish between two price histories of length $t \geq T_h$, so is type l. In other words, Π_h is a refinement of Π_l. Now consider a cell in Π_l, in which type l chooses to enter the market.

The long-run average price in the cell is at most $l-\varepsilon$. Therefore, the monopolist's long-run average profit conditional on that cell is at most $l-\varepsilon$. Similarly, consider a cell in Π_l, in which this type chooses not to enter the market. Since Π_h is a refinement of Π_l, we can partition the cell into sub-cells, where each sub-cell belongs to Π_h. If the average price in any of these sub-cells exceeds $h-\varepsilon$, type h's decision at that sub-cell is not to enter. Since the fraction of type l in the consumer population is $\frac{1}{2}$, the monopolist's long-run average profit conditional on a cell in which l chooses not to enter is at most $\frac{1}{2}(h-\varepsilon)$. We conclude that the monopolist's long-run average profit cannot exceed $\max[l-\varepsilon, \frac{1}{2}(h-\varepsilon)]$. Therefore, one of the two constant pricing strategies, $f(\omega)=h-\varepsilon$ for all ω, or $f(\omega)=l-\varepsilon$ for all ω, is necessarily optimal. $\square$

The intuition for this result is simple. The benefit from *not* confusing a consumer of type θ is that he can predict when the market price will exceed $\theta-\varepsilon$ and stay off the market in those periods. If the firm wants to enable type l to make perfect predictions, it will be able to profit no more than $l-\varepsilon$ from him when he enters the market. But when type h is more sophisticated than type l, type h will be able to make perfect price predictions whenever type l can. This means that in each period, the monopolist effectively faces a choice between charging $l-\varepsilon$ from both consumer types and charging $h-\varepsilon$ from type h. But this is the same dilemma that the monopolist faces when constrained to constant pricing strategies. Hence, the monopolist has no incentive to obfuscate.

Let us now turn to the opposite case, where the consumer type with the *lowest* willingness to pay is the more sophisticated type. One justification for this correlation is that a more sophisticated consumer has better outside options. It turns out that if entry costs are sufficiently high, the monopolist has an incentive to obfuscate. In particular, a DeBruijn sequence of prices is strictly better for the monopolist than a constant pricing strategy.

Proposition 8.3. *Let $T_h < T_l$ and $\varepsilon > \max(h-l, 2l-h)$. Then, a DeBruijn sequence of order T_l, consisting of the prices $L=l-\varepsilon$ and $H=(h-\varepsilon)+(h-l)$, generates a long-run average profit strictly above $\max[l-\varepsilon, \frac{1}{2}(h-\varepsilon)]$, hence it is strictly better for the monopolist than any constant pricing strategy.*

Proof. Let us see how the two consumer types respond to this pricing strategy. By the definition of a DeBruijn sequence of order T_l, type l is perfectly able to predict the price in each period, while type h is completely unable to predict prices and believes that in each period the price is equally likely to be L or H. Since $L=l-\varepsilon<H$, type l will enter only when the price is L. Because the long-run frequency of L is exactly $\frac{1}{2}$, the monopolist's long-run average profit from type l is $\frac{1}{2}L$. Since $\frac{1}{2}(L+H)=h-\varepsilon$, type h is willing to enter the market in every period. And since $L<H<h$, he will buy the product whenever he enters. As a result, the monopolist's long-run average profit from this type is $\frac{1}{2}(L+H)$. The monopolist's expected long-run profit is thus $\frac{1}{2}[\frac{1}{2}L+\frac{1}{2}(L+H)]$. By the assumption that $\varepsilon > 2l-h$, the optimal constant pricing strategy is to set $p=h-\varepsilon$ in every period. It immediately follows that the monopolist's total long-run profit exceeds the maximal profit it can generate from constant pricing strategies. □

The idea behind this construction is to use a price pattern that confuses type h only. The existence of DeBruijn sequences means that we can construct such a price pattern. Type l understands the price pattern and enters the market only when this is the optimal action. In contrast, type h fails to perceive any pattern and reasons only in terms of the long-run average price. The complex price pattern effectively achieves price discrimination. As in conventional models of second-degree price discrimination, there is inefficient trade with the "low" preference type—he enters the market only half the times—and his consumer surplus is fully extracted. This allows the monopolist to extract more surplus from efficient trade with the "high" preference type. What is unconventional is that the discrimination is implicit. The two consumer types face the *same* complex pricing strategy. However, their different abilities to perceive it turn it into a discriminatory device.

The assumption that the type with the highest willingness to pay is also the less sophisticated type is crucial for this unconventional form of price discrimination. Prices fluctuate between two levels, L and H. Type h is willing to pay both prices upon entry. If this type could perfectly predict the price, he would find that $H + \varepsilon > h$, and therefore would not enter when the price is H. However, since he cannot perceive any pattern in the price fluctuations, he compares the entry cost with the *expected* gain from entering, and finds that they are equal: $\varepsilon = h - \frac{1}{2}(L + H)$. The L realizations leave him with a surplus that compensates him for the loss he makes in the H realizations.

The existence of entry costs is also crucial for this analysis. It creates a wedge between the consumer's optimal decisions when he observes the price realization and the optimal decisions before he learns the price, when he has to make do with a prediction. This is what gives consumers with high predictive skills the advantage over less sophisticated consumers.

The strength of Proposition 8.3 is that even when the difference between the coarseness of the two consumer types' reasoning is minimal, obfuscation is optimal. However, the result is limited in two respects. First, it only works under particular restrictions on entry costs. Second, it merely shows that the monopolist has a strict incentive to obfuscate, but it does not derive the optimal obfuscatory pricing strategy, because in a DeBruijn price sequence, both L and H are realized with a long-run frequency of $\frac{1}{2}$. Suppose that the optimal pricing strategy induces a cyclic price sequence in which the long-run frequency of L is some arbitrary rational fraction α. Can we extend the notion of DeBruijn sequences to such pricing strategies, in a way that type l will be perfectly able to predict the price in each period, while type h will only be able to predict in each period that the probability of L is α? The answer is positive, as long as T_l is sufficiently large relative to T_h.

I omit the general proof of this claim. Instead, I illustrate it with a simple example. Let $T_h = 1$, and suppose that we wish to sustain $\alpha = \frac{2}{3}$. Extend the idea behind the construction of DeBruijn sequences as follows. Create two "copies" of the low price realization and denote them L_1 and L_2. The copies are relevant only for the monopolist's construction of the price sequence; consumers who observe the price sequence *cannot* distinguish between L_1 and L_2. Construct the complete directed graph in which the set of nodes is $\{L_1, L_2, H\}$, where each node a represents a price history in which the most recent realization is a. The total number of links in this graph is nine, because there is a link from every node to itself as well as the two other nodes. There is an Euler cycle through this complete graph, which generates the following cyclic pattern of price realizations: $(L_1, H, H, L_2, L_2, H, L_1, L_1, L_2)$. It is easy to verify that since $T_h = 1$, type h can only predict after every price history that the probability of L in the next period is $\frac{2}{3}$. In contrast, as long as $T_l \geq 4$, type l will be able to predict the price in every period.

Exercise 8.2. *Assume $T_h = 2$. Construct a cyclic price sequence, such that after every price history, type h can only predict that the probability of L is $\frac{2}{3}$. Find a value T_1^* such that whenever type* 1 *has $T_1 \geq T_1^*$, he can make a perfect prediction of the price in each period.*

Thus, if T_l is sufficiently high relative to T_h, we can reduce the problem of optimally discriminating between the two consumer types into the following constrained maximization problem:

$$\max_{\alpha, L, H} \quad \alpha L + [\alpha L + (1-\alpha)H]$$

subject to the constraints

$$\begin{aligned} L &\leq l - \varepsilon \\ \alpha L + (1-\alpha)H &\leq h - \varepsilon \\ H &\leq h \end{aligned}$$

Solving this problem is left for the reader as an exercise.

8.3 LIMITED UNDERSTANDING OF ADVERSE SELECTION

One of the most fundamental sources of market failure is the phenomenon of adverse selection—namely, the fact that in the presence of informational asymmetries between buyers and sellers, the sample of traded goods is biased against the interests of the uninformed side. For instance, consumers who choose to purchase health insurance tend to have higher health risks relative to the general population. Similarly, a used car is more likely to be in a bad mechanical state given that its owner is willing to sell it at the market price.

Adverse selection alone does not give rise to market failure. It is the sophistication of market agents, who are aware of the selection, that causes efficient trade to break down. In this section I pose the following question: How does this classical result change when consumers' understanding of the market equilibrium is less than perfect, in the sense of being "coarse" ? The primary pedagogical value of this exercise is that it improves our understanding of conventional adverse-selection logic. Instead of taking it wholesale as an "automatic" consequence of Bayesian Nash equilibrium, we will try to understand the types of market sophistication that it embodies.

One could argue that given the pervasiveness of adverse selection, consumers should be aware of this phenomenon and therefore display a "rational" understanding of its implications for product quality. For example, in the case of used cars, it would be hard to find real-life consumers without an intuitive perception of the problem. However, not all adverse-selection situations are equally transparent. Consumers—particularly inexperienced ones—are not aware of every incentive that pushes firms to provide one type of product rather than another. If consumers' feedback as they gain market experience is partial, they may end up having a coarse perception of the correlation between a firm's willingness to sell a product at a given price and the product's quality.

8.3.1 A Buyer-Seller Example

Consider the following market situation with adverse selection. For once in this book, I abandon the case in which consumers are price takers, and instead focus on a buyer-seller situation in which both parties interact on an ad-hoc basis (as, for example, in the case of online auctions for used articles). Nevertheless, the example is meant to capture in a pedagogically useful way the essence of competitive market situations in which consumers face a large number of firms that sell a product whose quality is known only to the firms.

A seller enters the market aiming to sell a single object. A buyer enters with the intent to buy the object. The seller's valuation of the object, denoted ω, is drawn uniformly from $\Omega = [0, 1]$ and represents the object's quality. The value of ω is the seller's private information. The buyer's valuation of the object is $v = \omega + b$, where $b \in (0, 1)$ is a parameter that measures the gains from trade. Since $b > 0$, trade is always efficient, independently of the state.

The market mechanism is a simple two-sided auction. The buyer and seller submit a bid p and an ask price a, respectively. If $p \geq a$, trade takes place at the price p. If $p < a$, trade does not occur. It follows that the seller has a weakly dominant strategy—namely, to submit an ask price equal to his valuation. From now on, we will assume that the seller follows this strategy (formally, $f(\omega) = \omega$) and focus on the buyer's considerations. The buyer's coarse reasoning is captured by a *perceptual partition* $\Pi = \{\Pi_1, \ldots, \Pi_K\}$ of Ω. The buyer's coarse representation of the seller's strategy f is a mixed strategy σ^{Π} defined as follows: for every $\omega \in [0, 1]$, σ^{Π} mixes uniformly over $\Pi(\omega)$.

Recall the learning-feedback interpretation of the perceptual partition, invoked earlier in this chapter. In the present context, a plausible interpretation of Π is that a state ω corresponds to a vector of multiple attributes, and the consumer's feedback from past buyer-seller interactions contains information about a subset of these attributes.

8.3.2 A Benchmark: A Bayesian-Rational Buyer

Let us first assume that the buyer understands the market model and knows the seller's strategy. That is, $\Pi(\omega) = \{\omega\}$ for all $\omega \in \Omega$. The buyer chooses his bid p to maximize the following expression:

$$\Pr(\omega \leq p) \cdot [E(v \mid \omega \leq p) - p]$$

where $\Pr(\omega \leq p)$ is the probability that trade takes place because it is the probability that the seller's ask price does not exceed the buyer's bid, and $E(v \mid \omega \leq p)$ is the buyer's expected valuation of the object conditional on trade taking place.

Since $v = \omega + b$ and ω is uniformly distributed over $[0, 1]$, $\Pr(\omega \leq p) = p$ and $E(v \mid \omega \leq p) = \frac{1}{2}p + b$. Therefore, the buyer chooses p to maximize $p(b - \frac{1}{2}p)$, yielding a solution $p_r^* = b$. We can see that adverse selection indeed causes market failure. While the efficient outcome is to have trade in every state, in this market equilibrium trade occurs only when the seller's valuation is weakly below b. As b tends to zero, the probability of trade converges to zero.

8.3.3 A "Coarse" Buyer

Let us now turn to the case of a "fully coarse" buyer, for whom the perceptual partition consists of a single cell: $\Pi(\omega) = \Omega$ for all $\omega \in \Omega$. The question is whether this failure to perceive the correlation between the seller's action and his private information prevents adverse selection from causing market failure.

Given that the seller's strategy is $f(\omega) = \omega$, the buyer's coarse perception of f is that the seller uniformly randomizes over the set of ask prices $[0, 1]$, independently of ω. One justification for this coarse belief in the context of the present buyer-seller example is that when the buyer enters the market, he has access to records of all the ask prices that were previously submitted by the seller (or his previous incarnations), but he does not have access to the records of the valuations that lay behind these ask prices. Following the "Occam's razor" principle, the buyer adopts the simplest theory that is consistent with the historical records, where simplicity here means that the theory is not allowed to depend on unobservable variables as long as it is consistent with the data.

The buyer chooses his bid p to maximize the following expression:

$$\Pr[f(\omega) \leq p] \cdot [E(v) - p]$$

where $\Pr[f(\omega) \leq p]$ is the probability that trade takes place because it is the probability that the seller's ask price does not exceed the buyer's bid, and $E(v)$ is the buyer's unconditional expected valuation of the object. Note that the buyer gets $\Pr[f(\omega) \leq p]$ right because it is only a function of the seller's marginal distribution over ask prices, and not of how the ask price depends on the underlying state.

It follows that the buyer chooses p to maximize $p \cdot (\frac{1}{2} + b - p)$, hence his best-reply to the "fully coarse" perception of f is $p_c^* = \frac{1}{2}(b + \frac{1}{2})$. Note that p_c^* is higher than the rational-consumer benchmark price p_r^* if and only if $b < \frac{1}{2}$. Thus, when the gains from trade are relatively small, the "fully coarse" buyer submits a higher bid and trades with higher probability than a Bayesian rational buyer. However, when the gains from trade are large, the comparison is reversed.

The reason for this ambiguity is that a fully coarse perception of the seller's strategy has two contradictory effects. On one hand, the coarse buyer's expected valuation of the object is higher than in the benchmark case because he fails to take adverse selection into account. This raises the buyer's bid relative to the benchmark. On the other hand, the buyer does not realize that if he raises his bid, this will enhance the expected quality of the traded object. This lowers the buyer's bid relative to the benchmark. When the gains from trade are small, the former consideration outweighs the latter.

When $b = 0$, the Bayesian-rational benchmark implies no trade. Nor is trade required by efficiency. In contrast, the fully coarse buyer trades with probability $\frac{1}{4}$. As in other models examined in this book, we see an example of a *spuriously active market* as a result of consumers' limited understanding of the market equilibrium.

The following pair of examples provide further demonstration of the subtle implications of coarse reasoning on the probability of trade. In these examples, we fix $b = 0$ and examine "partially coarse" perceptual partitions. First, suppose that the buyer's partition divides Ω into K equal intervals.

Proposition 8.4. *Let* $\Pi = \left\{\left[0, \frac{1}{K}\right), \left[\frac{1}{K}, \frac{2}{K}\right), \ldots, \left[\frac{K-1}{K}, 1\right]\right\}$. *The buyer's optimal action given this partition is* $p^* = \frac{1}{4K}$.

Proof. For every $k = 1, \ldots, K$, denote $\pi_k = \left[\frac{k-1}{K}, \frac{k}{K}\right)$. The seller's strategy is $f(\omega) = \omega$. The buyer's coarse perception of f is given by a mixed strategy $\sigma:[0, 1] \to \Delta[0, 1]$, where $\sigma(\omega)$ is the uniform distribution over $\Pi(\omega)$. It follows that conditional on trade at a price $p \in \pi_k$, the buyer believes that the object's expected value is $\frac{1}{2K}(2k - 1)$. The probability of trade as a function of the buyer's bid p, conditional on the state being in π_k, is:

$$\Pr[f(\omega) \le p \mid \pi_k] = \begin{cases} 1 & \text{if } p \ge \frac{k}{K} \\ pK - (k-1) & \text{if } p \in \pi_k \\ 0 & \text{if } p < \frac{k-1}{K} \end{cases}$$

The buyer's problem is thus to find a bid p that maximizes the following expression:

$$\frac{1}{K}\sum_{k=1}^{K} \Pr[f(\omega) \le p \mid \pi_k] \cdot \left[\frac{2k-1}{2K} - p\right]$$

To solve this problem, suppose that $p \in \pi_{l+1}$ for some integer $l > 0$. Then, the buyer's perceived expected payoff can be written as follows:

$$\frac{1}{K}\left[(pK-l)\left(\frac{2l+1}{2K}-p\right)+\sum_{k=1}^{l}\left(\frac{2k-1}{2K}-p\right)\right]$$
$$=\frac{1}{K}\left[(pK-l)\left(\frac{2l+1}{2K}-p\right)+\frac{l(l+1)}{2K}-l\left(p+\frac{1}{2K}\right)\right]$$
$$=\frac{1}{2K^2}[-2p^2K^2+pK(2l+1)-l(l+1)].$$

It is now straightforward to check that this expression is strictly negative for $l = 1, 2, \ldots$. It follows that $l = 0$. The objective function thus boils down to $p(\frac{1}{2K}-p)$, hence $p^* = \frac{1}{4K}$. □

Thus, given the buyer's optimal strategy, trade occurs if and only if $\omega \leq \frac{1}{4K}$. The probability of trade decreases with K and converges to zero as K tends to infinity. In this case, the amount of trade decreases monotonically with the buyer's level of sophistication: the finer his perception of the seller's strategy, the lower the probability of trade. The following exercise shows that this effect is not general.

Exercise 8.3. *Let $b = 0$ and $\Pi = \{[0, d), [d, 1]\}$. Show that if $d \in (0, \frac{1}{6})$, the buyer's optimal action given this partition is $p^* = \frac{1}{4}(1+d)$, such that trade occurs if and only if $\omega \leq \frac{1}{4}(1+d)$.*

Recall that when the buyer is rational, the probability of trade is zero. At the other extreme, when the buyer's perceptual partition is fully coarse, trade occurs if and only if $\omega \leq \frac{1}{4}$. The case in the exercise represents an intermediate case in which the buyer's perceptual partition is partially coarse. Nevertheless, the probability of trade is higher than in the case of a fully coarse buyer. Thus, the probability of trade can behave non-monotonically with respect to the buyer's level of coarseness when we fix the gain from trade.

8.3.4 Action-Dependent Feedback

Recall that one interpretation of our model of a coarse buyer was that the buyer's evaluation of various bidding strategies is based on feedback he receives about their past performance. Since the learning feedback is coarse, the buyer ends up evaluating the strategies as if he optimizes with respect to a coarse representation of the seller's strategy f. The coarse-feedback interpretation, however, relies on the implicit assumption that the learning feedback that the buyer's predecessors generate is independent of their actions. In this sub-section I describe a plausible feedback mechanism that violates this assumption, and therefore goes beyond the coarse reasoning framework adopted in this chapter.

As before, suppose that the buyer has access to a very large "database" containing historical records of previous buyer-seller interactions. These records include all past sellers' ask prices and buyers' bid prices. However, the records of past buyers' valuations of the object are incomplete: *they are available only for past periods in which trade actually took place*. When trade fails to take place, no feedback about the valuation is generated. Under this scenario, the buyer can correctly evaluate the probability of trade for every bid p—namely, $\Pr(\omega \leq p)$—because he has access to a large, unbiased sample of randomly generated ask prices. In contrast, the buyer's sample of past object valuations is biased, and the question is how he extrapolates from it.

Let us examine a "steady state" of the learning process, in which the buyer's predecessors all follow a constant bid p^*. Recall that the seller's strategy throughout the learning process is $f(\omega) = \omega$. Therefore, the object's expected value in the buyer's incomplete "database" is $E(v \mid \omega \leq p^*)$. We will assume that the buyer's extrapolation from this sample is *naive*, in the sense that he uses $E(v \mid \omega \leq p^*)$ to evaluate the object, independently of the bid p he considers. Therefore, the buyer chooses p to maximize

$$\Pr(\omega \leq p) \cdot [E(v \mid \omega \leq p^*) - p] = p\left[\frac{1}{2}p^* + b - p\right] \qquad (8.1).$$

The value of p that maximizes this expression is $p = \frac{1}{4}p^* + \frac{1}{2}b$. In order for p^* to be consistent with a steady state, it must be a solution to this maximization problem. In other words, p^* is a fixed point of the biased learning process that successive generations of buyers go through. It follows that $p^* = \frac{2}{3}b < p_r^*$.

The conclusion is that the buyer's bounded rationality exacerbates the market failure caused by adverse selection. The reason is that of the two effects highlighted in the previous sub-section, only one remains: the buyer's failure to perceive that if he changes his bid, the expected quality of the traded object will change. The other effect—namely, the buyer's failure to perceive adverse selection—is effectively eliminated, because the buyer's evaluation of the object's expected quality conditional on trade is correct, as it is based only on feedback from those periods in which the seller found it optimal to trade.

At a methodological level, this exercise demonstrates the limitations of the coarse reasoning framework developed in this chapter, as it does not allow the learning feedback that generates the consumer's representation of the firm's strategy to depend on the consumer's own actions.

8.4 SUMMARY

Let us summarize the lessons we can draw from the models examined in this chapter.

- When consumers differ in the degree of coarseness of their representation of a complex pricing strategy, firms may have an incentive to create complex price patterns as a discriminatory device. However, whether such obfuscation is profitable depends on the correlation between consumers' "preference type" and "cognitive type."

- When consumers have a coarse perception of pricing strategies in markets with asymmetric information, this gives rise to two opposing effects. On one hand, consumers underestimate the negative relation between the firms' displayed willingness to sell and the quality of its product. On the other hand, they underestimate the positive relation between product quality and their own displayed willingness to buy. This ambiguity means that the probability of Pareto-efficient trade is not monotone with respect to the magnitude of consumers' coarse reasoning, and it is not necessarily higher than in the rational-consumer benchmark.

8.5 BIBLIOGRAPHIC NOTES

The broad framework for analyzing consumer behavior in this chapter, which I presented rather informally at the beginning of this chapter, adapts the game-theoretic solution concept of "Analogy-Based Expectations Equilibrium" (ABEE) due to Jehiel (2005). (Jehiel's framework is also broader in the sense that it does allow the players' own actions to affect the learning feedback.) The price discrimination model is due to Piccione & Rubinstein (2003). For graph-theoretic background on DeBruijn sequences, see Bondy & Murty (1976).

The basic buyer-seller example of adverse selection under coarse reasoning is due to Eyster & Rabin (2005). My presentation of the example broadly follows Esponda (2008). Eyster & Rabin (2005) use the example to demonstrate a solution concept called "cursed equilibrium," which is closely related to ABEE. The case of partially coarse buyers follows Jehiel & Koessler (2008). The example of coarse reasoning with biased feedback is due to Esponda (2008), who uses it to motivate a solution concept, called "behavioral equilibrium," which captures the idea that agents' decisions influence the sample from which they derive their expectations, and that their lack of sophistication may affect how they extrapolate from their sample. The three solution concepts are applied to game-theoretic settings outside the realm of Industrial Organization, such as auctions with incomplete information and extensive-form games.

The idea that firms can use complex price schedules to discriminate between consumers with diversely coarse perceptions was first introduced by Rubinstein (1993). An important difference from the present chapter is that in Rubinstein (1993), only the cardinality of the perceptual partition is exogenously given to the consumer, but the actual partition is optimally designed ex ante, given a correct understanding of the firm's pricing strategy. In this sense, Rubinstein (1993) follows Dow (1991), who first presented the idea of an optimally designed coarse perception of a price distribution. The reason that I did not include this material in this chapter is that it has already received comprehensive treatment in Rubinstein (1998, pp. 87–107). The reader is encouraged to consult that book chapter as an important complement to the present chapter.

PART THREE

Reference Dependence

9 Loss Aversion

One of the main themes of this book is the instability and context dependence of consumer preferences. In Chapters 2–4, we examined the implications of instability of consumer preferences through the course of dynamic problems in which consumers first choose a price scheme and then make a consumption decision given the selected price scheme. In Part III we turn to market models in which consumer choice is sensitive to *reference points*.

A reference point is an action or a consequence (or some aspect of either) which acts as a "frame" of a choice problem and affects choices in a way that is ruled out by conventionally rational decision making. Because reference points can change over time and can also be manipulated by an outside party, they give rise to preference instabilities. The following are a few salient examples of types of reference points and their effects on choice.

Example 9.1 (Default options)**.** Decision makers often face choice problems in which one feasible alternative is singled out as a status quo or a default option. If the decision maker refrains from taking an active decision, the default will prevail. Even when the physical costs of switching away from the default are negligible, decision makers often exhibit a strong tendency to choose the default option. For example, when workers choose their retirement savings plan, they tend to follow default specifications (in terms of enrollment, contribution rates, and asset allocation).

Example 9.2 (Historical values)**.** In the context of dynamic choice under uncertainty, historical values affect decision makers' choices in various ways. For instance, a casino gambler whose current balance is in the red (relative to his initial endowment before entering the casino) is likely to take greater risks, such as "double or nothing" bets. Similarly, investors in financial markets tend to avoid selling shares whose price has dropped relative to the original purchase level.

Example 9.3 (Expectations)**.** When decision makers act in response to the realization of some exogenous uncertain event, their initial expectations often act as reference points that affect their response. For example, when a prospective worker receives a take-it-or-leave-it wage offer, he may be more likely to reject the offer if it is less generous than he initially expected.

Example 9.4 (Anchoring)**.** Many choice problems involve judgment of quantities (consider a judge who needs to determine a sentence for a convicted felon, or an administrator who needs to fix the size of a particular budget). An anchor is a value that is suggested to the decision maker by the designer of the choice problem or some other party, which exerts a pull on the decision maker's judgment even when it is arbitrary and devoid of any informational content. For instance, if the prosecutor in a trial demands a particular sentence of T years, the judge will tend to evaluate alternative sentences in relation to this demand, and his final judgment will tend to be closer to T than in the absence of such an anchor.

One of the greatest contributions of experimental psychology to economics has been the demonstration of the role of reference dependence in individual decision making, and the formulation of general principles and concepts that organize the plethora of reference-point effects. The most well-known of these concepts is *loss aversion*. It is based on two ideas. First, in some contexts, decision makers categorize outcomes as gains or losses relative to a reference point. Second, losses register more powerfully than gains in decision makers' psyche, so that when gains and losses are traded off, the latter are weighted more heavily than the former.

Loss aversion is a broad theoretical construct that is able to capture a variety of specific psychological phenomena. It was originally introduced to explain aspects of choices between lotteries over a unidimensional variable (typically monetary lotteries, as in the case of Example 9.2). The concept was later extended to encompass reference-point effects in riskless choice problems, as well as multidimensional outcomes.

In this chapter I study two market adaptations of loss aversion, in which consumers' *expectations* act as reference points (as in Example 9.3). In the first class of models, consumers evaluate the *price* of products offered on the market relative to a reference point, which is the price they expected upon entering the market. When the actual price exceeds the consumer's expectation, he records the difference as a loss, and this lowers his willingness to pay for the product. In the second class of models, *consumption quantity* is recorded as a gain or a loss relative to the consumer's expected consumption quantity.

9.1 EXPECTED PRICE AS A REFERENCE POINT: MONOPOLY PRICING

Let us turn to our first attempt to apply the theory of loss aversion to an Industrial Organization setting. This attempt is based on the idea that when consumers in retail markets are required to pay a price that exceeds their expectations, they experience a disutility that diminishes their willingness to pay for the product. This disutility may reflect the consumer's perception that unanticipatedly high prices constitute a departure from *fair pricing*. This interpretation is consistent with the observation that when firms raise prices, they often try to provide reasons for this move, which suggests a need to overcome consumers' perception that such a move is unfair. However, this element of justification will be absent from our model.

In Part II of the book we emphasized market situations in which price schemes are complex and hard to fathom, either because of the products' inherent complexity or as a result of firms' obfuscation tactics. In these situations, prices are unlikely to act as reference points because consumers are unclear about what prices to expect. Expected prices emerge as reference points more naturally when the product is inherently simple, firms do not obfuscate, and although prices may fluctuate, an individual price realization is clear and easy to recall.

I begin with a model of monopoly pricing. My objective is to explore the implications of consumer loss aversion for the way a monopolist adjusts its price to cost fluctuations. In the model, the monopolist sells a single unit of a product to a single consumer with uncertain willingness to pay. The monopolist's marginal cost c is distributed uniformly over some set of possible values C. Suppose for now that C is finite. Denote $|C| = m > 1$. Let c^h and c^l denote the highest and lowest cost values in C, $1 > c^h > c^l > 0$. Let $\bar{c}$ denote the average cost in C. The monopolist commits to a deterministic pricing strategy $P : C \to \mathbb{R}$, where $P(c)$ is the price the monopolist charges when the marginal cost is c. The randomness of the firm's marginal cost and the firm's pricing strategy induce a probability measure μ_P over prices, where

$$\mu_P(x) = \frac{|\{c \in C \mid P(c) = x\}|}{m}$$

The consumer's choice procedure is as follows. A reference price p^e is randomly drawn from μ_P. The consumer buys the product if and only if the actual price p satisfies $p \leq v - L(p, p^e)$, where v is the consumer's "raw" willingness to pay for the product, and L is a loss function that satisfies several properties: (i) it is weakly increasing in p and weakly decreasing in p^e; and (ii) $L(p, p^e) = 0$ whenever $p \leq p^e$. Assume that v is uniformly distributed over $[0, 1]$.

The assumption that p^e is randomly drawn from μ_P requires justification. Clearly, there are many other ways to model the formation of the reference points. For instance, one could assume that the expected price according to μ_P is the consumer's reference point. The interpretation of "sampling-based" reference point formation is that the consumer creates his expectations on the basis of his own past market experience, or on the basis of the random experience of another consumer he interacts with. This is in the spirit of the models of sampling-based reasoning explored in Chapters 6 and 7.

"Sampling-based" reference point formation implies that consumers can differ in two dimensions. First, they have different "raw" willingness to pay v for the product. Second, they have different market experiences that lead to different expectations and therefore different reference prices. Thus, the "sampling-based" formation of reference points enriches our conception of a consumer's "type"; moreover, it means that the firm can influence, through its pricing decision, the distribution of consumer types. By assumption, the two aspects of the consumer's type—his raw willingness to pay and his reference point—are independently distributed.

The monopolist's maximization problem can be written as follows. The firm chooses the function P to maximize

$$\Pi(P(c)_{c\in C}) = \frac{1}{m^2} \cdot \sum_{c} \sum_{c'} [P(c) - c] \cdot \max[0, 1 - P(c) - L(P(c), P(c'))]$$

To see why this is the objective function, note that for each realization of the marginal cost c, the firm is uncertain about the consumer's willingness to pay. Since his "raw" willingness to pay v is distributed uniformly over [0, 1], in the absence of loss aversion (i.e., if L always takes the value zero), the probability that the consumer buys the product at a price $P(c)$ is $1 - P(c)$. When loss aversion is incorporated, this probability drops to $\max[0, 1 - P(c) - L(P(c), P(c'))]$. Since the reference price $P(c')$ is drawn from μ_P, we need to sum over all possible values of c'.

When L always gets the value zero, the optimal pricing strategy is

$$P^0(c) = \frac{1+c}{2}, \quad \forall c \in C \tag{9.1}$$

In comparison, if the firm were restricted to charging a constant price for all cost values, the optimal price would be

$$\bar{p} = \frac{1+\bar{c}}{2} \tag{9.2}$$

independently of L (because by assumption $L(p, p) = 0$).

Note that in the presence of loss aversion, consumer demand is a function of the monopolist's price distribution. Specifically, given a pricing strategy P, the fraction of consumers who buy the firm's product at a price p is

$$D_P(p) = \frac{1}{m} \sum_{c'} \max[0, 1 - p - L(p, P(c'))] \tag{9.3}$$

Thus, aggregate consumer demand depends on the price distribution as a whole. When the firm changes its price in one cost state, this affects (probabilistically) the consumer's reference point, hence consumer demand at other prices. In this sense, local price changes have a global effect on consumer demand. Note that D_P is strictly decreasing, as usual, in the range in which it is strictly positive.

9.1.1 Reduced Price Variability

The following preliminary characterization of optimal pricing strategies will be useful in the sequel.

Lemma 9.1. *Let P be an optimal pricing strategy. Then, for every $c \in C$, $P(c) < 1$ and consumer demand (given by (9.3)) is strictly positive at $P(c)$.*

Proof. Let P be an optimal pricing strategy. Let us first show that $P(c) < 1$ for all $c \in C$. Assume, contrary to this claim, that $P(c^*) \geq 1$ for some $c^* \in C$. Then, consumer demand is zero at $P(c^*)$. Suppose that the firm deviates to a pricing strategy P' such that $P'(c^*) = 1 - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small, and $P'(c) = P(c)$ for all $c \neq c^*$. Following the deviation, consumer demand at $P'(c^*)$

is strictly positive, because $1 - P'(c^*) - L(P'(c^*), P'(c^*)) = \varepsilon > 0$. Furthermore, for every $c \neq c^*$,

$$\max[0, 1 - P'(c) - L(P'(c), P'(c^*))] = \max[0, 1 - P(c) - L(P(c), P(c^*))]$$

hence the deviation does not change consumer demand at any $c \neq c^*$. It follows that the deviation is profitable. Then, for every $c \in C$, $P(c) < 1$. Since $L(P(c), P(c)) = 0$, consumer demand is strictly positive at $P(c)$. □

The intuition for the result that aggregate consumer demand is always strictly positive is simple: for every price realization $p \in \{P(c)\}_{c \in C}$, there is positive probability that an individual consumer's reference price p^e is weakly higher, such that $L(p, p^e) = 0$. The next lemma relies on this result.

Lemma 9.2. *Every optimal pricing strategy P is weakly increasing in c and satisfies $P(c) \geq c$ for every $c \in C$.*

Proof. Let P be an optimal pricing strategy. Suppose that $c_1 > c_2$ and yet $P(c_2) > P(c_1)$. Denote $P(c_1) = p_1$ and $P(c_2) = p_2$. Suppose that the firm switches to a pricing strategy P' that coincides with P for all $c \neq c_1, c_2$, and $P'(c_1) = p_2$, $P'(c_2) = p_1$. The change in the firm's objective function as a result of the deviation is

$$\frac{1}{m} \sum_c [(P'(c) - c) \cdot D_{P'}(P'(c)) - (P(c) - c) \cdot D_P(P(c))]$$

The deviation has the property that it does not alter the induced price distribution; that is, $\mu_P = \mu_{P'}$. Therefore, the deviation does not change consumer demand: $D_{P'}(x) = D_P(x)$ for every price x. Accordingly, let us omit the subscript of the demand function. We can thus rewrite the above expression as follows:

$$\frac{1}{m} \sum_c \{[P'(c) - c] D(P'(c)) - [P(c) - c] D(P(c))\}$$

Since P and P' coincide over $c \neq c_1, c_2$, this expression is strictly positive if and only if

$$[p_2 - c_1] D(p_2) - [p_1 - c_1] D(p_1) + [p_1 - c_2] D(p_1) - [p_2 - c_2] D(p_2) > 0$$

which simplifies into

$$(c_2 - c_1)(D(p_2) - D(p_1)) > 0$$

Recall that by assumption, $c_1 > c_2$ and $p_2 > p_1$. Since D is strictly decreasing in the range in which it is strictly positive, and since we have established in the previous lemma that $D(P(c)) > 0$ for all $c \in C$, the inequality holds. Therefore, the deviation to P' is strictly profitable.

Let $C^* = \{c \in C \mid P(c) < c\}$. Suppose that C^* is non-empty. Consider a deviation to a pricing strategy P' that satisfies $P'(c) = c$ for all $c \in C^*$ and $P'(c) = P(c)$ for all $c \notin C^*$. For every $c \in C^*$, the firm's profit conditional on being chosen ceases to be strictly negative as a result of the deviation. For every $c \notin C^*$, the deviation weakly lowers the loss aversion term because it raises the consumers' reference price with positive probability. Since we have established that consumer demand is strictly positive at all cost states, the deviation is strictly profitable. Therefore, C^* must be empty. □

This result followed automatically from (9.1) in the absence of loss aversion. However, in the presence of loss aversion, it is a non-trivial result due to the sensitivity of consumer demand to the firm's overall pricing strategy.

We now turn to the result that characterizes the "reduced price variability" effect due to consumer loss aversion.

Proposition 9.1. *Let P be an optimal pricing strategy P. Then:*

$$P^0(c^l) \leq P(c^l) \leq P(c^h) \leq P^0(c^h)$$

***Proof*.** Let P be an optimal pricing strategy. By Lemma 9.2, P is weakly increasing and satisfies $P(c) \geq c$ for every $c \in C$, hence $P(c^h) \geq P(c^l) \geq c^l$ and $P(c^h) \geq c^h$.

(i) $P^0(c^l) \leq P(c^l)$.

Recall that $P^0(c^l) = \frac{1}{2}(1 + c^l)$. In the absence of loss aversion, the hump shape of the firm's objective function implies that for every $c \geq c^l$, the price $P^0(c^l)$ yields a higher profit than any $p' < P^0(c^l)$. Define c^0 to be the highest cost c for which $P(c) < \frac{1}{2}(1 + c^l)$. Since P is weakly increasing, $P(c) < \frac{1}{2}(1 + c^l)$ for all $c \leq c^0$. Suppose that the firm deviates to a pricing strategy P' that satisfies $P'(c) = \frac{1}{2}(1 + c^l)$ for $c \leq c^0$ and coincides with P for $c > c^0$. The "bare" profit excluding the loss aversion term goes up. As to the loss aversion component, because P' is flat over $c \leq c^0$, $L(P'(c_1), P'(c_2)) = 0$ whenever $c_1, c_2 \leq c^0$. Moreover, because $P'(c) > P(c)$ for $c \leq c^0$ and $P'(c) = P(c)$ for $c > c^0$, $L(P'(c_1), P'(c_2)) \leq L(P(c_1), P(c_2))$ whenever $c_2 \leq c^0 < c_1$. Since P and P' coincide at all $c > c^0$, $L(P'(c_1), P'(c_2)) = L(P(c_1), P(c_2))$ when $c_1, c_2 > c^0$. Finally, when $c_1 \leq c^0 < c_2$, $L(P'(c_1), P'(c_2)) = 0$ because by the definitions of c^0 and P', $P'(c_2) > P'(c_1)$. It follows that the deviation is profitable.

(ii) $P(c^h) \leq P^0(c^h)$.

Consider two cases. First, suppose that P is flat; that is, $P(c^l) = P(c^h) = \bar{p}$. We have seen that in this case, the optimal price $\bar{p}$ is given by (9.2), which is strictly below $\frac{1}{2}(1 + c^h)$. Second, suppose that $P(c^l) < P(c^h) = p^h$. Let C^* be the set of cost values c for which $P(c) = P(c^h) = p^h$. Since P is weakly increasing in c, there exists $c^* \in C$ such that $C^* = \{c \in C \mid c > c^*\}$. By (9.3), consumer demand at p given P is strictly lower than $1 - p$ for every $p > p^l$. Assume that $p^h > \frac{1}{2}(1 + c^h)$. Suppose that the firm switches to a pricing strategy P' such that $P'(c) = P(c) - \varepsilon$ for all

$c \in C^*$ and $P'(c) = P(c)$ for all $c \notin C^*$. If $\varepsilon > 0$ is sufficiently small, $P'(c) > P'(c')$ for every $c \in C^*$, $c' \notin C^*$. By the hump shape of the firm's profit function in the absence of loss aversion, this deviation strictly raises this "bare" profit in all states $c \in C^*$. In addition, the deviation weakly lowers the loss aversion term in those states, without changing the loss aversion term in the states outside C^*. Therefore, the deviation is strictly profitable. □

Thus, when the consumer is loss averse, the firm raises the price at the lowest level of marginal cost and lowers the price at the highest level of marginal cost, relative to the benchmark with no loss aversion. The firm does not want the consumer to experience large losses that will reduce his willingness to pay, and therefore shrinks the price range.

Example 9.5 (Extreme loss aversion)**.** Assume that $L(p, p^e) = \delta$ whenever $p > p^e$, where $\delta > 0$ is an exogenous parameter. This is an extreme case of loss aversion, in the sense that the loss function is discontinuous at $p = p^e$: the slightest price increase relative to the consumer's reference price generates a disutility. However, this disutility is insensitive to the magnitude of the price increase. This special case lends itself to a clean demonstration of the forces that influence reduced price variability.

Let us put aside the quest for an optimal pricing strategy, and instead compare two extreme strategies: the optimal, fully flexible pricing strategy in the absence of loss aversion, denoted P^0 and given by (9.1), and the optimal fully rigid (i.e., constant) pricing strategy given by (9.2). The latter eliminates the loss aversion term because it has no price variation.

When we calculate the loss in "bare" expected profits (i.e., ignoring the loss aversion term) as a result of switching from P^0 into the constant price $\bar{p}$, we see that it is

$$\frac{1}{4}\left[\left(\frac{1}{m}\sum_{c} c^2\right) - (\bar{c})^2\right]$$

which is thus proportional to the *variance* of c. On the other hand, by switching from P^0 into the optimal constant price $\bar{p}$, the firm eliminates the expected loss due to loss aversion. Since the probability of trade may be zero for some realizations of actual and expected prices, this expected loss is bounded from above by

$$\frac{1}{m^2}\sum_{c}\sum_{c'<c}(P^0(c) - c)\cdot\delta = \frac{1}{m^2}\sum_{c}\sum_{c'<c}\left(\frac{1-c}{2}\right)\cdot\delta <$$
$$\frac{1}{m^2}\cdot\frac{1-\bar{c}}{2}\cdot\frac{m(m-1)}{2}\cdot\delta = \frac{(m-1)(1-\bar{c})\cdot\delta}{4m}$$

Holding the parameters m, $\bar{c}$, and δ fixed, we can see that if the variance of c is sufficiently low, the firm will prefer the optimal constant price $\bar{p}$ to P^0.

The insight from this comparison is that the firm's incentive to introduce rigidity into its pricing strategy depends on two aspects of cost fluctuation: (i) the variance of costs, which measures the global spread of fluctuations; and (ii) the number of cost realizations, which captures the frequency of fluctuations. When the firm experiences a large number of small fluctuations, it will tend toward a rigid pricing strategy because the attempt to avoid antagonizing consumers will be the dominant consideration. On the other hand, when fluctuations are large and infrequent, the firm will tend toward a flexible pricing strategy, because the "bare" incentive to allow prices to respond to cost shocks outweighs the incentive to minimize unpleasant surprises for consumers.

9.1.2 Impact on Average Prices

What is the effect of loss aversion on the *expected* price that the monopolist charges in optimum? To address this problem, let us consider a special case, in which L is only a function of $\Delta = p - p^e$. This is a common specification in the literature due to its tractability. For simplicity, assume that loss aversion is not too strong, in the following sense:

$$L\left(\frac{1+c^h}{2}, \frac{1+c^l}{2}\right) < \frac{1-c^h}{2} \tag{9.4}$$

This restriction ensures that $1 - P(c) - L(P(c), P(c')) > 0$ for all c, c' under any optimal pricing strategy P. In other words, consumer demand is strictly positive, conditional on any realization of actual and reference prices. It turns out that under loss aversion, the average price that the firm charges is lower than in the benchmark model without loss aversion.

Proposition 9.2. *Under every optimal strategy, the average price the firm charges is weakly lower than* $\bar{p} = \frac{1}{2}(1 + \bar{c})$.

Proof. Let P be an optimal pricing strategy. If P is a constant function (i.e., $P(c) = \bar{p}$ for all c) then we have seen that $\bar{p} = \frac{1}{2}(1 + \bar{c})$. Assume that P is not a constant function. Since it maximizes the firm's expected profit, it satisfies the following inequality for every alternative strategy Q:

$$\begin{aligned}
&\sum_c \sum_{c'} [P(c) - c] \cdot \max[0, 1 - P(c) - L(P(c), P(c'))] \\
\geq &\sum_c \sum_{c'} [Q(c) - c] \cdot \max[0, 1 - Q(c) - L(Q(c), Q(c'))]
\end{aligned}$$

Define $Q(c) \equiv P(c) - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. Then, since L is only a function of the difference between the actual and expected price, the following inequality holds:

$$\sum_c \sum_{c'} [P(c) - c] \cdot \max[0, 1 - P(c) - L(P(c), P(c'))]$$
$$\geq \sum_c \sum_{c'} [P(c) - \varepsilon - c] \cdot \max[0, 1 - P(c) + \varepsilon - L(P(c), P(c'))]$$

Condition (9.4) ensures that $1 - P(c) + \varepsilon - L(P(c), P(c')) > 0$ for all c, c'. It follows that we can rewrite the above inequality as follows:

$$\sum_c \sum_{c'} \{[P(c) - c] \cdot [1 - P(c) - L(P(c), P(c'))]$$
$$- [P(c) - \varepsilon - c] \cdot [1 - P(c) + \varepsilon - L(P(c), P(c'))]\} \geq 0$$

This inequality is simplified into

$$2 \sum_c \sum_{c'} P(c) \leq \sum_c \sum_{c'} [1 + c + \varepsilon - L(P(c), P(c'))]$$

Since P is not a constant function, $\sum_c \sum_{c'} L(P(c), P(c')) > 0$. Therefore, if ε is sufficiently close to zero, we can write

$$2 \sum_c \sum_{c'} P(c) < \sum_c \sum_{c'} [1 + c]$$

which immediately implies the result. □

The rough intuition for this result is as follows. Since loss aversion diminishes willingness to pay, the firm effectively faces lower aggregate demand than in the absence of loss aversion, and this is a force that impels the firm to lower its price on average. Note that the average price does not decrease monotonically with the intensity of loss aversion, because when loss aversion is sufficiently strong, the firm's optimal pricing strategy is the constant price $\bar{p}$ given by (9.2), which is equal to the average price under P^0.

Exercise 9.1. *Characterize the optimal pricing strategy when* $C = \{c^l, c^h\}$ *and* $L = \lambda \cdot (p - p^e)$ *whenever* $p > p^e$, *where* $\lambda > 0$.

9.2 PRICE UNIFORMITY IN A DUOPOLY SETTING: "KINKED" DEMAND

In this section I continue to explore the implications of loss aversion when reference prices are formed by consumers' expectations. I focus on a different kind of price stickiness implied by the model: uniformity of prices across suppliers with heterogeneous costs. Just as a monopolistic firm may be reluctant to let its product price fluctuate too much, a firm in a duopolistic setting with differentiated consumer tastes may be reluctant to charge a price above its competitor. The same force that causes reduced price variability in the monopoly case can cause price uniformity across firms.

The following model captures this idea in a simple fashion. A market consists of two firms and a continuum of consumers. Each consumer is indexed by his location along the interval $[0, 1]$. Denote a location by $x = (x_1, x_2)$, where $x_1, x_2 \in [0, 1]$ and $x_1 + x_2 = 1$. The consumers' locations are uniformly distributed. A consumer whose location is $x = (x_1, x_2)$ derives a "raw" utility of x_i from firm i's product. Denote firm i's marginal cost by c_i, and assume that $1 > c_1 > c_2 \geq 0$. Firms choose prices simultaneously. For simplicity, assume that the consumer has no outside option: he is forced to choose one of the two firms. When costs are sufficiently small, this is without loss of generality.

Given the firms' price profiles (p_1, p_2) and the consumer's reference price p^e, a consumer of type x chooses firm i whenever the following inequality holds:

$$x_i - p_i - L(p_i, p^e) > x_j - p_j - L(p_j, p^e)$$

Given (p_1, p_2, p^e), the fraction of consumers who choose firm i is

$$\frac{1}{2}[1 - p_i - L(p_i, p^e) + p_j + L(p_j, p^e)]$$

Thus, firm i's payoff function is

$$\Pi(p_i, p_j, p^e) = \frac{1}{2}[p_i - c_i] \cdot [1 - p_i - L(p_i, p^e) + p_j + L(p_j, p^e)]$$

We will use the following popular specification of the loss function:

$$L(p, p^e) = \max[0, \lambda \cdot (p - p^e)]$$

where $\lambda > 0$ is an exogenous parameter.

To complete the model, we need to specify how the reference price p^e is formed. For our purposes, it will suffice to assume that if consumers initially expect the two firms to charge the same price p, then $p^e = p$. Note that if one firm deviates from the consumer's expectation, this does *not* change his reference price. I defer the discussion of how p^e is formed when the consumer expects the two firms to charge different prices.

In the absence of loss aversion (i.e., when $\lambda = 0$) there is a unique Nash equilibrium in the game, in which

$$p_i^* = 1 + \frac{1}{3}(2c_i + c_j)$$

for both $i = 1, 2, j \neq i$. Clearly, since $c_1 > c_2$, the firms charge different prices in equilibrium. Specifically, $p_1^* > p_2^*$.

Let us turn to the case in which $\lambda > 0$. A *uniform-price market equilibrium* is a price p^* that satisfies the following condition: when both firms charge the price p^*, neither firm has an incentive to deviate from this price given that $p^e = p^*$. The following result shows that if costs differences are sufficiently small, or loss aversion is sufficiently strong, a uniform-price market equilibrium exists.

Proposition 9.3. *If* $c_1 - c_2 < \frac{\lambda}{1+\lambda}$, *there exist uniform-price market equilibria* p^*, *where* p^* *can take any value in* $(\frac{1}{1+\lambda} + c_1, 1 + c_2)$.

***Proof*.** Suppose that both firms charge a price p^* and that $p^e = p^*$. When firm i contemplates deviating from p^* into $p_i > p^*$, its payoff following the deviation is

$$\Pi(p_i > p^*, p_j = p^*, p^e = p^*) = \frac{1}{2}[p_i - c_i][1 - (1+\lambda)(p_i - p^*)]$$

On the other hand, when firm i contemplates deviating from p^* into $p_i < p^*$, its payoff following the deviation is

$$\Pi(p_i < p^*, p_j = p^*, p^e = p^*) = \frac{1}{2}[p_i - c_i][1 - (p_i - p^*)]$$

The difference is that upward deviation relative to p^* generates a positive loss aversion term, while a downward deviation does not.

The following expression represents the derivative of firm i's profit function with respect to p_i in the range $p_i > p^*$ (and at $p_i = p^*$ it represents the right-hand derivative):

$$\frac{\partial \Pi(\cdot)}{\partial p_i}\Big|_{p_i > p^*} = \frac{1}{2}[-(1+\lambda)(p_i - c_i) + 1 - (1+\lambda)(p_i - p^*)]$$

Likewise, the following expression represents the derivative of firm i's profit function with respect to p_i in the range $p_i < p^*$ (and at $p_i = p^*$ it represents the left-hand derivative):

$$\frac{\partial \Pi(\cdot)}{\partial p_i}\Big|_{p_i < p^*} = \frac{1}{2}[-(p_i - c_i) + 1 - (p_i - p^*)]$$

Suppose that $c_1 - c_2 < \frac{\lambda}{1+\lambda}$ and $p^* \in (\frac{1}{1+\lambda} + c_1, 1 + c_2)$. Then:

$$1 > p^* - c_i > \frac{1}{1+\lambda}$$

for both $i = 1, 2$. Hence, the derivative of Π with respect to p_i is negative (positive) for $p_i > p^*$ ($p_i < p^*$), such that p^* is a best-reply to itself. □

The reason that a uniform-price equilibrium can exist when loss aversion is sufficiently strong relative to the cost differences is that loss aversion causes consumer demand to have a "kink" at the reference price. Specifically, residual consumer demand for firm i's product decreases more rapidly with p_i when $p_i > p^e$ than it does when $p_i < p^e$. The higher the loss aversion parameter λ, the more drastic the kink, and therefore, the greater the high-cost firm's reluctance to raise its price relative to the reference price.

Note that in all uniform-price equilibria, the price p^* is lower than the price that either firm charges in Nash equilibrium of the model without loss aversion. This effect resonates with Proposition 9.2 in the case of monopoly pricing.

Other equilibria

Is the uniform-price equilibrium the only possible market equilibrium? In order to discuss the possibility of asymmetric equilibria, we need to complete our specification of how the reference point is formed when consumers expect firms to charge different prices—that is, when $p_1^e \neq p_2^e$. One possible assumption is $p^e = \min(p_1^e, p_2^e)$. A market equilibrium is thus defined as a profile of prices (p_1^e, p_2^e) such that no firm wishes to deviate, given this specification of p^e.

Exercise 9.2. *Let $p^e = \min(p_1^e, p_2^e)$. Show that if λ is sufficiently large, the uniform-price equilibrium is the only market equilibrium.*

Should the reference price depend on consumers' expected choices?

One could argue that when $p_1^e \neq p_2^e$, a consumer's reference price should depend on his expectation of his own consumption decision. If he expects to choose firm i, then p_i^e should be the reference price. Thus, consumers with different locations may form different reference prices because they expect to make different consumption decisions. Since this complication of the model is irrelevant to our discussion of uniform-price equilibrium, I do not discuss it in detail here. However, the idea that consumers' expectations of their own future actions shape their reference points is explored further in the next section, in a different context.

9.3 EXPECTED CONSUMPTION AS A REFERENCE POINT: AN "ATTACHMENT EFFECT"

In the previous sections we analyzed a model in which the reference point was the price the consumer expected. Losses were registered whenever the actual price exceeded the expected price. In this section I explore the implications of loss aversion when losses are registered in another dimension. When the consumer expects to buy a product and ends up not buying it, he may experience a disappointment. To put it differently, his expectation to get hold of the product makes him emotionally attached to the product even before he actually buys it, and this de facto increases his willingness to pay for the product. This type of psychology seems to be more realistic when the product is a durable good that the consumer can expect to *own*, or a service he can expect to enjoy in the long run, rather than a perishable good that is consumed immediately upon purchase.

This "attachment effect" is a variant on the well-known experimental finding known as the "endowment effect." Recall Example 2.3 in Chapter 2. Suppose that before signing up for a service, you are willing to pay at most $20 for it. But once you have signed up for it, your willingness to pay jumps to $25. Suppose that at the time you are offered the service contract, the stated price is $20. After you sign up for it, the firm raises the price unexpectedly to $25. You are still able to cancel the deal at no cost. From your point of view before signing up, you would prefer to cancel. However, having signed up, you do not want to cancel.

This effect is called the "endowment effect" because it captures the idea that when a decision maker is initially endowed with an entitlement, the minimal amount of money he is willing to accept (WTA) in return for giving up on his entitlement is higher than the maximal amount he would be willing to pay (WTP) to acquire the entitlement in the first place. In other words, owning something increases its subjective value. While I believe in the existence of this psychological phenomenon, its experimental validation has been contested and intensely debated.

At any rate, the concept of loss aversion is useful for interpreting the endowment effect. When the consumer is initially endowed with an entitlement, the prospect of not having it is recorded as a loss, whereas when the consumer is initially without the entitlement, the prospect of having it is recorded as a gain. The WTA/WTP gap noted above is thus consistent with the idea that losses are weighted more heavily than gains in the consumer's deliberation.

Note that the endowment effect was originally observed in experiments where the decision maker's reference point was designed to be his initial actual endowment. In contrast, the attachment effect as presented above takes the reference point to be the endowment the consumer *expects* to have. Except for this re-specification of the reference point, the two effects are identical.

There is a methodological difference between the attachment effect and the loss-aversion effects studied in the previous sections, because the consumer's willingness to pay depends on his expectation of his *own consumption decision*, rather than on his expectation of a variable beyond his control, such as the product's price. Thus, we are led to extend the model of consumer choice under loss aversion, from a straightforward model of reference-dependent preferences in which choice is an outcome of a maximization problem, into a model in which choice is an outcome of some kind of internal *equilibrium*.

9.3.1 Personal Equilibrium

Assume for the moment that the consumer faces no exogenous uncertainty. Let X be a finite set of feasible consumption decisions for the consumer. Let P be a finite set of parameter values beyond the consumer's control (typically prices). Define a utility function $u : X \times X \times P \to \mathbb{R}$, with the interpretation that $u(x, x^e, p)$ is the consumer's payoff when the action he expects to take is x^e, the action he actually takes is x, and the parameter value is p. We say that a consumption decision x^* is a *personal equilibrium* for a given p if $u(x^*, x^*, p) \geq u(x, x^*, p)$ for every $x \in X$.

Taking a formal point of view, a personal equilibrium is nothing but a symmetric, pure-strategy Nash equilibrium in a two-person game with common interests. The interpretation is that the action x^* is what the consumer ends up choosing whenever he expects himself to choose x^*. It can easily be seen that this choice model can admit multiple equilibria. For example, suppose that $X = \{a, b\}$, $u(a, a) = 2$, $u(b, b) = 1$ and $u(a, b) = u(b, a) = 0$ (I hold the value of p fixed and therefore suppress it). Then, a and b are both personal equilibria. Likewise, a personal equilibrium may fail to exist at all. For instance, suppose that $X = \{a, b\}$, $u(a, a) = u(b, b) = 0$ and $u(a, b) = u(b, a) = 1$. I leave aside the question of whether mixed-strategy equilibria make sense in this environment.

To get some understanding of the behavioral content of personal equilibrium, conduct the following choice-theoretic exercise. Let $P(X)$ be the set of all choice problems (i.e., non-empty subsets of X). A choice correspondence $c : P(X) \to P(X)$ assigns a subset $c(A) \subseteq A$ to every choice problem A. We say that a utility function $U : X \times X \to R$ captures the *attachment effect* if for every $x, y \in X$, $U(x, y) \geq U(y, y)$ implies $U(x, x) > U(y, x)$. The first argument of U is interpreted as the action the consumer takes, while the second argument is interpreted as the action he expects to take. The definition thus captures the idea that when consumers expect to choose a particular market alternative, they become attached to it and thus their willingness to abandon it for a different market alternative drops. We say that a choice correspondence c is *consistent with personal equilibrium* if there exists a utility function $U : X \times X \to \mathbb{R}$ that captures the attachment effect, such that for every choice problem A, $c(A)$ is the set of elements $x^* \in A$ for which $U(x^*, x^*) \geq U(x, x^*)$ for every $x \in A$.

Proposition 9.4. *A choice correspondence c is consistent with personal equilibrium if and only if there exists a complete binary relation $\succeq$ over X, such that for every A, $c(A) = \{x \in A \mid x \succeq y$ for all $y \in A\}$.*

Proof. Let c be a choice correspondence. First, assume that c is consistent with personal equilibrium, with a utility function U. Define the following binary relation $\succeq$: $x \succeq y$ if $U(x, x) \geq U(y, x)$. That is, $x \succeq y$ if x is an action predicted by personal equilibrium when the choice problem is $\{x, y\}$. To see why $\succeq$ is complete, assume that $x \not\succeq y$; that is, $U(y, x) > U(x, x)$. By the assumption that U captures the attachment effect, $U(y, y) > U(x, y)$, hence $y \succeq x$. The result that $c(A) = \{x \in A \mid x \succeq y$ for all $y \in A\}$ follows immediately from the definition of $\succeq$.

Let us turn to the converse result. Assume there exists a complete binary relation $\succeq$ over X, such that for every A, $c(A) = \{x \in A \mid x \succeq y$ for all $y \in A\}$. Define U as follows. For every $x \in X$, let $U(x, x) = 0$. If $x \succeq y$ and $y \not\succeq x$, let $U(x, y) = 1$ and $U(y, x) = -1$. If $x \succeq y$ and $y \succeq x$, let $U(x, y) = U(y, x) = -1$. It can be verified that U captures the attachment effect. It is also straightforward to see that for every choice problem A, $c(A)$ coincides with the set of personal equilibria given A according to U. □

Thus, for a fixed market environment with no exogenous uncertainty, the concept of personal equilibrium is behaviorally equivalent to a choice procedure in which the consumer follows some complete binary relation, which may be intransitive, and chooses only those elements that are not dominated (according to the binary relation) by other elements in the choice set. (To obtain stronger behavioral implications of personal equilibrium, we would need to reintroduce uncertainty into the consumer's environment and impose structure on his reference point formation in such a stochastic environment.)

The following is a very simple application of personal equilibrium that demonstrates the potential implications of the attachment effect for monopoly pricing. Let $X = \{0, 1\}$, where $x = 1$ means that the consumer decides to buy a product and $x = 0$ means that he decides not to buy it. Let P be the set of *prices* that the consumer may face. Assume the following payoff function $u(x, x^e, p)$:

$$u(1, 0, p) = u(1, 1, p) = 1 - p$$

$$u(0, 0, p) = 0$$

$$u(0, 1, p) = -\lambda$$

where $\lambda > 0$. This parameter captures the attachment effect: the larger λ, the stronger the effect. It captures loss aversion in the sense that $u(1, x^e, p) - u(0, x^e, p)$ is larger when $x^e = 1$ than when $x^e = 0$. Note that the payoff function is independent of the price the consumer expects to pay. In other words, loss aversion affects the consumption quantity dimension, but not the price dimension.

Assume that a monopolist with zero costs sets a deterministic price p. When $p > 1 + \lambda$, the only personal equilibrium is $x^* = 0$. When $p < 1$, the only personal equilibrium is $x^* = 1$. When $p \in [1, 1 + \lambda]$, both $x^* = 0$ and $x^* = 1$ are equilibria. If the consumer believes that he will not buy the product, he finds the price weakly higher than his willingness to pay. If, however, the consumer believes that he will buy the product, he finds the price weakly lower than his willingness to pay, because his expectation has made him feel attached to the product. In order to predict the monopolist's pricing decision, we need to select among equilibria.

9.3.2 Price Randomization

Let us now extend the choice model of personal equilibrium to stochastic environments. This is particularly relevant when the consumer faces fluctuating product prices. As in the model studied at the beginning of this chapter, price fluctuations complicate the model of consumer choice because they cause reference points to fluctuate as well. However, in this section the transmission from fluctuating prices to fluctuating reference points is even subtler, since it is not prices but the consumer's consumption decisions that serve as reference points.

As before, let $u(x, x^e, p)$ be the consumer's payoff when the action he expects to take is x^e, the action he actually takes is x, and the parameter value he faces is p. Suppose that p is distributed according to a probability distribution $\mu \in \Delta(P)$.

For simplicity, assume that for every x^e, p, there is a unique x that maximizes $u(\cdot, x^e, p)$. Define the characteristic variable $\psi(x, x^e, p)$ as follows: $\psi(x, x^e, p) = 1$ if x maximizes $u(\cdot, x^e, p)$, and $\psi(x, x^e, p) = 0$ otherwise. We look for a notion of reference-dependent choice that reflects some kind of personal equilibrium, given the environment of fluctuating prices.

Definition 9.1. *A probability distribution $\alpha \in \Delta(X)$ is a personal sampling-based equilibrium if for every action x:*

$$\alpha(x) = \sum_{p \in P} \sum_{x^e \in X} \mu(p)\alpha(x^e)\psi(x, x^e, p)$$

This definition is, again, in the spirit of sampling-based reasoning. The consumer faces a price p, which is drawn from an underlying price distribution μ. At the same time, he draws his reference point x^e from the marginal distribution α over his own consumption decisions. Given the reference point and the realization p, he chooses an action that maximizes u. This in turn induces a marginal distribution over consumption decisions, which in equilibrium should coincide with α.

It can be shown (using standard fixed-point arguments) that a personal sampling-based equilibrium always exists. Note that when μ is degenerate (i.e., it assigns probability one to a single value of p), every probability distribution over the set of personal equilibria is a personal sampling-based equilibrium.

Armed with this model of reference-dependent choice, let us return to the monopoly pricing example of the previous sub-section. Consider the following random pricing strategy μ, which assigns positive probability to two price levels only, $p^L = 1 - \varepsilon$ and $p^H = 1 + \lambda$. When the price is p^L, the consumer's optimal action is $x = 1$ regardless of his reference point. In contrast, when the price is p^H, the consumer's optimal action is $x = 1$ if and only if the reference point he samples is $x^e = 1$. The condition for personal sampling-based equilibrium is thus:

$$\alpha(1) = \mu(p^L) \cdot 1 + \mu(p^H) \cdot \alpha(1)$$

The unique solution to this equation is $\alpha(1) = 1$. Thus, the only consumption decision that is consistent with equilibrium at the price p^H is buying the product. This gives the monopolist a rationale for probabilistic pricing, interpreted as temporary price reductions. Without randomization, we saw that both $x = 0$ and $x = 1$ were consistent with personal equilibrium.

Therefore, if the monopolist employs a pessimistic equilibrium selection criterion, it would strictly prefer to randomize. Note, however, that under this selection criterion, the unique equilibrium consumption decision is $x = 1$ whenever $\mu(p^L) > 0$ and $x = 0$ when $\mu(p^L) = 0$. Thus, the equilibrium selection correspondence is not continuous, and as a result there is no well-defined optimal pricing strategy.

■ 9.4 DISCUSSION

This section is devoted to variations, extensions, and criticisms of the models analyzed in this chapter.

9.4.1 Actual Prices as Reference Points

Throughout this chapter, we emphasized consumers' expectations in the formation of reference points. Moreover, expectations were consistent with the market environment, not in the sense of being "rational," but in the sense that they were drawn from the price distribution that prevails in the market. The idea of expectation-based reference points was central in the analysis. To gauge its importance, let us revisit the duopoly model analyzed in Section 9.2 and assume that the consumers' reference price is a function of the firms' actual, rather than expected, prices. For instance, suppose that for every price profile (p_1, p_2), the consumers' reference price is $p^e = \min(p_1, p_2)$. In this case, all uniform-price equilibria break down. The reason is as follows. Let $\varepsilon > 0$. Suppose that firms play $p_1 = p_2$. If firm 1 deviates to $p_1' = p_2 + \varepsilon$, a loss aversion term $\lambda\varepsilon$ dampens consumer demand for firm 1. In contrast, when firm 1 deviates to $p_1' = p_2 - \varepsilon$, consumer demand for firm 2 is dampened by the same term $\lambda\varepsilon$. However, since total demand for the firms always remains fixed, this means that demand for firm 1 is raised by $\lambda\varepsilon$. Thus, the loss aversion component affects consumer demand in the same way, regardless of whether the deviation is upward or downward. In other words, consumer demand for firm 1 is not "kinked," and for this reason we cannot sustain uniform pricing in market equilibrium.

Another possible specification of the reference point is $p^e = p_1$. The interpretation is that firm 1 is "prominent" in the market—either because it is an incumbent that has previously served the market as a monopolist while the rival firm is a new entrant, or because it advertises more intensively. At any rate, when one firm is prominent, it is natural for consumers to view its price as an anchor against which they evaluate the rival firm's price. Given a realization (p_1, p_2) of the firms' (possibly mixed) strategies, a consumer whose location is $x = (x_1, x_2)$ chooses firm 2 if and only if the following inequality holds:

$$x_2 - p_2 - L(p_2, p_1) > x_1 - p_1$$

Under this "prominence-based" specification, the demand function that the two firms face for any given price profile (p_1, p_2) has an interesting structure. Residual consumer demand for firm 2 (the less prominent firm) has a kink at $p_2 = p_1$: it decreases more rapidly with p_2 when $p_2 > p_1$ than it does when $p_2 < p_1$. But since total consumer demand is fixed, this means that firm 1's residual demand has a kink facing *the opposite direction* at $p_1 = p_2$. Thus, firm 1's residual demand function is convex while firm 2's residual demand is concave, where local convexity/concavity is strict only in the neighborhood of the kink. And since firm 1 directly controls the reference price, it effectively determines the location of the kink. This implies non-existence of pure-strategy equilibria, as the following exercise shows.

Exercise 9.3. *Assume that $c_1 = c_2 = 0$. Show that when $p^e = p_1$ for every price profile (p_1, p_2), the duopoly model has no pure-strategy equilibrium.*

In fact, it turns out that in Nash equilibrium, firm 1 plays a mixed strategy that randomizes over a high price and a low price, while firm 2 plays a pure, intermediate-level pricing strategy. In other words, the prominent firm employs sales strategically to manipulate consumers' reference point.

This example demonstrates that equilibrium analysis in models with consumer loss aversion are sensitive to the way reference points are formed. This is often viewed as a problematic lack of robustness. I do not share this criticism. The reference point is a component of the consumers' decision process, on a par with their preferences. Just as we would not deem a model non-robust because its predictions vary with assumptions on preferences, we should not criticize applications of loss aversion for depending on the specification of the reference point. However, it is true that the specification has to be well-motivated, and the question of whether it can be elicited from choice behavior is important to address. Market models that incorporate heterogeneity among consumers in this dimension would be especially interesting.

9.4.2 Pleasant Surprises

The loss aversion models examined in this chapter invariably assumed that consumers' willingness to pay reacts only to unpleasant surprises—that is, cases in which the actual prices exceed the expected price, or actual consumption quantity falls below the expected quantity. However, one could argue that in many real-life situations, consumers' willingness to pay also reacts to positive surprises. For instance, when the price they encounter is lower than expected, they view this as a "bargain" and their willingness to pay is spuriously enhanced. We will now see that our analysis in this chapter is sensitive to this effect.

Let us return to a simple version of the model of monopoly pricing studied in Section 9.1. The monopolist faces a single consumer whose "raw" willingness to pay is v. The monopolist incurs no costs. It can employ a random pricing strategy μ with finite support. In this case, the consumer's reference point is p^e with probability $\mu(p^e)$. Let $\lambda_g, \lambda_l \geq 0$. The consumer is willing to buy the product at a price p if $p \leq v + \lambda_g \cdot (p^e - p)$ when $p < p^e$, and if $p \leq v - \lambda_l \cdot (p - p^e)$ when $p > p^e$. Thus, the consumer's willingness to pay is affected by both pleasant and unpleasant surprises.

Assume, for simplicity, that $v = 1$ with certainty. If $\lambda_g = \lambda_l = 0$, the monopolist's optimal pricing strategy is $p^* = 1$. If $\lambda_g = 0$ and $\lambda_l > 0$, we are back with our original model (except that there are no costs and v is deterministic). The firm has a strict disincentive to randomize over prices in this case. To see why, observe that the consumer's willingness to pay cannot exceed one. If the firm assigns positive probability to prices above one, the consumer never buys at those prices, while if the firm assigns positive probability to prices below one, it earns by definition less than one (and in addition, the consumer may fail to buy the product, because even if $p < 1$, it is not necessarily the lowest price in the support of the price distribution, and can therefore constitute an unpleasant surprise that reduces the consumer's willingness to pay).

Exercise 9.4. *Assume that v is distributed according to a differentiable cdf F that satisfies the monotone hazard rate condition (i.e., the function $f(v)/(1 - F(v))$ is increasing in v). Show that when $\lambda_g = 0$ and $\lambda_l > 0$, the monopolist has a strict disincentive to randomize over prices.*

Now suppose that $\lambda_g > 0$, and consider a random pricing strategy that assigns probability α to p_1 and probability $1-\alpha$ to p_2, where p_1 and p_2 satisfy $p_2 > p_1 > 4$ and $1 + \lambda_g \cdot (p_2 - p_1) > p_1$. Then, the firm's expected profit is

$$\alpha \cdot p_1 \cdot [\alpha \cdot 0 + (1 - \alpha) \cdot 1] + (1 - \alpha) \cdot p_2 \cdot 0 = \alpha(1 - \alpha)p_1$$

Thus, since $p_1 > 4$, the firm's expected profit exceeds the level it can reach without randomization. Note that since there are only two price levels in the support of the price distribution, and since both price levels exceed the consumer's raw willingness to pay, the consumer's loss aversion parameter λ_l is irrelevant. The consumer experiences a loss when the expected price is p_1 and the actual price is p_2. But since p_2 exceeds the consumer's "raw" willingness to pay, he would not buy the product at this price even if we set the loss aversion parameter to $\lambda_l = 0$.

Two features in this example generate the strict incentive to randomize. First, L is unbounded. Second, the high price p_2 can be arbitrarily high. As a result, the contribution of the pleasant surprise to the consumer's willingness to pay is unbounded. However, the consumer does not buy the product at p_2. One could argue—in the spirit of "personal equilibrium"—that only prices that lead to trade with positive probability can serve as reference points. If we accept this critique, then the rationale for randomization in this example disappears.

9.5 SUMMARY

In this section we explored the implications of consumer loss aversion for pricing, mostly in monopolistic settings. We saw that these implications rely on the specification of loss aversion—namely, which dimensions of consumption outcomes does loss aversion affect, and how the reference point is formed. In particular:

- If consumers' willingness to pay for a product is dampened when its actual price exceeds the expected price, this gives rise to "price rigidity" effects. A monopolist curbs its price responses to cost shocks. If loss aversion is sufficiently strong, the monopolist employs an entirely flat pricing strategy. In a duopolistic setting with differentiated tastes, firms with different costs may charge the same price in equilibrium.

- When consumers' utility from not buying a product is lower if they expected to buy it, a monopolist may have an incentive to employ temporary price reductions in order to induce this "attachment effect."

9.6 BIBLIOGRAPHIC NOTES

The concept of loss aversion was introduced by Kahneman & Tversky (1979). For a collection of studies on loss aversion, see Kahneman & Tversky (2000). For early discussions in the economics literature regarding the negative effect of unanticipated high prices on willingness to pay, see Hall & Hitch (1939) and Okun (1980). Kahneman, Knetsch & Thaler (1986) provide experimental evidence in support of this hypothesis. For a more skeptical field study, see Courty & Pagliero (2008). The attachment effect is a variation on the well-known experimental "endowment effect" originally reported by Thaler (1980). There has been considerable controversy around this effect (see Plott & Zeiler (2005) and Isoni, Loomes & Sugden (2010) for recent examples). The concept of personal equilibrium is due to Kőszegi (2009). Kőszegi & Rabin (2006) adapted this concept to a model of decision making under loss aversion. The choice-theoretic characterization of personal equilibrium is based on Gul & Pesendorfer (2005).

Heidhues & Kőszegi (2005, 2008) pioneered the application of loss aversion to models of monopolistic and competitive pricing. The model of Section 9.1 follows Spiegler (2010*b*), which itself is a simple variation on Heidhues & Kőszegi (2005). The model of Section 9.2 is based on Heidhues & Kőszegi (2008), a model that was later taken up and extended by Karle & Peitz (2010). The treatment in this chapter differs from the original Kőszegi-Heidhues in two respects. First, for pedagogical purposes, I analyze implications of loss aversion in the price dimension and in the consumption quantity dimension separately, whereas Kőszegi and Heidhues combine the two. Second, the specification of how the reference point is formed given a stochastic market environment is different. Proposition 9.2 is due to Spiegler (2010*b*) (the proof is based on a suggestion by Ariel Rubinstein that supplanted a previous proof based on differentiability assumptions).

The prominence-based version of the duopoly model, discussed in Section 9.4.1, is due to Zhou (2009). The notion of personal sampling-based equilibrium combines the concept of personal equilibrium with a solution concept introduced by Osborne & Rubinstein (1998), which was also the pretext for the models studied in Chapters 6 and 7. The discussion of pleasant surprises in Section 9.4.2 is based on a suggestion made by Kfir Eliaz.

10 Inertia I: Price Competition

The next two chapters focus on a particular, yet pervasive kind of reference-dependent choice. Many consumption problems begin with one market alternative functioning as a status quo or default option. There are several reasons for such a scenario. The consumer may have inherited this alternative from his own prior consumption decisions. Alternatively, other consumers with whom he is in close touch may regularly consume this option, and therefore it is a natural starting point for the choice process. Often the default option is not to consume anything—that is, "do nothing." Sometimes there are institutional reasons for the existence of a default. For instance, the consumer's health insurance or retirement savings may be mediated by his employer, who assigns all employees to a particular option unless they specify otherwise. In all these cases, the consumer is stuck with the default unless he takes an active decision to the contrary.

There is an abundance of evidence that decision makers have a strong tendency to choose default options. The fraction of people who sign an organ donation card differs dramatically between countries, according to whether the default is opting in or opting out. Long-term savings mediated and subsidized by employers are sensitive—in total amount as well as portfolio choice—to the default option administered by the employer. The negative option offers discussed in Chapter 2 seem to be based on a default effect: once the consumer signs up for a service (often for a free trial period), renewal is often the default and the consumer needs to make an active decision in order to terminate the service. There is experimental and field evidence that the tendency to adhere to the default increases when the decision problem becomes more complex and when the consumer needs to justify his decision. This phenomenon is often referred to as *choice overload*.

Default bias can originate from a variety of psychological forces. It is useful to distinguish between two broad types of default bias.

Status quo preferences

The consumer may have an actual preference for not changing the way things are. In this case, choosing another alternative over the status quo carries an implicit switching cost. One psychological mechanism that generates status quo preferences is loss aversion. A loss-averse consumer will regard the status quo as a reference point, and will overweigh (underweigh) the new alternative's drawbacks (strengths) relative to the status quo.

Inertia

This type of default bias occurs at an earlier stage of the decision process, such that the consumer fails to get to the stage where he actually applies a preference ranking to the available alternatives. One reason for this inertia may be indecision.

In the face of a difficult decision problem, the consumer may simply "not know what to do." When there is a default option, the consumer clings to it as a way of "deciding not to decide," or at least postponing the decision. This prolongs the effective choice of the default. Another force that causes decision makers to procrastinate is a dynamically inconsistent evaluation of decision costs. Making an active decision is cognitively and emotionally taxing, and a consumer with a taste for immediate gratification may have a preference for postponing the decision. If the consumer is naive in the sense defined in Chapter 2, this postponement can go on "forever." Finally, when the choice set is large, the consumer may simplify his decision problem by applying initial screening criteria (which may partly overlap his preferences) that eliminate certain alternatives from being considered at all.

In this chapter, we will analyze firms' response to default bias in competitive settings. The market models we will examine assume that consumers' default bias falls into the second category, namely *inertia*. Therefore, I will refer to the tendency to adhere to the default as *default bias* or *inertia* interchangeably. Our primary questions are: What kind of pricing and marketing strategies arise in response to default bias? How does default bias affect the competitiveness of the market and consumer welfare?

Special focus will be attached to the role of *framing* in manipulating consumers' inertia. When consumers resist considering new alternatives or have difficulties doing so, the way these alternatives are presented to the consumer can magnify or mitigate his inertia. We will be interested in the interplay between firms' pricing and framing strategies, and how it affects the market outcome and the manifestation of consumers' default bias in market equilibrium.

10.1 PRICE COMPETITION UNDER CONSUMER INERTIA

In this section we begin our exploration of market implications of consumer inertia with a baseline model. Due to its utter simplicity, the model will be open to other interpretations than default bias. However, it will serve as a useful benchmark for extensions that capture default bias in a more distinctive manner.

Our market consists of two identical profit-maximizing firms and a continuum of consumers who are interested in buying one unit and have a willingness to pay of 1 for it. The two firms sell an intrinsically homogenous product at zero cost. They simultaneously choose a price $p \in [0, 1]$. Each consumer is initially assigned to a random firm. Each firm is assigned half the population of consumers. This initial assignment is interpreted as the consumer's *default* or status quo option. Conditional on being initially assigned to firm i, the consumer makes a price comparison with probability $\beta \in (0, 1)$. Conditional on making a price comparison, the consumer switches to firm j if and only if $p_j < p_i$. If we set $\beta = 1$, the model would collapse into standard Bertrand competition, such that both firms would charge $p = 0$ in Nash equilibrium.

For every price $p_i > 0$ that firm i considers charging, the firm's worst-case scenario is that firm j selects $p_j = 0$, and this leaves firm i with a market share of $\frac{1}{2}(1-\beta)$. It follows that the firm's max-min strategy is $p = 1$ and the max-min profit is $\frac{1}{2}(1-\beta) > 0$. Thus, the consumers' default bias means that the minimal profit that firms can earn in this market exceeds the competitive, rational-consumer benchmark.

Proposition 10.1. *There is a unique symmetric Nash equilibrium in this model, in which each firm plays a mixed strategy given by the cdf*

$$F(p) = 1 - \frac{1-\beta}{2\beta} \cdot \left(\frac{1}{p} - 1\right) \tag{10.1}$$

defined over the support

$$\left[\frac{1-\beta}{1+\beta}, 1\right]$$

Firms earn the max-min payoff $\frac{1}{2}(1-\beta)$ in equilibrium.

Proof. In symmetric Nash equilibrium, the equilibrium (mixed) strategy must be a continuous and strictly increasing *cdf* over an interval $[p^L, 1]$. The proof is essentially the same as in the case of Proposition 6.1 and therefore omitted here. This leaves us with the task of calculating the cdf F that represents the equilibrium strategy. The payoff from $p = 1$ is $\frac{1}{2} \cdot (1-\beta)$, because at that price the firm has only the non-rational consumers who are initially assigned to the firm (henceforth called loyal). By the standard indifference property of mixed-strategy Nash equilibrium, each price p in the support of F must be a best-reply, and therefore satisfy the following equation:

$$\frac{1}{2} \cdot (1-\beta) = p \cdot \left[\beta \cdot (1 - F(p)) + (1-\beta) \cdot \frac{1}{2}\right]$$

The logic behind the R.H.S. is that the firm's clientele at the price p consists of its loyal consumers as well as rational consumers in case the firm is the cheapest in the market. Solving this equation gives us expression (10.1). Plugging $F(p^L) = 0$ gives us p^L. □

The equilibrium has several noteworthy properties. First, it is a mixed-strategy equilibrium, which—recall our discussion in Chapter 6—can represent "cross-section" price dispersion, or temporal price variation. Second, firms earn equilibrium profits that are equal to their max-min payoff. In other words, although firms take advantage of consumers' default bias, competitive forces do prevail in that they push firms' profits to the minimal level they can secure given the consumers' bias. As a result, industry profits are $1-\beta$; that is, they are entirely given by the parameter that captures the consumers' default bias.

Third, the amount of equilibrium price dispersion (measured as the expected difference between the default price and the best available price) varies non-monotonically with β. When β is close to one (zero), almost all prices are close to the competitive (monopoly) level. In either case, there is little price dispersion. This means that price dispersion is maximal at some intermediate level of β. In particular, a regulatory intervention that lowers the fraction of "inert" consumers can raise price dispersion.

Choice overload

Suppose that the number of firms is $n > 2$. As before, each consumer is initially assigned to one firm, where each firm is assigned a fraction $\frac{1}{n}$ of the consumers. More importantly, let the probability that the consumer makes a price comparison depend on n, and denote it by β_n. Assume that β_n is *decreasing* in n. This captures the phenomenon of choice overload mentioned at the beginning of this chapter. The larger the set of market alternatives, the greater consumers' tendency to evade the choice problem and adhere to the default. Given that a consumer makes a price comparison, he chooses the cheapest alternative available from the entire set of n market alternatives. In case of a tie, he always prefers the default, and otherwise breaks the tie symmetrically.

As before, the max-min strategy is $p = 1$ and the max-min payoff is $\frac{1}{n}[1 - \beta_n]$. In symmetric Nash equilibrium, firms play a continuous and strictly increasing *cdf* over prices. The price $p = 1$ belongs to the support of the equilibrium strategy, and each firm earns its max-min payoff in equilibrium. Therefore, industry profits are equal to $1 - \beta_n$. This implies that equilibrium industry profits increase with the number of firms. This is simply because the larger number of market alternatives exacerbates choice overload.

Are there asymmetric Nash equilibria in the model? When $n = 2$, it can be shown that the equilibrium derived in Proposition 10.1 is unique—that is, there are no asymmetric equilibria. The situation is different when $n > 2$, as the following exercise demonstrates.

Exercise 10.1. *Let $n > 2$. Show that there exists an asymmetric equilibrium.*

Can the model be rationalized?

The basic model examined in this section is very stripped down, and thus open to multiple interpretations. We emphasized the aspect of default bias. However, the fact that some consumers fail to perform price comparisons may simply be the result of lack of information about the existence of other market alternatives, or of high search costs that make it sub-optimal to shop around. This rationalization breaks down under the extension to $n > 2$ which assumes that β_n is decreasing with n. In this case, the probability that the default alternative is chosen increases with the size of the choice set, and this is inconsistent with any utility maximization model.

Here is another way of thinking about this rationalization failure. When the prices of individual firms are independently drawn from some profile of probability distributions (as they are in Nash equilibrium), optimal consumer search would have a free disposal property: if searching through n alternatives is more costly than searching through $m < n$ alternatives, the consumer can always decide to stop his search after $m < n$ observations. Therefore, if searching for alternatives to the default is optimal when there are m alternatives, it should also be optimal when there are n alternatives. We should not expect a rational consumer with search costs to reduce his search effort when the number of market alternatives goes up.

■ 10.2 PRICE-FRAME COMPETITION

In this section I extend the basic model of the previous by allowing firms to choose strategically not only how to the price their product, but also how to present it to consumers—in other words, how to "frame" it. My objective is to construct a model that allows firms to use framing to influence the magnitude of consumer inertia.

I use the term "frame" in a broad sense that includes aspects of the products' presentation, which may be of no relevance to a consumer's utility and yet affect his propensity to make a price comparison between his default option and the new market alternative he faces. A frame can be a price format, a "language" in which a contract is written, or an aspect of the positioning of a product (e.g., a more-or-less arbitrary assignment of a food product into marketing categories: snacks, health food, etc.).

When the default and new market alternatives are presented in terms that are difficult to translate into one another, a consumer with default bias will tend to avoid making an active comparison and will adhere to the default. Similarly, if the default and new market alternatives are positioned differently (e.g., one is accompanied by an advertising campaign that cultivates a "healthy" image, while the other is promoted as a "fun" product), the consumer may fail to think of the new alternative as relevant to his consumption problem. The model of price-frame competition that I now present aims to capture these motives.

As in the baseline model, consider a market consisting of two identical, profit-maximizing firms and a continuum of consumers who are interested in buying one unit and have a willingness to pay of 1 for it. The two firms sell an intrinsically homogeneous product at zero cost. Let X be a finite set of product "frames." Firms simultaneously choose a pair (x, p), where $p \in [0, 1]$ is the price of the product and $x \in X$ is the product's frame.

Consumers choose as follows. As before, each consumer is randomly assigned to a default firm. Conditional on being initially assigned to firm i, the consumer makes a price comparison with probability $\pi(x_i, x_j)$, and then switches to firm j only if $p_j < p_i$. With probability $1 - \pi(x_i, x_j)$, the consumer does not make a price comparison and automatically adheres to the default i. Default bias is reflected by the following feature of the model: the consumer may choose his default firm when it is more expensive than the new alternative, but he never switches away from the default when the new alternative is at least as expensive.

Thus, given a strategy profile $(x_j, p_j)_{j=1,2}$ satisfying $p_2 < p_1$, the fraction of consumers who choose firm 1 is $\frac{1}{2}[1 - \pi(x_1, x_2)]$, and the fraction of consumers who choose firm 2 is $\frac{1}{2}[1 + \pi(x_1, x_2)]$. The firms' payoffs under such a strategy profile are thus:

$$U_1(x_j, p_j)_{j=1,2} = \frac{1}{2}p_1 \cdot [1 - \pi(x_1, x_2)]$$

$$U_2(x_j, p_j)_{j=1,2} = \frac{1}{2}p_2 \cdot [1 + \pi(x_1, x_2)]$$

When $p_1 = p_2$, each firm is chosen by half the population of consumers because no consumer ever switches away from his default. Firm i's payoff is thus $\frac{1}{2}p_i$.

The function $\pi : X \times X \to [0, 1]$ is a primitive of the consumers' choice procedure. It represents their limited propensity to make price comparisons, and how it is influenced by the way in which market alternatives are framed. The probabilistic aspect of π represents heterogeneity among consumers in this regard. The baseline model of the previous section is subsumed into this model as a special case in which $X = \{a\}$ and $\pi(a, a) = \beta$.

Throughout our discussion of this model, I restrict attention to the case of $X = \{a, b\}$, as it is the simplest departure from the baseline model that enables us to capture the role of framing in price competition with consumer inertia. Each of the two frames broadly represents a certain way of presenting a product.

I assume that π is symmetric—that is, $\pi(a, b) = \pi(b, a) = q$. The symmetry assumption is attractive when π captures how similar two frames are to each other. It will simplify our analysis because it implies that the probability the consumer compares two market alternatives does not depend on which of them is his default option. For all $x \in \{a, b\}$, denote $q_x = \pi(x, x)$. In order to avoid certain knife-edge cases, I assume that q_a, q_b, q are all *distinct* numbers *strictly* between 0 and 1. The role of these parameter restrictions will be addressed in exercises 10.2–10.4 at the end of the next section. Without loss of generality, let $q_a > q_b$.

A mixed strategy is a tuple $(\lambda, (F^x)_{x \in Supp(\lambda)})$, where $\lambda \in \Delta\{a, b\}$ is a mixed framing strategy and F^x is a mixed pricing strategy that is represented by a *cdf* over $[0, 1]$. The marginal pricing strategy F induced by $(\lambda, (F^x)_{x \in Supp(\lambda)})$ is

$$F = \lambda(a)F^a + \lambda(b)F^b$$

The assumption that $\pi(x, y) < 1$ for all $x, y \in \{a, b\}$ ensures that the firms' max-min payoff is strictly positive. The reason is that for any strategy (x, p) with $p > 0$ that a firm may play, a strictly positive fraction of the consumers will choose that firm, regardless of what the rival firm does. The fact that the max-min payoff is positive means that any equilibrium outcome must depart from the competitive, rational-consumer benchmark.

10.2.1 Nash Equilibrium

The restriction to two frames is of particular interest because different interpretations of the model are captured by two different parameter restrictions, which turn out to be crucial for equilibrium characterizations. In what follows, I construct a symmetric Nash equilibrium for each of these two cases and discuss its properties. In fact, it can be shown that there exist no other symmetric equilibria.

Case 1: Frames are ranked by comparability

In some situations, it is plausible to assume that one way of presenting a product is unambiguously better at fostering a price comparison. For instance, we may wish to interpret the frames a and b as simple and complex price formats, respectively. The former facilitates comparison and therefore curbs consumer inertia, while the latter obfuscates and therefore magnifies inertia. In this case, it makes sense to assume that a dominates b in terms of comparability: $\pi(a, x) > \pi(b, x)$ for every $x \in \{a, b\}$. This is equivalent to the following parameter restriction:

$$(q_a - q)(q_b - q) < 0$$

Since the frame a dominates the frame b in terms of comparability, it is intuitive that when a firm charges a low price, it will prefer to adopt the frame a because it wants consumers who are initially assigned to the rival firm to make a price comparison. In contrast, when a firm charges a high price, it will prefer to adopt the frame b because it does not want its own initial clientele to make a comparison with the rival firm.

We will now construct a mixed-strategy equilibrium strategy $(\lambda, (F^x)_{x\in\{a,b\}})$ that captures this intuition. The conditional pricing strategies F^a and F^b will be continuous and strictly increasing *cdf*s with supports that lie on top of one another, such that the frames a and b are unambiguously associated with low and high prices, respectively. Let us formalize this claim.

Proposition 10.2. *When $(q_a - q)(q_b - q) < 0$, there exists a symmetric mixed-strategy equilibrium strategy $(\lambda, (F^x)_{x\in\{a,b\}})$ with the following features. There exist prices p^l, p^m, where $0 < p^l < p^m < 1$, such that F_a is continuous and strictly increasing over the support $[p^l, p^m]$, while F^b is continuous and strictly increasing over the support $[p^m, 1]$. The probability that the marginal pricing strategy F assigns to $p < p^m$ is thus $F(p^m) = \lambda(a)$, and the probability it assigns to $p > p^m$ is $1 - F(p^m) = \lambda(b)$.*

In fact, this is the unique symmetric equilibrium under the parameter restriction of case 1. The uniqueness proof is omitted. Let us verify the existence of this equilibrium in a sequence of steps.

Step 1: Establishing an expression for equilibrium payoffs

We have assumed that the pure strategy $(b, 1)$ is in the support of $(\lambda, (F^x)_{x\in\{a,b\}})$. This pure strategy generates a payoff of

$$\frac{1}{2}\cdot 1\cdot [1-\lambda(a)q-\lambda(b)q_b] \tag{10.2}$$

The reason is that when a firm plays $(b, 1)$, it is compared to the rival firm with an overall probability of $\lambda(a)\pi(b, a) + \lambda(b)\pi(b, b)$. Since $p = 1$ is the highest price in the equilibrium price distribution, the firm loses its initial clientele whenever they make a price comparison, and it never gains new clients.

Step 2: Indifference conditions

In mixed-strategy Nash equilibrium, firms are indifferent among all pure strategies in the support of the equilibrium strategy. Thus, firms are indifferent among all strategies (b, p) and (a, p'), where $p \in [p^m, 1]$ and $p' \in [p^l, p^m]$. This translates into two systems of equations. For every $p \in [p^m, 1]$ and $p' \in [p^l, p^m]$:

$$\frac{1}{2}\cdot p\cdot [1-\lambda(a)q+\lambda(b)q_b(1-2F^b(p))] = \frac{1}{2}\cdot 1\cdot [1-\lambda(a)q-\lambda(b)q_b] \tag{10.3}$$

$$\frac{1}{2}\cdot p'\cdot [1+\lambda(a)q_a(1-2F^a(p'))+\lambda(b)q] = \frac{1}{2}\cdot 1\cdot [1-\lambda(a)q-\lambda(b)q_b] \tag{10.4}$$

In particular, p^l and p^m are given by setting $F^a(p^l) = 0$, $F^a(p^m) = 1$, $F^b(p^m) = 0$.

Step 3: Verifying the optimality of framing decisions

It is clear that no firm would ever want to deviate to a strategy (x, p) with $p < p^l$, since it will generate the same market share as the strategy (x, p^l). Thus, to verify that we are indeed in equilibrium, it only remains to check that the firms' choices of frames are optimal for each price $p \in [p^l, 1]$.

In order for the frame b to be optimal in the price range $[p^m, 1]$, it must generate a weakly higher market share than a in this price range. Similarly, in order for the frame a to be optimal in the price range $[p^l, p^m]$, it must generate a weakly higher market share than b in this price range. This translates into two systems of inequalities. For every $p \in [p^m, 1]$ and $p' \in [p^l, p^m]$:

$$\lambda(b)q_b(1-2F^b(p))-\lambda(a)q \geq \lambda(b)q(1-2F^b(p))-\lambda(a)q_a \tag{10.5}$$

$$\lambda(a)q_a(1-2F^a(p'))+\lambda(b)q \geq \lambda(a)q(1-2F^a(p'))+\lambda(b)q_b \tag{10.6}$$

The intuition behind these inequalities is that when a firm charges an intermediate price $p \in (p^l, 1)$, it trades off the incentive to force a comparison with more expensive rival firms, and the incentive to avoid a comparison with cheaper firms. When the price is above p^m, the first force should dominate and the firm should prefer the frame b, whereas when the price is below p^m, the second force should dominate and the firm should prefer the frame a.

Step 4: Pinning down the framing strategy

The cutoff price p^m is exactly where the two forces are meant to offset each other, such that each firm is indifferent between the two frames conditional on this price. This means that inequalities (10.5) and (10.6) are supposed to hold with equality at p^m. Since $F^a(p^m) = 1$, $F^b(p^m) = 0$ and $\lambda(a) + \lambda(b) = 1$, these equations lead to an exact solution for the equilibrium framing strategy:

$$\lambda(a) = \frac{q - q_b}{q_a - q_b} \tag{10.7}$$

Note that our parameter restriction ensures that $\lambda(a)$ is strictly between 0 and 1. In addition, it is easy to verify that inequalities (10.5) and (10.6) do indeed hold.

Step 5: Pinning down the conditional pricing strategies and equilibrium payoffs

We can now plug the expression for $\lambda(a)$ into equations (10.3) and (10.4) and get a closed solution for the equilibrium strategy in terms of the parameters q_a, q_b and q. The firms' equilibrium profit can be obtained by plugging (10.7) into (10.2), yielding:

$$\frac{1}{2} \cdot [1 - \frac{q - q_b}{q_a - q_b} \cdot q - \frac{q_a - q}{q_a - q_b} \cdot q_b] \tag{10.8}$$

We will later use this expression for comparative statics exercises that capture regulatory market interventions of interest.

Case 2: Frames are not ranked by comparability

This is the opposite of case 1—that is, no frame dominates the other in terms of comparability. Equivalently:

$$(q_a - q)(q_b - q) > 0$$

In particular, when we wish to interpret a and b as two ways of positioning an intrinsically homogeneous product, it makes sense to assume that $q < q_b, q_a$. That is, the consumer is less likely to compare his default product to a new product if the two are positioned differently.

Since no frame dominates another, it is possible to find a framing strategy λ^* that equalizes the probability of a price comparison across frames. In other words, λ^* satisfies the following equation:

$$\lambda^*(a)\pi(a, a) + \lambda^*(b)\pi(a, b) = \lambda^*(a)\pi(b, a) + \lambda^*(b)\pi(b, b)$$

Since $\lambda(a) + \lambda(b) = 1$, the solution to this equation is

$$\lambda^*(a) = \frac{q_b - q}{(q_a - q) + (q_b - q)} \tag{10.9}$$

such that the frames a and b generate the same probability of a price comparison

$$v^* = \frac{q_a q_b - q^2}{(q_a - q) + (q_b - q)} \tag{10.10}$$

When one firm plays the framing strategy λ^*, the other firm is indifferent between the two frames, regardless of the price it charges, because the two frames induce the same probability of a price comparison. In other words, λ^* neutralizes the relevance of framing. This enables us to construct an equilibrium with very different features from those we devised for case 1. In fact, there is a unique symmetric Equilibrium in case 2. The uniqueness proof is omitted.

Proposition 10.3. *When* $(q_a - q)(q_b - q) > 0$, *there exists a symmetric mixed-strategy equilibrium strategy* $(\lambda, (F^x)_{x \in \{a,b\}})$ *with the following features. First,* $\lambda = \lambda^*$ *as given by* (10.9). *Second,* $F^a = F^b = F$ *is given by the cdf* (10.1), *where* $\beta = v^*$ *as in expression* (10.10).

In this equilibrium, firms play the framing strategy λ^*, and *independently* play the same equilibrium pricing strategy they would play in the baseline model with $\beta = v^*$.

Verifying that this constitutes an equilibrium is straightforward. When one firm plays the framing strategy λ^*, it is optimal for the other firm to play λ^* as well, regardless of the price it charges. This reduces the model to the baseline model of the previous section, in which the probability of a price comparison is exogenously fixed at v^*. This immediately implies that the firms' pricing strategy is the *cdf* (10.1) with $\beta = v^*$. It also means that the firms' equilibrium payoff is $\frac{1}{2}(1 - v^*)$, just as in the baseline model.

I conclude this section with a sequence of exercises that illustrate the role of our assumptions on π in the equilibrium analysis. The first two exercises simply deal with knife-edge cases that were assumed away by the assumption that q_a, q_b, q are distinct numbers in $(0, 1)$, and they give rise to an equilibrium characterization that is in harmony with the above analysis. In contrast, the third exercise relaxes the assumption that π is symmetric—namely, that the probability a consumer makes a price comparison between two market alternatives does not depend on which of them is his default option. This leads to a significantly different equilibrium analysis.

Exercise 10.2. *Let* $q_a = 1$, $q_b = 0$, $q \in (0, 1)$. *Find a symmetric Nash equilibrium. (Hint: The marginal pricing strategy is continuous over* $[p^l, 1]$, *but has an atom on* $p = 1$.*)*

Exercise 10.3. *Let* $q_a, q_b, q \in (0, 1)$. *Assume that* $q_a \neq q_b$ *and* $q \in \{q_a, q_b\}$. *Find a symmetric Nash equilibrium.*

Exercise 10.4. *Relax the assumption that* $\pi(a, b) = \pi(b, a)$. *In particular, let* $q_a = q_b = 1$, $\pi(a, b) = q \in (0, 1)$, *and* $\pi(b, a) = 0$. *Show that in symmetric Nash equilibrium, the supports of* F^a *and* F^b *contain one another. Compare the firms' equilibrium profits to the max-min level.*

10.2.2 Equilibrium Properties

The equilibria we constructed for the two cases have several properties of interest.

Spurious product differentiation

In this model, the product that the firms provide is intrinsically homogeneous. In particular, consumers are willing to pay 1 for the product, independently of its frame. In both cases 1 and 2, each of the frames a and b is adopted with positive probability. This can be viewed as a kind of product differentiation. As in the model of Section 6.3, this differentiation is spurious: it has nothing to do with consumer preferences and entirely to do with the firms' response to consumer inertia, which implies that only a fraction of consumers make active price comparisons.

Correlation between prices and frames

In case 1, where the frame a dominates the frame b in terms of comparability, the two frames always generate different probabilities of a price comparison, and therefore the framing decision is relevant in equilibrium. As a result, firms strictly prefer to adopt the frame a (b) when they charge low (high) prices. Thus, the firms' pricing and framing decisions are correlated. In contrast, in case 2, where no frame dominates the other in terms of comparability, we saw that it is possible to find a framing strategy λ^* that neutralizes the relevance of framing. This allowed us to construct an equilibrium in which the firms' pricing and framing decisions are statistically independent. Thus, whether prices and frames are correlated in equilibrium depends on whether we can rank the two frames according to their comparability.

The possibility of collusive profits

An interesting question is whether competitive forces in the model of price-frame competition are strong enough to push firms' profits to the minimal level they can secure given the consumers' bounded rationality—namely, the max-min level. It turns out that the answer to this question, too, depends on whether the two frames can be ranked according to their comparability.

In case 1, the max-min payoff is $\frac{1}{2}(1 - q)$. The reason is that a firm's worst-case scenario is where the opponent plays the pure strategy $(a, 0)$. The best-reply against this scenario is the pure strategy $(b, 1)$, such that the probability the consumer makes a price comparison is q. But now look at expression (10.8) for the firms' equilibrium payoff. Since $q_b < q$, this expression is strictly above the max-min level. The logic behind this observation is that although each firm adopts the frame b when it charges $p = 1$ in equilibrium, the rival firm does *not* play the frame a with probability one. The equilibrium probability that a firm playing $(b, 1)$ faces a price comparison is a weighted average of q and q_b, which is below q. This gives firms some market power relative to the max-min.

Let us turn to case 2. When a firm plays a framing strategy λ such that $\lambda(a) > \lambda^*(a)$ ($\lambda(a) < \lambda^*(a)$), the worst-case scenario is that the rival firm plays $(a, 0)$ ($(b, 0)$). In both cases, the probability of a price comparison is above v^* (as given by (10.10)). The max-min strategy is thus to charge $p = 1$ and play the framing strategy λ^*. This ensures that the probability of a price comparison is v^*, such that the max-min payoff is $\frac{1}{2}(1 - v^*)$. But this is exactly the payoff that firms earn in the equilibrium we constructed. Thus, firms earn max-min payoffs in equilibrium.

To summarize, in the equilibria we constructed in the two cases, both price-frame correlation and the possibility of collusive equilibrium profits are linked to the question of whether one frame dominates another in terms of comparability. We will discuss the generality of this observation towards the end of this chapter.

10.2.3 Two Market Interventions

The equilibrium characterization of the two-frame case allows us to examine the implications of two stylized regulatory interventions.

Enhancing the transparency of price formats

The interpretation we proposed for case 1 was that a represents a simple, transparent price format, while b represents a complex, obscure format. Imagine a regulator who wishes to improve "market transparency" to foster price comparisons. Suppose that the regulator's intervention has the effect of increasing q_a. Then, it is easy to see from expression (10.8) that the firms' equilibrium payoff (equivalently, expected equilibrium price) *increases* as a result of the intervention.

This is a surprising finding. When we increase the values of q_a, q_b, or q, there is a sense in which we make the consumer "more rational": for any fixed strategy profile, he is more likely to choose the cheaper market alternative. Nevertheless, a higher value of q_a implies higher equilibrium prices and industry profits.

The intuition for this effect is that a higher q_a strengthens an expensive firm's incentive to adopt the complex format b. As a result, the equilibrium fraction of firms that adopt this frame goes up. Now, recall that adopting the frame b is correlated with charging a high price in equilibrium. The probability that these expensive firms face a price comparison is a weighted average of q and q_b, where the weight on q_b is equal to the fraction of firms that adopt b. But since this fraction goes up as a result of the increase in q_a, the probability that expensive firms face a price comparison goes down, toward q_b. This gives expensive firms greater market power and as a result, equilibrium profits go up.

Note that by contrast, when the regulator's intervention entails an increase in q_b, this lowers equilibrium profits, because it reduces the fraction of firms that charge a high price and adopt the frame b, as well as increases the probability that consumers compare between such firms. Thus, when a and b represent simple and complex price formats, an intervention that simplifies the relatively complex format is beneficial for consumers, whereas an intervention that further simplifies the relatively simple format is counterproductive in this respect.

Introducing new brands

Assume that a regulator considers consumers' benefit from introducing a new brand category into the market. A standard approach that views product differentiation as a market response to true diversity in tastes may welcome this addition. However, if we take the approach that the differentiation is spurious and merely involves a new way of marketing a homogeneous product, the intervention may diminish consumer welfare. In particular, suppose that a and b stand for two brand categories. If q (the parameter that measures the comparability between the two brand categories) is lower than q_b and q_a (which measure comparability within each brand category), equilibrium profits and prices are higher than if we eliminated one of the two categories.

This pair of examples demonstrates that when firms use framing to respond to and manipulate consumer inertia, the equilibrium implications of conventional competition or consumer protection policies may be counterproductive.

■ 10.3 CONSUMER SWITCHING

Given that consumer inertia is our focus in this chapter, an interesting question is how frequently consumers actually get stuck with their default option in equilibrium, and how this quantity is related to the competitiveness of the market outcome. For any symmetric Nash equilibrium, define the *switching rate* to be the probability that a consumer switches away from the default in equilibrium. Define the *conversion rate* to be the rate of switching *conditional on* the event that the consumer has made a price comparison.

In the basic model of Section 10.1, the probability that consumers make a price comparison is exogenously fixed to be β. Given that firms play a symmetric equilibrium, the conversion rate is $\frac{1}{2}$ because each firm is equally likely to be the cheaper market alternative. Therefore, the switching rate is $\frac{1}{2}\beta$. It follows that weaker inertia leads to a more competitive equilibrium outcome and a greater frequency of consumer switching.

In the extended model of price-frame competition, the probability that a consumer compares his default option to the other market alternative is determined by the firms' endogenous framing decisions. However, the conversion rate in any symmetric equilibrium remains $\frac{1}{2}$, for the following simple reason. Conditional on making a price comparison, the consumer faces some symmetric posterior probability distribution over price profiles (p_1, p_2). (The symmetry of π ensures the symmetry of this posterior distribution.) Because the marginal pricing strategy is continuous, $p_1 \neq p_2$ with probability one. But then consumers switch suppliers if and only if they were initially assigned to the more expensive firm. Because of the symmetry of the consumers' initial assignment to firms and the symmetry of the posterior probability distribution over prices, the fraction of consumers who switch away from the default, conditional on making a price comparison, is $\frac{1}{2}$. It follows that the switching rate is half the probability that consumers make a price comparison in equilibrium.

In contrast, the probability that consumers make a price comparison turns out to depend on the underlying parameters in a rather nuanced way. We saw in case 2 that in symmetric equilibrium, firms play independent pricing and framing strategies, and that the probability of a price comparison is given by expression (10.10). Therefore, the switching rate is half that expression. Since equilibrium payoffs are equal to the max-min level in this range of parameter values, it follows that the switching rate unambiguously rises as equilibrium profits fall.

In contrast, in case 1, the equilibrium probability of a price comparison is

$$[\lambda(a)]^2 q_a + 2\lambda(a)\lambda(b)q + [\lambda(b)]^2 q_b$$

where λ is given by (10.7). This expression can lie above or below q, depending on whether or not $q > \frac{1}{2}(q_a + q_b)$. Furthermore, its comovement with the competitiveness of the market outcome is ambiguous, because as we saw, equilibrium profits in case 1 increase with q_a and decrease with q_b. Thus, when prices and frames are correlated in equilibrium, the clear positive link between the switching rate and the competitiveness of the market outcome breaks down.

10.4 ASYMMETRIC DEFAULT ASSIGNMENT

So far, we analyzed a symmetric model, in which each firm plays the role of a default option for exactly half the population of consumers. Now consider an asymmetric variant on the model of price-frame competition, in which all consumers are initially assigned to firm 1.

This version turns out to be considerably simpler than the case we have analyzed so far. The reason is that, unlike the case of symmetric default assignment, here firm 1 (2) has an unambiguous incentive to choose a frame that minimizes (maximizes) the probability of a price comparison, independently of the price it charges. Therefore, firms choose their framing strategies as if they play a Nash equilibrium in a zero-sum game, in which the players are firms 1 and 2, the action space is X for both players, and firm 2's payoff function is π. Let v^* denote the *value* of this zero-sum game. Then, firms choose their marginal pricing strategies as if they play the baseline model of this chapter, except that the default assignment is asymmetric. It is straightforward to show that the firms' marginal equilibrium pricing strategies are defined over the interval $[1 - v^*, 1)$ as follows:

$$F_1(p) = 1 - \frac{1 - v^*}{p}$$

$$F_2(p) = \frac{1}{v^*} \cdot F_1(p)$$

In addition, F_1 has an atom of size $1 - v^*$ at $p = 1$. Firm 1's equilibrium payoff is $1 - v^*$, while firm 2's equilibrium payoff is $v^*(1 - v^*)$.

Note that firm 2's equilibrium payoff does not rise monotonically with the probability of a price comparison v^*. The intuition is that a high v^* means that competitive forces are strong and thus prices are close to zero, whereas when v^* is low, competitive forces are weak and thus most consumers do not switch away from firm 1.

Industry profits in Nash equilibrium are equal to $1 - (v^*)^2$. A comparison with the symmetric default assignment case is not trivial, because as we saw in the two-frame case, the equilibrium characterization relies on whether frames can be ranked in terms of comparability. The comparison is straightforward when π fits into case 2, because as we saw, equilibrium characterization reduces the model to the symmetric baseline model with $\beta = v^*$, where industry profits are equal to $1 - v^*$. Therefore, in this case, asymmetric default assignment generates a less competitive equilibrium market outcome. It can be shown that this conclusion extends to case 1 as well. However, in this case the comparison is more cumbersome and probably less amenable to generalizations.

10.5 A FEW GENERAL REMARKS

I conclude this chapter with a few general comments that are meant to deepen our understanding of the models analyzed in this chapter.

10.5.1 More Than Two Frames

Can we generalize the insights obtained in the model of price-frame competition under the restriction to two frames? In particular: (i) What is the general condition for the equilibrium outcome to coincide with the competitive, rational-consumer benchmark? (ii) Is there a proper extension of the distinction between cases 1 and 2 in the two-frame model? In this sub-section I make do with a brief discussion of the model for an arbitrary finite set of frames X, and an arbitrary function $\pi : X \times X \to [0, 1]$ satisfying $\pi(x, x) > 0$ and $\pi(x, y) = \pi(y, x)$ for all $x, y \in X$.

Conditions for a competitive equilibrium outcome

As already mentioned, when $\pi(x, y) = 1$ for all $x, y \in X$, the consumer's behavior is rational, in the sense that he makes a price comparison and chooses the cheapest firm with probability one, independently of the firms' framing strategies. In this case, firms play $p = 0$ and an arbitrary framing strategy in Nash equilibrium. However, a competitive outcome prevails in equilibrium even if consumer choice falls short of full rationality.

Proposition 10.4. *Firms play $p = 0$ in Nash equilibrium if and only if there exists $x^* \in X$ such that $\pi(y, x^*) = 1$ for all $y \in X$.*

I omit the proof. The intuition is that x^* is a frame that enforces a price comparison with a more expensive firm, and competitive forces imply that each firm has an incentive to lower its price and adopt this frame. Note that this incentive is not clear-cut a priori, because the deviation may increase the probability of a comparison with *cheaper* firms. This is what makes the proof non-trivial.

Frame neutrality

The essence of the distinction between cases 1 and 2 in the two-frame case was the possibility of frame neutrality. In case 1, since one frame dominated the other in terms of comparability, framing was always relevant and therefore correlated with the firms' pricing decisions in equilibrium. In contrast, in case 2 it was possible to find a framing strategy that equalizes the probability of a price comparison across frames and thus renders framing irrelevant. The following is a generalization of this notion of frame neutrality.

We say that π is *frame-neutral* if there exist a framing strategy $\lambda^* \in \Delta(X)$ and a number $v^* \in [0, 1]$ such that

$$\sum_{y \in X} \lambda^*(y)\pi(x, y) = v^*$$

for every $x \in X$. We then say that π is *frame-neutralized* by λ^*.

This concept of frame neutrality turns out to be the right one for generalizing the two-frame analysis.

Proposition 10.5. In symmetric Nash equilibrium, firms earn max-min payoffs if and only if π is frame-neutral.

Again, the proof is omitted. This result is important because it establishes a tight link between the question of whether framing can be neutralized and the question of whether firms can earn collusive profits. Indeed, if π is frame-neutral, firms play a framing strategy that neutralizes π in any symmetric equilibrium.

As to correlation between firms' equilibrium pricing and framing decisions, here again frame neutrality proves to be a relevant concept. If π is not frame-neutral, prices and frames must be correlated in any symmetric equilibrium. In particular, firms adopt different framing strategies when they charge the highest and lowest prices in the equilibrium price distribution.

The converse is not true: frame neutrality does not imply statistical independence between firms' pricing and framing decisions. Instead, it implies a weaker notion of independence: if π is frame-neutral, the probability of a price comparison in symmetric equilibrium is independent of the firms' pricing decisions. Thus, even if firms vary the frames they adopt with the prices they charge, they do so in a way that does not affect the comparison probability.

10.5.2 Revealed Preferences

The nature of consumers' departure from rational choice in the model of price-frame competition can be illuminated by a brief choice-theoretic exercise. Specifically, let us write down the revealed preference relation induced by the consumers' choice procedure and examine its rationality properties.

A consumption problem with a default option can be described as a pair (A, d), where A is the set of available alternatives and $d \in A$ is the default option. The consumer's revealed (strict) preference relation $\succ$ over alternatives can be defined as follows: $z \succ z'$ if the consumer chooses z in the choice problem $(\{z, z'\}, z')$. The revealed indifference relation $\sim$ is therefore defined as follows: $z \sim z'$ if $z \not\succ z'$ and $z' \not\succ z$. Default bias has an intuitive definition in terms of the revealed preference relation: if the consumer chooses z in $(\{z, z'\}, z')$, then he necessarily chooses z in $(\{z, z'\}, z)$.

Recall that in the model of price-frame competition, an alternative z is a pair (x, p). Ignore consumer heterogeneity, such that the function $\pi(x, y) \in \{0, 1\}$ for all x, y. If the consumer chooses (x, p) over (x', p') when the latter alternative is the default, then it must be the case that $p < p'$. But this means that the consumer will definitely choose (x, p) over (x', p') when the former alternative is the default. Thus, consumer behavior indeed satisfies the choice-theoretic definition of default bias.

The revealed preference relation over price-frame pairs is generally intransitive. This holds both for weak and strict preferences. To illustrate the latter, let $X = \{a, b, c\}$, and assume that $\pi(a, c) = \pi(c, a) = 0$ and $\pi(x, y) = 1$ for every other pair of frames. Let $p < p' < p''$. When faced with the strategy profile $((a, p), (b, p'))$, the consumer makes a price comparison and chooses the cheaper alternative (a, p), independently of which of the two alternatives is the default. Similarly, when faced with the strategy profile $((b, p'), (c, p''))$, the consumer always chooses the cheaper alternative (b, p'). However, when faced with the strategy profile $((a, p), (c, p''))$, the consumer sticks to his default alternative because he fails to make a price comparison.

Exercise 10.5. *Let $X = \{a, b\}$. Construct a function $\pi : \{a, b\}^2 \to \{0, 1\}$ such that the revealed indifference relation induced by the consumers' choice procedure in the model of price-frame competition will be intransitive.*

The intransitivities demonstrated here imply that, in general, the consumer's choice behavior in the model of price-frame competition cannot be accounted for by maximization of a utility function.

10.6 SUMMARY

In this chapter we examined price competition models in which consumers exhibit inertia. In particular, we examined two forces that can affect the magnitude of the consumers' bias: the number of market alternatives and their framing. We can summarize the lessons we drew as follows.

- When some consumers exhibit inertia, this gives firms partial market power that gives rise to price variation. If a larger number of alternatives strengthens inertia, then industry profits increase with the number of firms.
- New effects arise when firms can use framing to influence consumer inertia. In a two-firm, two-frame model, if one frame a unambiguously induces a weaker bias than the other frame b, then firms' pricing and framing decisions are correlated in equilibrium, such that low (high) prices are unequivocally associated with a (b). In this equilibrium, firms earn profits above the max-min level. In contrast, when no frame dominates another in terms of its impact on inertia, pricing and framing are independent in equilibrium. Indeed, the equilibrium strategy induces a frame-independent probability of a price comparison. Firms earn max-min profits in equilibrium.
- Weakening consumer inertia can perversely give rise to a less competitive equilibrium outcome. This means that well-meaning regulatory interventions can be counter-productive in terms of consumer welfare.
- In general, the frequency with which consumers switch suppliers in equilibrium is ambiguously related to the competitiveness of the equilibrium outcome. However, when framing is irrelevant (either by assumption or endogenously), a more competitive outcome goes hand in hand with more switching.

10.7 BIBLIOGRAPHIC NOTES

For psychological evidence on the tendency to choose default options, see Samuelson & Zeckhauser (1988), Tversky & Shafir (1992), Ritov & Baron (1992) and Anderson (2003). The phenomenon of choice overload was demonstrated by Iyengar & Lepper (2000) in an experiment that generated some controversy (see Scheibehenne, Greifender & Todd (2010)), and more recently by Iyengar & Kamenica (2008). Default bias turns out to be particularly relevant for participation in defined contribution pension plans—see Choi, Laibson, Madrian & Metrick (2010), Huberman, Iyengar & Jiang (2007), and Carroll, Choi, Laibson, Madrian & Metrick (2009) for representative empirical studies. For popular discussions of default bias and its economic implications, see Iyengar (2010) and Thaler & Sunstein (2008).

The baseline model in Section 10.1 is due to Varian (1980). Fershtman & Fishman (1994) and Armstrong, Vickers & Zhou (2009) endogenize the fraction of consumers who make price comparisons as the outcome of conventional search decisions. In these models, making price comparisons carries a cost. In market equilibrium, consumers' decision whether to make a comparison is optimal given correct knowledge of the firms' equilibrium pricing strategy. In this environment, price controls can have a perverse effect. Suppose that a regulator imposes a price cap $\bar{p} < 1$. This puts an upper bound on the amount of price dispersion that can exist in the market, and therefore lowers consumers' incentive to incur the comparison cost. As a result, it is possible that the new equilibrium value of β will go down to such an extent that industry profits will be higher than in the absence of a price cap. Carlin (2009) endogenizes the fraction of consumers who make a comparison in a different way. In his model, it is the firms who are able to take actions (interpreted as obfuscation) that will affect the fraction of "inert" consumers.

The basic modeling idea that framing can influence the probability that consumers fail to consider new alternatives is based on Eliaz & Spiegler (2010*a*). The model of price-frame competition closely follows Piccione & Spiegler (2009). In particular, the two-frame case is a simplified version of a specification of the Piccione-Spiegler model called "bi-symmetry." Chioveanu & Zhou (2009) develop further the two-frame model in an environment with more than two firms. However, their model of consumer choice is somewhat different, as consumers enter the market without a default alternative. The firms' framing decisions influence the subset of market alternatives that consumers end up considering in such a way that when there are only two firms, their model formally collapses to case 1 of the two-frame model studied here. They show that for certain natural specifications of their model, almost all firms adopt the frame that minimizes the probability of a price comparison, and industry profits do not converge to zero when the number of firms tends to infinity.

This chapter focused on the type of default bias that originates from a failure to give new alternatives serious consideration. The other approach to modeling default bias is based on the notion of status quo preferences, where choosing a new alternative over the status quo carries a "psychological" switching cost, was studied choice-theoretically by Masatlioglu & Ok (2005). Dean (2008) conducts a choice-theoretic and experimental comparison between the two approaches. Within the Industrial Organization literature, there has been some work on price competition in the presence of switching costs—interpreted as physical or psychological in origin—for example, see Klemperer (1995). However, this literature has mostly focused on multi-period models in which firms may create switching costs as part of their dynamic competitive strategy.

11 Inertia II: Costly Marketing

In this chapter we continue our investigation of market competition under consumer inertia. We will examine a model in which firms can employ costly marketing devices in order to overcome consumer inertia. The role of marketing in this model is to force consumers' attention away from their default and toward new market alternatives, and to persuade consumers to consider them seriously.

In the model we shall study, each firm offers a *collection* of products. Consumers have limited ability to pay attention to every product. Naturally, some products may be better at attracting attention than others. This may be due to the fact that some options may be better than others, along some salient dimension. For example, when searching for a laptop, a very low price or a very light weight will most likely draw one's attention; when flipping through TV channels in search of a program to view, one may pay greater attention to a sensational news report, or to a special guest appearance by a celebrity on a situation comedy.

Thus, the mere offering of a particular item can have an indirect effect on a firm's market share by drawing attention to the firm and other items it offers. For instance, the items that stores display on their shop front and that web retailers put on their homepage can exert a positive externality on other items, by persuading consumers to enter the store/web site and browse its selection. Similarly, the shows and news items that a TV network chooses to broadcast may persuade viewers to stay tuned to that channel and therefore become exposed to other items it offers. As a result, consumers whose attention is initially attracted to a firm because of a particular item may end up consuming another. Firms may take this indirect marketing effect into account when designing a "product line." Specifically, they may introduce an item even when the direct demand for this item fails to cover its cost. Each firm faces a trade-off between the cost of adding "attention grabbers" and the benefit of the extra market share they may generate.

Because of the fixed cost component of the model, it will be technically easier to analyze a competitive model that abstracts from price competition and focuses on the trade-off between market share and marketing costs. Given this focus, it will be natural to consider media markets in which firms derive their revenues from advertising and do not charge prices from consumers. In this environment, the assumption that firms care only about their market share and their (fixed) costs is realistic.

Therefore, we will cast the model in terms of a highly stylized broadcast television market, and examine the TV networks' strategic programming, taking into account their attempt to overcome viewers' tendency to stick to their default channel. The assumption of consumer inertia is realistic in this context. TV viewers are known to have a tendency to adopt a default channel and use it as a "home base" for occasional channel-flipping cruises. For the competing channel, the challenge is first to attract the viewer's attention, and then to convince him to continue watching it. The competing channel's programming strategy will take this motive into account: it may wish to introduce sensational shows because of their attention-grabbing value, even if this value quickly wears off and the consumer would not have been willing to pay extra for watching these shows.

There are other examples for the use of products as attention grabbers. For instance, in 2004, McDonald's enriched their menus with "healthy" options such as salads and fresh fruit, in an attempt to appeal to health-conscious customers. One may argue that the rationale was not so much to generate large direct revenues from the healthy options, but to create a more health-conscious image that would induce a segment of the consumer population to consider McDonald's restaurants seriously. Once at the restaurant, these consumers will not necessarily choose the healthiest items on the menu, and their consumption decision at the restaurant will involve other motives (such as price, or how filling the meal is).

Alternatively, think of a consumer who wants to buy a new laptop computer. He initially considers a particular model x, possibly because it is his current machine. The consumer may then notice that a computer store offers a model y that is significantly lighter than x. This triggers the consumer's curiosity and gives him a sufficient reason to consider y in addition to x. Upon closer inspection, the consumer realizes that he does not like y as much as he does x. However, since he is already inside the store, he may browse the other laptop computers on offer and find a model z that he ranks above both x and y. Thus, although few consumers may actually buy y, this model functions as a "door opener" that draws consumers' serious attention to the other products offered by the store. At any rate, in this chapter we will adhere to the broadcast TV metaphor and interpret the model and the results accordingly.

■ 11.1 A MODEL OF COMPETITIVE MARKETING

Let X be a finite set of "items." Denote $|X| = n$. A menu is a non-empty subset of X. Let $P(X)$ be the set of all menus. Two firms, interpreted as TV networks, compete for a continuum of identical consumers (viewers) having a strict preference relation $\succ^*$ over items. For every menu M, let $b(M)$ denote the (unique) $\succ^*$-maximal item in M. Denote $x^* = b(X)$. Consumers have a preference relation $\square$ over menus. These preferences are "max-max"—that is, $M \succ M'$ if and only if $b(M) \succ^* b(M')$. Thus, when consumers evaluate a menu M, they only consider their favorite item on the menu, $b(M)$. The interpretation is that $b(M)$ is *the only item the consumers regularly consume from the menu* M. Both consumer homogeneity and max-max preferences are highly restrictive assumptions that are made for simplicity.

The two firms play a simultaneous-move game in which they choose menus. A menu is interpreted as the TV network's programming strategy. A mixed strategy is a probability distribution $\sigma \in \Delta(P(X))$. Given a mixed strategy σ, define

$$\beta_\sigma(x) = \sum_{b(M)=x} \sigma(M)$$

to be the probability that the strategy assigns to menus in which the consumers' favorite item is x. Let $S(\sigma)$ denote the support of σ.

Each menu M carries a fixed cost $c(M) = \sum_{x \in M} c_x$, where $c_x > 0$ is the fixed cost associated with the item x. The cost structure is identical for both firms. Assume that $c_x > c_y$ if and only if $x \succ^* y$. That is, it is more costly to produce an item that the consumer likes better. This assumption enables us to interpret $\succ^*$ as a *quality ranking*. From now on, we will adopt this interpretation.

The consumers' choice procedure is based on two primitives, which are two binary relations over X. The first is the preference relation $\succ^*$ over items we have already encountered. As we saw, $\succ^*$ underlies the consumers' (max-max) preferences over menus. The second primitive is a *consideration relation* R. The interpretation of xRy is that x attracts attention away from y. Given a profile of the firms' chosen menus (M_1, M_2), consumers choose as follows. Each consumer is initially assigned to a firm $i = 1, 2$. The fraction of consumers who are initially assigned to each firm is identical. The initial assignment represents the consumer's default. We say that M_j beats M_i if two conditions hold: (i) $yRb(M_i)$ for some $y \in M_j$; and (ii) $M_j \succ M_i$. The consumer switches from his default firm i to firm $j \neq i$ if and only if M_j beats M_i.

The interpretation is as follows. Because of inertia, consumers consider a TV channel other than their default only if some item on the new channel attracts their attention away from the item they regularly consume at the default channel. Having considered the new channel, the consumer will switch to it if he finds it strictly superior to his default menu according to his true, underlying preferences.

The consumers' choice procedure induces the following payoff function in the symmetric, complete-information game played by the two firms:

$$U_i(M_1, M_2) = \frac{1}{2}[1 + f_i(M_1, M_2)] - c(M_i)$$

where $f_i(M_1, M_2) = 1$ if M_i beats M_j, $f_i(M_1, M_2) = -1$ if M_j beats M_i, and $f(M_1, M_2) = 0$ if neither menu beats the other.

Example 11.1. Let $X = \{x, y\}$, and assume $x \succ^* y$, xRx, yRy, yRx, $x\neg Ry$. Then, if a consumer's default firm offers the menu $\{y\}$ while the rival firm offers the menu $\{x\}$, the consumer will stick to his default firm. However, if the rival firm offers $\{x, y\}$, the consumer will switch to the new firm. Note that in this example, the inclusion of y in $\{x, y\}$ affects consumer choice, even though $y \neq b\{x, y\}$. The item y is irrelevant for the consumers' evaluation of the menu $\{x, y\}$, and is only relevant as an attention grabber. In general, I will refer to any item $x \in M \backslash \{b(M)\}$ as a "pure attention grabber."

From now on, unless otherwise specified, we shall assume that R is *reflexive, transitive, and antisymmetric*. Such a consideration relation captures the "sensation value" of various items, or the ranking of items along some salient preference criterion (which may be incorporated somehow into the consumers' overall preference ranking of items $\succ^*$). I do not impose any restrictions concerning the overlap between the two primitive relations, $\succ^*$ and R. For every menu M, let $r(M)$ denote the (unique) R-maximal item in M. Denote $r^* = r(X)$. Thus, r^* is the best attention grabber in X, whereas x^* is the highest-quality item in X. Assume that $c\{x^*, r^*\} < \frac{1}{2}$. That is, if a firm offers the highest-quality item in conjunction with the best attention grabber, its cost is lower than the value of 50% market share.

If consumers had unlimited attention and thus always chose according to their preferences $\sqsupset$ over menus, the beating relation would be simple: M would beat M' if and only if $b(M) \succ^* b(M')$. In Nash equilibrium, both firms would offer $\{x^*\}$ and earn a payoff of $\frac{1}{2} - c_{x^*}$. In contrast, under the two-stage procedure given by $(\succ^*, R)$, we will see that Nash equilibrium typically departs from the rational-consumer benchmark. However, the menu $\{x^*\}$ continues to play an important role in our model. It is the firms' max-min strategy, and $\frac{1}{2} - c_{x^*}$ is their max-min payoff. To see why, note that regardless of a firm's strategy, its worst-case scenario is when the opponent plays any menu that contains $\{x^*, r^*\}$. The best-reply against this strategy is $\{x^*\}$, because it is the cheapest strategy that generates a market share of $\frac{1}{2}$ in the worst-case scenario. All other strategies either generate the same market share at a greater cost, or induce a market share of zero.

Is consumer choice rational?

The consumers' revealed preference relation over menus can be defined in the same way as in Chapter 10. The menu M is revealed to be strictly preferred to M' if a consumer who is initially assigned to M' switches to M—that is, if M *beats* M'. When neither M nor M' beat one another, the consumer is revealed to be indifferent between them. Example 11.1 demonstrates the intransitivity of the revealed indifference relation: $\{x, y\}$ does not beat $\{x\}$ and vice versa; $\{x\}$ does not beat $\{y\}$ and vice versa; and yet $\{x, y\}$ beats $\{y\}$. The following exercise demonstrates that the strict revealed preference (i.e., beating) relation over menus need not be transitive as well. Thus, consumer choice is highly inconsistent with utility maximization.

Exercise 11.1. *Let $X = \{x, y, z\}$, $x \succ^* y \succ^* z$. Construct a consideration relation R such that the beating relation violates transitivity.*

An alternative interpretation: Dynamically inconsistent preferences

Our welfare analysis will treat $\sqsupset$ as the consumers' true preferences over menus. However, the model admits an alternative interpretation, in the spirit of the Part I, which may suggest different welfare criteria. According to this interpretation, the rationale that consumers use to rank menus differs from the rationale they use when ranking items *within* a given menu. Thus, the binary relation R represents the preferences over items of the consumer's "first-period self," whereas $\succ^*$ represents the preference over items of his "second-period self." The consumer is naive in the sense of Chapter 2: when he chooses between menus, he erroneously believes that he will use his first-period self's preference relation R to make his second-period choice, whereas in actuality he uses his second-period self's preference relation $\succ^*$.

When economists study such two-stage, multi-self choice models with naive decision makers, they often use the first-period self's preference relation as the normative welfare criterion, because it tends to represent cool deliberation, whereas the second-period self's preference relation captures visceral urges that are inconsistent with long-run well-being. It follows that if we adopted this alternative interpretation of the model, we would be led to conduct a welfare analysis that replaces $\succ^*$ with R as a welfare criterion.

It should be emphasized that the dynamic-inconsistency reinterpretation disappears in more general versions of the model, which involve different preferences over menus and consideration relations.

Connection with the model of price-frame competition

The consumers' choice procedure in the present model is closely related to the two-stage procedure utilized in the model of price-frame competition in Section 10.2. In both cases, consumers face two market alternatives, where each alternative can be broken into a payoff-relevant component and a frame. In Section 10.2, this decomposition was explicit: each market alternative was described as a pair (x, p), where p was the price of a product and x was its frame. In the case of the present model, the decomposition is implicit: for every menu M, the payoff-relevant component is $b(M)$ and the "frame" is $M\backslash\{b(M)\}$. Note, however, that the decomposition relies on the consumer's preference relation $\succ^*$ over items, such that two consumers with different preferences will have different decompositions. In both models, the only role of the "frames" is to determine whether the consumer will apply his preference ranking over the (payoff-relevant components of) available market alternatives, or exhibit inertia and adhere to his default option. However, in the model of Chapter 10, this was determined by the profile of frames alone, whereas in the current model, it is a function of the payoff-relevant component of the default option and both components of the other market alternative.

11.2 NASH EQUILIBRIUM

We have already noted that if consumers have unlimited attention and therefore choose rationally according to □, the Nash equilibrium outcome is for both firms to offer $\{x^*\}$. However, unlimited attention is not a necessary condition for a competitive equilibrium outcome, as the following result demonstrates.

Proposition 11.1. *Firms play $\{x^*\}$ in symmetric Nash equilibrium if and only if $x^* = r^*$.*

Proof. (i) Assume that both firms play $\{x^*\}$. If $x^* \neq r^*$, and a firm deviates into $\{r^*\}$, its payoff will be $\frac{1}{2} - c_{r^*}$, because the menus $\{x^*\}$ and $\{r^*\}$ fail to beat one another. Since $c_{r^*} < c_{x^*}$, the deviation is profitable.

(ii) Assume that $x^* = r^*$. Let us first show that $\{x^*\}$ is a symmetric Nash equilibrium strategy. Suppose that one firm deviates to $M \neq \{x^*\}$. If $b(M) = x^*$, then $c(M) > c_{x^*}$ and the firm has a market share of $\frac{1}{2}$ both before and after the deviation. If $b(M) \neq x^*$, then $\{x^*\}$ beats M and therefore the deviating firm earns a market share of 0. In either case, the deviation is unprofitable. It remains to show that there are no other symmetric equilibria. Consider an equilibrium (mixed) strategy σ, and let M be a $\sqsubset$-minimal menu in $S(\sigma)$. Then, M is a best-reply to σ. Suppose that $b(M) = x^*$. Then, there exists such a menu M that contains other items. It if therefore profitable for a firm to deviate to $\{x^*\}$, because the firm's market share will remain unchanged while its costs will go down. Therefore, $b(M) \neq x^*$. If a firm deviates to $\{x^*\}$, it gains a market share of at least $\frac{1}{2}\beta_\sigma(x^*) + \frac{1}{2}[1 - \beta_\sigma(x^*)] = \frac{1}{2}$, because the deviation prevents consumers who are initially assigned to the firm from switching to menus M' with $b(M') = x^*$, while allowing the firm to attract all consumers who are initially assigned to a firm offering a menu M' with $b(M') \neq x^*$. By assumption, the gain in market share exceeds $c_{x^*} - c(M)$, and therefore the deviation is profitable. □

Thus, equilibrium outcomes coincide with the rational-consumer benchmark if and only if the highest-quality item is also the best attention grabber. The result holds for arbitrary Nash equilibria, and the restriction to symmetric equilibria is only for expositional purposes.

In the remainder of this section, we will assume that $x^* \neq r^*$. The following example illustrates the structure of symmetric Nash equilibria in this case. Suppose that $(X, \succ^*, R)$ are as in Example 11.1. Note that in this example, $x^* = x$ and $r^* = y$. There is a unique symmetric Nash equilibrium in this case, where the mixed equilibrium strategy σ is

$$\begin{aligned} \sigma\{y\} &= 2c_y \\ \sigma\{x\} &= 1 - 2c_x \\ \sigma\{x, y\} &= 2c_x - 2c_y \end{aligned}$$

It is easy to verify that all three menus generate a payoff of $\frac{1}{2} - c_x$ against σ. Since σ has full support on $P(X)$ in this example, this immediately implies that there are no profitable deviations.

This equilibrium has several noteworthy features:

- The equilibrium outcome departs from the rational-consumer benchmark, in which both firms play $\{x\}$ with probability one.
- The equilibrium strategy employs pure attention grabbers with positive probability, as the item y functions as a pure attention grabber in the menu $\{x, y\}$.

- The probability that the consumer is offered a menu that maximizes his utility is $\sigma\{x\} + \sigma\{x, y\} = 1 - 2c_y$; hence, it is entirely determined by the cost of the best attention grabber in X.
- Firms earn the rational-consumer (max-min) payoff $\frac{1}{2} - c_x$ in equilibrium.

Our objective is to investigate the generality and economic significance of these observations.

Proposition 11.2. *Let σ be a symmetric Nash equilibrium strategy. Then:*

(i) $\beta_\sigma(x^*) \in (0, 1)$;
(ii) there exists $M \in S(\sigma)$ such that $|M| > 1$.

Proof. (i) Let σ be a symmetric Nash equilibrium strategy. Suppose $\beta_\sigma(x^*) = 0$. Consider a $\square$-minimal menu M in $S(\sigma)$. The menu M is a best-reply to σ. It beats no menu in $S(\sigma)$ and therefore generates a market share of at most $\frac{1}{2}$. If a firm deviates from M into $\{x^*, r^*\}$, it beats every menu in $S(\sigma)$, and therefore raises its market share by at least $\frac{1}{2}$, while increasing its cost by $c_{x^*} + c_{r^*} - c(M)$, which is lower than $\frac{1}{2}$ by assumption. Now suppose that $\beta_\sigma(x^*) = 1$. Then, both firms must offer $\{x^*\}$ with probability one—otherwise, deviating to $\{x^*\}$ preserves the firm's market share while lowering costs. However, as we saw in Proposition 11.1, this is inconsistent with equilibrium when $x^* \neq r^*$.

(ii) Assume the contrary—that is, all menus in $S(\sigma)$ are singletons. By (i), $\beta_\sigma(x^*) > 0$, hence $\beta_\sigma(x^*) = \sigma\{x^*\} > 0$. Let $\mathcal{M}_1$ denote the set of menus in $S(\sigma)$ that $\{x^*\}$ beats, and let $\mathcal{M}_0$ denote the set of menus $M \in S(\sigma)$ with $b(M) \neq x^*$, which are not beaten by $\{x^*\}$. If $\mathcal{M}_1$ is empty, then $\{x^*\}$ generates a payoff of $\frac{1}{2} - c_{x^*}$. If a firm deviates into $\{r^*\}$, it will not be beaten by any menu, and therefore earn $\frac{1}{2} - c_{r^*} > \frac{1}{2} - c_{x^*}$. It follows that $\mathcal{M}_1$ is non-empty. Let M_* be some $\square$-minimal menu in $\mathcal{M}_1$. By definition, this menu does not beat any menu in $\mathcal{M}_1$. Since $S(\sigma)$ consists of singletons only, the beating relation is transitive when restricted to $S(\sigma)$. Therefore, M_* does not beat any menu in $S(\sigma)$. The menu M_* is a best-reply to σ. Suppose that a firm deviates from M_* into $\{x^*\}$. This deviation is unprofitable only if the following inequality holds:

$$\frac{1}{2}\sigma\{x^*\} + \frac{1}{2}\sum_{M \in \mathcal{M}_1} \sigma(M) - (c_{x^*} - c(M_*)) \leq 0 \tag{11.1}$$

The menu $\{x^*\}$ is in $S(\sigma)$, hence it is a best-reply to σ. Suppose that a firm deviates from $\{x^*\}$ into $\{x^*, r^*\}$. This deviation is unprofitable only if the following inequality holds:

$$\frac{1}{2}\sum_{M \in \mathcal{M}_0} \sigma(M) - c_{r^*} \leq 0 \tag{11.2}$$

Note that $S(\sigma) = \{\{x^*\}\} \cup \mathcal{M}_0 \cup \mathcal{M}_1$. Therefore, adding up the two inequalities yields the inequality $\frac{1}{2} \leq c_{x^*} + c_{r^*} - c(M_*)$, a contradiction. □

Thus, when $x^* \neq r^*$, the outcome of symmetric Nash equilibrium departs from the rational-consumer benchmark (in the sense that menus that the consumers find sub-optimal are offered with positive probability), and pure attention grabbers are offered with positive probability. Since a pure attention grabber is costly to offer and makes no difference for consumer welfare, the equilibrium use of pure attention grabbers is socially wasteful. The rationale for their use is that they exert a positive externality on other items on the firm's menu; they overcome consumers' inertia by drawing their attention to items of higher quality than what they consume from their default menu, thus increasing the firm's market share.

Note that if σ is a symmetric Nash equilibrium strategy, then $|M| \leq 2$ for every $M \in S(\sigma)$. That is, firms never include more than one pure attention grabber in a menu. More specifically, for all $M \in S(\sigma)$, $M = \{b(M), r(M)\}$. The reason is simple. If $M \in S(\sigma)$ contains an item x such that $x \neq b(M)$ and $x \neq r(M)$, then removing this item from the menu lowers costs without changing the firm's market share. Therefore, such a deviation is profitable.

The following lemma will be useful for the characterization of symmetric Nash equilibria. For every menu M, let $B_\sigma(M)$ denote the set of menus $M' \in S(\sigma)$ that M beats.

Lemma 11.1. *Let σ be a symmetric Nash equilibrium strategy. Then, $\{x^*\} \in S(\sigma)$. Furthermore, $\{x^*\}$ does not beat any menu in $S(\sigma)$.*

***Proof*.** Define $\mathcal{M}_\sigma = \{M \in S(\sigma) \mid b(M) = x^*\}$. By Proposition 11.2, $\sum_{M \in \mathcal{M}_\sigma} \sigma(M) = \beta_\sigma(x^*) \in (0, 1)$. Suppose that $B_\sigma(M)$ is empty for some menu $M \neq \{x^*\}$ in $\mathcal{M}_\sigma$. Therefore, M generates a market share of $\frac{1}{2}$ and costs more than c_{x^*}, yielding a payoff strictly below the max-min level $\frac{1}{2} - c_{x^*}$, a contradiction.

It follows that in order to prove the result, we only need to rule out the possibility that $B_\sigma(M)$ is non-empty for all $M \in \mathcal{M}_\sigma$. List the menus in $\mathcal{M}_\sigma$ as $M_1, \ldots, M_K$, $K \geq 1$, such that

$$r(M_K)Rr(M_{K-1})R \cdots Rr(M_1)$$

Let $\tilde{M}_1$ be some $\sqsupset$-minimal menu in $B_\sigma(M_1)$. Therefore, $r(M_1)Rb(\tilde{M}_1)$. By transitivity of R, it follows that for every $k = 2, \ldots, K$, $r(M_k)Rb(\tilde{M}_1)$; that is, $\tilde{M}_1$ is beaten by every menu in $\mathcal{M}_\sigma$. Our next step is to show that $B_\sigma(\tilde{M}_1)$ is empty. Assume the contrary—that $\tilde{M}_1$ beats some $M \in S(\sigma)$. That is, $r(\tilde{M}_1)Rb(M)$ and $b(\tilde{M}_1) \succ^* b(M)$. Let us distinguish between two cases.

Case 1: $r(\tilde{M}_1) = b(\tilde{M}_1)$.

This means that $\tilde{M}_1 = \{b(\tilde{M}_1)\}$. Recall that $r(M_1)Rb(\tilde{M}_1)$. By the transitivity of R, $r(M_1)Rb(M)$. It follows that M_1 beats M, contradicting the definition of $\tilde{M}_1$ as a $\sqsupset$-minimal menu in $B_\sigma(M_1)$.

Case 2: $r(\tilde{M}_1) \neq b(\tilde{M}_1)$.

Recall that $\tilde{M}_1 = \{b(\tilde{M}_1), r(\tilde{M}_1)\}$. It must be the case that $r(M_1)\neg Rr(\tilde{M}_1)$—otherwise, by transitivity, $r(M_1)Rr(M)$, and since $b(M_1) \succ b(M)$, M_1 beats M, contradicting the definition of $\tilde{M}_1$. Let B^* denote the set of menus in $S(\sigma)$ that are beaten by $\tilde{M}_1$ and are not beaten by $\{b(\tilde{M}_1)\}$. Note that by the definition of $\tilde{M}_1$, M_1 does not beat any menu in B^*. Since $\tilde{M}_1$ is in the support of $S(\sigma)$, it is a best-reply to σ. From the firms' decision not to deviate to $\{b(\tilde{M}_1)\}$, we conclude that

$$\frac{1}{2}\sum_{M\in B^*}\sigma(M) - c_{r(\tilde{M}_1)} \geq 0$$

Similarly, since M_1 is in the support of $S(\sigma)$, it is a best-reply to σ. From the firms' decision not to deviate to $\{b(M_1), r(\tilde{M}_1)\}$, we conclude that

$$\frac{1}{2}\sum_{M\in B^*}\sigma(M) - c_{r(\tilde{M}_1)} + c_{r(M_1)} \leq 0$$

The two inequalities contradict each other.

We have thus established that $\tilde{M}_1$ is a menu that beats no menu in $S(\sigma)$ and is beaten by every menu in M_σ. If a firm deviates from $\tilde{M}_1$ to $\{x^*, r^*\}$, it stops being beaten by menus in M_σ, and it beats all menus outside M_σ. As a result, the firm increases its market share by at least $\frac{1}{2}\beta_\sigma(x^*) + \frac{1}{2}(1 - \beta_\sigma(x^*)) = \frac{1}{2}$, which is by assumption strictly higher than the change in menu costs. Therefore, the deviation is profitable, a contradiction. □

Thus, the max-min strategy $\{x^*\}$ is necessarily played with positive probability in symmetric equilibrium. The fact that it beats no menu in equilibrium has strong implications for the structure of equilibrium.

We are now ready for a characterization of symmetric Nash equilibria in the model.

Proposition 11.3. *Let σ be a symmetric Nash equilibrium strategy. Then:*

(i) Firms earn the max-min payoff $\frac{1}{2} - c_{x^*}$.
(ii) For every $M \in S(\sigma)$, $|M| = 2$ only if $b(M) = x^*$.
(iii) $\beta_\sigma(x^*) = 1 - 2c_{r^*}$.

***Proof*.** (i) This follows immediately from Lemma 11.1. Since $\{x^*\}$ belongs to $S(\sigma)$ and beats no menu in $S(\sigma)$, it generates a market share of $\frac{1}{2}$ and therefore yields a payoff of $\frac{1}{2} - c_{x^*}$.

(ii) Assume that there exists a menu $M \in S(\sigma)$ such that $b(M) \neq x^*$ and $b(M) \neq r(M)$. From the firms' decision to include $r(M)$ in the menu, we conclude that

$$\frac{1}{2}[\sum_{M'\in B_\sigma(M)}\sigma(M') - \sum_{M''\in B_\sigma(\{b(M)\})}\sigma(M'')] - c_{r(M)} \geq 0$$

Now consider the menu $\{x^*, r(M)\}$. This menu beats every menu $M' \in S(\sigma)$ that is beaten by M. In addition, by construction, the menu $\{x^*, r(M)\}$ beats M.

By Lemma 11.1, the menu $\{x^*\}$ is a best-reply to σ and it beats no menu in $S(\sigma)$. Therefore, if a firm deviates to $\{x^*, r(M)\}$, it gains at least

$$\frac{1}{2}[\sigma(M) + \sum_{M' \in B_\sigma(M)} \sigma(M')] - c_{r(M)} > 0$$

The deviation is therefore profitable, a contradiction.

(iii) By Lemma 11.1, the menu $\{x^*\}$ is a best-reply to σ and it beats no menu in $S(\sigma)$. Therefore, in order for a deviation from $\{x^*\}$ into $\{x^*, r^*\}$ to be unprofitable, it must be that $c_{r^*} \geq \frac{1}{2}[1 - \beta_\sigma(x^*)]$, hence $\beta_\sigma(x^*) \geq 1 - 2c_{r^*}$. Assume that the inequality is strict. By part (ii), there exists $M \in S(\sigma)$ such that $b(M) = x^*$ and $r(M) \neq x^*$. The set $B_\sigma(M)$ must be non-empty—otherwise, M generates a payoff strictly below the max-min level $\frac{1}{2} - c_{x^*}$, a contradiction. By part (i), M generates a payoff of $\frac{1}{2} - c_{x^*}$ against σ. Therefore:

$$\frac{1}{2} - c_{x^*} = \frac{1}{2} - c(M) + \frac{1}{2} \sum_{M' \in B_\sigma(M)} \sigma(M')$$

By definition,

$$\sum_{M' \in B_\sigma(M)} \sigma(M') \leq 1 - \beta_\sigma(x^*)$$

Therefore,

$$c(M) - c_{x^*} \leq \frac{1}{2}[1 - \beta_\sigma(x^*)] < c_{r^*}$$

It follows that $c(M) < c_{x^*} + c_{r^*}$, hence $r^* \notin M$. By part (ii), if $r^* \in M'$ for some $M' \in S(\sigma)$, then $M' = \{r^*\}$. If a firm deviates to $\{r^*\}$, it is not beaten by any menu in $S(\sigma)$, and therefore generates a payoff of at least $\frac{1}{2} - c_{r^*} > \frac{1}{2} - c_{x^*}$, a contradiction. Therefore, $\beta_\sigma(x^*) = 1 - 2c_{r^*}$. □

Thus, symmetric Nash equilibria in this model have several strong properties. First, although the equilibrium outcome departs from the rational-consumer benchmark, firms' profits are equal to the max-min level, which, as we saw, coincides with the rational-consumer benchmark. Second, the use of pure attention grabbers is restricted to menus that also feature the highest-quality item x^*. Although pure attention grabbers are socially wasteful, they help to draw consumers' attention to the highest-quality item; in equilibrium, they are never used to promote inferior-quality items. Finally, the probability that utility-maximizing menus are offered is entirely determined by the cost of the best attention grabber. As creating attention-grabbing sensations becomes more costly, consumers are less likely to be offered their most desirable item x^*.

The following exercise shows that the menu consisting of the highest-quality item and the best attention grabber must be offered with positive probability in symmetric equilibrium.

Exercise 11.2. *Let σ be a symmetric Nash equilibrium strategy. Show that* $\sigma\{x^*, r^*\} > 0$.

Comment: Attracting attention away from pure attention grabbers

The consumers' choice procedure assumes that in order for a new menu M to attract the consumers' attention away from their default menu M', M must include an item x such that $xRb(M')$. Note that x is not required to have a higher sensation value than *all* items in M'. The interpretation is that $b(M')$ is the only alternative in M' that is consumed on a regular basis, and therefore attention needs to be drawn away from $b(M')$. An alternative specification of the attention-grabbing process would be the following: the consumer pays attention to M if and only if M includes an item x such that xRy for all $y \in M'$.

All the results of this section continue to hold under this alternative specification. In fact, it is simpler to analyze, because it implies that the beating relation—namely, consumers' revealed strict preference relation over menus—is transitive. (However, the revealed indifference relation generally violates transitivity.) I leave this alternative model to the interested reader.

The following exercise addresses the implication of introducing rational consumers into the market.

Exercise 11.3. *Suppose that X and $\succ^*$ are as in Example 11.1. However, there is heterogeneity among consumers in the consideration relation. For a fraction λ of the population, R is as given by Example 11.1. The remaining consumers are rational—that is, $x'Rx''$ for all $x', x'' \in X$. Characterize symmetric Nash equilibrium. In particular, how does the probability that the boundedly rational consumers end up consuming their favorite item x depend on λ?*

11.3 THE EFFECTIVE MARKETING PROPERTY

In this section I relax the assumption that R is a complete, transitive, and antisymmetric relation, and allow R to be an arbitrary binary relation over X, which satisfies the following property: for every $x \in X$ there exists $y \in X$ such that yRx. This assumption implies that if a firm offers the grand set X, it ensures that it will attract consumers' attention. Assume further that $c(X) < \frac{1}{2}$. These assumptions guarantee that the max-min strategy remains $\{x^*\}$ and the max-min payoff remains $\frac{1}{2} - c_{x^*}$.

When R is not complete and transitive, the interpretation of yRx cannot be that y has a higher sensation value than x. Instead, the interpretation could be that x reminds the consumer of y, or that y resembles x. Introducing items that are similar to things the consumer is familiar with is often an effective way of attracting attention. For instance, think of a TV viewer on a channel-flipping cruise who stumbles on a show he recognizes from his default channel; he is likely to pause and pay more attention to the channel that is currently broadcasting this show.

An interesting feature of the equilibrium characterized in Proposition 11.3 was that pure attention grabbers were offered only in conjunction with the consumers' favorite item x^*. The following exercise shows that this property does not hold for general consideration relations.

Exercise 11.4. *Let $X = \{x, y, z\}$. Assume that $x \succ^* y \succ^* z$. Assume that R is the identity relation: $x'Rx''$ if and only if $x' = x''$. Construct an example of a symmetric Nash equilibrium in which firms offer with positive probability a menu M such that $b(M) \neq x$ and $|M| > 1$.*

Even though pure attention grabbers can accompany items of inferior quality in equilibrium, it turns out that some form of correlation between menu quality and the use of pure attention grabbers does hold in symmetric equilibrium. Suppose that firms earn the rational-consumer payoff in equilibrium. Then, whenever a consumer considers a firm thanks to a pure attention grabber on its menu, he switches to the firm with probability one (unless his default menu includes x^*). A priori, the fact that a pure attention grabber causes a consumer to consider a menu does not guarantee that he will *choose* that menu over his default. However, if the equilibrium is competitive in the sense that firms earn rational-consumer payoffs, there is such a guarantee.

Proposition 11.4 (Effective Marketing Property). *Suppose that a symmetric Nash equilibrium strategy σ induces the max-min payoff $\frac{1}{2} - c_{x^*}$. Let M and M' be two menus in $S(\sigma)$ that satisfy the following properties: (1) $b(M') \neq x^*$; (2) $xRb(M')$ for some $x \in M$; (3) $b(M) \neg R b(M')$. Then, M beats M'.*

Proof. Assume the contrary—that is, there exist menus $M, M' \in S(\sigma)$ that satisfy properties 1–3 above, and yet $M \not\succ M'$. Let B denote the set of menus in $S(\sigma)$ that are beaten by M and not by $\{b(M)\}$. Note that $M' \notin B$. Since $M, M' \in S(\sigma)$, both M and M' are best-replies to σ. From the firms' decision not to deviate from M into $\{b(M)\}$, we conclude that

$$\frac{1}{2} \sum_{\tilde{M} \in B} \sigma(\tilde{M}) - c(M \backslash \{b(M)\}) \geq 0$$

The reason is that when a firm adds a pure attention grabber to a menu it offers, this can change the set of menus that the firm beats, but not the set of menus the firm is beaten by.

By assumption, firms earn $\frac{1}{2} - c_{x^*}$ in equilibrium. Therefore, the menu $\{x^*\}$ is a best-reply to σ and it does not beat any menu in $S(\sigma)$. Now suppose that a firm deviates to the menu $\{x^*\} \cup (M \backslash \{b(M)\})$. In order for the deviation to be unprofitable, the following inequality must hold:

$$\frac{1}{2} \sum_{\tilde{M} \in B} \sigma(\tilde{M}) + \frac{1}{2} \sigma(M') - c(M \backslash \{b(M)\}) \leq 0$$

The reason is that adding $M \setminus \{b(M)\}$ to $\{x^*\}$ allows a firm to beat not only all the menus in B, but also the menu M'. The two inequalities we derived contradict each other. □

Thus, if firms earn max-min payoffs in equilibrium, there is a strong connection between menu quality and the use of pure attention grabbers as a device for overcoming consumer inertia. Conditional on considering a firm thanks to a pure attention grabber on its menu, the consumer switches to this firm with probability one (unless his default menu offers the highest-quality item to begin with).

The antecedent to Proposition 11.4 is that firms earn the max-min payoff in equilibrium. As we saw in the previous section, this antecedent holds for an interesting class of consideration relations under a relatively weak assumption on menu costs. The next exercise shows that firms earn max-min payoffs in equilibrium for any consideration relation that satisfies the condition imposed in this section, provided that menu costs are sufficiently small.

Exercise 11.5. *Show that there exists c^* such that if $c_{x^*} < c^*$, firms earn the max-min payoff in any symmetric Nash equilibrium.*

Conversion and switching rates

The effective marketing property means that the consumers' conversion rate (namely, the probability that they switch away from the default conditional on making an active comparison between all market alternatives) is essentially one. The (unconditional) equilibrium switching rate is more sensitive to the model's parameters. For instance, recall the symmetric equilibrium derived for the specification of R given in Example 11.1. In this equilibrium, the fraction of consumers who switch suppliers is

$$\sigma\{y\} \cdot \sigma\{x, y\} = 4c_y(c_x - c_y)$$

Thus, the switching rate behaves non-monotonically with c_y. In contrast, the switching rate unambiguously increases with c_x. Recall that the equilibrium payoff is at the max-min level $\frac{1}{2} - c_x$. Thus, as the cost of the highest-quality item increases, industry profits go down while the switching rate goes up.

11.4 DISCUSSION

I conclude with a brief discussion of two aspects of real-life media markets that are absent from the highly stylized model developed in this chapter.

Preference for diversity

The assumption that consumers have max-max preferences over menus is unrealistic. Typically, consumers do not consume a single, "most favored" item from a menu of content items. A more realistic assumption is that consumers seek diversity, and therefore prefer larger menus (whereas max-max preferences imply that for every menu M there is $x \in M$ such that the consumer is indifferent between M and $\{x\}$).

The following is an extension of the model, which captures a taste for diversity. Assume that the consumers' preference relation $\square$ over $P(X)$ is monotonic: $M \subset M'$ implies $M' \square M$. Assume further that for every menu M, there is a unique subset $L(M) \subseteq M$ such that for every $M' \subseteq M$ for which $M' \sim M$, $L(M) \subseteq M'$. The interpretation is that for every menu M, we can uniquely identify the set of items that the consumer regularly consumes from M, and this set is $L(M)$. Every item in $M \backslash L(M)$ is therefore a pure attention grabber. The consumers' choice procedure is modified accordingly. Each consumer is initially assigned to a firm i offering a menu M_i. The consumer switches to firm j if and only if two conditions hold: (i) there exists $x \in M_j$ such that xRy for all $y \in L(M_i)$; and (ii) $M_j \succ M_i$. This extension preserves the main results of this chapter, under suitable modification; see Eliaz & Spiegler (2010*b*).

Diversity of preferences

Some of the main insights of this chapter persist if we allow for heterogeneous preferences. For instance, assume that there are $m < n$ consumer types, uniformly distributed in the population of consumers. Each consumer type i has some strict preference relation over X, with a distinct top-ranked item x_i^*. Denote $A = \{x_i^*\}_{i=1,\dots,m}$. The $n - m$ items in $X \backslash A$ are not top-ranked by any consumer type. Assume that all items have the same fixed cost $c < \frac{1}{2m}$.

In this model, if consumers have unlimited attention, both firms offer A in Nash equilibrium. In contrast, assume that consumers follow our model with a homogeneous consideration relation R, which is reflexive, transitive, and antisymmetric. Assume that $r^* \notin A$. That is, the best attention grabber is not top-ranked by any consumer type. It can be shown that the following is a symmetric Nash equilibrium strategy:

$$\sigma(\{r^*\}) = 2c$$

$$\sigma(A \cup \{r^*\}) = 2(m-1)c$$

$$\sigma(A) = 1 - 2mc$$

Firms earn the same profits as in the benchmark with rational consumers having unlimited attention. Consumers switch only from $\{r^*\}$ to $A \cup \{r^*\}$. This equilibrium looks exactly as if firms faced a single "representative consumer," whose utility is maximized by menus that (weakly) contain A. In particular, the pure attention grabber r^* is employed only to promote the menu A that maximizes the representative consumer's utility. Not only firms' behavior, but also observed consumer behavior and industry profits mimic the representative-consumer scenario.

11.5 SUMMARY

In this chapter we left the world of price competition and examined a different sort of market interaction, in which firms trade off market share and fixed marketing costs. In the context of a stylized model of competition between two media platforms, we derived the following lessons:

- Firms incur socially wasteful fixed costs in items that are irrelevant for consumer welfare, in order to overcome consumers' inertia and draw their attention away from default options of inferior quality.
- In a number of settings, firms' equilibrium profits are equal to their max-min payoff, which in this model coincides with the rational-consumer benchmark. Thus, while consumers are worse off compared with the case of no inertia, firms neither lose nor gain from the bias in equilibrium.
- Max-min equilibrium payoffs have an implication for the correlation between payoff-relevant and payoff-irrelevant features of the firms' strategies. Conditional on the event in which a consumer considers a firm thanks to a pure attention grabber, he switches to that firm essentially with probability one. Thus, once the firm passes the hurdle of persuading the consumer to overcome his inertia and consider new alternatives, persuading him to switch follows automatically. When items can be ordered according to how well they overcome inertia, this correlation takes a starker form: only the highest-quality menus are accompanied by irrelevant alternatives.

11.6 BIBLIOGRAPHIC NOTES

This chapter is based on Eliaz & Spiegler (2010*b*). For a related model of competitive costly marketing in the presence of consumer inertia, see Eliaz & Spiegler (2010*a*). For empirical and experimental evidence for the relevance of inertia for marketing, particularly in the case of media consumption, see Nedungadi (1990), Markman & Zhang (1998), Bucklin & Sismeiro (2003), Zauberman (2003), and Meyer & Muthaly (2008).

PART FOUR
Discussion

12 Recurring Themes

In the final two chapters, I adopt a global view of the market models examined in this book. In the next chapter, I will address one nagging methodological issue that has accompanied us throughout this journey, namely the need to compare market models based on bounded consumer rationality with more conventional models that do not depart from the rationality paradigm. In this chapter, I will try to distill a few general themes that run through many of the models, taking a broad view of these themes, rather than a view that is entirely based on a particular model. While these cannot be regarded as "bottom lines" or generally valid lessons, they do point out that certain distinctions and questions are persistent in the literature on bounded rationality and industrial organization. As usual in economic theory, it is hard to get robust theoretical answers, and the value of the theory is to a large extent that it helps formulating the "right" questions.

12.1 COMPLEX PRICING STRATEGIES

One virtue of market models based on consumers' bounded rationality is that they naturally give rise to complex pricing strategies in many monopolistic or competitive settings. Let us review our findings.

- Two-part tariffs cease to be optimal when consumers have biased beliefs of their future preferences, and firms have an incentive to introduce prices that change with the consumption quantity, for example, three-part tariffs (Chapters 3, 5).
- Subtle contract renewal policies such as negative options are an optimal response to consumer naivete regarding the future costs of switching or cancelling (Chapter 2).
- Highly competitive pricing of basic products coupled with monopolistic pricing of add-ons emerges naturally when some consumers are unaware of contingencies that create a demand for the add-ons (Chapter 5).
- Prices that change erratically over time or across product attributes are an optimal/equilibrium response of firms to consumers' limited ability to evaluate such objects, possibly as an implicit device for discriminating between consumers who differ in this regard (Chapters 7, 8).
- Strategic use of incommensurable price formats is an equilibrium response to consumers' limited propensity to compare new alternatives to their default option (Chapter 10).

Importantly, these complex pricing strategies emerge in market models with minimal inherent complexity. If one insisted on full consumer rationality, one would have to complicate the model of the environment substantially in order to reproduce them, if at all. In this sense, consumers' bounded rationality can be viewed as a natural source of price complexity. The welfare implications of complex prices in the above-mentioned models are enhanced exploitation of boundedly rational consumers, and often ex post inefficiency.

Our analysis of optimal pricing for sophisticated consumers with dynamically inconsistent preferences (Chapters 2, 3) and for consumers with preferences that exhibit loss aversion (Chapter 9) provided exceptions to this theme. In the former case, firms offer commitment devices that minimize choice, and two-part tariffs can function as effective commitment devices. In the latter case, pricing strategies tend to be unresponsive to shocks, and in this sense, simpler. Significantly, these are examples in which consumers' departure from the conventional model lies in their context-sensitive preferences, rather than in any bias or cognitive limitation.

12.2 SPURIOUS VARIETY

In several models analyzed in this book, the market response to consumers' bounded rationality involved a proliferation of alternatives. This enhanced variety of products is spurious, in the sense that it is not genuinely desired by consumers and would disappear if consumers were fully rational. Spurious variety has taken two forms in this book: horizontal product differentiation and the use of irrelevant alternatives.

Horizontal product differentiation

Even when the product traded in a market is inherently homogeneous, firms facing boundedly rational consumers may have an incentive to create apparent differentiation, in order to ease competitive pressures. If consumers were rational, they would be able to see through the spurious differentiation tactics and continue to regard the product as homogeneous. Boundedly rational consumers can be misled by the spurious differentiation. Let us review our findings.

- When consumers use naive sampling or anecdotal evidence to evaluate alternatives, firms in industries that involve soft expertise (alternative medicine, consulting, money management) have an incentive to differentiate themselves by essentially betting against each other (Chapter 6).
- The proliferation of frames in Chapter 10 can be viewed as a form of differentiation, involving multiple ways to present the same underlying product.
- Multiplicity of price plans in Chapters 2 and 4 is a way of discriminating between consumers according to their degree of naivete. If all consumers had correct beliefs of their future preferences, it would disappear.

Irrelevant alternatives

An alternative that belongs to a consumer's feasible set is irrelevant if it is dominated, or appears to be dominated by another feasible alternative. In conventional market models based on consumer rationality, there is no role for irrelevant alternatives. (If, however, having a large product line signals a higher quality to uninformed consumers, firms may want to inflate their product line, as in Kamenica (2008).) When consumers are boundedly rational, firms may introduce irrelevant alternatives as a means of manipulating consumer choice. Let us review our findings.

- Firms may wish to introduce irrelevant alternatives into a price plan in order to help discrimination between consumers with different patterns of dynamically inconsistent preferences. The irrelevant alternative tempts different consumers to a different degree, and therefore deters only some of them from choosing the price plan. This relaxes incentive compatibility constraints and enhances the power of price discrimination (Chapter 2).[1]
- When firms in a competitive environment wish to distinguish between naive and sophisticated consumers, they may offer price schemes that seem to dominate one another. In this case, the irrelevance of the dominated price scheme is apparent because sophisticates actually choose it (Chapter 2).
- Firms may want to offer alternatives because of their ability to attract attention, even if they are irrelevant in the sense that they do not change consumers' overall evaluation of the menu.

As in the case of price complexity, spurious variety has adverse welfare implications for consumers, because it is associated with weak competitive pressures, exploitative contracts or fixed costs that end up being borne by consumers—without the added value that variety brings in conventional models by catering to genuine preference niches.

12.3 MARKET TRANSACTIONS AS A FORM OF SPECULATIVE TRADE

In some of the market models examined in this book, market transactions can be viewed as forms of speculative trade between consumers and firms.

- When consumers use anecdotal reasoning to evaluate market alternatives, they may enter "markets for quacks" and reward firms for outcomes that are due to sheer luck. Industries such as alternative medicine and actively managed mutual funds arguably thrive thanks to consumers' inference errors. In these extreme cases, market transactions are nothing but a transfer from consumers to firms, hence they are a form of speculative trade (Chapter 6).
- Consumers and firms may have different prior beliefs regarding the consumer's future preferences, and this directly creates a mutual desire to engage in speculative trade. The complex pricing strategies discussed

[1] At the end of Chapter 11 we discussed another channel through which irrelevant alternatives enhance price discrimination, based on Ok, Ortoleva & Riella (2009).

above are implicit bets on the consumer's future consumption decision (Chapters 2–5).

As in the case of product differentiation, the bounded-rationality perspective has important welfare implications, as it calls into question the added value from certain market transactions (extending at times to entire market segments).

12.4 HOW EFFECTIVE ARE COMPETITION AND CONSUMER PROTECTION POLICIES?

Given that consumers' bounded rationality often exposes them to exploitation by firms, we often wondered whether competition and consumer protection policies could remedy these exploitative effects. The answer was negative in many cases.

- When consumers have dynamically inconsistent preferences and differ in their degree of naivete, introducing competition into the market may increase the ubiquity of exploitative, speculative contracts, even as it lowers the magnitude of exploitation (Chapter 4).
- When consumers use anecdotal reasoning to evaluate market alternatives of low intrinsic value, increasing the number of firms can enhance consumers' total welfare loss from market transactions, because it spuriously increases aggregate demand. When firms can obfuscate by introducing noise into their pricing strategies, increasing the number of firms magnifies obfuscation without changing expected prices, and it may even result in productive inefficiencies in the form of excessive quality (Chapters 6, 7).
- Certain regulatory interventions that are meant to simplify the consumers' decision problem may have anti-competitive consequences, once the firms' strategic response to the intervention is taken into account. Thus, introducing a simple, "plain vanilla" option can merely enhance the firms' obfuscatory tactics (Chapter 7). Similarly, interventions that are meant to improve the comparability of different price formats may strengthen firms' incentive to adopt complex formats in a way that weakens competitive pressures and causes prices to go up (Chapter 10). This can be viewed as a "Lucas critique" of certain consumer protection policies.

Another market intervention implicit in many discussions was the regulation of disclosure. The mirror image of firms' incentive to confuse boundedly rational consumers is their lack of incentive to correct consumers' decision errors. This disincentive to "educate" consumers persists in competitive environments. This implies a scope for mandatory disclosure of aspects of prices and product characteristics, which essentially means forcing firms to "educate" consumers. Putting aside the important question of how to implement disclosure policies and how they affect rational consumers, this type of intervention—unlike the other interventions mentioned above—does seem to directly address some of the market failures resulting from consumers' bounded rationality.

12.5 EXTERNALITIES BETWEEN RATIONAL AND BOUNDEDLY RATIONAL CONSUMERS

A priori, one might expect that injection of rational, unbiased consumers into the market would exert a positive externality on boundedly rational consumers. However, as we saw in this book, the externality can go either way, depending on the details of the market model.

- In market environments that allow for price discrimination, firms may be able to perfectly screen the rational types, who therefore exert no informational externality on boundedly rational ones (Chapters 2, 4). Discrimination does not have to be explicit. For instance, sufficiently complex pricing strategies can be correctly perceived by rational consumers alone, like a cipher that can be cracked only by receivers in possession of the code (Chapter 8).
- In some cases, injection of rational consumers has adverse welfare implications for their boundedly rational fellows. For instance, consumers who anticipate their future demand for add-ons are cross-subsidized by those who do not (Chapter 5). Likewise, introducing consumers with unlimited ability to consider all market alternatives weakens the firms' incentive to tailor their competitive strategy to consumers who are bounded in this respect, and this may result in a worse equilibrium outcome for the latter (Chapter 11).

It appears that the basic intuition regarding the positive externalities of consumer rationality holds in models where firms are unable to distinguish between rational and boundedly rational consumers, and where the set of market alternatives that consumers consider can be viewed as fixed. In such models, I expect that a larger fraction of rational consumers will generally increase the firms' incentive to offer more attractive market alternatives, thus benefiting boundedly rational consumers. Formulating abstract properties that characterize the sign of rational consumers' externality on their boundedly rational fellows is an interesting theoretical problem.

12.6 CONCLUSION

Although the running themes highlighted in this chapter do not amount to "general lessons," let alone watertight "policy implications," they do suggest that some of the ideas that emerge from our models of markets with boundedly rational consumers are stubborn. As economic theorists continue to formulate models of consumer choice that abstract from specific psychological effects and uncover their more general behavioral properties, it will be possible to appreciate better how robust some of the recurring themes are.

13 But Can't We Get the Same Thing with a Standard Model?

The previous chapter was devoted to economic themes that run through this book. There is, however, a methodological concern that has been with us from the outset, namely the need to compare market models with boundedly rational consumers to more conventional models based on full consumer rationality. When the theorist does not feel this need internally, he can typically count on others—referees, seminar audiences, coffee-machine conversation partners—to fill the gap. I have decided to devote the final chapter in this book to a systematic discussion of this subject.

Let us adopt for a moment the critic's position:

> Although bounded-rationality models may shed some light on economic phenomena, in many cases one can think of a rational-choice model that could account for them just as well. And if we can "get the same thing" with a standard model, why should we depart from the rational-choice paradigm? Moreover, since rational-choice and bounded-rationality models tend to have dramatically different welfare implications, a switch from rational-choice to bounded-rationality models is not only methodologically problematic, but also carries a significant cost in terms of its implied policy prescriptions.

The methodological issue behind this criticism is fundamentally *theory selection*: How should we choose from a number of competing models that provide different accounts for a given phenomenon? The need to choose is only magnified by the models' diverging welfare implications. The normatively scientific way of making this choice is to tease out cases in which the models generate different predictions, and subject these predictions to an empirical test. However, economics being the dismal science that it is, such empirical tests are difficult and rare. Indeed, the whole point of the "can't we get the same thing with a standard model" critique is that since the two types of explanations are empirically hard to distinguish, the conventional rational-choice explanation should be given priority. (The reasons for the preference for "rational explanations" will be discussed in the final section.)

Therefore, for the purpose of our discussion here, I will set aside empirically driven criticisms of models of the kind presented in this book. I will take it for granted that the accounts these models provide for certain market phenomena (such as the departures from marginal-cost pricing derived in Chapter 3, or the effect of competition policies on equilibrium price distributions in Chapter 7) are sound: the "story" they tell "rings true"; their behavioral assumptions seem to fit generally known psychological principles as well as the market situation in question; and their predictions are broadly consistent with known (stylized) facts.

What the "can't we get the same thing with a standard model" critique maintains is that for the purpose of making these predictions, one does not have to abandon conventional behavioral assumptions, and therefore one ought not to; even if there is some truth in the bounded-rationality story, the same truth could be captured equally well by a rational-choice model. I refer to such a rational-choice that is offered in refutation of a bounded-rationality model as a *rationalization*, or as a *rationalizing model*.

My discussion so far may have given the impression that every bounded-rationality model faces a single, well-defined rationalization. This is obviously not the case. The rational-choice paradigm is famously flexible, and there is a variety of conventional models that can be offered in refutation of any given bounded-rationality model. Rationalizing models tend to come in one of the following three forms:

Rationalization via modified information

The rationalizing model modifies the bounded-rationality model by replacing the boundedly rational agents with conventionally rational agents who happen to have different information.

Rationalization via modified preferences

The rationalizing model modifies the bounded-rationality model by replacing the boundedly rational agents with conventionally rational agents who happen to have different preferences.

Rationalization via endogenization

The rationalizing model refuses to take the behavioral rule assumed by the bounded-rationality model as truly exogenous, and instead *derives* it as a rational equilibrium response in a larger model that introduces frictions not explicitly included in the original model.

I could illustrate these forms with any number of models from this book. For expositional effectiveness, I will take a case-study approach and use the model of Section 6.2 (10.2) to demonstrate the first two forms (third form) of rationalization. We will see that the "can't we get the same thing with a standard model" criticism is not as simple as it may seem at first glance, and it is not at all clear what it means for a rationalizing model to count as a successful refutation of a bounded-rationality model.

13.1 RATIONALIZATION VIA MODIFIED INFORMATION

Replacing imperfect rationality with imperfect information is perhaps the most immediate and common of traditional responses to models of bounded rationality. The idea is to replace what seems like a decision error resulting from bounded rationality with a rational response to limited information. For instance, choosing a low-quality product over an identically priced, high-quality product can be interpreted as evidence of imperfect information about product characteristics.

Consider the model of Section 6.2, under the specification in which all market alternatives—including the outside option—have the same success rate α. In this case, we referred to the market as a "market for quacks." Rationalization via modified information is very naturally suggested by the sampling-based procedure itself. Instead of viewing the samples as part of the choice procedure in a complete-information model, we can reinterpret the samples as information sets in a model in which rational consumers are imperfectly informed. The rationalization turns the model into an incomplete-information extensive-form game: firms move first (making simultaneous pricing decisions) and consumers move second, after receiving partially informative signals of the firms' success rates. The model's predictions are given by applying the solution concept of sequential equilibrium to the incomplete-information game. In the bounded-rationality model, consumers confront their market environment with a decision procedure that generates systematic inference errors. In contrast, the imperfect-information rationalizing model rules out systematic inference errors because the solution concept of sequential equilibrium embodies "rational expectations."

Although this rationalization sounds plausible, it suffers from a number of difficulties.

Changed assumptions about the external environment

In order for consumers' imperfect information to have any relevance, we must assume that firms have multiple payoff-relevant *types*—for examples, a high-quality firm and a low-quality firm. Thus, in order to apply the rationalization, we also need to modify our assumptions about the *external* market environment.

This is not an innocuous modification. For example, consider the market for homeopaths. A homeopathic medicine is based on a solution so diluted that it is, to an excellent approximation, pure water. To claim that there are high-quality and low-quality homeopaths is to claim that different types of water have different therapeutic properties.[1]

In another context, if we consider the money-management application of the model, the assumption that there are high-quality and low-quality money

[1] Alternatively, a higher-quality homeopath could be regarded as one who is better at generating a placebo effect. However, admitting placebo effects introduces irrationality through the back door, and therefore I ignore this possibility.

managers means that some managers can systematically beat the market. This is an important substantive assumption, which is not taken lightly by financial economists. One should continue not to take it lightly when using it to rationalize models of money management markets with boundedly rational investors.

New unobservable parameters

The incomplete-information game designed to rationalize the market-for-quacks model requires us to introduce new parameters that describe the distribution of firm types and the consumers' signal structure. The following specification is minimalistic in this regard. Each market alternative (the firms as well as the outside option) has a type $t \in \{L, H\}$. The prior probability of L is λ, independently across market alternatives. When a consumer chooses an alternative of type t, his need is satisfied with probability α_t, where $1 \geq \alpha_H > \alpha_L > 0$. Thus, each alternative's ex-ante success rate is $\alpha = \lambda\alpha_L + (1 - \lambda)\alpha_H$. Each consumer observes a signal $s_i \in \{L, H\}$ about each alternative. The signals are distributed independently across market alternatives and across consumers. Let q_{ts} denote the probability that the consumer observes the signal s conditional on the alternative's type being t. Assume that $q_{LL} > q_{LH}$ and $q_{HH} > q_{HL}$; that is, signals have some informational content.

Notice how many new parameters have been introduced, even in such a minimalistic two-type, two-signal rationalization: $\alpha_L, \alpha_H, q_{LL}, q_{HH}$ (λ is not an independent parameter, as it is determined by α_L, α_H and α.) In contrast, the market-for-quacks model essentially had a single parameter, namely the firms' ex-ante success rate α. Also, note that if $\alpha_H = \alpha_L$, consumers know with certainty that they are in a market for quacks, and the model collapses to what we referred to as the rational-consumer benchmark in Chapter 6. Therefore, in what follows we must insist that $\alpha_H > \alpha_L$. In addition, we need to specify the firms' information. Assume that firms are only informed of their own type.[2]

Does the rationalizing model replicate the original model's key predictions?

Let us explore sequential equilibria in this incomplete-information game, and compare them to the unique Nash equilibrium in the market-for-quacks model. Formal near-equivalence between the original model and its rationalization is attained for the following parameter values: $\alpha_H = 1, \alpha_L = 0$, and $q_{HH} = q_{LL} = 1$. That is, high-quality (low-quality) alternatives satisfy the consumer's need with probability one (zero), and the consumer is perfectly informed of the alternative's type. The equilibrium strategy for high-type firms is the same as in the basic model. Low types are always recognized as such and are never chosen, and so their pricing strategy is indeterminate as well as irrelevant for the market outcome.

[2]This assumption is made for simplicity; alternative specifications would be harder to analyze and give rise to new problems.

The reason I refer to this as "near-equivalence" is twofold. First, we should have compared the firms' ex-ante pricing strategy in the rationalizing model to the equilibrium strategy in the original market-for-quacks model. Instead, we compared the latter to the equilibrium behavior of *high-quality* firms in the rationalizing model. Second, and more importantly, the two models generate different consumer behavior. In the rationalizing model, all consumers make the same decision in equilibrium. They are all informed of the firms' types, and since firms play continuous pricing strategies, price ties occur with probability zero. Therefore, all consumers make the same choice: they select the cheapest high-quality alternative (or the outside option, if no high-quality alternative is available). In contrast, recall that a salient feature of the market-for-quacks model was that each firm—including the most expensive one—had a positive clientele. Here, the most expensive firm ends up with either no clients or with all clients. In light of this discrepancy in the predictions that these two models make regarding consumer behavior, should we view this rationalization as successful?

The parameter values in this specification of the rationalizing model are also problematic in terms of interpretation, as they imply that consumers are perfectly informed of the types of all market alternatives. Recall that our motivation was to replace imperfect rationality with imperfect information about firms' types, yet consumers turn out to be perfectly informed under these parameter values.

When we turn to more plausible parameter specifications, the rationalizing model fails to reproduce salient features of firm behavior in the original model. Recall that in the market-for-quacks model, the firms' equilibrium pricing strategy is a continuously increasing *cdf* over the interval $[(1-\alpha)^{n-1}, 1]$. That is, firms charge prices that range all the way up to consumers' willingness to pay for guaranteed satisfaction of their need, and these prices generate a positive clientele. Can sequential equilibrium in the rationalizing model reproduce this effect?

When consumers are imperfectly informed of the firms' types, the firms' pricing strategy in equilibrium is independent of their type, because there is nothing in the incentive structure in the model that enables firms to signal their type. In other words, equilibrium must be *pooling*.

(Note, however, that in other cases, rationalization via modified information does introduce signaling issues that give rise to multiple equilibria. In this case, the rationalizing model's ability to replicate the predictions of the target bounded-rationality model relies on suitable equilibrium selection, thus raising a burden-of-proof problem similar to the parameter selection problem already discussed.)

By Bayes' rule:

$$\Pr(t = H \mid s = H) = \frac{\lambda q_{HH}}{\lambda q_{HH} + (1-\lambda) q_{LH}}$$

Therefore, when a consumer observes a good signal about an alternative, the alternative's posterior success rate is

$$\bar{\alpha} \mid H = \frac{\lambda q_{HH}\alpha_H + (1-\lambda) q_{LH}\alpha_L}{\lambda q_{HH} + (1-\lambda) q_{LH}}$$

Similarly, when a consumer observes a bad signal about an alternative, the alternative's posterior success rate is

$$\bar{\alpha} \mid L = \frac{\lambda q_{HL}\alpha_H + (1-\lambda)q_{LL}\alpha_L}{\lambda q_{HL} + (1-\lambda)q_{LL}}$$

In order for a consumer to be willing to pay a positive price for a firm, it must be the case that he received a bad signal about the outside option. This is just as in the market-for-quacks model. The reason is that all market alternatives are symmetric in terms of ex-ante quality, but the outside option comes free, whereas firms charge positive prices. Therefore, the maximal price that consumers are willing to pay to firms is $(\bar{\alpha} \mid H) - (\bar{\alpha} \mid L)$. This has to be the maximal price in the support of the marginal equilibrium pricing strategy. It is easy to see that $(\bar{\alpha} \mid H) - (\bar{\alpha} \mid L) \leq 1$. This inequality is strict unless $q_{HH} = \alpha_H = 1$ and $q_{HL} = \alpha_L = 0$, which is the case we already covered above. Thus, the price range cannot be replicated under reasonable assumptions on the signal structure.

It could be argued that the range of equilibrium prices is not a key prediction of the original model, because of the difficulty of observing the consumers' underlying willingness to pay. But the rationalizing model also fails to reproduce the market-for-quacks model's comparative statics. As α_L and α_H go down, it is easy to see that since $q_{LL} > q_{LH}$ and $q_{HH} > q_{LH}$, $(\bar{\alpha} \mid H) - (\bar{\alpha} \mid L)$ decreases as well. Therefore, the maximal price that consumers are willing to pay in the rationalizing model goes down. In the limit, as α_L and α_H tend to zero, the maximal price converges to zero. This is in marked contrast to the effect of lowering ex-ante success rates on equilibrium prices in the original market-for-quacks model.[3]

To summarize, in the zoo of new parameters that are needed to specify the rationalizing model, there is a configuration of parameter values that roughly replicates the firms' equilibrium behavior in the market-for-quacks model. However, this configuration is inconsistent with the motivation of imperfect informed consumers. Indeed, it has a difficult-to-interpret property that consumers are fully informed of firms' quality (while firms were assumed to be uninformed of their opponents' types). Furthermore, consumer behavior in equilibrium differs from the market-for-quacks model. For all other configurations of parameter values, the rationalizing model fails to replicate the original model's range of equilibrium prices, and the comparative statics of expected prices with respect to the ex-ante success rate are diametrically opposed to what the original model predicts.

Summary

Our analysis has raised several questions regarding the burden of proof we wish to impose on the rationalizing model. How should we evaluate a rationalization when it requires us to modify assumptions about the external environment, particularly

[3]There are other ways to manipulate the rationalizing model's parameters in a way that lowers the ex ante success rate. For instance, we can reduce λ without changing α_L or α_H. This would have a similar effect.

when these are essential to the "moral" of the original story? Should we discount the rationalizing model if it forces us to introduce a number of new parameters? Is it enough to replicate the firms' behavior, or do we need to reproduce consumer behavior as well, in order for the rationalization to count as a success? Is it enough to find particular parameter values that replicate certain aspects of the original model's equilibrium? Or should the replication hold for a large range of parameter values? What is the interpretational burden on the parameter values that are used for replicating the original model's predictions? Is it enough to replicate only some aspects of the original equilibrium? If so, how shall we decide which aspects are crucial and which are incidental?

13.2 RATIONALIZATION VIA MODIFIED PREFERENCES

When a certain choice pattern appears like a decision error that results from bounded rationality, we should entertain the possibility that it is in fact consistent with a perfectly rational decision, and the only reason it seems erroneous is that we, as outside observers, attribute the wrong preferences to the agent. For instance, when a manager chooses a big project over a smaller, yet more profitable one, this can be interpreted as evidence of a career concern. Replacing a behavioral model based on boundedly rational reasoning with a rational-choice model in which the consumer's preferences are re-specified is another common form of rationalizing bounded-rationality models.

Unlike the rationalization via modified information, rationalization via modified preferences turns out to be quite effective in the case of the market-for-quacks model. In fact, we have already provided such a rationalization in Section 6.2.1. Drop the assumption that consumers are interested in the firms' products only insofar as they expect it to satisfy a given need. Instead, assume that there is an idiosyncratic value for each consumer-firm match. Specifically, a consumer's evaluation of each alternative $i \in \{0, 1, \ldots, n\}$ is $u_i - p_i$, where u_i gets the values 1 or 0, with probabilities α and $1 - \alpha$, independently across alternatives and consumers. Each consumer is perfectly informed of the realizations of u_i for all i.

Rational consumers with this specification of independent private values behave *exactly* like boundedly rational consumers who evaluate alternatives according to the sampling procedure. Therefore, the rationalizing model is formally—and therefore behaviorally—equivalent to the market-for-quacks model. This is an extreme case of the methodological dilemma that motivated this chapter. The formal equivalence between the two models means that every prediction about market outcomes that we make in one model is *perfectly* mimicked by the other. However, the welfare implications are radically different. The bounded-rationality model implies that in a "market for quacks" (where the success rate of any product traded in the market is no different from the outside option of doing nothing), industry profits are a pure welfare loss for consumers. Moreover, this loss grows with n in a certain range. In contrast, in the rationalization, the market serves genuine consumer needs that the outside option fails to satisfy. It is welfare enhancing, and a greater number of firms is unambiguously better for consumers because it gives consumers access to a greater set of alternatives to choose from, while lowering their prices.

How should we compare these two accounts?

Prior plausibility of behavioral assumptions

I do not see any escape from the need to judge the prior plausibility of the behavioral assumptions that underlie two, formally equivalent models, in the context of their intended application. For instance, when the market in question is for forecasting services, assuming independent private values makes little sense: every rational consumer should prefer a forecaster who makes more accurate predictions. In contrast, when the market in question is for self-help guides, both explanations are plausible. Independent private values make sense because different self-help guides may contain different pieces of advice that fit different people. On the other hand, casual observation suggests that people extrapolate naively from anecdotal evidence to evaluate self-help guides.

This is an obvious point, which I already made in Section 6.2.1. The only reason I reiterate it is that there is a strong tradition in economic methodology (following Friedman (1953)) that is opposed to a priori judgments of behavioral assumptions and preaches that we should evaluate models exclusively by their predictive success. However, when we need to choose between two formally equivalent models having different welfare implications, Friedman's positivistic criterion is too weak, and it seems clear to me that we should favor the model that is based on behavioral assumptions that make better sense in the relevant context.

How should we evaluate distinctions that emerge in extended models?

Even when two models appear equivalent, they may differ when we move outside the original environment for which they were formulated. That is because different models tend to suggest different extensions. For example, in Chapter 7 we extended the model of price competition with "sampling" consumers by allowing firms to randomize over prices. The extension is based on a simple idea: the same element of bounded rationality that makes consumers extrapolate naively from small samples when evaluating the random performance of credence goods is going to make them extrapolate naively from small samples to evaluate random pricing strategies. We saw that this behavioral model implies a strict incentive for firms to randomize over prices. Moreover, a greater number of competitors results in a mean-preserving spread in the equilibrium price distribution that firms adopt. It is hard to think of an organic extension of the differentiated-taste rationalization of the market-for-quacks model that will generate these effects. Should the observation that the bounded-rationality model and its rationalization become behaviorally distinct in an enlarged domain affect our judgment of the rationalizing model in the original domain?

This dilemma becomes more acute when the distinctions between the two model extensions are restricted to off-equilibrium outcomes. Consider the extension of the market-for-quacks model examined in Section 6.4, where firms choose not only prices but also (simultaneously) whether to disclose their success rates to consumers. Disclosure is meaningless under the differentiated-taste rationalization, because its premise is that consumers are better informed than firms, and not the other way around. One could argue that this by itself provides a meaningful distinction between the sampling-based model and its differentiated-taste rationalization. However, recall that the equilibrium prediction of the extended model is that firms choose *not* to disclose their success rates. Therefore, as far as equilibrium behavior is concerned, the two models are equivalent after all. In the sampling-based model, disclosure is meaningful but fails to occur in equilibrium, while in the differentiated-taste rationalization, disclosure does not occur because it is meaningless in the first place. Can we say that the differentiated-taste rationalization replicates the sampling-based model's predictions in the extended domain that includes disclosure?

Summary

In this section we examined a rationalization that looks perfect at first glance, as it is formally equivalent to the original bounded-rationality model. However, we identified two burden-of-proof issues. First, the behavioral assumptions underlying the rationalizing model may be implausible in the context of the original model's intended domain of applications. Second, determining the relevant domain itself is not straightforward, because the original model has natural extensions that are either nonsensical from the point of view of the rationalizing model, or generate predictions that are distinct from those of an analogous similar extension of the rationalizing model. How should we evaluate the rationalizing model in light of these observations?

13.3 RATIONALIZATION VIA ENDOGENIZATION

Another way of rationalizing a bounded-rationality model is to argue that the behavior it generates appears to be non-rational only because the model leaves out certain costs associated with the decision process. Once these are explicitly incorporated into the model, rationality is restored. In the extended model, the consumer chooses how much mental resources to spend on the decision problem, on the basis of "rational expectations" of the benefits of information processing. Note that it is not so much the friction itself as its formal treatment that is conventional. Decision costs are rarely incorporated into standard economic models. However, the type of extended model described here is conventional in that it treats decision costs as if they were search costs, or costs of acquiring information.

The following example has become almost canonical in methodological discussions of bounded-rationality models (see Caplin & Schotter (2008)). An American tourist visits London (the tourist is invariably American in tellings of this story). Before crossing a street, he looks left, sees that the road is clear, starts walking and gets hit by a car coming from his right. The bounded-rationality interpretation of the tourist's behavior is that he does not deliberate over the problem of deciding when to cross the street. Instead, he follows an automatic rule that was optimal in his home environment. A rationalization that incorporates decision costs would proceed as follows. The tourist realizes that he is on foreign soil and that he needs some time to remember which side the cars are coming from. However, spending time on this mental task is costly. The tourist rationally trades off this cost against the benefit of safe crossing.

As this example is only meant to illustrate a methodological dilemma, I will not get into a detailed discussion of the plausibility of the rationalization of the tourist's behavior. I will only comment that the two explanations of the tourist's behavior are in principle distinguishable. For instance, one could put up a sign for pedestrians saying "don't cross the street without thinking first." This intervention should have an impact only under the bounded-rationality interpretation.

Turning back to Industrial Organization, consider the model presented in Section 10.2, in which firms simultaneously choose a price and a frame for their products. In the model, the probability that a consumer compares his default option to the other market alternative is $\pi(x, y)$, where x and y are the frames of the default option and the other market alternative, respectively. The function π is a primitive of the model, capturing the comparability of frames. However, one could argue that the act of comparing two alternatives is a result of a deliberate choice that takes information-processing costs into account. Making a comparison between the frames x and y carries a mental cost $c(x, y) > 0$. In the rationalizing model, the probability $\pi(x, y)$ should result from the consumer's equilibrium information-processing (mixed) strategy, which in turn is based on a correct belief of the expected benefit from being able to compare two alternatives framed by x and y.

I find this extended model interesting and worth exploring. However, it is not clear why the possibility of analyzing this model should be viewed as a criticism against a model that takes π to be exogenous. I list the reasons for my bafflement.

When should we endogenize informational constraints?

Any informational constraint in any economic model could be endogenized, by enabling agents to invest resources that help them relax their constraint. Economists address this endogeneity only when they wish to focus on the information acquisition process, and in most applications they are happy to treat the informational limitations as exogenous, because this is a good modeling strategy. The same standard should hold for rationality constraints. If the modeler has good reasons for certain restrictions on π, and a much fuzzier notion of the restrictions that could be imposed on the cost function c, then it is a good modeling strategy to take π as primitive.

At which scale should we endogenize informational constraints?

Even if the limited comparability between frames is a result of some optimizing process that incorporates information-processing costs, the optimization often takes place not on a market-by-market basis, but at a "general equilibrium" level, or on an "evolutionary" time scale. Heuristics and calculational short-cuts evolve to work well on average across a large number of market (as well as non-market) situations. For the kind of partial equilibrium analysis that economists apply in Industrial Organization, taking the consumer's calculational short-cuts as given is a good approximation.

Should the endogenization of informational constraints be based on rational expectations?

In order for rationalization via endogenization to be operational, the consumer's optimization must be made on the basis of correct knowledge of the market equilibrium. Otherwise, we are not dealing with a rationalization, but merely shifting the element of bounded rationality to another level. It turns out that under this "rational expectations" restriction, some functions π cannot be endogenized as equilibrium outcomes of optimal information-processing decisions.

Consider the two-frame case analyzed in Section 10.2, and suppose that the specification of π gives rise to a cutoff equilibrium: there is a cutoff price p^* such that firms adopt the frame a conditional on $p < p^*$ and the frame b conditional on $p > p^*$. In particular, recall that $\pi(a, b) > 0$. Under the rationalizing model, the consumer incurs the cost $c(a, b)$ if and only if he chooses to compare the two frames. Since in equilibrium the price associated with b is always higher than the price associated with a, consumers should not incur this cost because their equilibrium knowledge is a perfect substitute for active comparison between the two frames. Therefore, $\pi(a, b)$ should be equal to zero, a contradiction. It follows that the function π cannot be justified by the rationalizing model.

One point of view is that this failure to rationalize π is in fact a *criticism* of the bounded-rationality model, akin to the famous *Lucas critique* of traditional Keynesian macroeconomic models by Lucas (1976). According to this interpretation, the fact that certain specifications of π cannot be justified as equilibrium responses in a larger model that incorporates explicit information-processing costs means that these specifications are illegitimate. Note the rhetorical cunning at play here. So far, we have viewed success at rationalizing bounded-rationality models as a vindication of the "can't we get the same thing with a standard model" critique. Now, the "Lucas critique" turns the rationalization program on its head, and sees its failure as a reason to detract the bounded-rationality model.

Nothing better illustrates the tricky burden-of-proof questions that surround the debate over bounded-rationality models.

13.4 DISCUSSION

This chapter has been concerned with the following dilemma: What is the burden of proof that should be imposed on a rational-choice model (referred to as R) that is offered in refutation of a given bounded-rationality model (referred to as BR)? The following scenarios abstract from the details of the specific models we examined. Each scenario raises a difficulty in the evaluation of R as a successful refutation of BR.

- R mimics key predictions of BR, but R differs from BR not only in behavioral assumptions, but also in assumptions about the external environment.
- R introduces new parameters that were not included in BR. Whether R mimics key predictions of BR depends on a suitable selection of parameter values.
- BR and R are observationally equivalent in a certain domain of market situations, but BR is based on behavioral assumptions that appear more plausible in this particular domain.
- BR and R are observationally equivalent in a certain domain of market situations, but become distinct when we extend (in a "natural" way) the behavioral models underlying BR and R to a broader domain.
- BR and R are observationally distinct out of equilibrium, but they imply identical equilibrium behavior. An extreme case is where a certain action is meaningful in BR yet not taken in equilibrium, while the same action is meaningless and inconceivable a priori in R.
- R has multiple equilibria, whereas BR has a unique equilibrium. Whether R mimics the predictions of BR depends on a suitable equilibrium selection.

I do not aim to resolve these burden-of-proof questions, as my intention here is merely to problematize rationalization as a mode of criticizing bounded-rationality models. However, these difficulties should make us wary of the "can't we get the same thing with a standard model" critique. It would be silly to suggest that economists give up on attempts to provide "rational explanations" for apparently non-rational behavior; this is part of what defines economic research, and it has led to numerous successes. However, "rational explanations" that are offered in refutation of a bounded-rationality model should be subjected to some burden-of-proof requirements, particularly when the bounded-rationality model is based on behavioral assumptions that seem plausible in the relevant domain, generates predictions that appear consistent with known (stylized) facts, and tells a story that "rings true."

At the very least, the burden-of-proof criteria that we use to evaluate rationalizing models should not differ from those that we use to evaluate two rational-choice explanations against each other. If a certain prediction of a given model is qualified by a particular equilibrium refinement, a particular choice of parameter values, or a particular assumption about the external environment, we should not discard these qualifications simply because the model is offered in an attempt to refute a bounded-rationality model.

We should also be reluctant to criticize assumptions of a bounded-rationality model that concern the *domain* of market situations that agents face. Whether a model is static or dynamic, whether its informational constraints are endogenous or exogenous, whether agents are assumed to be homogeneous or heterogeneous—these are all modeling choices that the theorist makes to *define* the limits of the theoretical exercise he wishes to pursue. We are not forced to respect these assumptions when discussing the model's merits and drawbacks, but we should try to accept them as given when advancing alternative "rational explanations."

13.5 EPILOGUE

Economists' methodological preference for rational-choice models is not a mere habit of thought. One good reason is that the rational-choice model offers a coherent analytical framework with enormous scope. It gives the impression that any conceivable human interaction can be written down as an extensive-form game with incomplete information and be subjected to equilibrium analysis.[4] In contrast, the bounded-rationality models we have seen in this book were defined for very limited domains, and were seldom presented as specifications of a single, general framework. In other words, bounded-rationality models tend to appear extremely *ad hoc* relative to their rational-choice counterparts, and their behavioral assumptions seem tailored to specific applications.

I agree with this criticism to some extent. However, we should not confuse between general frameworks and specific models. The standard rational-choice model is a framework that leaves the consumer's preferences, action space, and information free to be specified by the economic theorist. Any specification of the rational-choice model could be attacked as arbitrary and ad hoc, given the considerable freedom that the economic theorist has when specifying the agents' preferences and constraints.

What mutes the ad-hockery criticism in the case of the rational-choice model is the existence of a coherent analytical framework, in which all standard economic models are embedded. A comparable abstract framework is, in my opinion, what is missing the most from the current bounded rationality and behavioral economics literature. People display various forms of boundedly rational reasoning, just as they vary in their preferences or information. Embedding some of these patterns of reasoning in a "general" language will not eliminate the need for specific modeling choices for the sake of specific applications, but it will enable us to see

[4]This impression can be misleading. For instance, Piccione & Rubinstein (1997) demonstrated that the formalism is unable to describe without ambiguities decision problems with imperfect recall. The heated debate that ensued (see the *Games and Economic Behavior* issue in which the original Piccione-Rubinstein paper was published) attests to the widespread belief among economists that the rational-choice framework has universal applicability.

the connections between them, and thus render them less arbitrary. Hopefully, future theoretical research in bounded rationality will rise to this challenge.

13.6 BIBLIOGRAPHIC NOTES

This chapter is based on Spiegler (2010*a*). For a discussion of closely related issues, see Rabin (2002).

A Appendix to Part I: A Decision-Theoretic Perspective

Chapters 2–4 introduced two related models of temptation-driven behavior in two-period decision problems. In these models, consumers choose a menu of alternatives in period 1, and then select an alternative from the chosen menu in period 2. We analyzed some implications of these decision models for pricing behavior in monopolistic or competitive markets. This appendix is devoted to a deeper characterization of the decision models themselves. The decision-theoretic analysis is not crucial for the industrial-organization analysis, but it helps in articulating the essential differences and similarities between the various decision models.

Let X be a grand set of alternatives. Throughout the Appendix, X is finite, except when explicitly stated otherwise. The set $P(X)$ is the set of all menus—that is, non-empty subsets of X. In the market models we examined, an alternative was typically an action-payment pair, and a menu corresponded to a price scheme. The consumer's first-period choices between available price schemes could be summarized by a preference relation $\succsim^*$ over $P(X)$, such that for every collection of menus $\mathcal{A} \subseteq P(X)$, the consumer chose a menu $A \in \mathcal{A}$ satisfying $A \succsim^* A'$ for all $A' \in \mathcal{A}$. The decision-theoretic analysis in the following sections is devoted to characterizing properties of preference relations over menus that describe consumers' first-period behavior under a variety of two-period decision models. The focus on first-period choices alone can be motivated by a realistic scenario, in which an economist can observe how the consumer chooses a price scheme among those available in the marketplace, but he lacks data about the consumer's subsequent consumption quantity choices.

A.1 THE MULTI-SELVES MODEL

Let us begin with a characterization of first-period choices between menus induced by the multi-selves model, introduced in Chapter 2. We have already noted that sophisticated multi-selves consumers generally have a preference for commitment—which in the present context means a preference for smaller menus. Our goal here is to characterize the exact structure of this preference for commitment. It turns out that when X is finite, we are able to fully characterize it with a simple axiom.

Let $\succsim^*$ be a preference relation over $P(X)$. As described above, this preference relation summarizes the way consumers choose between menus in period 1. The following axiom imposes a restriction on $\succsim^*$.

Axiom A.1. *For every $A, B \in P(X)$, $A \sim^* A \cup B$ or $B \sim^* A \cup B$.*

Note that this axiom accommodates a taste for commitment, because the large menu $A \cup B$ is never ranked strictly above *both* smaller menus A and B. However, the axiom imposes a lot more structure on the taste for commitment, as it requires $A \cup B$ to be equivalent in terms of preferences to A or B.

Proposition A.1. *A preference relation $\succsim^*$ over $P(X)$ satisfies Axiom A.1 if and only if there exist two numerical functions, $u : X \to \mathbb{R}$ and $v : X \to \mathbb{R}$, such that for every $A, B \in P(X)$,*

$$A \succ^* B \text{ if and only if } \max_{x \in \arg\max_{z \in A} v(z)} u(x) > \max_{y \in \arg\max_{z \in B} v(z)} u(y)$$

Thus, Axiom A.1 completely characterizes the first-period behavior induced by the two-period multi-selves model. However, there is a caveat. When $\arg\max_{z \in A} v(z)$ is not a singleton, the representation resolves the tie according to the first-period self's preferences. In contrast, our practice in Chapters 2–4 was to resolve second-period ties in favor of the firm interacting with the consumer. Instead of proving Proposition A.1, let us prove a slightly easier result, which evades the issue of second-period indifferences. Note that Axiom A.1 immediately implies that for every menu A, there exists some $x \in A$ such that $A \sim^* \{x\}$. The following axiom strengthens this property.

Axiom A.2. *For every $A \in P(X)$, there is a **unique** element $x \in A$, such that $A \sim^* \{x\}$.*

The following is a characterization of the multi-selves model, in which the two numerical functions u and v are one-to-one.

Proposition A.2. *A preference relation $\succsim^*$ over $P(X)$ satisfies Axioms A.1 and A.2 if and only if there exist two **one-to-one** numerical functions, $u : X \to \mathbb{R}$ and $v : X \to \mathbb{R}$, which are unique up to monotone transformations, such that for every $A, B \in P(X)$,*

$$A \succ^* B \text{ if and only if } u[\arg\max_{x \in A} v(x)] > u[\arg\max_{x \in B} v(x)]$$

***Proof*.** First, let us show that the representation implies Axioms A.1 and A.2. To see that Axiom A.1 is satisfied, note that $\arg\max_{x \in A \cup B} v(x)$ belongs to A or to B. Therefore, $\arg\max_{x \in A \cup B} v(x) = \arg\max_{x \in A} v(x)$ or $\arg\max_{x \in A \cup B} v(x) = \arg\max_{x \in B} v(x)$, hence $A \cup B \sim^* A$ or $A \cup B \sim^* B$. Axiom A.2 follows immediately from the assumption that u is a one-to-one function.

Second, assume that $\succ^*$ satisfies Axioms A.1 and A.2, and let us construct the one-to-one functions u and v, such that $A \succ^* B$ if and only if $u[\arg\max_{x \in A} v(x)] > u[\arg\max_{x \in B} v(x)]$. Construct u as follows: $u(x) \geq u(y)$ if $\{x\} \succsim^* \{y\}$. Because $\succsim^*$

is a preference relation, u is well-defined. By Axiom A.2, for every distinct $x, y \in X$, $\{x, y\} \sim^* \{x\}$ if and only if $\{x, y\} \nsim^* \{y\}$, hence u is one-to-one.

Let us turn to constructing v. To do so, define the following choice function c: for every menu $A \subseteq X$, $c(A)$ is the element $x \in A$ for which $\{x\} \sim^* A$. By Axiom A.2, there is exactly one such element. Therefore, c is a well-defined choice function. To see that c satisfies IIA, let $c(A) = x$, and consider a subset $B \subset A$ which contains x. Let us show that $\{x\} \sim^* B$. Assume the contrary. Then, $A \nsim^* B$. By Axiom A.1, $A \sim^* A \backslash B$. By Axiom A.2, there exists an element $y \in A \backslash B$ such that $\{y\} \sim^* A \backslash B$. Therefore, $\{y\} \sim^* A$. But since by assumption $c(A) = x \neq y$, and since by definition this means that $\{x\} \sim^* A$, we get a violation of Axiom A.2. Therefore, $\{x\} \sim^* B$, hence $c(B) = x$. We have established that c satisfies IIA. Therefore, c can be rationalized by a one-to-one utility function v over X.

Suppose that $A \succ^* B$. By the definition of v, $A \sim^* \{\arg\max_{x \in A} v(x)\}$ and $B \sim^* \{\arg\max_{x \in B} v(x)\}$. By the definition of u, $\{\arg\max_{x \in A} v(x)\} \succ^* \{\arg\max_{x \in B} v(x)\}$ if and only if $u[\arg\max_{x \in A} v(x)] > u[\arg\max_{x \in B} v(x)]$. Finally, since u was constructed to represent a complete and transitive relation over X, while v was constructed to rationalize a well-defined choice function, a basic result in choice theory ensures that these utility functions are unique up to monotone transformations. □

The proof shows us how to elicit the first- and second-period preferences over alternatives from first-period preferences over menus. Self 1's preferences over alternatives are given by his commitment rankings (i.e., preferences over singleton menus). Self 2's preferences over alternatives are constructed to rationalize an implicit second-period choice function, which is elicited from the consumer's indifference between a menu and a singleton contained in the menu.

Note that the ordinal uniqueness of v in the representation given by Proposition A.2 relies on Axiom A.2, and need not hold in the weaker Proposition A.1. The reason is that when Axiom A.2 is relaxed, we are unable to fully elicit the second-period self's preferences from $\square^*$. For instance, suppose that $\{x\} \sim^* \{y\} \sim^* \{x, y\}$. By the construction of u, $u(x) = u(y)$. But we are entirely unable to tell the v ranking of x and y, simply because the first-period self does not care whether he will choose x or y from $\{x, y\}$ in period 2. In contrast, ordinal uniqueness of u holds under both versions of the representation theorem, because u is completely defined by preferences over singleton menus. Since these preferences are complete and transitive, they admit an ordinally unique utility representation in the usual, utility-theory-for-dummies sense.

A.2 SELF-CONTROL PREFERENCES

Let us now turn to the model of self-control preferences presented in Chapter 3. In this model, we defined preferences over decision paths (A, x), whereas in the multi-selves model we defined first- and second-period preferences over alternatives x. The utility function we used to express self-control preferences over decision paths was[1]

$$U(A, x) = u(x) - [\max_{y \in A} v(y) - v(x)]$$

This means that the induced first-period preference relation over menus $\square^*$ is represented by the indirect utility function

$$W(A) = \max_{x \in A}[u(x) + v(x)] - \max_{y \in A} v(y) \tag{A.1}$$

This decision model can give rise to first-period choice patterns such as $\{x\} \succ^* \{x, y\} \succ^* \{y\}$. Note that this preference ranking holds only if $u(x) > u(y)$ and $v(y) > v(x)$ and $u(x) + v(x) > u(y) + v(y)$. We interpreted this pattern as follows: y is a more tempting alternative than x, whereas x is more desirable ex ante. When both alternatives are available, the consumer exerts self-control and chooses the ex-ante preferred element x, but exerting self-control is costly; therefore, the menu $\{x, y\}$ is ranked between the two singleton menus. The preference ranking $\{x\} \succ^* \{x, y\} \sim^* \{y\}$ was interpreted as evidence that y is overwhelmingly tempting, such that no self-control was exerted.

Let us try to generalize this choice pattern. We say that a preference relation $\square^*$ over $P(X)$ satisfies *betweenness* if for every $A, B \in P(X)$, $A \;\square^*\; B$ implies $A \;\square^*\; A \cup B \;\square^*\; B$.

Remark A.1. *If $\square^*$ is represented by W, then $\square^*$ satisfies betweenness.*

Proof. For every menu A, define $w^*(A) = \max_{x \in A}[u(x) + v(x)]$ and $v^*(A) = \max_{y \in A} v(y)$. Then, $W(A) = w^*(A) - v^*(A)$. Note that $v^*(A \cup B) = \max(v^*(A), v^*(B))$ and $w^*(A \cup B) = \max(w^*(A), w^*(B))$. Assume, without loss of generality, that $A \;\square^*\; B$. Let us distinguish between two cases. First, suppose that $v^*(A) \geq v^*(B)$. Then, it must be the case that $w^*(A) \geq w^*(B)$, hence $W(A \cup B) = W(A)$. Second, suppose that $v^*(A) < v^*(B)$. This implies $W(A \cup B) = w^*(A \cup B) - v^*(B) \geq w^*(B) - v^*(B) = W(B)$. If $w^*(B) \geq w^*(A)$, then $w^*(A \cup B) = w^*(B)$, and therefore $W(A \cup B) = w^*(B) - v^*(B) = W(B)$. If $w^*(B) < w^*(A)$, then $W(A \cup B) = w^*(A) - v^*(B) < w^*(A) - v^*(A) = W(A)$. We have thus established that $W(B) \leq W(A \cup B) \leq W(A)$. □

Now let us assume that a preference relation over menus satisfies betweenness, and see what this implies for the structure of any utility function over menus that represents it.

Proposition A.3. *Let V be a utility function that represents $\square^*$. If $\square^*$ satisfies betweenness, then:*

$$V(A) = \max_{x \in A} \min_{y \in A} V(\{x, y\})$$

[1]For notational simplicity, in this appendix I omit the superscript * from u and v.

***Proof*.** For every $x \in A$, define $y^*(x) = \arg\min_{y \in A} V\{x, y\}$. If $V\{x, y^*(x)\} > V(A)$ for some x, then $V\{x, y\} > V(A)$ for all y. By iterated application of betweenness, $V(\cup_{y \in A}\{x, y\}) > V(A)$. But $\cup_{y \in A}\{x, y\} = A$, a contradiction. It follows that $V\{x, y^*(x)\} \leq V(A)$ for all $x \in A$. If this inequality is strict for all $x \in A$, then again by iterated application of betweenness, $V(\cup_{x \in A}\{x, y^*(x)\}) < V(A)$. But $\cup_{x \in A}\{x, y^*(x)\} = A$, a contradiction. It follows that

$$\max_{x \in A} V\{x, y^*(x)\} = \max_{x \in A} \min_{y \in A} V\{x, y\} = V(A)$$

$\square$

Thus, when preferences over menus satisfy betweenness, each menu is equivalent in terms of preferences to a sub-menu of (at most) size two. This is consistent with the interpretation that one of these elements is the chosen element and the other is the maximally tempting element (the two can coincide). For each potential choice x, the element $y^*(x)$ is interpreted as the maximally tempting element *relative to* x. Then, the actual choice is the element x for which the pair $(x, y^*(x))$ is most desirable. This interpretation does not require the menu to contain a single element that is maximally tempting in relation to all other elements in the menu.

The betweenness property does *not* imply that $\square^*$ can be represented by the functional W. For example, let $X = \{x, y, z\}$ and assume the following preferences over menus:

$$\{x\} \succ^* \{x, y\} \succ^* \{y\} \succ^* \{y, z\} \sim^* \{x, y, z\} \succ^* \{x, z\} \sim^* \{z\}$$

This preference relation satisfies betweenness. Suppose that it could be accounted for by the functional W. Then, the ranking $\{x, y\} \succ^* \{y\}$ would reveal that $u(x) + v(x) > u(y) + v(y)$. Similarly, the ranking $\{y, z\} \succ^* \{z\}$ would reveal that $u(y) + v(y) > u(z) + v(z)$. Therefore, $\max(u + v)$ must be the same for $\{x, y, z\}$ and $\{x, z\}$. By definition, $\max(v)$ must be weakly higher for $\{x, y, z\}$ than for $\{x, z\}$. By the functional W, this implies that $\{x, z\}$ is weakly preferred to $\{x, y, z\}$, a contradiction.

In order to establish a tight link between the betweenness property and the functional W, additional structure is needed. Let Z be some finite set of prizes, and define $X = \Delta(Z)$. Thus, $\square^*$ is a preference relation over menus of lotteries. Assume that $\square^*$ satisfies betweenness. In addition, impose a continuity assumption: if $A \succ^* B$, and A' and B' are in some sense "close" to A and B, respectively, then $A' \succ^* B'$.[2] For every pair of menus A, B, and every $\alpha \in (0, 1)$, define $\alpha A + (1 - \alpha)B = \{x \in X \mid x = \alpha x' + (1 - \alpha)x'' \text{ for some } x' \in A, x'' \in B\}$. The interpretation of this mixture of menus is as follows. Suppose that we randomize between two menus, A and B, such that the former (latter) becomes available with probability α $(1 - \alpha)$. Prior to the realization of this lottery, the decision maker plans a contingent choice of a single lottery x from A and a single lottery y from B. Essentially, the

[2]More specifically, A' is close to A if the maximal "distance" between an element in A and an element in A' is small.

decision maker chooses the lottery $\alpha x + (1 - \alpha)y$. The menu $\alpha A + (1 - \alpha)B$ is the collection of these contingent plans. We say that $\succsim^*$ satisfies *independence* if for every $A, B, C \in P(X)$, $A \succ^* B$ implies $\alpha A + (1 - \alpha)C \succ^* \alpha B + (1 - \alpha)C$ for every $\alpha \in (0, 1)$.

Theorem A.1. *A preference relation $\succsim^*$ over $P(X)$ satisfies betweenness, continuity, and independence if and only if there exist expected utility functions $u : X \to \mathbb{R}$, $v : X \to \mathbb{R}$, such that the function W represents $\succsim^*$.*

The interpretation of continuity

The continuity requirement is not merely technical: it rules out an interpretation of W that is consistent with the multi-selves approach. Consider a decision maker for whom u represents his commitment ranking of elements and v represents his temptation ranking. However, the decision maker's second-period choice is made entirely according to v, as in the multi-selves model. Therefore, the decision maker will evaluate sets by the maximal u-value of the v-maximal elements. In other words:

$$W(A) = \max_{x \in \arg\max_{y \in A} v(y)} u(x)$$

This behavior can violate continuity. To see why, consider three alternatives x, y, y', such that y and y' are very "close" to each other, while x is "far" from both y and y'. Assume that $u(x) \gg u(y), u(y')$ and $v(x) = v(y) < v(y')$. In this case, the multi-selves model implies the following ranking over menus:

$$\{x, y\} \sim^* \{x\} \succ^* \{y'\} \sim^* \{x, y'\}$$

The menus $\{x, y\}$ and $\{x, y'\}$ can be arbitrarily "close," yet $W\{x, y'\}$ is bounded away from $W\{x, y\}$.

Thus, although the preferences over menus induced by the multi-selves model satisfy betweenness, the multi-selves model is not a special case of the functional W because it does not satisfy the continuity property that characterizes the latter. It is, however, a limit case of the functional W. When the scale of the temptation utility v is much larger than the scale of the commitment utility u, the consumer behaves approximately as if he chooses according to v in the second period, and therefore evaluates menus according to the u-value of their v-maximal element, just as a multi-selves consumer does.

The interpretation of independence

Superficially, the independence axiom looks like its classical analogue in Expected Utility Theory.[3] But does it carry the same normative baggage? Suppose that the

[3] The classical independence axiom in Expected Utility Theory is as follows: for every triple of lotteries $x, y, z \in X$, $x \succ y$ implies $\alpha x + (1 - \alpha)z \succ \alpha y + (1 - \alpha)z$ for every $\alpha \in (0, 1)$.

set of prizes Z includes a tempting alternative, say ice cream. All other prizes are equally tempting (and less so than ice cream). Let $x \in A$ and $y \in B$ assign a probability of 0.2 and 0.4 to ice cream, respectively. Assume that these are the only lotteries in A and B that assign positive probability to ice cream. Assume further that $c(A) \neq x$ and $c(B) \neq y$, as each of the two menus contains lotteries with a sufficiently high commitment value.

Let C be a singleton menu consisting of a degenerate lottery that assigns probability one to ice cream. Now let $\alpha = 0.8$. The menu $\alpha A + (1 - \alpha)C$ includes a lottery that assigns probability 0.36 to ice cream, while $\alpha B + (1 - \alpha)C$ includes a lottery that assigns probability 0.52 to ice cream. All other lotteries in the two menus assign probability 0.2 to ice cream, and are therefore less tempting. By Independence, if $B \succ^* A$ then $0.8B + 0.2C \succ^* 0.8A + 0.2C$. However, from a normative point of view, nothing prevents the consumer from regarding a lottery that assigns a probability below 0.5 to ice cream as mildly tempting, while at the same time perceiving a lottery that assigns a probability above 0.5 to ice cream as highly tempting. As a result, he may display the preference rankings $B \succ^* A$ and $0.8A + 0.2C \succ^* 0.8B + 0.2C$.

The reason that the independence axiom is less attractive here than in conventional Expected Utility Theory is that in the present context, choice behavior relies on the notion of "temptation utility." Since this notion is meant to capture a primarily visceral response, there is no reason for it to satisfy normative criteria, such as linearity in the probability of prizes. In contrast, in Expected Utility Theory, the considerations that guide decisions are meant to be entirely rational.

A comment on revealed preferences

Proposition A.1 and Theorem A.1 are characterizations of first-period preferences over menus. These are revealed-preference characterizations, in the sense that utility rankings over menus are meant to represent observed choices between menus. In contrast, the interpretation of $\arg\max_{x \in A}[u(x) + v(x)]$ in Theorem A.1 or $\arg\max_{x \in A} v(x)$ in Proposition A.1 as the consumer's second-period choice from the menu A goes beyond revealed preferences, because the representation theorems do not address second-period choice. Thus, in the case of Theorem A.1, although it makes perfect sense to interpret $u + v$ as a representation of second-period choice and to interpret v as a representation of how tempting alternatives are, we should not view the theorem as providing a decision-theoretic, revealed-preference foundation for this interpretation. The same holds for our interpretation of v in the case of Proposition A.1. The next section makes a point that emphasizes this limitation of utility representations of preferences over menus.

A.3 THE RELATION BETWEEN SELF-CONTROL PREFERENCES AND THE MULTI-SELVES MODEL

So far, our comparison between the multi-selves model and the model of self-control preferences assumed that in the former case, the consumer has no uncertainty regarding his second-period preferences—as in the case of fully sophisticated consumers. It turns out that when we enrich the multi-selves model by allowing the consumer to hold probabilistic beliefs regarding his second-period preferences (as in the case of partial frequency naivete explored in Chapter 4), the relation between the two models becomes tighter.

Proposition A.4. *Suppose that a preference relation* $\sqsupset^*$ *over* $P(X)$ *can be represented by the functional* W *with* (u, v)*. Then,* $\sqsupset^*$ *is consistent with a multi-selves model in which the first-period self is uncertain about the second-period self's preferences. Moreover, the first-period self's preferences over* X *are represented by* u*.*

Proof**.** Suppose that $\sqsupset^*$ over $P(X)$ is represented by (A.1) with the commitment and temptation utilities being u and v. Let us construct a multi-selves model, in which the first-period self's preferences over X are represented by u, and there is uncertainty about the second-period self's preferences. Specifically, the second-period self's preferences over X are represented by a utility function $ku + v$, where $k \sim U[0, 1]$. We will now show that the first-period preferences over menus induced by this multi-selves model can be represented by (A.1).

Fix a menu A, and let $x^*(k)$ denote any member of A that maximizes u over the set $\arg\max_{x \in A}[ku(x) + v(x)]$. Define $\bar{u}(k) = u[x^*(k)]$, $\bar{v}(k) = v[x^*(k)]$, $w(k) = k\bar{u}(k) + \bar{v}(k)$. Thus, the first-period self's induced preferences over menus are represented by the indirect expected utility from second-period choice, given by the functional

$$V(A) = \int_0^1 \bar{u}(k)dk$$

By the definition of $x^*(k)$, $ku[x^*(k)] + v[x^*(k)] \geq ku(x) + v(x)$ for every $x \in A$. In particular, this means that $k\bar{u}(k) + \bar{v}(k) \geq k\bar{u}(k') + \bar{v}(k')$ for every $k' \in [0, 1]$. Thus, $w(k)$ is a maximum over a collection of affine functions, and therefore it is convex and differentiable almost everywhere over $[0, 1]$, where $w'(k) = \bar{u}(k)$. We can write $w(1)$ as follows:

$$w(1) = w(0) + \int_0^1 w'(k)dk$$

But since $w(1) = \max_{x \in A}[u(x) + v(x)]$ and $w(0) = \max_{y \in A} v(y)$:

$$V(A) = \max_{x \in A}[u(x) + v(x)] - \max_{y \in A} v(y)$$

which is exactly the functional W with (u, v). □

This result is attractive not only because of the elegance of its proof, but also because it provides an alternative interpretation of the betweenness axiom. The idea that a consumer who is uncertain about his second-period preferences will rank $A \cup B$ between A and B is intuitive, because it reflects some averaging of the possible second-period preferences. Of course, the multi-selves and the self-control models differ in the second-period behavior, and consequently the distinction between the models is important for the firms that interact with the consumer. Without observing the firms' behavior or the consumer's second-period behavior, we cannot distinguish between the two models.[4]

The multi-selves model with partial frequency naivete discussed in Chapter 4 did not involve a continuum of second-period possible preference relations, but only two, one of which coincides with the first-period self's preferences. Chatterjee & Krishna (2009) provide an axiomatization of the first-period preferences over menus that are induced by such a model. It typically violates the betweenness axiom, hence it generally fails to produce the same first-period choices over menus as the model of self-control preferences.

■ A.4 OTHER CLASSES OF TEMPTATION-DRIVEN PREFERENCES

The model of self-control preferences is a special case of a model of preferences over menus that captures a taste for commitment due to the presence of temptations. In this section I present two generalizations. The functional W belongs to a family called "finite additive expected-utility (EU) representations." Specifically, let $X = \Delta(Z)$, where Z is some finite set of prizes. We say that a preference relation $\succsim^*$ over $P(X)$ has a finite additive EU representation if there exists a pair of finite collections of expected utility functions over X, $(w_i)_{i=1,\dots,I}$ and $(v_j)_{j=1,\dots,J}$, such that the function

$$V(A) = \sum_{i=1}^{I} \max_{x \in A} w_i(x) - \sum_{j=1}^{J} \max_{x \in A} v_j(x)$$

represents $\succsim^*$. In the model of self-control preferences, $I = J = 1$. Throughout this section, assume that $\succsim^*$ has a finite additive EU representation. The class of preferences over menus of lotteries that possess such a representation is characterized by the independence axiom discussed above and a suitable continuity property.

One generalization of the model of self-control preferences is based on the idea that temptation can occur on many dimensions. For instance, one food product can be tempting because of its taste, another because of its design, and so forth. The following functional captures this idea:

$$V^*(A) = \max_{x \in A}[u(x) - s(A, x)]$$

[4] This finding does not contradict the distinction we made earlier, between the continuity of self-control preferences and the discontinuity of preferences over menus induced by the (deterministic) multi-selves model. The latter is "'smoothed out" by the uncertainty regarding second-period preferences.

where

$$s(A, x) = \sum_{j=1}^{J} [\max_{y \in A} v_j(y) - v_j(x)]$$

is the cost of self-control involved in choosing x from the menu A. The following result provides an axiomatic characterization of V^*.

Proposition A.5. *The functional V^* represents $\succsim^*$ if and only if for every $A, B \in P(X)$, $A \succsim^* B$ implies $A \succsim^* A \cup B$.*

Another generalization is based on the idea that the consumer may be uncertain about the intensity of temptations. The following functional captures this idea:

$$V^{**}(A) = \sum_i q_i \max_{x \in A}[u(x) - \gamma_i s(A, x)]$$

where $q_i > 0$ for all i, $\sum_i q_i = 1$, $\gamma_i \geq 0$ for all i, and

$$s(A, x) = [\max_{y \in A} v(y)] - v(x)$$

In this representation, temptation utility is always v, but the exact trade-off between temptation and commitment utilities is random. The following result provides an axiomatic characterization of V^{**}.

Proposition A.6. *The functional V^{**} represents $\succsim^*$ if and only if the following conditions hold:*

(i) for every $A \in P(X)$ there exists $x \in A$ such that $\{x\} \succsim^* A$;
(ii) for every $A, B \in P(X)$, $A \succsim^* B$ implies $A \cup B \succsim^* B$.

Note that each functional is characterized by a different weakening of the betweenness axiom that imposes a different structure on the taste for commitment. If there is no taste for commitment—that is, if $A \subset B$ implies $B \succsim^* A$ for every $A, B \in P(X)$—it can be shown that $J = 0$ in the finite additive EU representation of $\succsim^*$.

A.5 BIBLIOGRAPHIC NOTES

The characterization of the multi-selves model is a simplified version of an axiomatization given by Gul & Pesendorfer (2001). The model of self-control preferences and its axiomatization are also due to Gul & Pesendorfer (2001). The result that the two models are formally linked is due to Dekel & Lipman (2010). As mentioned above, Chatterjee & Krishna (2009) axiomatize the two-state version of the multi-selves model with uncertain second-period preferences. Dekel, Lipman & Rustichini (2009) characterize the other classes of temptation-driven preferences, basing it on a previous characterization of additive expected-utility representations of preferences over menus (Dekel, Lipman & Rustichini (2001), Dekel, Lipman, Rustichini & Sarver (2007)). Gul & Pesendorfer (2004) extend the model of self-control preferences to infinite-horizon consumption problems. Gul & Pesendorfer (2005) extend the axiomatic characterization of the multi-selves model to arbitrary finite-horizon decision problems.

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