Exam MSc Macroeconomics, EconG022
2012

Instructions. This exam consists of six pages with four questions in total. You have THREE HOURS to answer all four questions. There are 100 points to be earned. Please write clearly and hand in your answers ordered numerically.

Question 1 (25 points)
Consider the statements below. Do you agree or disagree? Explain your answers.

a. In the Solow model, the Golden Rule rate of savings maximizes consumption at all times.

b. If the Blanchard-Kahn conditions are satisfied in a linear(ized) dynamic rational expectations model, then sunspot equilibria do not occur.

c. The labour wedge is a measure of distortions in the labour market, as it measures the ratio of the marginal product of labour to the intertemporal marginal rate of substitution.

d. In an overlapping generations model with two generations, a strictly positive transfer from the old generation to the young generation cannot constitute a Pareto improvement.

e. In models with money in the utility function, the growth rate of the money supply is irrelevant for welfare when money is superneutral.

Question 2 (30 points)

Consider an economy with a continuum of identical households, of measure one in total. Discounted lifetime utility at time zero, U_0, is given by:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}, \ \sigma > 1$$

where c_t denotes the amount of consumption in period t and $\beta \in (0,1)$ is the discount factor. Households inelastically supply one unit of labour and they own the stock of capital, which they rent out to firms. The stock of capital, k_t, evolves according to

$$k_{t+1} = (1-\delta)k_t + i_t$$

where i_t denotes the amount of investment as chosen by the household in period t and $\delta \in (0,1)$ is the rate of capital depreciation. Households own the firms and receive their profits. The budget constraint of a household is given by:

$$c_t + i_t = r_t k_t + w_t + \pi_t$$

where r_t denotes the rental rate of capital, w_t is the wage per unit of labour and π_t denotes total firm profits. In each period t, households optimally chose c_t, i_t and k_{t+1}. An initial capital stock k_0 is taken as given.

a. Set up the maximization problem of the households and derive the first-order conditions for c_t and k_{t+1}. Provide an interpretation for the first-order condition for k_{t+1}.

There is a large number of identical and competitive firms, who rent capital and labour and maximize profits. Firms are indexed by j and of measure one in total. Firm j produces $y_{j,t}$ goods and operates on the following production function:

$$y_{j,t} = k_{j,t}^{\alpha} k_{agg,t}^{\gamma} l_{j,t}^{1-\alpha-\gamma}, \tag{1}$$

where $\alpha \in (0,1)$, $\gamma \in (0,1)$ and $\alpha + \gamma \in (0,1)$. In the above equation, $k_{j,t}$ is the stock of capital rented by firm j, $k_{agg,t}$ is the *aggregate* stock of capital and $l_{j,t}$ is the amount of labour rented by firm j. The reason why the aggregate stock of capital enters the production function is that capital accumulation generates public knowledge, available to all firms.

b. Do the competitive equilibrium allocations in this model coincide with the allocations that would be chosen by a benevolent social planner? Explain your answer.

c. Set up the profit-maximization problem of the firms. Derive expressions for the wage and the rental rate of capital. Do firms make profits in this model? Derive an equation for profits to support your answer.

d. Write down the market clearing conditions and define a competitive equilibrium, referring explicitly to the model's equations.

e. Derive an expression for the steady-state level of capital that would prevail in a competitive equilibrium.

Consider an alternative economy with the following production function:

$$y_{j,t} = k_{j,t}^{\alpha+\gamma} l_{j,t}^{1-\alpha-\gamma}. \tag{2}$$

f. Derive an expression for the steady-state level of capital for this economy. Consider the competitive equilibria outcomes under production functions (1) and (2). Which of the two production function results in the highest steady-state level of capital?

Question 3 (25 points)

Consider an overlapping generations economy. During each period, two generations exist, both of unit measure. Young agents inelastically supply one unit of labour in exchange for a wage w_t. They also invest in capital, which they rent out in the second period of their lives at a rate r_{t+1}. Old agents do not supply labour. Young agents solve the following optimization problem:

$$\begin{aligned}
&\max_{c_{y,t},c_{o,t+1},k_{t+1}} \ln c_{y,t} + \beta \ln c_{o,t+1} \\
&s.t. \\
&c_{y,t} + k_{t+1} = w_t \\
&c_{o,t+1} = (1 - \delta + r_{t+1})\, k_{t+1}
\end{aligned}$$

where $c_{y,t}$ and $c_{o,t}$ are consumption of the young and the old agents in period t, respectively, k_t is the capital stock in period t, $\beta \in (0,1)$ is the discount factor and $\delta \in (0,1)$ is the depreciation rate of capital. There is a representative and competitive firm that rents capital and labour, the latter being denoted by l_t. The firm operates on a production function $y_t = k_t^{\alpha} l_t^{1-\alpha}$ and maximizes profits, given by $\pi_t = y_t - w_t l_t - r_t k_t$. Given that aggregate labour supply equals one, goods market clearing requires that

$$c_{y,t} + c_{o,t} + k_{t+1} = k_t^{\alpha} + (1 - \delta)\, k_t.$$

a. Derive the first-order conditions for the optimization problems of the young agents and the firms.

b. Use your results from question 3a., as well as the budget constraints, to derive an expression for the capital stock in the steady state as a function of the model's parameters. Also, show that the rental rate of capital in the steady state is given by $r = \frac{\alpha}{1-\alpha}\frac{1+\beta}{\beta}$.

c. Suppose that $\alpha = 0.2$, $\delta = 0.6$, and $\beta = 1$. Do we know if the steady state of the economy Pareto efficient? Relate your answer to the Golden Rule.

Suppose the government sets up a fully funded pension system. That is, the government collects a lump-sum amount $\widetilde{x}_t$ from any young agent (not exceeding w_t) in period t. The government invests what it collects in capital, and transfers the gross revenues back to the old agents in period $t + 1$ (the generation that is young in period t).

d. Adjust the young agents' optimization problem to allow for this pension system. Show that the equilibrium outcomes of the economy with and without a fully funded pension system are the same.

e. Could a pay-as-you-go pension scheme constitute a Pareto improvement over the economy without pensions? You don't need to derive anything, but make explicit what the intuition is.

Question 4 (20 points)
This question consists of two separate parts. Answer both.

Part one
Consider the following model:

$$\begin{aligned} x_t^2 &= \gamma x_{t-1} \\ x_t &> 0 \end{aligned} \qquad \text{for } t = 1, 2, 3...$$

with $\gamma > 0$ and $x_0 > 0$ given.

a. Compute the permissible steady state of this model and log-linearize the model around this steady state.[1]

b. Derive an expression for $\widehat{x}_t$ as a function of $\widehat{x}_0$, where hatted variables denote log-deviations from the steady state. Does the log-linearized model have a unique stable solution?

Part two
Consider a Real Business Cycle model in which households have the following utility function:

$$U(c_t, h_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \mu (T - h_t)^{\alpha}$$

with $\alpha \in [0, 1]$ and $\mu, \sigma > 0$. Here, c_t denotes consumption, h_t denotes hours worked and $T > 0$ is the time endowment of the households.

c. Without deriving anything, provide a key restriction on one of the parameters that could make the model observationally equivalent to a model of indivisible labour. Explain your answer.

d. Derive the Frisch elasticity of labour supply in this model.

[1] When log-linearizing the model, you can ignore the constraint that $x_t > 0$ for all t. It can be verified that as long as $x_0 > 0$, x_t remains strictly positive.

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Question 1: answer

a. Not true. The Golden Rule rate of savings only maximizes long-run (steady-state) consumption. A savings rate that is lower than the Golden Rule rate will lead to higher consumption in the short-run.

- 5 points for the correct answer and explanation
- 3 points if the correct definition of the Golden Rule is given, but if an incorrect explanation is given.
- 1 point for the correct answer but no correct explanation
- 0 points otherwise

b. True. (i) When the Blanchard-Kahn conditions are satisfied there is a unique stable equilibrium. (ii) For sunspot equilibria to occur, multiple stable equilibria need to exist.

- 5 points for the correct answer and explanation
- 3 points if only (i) or only (ii) is correctly mentioned
- 1 point for the correct answer but no incorrect explanation
- 0 points otherwise

c. Not true. (i) The labour wedge is defined as the ratio of the marginal product of labour to the intratemporal marginal rate of substitution *between leisure and consumption*, not the intertemperal marginal rate of substitution. (ii) It is true that the labour wedge is a measure of distortions in the labour market. In an economy without frictions, the labour wedge would be one. When the labour wedge rises above one, aggregate labour supply is lower than in a world without frictions. (iii) An increase in the labour wedge is observationally equivalent to an increase in a (distortionary) labour income tax.

- 5 points for the correct answer
- 2 points if the answer is incorrect, but if (ii) or (iii) is correctly mentioned.
- 0 points otherwise

d. True. (i) A necessary condition for transfer scheme to constitute a Pareto improvement is that all agents are at least as well off as without a transfer. (ii) A transfer from old agents to young agents would always make the initially old generation strictly worse off, as they will no longer exist in subsequent periods and thus cannot be compensated in a later period.

- 5 points for the correct answer and explanation
- 3 points if only (i) or (ii) is correctly mentioned
- 1 point for the correct answer but no correct explanation

e. Not true. (i) superneutrality of money means that in the long run, real activity variables like consumption, output and capital do not depend on the growth rate of money. (ii) However, even under superneutrality the steady-state level of real money balances does depend on the growth rate of money. (iii) Since real money balances enter the utility function, the growth rate of money is relevant for welfare.

- 5 points for the correct answer and explanation
- 4 points when (i) and (ii) are correctly mentioned
- 3 points when only (i) is correctly mentioned
- 2 points when it mentioned that superneutrality means that money growth is irrelevant for steady-state outcomes, without making an exception for real money balances.

- 1 point for the correct answer but no correct explanation

Question 2: answer

a. (5 points) First note that we can substitute out i_t.Let λ_t be the Lagrange multiplier on the budget constraint. The household's first-order conditions are:

$$\begin{aligned} c_t^{-\sigma} &= \lambda_t \\ \lambda_t &= \beta\lambda_{t+1}\left(1-\delta+r_{t+1}\right) \end{aligned}$$

Left-hand side Euler equation: utility cost of saving a marginal unit of resources. Right-hand side Euler equation: the discounted utility benefit of saving a marginal unit. The term between bracket is the benefit expressed in units of goods and is equal to the undepreciated part of the capital purchased plus the income from renting out the capital.

b. (5 points) The decentralized solution and the social planner solution do not coincide. The reason is that there is an externality since the aggregate amount of capital enters the production function. The intuition is that when aggregate capital matters for output, the rental rate of capital does not reflect the full social marginal product of capital. Hence, households who purchase capital do not fully internalize the benefits. One can thus expect capital accumulation to be lower in the decentralized world than what a social planner would choose.

c. (5 points) The firm problem:

$$\max_{k_{j,t},l_{j,t}} \pi_{j,t} = k_{j,t}^{\alpha}k_{agg,t}^{\gamma}l_{j,t}^{1-\alpha-\gamma} - r_t k_{j,t} - w_t l_{j,t}$$

First-order conditions:

$$\begin{aligned} r_t &= \alpha k_{j,t}^{\alpha-1}k_{agg,t}^{\gamma}l_{j,t}^{1-\alpha-\gamma} \\ w_t &= \left(1-\alpha-\gamma\right)k_{j,t}^{\alpha}k_{agg,t}^{\gamma}l_{j,t}^{-\alpha-\gamma} \end{aligned}$$

We can now evaluate firm profits, which turn out to be zero:

$$\pi_{j,t} = k_{j,t}^{\alpha}k_{agg,t}^{\gamma}l_{j,t}^{1-\alpha-\gamma} - \alpha k_{j,t}^{\alpha}k_{agg,t}^{\gamma}l_{j,t}^{1-\alpha-\gamma} - \left(1-\alpha-\gamma\right)k_{j,t}^{\alpha}k_{agg,t}^{\gamma}l_{j,t}^{1-\alpha-\gamma}$$

By symmetry $k_{j,t} = k_{agg,t}$ so we get

$$\pi_{j,t} = \gamma k_{j,t}^{\alpha+\gamma}l_{j,t}^{1-\alpha-\gamma}$$

Hence, firms do make profits in this model.

d. (5 points) Labor market clearing and symmetry implies that

$$l_t = 1$$

capital market clearing and symmetry implies that:

$$k_{agg,t} = k_{j,t} = k_t$$

By substituting out l_t and $k_{agg,t}$, clearing in these two markets is imposed. Goods market clearing implies that:

$$c_t + i_t = k_t^{\alpha+\gamma}$$

A competitive equilibrium is a sequence of allocations $\{c_t, i, k_t\}_{t=0}^{\infty}$ and prices $\{r_t, w_t\}_{t=0}^{\infty}$ such that in each period the following conditions are satisfied: (i) the household's Euler equation, (ii) the household's capital accumulation equation, (iii) the firm's first-order condition for capital, (iv) the firm's first-order condition for labour, and (v) the goods market clears. Because of Walras' Law, the household's budget constraint is then also satisfied.

e) (5 points) From the Euler equation and the firm's optimality condition we get, imposing labour and capital market clearing as well as symmetry:

$$1 = \beta \left(1 - \delta + \alpha k^{\gamma+\alpha-1}\right)$$

We get the following solution for steady-state capital:

$$\begin{aligned} k &= \left(\frac{1}{\alpha\beta} - \frac{1-\delta}{\alpha}\right)^{\frac{1}{\gamma+\alpha-1}} \\ &= \alpha^{\frac{1}{1-\gamma-\alpha}} \left(\frac{1}{\beta} - (1-\delta)\right)^{\frac{1}{\gamma+\alpha-1}} \end{aligned}$$

In the Ramsey model we would get:

$$\begin{aligned} k^R &= \left(\frac{1}{(\alpha+\gamma)\beta} - \frac{1-\delta}{(\alpha+\gamma)}\right)^{\frac{1}{\gamma+\alpha-1}} \\ &= (\alpha+\gamma)^{\frac{1}{1-\gamma-\alpha}} \left(\frac{1}{\beta} - (1-\delta)\right)^{\frac{1}{\gamma+\alpha-1}} \end{aligned}$$

Since $(\alpha+\gamma)^{\frac{1}{1-\gamma-\alpha}} > \alpha^{\frac{1}{1-\gamma-\alpha}}$ we have $k^R > k$. Hence, there is indeed underaccumulation of capital when aggregate capital enters the production function.

Question 3: answer

a. (5 points) Let $\lambda_{1,t}$ and $\lambda_{2,t}$ be the Lagrange multipliers on the first and second budget constraint, respectively. The first-order conditions are:

$$\begin{aligned} \frac{1}{c_{y,t}} &= \lambda_{1,t} \\ \frac{1}{c_{o,t+1}} &= \lambda_{2,t} \\ \lambda_{1,t} &= \beta\left(1-\delta+r_{t+1}\right)\lambda_{2,t} \end{aligned}$$

Substituting out the Lagrange multipliers gives:

$$\frac{1}{c_{y,t}} = \beta \frac{1-\delta+r_{t+1}}{c_{o,t+1}}$$

The firm profit maximization problem is:

$$\max_{k_t, l_t} k_t^{\alpha} l_t - r_t k_t - w_t l_t$$

and it follows that

$$\begin{aligned} r_t &= \alpha k_t^{\alpha-1} l_t^{1-\alpha} \\ w_t &= (1-\alpha) k_t^{\alpha} l_t^{-\alpha} \end{aligned}$$

and note that $l_t = 1$.

b. (5 points) Using the budget constraints we can express the steady-state Euler equation as:

$$\frac{1}{(1-\alpha)k^{\alpha} - k} = \beta \frac{1-\delta+r}{(1-\delta+r)k}$$

from which it follows that

$$k = \left(\frac{1+\beta}{\beta(1-\alpha)}\right)^{\frac{1}{\alpha-1}}$$

and

$$r = \frac{\alpha}{1-\alpha}\frac{1+\beta}{\beta}$$

c. (5 points) We get $r = \frac{1}{2}$. The Golden rule prescribes $r = \delta = 0.6 > \frac{1}{2}$. Given that labor supply is fixed, the rental rate of capital is decreasing in the amount of capital. Hence, the level of capital in the steady state of the OLG model is higher than under the Golden Rule. It follows that by lowering the aggregate savings rate, consumption can increase both in the current periods and in future periods. Hence, all generations can benefit from a decline in the aggregate savings rate. Thus, the steady-state of the OLG economy is Pareto inefficient.

Question 4: answer

a. (5 points) In the steady state it must hold that

$$x^2 = \gamma x.$$

Thus, $x = 0$ and $x = \gamma$ are steady-state values, but $x = 0$ is ruled out by assumption. Log-linearization gives

$$\widehat{x}_t = \frac{1}{2}\widehat{x}_{t-1}.$$

b. (5 points) Iterating on the above equation, we get

$$\widehat{x}_t = \left(\frac{1}{2}\right)^t \widehat{x}_0.$$

Because $\widehat{x}_0$ is given, the solution to the model is unique. Moreover, the solution is stable since $\lim_{t\to\infty} \widehat{x}_t = 0$, implying that the model converges to the steady state. This also follows immediately from the fact that the system can be written as $\widehat{x}_t = A\widehat{x}_{t-1}$ with $A = \frac{1}{2}$. Since the eigenvalue of A is simply equal to $\frac{1}{2} < 1$ and we have one predetermined variable, the BK-conditions are satisfied. Hence, there is a unique stable solution.

c. (5 points) Key restriction: $\alpha = 1$. As shown during the lectures, a model of indivisible labour is observationally equivalent to a model with a representative agent whose utility is linear in hours worked. By setting $\alpha = 1$ the utility function in the model under consideration becomes linear in hours worked.

d. (5 points) The Frisch elasticity is given by:

$$\begin{aligned}
\frac{U_h}{U_{hh}h} &= \frac{-\alpha\mu\left(T-h_t\right)^{\alpha-1}}{\alpha\left(\alpha-1\right)\mu\left(T-h_t\right)^{\alpha-2}h_t} \\
&= \frac{-\left(T-h_t\right)}{\left(\alpha-1\right)h_t} \\
&= \frac{1}{\left(1-\alpha\right)}\frac{\left(T-h_t\right)}{h_t}
\end{aligned}$$

and note that by setting $\alpha = 1$ the Frisch elasticity becomes infinite.

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Question 1 (40 points)

Consider **four** out of the six statements below. Do you agree or disagree? Explain your answers.

(**a**) In the Ramsey growth model, the Golden Rule rate of savings typically does not maximize steady-state social welfare.

(**b**) If a linearized dynamic model features sunspot equilibria, then agents do not have rational expectations.

(**c**) In an overlapping generations model with capital, the existence of a pay-as-you-go pension system may increase social welfare.

(**d**) The Kaldor growth facts imply that real wages remain roughly constant over time.

(**e**) In the Diamond-Mortensen-Pissarides model, equilibrium wages are determined by the marginal product of labor.

(**f**) The Friedman rule prescribes that the nominal interest rate should be equal to the rate of inflation.

Question 2 (30 points)

Consider an economy with a continuum of identical households, of measure one in total. Discounted lifetime utility at time zero, U_0, is given by:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left(\ln c_t + \eta \ln \left(T - h_t \right) \right)$$

where c_t denotes consumption in period t, h_t denotes hours worked, $\beta \in (0,1)$ is the households' discount factor and $\eta > 0$ is a parameter capturing the utility of leisure, and T is a time endowment. Households own the capital stock, which they rent out to firms at a rate $r_{k,t}$. They also supply labor to the firms, at a rate w_t per hour. The households' budget constraint for period t is given by:

$$c_t + k_{t+1} = \left(1 - \delta + r_{k,t}\right) k_t + w_t h_t$$

where $\delta \in (0,1)$ is the rate of capital depreciation. In each period t, households optimally choose c_t, k_{t+1} and h_t, taking as given the wage, the rental rate of capital and an initial capital stock k_0, and respecting the constraint that $k_t \geq 0$.

(**a**) Set up the maximization problem of the households and derive the first-order necessary conditions for c_t, k_{t+1} and h_t.

Let $l_t \equiv T - h_t$ denote the amount of leisure enjoyed by a household, where T is an exogenous time endowment.

(**b**) Using the first-order conditions, show that the marginal rate of substitution between leisure and consumption, defined as $MRS_t \equiv \frac{\partial U_0}{\partial l_t} / \frac{\partial U_0}{\partial c_t}$, is equal to the real wage.

(**c**) Through what two channels does a change in the real wage affect the household's consumption choice? Refer to the appropriate model equations.

There is a large number of identical firms, of measure one in total. Firms choose capital and labor inputs to maximize profits, operating on the following production function:

$$y_t = k_t^{\alpha} h_t^{1-\alpha}, \ \alpha \in (0,1)$$

Firm profits are given by:

$$\Pi_t = y_t - r_{k,t} k_t - (1 + \tau) w_t h_t$$

where $\tau \in (0,1)$ is a *tax rate* on wage expenditures. The tax revenues are used for government purchases, denoted g_t. The government runs a balanced budget, which implies that $g_t = \tau w_t h_t$.

(**d**) Set up the firms' profit maximization problem and derive the first-order conditions.

(**e**) Show that the firms make zero profits.

(**f**) Derive an expression for the steady-state capital-labor ratio, $\frac{k}{l}$, in terms of the parameters of the model. Also, show that $\frac{k}{l}$ is not affected by the labor tax. Hint: use the household's Euler Equation as well as the firm's first-order condition for capital.

Question 3 (15 points)

Consider an economy with a continuum of identical households of measure one. Households enjoy utility from consumption and real money balances. They solve the following optimization problem:

$$\max_{c_t, M_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \frac{x_t^{1-\sigma}}{1-\sigma}, \ \beta \in (0,1), \sigma > 1$$
$$s.t$$
$$x_t = c_t^{\alpha} \left(M_t/P_t\right)^{1-\alpha}, \ \alpha \in (0,1)$$
$$P_t c_t + M_t + P_t k_{t+1} = M_{t-1} + P_t \left(1-\delta\right) k_t + P_t k_t^{\alpha} + T_t$$

Here, c_t denotes consumption, k_t is the capital stock, M_t is the nominal stock of money, P_t is the price level, x_t is a Cobb-Doublas basket of consumption and real money balances, and T_t is a government transfer. Money grows at an exogenous net rate θ. Seigniorage revenues are transferred directly back to the households, i.e. $T = M_t - M_{t-1}$. All markets clear. Note that the budget constraint can be expressed in real terms as

$$c_t + m_t + k_{t+1} = \frac{1}{1+\pi_t} m_{t-1} + \left(1-\delta\right) k_t + k_t^{\alpha} + \tau_t$$

where $m_t \equiv \frac{M_t}{P_t}$ is real money balances, $\pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}}$ is the net rate of inflation and $\tau_t \equiv \frac{T_t}{P_t}$ is the seigniorage transfer expressed in real terms.

(**a**) Derive the first-order conditions for c_t, m_t and k_{t+1} and define a competitive equilibrium, refering explicitly to the appropriate conditions.

(**b**) Show that in a steady state with constant real money balances and a constant inflation rate, the net rate of money growth equals the rate of inflation, i.e. $\pi = \theta$.

(**c**) Show that (i) money is superneutral in the steady state of this economy and that (ii) steady-state social welfare is decreasing in the rate of money growth θ.

Question 4 (15 points)

Consider an infinitely-lived worker who maximizes expected lifetime utility, given by:

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \ \ \beta \in (0,1)$$

where $u(c_t) = c_t$ is the utility function, c_t denotes consumption and $\mathbb{E}_t$ is the conditional expectations operator. There are no savings opportunities so the worker simply consumes her income in any period. At time zero, the worker has a job paying a wage w_1. The job is safe, in the sense that the worker cannot be fired from it. At the beginning of each period, the worker receives an offer for a *new* job, which may pay a better wage. However, the new job does not offer firing protection and is lost with a time-invariant probability $\rho_x \in (0,1)$ in any given period. If the worker is fired, she can never regain employment and forever receives a time-invariant unemployment benefit $b \in (0, w_1)$. Job offers come with associated which offers, denoted w_2, which are independently drawn from a distribution with support $[0, w_2^{\max}]$, where $w_2^{\max} > w_1$. Once the second job is accepted no further offers are obtained. For both the initial job and the second job it is the case that the wage remains constant over the duration of the job.

Let $V^{e1}(w_1)$ denote the value of an agent in its initial job, and $V^{e2}(w_2)$ the value of the agent once it has accepted a second job. Finally, let V^u denote the value of the agent once it has reached the unemployment state.

(**a**) Show that $V^u = \frac{b}{1-\beta}$.

(**b**) Write down the Bellman equations for $V^{e1}(w_1)$ and $V^{e2}(w_2)$, under the assumption that there exists a threshold $\overline{w}_2$ such that the agent accepts the second job if $w_2 \geq \overline{w}_2$.

If the agent's initial wage is sufficiently high, the agent will never accept a new job. Let $\widetilde{w}_1$ denote a threshold on the agent's initial wage, such that the agent for sure will never accept a second job if $w_1 > \widetilde{w}_1$.

(**c**) Find an expression for $\widetilde{w}_1$ in terms of β, ρ_x, b and $w_2^{\max}$ and note that $\frac{\partial \widetilde{w}_1}{\partial b} > 0$. Explain intuitively why $\widetilde{w}_1$ is increasing in the unemployment benefit.

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Question 1 (40 points)

Consider **four** out of the six statements below. Do you agree or disagree? Explain your answers.

(**a**) In the Ramsey growth model, the Golden Rule rate of savings typically does not maximize steady-state social welfare.

(**b**) If a linearized dynamic model features sunspot equilibria, then agents do not have rational expectations.

(**c**) In an overlapping generations model with capital, the existence of a pay-as-you-go pension system may increase social welfare.

(**d**) The Kaldor growth facts imply that real wages remain roughly constant over time.

(**e**) In the Diamond-Mortensen-Pissarides model, equilibrium wages are determined by the marginal product of labor.

(**f**) The Friedman rule prescribes that the nominal interest rate should be equal to the rate of inflation.

Solution Question 1 (40 points)

(**a**) brief answer: TRUE. The socially optimal rate of savings is given by the *modified* golden rule, which generally does not coincide with the golden rule because agents' discounting.

(**b**)brief answer: FALSE: sunspot equilibria can arise in a linearized rational expectations model, when the equilibrium is not unique.

(**c**) brief answer TRUE: Without a pension system, an inefficiently large degree of capital accumulation may arise because of agents' desire to save for retirement, combined with the violation of the welfare theorems that is inherent to the OLG model. A pay-as-you-go pension system reduces the saving motive, and may increase social welfare by reducing aggregate capital accumulation.

(**d**)brief answer: FALSE: This is not one of the Kaldor facts. The Kaldor facts state that there is no trend growth in the labor share of income $\frac{wL}{Y}$ and that output per worker, $\frac{Y}{L}$ grows at a constant rate. As a direct implication, the real wage grows at a constant rate.

(**e**)brief answer: FALSE: In the DMP model, workers and firms are in a bilateral monopoly position, and any wage that is in the bargaining set is consistent with equilibrium. Hence, equilibrium wages are generally not equal to the marginal product of labor in the DMP model. Often, the model is closed by assuming that wages are determined by Nash bargaining.

(**f**)brief answer: FALSE: the Friedman rule prescribes a zero nominal interest rate, as this level would equate the social cost of printing money (zero) to the opportunity cost of holding money (the nominal interest rate)

Question 2 (30 points)

Consider an economy with a continuum of identical households, of measure one in total. Discounted lifetime utility at time zero, U_0, is given by:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left(\ln c_t + \eta \ln \left(T - h_t\right)\right)$$

where c_t denotes consumption in period t, h_t denotes hours worked, $\beta \in (0,1)$ is the households' discount factor and $\eta > 0$ is a parameter capturing the utility of leisure, and T is a time endowment. Households own the capital stock, which they rent out to firms at a rate $r_{k,t}$. They also supply labor to the firms, at a rate w_t per hour. The households' budget constraint for period t is given by:

$$c_t + k_{t+1} = \left(1 - \delta + r_{k,t}\right) k_t + w_t h_t$$

where $\delta \in (0,1)$ is the rate of capital depreciation. In each period t, households optimally choose c_t, k_{t+1} and h_t, taking as given the wage, the rental rate of capital and an initial capital stock k_0, and respecting the constraint that $k_t \geq 0$.

(**a**) Set up the maximization problem of the households and derive the first-order necessary conditions for c_t, k_{t+1} and h_t.

Let $l_t \equiv T - h_t$ denote the amount of leisure enjoyed by a household.

(**b**) Using the first-order conditions, show that the marginal rate of substitution between leisure and consumption, defined as $MRS_t \equiv \frac{\partial U_0}{\partial l_t} / \frac{\partial U_0}{\partial c_t}$, is equal to the real wage.

(**c**) Through what two channels does a change in the real wage affect the household's consumption choice? Refer to the appropriate model equations.

There is a large number of identical firms, of measure one in total. Firms choose capital and labor inputs to maximize profits, operating on the following production function:

$$y_t = k_t^{\alpha} h_t^{1-\alpha}, \;\; \alpha \in (0,1)$$

Firm profits are given by:

$$\Pi_t = y_t - r_{k,t} k_t - (1 + \tau)\, w_t h_t$$

where $\tau \in (0,1)$ is a *tax rate* on wage expenditures. The tax revenues are used for government purchases, denoted g_t. The government runs a balanced budget, which implies that $g_t = \tau w_t h_t$.

(**d**) Set up the firms' profit maximization problem and show that the first-order conditions for the choice of capital and labour can be written, respectively, as

$$\begin{aligned} r_{k,t} &= \alpha \frac{y_t}{k_t}, \\ (1 + \tau)\, w_t &= (1 - \alpha) \frac{y_t}{h_t}. \end{aligned}$$

(**e**) Show that the firms make zero profits.

It turns out that the aggregate steady-state capital-labor ratio, $\frac{k}{h}$, does not depend on the labour tax.

(**f**) Verify the above statement by deriving an expression for $\frac{k}{h}$ in terms of the parameters of the model, using the household's Euler Equation as well as the firm's first-order condition for capital.

Solution Question 2 (30 points)

(**a**) Using the Lagrange method, the optimization problem can be expressed as:

$$\mathcal{L} = \max_{c_t, k_{t+1}, h_t} \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \eta \ln (T - h_t) - \lambda_t \left(\left(1 - \delta + r_{k,t}\right) k_t + w_t h_t - c_t - k_{t+1} \right) \right\}$$

for $t = 0, 1, 2, ...$, where λ_t is the Lagrange multiplier on the budget constraint. The first-order conditions are:

$$\begin{aligned} \frac{1}{c_t} &= \lambda_t \\ \lambda_t &= \beta \lambda_{t+1} \left(1 - \delta + r_{k,t+1}\right) \\ \frac{\eta}{T - h_t} &= w_t \lambda_t = \frac{w_t}{c_t} \end{aligned}$$

Let $l_t \equiv T - h_t$ denote the amount of leisure enjoyed by the agent, where T is an exogenous time endowment.

(**b**) The marginal utility of consumption and leisure are, respectively, given by:

$$\begin{aligned} \frac{\partial U_0}{\partial c_t} &= \frac{1}{c_t} \\ \frac{\partial U_0}{\partial l_t} &= U_l = \frac{\eta}{l_t} = \frac{\eta}{T - h_t} \end{aligned}$$

- It follows that $\frac{\partial U_0}{\partial l_t} / \frac{\partial U_0}{\partial c_t} = \frac{c_t \eta}{T - h_t} = w_t$, where the second equality follows from the first-order condition for h_t.

(**c**) (i) A change in the wage affects consumption through the *substitution effect* which operates primarily through the first-order condition for labor, in which the wage enters. The second effect is the income effect which operates primarily through the budget constraints, in which the wage enters as well.

(**d**) The firm problem can be written as:

$$\max_{k_t, h_t} \Pi_t = k_t^{\alpha} h_t^{1-\alpha} - r_{k,t} k_t - (1 + \tau) w_t h_t$$

The first-order conditions for the choice of capital and labour are:

$$\begin{aligned} r_{k,t} &= \alpha k_t^{\alpha - 1} h_t^{1-\alpha} = \alpha \frac{y_t}{k_t} \\ (1 + \tau_t) w_t &= (1 - \alpha) k_t^{\alpha} h_t^{-\alpha} = (1 - \alpha) \frac{y_t}{h_t} \end{aligned}$$

(**e**) Profits can be written as:

$$\begin{aligned} \Pi_t &= y_t - r_{k,t} k_t - (1 + \tau) w_t h_t \\ &= y_t - \alpha y_t - (1 - \alpha) y_t \\ &= 0 \end{aligned}$$

where the second equality follows from the first-order conditions.

(**f**) The first-order condition for capital chosen by the firm, evaluated at the steady state is:

$$r_k = \alpha \left(\frac{k}{h}\right)^{\alpha-1},$$

which we can rewrite as:

$$\frac{k}{h} = \left(\frac{r_k}{\alpha}\right)^{\frac{1}{\alpha-1}}$$

The household's first order condition for capital, which in the steady state reduces to:

$$1 = \beta \left(1 - \delta + r_k\right)$$

which we can write as:

$$r_k = \frac{1}{\beta} - 1 + \delta$$

Combining the two expressions gives:

$$\frac{k}{h} = \left(\frac{\frac{1}{\beta} - 1 + \delta}{\alpha}\right)^{\frac{1}{\alpha-1}}$$

Given that this expression does not depend on τ, it follows directly that $\frac{k}{h}$ is not affected by the tax.

Question 3 (15 points)

Consider an economy with a continuum of identical households of measure one. Households enjoy utility from consumption and real money balances. They solve the following optimization problem:

$$\max_{c_t, M_t, k_{t+1}, x_t} \sum_{t=0}^{\infty} \beta^t \frac{x_t^{1-\sigma}}{1-\sigma}, \; \beta \in (0,1), \sigma > 1$$

$$s.t$$

$$x_t \equiv c_t^{\alpha} \left(M_t / P_t\right)^{1-\alpha}, \; \alpha \in (0,1)$$

$$P_t c_t + M_t + P_t k_{t+1} = M_{t-1} + P_t \left(1-\delta\right) k_t + P_t k_t^{\alpha} + T_t$$

Here, c_t denotes consumption, k_t is the capital stock, M_t is the nominal stock of money, P_t is the price level, x_t is a Cobb-Doublas basket of consumption and real money balances, and T_t is a government transfer. The nominal stock of money grows at an exogenous net rate θ, i.e. $M_t = (1+\theta) M_{t-1}$. Seigniorage revenues are transferred directly back to the households, i.e. $T = M_t - M_{t-1}$. All markets clear. Note that the budget constraint can be expressed in real terms as

$$c_t + m_t + k_{t+1} = \frac{1}{1+\pi_t} m_{t-1} + \left(1-\delta\right) k_t + k_t^{\alpha} + \tau_t$$

where $m_t \equiv \frac{M_t}{P_t}$ is real money balances, $\pi_t \equiv \frac{P_t - P_{t-1}}{P_t}$ is the net rate of inflation and $\tau_t \equiv \frac{T_t}{P_t}$ is the seigniorage transfer expressed in real terms.

(**a**) Use the definitions of m_t and π_t to show that in a steady-state equilibrium with constant real money balances and inflation, the net rate of money growth equals the rate of inflation, i.e. $\pi = \theta$.

(**b**) Derive the household's first-order condition for c_t and k_{t+1} and show that they can be combined to get:

$$\frac{x_t^{1-\sigma}}{c_t} = \beta \frac{x_{t+1}^{1-\sigma}}{c_{t+1}} \left(1 - \delta + \alpha k_{t+1}^{\alpha-1}\right).$$

(**c**) Use the above equation as well as the goods market clearing condition to argue that money is superneutral in a steady state with constant real money balances, capital and consumption.

Solution Question 3

(**a**) Using that $1+\pi_t = \frac{P_t}{P_{t-1}}$One can write:

$$m_t \equiv \frac{M_t}{P_t} = \frac{(1+\theta)\,M_{t-1}}{P_t}\frac{P_{t-1}}{P_{t-1}} = \left(\frac{1+\theta}{1+\pi_t}\right)m_{t-1}.$$

In a steady state with it therefore holds that

$$m = \left(\frac{1+\theta}{1+\pi}\right)m$$

and it follows directly that $\pi = \theta$.

(**b**) The first-order conditions for consumption and capital are:

$$\begin{aligned}
\lambda_t &= \left(c_t^{\alpha}m_t^{1-\alpha}\right)^{-\sigma}\alpha c_t^{\alpha-1}m_t^{1-\alpha} \\
&= \alpha x_t^{-\sigma}\frac{x_t}{c_t} \\
&= \alpha\frac{x_t^{1-\sigma}}{c_t} \\
\lambda_t &= \beta\lambda_{t+1}\left(1-\delta+\alpha k_t^{\alpha-1}\right)
\end{aligned}$$

Combining the two equations gives

$$\frac{x_t^{1-\sigma}}{c_t} = \beta\frac{x_{t+1}^{1-\sigma}}{c_{t+1}}\left(1-\delta+\alpha k_{t+1}^{\alpha-1}\right).$$

(**c**) The market clearing condition for goods implies that

$$c_t + k_{t+1} = (1-\delta)\,k_t + k_t^{\alpha}$$

which in the steady state reduces to

$$c + \delta k = k^{\alpha}$$

The combined first-order conditions, evaluated at in the steady state imply:

$$1 = \beta\left(1-\delta+\alpha k^{\alpha-1}\right)$$

These are two equations in two unknowns (c and k) which are unaffected by the rate of money growth and inflation.

Question 4 (15 points)

Consider an infinitely-lived worker who maximizes expected lifetime utility, given by:

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \ \beta \in (0,1)$$

where $u(c_t) = c_t$ is the utility function, c_t denotes consumption and $\mathbb{E}_t$ is the conditional expectations operator. There are no savings opportunities so the worker simply consumes her income in any period. At time zero, the worker has a job paying a wage w_1. The job is safe, in the sense that the worker cannot be fired from it. At the *end* of each period, the worker receives an offer for a *new* job, which may pay a better wage and which she has to accept or reject immediately. This new job, however, does not offer firing protection and is lost with a time-invariant probability $\rho_x \in (0,1)$ in any given period. If the worker is fired, she can never regain employment and forever receives a time-invariant unemployment benefit $b \in (0, w_1)$. Job offers come with associated which offers, denoted w_2, which are independently drawn from a distribution with CDF $F(w_2)$ with $w_2 \in [0, w_2^{\max}]$, where $w_2^{\max} > w_1$. Once the second job is accepted no further offers are obtained. For both the initial job and the second job it is the case that the wage remains constant over the duration of the job.

Let $V^{e1}(w_1)$ denote the value of an agent in its initial job at the *beginning of the period*, that is, before receiving the wage w_1 and before receiving the offer for a new job. Also, let $V^{e2}(w_2)$ be the beginning-of period value of the agent once she has accepted a second job paying wage w_2. Finally, let V^u denote the beginning-of-period value of the agent once the unemployment state has been reached.

(**a**) Show that $V^u = \frac{b}{1-\beta}$.

(**b**) Write down the Bellman equations for $V^{e1}(w_1)$ and $V^{e2}(w_2)$, under the assumption that there exists a threshold $\overline{w}_2$ such that the agent accepts the second job if $w_2 \geq \overline{w}_2$.

If the agent's initial wage is sufficiently high, the agent will never accept a new job. Let $\widetilde{w}_1$ denote a threshold on the agent's initial wage, such that the agent for sure will never accept a second job if $w_1 > \widetilde{w}_1$.

(**c**) Show that one can express $\widetilde{w}_1$ as

$$\widetilde{w}_1 = (1-\beta) \frac{w_2^{\max} + \beta \rho_x \frac{b}{1-\beta}}{1 - \beta(1-\rho_x)}$$

and note that $\frac{\partial \widetilde{w}_1}{\partial b} > 0$. Also, explain intuitively why $\widetilde{w}_1$ is increasing in the size of the unemployment benefit.

Solution question 4

(**a**) Once unemployed, the agent forever receives utility b. As a result the value of the agent once unemployed is

$$\begin{aligned} V^u &= \sum_{t=0}^{\infty} \beta^t b \\ &= \frac{b}{1-\beta} \end{aligned}$$

(**b**) The Bellman equations can be written as:

$$\begin{aligned} V^{e1}(w_1) &= w_1 + \beta \int_0^{w_2^{\max}} \max\left\{V^{e1}(w_1), V^{e2}(w_2)\right\} dF(w_2) \\ &= w_1 + \beta F(\overline{w}_2) V^{e1}(w_1) + \beta \int_{\overline{w}_2}^{w_2^{\max}} V^{e2}(w_2)\, dF(w_2) \\ V^{e2}(w_2) &= w_2 + \beta(1-\rho_x) V^{e2}(w_2) + \beta \rho_x V^u \\ &= \frac{w_2 + \beta \rho_x \frac{b}{1-\beta}}{1-\beta(1-\rho_x)} \end{aligned}$$

(**c**) If $w_1 > \widetilde{w}_1$ the agent rejects even the highest possible wage offer, preferring to keep her initial job forever. Doing so gives value $\frac{w_1}{1-\beta}$. It follows that at the threshold level $\widetilde{w}_1$, the agent is exactly indifferent between keeping the initial job forever and accepting the second job with wage $w_2^{\max}$, which implies $V^{e2}(w_2^{\max}) = \frac{\widetilde{w}_1}{1-\beta}$. Hence, the threshold is given by

$$\widetilde{w}_1 = (1-\beta) \frac{w_2^{\max} + \beta \rho_x \frac{b}{1-\beta}}{1-\beta(1-\rho_x)}$$

Given that $1-\beta(1-\rho_x) \in (0,1)$ and $(1-\beta)\beta\rho_x \in (0,1)$ it holds that $\frac{\partial \overline{w}_1}{\partial b} > 0$.

Exam MSc Macroeconomics, EconG022

2014

Instructions. This exam consists of five pages with four questions in total. You have THREE HOURS to answer all four questions. There are 100 points to be earned. Please write clearly and hand in your answers ordered numerically.

Question 1 (40 points)

Consider **four** out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) The Kaldor growth facts imply that the capital share of income remains roughly constant over time.

(**b**) In the Solow growth model without technological progress, per-capita output can be increased without limit by choosing a high enough aggregate savings rate.

(**c**) In the Real Business Cycle model, real wage growth is partly predictable.

(**d**) Robert Hall's version of Permanent Income Hypothesis implies that current consumption levels are helpful in predicting future consumption levels.

(**e**) In the overlapping generations model with two-period lives and exogenous endowments, parameter values determine whether there exists a monetary equilibrium.

(**f**) In the Diamond-Mortensen-Pissarides model, the generosity of unemployment benefits does not affect equilibrium unemployment because labor supply is fixed.

[1] In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 2 (30 points)

Consider an economy with a continuum of identical and infinitely-lived households, of measure one in total. Discounted lifetime utility at time zero, U_0, is given by:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma} - 1}{1-\sigma} - \eta \frac{h_t^{1+\kappa}}{1+\kappa} \right)$$

where c_t denotes consumption in period t, h_t denotes hours worked, $\beta \in (0,1)$ is the households' discount factor, $\sigma > 0$ is the coefficient of relative risk aversion and $\eta, \kappa > 0$ are preference parameters pertaining to the supply of labor. Households own the capital stock, which they rent out to firms at a rate $r_{k,t}$. They also supply labor to the firms, at a rate w_t per hour. The households' budget constraint for period t is given by:

$$c_t + k_{t+1} = (1 - \delta + r_{k,t})\, k_t + w_t h_t$$

where $\delta \in (0,1)$ is the rate of capital depreciation. In each period t, households optimally choose c_t, k_{t+1} and h_t, taking as given the wage, the rental rate of capital and an initial capital stock $k_0 > 0$. You may ignore the constraint that $k_t \geq 0$ in every period.

(**a**) Set up the maximization problem of the households and derive the first-order conditions for c_t, k_{t+1} and h_t.

Taking natural logs on both sides of the first-order condition for h_t gives:

$$\ln w_t + \ln \lambda_t = \ln \eta + \kappa \ln h_t$$

where λ_t is the Lagrange multiplier on the budget constraint for period t.

(**b**) Derive an expression for the Frisch elasticity of labor supply in terms of only one model parameter.[2]

(**c**) Discuss how the coefficient of risk aversion σ affects the labor supply in response to a change in wage income.

There is a large number of identical firms, of measure one in total. Firms choose capital and labor inputs to maximize profits, operating on the following production function:

$$y_t = A k_t^{\alpha} h_t^{1-\alpha}, \;\; \alpha \in (0,1)$$

where y_t denotes output, $A > 0$ is a parameter pinning down the level of Total Factor Productivity. Like households, firms take w_t and r_t as given.

(**d**) Set up the firms' optimization problem and derive the first-order conditions.

(**e**) Write down the aggregate resource constraint and define a competitive equilibrium, referring to the model's equations.

(**f**) Use the firm's first-order conditions to show that the steady-state labor share of income does not depend on the productivity level A.

[2] Remember that the Frisch elasticity is given by $\left.\frac{dh_t/h_t}{dw_t/w_t}\right|_{\lambda_t}$.

Question 3 (20 points)

Consider an economy with an infinitely-lived representative household facing a cash-in-advance constraint. In each period, the household chooses consumption, c_t, hours worked, h_t, and nominal money balances, M_t, solving the following optimization problem:

$$\max_{\{c_t, M_t, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \eta \frac{h_t^{1+\kappa}}{1+\kappa} \right), \ \eta, \kappa > 0, \ \beta \in (0,1)$$

$$\begin{aligned}
&s.t \\
&P_t c_t + M_t = M_{t-1} + W_t h_t + T_t \\
&P_t c_t \leq M_{t-1} + T_t \\
&M_0 \text{ given}
\end{aligned}$$

Here, P_t is the price level, T_t is a government transfer and W_t is the nominal wage. The second constraint is the cash-in-advance constraint.

The nominal stock of money grows at an exogenous net rate θ. Seigniorage revenues are transferred directly back to the households, i.e. $T_t = M_t - M_{t-1}$.

A representative firm maximizes profits and has a production technology $y_t = h_t$, where y_t denotes the quantity of goods produced. All markets clear and agents take prices and wages as given.

(**a**) Rewrite the household's optimization problem in *real* terms, i.e. such that the household's choice variables are c_t, h_t, and $m_t \equiv \frac{M_t}{P_t}$.

(**b**) Derive the household's first-order conditions for consumption, labor supply and real money balances.[3]

(**c**) Discuss how inflation distorts the labor supply decision in the steady state with constant consumption, hours worked and real money balances.

(**d**) What is the optimal steady-state rate of money growth? Support your answer with a derivation and provide intuition.

[3] When deriving the first-order conditions you may take as given that the CIA constraint always binds, i.e. $P_t c_t = M_{t-1} + T_t$.

Question 4 (10 points)

Consider the following extension of McCall's search model. A risk-neutral household maximizes the expected present value of income given by $E_0 \sum_{y=0}^{\infty} \beta^t y_t$, where $\beta \in (0,1)$ is the household's discount factor, y_t denotes income in period t, and E_t is the expectations operator conditional on information available in period t. Job offers are associated with a per-period wage w, which remains fixed over the duration of the job and which is randomly drawn from a distribution with CDF $F(W) = \Pr(w < W)$ and support $w \in [0, B]$. In each period of unemployment, the household receives an unemployment benefit $c \geq 0$. Job offers are made at the beginning of each period and the household has to decide immediately whether to accept or reject an offer.

Unlike in the standard version of McCall's model, the household also receives job offers when employed, although less frequently than when unemployed. In particular, an unemployed household receives a job offer in every period while an employed households receives a job offer with probability $\theta \in [0,1]$ in a given period. If a household accepts a new job while being employed, it loses the current job. Note that under $\theta = 0$ the model reduces to the standard version of McCall's model.

The value of an unemployed household with wage offer w at hand can be expressed as $V = \max\{V^e(w), V^u\}$, where $V^e(w)$ and V^u are, respectively, the values conditional upon accepting and rejecting the offer.

(**a**) Write down the Bellman equations for V^u and $V^e(w)$.

(**b**) Use the Bellman equations to show that when $\theta = 1$, the reservation wage of an unemployed agent is equal to the unemployment benefit. Also, explain intuitively why this would not be the case if $\theta < 1$.

Exam MSc Macroeconomics, EconG022

2014

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Question 1 (40 points)

Consider **four** out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) The Kaldor growth facts imply that the capital share of income remains roughly constant over time.

(**b**) In the Solow growth model without technological progress, per-capita output can be increased without limit by choosing a high enough aggregate savings rate.

(**c**) In the Real Business Cycle model, real wage growth is partly predictable.

(**d**) Robert Hall's version of Permanent Income Hypothesis implies that current consumption levels are helpful in predicting future consumption levels.

(**e**) In the overlapping generations model with two-period lives and exogenous endowments, parameter values determine whether there exists a monetary equilibrium.

(**f**) In the Diamond-Mortensen-Pissarides model, the generosity of unemployment benefits does not affect equilibrium unemployment because labor supply is fixed.

[1] If you choose more than four statements, your first four answers will be graded.

Question 1 - Solution

(**a**) *True.* One Kaldor fact is that $\frac{K}{Y}$ is constant in the long run and another is that r is constant. Together these imply that the capital share of income, $\frac{rK}{Y}$ is constant.

(**b**) *False.* Even under a 100% savings rate there is a well-defined finite level of capital and hence output. This can be seen more explicitly as follows. The steady-state capital accumulation equation is:

$$\delta k = sk^{\alpha}$$

where $\alpha \in (0, 1)$ which implies that steady-state output is given by

$$y = k^{\alpha} = \left(\frac{\delta}{s}\right)^{\frac{\alpha}{\alpha-1}}$$

Here, y is increasing in s, but given that $s \leq 1$ there is a well-defined maximum. Alternatively, one can make the same argument graphically.

(**c**) *True.* In equilibrium, prices and wages are a function of the state of the economy, consisting of capital and the level of TFP. There is predictability in TFP inherent to the assumed AR(1) stochastic process. Also, capital has a tendency to revert back to its steady-state level, creating a second source of predictability.
(**d**) *True.* Hall's PIH states that consumption follows a random walk (or a Martingale to be precise). Hence the only variable that is helpful in predicting future consumption levels is the current level of consumption.

(**e**) *True.* Whether a monetary equilibrium exists depends crucially on the intertemporal Marginal Rate of Substitution under autarchy. Endowment levels, treated as parameters, crucially pin down the MRS.

(**f**) *False.* Unemployment benefits affect the outside option of workers, which in turn influence the bargained wage. Higher benefits increase the worker's outside value, driving up wages. This in turn discourages the firm's surplus, discouraging vacancy posting and increasing unemployment.

Question 2 (30 points)

Consider an economy with a continuum of identical and infinitely-lived households, of measure one in total. Discounted lifetime utility at time zero, U_0, is given by:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma} - 1}{1-\sigma} - \eta \frac{h_t^{1+\kappa}}{1+\kappa} \right)$$

where c_t denotes consumption in period t, h_t denotes hours worked, $\beta \in (0,1)$ is the households' discount factor, $\sigma > 0$ is the coefficient of relative risk aversion and $\eta, \kappa > 0$ are preference parameters pertaining to the supply of labor. Households own the capital stock, which they rent out to firms at a rate $r_{k,t}$. They also supply labor to the firms, at a rate w_t per hour. The households' budget constraint for period t is given by:

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where $\delta \in (0,1)$ is the rate of capital depreciation. In each period t, households optimally choose c_t, k_{t+1} and h_t, taking as given the wage, the rental rate of capital and an initial capital stock $k_0 > 0$. You may ignore the constraint that $k_t \geq 0$ in every period.

(**a**) Set up the maximization problem of the households and derive the first-order conditions for c_t, k_{t+1} and h_t.

Taking natural logs on both sides of the first-order condition for h_t gives:

$$\ln w_t + \ln \lambda_t = \ln \eta + \kappa \ln h_t$$

where λ_t is the Lagrange multiplier on the budget constraint for period t.

(**b**) Derive an expression for the Frisch elasticity of labor supply in terms of only one model parameter.[2]

(**c**) Discuss how the coefficient of risk aversion σ affects the labor supply in response to a change in wage income.

There is a large number of identical firms, of measure one in total. Firms choose capital and labor inputs to maximize profits, operating on the following production function:

$$y_t = A k_t^{\alpha} h_t^{1-\alpha}, \ \alpha \in (0,1)$$

wherey_t denotes output $A > 0$ is a parameter pinning down the level of Total Factor Productivity. Like households, firms take w_t and r_t as given.

(**d**) Set up the firms' optimization problem and derive the first-order conditions.

(**e**) Write down the aggregate resource constraint and define a competitive equilibrium, referring to the model's equations.

(**f**) Use the firm's first-order conditions to show that the steady-state labor share does not depend on the productivity level A.

[2] Remember that the Frisch elasticity is given by $\left.\frac{dh_t/h_t}{dw_t/w_t}\right|_{\lambda_t}$.

Question 2 - solution

(**a**) FOCs for c_t, k_t and h_t:

$$\begin{aligned} c_t^{-\sigma} &= \lambda_t \\ c_t^{-\sigma} &= \beta\left(1-\delta+r_{k,t+1}\right)c_{t+1}^{-\sigma} \\ \lambda_t w_t &= \eta h_t^{\kappa} \end{aligned}$$

(**b**) Total differentiation, keeping λ_t fixed, gives:

$$\frac{dw_t}{w_t} = \kappa\frac{dh_t}{h_t}$$

from which it follows that:

$$\left.\frac{dh_t/h_t}{dw_t/w_t}\right|_{\lambda_t} = \frac{1}{\kappa}$$

(**c**) The *income* effect of an increase in wages affects labor supply through it's effect on the marginal utility of wealth λ_t. Note that

$$\ln\lambda_t = -\sigma\ln c_t.$$

In words, $-\sigma$ is the elasticity of the Lagrange multiplier with respect to the level of consumption. Thus, to the extent that a change in wage income affects consumption, the risk aversion coefficient σ determines the strength of the income effect on labor supply.

(**d**) Profit maximization:

$$\max_{k_t,h_t} Ak_t^{\alpha}h_t^{1-\alpha} - w_t h_t - r_t k_t$$

FOCs:

$$\begin{aligned} r_t &= \alpha A k_t^{\alpha-1}h_t^{1-\alpha} \\ w_t &= (1-\alpha)Ak_t^{\alpha}h_t^{-\alpha} \end{aligned}$$

(**e**) The aggregate resource constraint is:

$$c_t + k_{t+1} = Ak_t^{\alpha}h_t^{1-\alpha} + (1-\delta)k_t.$$

A competitive equilibrium is a sequence of prices $\{r_t, w_t\}_{t=0}^{\infty}$ and allocations $\{k_t, h_t, c_t\}_{t=0}^{\infty}$ such that the household's FOCs for k_t and h_t are satisfied, the firm's FOCs are satisfied and the aggregate resource constraint is satisfied (5 equations) at any period in time.

(**f**) From the firm's first-order condition we know that $w_t = (1-\alpha)\frac{y_t}{h_t}$. It follows immediately that

$$\frac{w_t h_t}{y_t} = 1-\alpha.$$

which does not depend on A.

Question 3 (20 points)

Consider an economy with an infinitely-lived representative household facing a cash-in-advance constraint. In each period, the household chooses consumption, c_t, hours worked, h_t, and nominal money balances, M_t, solving the following optimization problem:

$$
\begin{aligned}
&\max_{\{c_t, M_t, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \eta \frac{h_t^{1+\kappa}}{1+\kappa} \right), \quad \eta, \kappa > 0, \ \beta \in (0,1) \\
&s.t \\
&P_t c_t + M_t = M_{t-1} + W_t h_t + T_t \\
&P_t c_t \leq M_{t-1} + T_t \\
&M_0 \text{ given}
\end{aligned}
$$

Here, P_t is the price level, T_t is a government transfer and W_t is the nominal wage. The second constraint is the cash-in-advance constraint.

The nominal stock of money grows at an exogenous net rate θ. Seigniorage revenues are transferred directly back to the households, i.e. $T_t = M_t - M_{t-1}$.

A representative firm maximizes profits and has a production technology $y_t = h_t$, where y_t denotes the quantity of goods produced. All markets clear and agents take prices and wages as given.

(**a**) Rewrite the household's optimization problem in *real* terms, i.e. such that the household's choice variables are c_t, h_t, and $m_t \equiv \frac{M_t}{P_t}$.

(**b**) Derive the household's first-order conditions for consumption, labor supply and real money balances.[3]

(**c**) Discuss how inflation distorts the labor supply decision in the steady state with constant consumption, hours worked and real money balances.

(**d**) What is the optimal steady-state rate of money growth? Support your answer with a derivation and provide intuition.

[3] When deriving the first-order conditions you may take as given that the CIA constraint always binds, i.e. $P_t c_t = M_{t-1} + T_t$.

Question 3 - solution

(**a**) Maximization problem in real terms:

$$\max_{\{c_t,m_t,h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \eta \frac{h_t^{1+\kappa}}{1+\kappa} \right)$$
$$s.t$$
$$c_t + m_t = \frac{m_{t-1}}{1+\pi_t} + w_t h_t + \tau_t$$
$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t$$

where $\pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}}$ is the net rate of inflation.
(**b**) FOCs for c_t, h_t and m_t:

$$\begin{aligned} \lambda_t &= \frac{1}{c_t} - \mu_t \\ w_t \lambda_t &= \eta h_t^{\kappa} \\ \lambda_t &= \beta \frac{\lambda_{t+1} + \mu_{t+1}}{1+\pi_{t+1}} \end{aligned}$$

where λ_t is the Lagrange multiplier on the budget constraint and μ_t is the Lagrange multiplier on the cash-in-advance constraint.
(**c**) From the last FOC it follows that in the steady-state

$$\lambda = \beta \frac{\lambda + \mu}{1+\pi}$$

or

$$\mu = \lambda \frac{1 - \beta + \pi}{\beta}$$

Substituting this into the steady-state FOC for consumption gives:

$$\lambda = \frac{\beta}{c\,(1+\pi)}$$

The steady-state FOC for labor becomes:

$$w \frac{\beta}{c\,(1+\pi)} = \eta h^{\kappa}$$

The term $\frac{\beta}{1+\pi}$ introduces a wedge in the labor supply decision: inflation reduces the marginal utility of consumption, pushing down labor supply. Intuitively, in order to consume the agent needs to hold money balances, but pays an inflation tax on holding money which thus acts as a consumption tax, encouraging households to substitute towards leisure.

(**d**) First: in the steady-state $m_t = \frac{M_t}{P_t}$ is constant. It follows immediately that the net rate of inflation, given by π, is equal to the rate of money growth θ in the steady state. Consequenty, allocations of a version of the economy without CIA constraint can be replicated by setting $\pi = \theta = \beta - 1$. Indeed, this implies $\frac{\beta}{1+\pi} = 1$ and hence the first-order condition for labor becomes identical to the one in a version of the economy without CIA constraint: $\frac{w}{c} = \eta h^{\kappa}$. In the economy without CIA constraint there is no market imperfection and the decentralized outcome coincides with the social planner solution. Replicating the allocations of the economy without CIA constraint thus achieves the social optimum. Intuitively, eliminating the inflation tax on holding money eliminates the consumption-leisure distortion introduced by inflation.

Question 4 (10 points)

Consider the following extension of McCall's search model. A risk-neutral household maximizes the expected present value of income given by $E_0 \sum_{y=0}^{\infty} \beta^t y_t$, where $\beta \in (0,1)$ is the household's discount factor, y_t denotes income in period t, and E_t is the expectations operator conditional on information available in period t. Job offers are associated with a per-period wage w, which remains fixed over the duration of the job and which is randomly drawn from a distribution with CDF $F(W) = \Pr(w < W)$ and support $w \in [0, B]$. In each period of unemployment, the household receives an unemployment benefit $c \geq 0$. Job offers are made at the beginning of each period and the household has to decide immediately whether to accept or reject an offer.

Unlike in the standard version of McCall's model, the household also receives job offers when employed, although less frequently than when unemployed. In particular, an unemployed household receives a job offer in every period while an employed households receives a job offer with probability $\theta \in [0,1]$ in a given period. If a household accepts a new job while being employed, it loses the current job. Note that under $\theta = 0$ the model reduces to the standard version of McCall's model.

The value of an unemployed household with wage offer w at hand can be expressed as $V = \max\{V^e(w), V^u\}$, where $V^e(w)$ and V^u are, respectively, the values conditional upon accepting and rejecting the offer.

(**a**) Write down the Bellman equations for V^u and $V^e(w)$.

(**b**) Use the Bellman equations to show that when $\theta = 1$, the reservation wage of an unemployed agent is equal to the unemployment benefit. Also, explain intuitively why this would not be the case if $\theta < 1$.

(**a**) The Bellman equations are:

$$\begin{aligned} V^u &= c + \beta \int_0^B \max\{V^e(w'), V^u\}\, dF(w') \\ V^e(w) &= w + \beta\theta \int_0^B \max\{V^e(w'), V^e(w)\}\, dF(w') + \beta(1-\theta) V^e(w) \end{aligned}$$

(**b**) The Bellman equations can be written as:

$$\begin{aligned} V^u &= c + \beta \int_0^{\overline{w}} V^u dF(w') + \beta \int_{\overline{w}}^B V^e(w') dF(w') \\ V^e(w) &= w + \beta\theta \int_0^{\widetilde{w}(w)} V^e(w) dF(w') + \beta\theta \int_{\widetilde{w}(w)}^B V^e(w') dF(w') + \beta(1-\theta) V^e(w) \end{aligned}$$

where $\overline{w}$ is the reservation wage of an unemployed worker and $\widetilde{w}(w)$ is the reservation wage of an employed worker with current wage w. For an employed worker the reservation wage is equal to the current wage, i.e. $\widetilde{w}(w) = w$, because the reservation wage solves $V(w) = V(\widetilde{w})$. Using this and setting $\theta = 1$ the Bellman equations become:

$$\begin{aligned} V^u &= c + \beta \int_0^{\overline{w}} V^u dF(w') + \beta \int_{\overline{w}^u}^B V^e(w') dF(w') \\ V^e(w) &= w + \beta \int_0^{w} V^e(w) dF(w') + \beta \int_{w}^B V^e(w') dF(w') \end{aligned}$$

The reservation wage solves $V^u = V^e(\overline{w})$, i.e.

$$c + \beta \int_0^{\overline{w}} V^u dF(w') + \beta \int_{\overline{w}}^B V^e(w') dF(w') = \overline{w} + \beta \int_0^{\overline{w}} V^e(\overline{w}) dF(w') + \beta \int_{\overline{w}}^B V^e(w') dF(w')$$

or

$$c + \beta \int_0^{\overline{w}} V^u dF(w') = \overline{w} + \beta \int_0^{\overline{w}} V^e(\overline{w}) dF(w').$$

Given $V^u = V^e(\overline{w})$ it follows that $c = \overline{w}$. Intuitively, future job offers are independent of the acceptance decision when $\theta = 1$, so the agent will accept any wage higher than the unemployment benefit. If $\theta < 1$ accepting a job as an unemployed worker has a cost in the sense that it reduces the arrivial rate of new job offers, so not any wage above the unemployment rate will be accepted by the worker.

Exam MSc Macroeconomics, EconG022

2015

Instructions. This exam consists of five pages with four questions in total. You have THREE HOURS to answer all four questions. There are 100 points to be earned. Please write clearly and hand in your answers ordered numerically.

Question 1 (40 points)

Agree or disagree?
Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) The Kaldor facts imply that, in the long run, real wages grow at a constant rate.

(**b**) In the Ramsey growth model, a benevolent social planner will choose the capital stock to be constant and equal to the Modified Golden Rule level.

(**c**) The labour wedge is a measure of distortions in the labour market.

(**d**) The Friedman rule prescribes zero inflation.

(**e**) "Superneutrality" of money means that the real capital stock, real output, and real money balances do not depend on growth rate of money in the long run.

(**f**) Robert Hall's result that consumption follows a random walk breaks down when there are fluctuations in the real interest rate.

[1] In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 2 (10 points)

Money in the utility function.
Consider an infinite-horizon economy, populated by a representative household whose utility depends on non-durable consumption and holdings of real money balances. The household can also purchase one-period nominal bonds. There is no uncertainty. The household solves the following decision problem:

$$\max_{\{c_t, b_t, m_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - \alpha \left(m_t - \gamma \right)^2 \right\},$$

subject to

$$c_t + b_t + m_t = y + \frac{1 + r_{t-1}}{1 + \pi_t} b_{t-1} + \frac{1}{1 + \pi_t} m_{t-1} + \tau_t,$$

where $\beta \in (0, 1)$ is the household's subjective discount factor, $\alpha > 0$ and $\gamma > 0$ are preference parameters, c_t is non-durable consumption, m_t is real money balances, b_t is the real value of bond holdings, r_t is the nominal interest rate on bonds, π_t is the net rate of inflation, $y > 0$ is an exogenous and time-invariant flow of income, and τ_t is a real government transfer. The household takes prices and interest rates as given. The household's initial holdings of money and bonds are equal to zero (i.e. $m_{-1} = b_{-1} = 0$).

The government controls the nominal money supply and transfers any seigniorage revenues directly to the household. Clearing of the goods market and the bond market requires, respectively, that $c_t = y$ and $b_t = 0$.

(**a**) Derive the first-order conditions for the household's decision problem.

(**b**) Show that the socially optimal steady-state rate of inflation is given by $\pi = \beta - 1$ and that the associated level of real money balances is given by $m = \gamma$.

Question 3 (30 points)

A model with consumer durables.

Consider a perfect-foresight economy with a continuum of identical and infinitely-lived households, of measure one in total. Households enjoy utility from the consumption of non-durable *and* durable goods. They also suffer disutility from supplying labour. Discounted lifetime utility at time zero, U_0, is given by:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln d_t - \eta \frac{h_t^{1+\kappa}}{1+\kappa} \right\},$$

where c_t denotes consumption of non-durables in period t, d_t is the *stock* of durables owned in period t, h_t denotes hours worked, $\beta \in (0,1)$ is the households' subjective discount factor and $\eta, \kappa, \gamma > 0$ are other preference parameters. Households supply labor to the firms, at a rate w_t per hour. Households own the stock of durables they consume and durables depreciate at a rate $\delta \in (0,1)$. The budget constraint of the households in period t is given by:

$$c_t + q_t d_t = (1-\delta) q_t d_{t-1} + w_t h_t,$$

where q_t is the relative price of a unit of durables, expressed in units of non-durables. In each period t, households choose c_t, d_t and h_t to maximize their discounted life-time utility, taking as given prices, wages and an initial stock of durables $d_{-1} > 0$.

(**a**) Formulate the maximization problem of the households and derive the first-order conditions for c_t, d_t and h_t. You may ignore the constraints that these variables should be positive-valued. Show that the first-order conditions for d_t can be written as:

$$\frac{q_t}{c_t} = \frac{\gamma}{d_t} + \beta(1-\delta)\frac{q_{t+1}}{c_{t+1}}.$$

(**b**) Use the above equation to show that the following steady-state relation holds:

$$\frac{c}{q\delta d} = \frac{1-\beta(1-\delta)}{\delta\gamma}.$$

(**c**) Explain why the left-hand side of the above equation is the steady-state ratio of aggregate expenditures on non-durables to aggregate expenditures on new durables. Also, explain intuitively how an increase in β affects this ratio.

On the supply side of the economy, there is a continuum of identical and competitive firms, of measure one in total. They operate a linear technology:

$$y_t = h_t,$$

where y_t denotes output and h_t is the firm's choice of labor inputs, expressed in hours. Durables and non-durables are produced with the same technology. Accordingly, the aggregate resource constraint is given by:

$$c_t + d_t = (1-\delta) d_{t-1} + h_t.$$

(**d**) Formulate the firms' profit maximization problem and derive the first-order condition for h_t.

(**e**) Define a competitive equilibrium, referring explicitly to the model's equations.

(**f**) Show that in the equilibrium (i) firms do not make any profits, and (ii) $q_t = 1$.

Question 4 (20 points)

The Diamond-Mortensen-Pissarides model.
Consider the Diamond-Mortensen-Pissarides model of the labour market, as discussed in the lecture but without aggregate uncertainty. Let $\overline{A}$ be a parameter capturing the productivity of a worker and let the wage of a worker be fixed and given by a parameter $\overline{w} \in \left(\overline{b}, \overline{A}\right)$, where $\overline{b}$ is the unemployment benefit. Further, let the aggregate matching function be given by $m_t = u_t^\gamma v_t^{1-\gamma}$, where m_t is the number of newly formed worker-firm pairs, u_t is the unemployment rate, v_t is the number of vacancies, and $\gamma \in (0,1)$ is a parameter. New worker-firm pairs can start producing one period after they meet. Furthermore, let labour market tightness, θ_t, be defined as the ratio of vacancies to unemployment, i.e. $\theta_t \equiv \frac{v_t}{u_t}$.

Consider the steady state of the economy, indicated by variables without a time subscript. The steady-state job creation condition (or "free-entry condition") of the model can be written as:

$$\vartheta \theta^\gamma = \beta \frac{\overline{A} - \overline{w}}{1 - \beta(1-\rho)},$$

where $\vartheta > 0$ is the cost of posting a vacancy, $\beta \in (0,1)$ is a discount factor, and $\rho \in (0,1)$ is the rate at which workers and firms separate.

(**a**) Provide an interpretation to both the left- and the right-hand side of the job creation condition.

(**b**) Express the steady-state job finding rate, defined as $f \equiv \frac{m}{u}$, as a function of the model's parameters only.

(**c**) Explain intuitively through what channels a permanent increase in the vacancy cost affects f.

(**d**) Show formally that the elasticity of f with respect to ϑ is negative.

∞Exam MSc Macroeconomics, EconG022

2015

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Agree or disagree?
Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) The Kaldor facts imply that, in the long run, real wages grow at a constant rate.

(**b**) In the Ramsey growth model, a benevolent social planner will choose the capital stock to be constant and equal to the Modified Golden Rule level.

(**c**) The labour wedge is a measure of distortions in the labour market.

(**d**) The Friedman rule prescribes zero inflation.

(**e**) "Superneutrality" of money means that the real capital stock, real output, and real money balances do not depend on growth rate of money in the long run.

(**f**) Robert Hall's result that consumption follows a random walk breaks down when there are fluctuations in the real interest rate.

[1]In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 1 (40 points) - solution

(**a**) True. One Kaldor fact is that the labour share of income $\frac{wL}{Y}$ is constant in the long run. Another Kaldor fact is that GDP per worker, $\frac{Y}{L}$ grows at a constant rate in the long run. Hence, $\frac{L}{Y}$ declines at a constant rate. For $\frac{wL}{Y}$ to be constant it then has to be the case that w grows at a constant rate.

(**b**) False. The planner chooses the Modified Golden Rule (MGR) level of capital only if the initial level of capital is already equal to the MGR level. For any other initial level of capital, the social planner will choose a transition path at which the capital will eventually arrive at the MGR level, but is different from this level along the transition path.

(**c**) True. The labour wedge captures the discrepancy between the Marginal Rate of Substitution and the Marginal Product of Labour, which in a frictionless labour market would be equalized.

(**d**) False. The Friedman rule prescribes a zero nominal interest rate, in order to set the opportunity cost of money to zero. The associated level of inflation is generally negative.

(**e**) False. Superneutrality of money does imply that real economic activity (the real capital stock and real output) does not depend on the growth rate of the money stock, in the long run. However, it does not mean that real money balances are not affected by the growth rate of the money stock. In the Money-In-the-Utility model, there is in fact an optimal rate of money growth, which sets the level of real money balances such that the marginal utility of money is zero. This would not be possible if the level of real money balances was not affected by the growth rate of the money stock.

(**f**) True. The Euler equation that leads to Hall's random walk result is:

$$u_{c,t} = \beta\left(1+r\right)E_t u_{c,t+1}$$

where β is the subjective discount factor, r is the real interest rate $u_{c,t}$ is the marginal utility of consumption which under Hall's assumption is equal to $1 - ac_t$, where a is a parameter. It then follows that:

$$c_t = \beta\left(1+r\right)E_t c_{t+1}.$$

If $\beta\left(1+r\right) = 1$ then $c_t = E_t c_{t+1}$, i.e. consumption follows a random walk. However, if r fluctuates over time then r should also help us forecast c_{t+1}. Thus, current consumption is no longer the only variable that is helpful in predicting future consumption and the random walk result breaks down.

Question 2 (10 points)

Money in the utility function.
Consider an infinite-horizon economy, populated by a representative household whose utility depends on non-durable consumption and holdings of real money balances. The household can also purchase one-period nominal bonds. There is no uncertainty. The household solves the following decision problem:

$$\max_{\{c_t,b_t,m_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\beta^t \left\{\ln c_t - \alpha\left(m_t-\gamma\right)^2\right\},$$

subject to

$$c_t + b_t + m_t = y + \frac{1+r_{t-1}}{1+\pi_t}b_{t-1} + \frac{1}{1+\pi_t}m_{t-1} + \tau_t,$$

where $\beta \in (0,1)$ is the household's subjective discount factor, $\alpha > 0$ and $\gamma > 0$ are preference parameters, c_t is non-durable consumption, m_t is real money balances, b_t is the real value of bond holdings, r_t is the nominal interest rate on bonds, π_t is the net rate of inflation, $y > 0$ is an exogenous and time-invariant flow of income, and τ_t is a real government transfer. The household takes prices and interest rates as given. The household's initial holdings of money and bonds are equal to zero (i.e. $m_{-1} = b_{-1} = 0$).

The government controls the nominal money supply and transfers any seigniorage revenues directly to the household. Clearing of the goods market and the bond market requires, respectively, that $c_t = y$ and $b_t = 0$.

(**a**) Derive the first-order conditions for the household's decision problem.

(**b**) Show that the socially optimal steady-state rate of inflation is given by $\pi = \beta - 1$ and that the associated level of real money balances is given by $m = \gamma$.

Question 2 - solution
Money in the utility function

a) The first-order conditions for money and bonds are, respectively:

$$\begin{aligned}\frac{1}{c_t} &= -2\alpha\left(m_t-\gamma\right)+\frac{\beta}{1+\pi_{t+1}}\frac{1}{c_{t+1}}\\ \frac{1}{c_t} &= \beta\frac{1+r_t}{1+\pi_{t+1}}\frac{1}{c_{t+1}}\end{aligned}$$

b) In a steady state, the FOC for money can be written as:

$$\left(1-\frac{\beta}{1+\pi}\right)=-2\alpha c\left(m-\gamma\right)$$

The social welfare criterion is $U=\ln c-\alpha\left(m-\gamma\right)^2$. The first term, $\ln c$, follows directly from the goods market clearing condition and is therefore not affected by any government policy. Maximizing $-\alpha\left(m-\gamma\right)^2$ requires[2] setting $2\alpha\left(m-\gamma\right)=0$, or $m=\gamma$. From the above first-order condition it then follows that the optimal rate of inflation satisfies $\frac{\beta}{1+\pi}=1$, or $\pi=\beta-1$.

[2]Note that $-\alpha\left(m-\gamma\right)^2$ is a concave function and thus has a maximum.

Question 3 (30 points)

Consider a perfect-foresight economy with a continuum of identical and infinitely-lived households, of measure one in total. Households enjoy utility from the consumption of non-durable *and* durable goods. They also suffer disutility from supplying labour. Discounted lifetime utility at time zero, U_0, is given by:

$$U_0 =_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln d_t - \eta \frac{h_t^{1+\kappa}}{1+\kappa} \right\}$$

where c_t denotes consumption of non-durables in period t, d_t is the *stock* of durables owned in period t, h_t denotes hours worked, $\beta \in (0,1)$ is the households' discount factor and $\eta, \kappa, \gamma > 0$ are other preference parameters. Households supply labor to the firms, at a rate w_t per hour. Households own the stock of durables they consume and durables depreciate at a rate $\delta \in (0,1)$. The budget constraint of the households in period t is given by:

$$c_t + q_t d_t = (1-\delta)\, q_t d_{t-1} + w_t h_t$$

where q_t is the relative price of a unit of durables, expressed in units of non-durables.

In each period t, households choose c_t, d_t and h_t to maximize their discounted life-time utility, taking as given prices, wages and an initial stock of durables $d_{-1} > 0$.

(**a**) Formulate the maximization problem of the households and derive the first-order conditions for c_t, d_t and h_t. You may ignore the constraints that these variables should be positive-valued. Show that the first-order conditions for d_t can be written as:

$$\frac{q_t}{c_t} = \frac{\gamma}{d_t} + \beta(1-\delta)\frac{q_{t+1}}{c_{t+1}}$$

(**b**) Use the above equation to show that the following steady-state relation holds:

$$\frac{c}{q\delta d} = \frac{1-\beta(1-\delta)}{\delta\gamma}$$

(**c**) Explain why the left-hand side of the above equation is the steady-state ratio of aggregate expenditures on non-durables to aggregate expenditures on new durables. Also, explain intuitively how an increase in β affects this ratio.

The supply side of the economy is given as follows. There is a large number of identical, profit-maximizing firms, of measure one in total. Firms operate a linear technology:

$$y_t = h_t$$

where y_t denotes output and h_t denotes labor inputs, expressed in hours. Like households, firms take prices and wages as given. Moreover, durables and non-durables are produced using the same technology. Accordingly, the aggregate resource constraint is given by:

$$c_t + d_t = (1-\delta)\, d_{t-1} + h_t$$

(**d**) Formulate the firms' profit maximization problem and derive the first-order condition for the optimal choice of labour inputs.

(**e**) Define a competitive equilibrium, referring explicitly to the model's equations.

(**f**) Show that in the equilibrium (i) firms do not make any profits, and (ii) $q_t = 1$.

Question 3 (30 points) - solution

(**a**) The first-order conditions for c_t, d_t and h_t are:

$$\begin{aligned}
\frac{1}{c_t} &= \lambda_t \\
q_t\lambda_t &= \frac{\gamma}{d_t} + \beta\,(1-\delta)\,q_{t+1}\lambda_{t+1} \\
w_t\lambda_t &= \eta h_t^{\kappa}
\end{aligned}$$

where λ_t is the Lagrange multiplier on the budget constraint. After substituting out λ_t the first-order conditions simplify to:

$$\begin{aligned}
\frac{q_t}{c_t} &= \frac{\gamma}{d_t} + \beta\,(1-\delta)\,\frac{q_{t+1}}{c_{t+1}} \qquad (1)\\
\frac{w_t}{c_t} &= \eta h_t^{\kappa} \qquad (2)
\end{aligned}$$

(**b**) In a steady-state the first-order condition for durables becomes:

$$\frac{q}{c} = \frac{\gamma}{d} + \beta\,(1-\delta)\,\frac{q}{c}$$

or

$$\frac{c}{qd} = \frac{1-\beta\,(1-\delta)}{\gamma}$$

It follows that

$$\frac{c}{q\delta d} = \frac{1-\beta\,(1-\delta)}{\delta\gamma}$$

(**c**) In the numerator of the left-hand side, c is the amount of non-durable expenditures in the steady state. In the denominator, δd is the amount of new durables purchases in the steady state, which exactly makes up for depreciation. Hence, $q\delta d$, is the total amount spent on new durable purchases in the steady state. Thus, the left-hand side of the equation is the ratio of non-durable expenditures to expenditures of new durables in the steady state.

An increase in β means that households are more patient. Purchasing durables is partly a form of saving and how much agents value savings depends on how patient they are. Thus, an increase in β will make households allocate more of their expenditures to durables, at the expense of non-durable purchases. Therefore, the expenditure ratio falls.

(**d**) Let Π_t denote a firm's profit in period t. The profit maximization problem reads:

$$\max_{h_t} \Pi_t = h_t - w_t h_t.$$

The first-order condition is:

$$w_t = 1.$$

(**e**) A competitive equilibrium is a sequence of allocations $\{c_t, d_t, h_t\}_{t=0}^{\infty}$ and prices $\{w_t, q_t\}_{t=0}^{\infty}$ which satisfy the initial condition d_0 and the following equations in each period t:

- the households' first-order conditions for d_t and h_t(Equations (1) and (2)),

- the firms' first-order condition ($w_t = 1$)
- the households' budget constraint
- the aggregate resource constraint.

To be 100% complete, one could replace h_t by separate variables for labour supply and labour demand, say h_t^s and h_t^d and add a labour market clearing condition $h_t^s = h_t^d$.

(**f**) (i) Substituting $w_t = 1$ into $\Pi_t = h_t - w_t h_t$ delivers $\Pi_t = 0$. (ii) Combining the resource constraint and the households' budget constraint, using that $w_t = 1$, immediately delivers $q_t = 1$.

Question 4 (20 points)

The Diamond-Mortensen-Pissarides model.
Consider the Diamond-Mortensen-Pissarides model of the labour market, as discussed in the lecture but without aggregate uncertainty. Let $\overline{A}$ be a parameter capturing the productivity of a worker and let the wage of a worker be fixed and given by a parameter $\overline{w} \in \left(\overline{b}, \overline{A}\right)$, where $\overline{b}$ is the unemployment benefit. Further, let the aggregate matching function be given by $m_t = u_t^{\gamma} v_t^{1-\gamma}$, where m_t is the number of newly formed worker-firm pairs, u_t is the unemployment rate, v_t is the number of vacancies, and $\gamma \in (0,1)$ is a parameter. New worker-firm pairs can start producing one period after they meet. Furthermore, let labour market tightness, θ_t, be defined as the ratio of vacancies to unemployment, i.e. $\theta_t \equiv \frac{v_t}{u_t}$.

Consider the steady state of the economy, indicated by variables without a time subscript. The steady-state job creation condition (or "free-entry condition") of the model can be written as:

$$\vartheta \theta^{\gamma} = \beta \frac{\overline{A} - \overline{w}}{1 - \beta(1-\rho)},$$

where $\vartheta > 0$ is the cost of posting a vacancy, $\beta \in (0,1)$ is a discount factor, and $\rho \in (0,1)$ is the rate at which workers and firms separate.

(**a**) Provide an interpretation to both the left- and the right-hand side of the job creation condition.

(**b**) Express the steady-state job finding rate, defined as $f \equiv \frac{m}{u}$, as a function of the model's parameters only.

(**c**) Explain intuitively through what channels a permanent increase in the vacancy cost affects f.

(**d**) Show formally that the elasticity of f with respect to ϑ is negative.

Question 4 (20 points) - solution

(**a**) Start with the right-hand side. The term $\overline{A} - \overline{w}$ is the flow profit to a firm, and hence the total right-hand side is the present value of future profits, accounting for the subjective discount rate β and the fact that a match breaks up with probability ρ in each period. Thus, the right-hand side captures the total expected benefit to a firm of hiring a new worker. The left hand side captures the expected cost of hiring a new worker. This can be seen as follows. Note that the probability of filling a vacancy is given by $g \equiv \frac{m}{v} = u^{\gamma} v^{-\gamma} = \theta^{-\gamma}$. Given that ϑ is the cost per vacancy, the expected cost of hiring a worker is $\frac{\vartheta}{g} = \frac{\vartheta}{\theta^{-\gamma}} = \vartheta\theta^{\gamma}$.

(**b**) First re-write the job creation condition as $\theta = \left(\frac{\beta}{\vartheta}\frac{\overline{A}-\overline{w}}{1-\beta(1-\rho)}\right)^{\frac{1}{\gamma}}$. Using the definition of the job finding rate we obtain

$$\begin{aligned} f &= \frac{u^{\gamma} v^{1-\gamma}}{u} \\ &= \theta^{1-\gamma} \\ &= \left(\frac{\beta}{\vartheta}\frac{\overline{A}-\overline{w}}{1-\beta\left(1-\rho\right)}\right)^{\frac{1-\gamma}{\gamma}} \end{aligned}$$

(**c**) An increase in the vacancy cost discourages firms to post vacancies. Note that a change in the vacancy does not affect the present value of profits of a new match (the right-hand side of the job creation (JC) condition). Expressing the left-hand side of the JC condition as $\frac{\vartheta}{g}$ makes clear that in equilibrium the vacancy filling probability must in fact increase one for one with an increase in the vacancy cost. Intuitively, as the number of vacancies falls, the probability of filling and vacancy increases, up to the point that the expected cost of hiring a worker are again equal to the benefits.

In turn, an increase in the probability of filling a vacancy requires a decline in labour market tightness, which in turn leads to a decline in the job finding probability. From the matching function it follows that $f = u^{\gamma-1} v^{1-\gamma} = \theta^{1-\gamma}$, and hence a reduction in v and θ is associated with a decline in f. Intuitively, as fewer vacancies are posted, it becomes less likely for an individual unemployed worker to match with a firm. Further, a reduction in f will increase unemployment. Ceteris paribus, an increase in u reduces the job finding rate. Note that both an increase in u and a decline in v contribute to a decline in tightness, since $\theta = \frac{v}{u}$. As an unemployed worker competes with more job searchers, the probability of finding a job is pushed down further.

The above effects can be illustrated graphically by drawing the Beveridge curve and the job creation curve, with an increase in ϑ shifting down the job creation curve.

(**d**) Taking logs we can write the above expression as:

$$\ln f = \frac{\gamma-1}{\gamma}\ln\vartheta + \frac{1-\gamma}{\gamma}\ln\left(\beta\frac{\overline{A}-\overline{w}}{1-\beta\left(1-\rho\right)}\right).$$

Given that the second term on the right-hand side is a function of only parameters, not including ϑ, it is unaffected by a change in the cost of a vacancy. Differentiation then gives the elasticity of f with respect to ϑ:

$$\epsilon_{f,\vartheta} \equiv \frac{\mathrm{d}\ln f}{\mathrm{d}\ln\vartheta} = \frac{\gamma - 1}{\gamma} < 0.$$

Exam MSc Macroeconomics, EconG022
2016

Instructions. This exam consists of five pages with four questions in total. You have THREE HOURS to answer all four questions. There are 100 points to be earned. Please write clearly and hand in your answers ordered numerically.

Question 1 (40 points)

Agree or disagree?
Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) An upward trend in the capital share of income is inconsistent with the Kaldor facts.

(**b**) In the Ramsey growth model, a benevolent social planner will always choose the level of consumption to be equal to the "modified" Golden Rule level.

(**c**) In the Real Business Cycle model, consumption follows a random walk.

(**d**) In practice, central banks often do not follow the Friedman rule.

(**e**) Rational expectations imply that prediction errors are functions of economic fundamentals only.

(**f**) In OverLapping Generations models, equilibria are generally inefficient without government intervention.

[1] In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 2 (15 points)

Consumption of durables and non-durables
Consider an infinitely-lived household who consumes both durables and non-durables. Discounted lifetime utility at time zero is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma d_t \right\},$$

where c_t denotes consumption of non-durables in period t, d_t is the *stock* of durables owned by the household in period t, $\beta \in (0,1)$ is the household's subjective discount factor, $\gamma > 0$ is a preference parameter, and E_t is the expectations operator conditional upon information available in period t. The price of both durables and non-durables is one. The budget constraint of the household in period t is given by:

$$c_t + d_t = y_t + (1-\delta) d_{t-1}, \;\; t = 0, 1, 2, 3..$$

where $\delta \in (0,1)$ is the depreciation rate of durables and $y_t > 0$ is an exogenous income variable, which follows a stochastic but stationary process. In each period t, the household chooses c_t and d_t such as to maximize the expected present value of lifetime utility.

(**a**) Derive the first-order optimality conditions for the household's decision problem.

(**b**) Show that the household chooses a constant consumption level, given by $c_t = \frac{1-\beta(1-\delta)}{\gamma}$, in any period t.

(**c**) Explain intuitively why the agent avoids any fluctuations in non-durable consumption.

Question 3 (35 points)

A model with real and nominal variables

Consider an infinite-horizon economy, populated by a representative household and a representative firm, who take prices and wages as given. There is no uncertainty. The household inelastically supplies one unit of labour to the firm and can invest in nominal one-period bonds and in capital. Capital is rented out to the firm at a rate $r_{k,t}$ in period t, and depreciates at a rate δ. The household's decision problem can be expressed, in real terms,as:

$$\max_{\{c_t,k_{t+1},b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

subject to

$$c_t + k_{t+1} + b_t = w_t + \frac{1 + r_{b,t-1}}{1 + \pi_t} b_{t-1} + (1 - \delta + r_{k,t})\, k_t, \;\; t = 0, 1, 2, 3..$$

k_0, b_0 given

where $\beta \in (0, 1)$ is the subjective discount factor, c_t is non-durable consumption, k_{t+1} is capital purchased in period t, b_t is the real value of bonds purchased in period t, w_t is the wage, $r_{b,t}$ is the net nominal interest rate, and π_t is the net inflation rate.

(**a**) Derive the first-order optimality conditions for the household's decision problem and show that $1 - \delta + r_{k,t+1} = \frac{1+r_{b,t}}{1+\pi_{t+1}}$ in any period. What is the interpretation of this equation?

The firm operates a Cobb-Douglas technology, with output given by $y_t = k_t^{\alpha} l_t^{1-\alpha}$, with $\alpha \in (0, 1)$ and where l_t denotes the amount of labour used by the firm. Output can be used either as a consumption good or as a capital good. The goods and labour market are perfectly competitive. As a result, the real wage and the rental rate of capital are determined as $w_t = (1 - \alpha)\, k_t^{\alpha}$ and $r_{k,t} = \alpha k_t^{\alpha-1}$ and the firm makes no profits. The nominal interest rate, $r_{b,t}$, is determined by a central bank. The policy of the central bank is to set $r_{b,t} = 0$ at all times. Clearing of the bond market and of the labour market requires, respectively, that:

$$\begin{aligned} b_t &= 0, \;\; t = 0, 1, 2, 3.. \\ l_t &= 1, \;\; t = 0, 1, 2, 3... \end{aligned}$$

(**b**) Show that from the above it follows that the goods market clears, i.e. aggregate output is the sum of aggregate consumption and aggregate investment.

(**c**) Consider a steady state in which real variables remain constant over time. Derive an expression for the steady-state level of capital, denoted $\widetilde{k} > 0$, in terms of the model parameters.

(**d**) Show that goods prices and nominal wages decline in the steady state.

(**e**) Consider a transition towards the steady state, during which the capital stock gradually increases, i.e. $\widetilde{k} > k_{t+1} > k_t$ in any period t. Show that, along the transition path, the rate of inflation increases, i.e. $\pi_{t+1} > \pi_t$. Explain the economic mechanism behind this result.

(**f**) Now consider a version of the model with full capital depreciation, i.e. $\delta = 1$. Show that in the equilibrium $k_{t+1} = \alpha\beta k_t^{\alpha}$.

(**g**) Show that, when $\delta = 1$, nominal wages grow at a constant net rate given by $\beta - 1 < 0$.

Question 4 (10 points)

The Diamond-Mortensen-Pissarides model.
Consider the Diamond-Mortensen-Pissarides model of the labour market, as discussed in the lecture but without aggregate uncertainty. Let $\overline{A}$ be a parameter capturing the productivity of a worker and let the real wage of a worker be fixed and given by a parameter $\overline{w} \in \left[\overline{b}, \overline{A}\right]$, where $\overline{b}$ is the unemployment benefit. Further, let the aggregate matching function be given by $m_t = \overline{m}\sqrt{u_t v_t}$, where m_t is the number of newly formed worker-firm pairs, u_t is the unemployment rate, v_t is the number of vacancies, and $\overline{m}$ is a parameter. New worker-firm pairs can start producing one period after they meet. Furthermore, let $f_t \equiv \frac{m_t}{u_t}$ be the job finding rate among job searchers.

Consider the steady state of the economy, indicated by variables without a time subscript. The job creation condition (or "free-entry condition") of the model can be written as follows:

$$\frac{\vartheta f}{\overline{m}^2} = \beta \frac{\overline{A} - \overline{w}}{1 - \beta\left(1 - \rho\right)}.$$

where $\rho > 0$ is an exogenous rate of job loss and $\vartheta > 0$ is the cost for firms to post a vacancy.

(**a**) Show that the elasticity of f with respect to a unanticipated and permanent change in $\overline{A}$ (with $\overline{w}$ remaining fixed) is given by $\frac{\overline{A}}{\overline{A} - \overline{w}}$.

(**b**) Show that the rate at which vacancies are filled is given by $\overline{m}^2/f_t$ and use the job creation condition to explain intuitively why the job finding rate increases after a permanent and unanticipated increase in $\overline{A}$.

Exam MSc Macroeconomics, EconG022

2016

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Question 1 (40 points)

Agree or disagree?
Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) An upward trend in the capital share of income is inconsistent with the Kaldor facts.

SHORT ANSWER: True. Two Kaldor facts are that, in the long run, the rental rate of capital r is constant and the capital-to-output ratio K/Y is constant as well. As a result, the capital share of income $\frac{rK}{Y}$ should not have a trend according to the Kaldor facts.

(**b**) In the Ramsey growth model, a benevolent social planner will always choose the level of consumption to be equal to the "modified" Golden Rule level.

SHORT ANSWER: False, this is true in the steady state, but not generally along a transition path. In fact, it could be the case that the initial level of capital is so low that there are not enough resources available to reach the modified Golden Rule level of consumption, even if all resources were used for consumption.

(**c**) In the Real Business Cycle model, consumption follows a random walk.

SHORT ANSWER: False. The fact that the model has persistent state variables (capital, productivity) implies that changes in consumption, which are determined as a function of the state variables, are partly predictable. Further, the random-walk result of Hall (1978) is derived under a special case with quadratic utility. RBC models typically assume some other utility functions, such as isoelastic (CRRA) utility.

(**d**) In practice, central banks often do not follow the Friedman rule.

SHORT ANSWER: True. The friedman rule prescribes a zero nominal interest rate. In reality, central banks often target positive interest rates. Recent years are an exception, as the zero lower bound on the nominal interest rate became binding in several countries.

(**e**) Rational expectations imply that prediction errors are functions of economic fundamentals only.

SHORT ANSWER: False. Rational expectations models can feature multiple equilibria, which gives rise to non-fundamental "sunspot" fluctuations. In that case, prediction errors can be the result of non-fundamental sunspot shocks.

(**f**) In OverLapping Generations models, equilibria are generally inefficient without government intervention.

SHORT ANSWER: False. Equilibria can be inefficient in OLG models, but whether this is the case typically depends on whether the intertemporal Marginal Rate of Substitution without government intervention is bigger or smaller than one. Back-of-the envelope calculations suggest that reasonable calibrations tend to imply efficiency without government intervention.

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Question 2 (15 points)

Consumption of durables and non-durables
Consider an infinitely-lived household who consumes both durables and non-durables. Discounted lifetime utility at time zero is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{\ln c_t + \gamma d_t\},$$

where c_t denotes consumption of non-durables in period t, d_t is the *stock* of durables owned by the household in period t, $\beta \in (0,1)$ is the household's subjective discount factor, $\gamma > 0$ is a preference parameter, and E_t is the expectations operator conditional upon information available in period t. The price of both durables and non-durables is one. The budget constraint of the households in period t is given by:

$$c_t + d_t = y_t + (1-\delta) d_{t-1}, \ t = 0, 1, 2, 3..$$

where $\delta \in (0,1)$ is the depreciation rate of durables and $y_t > 0$ is an exogenous income variable, which follows a stochastic but stationary process.

(**a**) Derive the first-order optimality conditions for the household's decision problem.

SHORT ANSWER: Let λ_t be the Lagrange multiplier on the budget constraint. The first-order conditions for c_t and d_t are, respectively:

$$\begin{aligned} \lambda_t &= \frac{1}{c_t}, \ t = 0, 1, 2, 3... \\ \lambda_t &= \gamma + \beta (1-\delta) E_t \lambda_{t+1}, \ t = 0, 1, 2, 3... \end{aligned}$$

(**b**) Show that the household chooses a constant consumption level $c_t = \frac{1-\beta(1-\delta)}{\gamma}$ in any period t.

SHORT ANSWER: Combining the first-order conditions gives:

$$\begin{aligned} \frac{1}{c_t} &= \gamma + \beta (1-\delta) E_t \frac{1}{c_{t+1}}, \\ &= E_t \textstyle\sum_{t=0}^{\infty} \beta^t (1-\delta)^t \gamma, \\ &= \frac{\gamma}{1-\beta(1-\delta)}. \end{aligned}$$

From this the answer follows directly.

(**c**) Explain intuitively why the agent avoids any fluctuations in non-durable consumption.

SHORT ANSWER: The agent's utility function is concave in non-durables, but linear in durables. Hence, marginal utility with respect to durables is constant, whereas the marginal utility with respect to non-durables is decreasing in the level of non-durables. Intuitively, the agent dislikes fluctuations in non-durables but is indifferent with respect to fluctuations in non-durables. Consequently, the agent optimally adjusts durable purchases such as to fully "insure" away any fluctuations in non-durable consumption.

Question 3 (35 points)

A model with real and nominal variables

Consider an infinite-horizon economy, populated by a representative household and a representative firm, who take prices and wages as given. There is no uncertainty. The household inelastically supplies one unit of labor to the firm and can invest in nominal one-period bonds and in capital. Capital is rented out to firms at a rate $r_{k,t}$ in period t, and depreciates at a rate δ. The household's decision problem can be expressed as:

$$\max_{\{c_t,k_{t+1},b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

subject to

$$c_t + b_t + k_{t+1} = w_t + \frac{1+r_{b,t-1}}{1+\pi_t} b_{t-1} + (1-\delta + r_{k,t})\, k_t, \;\; t = 0,1,2,3..$$

$k_0, \; b_{-1}$ given

where $\beta \in (0,1)$ is the household's subjective discount factor, c_t is non-durable consumption, b_t is the real value of bonds purchased in period t, k_{t+1} is the amount of capital purchased in period t, w_t is the wage, $r_{b,t}$ is the net nominal interest rate on bonds, π_t is the net inflation rate.

(**a**) Derive the first-order optimality conditions for the household's decision problem and show that $1-\delta+r_{k,t+1} = \frac{1+r_{b,t}}{1+\pi_{t+1}}$ in any period. What is the interpretation of this equation?

SHORT ANSWER: Let λ_t be the Lagrange multiplier on the budget constraint. The first-order conditions for c_t, k_{t+1}, and b_{t+1} are, respectively:

$$\begin{aligned}
\frac{1}{c_t} &= \lambda_t, \;\; t = 0,1,2,3.. \\
\frac{1}{c_t} &= (1-\delta + r_{k,t+1})\, \beta \frac{1}{c_{t+1}}, \;\; t = 0,1,2,3.. \\
\frac{1}{c_t} &= \frac{1+r_{b,t}}{1+\pi_{t+1}} \beta \frac{1}{c_{t+1}}, \;\; t = 0,1,2,3...
\end{aligned}$$

Combining the last two equations gives

$$(1-\delta + r_{k,t+1})\, \beta \frac{1}{c_{t+1}} = \frac{1+r_{b,t}}{1+\pi_{t+1}} \beta \frac{1}{c_{t+1}},$$

from which the result directly follows. The equation is a no-arbitrage/indifference condition which states that bonds and capital must pay the same net returns.

The firm operates a Cobb-Douglas technology, with output given by $y_t = k_t^{\alpha} l_t^{1-\alpha}$, with $\alpha \in (0,1)$ and where l_t denotes the amount of labor demanded by the firm. Output can be used either as a consumption good or as a capital good. The goods and labor market are perfectly competitive. As a result, the real wage and rental rate of capital are determined as $w_t = (1-\alpha)\, k_t^{\alpha}$ and $r_{k,t} = \alpha k_t^{\alpha-1}$ and the representative firm makes no profits. The nominal interest rate, $r_{b,t}$, is determined by a central bank. The policy of the central bank is to set $r_{b,t} = 0$ at all times. Clearing of the bond market and the labor market requires, respectively, that:

$$\begin{aligned} b_t &= 0, \\ l_t &= 1. \end{aligned}$$

(**b**) Show that from the above it follows that the goods market clears, i.e. aggregate output is the sum of aggregate consumption and aggregate investment.

SHORT ANSWER: Plugging $w_t = (1-\alpha) k_t^\alpha$ and $r_{k,t} = \alpha k_t^{\alpha-1}$ in the budget constraint and imposing $b_t = 0$ at all times gives:

$$\begin{aligned} c_t + k_{t+1} &= (1-\alpha) k_t^\alpha + \left(1 - \delta + \alpha k_t^{\alpha-1}\right) k_t \\ &= (1-\alpha) k_t^\alpha + \alpha k_t^\alpha + (1-\delta) k_t \\ &= k_t^\alpha + (1-\delta) k \\ &= y_t + (1-\delta) k \end{aligned}$$

where the final equality uses the labor market clearing condition and implies that $y_t = c_t + k_{t+1} - (1-\delta) k_t$, where c_t is aggregate consumption and $k_{t+1} - (1-\delta) k_t$ is aggregate investment.

(**c**) Consider a steady state in which real variables remain constant over time. Derive expressions for the steady-state level of capital, denoted $\widetilde{k} > 0$, in terms of the model parameters only.

SHORT ANSWER: The steady-state condition $c_t = c_{t+1}$ and the Euler equation for capital imply that:

$$\frac{1}{\beta} = 1 - \delta + \alpha \widetilde{k}^{\alpha-1}$$

$\Leftrightarrow$

$$\widetilde{k} = \left(\frac{1}{\alpha\beta} - \frac{1-\delta}{\alpha}\right)^{\frac{1}{\alpha-1}}$$

(**d**) Show that goods prices and nominal wages decline in the steady state.

SHORT ANSWER: From the Euler equation for bonds and the monetary policy rule it follows that in the steady state:

$$\frac{1}{1+\pi_{t+1}} = \frac{1}{\beta} > 1$$

and hence $\pi_{t+1} = \beta - 1 < 0$. Thus, the price level declines in the steady sate (i.e. there is deflation). Next, note that the steady-state real wage is given by a constant $\widetilde{w} = (1-\alpha) \widetilde{k}^\alpha$. Let the nominal wage denoted $W_t = P_t W_t$, where P_t is the price level. Nominal wage growth is given by:

$$\frac{W_{t+1}}{W_t} = \frac{w_{t+1}}{w_t} \frac{P_{t+1}}{P_t}.$$

Since in the steady state real wages remain constant, it follows that nominal wage growth, $\frac{W_{t+1}}{W_t}$, equals the inflation rate, $\frac{W_{t+1}}{W_t} = \frac{P_{t+1}}{P_t} = 1 + \pi_{t+1} < 1$.

(**e**) Consider a transition towards the steady state, during which the capital stock gradually increases, i.e. $\widetilde{k} > k_{t+1} > k_t$ in any period t. Show that, along the transition path, the rate of inflation increases, i.e. $\pi_{t+1} > \pi_t$. Explain the economic mechanism behind this result.

SHORT ANSWER: First note that along the transition path $r_{k,t+1} < r_{k,t}$, since $r_{k,t} = \alpha k_t^{\alpha-1}$ is a decreasing function of k_t. From the condition $1 - \delta + r_{k,t+1} = \frac{1+r_{b,t}}{1+\pi_{t+1}} = \frac{1}{1+\pi_{t+1}}$ it then follows that along the transition path:

$$\frac{1}{1+\pi_{t+1}} < \frac{1}{1+\pi_t},$$

$\Longleftrightarrow$

$$\pi_{t+1} > \pi_t,$$

Economic mechanism: along the transition path, the capital stock gradually increases, and as a result the marginal product of capital, and thus the rental rate of capital, declines. No arbitrage between bonds and capital investment requires that the real interest rate falls along the transition path. Given that the nominal interest rate is zero at all times, an decrease in the real interest rate requires an increase in the inflation rate.

(**f**) Now consider a version of the model with full capital depreciation, i.e. $\delta = 1$. Show that in the equilibrium $k_{t+1} = \alpha\beta k_t^\alpha$.

SHORT ANSWER: Guess a policy rule of the form $k_{t+1} = \gamma k_t^\alpha$. The resource constraint then implies $c_t = (1-\gamma) k_t^\alpha$. Substituting out consumption in the Euler equation gives

$$\begin{aligned} \frac{1}{(1-\gamma) k_t^\alpha} &= \beta \frac{\alpha k_{t+1}^{\alpha-1}}{(1-\gamma) k_{t+1}^\alpha}, \\ &= \beta \frac{\alpha}{(1-\gamma) k_{t+1}}. \end{aligned}$$

$\Longleftrightarrow$

$$k_{t+1} = \alpha\beta k_t^\alpha.$$

(**g**) Show that, when $\delta = 1$, nominal wages grow at a constant net rate given by $\beta - 1 < 0$.

SHORT ANSWER: The gross rate of nominal wage growth satisfies:

$$\begin{aligned} \frac{W_{t+1}}{W_t} &= \frac{w_{t+1}}{w_t} (1 + \pi_{t+1}), \\ &= \frac{k_{t+1}^\alpha}{k_t^\alpha} (1 + \pi_{t+1}), \\ &= \alpha\beta k_{t+1}^{\alpha-1} \frac{1}{r_{k,t+1}}, \\ &= \alpha\beta k_{t+1}^{\alpha-1} \frac{1}{\alpha k_{t+1}^{\alpha-1}}, \\ &= \beta. \end{aligned}$$

and hence the net rate of nominal wage growth is $\frac{W_{t+1}}{W_t} - 1 = \beta - 1$.

Question 4 (10 points)

The Diamond-Mortensen-Pissarides model.
Consider the Diamond-Mortensen-Pissarides model of the labour market, as discussed in the lecture but without aggregate uncertainty. Let $\overline{A}$ be a parameter capturing the productivity of a worker and let the real wage of a worker be fixed and given by a parameter $\overline{w} \in \left[\overline{b}, \overline{A}\right]$, where $\overline{b}$ is the unemployment benefit. Further, let the aggregate matching function be given by $m_t = \overline{m}\sqrt{u_t v_t}$, where m_t is the number of newly formed worker-firm pairs, u_t is the unemployment rate, v_t is the number of vacancies, and $\overline{m}$ is a parameter. New worker-firm pairs can start producing one period after they meet. Furthermore, let $f_t \equiv \frac{m_t}{u_t}$ be the job finding rate among job searchers.

Consider the steady state of the economy, indicated by variables without a time subscript. The job creation condition (or "free-entry condition") of the model can be written as follows:

$$\frac{\vartheta f}{\overline{m}^2} = \beta \frac{\overline{A} - \overline{w}}{1 - \beta(1-\rho)}.$$

where $\rho > 0$ is an exogenous rate of job loss and $\vartheta > 0$ is the cost for firms to post a vacancy.

(**a**) Show that the elasticity of f with respect to a unanticipated and permanent change in $\overline{A}$ (with $\overline{w}$ remaining fixed) is given by $\frac{\overline{A}}{\overline{A}-\overline{w}}$.

SHORT ANSWER: Taking logs gives:

$$\ln \vartheta + \ln f - 2\ln \overline{m} = \ln \frac{\beta}{1-\beta(1-\rho)} + \ln(\overline{A} - \overline{w}).$$

Given that ϑ, β, $\overline{m}$ and ρ are time-invariant parameters, total differentiation gives:

$$\begin{aligned} \frac{df}{f} &= \frac{d\overline{A}}{\overline{A}-\overline{w}} \\ &= \frac{d\overline{A}}{\overline{A}} \frac{\overline{A}}{\overline{A}-\overline{w}} \end{aligned}$$

It follows that the elasticity is given by

$$\frac{df/f}{d\overline{A}/\overline{A}} = \frac{\overline{A}}{\overline{A}-\overline{w}}$$

(**b**) Show that the rate at which vacancies are filled is given by $\overline{m}^2/f_t$ and use the job creation condition to explain intuitively why the job finding rate increases after a permanent increase in $\overline{A}$.

SHORT ANSWER: The vacancy filling rate is given by

$$q_t = \frac{m_t}{v_t} = \frac{\overline{m}\sqrt{u_t v_t}}{v_t} = \overline{m}\sqrt{\frac{u_t}{v_t}}$$

$\Leftrightarrow \frac{u_t}{v_t} = \frac{q_t^2}{\overline{m}^2}$ The job finding rate is given by

$$\begin{aligned} f_t &= \frac{\overline{m}\sqrt{u_t v_t}}{u_t} \\ &= \overline{m}\sqrt{\frac{v_t}{u_t}} \\ &= \overline{m}\sqrt{\frac{\overline{m}^2}{q_t^2}} \\ &= \overline{m}^2/q_t. \end{aligned}$$

which implies $q_t = \frac{\overline{m}^2}{f_t}$.

The right-hand side of the job creation condition captures the expected present value of profits that a worker generates to a firm, gross of vacancy cost. The left hand side can be written as ϑ/q_t and hence captures the expected vacancy cost of hiring a worker. The condition thus states that the net profit generate by posting a vacancy, $\vartheta/q_t - \beta\frac{\overline{A}-\overline{w}}{1-\beta(1-\rho)}$, equals zero. After an increase in $\overline{A}$, then the gross profit generated by employees increases (given that the wage remains unchanged). The job creation condition states that the expected vacancy cost of hiring a worker, ϑ/q_t, must increase in order to restore equilibrium. This happens through an increase in market tightness, i.e. in the number of vacancies per unemployed workers $\frac{v_t}{u_t}$, which implies that the job finding rate increases.

Exam MSc Macroeconomics, EconG022

2017

Instructions. This exam consists of five pages with four questions in total. You have THREE HOURS to answer all four questions. There are 100 points to be earned. Please write clearly and hand in your answers ordered numerically.

Question 1 (40 points)

Agree or disagree?
Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) In the data, the intensive margin (hours per employed worker) accounts for most of the fluctuations in aggregate hours worked.

(**b**) In an OverLapping Generations model with capital, a public pension system may be desirable from a social welfare perspective.

(**c**) In the Solow growth model, a higher aggregate savings rate always increases consumption in the long run.

(**d**) If the Kaldor facts hold *and* the *nominal* return on capital is stable in the long-run, then inflation is also stable in the long run.

(**e**) In the Money-in-the-Utility-Model, it is socially optimal to keep the money supply constant in the long run.

(**f**) In the Diamond-Mortensen-Pissarides model, the introduction of a minimum wage may lead to greater income inequality.

[1] In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 2 (35 points)

Consider an economy with a continuum of identical and infinitely-lived households, of measure one in total. Discounted lifetime utility at time zero, U_0, is given by:

$$U_0 = \sum_{s=0}^{\infty} \beta^s \left(\frac{c_s^{1-\sigma} - 1}{1-\sigma} - \frac{h_s^{1+\kappa}}{1+\kappa} \right),$$

where c_s denotes consumption in period s, h_s denotes hours of work, $\beta \in (0,1)$ is the households' discount factor, and $\sigma, \kappa \geq 0$ are preference parameters. Households own the capital stock, which they rent out to firms at a rate $r_{k,s}$. They also supply labor to the firms, at a rate w_s per hour. The households' budget constraint for period s is given by:

$$c_s + k_{s+1} = (1 - \delta + r_{k,s})\, k_s + w_s h_s + \Pi_s,$$

where $\delta \in (0,1)$ is the rate of capital depreciation and Π_s denote profits of firms, who are owned by the household. In each period s, households optimally choose c_s, k_{s+1} and h_s, taking as given the wage, profits, the rental rate of capital and an initial capital stock $k_0 > 0$. You may ignore the constraint that $c_s, k_s \geq 0$ in every period.

(**a**) Set up the maximization problem of the households and derive the first-order conditions.

(**b**) Show that the Frisch elasticity of labour supply equals $\frac{1}{\kappa}$.

(**c**) Use the first-order condition for labour to show that, if the Frisch elasticity were infinite ($\kappa = 0$), the elasticity of consumption with respect to the wage, $\frac{dc_s/c_s}{dw_s/w_s}$, would equal $\frac{1}{\sigma}$.

(**d**) Suppose you were to have time-series observations for $\ln c_s$, $\ln h_s$, and $\ln w_s$. The series are generated from the above model (with $\kappa, \sigma \geq 0$), but are all measured with i.i.d. noise. Describe how you could use the households' first-order condition for labour to obtain estimates for σ and κ from these time series (Hint: taking logs gives $\ln w_s = \sigma \ln c_s + \kappa \ln h_s$).

On the production side of the economy, there is a large number of identical firms, of measure one in total. Firms choose capital and labor inputs to maximize profits, operating the following production function:

$$y_s = k_s^{\alpha} h_s^{1-\alpha}, \ \alpha \in (0,1),$$

where y_s denotes output. Like households, firms take w_s and $r_{k,s}$ as given. The government taxes firms' wage payments at a rate $\tau \in [0,1]$. In each period, the government balances its budget by using tax revenues to provide firms with a lump-sum subsidy, denoted Ω_s. Firm profits are therefore given by:

$$\Pi_s = y_s - w_s (1+\tau)\, h_s - r_{k,s} k_s + \Omega_s.$$

(**e**) Set up the firms' profit maximization problem and derive the first-order conditions.

(**f**) Derive an expression for the amount of profits made by firms in equilibrium.

(**g**) Suppose now that $\kappa = \sigma = 0$. Use the first-order conditions to show that the economy's capital-labor ratio, $\frac{k_s}{h_s}$, is increasing in τ. What is the intuition?

Question 3 (10 points)

Consider the Diamond-Mortensen-Pissarides model of the labour market, as discussed in the lecture but without aggregate uncertainty. Let $\overline{A}$ be a parameter capturing the productivity of a worker and let the wage of a worker be fixed and given by a parameter $\overline{w} \in \left(\overline{b}, \overline{A}\right)$, where $\overline{b}$ is the unemployment benefit. Further, let the aggregate matching function be given by $m_t = \phi\sqrt{u_t v_t}$, where m_t is the number of newly formed worker-firm pairs in time period t, u_t is the unemployment rate, v_t is the number of vacancies, and $\phi > 0$ is a scaling's parameter. New worker-firm pairs can start producing one period after they meet. The unemployment rate evolves as:

$$u_{t+1} = (1 - f_t)\, u_t + \rho\, (1 - u_t)\,,$$

where $f_t \equiv \frac{m_t}{u_t}$ is the job finding rate, which is endogenous, and $\rho \in (0, 1)$ is the rate at which workers and firms separate, which is an exogenous parameter. Let $q_t \equiv \frac{m_t}{v_t}$ be the vacancy filling rate.

Consider now the deterministic steady state of the model, denoted by the absence of time subscripts on the variables. The steady-state job creation condition (or "free-entry condition") can be written as:

$$\frac{\vartheta}{q} = \beta \frac{\overline{A} - \overline{w}}{1 - \beta\,(1 - \rho)},$$

where $\vartheta > 0$ is the cost of posting a vacancy, and $\beta \in (0, 1)$ is the agents' discount factor.

(**a**) Express the steady-state unemployment rate u as a function of ρ and f. Also, express f as a function of model parameters only.

(**b**) Consider an unexpected and permanent increase in the separation rate ρ. Via which channels does this affect the unemployment rate in the new steady state? Describe the intuition and also refer to your answers under (**a**).

Question 4 (15 points)

Consider the following OverLapping Generations economy with housing. During each period, two generations exist, both of unit measure. Within each generation, agents are identical. Young agents receive an endowment $e_y > 0$ whereas old agents receive an endowment $e_o > 0$. Both young and old agents derive utility from non-durable consumption, whereas only young agents enjoy housing (old agents cannot own housing). Young agents solve the following utility maximization problem:

$$\begin{aligned}
&\max_{c_{y,t},h_{y,t},c_{o,t+1}} \quad \ln c_{y,t} + \eta \ln h_{y,t} + \ln c_{o,t+1}, \\
&s.t. \\
&c_{y,t} + q_t h_{y,t} = e_y, \\
&c_{o,t+1} = q_{t+1} h_{y,t} + e_o,
\end{aligned}$$

where $c_{y,t}$ and $c_{o,t}$ are, respectively, non-durable consumption of the young and the old agents in period t, $h_{y,t}$ is the amount of housing consumed by the young, q_t is the price of housing in units of non-durables, $\eta \geq 0$ is a housing preference parameter. The total supply of housing is fixed and equal to $H > 0$.

(**a**) Derive the first-order conditions for the optimization problems of the young agents and provide intuition.

Consider now the case with $\eta = 0$, i.e. households derive no intrinsic utility from housing. Also, assume that $e_y > e_o$ from now on.

(**b**) Show that there is a "bubble" equilibrium in which housing has a constant value given by $q = \frac{e_y - e_o}{2H} > 0$.

In addition to equilibrium mentioned under (**b**) there is also a "fundamental" equilibrium in which housing has no value.

(**c**) What happens to non-durables consumption of the young and old agents when the housing bubble bursts, i.e. when the economy unexpectedly moves from the bubble equilibrium to the fundamental equilibrium?

$\in$**Exam MSc Macroeconomics, EconG022**

2017

Instructions. This exam consists of five pages with four questions in total. You have THREE HOURS to answer all four questions. There are 100 points to be earned. Please write clearly and hand in your answers ordered numerically.

Question 1 (40 points)

Agree or disagree?
Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) In the data, the intensive margin (hours per employed worker) accounts for most of the fluctuations in aggregate hours worked.

SHORT ANSWER: FALSE. Fluctuations in aggregate hours worked may be decomposed into the contribution of the intensive margin (hours per employed person) and the extensive margin (number of employed persons). In the data, fluctuations in the number of employed persons are much larger than the number of hours per employed person. Hence, the extensive margin accounts for most of the fluctuations in aggregate hours worked.

(**b**) In an OverLapping Generations model with capital, a public pension system may be desirable from a social welfare perspective.

SHORT ANSWER: TRUE. In the OLG model with capital, the steady-state equilibrium may be inefficient due to overaccumulation of capital. A public pension system may help to reduce capital aggregate accumulation. However, not all pension systems have this effect. In particular, under a fully funded pension system, private saving is reduced, but is exactly compensated for by an increase in public saving. Hence, aggregate saving does not change. Under a pay-as-you-go system, by contrast, there is no increase in public saving, while private saving is reduced. Hence, aggregate saving in capital can be reduced via a pay-as-you-go system, alleviating inefficient capital accumulation and thereby improving welfare.

(**c**) In the Solow growth model, a higher aggregate savings rate always increases consumption in the long run.

SHORT ANSWER: FALSE. In the Solow Model, steady-state consumption is a hump-shaped function of the savings rate, reaching a maximum at the golden rule level. If the aggregate savings rate is already above the golden rule level, a further increase in the saving rate reduces steady-state consumption. [can be illustrated with a diagram].

(**d**) If the Kaldor facts hold *and* the *nominal* return on capital is stable in the long-run, then inflation is also stable in the long run.

SHORT ANSWER: TRUE. One of the Kaldor facts states that the real return on capital, r, is constant in the long run. The nominal return on capital can be expressed as $r^{nom} = r(1+\pi)$, where π is the inflation rate. It follows that if both r and r^{nom} is constant in the long run, then π is constant in the long run.

(**e**) In the Money-in-the-Utility-Model, it is socially optimal to keep the money supply constant in the long run.

[1] In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

SHORT ANSWER: FALSE. The optimal long-run policy, as given by the Friedman rule, prescribes that money supply growth sets the net nominal interest rate r^{nom} equal to zero. The associated inflation rate is given by $\pi = \beta - 1 < 0$, where $\beta \in (0,1)$ is the stochastic discount factor. The latter follows from the steady-state Euler eqation for nominal bonds, which reads $1 = \beta \frac{1+r^{nom}}{1+\pi}$. Recall further that in the steady state of the MIU model, the inflation rate, π, equals the growth rate of the money supply θ. It follows that the money growth rate is optimally set to $\theta = \beta - 1 < 0$. Thus, the optimal monetary policy prescribes the money supply to shrink over time.

(**f**) In the Diamond-Mortensen-Pissarides model, the introduction of a minimum wage may lead to greater income inequality.

SHORT ANSWER: TRUE. One can think of two channels. First, a (binding) minimum wage increases the difference in income between those who are employed and those who are unemployed, provided that unemployment benefits are not increased by the same amount as the wage. Second, the minimum wage reduces firm profits and hence may lead to a decline in vacancy posting, thereby increasing the unemployment rate. For moderate levels of the unemployment rate, an increase in unemployment increases income inequality.[2]

[2] *Formally (not required) one could show that the cross-sectional variance of income is give by $(w-b)^2 u(1-u)$, where w is the wage, b is the unemployment benefit and u* is the unemployment rate. *The first channel increases $(w-b)^2$ via an increase in w. The second channel increases $u(1-u)$ via an increase in u, provided that u is below 50 percent.*

Question 2 (35 points)

Consider an economy with a continuum of identical and infinitely-lived households, of measure one in total. Discounted lifetime utility at time zero, U_0, is given by:

$$U_0 = \sum_{s=0}^{\infty} \beta^s \left(\frac{c_s^{1-\sigma} - 1}{1-\sigma} - \frac{h_s^{1+\kappa}}{1+\kappa} \right)$$

where c_s denotes consumption in period s, h_s denotes hours of work, $\beta \in (0,1)$ is the households' discount factor, and σ, $\kappa \geq 0$ are preference parameters. Households own the capital stock, which they rent out to firms at a rate $r_{k,s}$. They also supply labor to the firms, at a rate w_s per hour. The households' budget constraint for period s is given by:

$$c_s + k_{s+1} = (1 - \delta + r_{k,s}) k_s + w_s h_s + \Pi_s$$

where $\delta \in (0,1)$ is the rate of capital depreciation and Π_s denote profits of firms, who are owned by the household. In each period s, households optimally choose c_s, k_{s+1} and h_s, taking as given the wage, the rental rate of capital and an initial capital stock $k_0 > 0$. You may ignore the constraint that $k_s \geq 0$ in every period.

(**a**) Set up the maximization problem of the households and derive the first-order conditions.

SHORT ANSWER: The Lagrangian is given by:

$$\boldsymbol{L} = \max_{\{c_s, k_{s+1}, h_s\}} \sum_{s=0}^{\infty} \left[\beta^s \left(\frac{c_s^{1-\sigma} - 1}{1-\sigma} - \frac{h_s^{1+\kappa}}{1+\kappa} \right) + \lambda_s \left[(1 - \delta + r_{k,s}) k_s + w_s h_s - c_s - k_{s+1} \right] \right]$$

First-order conditions (after substituting out the Lagrange multiplier λ_s):

$$\begin{aligned} w_s c_s^{-\sigma} &= h_s^{\kappa} \\ c_s^{-\sigma} &= (1 - \delta + r_{k,s+1}) c_{s+1}^{-\sigma} \end{aligned}$$

for $s = 0, 1, 2, ...$

(**b**) Show that the Frisch elasticity of labour supply equals $\frac{1}{\kappa}$.

SHORT ANSWER: Recall that the Frisch elasticity of labour supply is given by $\frac{dh_s/h_s}{dw_s/w_s}$, keeping $\lambda_s = c_s^{-\sigma}$ constant. Taking logs on both sides of the first-order condition for labour supply and totally differentiating, keeping $c_s^{-\sigma}$ constant, gives:

$$\frac{dw_s}{w_s} = \kappa \frac{dh_s}{h_s}.$$

It now follows directly that the Frisch labour supply elasticity is given by $\frac{1}{\kappa}$. Alternatively, could arrive at the same answer by differentiating without taking logs:

$$c_s^{-\sigma} dw_s = \kappa h_s^{\kappa - 1} dh_s$$

$\Rightarrow$

$$\frac{dh_s/h_s}{dw_s/w_s} = \frac{\frac{c_s^{-\sigma} dw_s}{\kappa h_s^{\kappa-1} h_s}}{dw_s/w_s} = \frac{\frac{c_s^{-\sigma} dw_s}{\kappa h_s^{\kappa}}}{dw_s/w_s} = \frac{\frac{c_s^{-\sigma}}{\kappa h_s^{\kappa}}}{1/w_s} = \frac{w_s c_s^{-\sigma}}{\kappa h_s^{\kappa}} = \frac{1}{\kappa}$$

Yet another alternative is to directly apply the formula for the Frisch elasticity:

$$\frac{dh_s/h_s}{dw_s/w_s} = \frac{U_h(h_s, c_s)}{U_{hh}(h_s, c_s)h_s} = \frac{h_s^{\kappa}}{\kappa h_s^{\kappa-1} h_s} = \frac{1}{\kappa}$$

(**c**) Show that, if the Frisch elasticity were infinite ($\kappa = 0$), the elasticity of consumption with respect to the wage, $\frac{dc_s/c_s}{dw_s/w_s}$, equals $\frac{1}{\sigma}$.

SHORT ANSWER: First note that when the Frisch elasticity is infinite then $\kappa = 0$. In that case, the first-order condition for labour reduces to:

$$w_s = c_s^{\sigma}.$$

Taking logs a differentiating gives $\frac{dw_s}{w_s} = \sigma \frac{dc_s}{c_s}$ and it follows that the elasticity of consumption w.r.t. the wage is given by $\frac{dc_s/c_s}{dw_s/w_s} = \frac{1}{\sigma}$. Atlernatively, one can totally differentiate without taking logs:

$$dw_s = \sigma c_s^{\sigma-1} dc_s$$

and hence

$$\frac{dc_s/c_s}{dw_s/w_s} = \frac{dc_s}{dw_s}\frac{w_s}{c_s} = \frac{w_s}{\sigma c_s^{\sigma}} = \frac{1}{\sigma}.$$

(**d**) Suppose you were to have time-series observations for $\ln c_s$, $\ln h_s$, and $\ln w_s$. The series are generated from the above model (with $\kappa, \sigma \geq 0$), but are all measured with i.i.d. noise. Describe how you could use the households' first-order condition for labour to obtain estimates for σ and κ from these time series (Hint: taking logs gives $\ln w_s = \sigma \ln c_s + \kappa \ln h_s$).

SHORT ANSWER: Taking logs on both sides of the labour supply equation and re-writing gives:

$$\ln w_s = \sigma \ln c_s + \kappa \ln h_s$$

Let $\widehat{w}_s = \ln w_s + \epsilon_{w,s}$, $\widehat{c}_s = \ln c_s + \epsilon_{c,s}$, and $\widehat{h}_s = \ln h_s + \epsilon_{h,s}$ be the measured time series variables, where $\epsilon_{w,s}$, $\epsilon_{c,s}$ and $\epsilon_{h,s}$ are the i.i.d. noise components. We can now re-write the labour supply condition as:

$$\widehat{w}_s = \sigma \widehat{c}_s + \kappa \widehat{h}_s + \epsilon_{w,s} - \sigma \epsilon_{c,s} - \kappa \epsilon_{h,s}.$$

Note that $\epsilon_{w,s}$-$\sigma\epsilon_{c,s}$-$\kappa\epsilon_{h,s}$ is itself an i.i.d. noise variable. It follows that we can obtain estimates by running a linear regression of $\widehat{w}_s$ on $\widehat{c}_s$ and $\widehat{h}_s$.

On the production side of the economy, there is a large number of identical firms, of measure one in total. Firms choose capital and labor inputs to maximize profits, operating the following production function:

$$y_s = k_s^{\alpha} h_s^{1-\alpha}, \ \alpha \in (0, 1),$$

where y_s denotes output. Like households, firms take w_s and $r_{k,s}$ as given. The government taxes firms' wage payments at a rate $\tau \in [0, 1]$. In each period, the government balances its budget by providing firms with a lump-sum subsidy, denoted Ω_s. Firm profits are therefore given by:

$$\Pi_s = y_s - w_s (1 + \tau) h_s - r_{k,s} k_s + \Omega_s.$$

(**e**) Set up the firms' profit maximization problem and derive the first-order conditions.

SHORT ANSWER:

$$\max_{h_s, k_s} k_s^\alpha h_s^{1-\alpha} - w_s (1 + \tau) h_s - r_{k,s} k_s + \Omega_s$$

The first-order conditions can be written as:

$$\begin{aligned} w_s &= \frac{1-\alpha}{1+\tau} \frac{y_s}{h_s}, \\ r_{k,s} &= \alpha \frac{y_s}{k_s}. \end{aligned}$$

(**f**) Derive an expression for the amount of profits made by firms in equilibrium.

SHORT ANSWER: Balancing of the government budget constraint implies $\Omega_s = \tau w_s h_s$. *We can now write firm profits as:*

$$\begin{aligned} \Pi_s &= y_s - w_s (1 + \tau) h_s - r_{k,s} k_s + \tau w_s h_s \\ &= y_s - (1 - \alpha) y_s - \alpha y_s + \tau w_s h_s \\ &= \tau w_s h_s \end{aligned}$$

(**g**) Suppose now that $\kappa = \sigma = 0$. Use the first-order conditions to show that the economy's capital-labor ratio, $\frac{k_s}{h_s}$, is increasing in τ. What is the intuition?
SHORT ANSWER: The first-order condition for labour supply becomes:

$$w_s = 1$$

Plugging this result into the first-order condition for labor demand gives:

$$1 = \frac{1-\alpha}{1+\tau} \left(\frac{k_s}{h_s} \right)^\alpha$$

or

$$\frac{k_s}{h_s} = \left(\frac{1+\tau}{1-\alpha} \right)^{\frac{1}{\alpha}}$$

Given that $\tau, \alpha \in (0, 1)$ and this an increasing function of τ. Intuitively, a tax on labour encourages firms to substitute towards capital.

Question 3 (10 points)

Consider the Diamond-Mortensen-Pissarides model of the labour market, as discussed in the lecture but without aggregate uncertainty. Let $\overline{A}$ be a parameter capturing the productivity of a worker and let the wage of a worker be fixed and given by a parameter $\overline{w} \in \left(\overline{b}, \overline{A}\right)$, where $\overline{b}$ is the unemployment benefit. Further, let the aggregate matching function be given by $m_t = \phi\sqrt{u_t v_t}$, where m_t is the number of newly formed worker-firm pairs in time period t, u_t is the unemployment rate, v_t is the number of vacancies, and $\phi > 0$ is a scaling's parameter. New worker-firm pairs can start producing one period after they meet. The unemployment rate evolves as:

$$u_{t+1} = (1 - f_t)\, u_t + \rho\,(1 - u_t)\,,$$

where $f_t \equiv \frac{m_t}{u_t}$ is the job finding rate, which is endogenous, and $\rho \in (0,1)$ is the rate at which workers and firms separate, which is an exogenous parameter. Let $q_t \equiv \frac{m_t}{v_t}$ be the vacancy filling rate.

Consider now the deterministic steady state of the model, denoted by the absence of time subscripts on the variables. The steady-state job creation condition (or "free-entry condition") can be written as:

$$\frac{\vartheta}{q} = \beta\frac{\overline{A} - \overline{w}}{1 - \beta\,(1 - \rho)},$$

where $\vartheta > 0$ is the cost of posting a vacancy, and $\beta \in (0,1)$ is the agents' discount factor.

(**a**) Express the steady-state unemployment rate u as a function of ρ and f. Also, express f as a function of model parameters only.

SHORT ANSWER: Using the first equation, the steady-state unemployment rate can be written as:

$$u = \frac{\rho}{f + \rho}$$

Next, note that $q = \frac{m}{v} = \phi\sqrt{\frac{u}{v}}$. *It follows that* $\frac{v}{u} = \left(\frac{\phi}{q}\right)^2$ *and hence* $f = \frac{m}{u} = \phi\sqrt{\frac{v}{u}} = \frac{\phi^2}{q}$. *We can now re-write the free-entry condition as:*

$$\frac{\vartheta f}{\phi^2} = \beta\frac{\overline{A} - \overline{w}}{1 - \beta\,(1 - \rho)},$$

$$\Longleftrightarrow$$

$$f = \frac{\phi^2\beta}{\vartheta}\frac{\overline{A} - \overline{w}}{1 - \beta\,(1 - \rho)}.$$

(**b**) Consider an unexpected and permanent increase in the separation rate ρ. Via which channels does this affect the unemployment rate in the new steady state? Describe the intuition and also refer to your answers under (**a**).

SHORT ANSWER: A first channel is that the increase in separation rate directly increase the size of the inflow into unemployment. This effect comes in via the steady-state expression

$u = \frac{\rho}{f+\rho}$ *which is increasing in* ρ *given that* $f > 0$*. The second channel is more subtle and comes in via the expression for the job finding rate* f*. The right-hand side of the above expression for* f is decreasing in ρ*. An increase in* ρ *means that in expectation, matches are shorter-lived. This discourages vacancy posting and free entry dictates that in equilibrium the vacancy filling rate,* q*, must go up. This in turn implies that the labour market becomes less tight and the job finding rate goes down.*

Question 4 (15 points)

Consider the following OverLapping Generations economy with housing. During each period, two generations exist, both of unit measure. Within each generation, agents are identical. Young agents receive an endowment $e_y > 0$ whereas old agents receive an endowment $e_o > 0$. Both young and old agents derive utility from non-durable consumption, whereas only young agents enjoy housing (old agents cannot own housing). Young agents solve the following utility maximization problem:

$$\begin{aligned}
&\max_{c_{y,t},h_{y,t},c_{o,t+1}} \quad \ln c_{y,t} + \eta \ln h_{y,t} + \ln c_{o,t+1}, \\
&s.t. \\
&c_{y,t} + q_t h_{y,t} = e_y, \\
&c_{o,t+1} = q_{t+1} h_{y,t} + e_o,
\end{aligned}$$

where $c_{y,t}$ and $c_{o,t}$ are, respectively, consumption of the young and the old agents in period t, $h_{y,t}$ is the amount of housing consumed by the young, q_t is the price of housing in units of non-durables, $\eta \geq 0$ is a housing preference parameter. The total supply of housing is fixed and equal to $H > 0$.

(**a**) Derive the first-order conditions for the optimization problems of the young agents and provide intuition for the equation.

SHORT ANSWER: The (combined) first-order conditions are:

$$\frac{q_t}{c_{y,t}} = \frac{\eta}{h_{y,t}} + \frac{q_{t+1}}{c_{o,t+1}}$$

Intuition: the left-hand side captures the marginal utility cost of owning more housing, which equals the price of housing times the marginal utility of non-durable consumption. The right-hand side captures the benefits, which consist of two components. The first, $\frac{\eta}{h_{y,t}}$*, is the marginal utility from housing. The second,* $\frac{q_{t+1}}{c_{o,t+1}}$*, is the marginal utility from owning more housing wealth when going into the second life-cycle stage.*

Consider now the case with $\eta = 0$, i.e. households derive no intrinsic utility from housing. Also assume that $e_y > e_o$ from now on.

(**b**) Show that there is a "bubble" equilibrium in which housing has a constant value given by $q = \frac{e_y - e_o}{2H} > 0$.

SHORT ANSWER: First note that in equilibrium, each young agents holds H *units of housing. This follows from housing market clearing. When* $\eta = 0$ *the budget constraints can be used to write the first-order condition as:*

$$\frac{e_o + q_{t+1}H}{e_y - q_t H} = \frac{q_{t+1}}{q_t}$$

Consider now an equilibrium with a positive but constant house price i.e. $q_{t+1} = q_t = q > 0$*. The first-order condition then becomes*

$$e_o + qH = e_y - qH,$$

$\Leftrightarrow$

$$q = \frac{e_y - e_o}{2H}.$$

In addition to equilibrium mentioned under (b) there is also a "fundamental" equilibrium in which housing has no value.

(**c**) What happens to non-durables consumption of the young and old agents when the housing bubble bursts, i.e. when the economy unexpectedly moves from the bubble equilibrium to the fundamental equilibrium?

SHORT ANSWER: In the fundamental equilibrium households live in autarky, i.e. $c_{y,t} = e_y$ *and* $c_{o,t} = e_o$*. In the bubble equilibrium it holds that* $c_{y,t} < e_y$ *and* $c_{o,t} > e_o$*, since the agents purchase housing when young and sell it when old. It follows that when the bubble bursts, consumption of the young increases, whereas consumption of the old declines.*

Exam MSc Macroeconomics, EconG022
2018

Instructions. This exam consists of five pages with four questions in total. You have THREE HOURS to answer all four questions. There are 100 points to be earned. Please write clearly and hand in your answers ordered numerically.

Question 1 (40 points)

Agree or disagree?
Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) In the Ramsey growth model, a benevolent social planner will choose consumption to be constant over time.

(**b**) The Kaldor facts imply that aggregate labor income and aggregate total income grow at the same rate in the long run.

(**c**) In the Diamond-Mortensen-Pissarides model of the labour market, there is no investment.

(**d**) In the Real Business Cycle model, fluctuations in aggregate hours worked are socially efficient.

(**e**) In an OverLapping Generations model with capital, the introduction of a pay-as-you-go pension system may increase steady-state consumption.

(**f**) The consumption Euler equation implies that in economies with high consumption growth, nominal interest rates are high.

[1] In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 2 (35 points)

Consider an economy with a continuum of identical and infinitely-lived households, of measure one in total. Households consume two types of non-durable goods: goods produced by firms on the market and home-produced goods. Discounted lifetime utility at time t is given by $U_t = \sum_{s=t}^{\infty} \beta^s (\ln c_{m,s} + \eta \ln c_{h,s})$, where $c_{m,s}$ denotes market produced goods at time s, $c_{h,s}$ denotes home-produced consumption goods, $\eta > 0$ is a preference parameter, and $\beta \in (0,1)$ is the subjective discount factor. Households have a total time endowment denoted by T hours per period and supply labour on a competitive labor market in return for an hourly wage w_s. Let $l_s \leq T$ denote hours worked in the market sector. The remaining time $T - l_s$ is spent producing goods at home. The home production function is given by $c_{h,s} = d_s^{\alpha} (T - l_s)^{1-\alpha}$, with $\alpha \in (0,1)$, where d_s is a durable good used in home production, which depreciates at a rate $\delta \in (0,1)$. The durable good is produced by firms on the market, using the same technology as for the non-durable market good. The flow budget constraint of the household is given by:

$$c_{m,s} + d_s = (1-\delta)\, d_{s-1} + w_s l_s + \Pi_s.$$

where Π_s denotes profits of firms, which are owned by the household. The household starts with an initial durables stock d_{t-1}. In each period, the household chooses $c_{m,s}$, $c_{h,s}$, d_s and l_s.

(**a**) Discuss why the model can be re-interpreted as one in which households derive utility from consuming non-durables ($c_{m,s}$), durables (d_s) and leisure ($T - l_s$), with a flow utility function given by $\ln c_{m,s} + \eta\alpha \ln d_s + \eta (1-\alpha) \ln (T - l_s)$.

(**b**) Set up the maximization problem of the households and derive the first-order optimality conditions for $c_{m,s}$, d_s and l_s.

(**c**) Show that the Frisch elasticity of labor supply is given by $\frac{T-l_s}{l_s}$.

The market goods are produced by competitive firms which produce only labor, operating a production linear technology $y_s = l_s$.

(**d**) Show that (i) the equilibrium wage is given by $w_s = 1$, and (ii) firms make no profits, i.e. $\Pi_s = 0$.

(**e**) Define the competitive equilibrium.

(**f**) Let $i_s \equiv d_s - (1-\delta)\, d_{s-1}$ denote investment in durables. Show that in the steady-state equilibrium, $\frac{c_m}{i}$, the ratio of expenditures on market-produced non-durables to investment in durables, is given by $\frac{c_m}{i} = \frac{1-\beta(1-\delta)}{\delta\eta\alpha}$. Explain intuitively why this ratio is decreasing in β.

During recent years, aggregate hours worked by men in the United States have declined substantially. Some commentators have suggested that the increased quality of video games had made it more attractive for men to stay at home instead of work. To think about this narrative through the lens of the model, let us extend the home production function as

$c_{h,s} = (Ad_s)^{\alpha} (T - l_s)^{1-\alpha}$, where A is a parameter which captures the efficiency of durables in home production.

(**g**) What does the model say about the effect of an increase in A on aggregate hours worked and social welfare? Hint: revisit your answer under (a).

Question 3 (15 points)

Consider the following infinite-horizon OverLapping Generations economy with bitcoins, denoted by b, with a unit price denoted by p. During each period, two generations exist, both of unit measure. Within each generation, agents are identical. Young agents receive an endowment $e_y > 0$ whereas old agents receive an endowment $e_o \in (0, e_y)$. Young agents solve the following utility maximization problem:

$$\begin{aligned}
&\max_{c_{y,t}, b_{y,t}, c_{o,t+1}} \quad u\left(c_{y,t}\right) + u\left(c_{o,t+1}\right), \\
&s.t. \\
&c_{y,t} + p_t b_{y,t} = e_y, \\
&c_{o,t+1} = p_{t+1} b_{y,t} + e_o,
\end{aligned}$$

where $c_{y,t}$ and $c_{o,t}$ are, respectively, non-durable consumption of the young and the old agents in period t, $b_{y,t}$ are bitcoins purchased when young, and $u\left(c\right) = \frac{c^{1-\gamma}}{1-\gamma}$, $\gamma > 0$, is a utility function. For simplicity, we assume that the aggregate supply of bitcoins is constant and given by B.

(**a**) Derive the first-order conditions for the optimization problems of the young agents.

(**b**) Show that, in an equilibrium in which the price of the bitcoin is positive and constant over time, the price of the bitcoin is given by $p = \frac{e_y - e_o}{2B}$. Explain intuitively why p is increasing in $e_y - e_o$.

Suppose that when the bitcoin is first introduced, the supply is owned by the then old generation.

(**c**) Show that in the equilibrium mentioned under (b), any generation is better off than in an economy without the introduction of bitcoins (i.e. $B = 0$). Does this mean that bitcoins generally improve social welfare *within this model*?

Question 4 (10 points)

Consider an economy with a number of firms indexed by i, operating production technologies $y_i = A_i k_i^{\alpha} l_i^{\gamma}$, $\alpha + \gamma < 1$, where A_i is a firm-specific Total Factor Productivity (TFP) level, k_i is the firm's capital and l_i is its labour input. Firms all produce the same goods but differ in their levels of TFP (and as a result their levels of capital and labour). Firms rent capital and labor on competitive markets, at prices given by r and w. Note that r and w are the same across firms. They choose capital and labour to maximize profits, which are given by $\Pi_i = y_i - rk_i - wl_i$.

(**a**) Show that labor productivity, i.e. output per unit of labor input, is the same across all firms, despite the heterogeneity in TFP.

In the real-world data, there are large differences in labor productivity across firms. Based on models like the one discussed above, some economists have interpreted this observation as a sign of misallocation of labor. The underlying idea is that some firms face distortions in hiring labor, and these distortions may be larger for some firms than for others.

(**b**) How would you modify the model, in a simple way, to think about the possibility of labor misallocation and its connection with dispersion in labor productivity? Hint: introduce labor wedges.

Exam MSc Macroeconomics, EconG022
2018

Instructions. This exam consists of four questions in total. You have THREE HOURS to answer all four questions. There are 100 points to be earned. Please write clearly and hand in your answers ordered numerically.

Question 1 (40 points)

Agree or disagree?
Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) In the Ramsey growth model, a benevolent social planner will choose consumption to be constant over time.

(**b**) The Kaldor facts imply that aggregate labor income and aggregate total income grow at the same rate in the long run.

(**c**) In the Diamond-Mortensen-Pissarides model of the labour market, there is no investment.

(**d**) In the Real Business Cycle model, fluctuations in aggregate hours worked are socially efficient.

(**e**) In an OverLapping Generations model with capital, the introduction of a pay-as-you-go pension system may increase steady-state consumption.

(**f**) The consumption Euler equation implies that in economies with high consumption growth, nominal interest rates are high.

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Question 1 Solution

(**a**) False. Consumption may be constant in a steady state, but given some arbitrary initial level of capital, the path of consumption varies along the transition path towards the steady state. This relates to the fact that the marginal product of capital, which affects the incentive to invest and hence the consumption-saving decision, changes along the transition path. For example, with a low initial level of capital, the marginal product of capital is high, creating a strong incentive to invest, at the expense of less consumption. As the capital stock grows over time, the incentive weakens while the availability of resources increases, pushing up consumption.

(**b**) True. One of the Kaldor facts is that the labor share of income $\frac{wL}{Y}$ is constant in the long run. The numerator is aggregate labor income whereas the denominator is aggregate total income. If the ratio is constant over time then both must grow at the same rate.

(**c**) False, there may not be capital investment, but vacancies are a form of investment in the DMP model. A vacancy leads to a match with a worker, which lasts for a certain amount in time, generating profits for the firm in the future. At the same time, posting a vacancy in the DMP model requires an upfront cost. Hence, it can be thought of as an investment, requiring a cost today, but generating profits in the future.

(**d**) True. It is an infinite horizon model without market imperfections. The two welfare theorems imply that the equilibrium is efficient, i.e. the social planner outcome coincides with the decentralized outcome. The equilibrium is thus socially efficient, even though macroeconomic variables fluctuate along the equilibrium path due to technology shocks.

(**e**) True. The PAYG system crowds out private savings without increasing public savings, and hence aggregate savings decline. As a result, the steady-state capital stock declines. Recall that steady-state consumption is non-monotonic in the capital stock. If the steady-state level of capital without PAYG is above the Golden Rule level, this means that steady-state consumption increases.

(**f**) False. The Euler equation implies that high consumption growth is associated with high real interest rates (agents like to borrow and smooth consumption). However, a higher real interest rate does not necessarily imply a high nominal interest rate, since lower inflation also implies a higher real interest rate.

Question 2 (35 points)

Consider an economy with a continuum of identical and infinitely-lived households, of measure one in total. Households consume two types of non-durable goods: goods produced by firms on the market and home-produced goods. Discounted lifetime utility at time t is given by $U_t = \sum_{s=t}^{\infty} \beta^s (\ln c_{m,s} + \eta \ln c_{h,s})$, where $c_{m,s}$ denotes market produced goods at time s, $c_{h,s}$ denotes home-produced consumption goods, $\eta > 0$ is a preference parameter, and $\beta \in (0,1)$ is the subjective discount factor. Households have a total time endowment denoted by T hours per period and supply labour on a competitive labor market in return for an hourly wage w_s. Let $l_s \leq T$ denote hours worked in the market sector. The remaining time $T - l_s$ is spent producing goods at home. The home production function is given by $c_{h,s} = d_s^{\alpha} (T - l_s)^{1-\alpha}$, with $\alpha \in (0,1)$, where d_s is a durable good used in home production, which depreciates at a rate $\delta \in (0,1)$. The durable good is produced by firms on the market, using the same technology as for the non-durable market good. The flow budget constraint of the household is given by:

$$c_{m,s} + d_s = (1-\delta) d_{s-1} + w_s l_s + \Pi_s.$$

where Π_s denotes profits of firms, which are owned by the household. The household starts with an initial durables stock d_{t-1}. In each period, the household chooses $c_{m,s}$, $c_{h,s}$, d_s and l_s.

(**a**) Discuss why the model can be re-interpreted as one in which households derive utility from consuming non-durables ($c_{m,s}$), durables (d_s) and leisure ($T - l_s$), with a flow utility function given by $\ln c_{m,s} + \eta\alpha \ln d_s + \eta (1-\alpha) \ln (T - l_s)$.

(**b**) Set up the maximization problem of the households and derive the first-order optimality conditions for $c_{m,s}$, d_s and l_s.

(**c**) Show that the Frisch elasticity of labor supply is given by $\frac{T-l_s}{l_s}$.

The market goods are produced by competitive firms which produce only labor, operating a production linear technology $y_s = l_s$.

(**d**) Show that (i) the equilibrium wage is given by $w_s = 1$, and (ii) firms make no profits, i.e. $\Pi_s = 0$.

(**e**) Define the competitive equilibrium.

(**f**) Let $i_s \equiv d_s - (1-\delta) d_{s-1}$ denote investment in durables. Show that in the steady-state equilibrium, $\frac{c_m}{i}$, the ratio of expenditures on market-produced non-durables to investment in durables, is given by $\frac{c_m}{i} = \frac{1-\beta(1-\delta)}{\delta\eta\alpha}$. Explain intuitively why this ratio is decreasing in β.

During recent years, aggregate hours worked by men in the United States have declined substantially. Some commentators have suggested that the increased quality of video games had made it more attractive for men to stay at home instead of work. To think about this narrative through the lens of the model, let us extend the home production function as

$c_{h,s} = (Ad_s)^{\alpha}(T - l_s)^{1-\alpha}$, where A is a parameter which captures the efficiency of durables in home production.

(**g**) What does the model say about the effect of an increase in A on aggregate hours worked and social welfare? Hint: revisit your answer under (a).

Question 2 Solution

(a) Substituting out $c_{h,s}$ in the utility function (using the home production function) immediately gives $\ln c_{m,s} + \eta\alpha \ln d_s + \eta(1-\alpha)\ln(T-l_s)$. This is precisely the utility function of a household who derives utility directly from non-durables ($c_{m,s}$), from stock of durables (d_s), and leisure ($\ln(T-l_s)$), with utility that is additive in logs. This leads to the aforementioned re-interpretation, since none of the other aspects of the model (budget constraint, firms, market clearing condition) are affected by the substitution.

(b) The present-value utility can be written as:

$$U_t = \sum_{s=t}^{\infty} \beta^s \left(\ln c_{m,s} + \eta\alpha \ln d_s + \eta(1-\alpha)\ln(T-l_s)\right)$$

The household maximizes U_t subject to the budget constraint. The first-order conditions for $c_{m,s}$, d_s and l_s are, respectively, given by:

$$\begin{aligned}
\frac{1}{c_{m,s}} &= \lambda_s, \\
\frac{1}{c_{m,s}} &= \frac{\eta\alpha}{d_s} + \beta(1-\delta)\frac{1}{c_{m,s+1}}, \\
\frac{w_s}{c_{m,s}} &= \frac{\eta(1-\alpha)}{T-l_s},
\end{aligned}$$

where λ_s is the Lagrange multiplier on the budget constraint.

(c) As derived in class, the Frisch elasticity at time s is given by $\frac{U_{l,s}}{U_{ll,s}l_s}$. Note that here $U_{l,s} = -\frac{\eta(1-\alpha)}{T-l_s}$ and $U_{ll,s} = -\frac{\eta(1-\alpha)}{(T-l_s)^2}$. Hen we get $\frac{U_{l,s}}{U_{ll,s}l_s} = \frac{-\frac{\eta(1-\alpha)}{T-l_s}}{-\frac{\eta(1-\alpha)}{(T-l_s)^2}l_s} = \frac{T-l_s}{l_s}$.

(d) Firms choose l_s to maximize $\Pi_s = l_s(1-w_s)$. It follows directly that $w_s = 1$ and $\Pi_s = 0$.

(e) A competitive equilibrium is a sequence of allocations $\{c_{m,s}, d_s, c_{h,s}, l_s\}_{s=t}^{\infty}$ and a sequence of wages $\{w_s\}_{s=t}^{\infty}$ such that (i) the household maximizes lifetime utility subject to its budget constraint and production function for home goods, and an initial stock of durables (ii) firms maximize profits, and (iii) the goods and markets clear.

(f) In the steady state we get from the second first-order condition that $c_m = \frac{1-\beta(1-\delta)}{\eta\alpha}d$. The resource constraint in the steady state is $c_m + \delta d = l$, and steady-state expenditures on durables are given by $i = \delta d$. It now follows that:

$$\frac{c_m}{\delta d} = \frac{1-\beta(1-\delta)}{\delta\eta\alpha}.$$

To understand why the ratio is decreasing in β, consider the first-order condition for durables. The benefits of investing more in durables are captured by the right-hand side and is the sum of the additional current utility the durable delivers and the discounted resale value in the next period, which captures the fact that buying durables is a form of saving. When the household is more patient, i.e. when she has a higher discount factor β, the household values this savings aspect more. This will tilt the household's choice towards more durables, at the expense of less non-durables, i.e. it decreases the non-durables to durables ratio.

(g) Under the new home production function we can express lifetime utility as

$$U_t = \sum_{s=t}^{\infty} \beta^s \left(\ln c_{m,s} + \eta\alpha \ln A + \eta\alpha \ln d_s + \eta \left(1-\alpha\right) \ln \left(T - l_s\right) \right)$$

No other parts of the model are affected. Note now that the term $\eta\alpha \ln A$ enters utility additively, so it is not affected by any of the choices of the households, i.e. it will not affect any of the first-order conditions. Hence, none of the equilibrium outcomes are affected. Hence, the increase in the quality of durables has no effect on equilibrium hours worked. However, it does directly increase utility, i.e. social welfare.

Question 3 (15 points)

Consider the following infinite-horizon OverLapping Generations economy with bitcoins, denoted by b, with a unit price denoted by p. During each period, two generations exist, both of unit measure. Within each generation, agents are identical. Young agents receive an endowment $e_y > 0$ whereas old agents receive an endowment $e_o \in (0, e_y)$. Young agents solve the following utility maximization problem:

$$\begin{aligned}
&\max_{c_{y,t}, b_{y,t}, c_{o,t+1}} \quad u(c_{y,t}) + u(c_{o,t+1}), \\
&s.t. \\
&c_{y,t} + p_t b_{y,t} = e_y, \\
&c_{o,t+1} = p_{t+1} b_{y,t} + e_o,
\end{aligned}$$

where $c_{y,t}$ and $c_{o,t}$ are, respectively, non-durable consumption of the young and the old agents in period t, $b_{y,t}$ are bitcoins purchased when young, and $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, $\gamma > 0$, is a utility function. For simplicity, we assume that the aggregate supply of bitcoins is constant and given by B.

(**a**) Derive the first-order conditions for the optimization problems of the young agents.

(**b**) Show that, in an equilibrium in which the price of the bitcoin is positive and constant over time, the price of the bitcoin is given by $p = \frac{e_y - e_o}{2B}$. Explain intuitively why p is increasing in $e_y - e_o$.

Suppose that when the bitcoin is first introduced, the supply is owned by the then old generation.

(**c**) Show that in the equilibrium mentioned under (b), any generation is better off than in an economy without the introduction of bitcoins (i.e. $B = 0$). Does this mean that bitcoins generally improve social welfare *within this model*?

Question 3 Solution

(**a**) After substituting out Lagrange multipliers the first-order conditions can be combined as:

$$c_{y,t}^{-\gamma} = \beta \frac{p_{t+1}}{p_t} c_{o,t+1}^{-\gamma}$$

(**b**) The Euler equation in this equilibrium can be expressed as

$$(e_y - pB)^{-\gamma} = (e_o + Bp)^{-\gamma}$$

and it follows that

$$2Bp = e_y - e_o$$

or

$$p = \frac{e_y - e_o}{2B}$$

Intuition: the higher is $e_y - e_o$, then the larger is the decrease in endowment upon retirement, i.e. the stronger is the desire to save (note that households are risk averse here and like to smooth consumption). Given that bitcoins are a way of saving, this pushes up the price of a bitcoin.

(**c**) First consider generations born when the bitcoin is introduced, or after, and note that total consumption over the lifetime is not affected by the introduction of the bitcoin. However, without bitcoin the households consume more in the first period than in the second, whereas with bitcoin consumption is equalized across the two periods. Given that the household is risk averse (the utility is concave) the latter outcome gives more lifetime utility. The initially old generation is also better off since they get to consume the proceeds of the initial bitcoin sale. However, this is only one equilibrium. There may be other equilibria, involving e.g. the burst of a bitcoin value, which may hurt some generations and are not generally welfare enhancing.

Question 4 (10 points)

Consider an economy with a number of firms indexed by i, operating production technologies $y_i = A_i k_i^\alpha l_i^\gamma$, $\alpha + \gamma < 1$, where A_i is a firm-specific Total Factor Productivity (TFP) level, k_i is the firm's capital and l_i is its labour input. Firms all produce the same goods but differ in their levels of TFP (and as a result their levels of capital and labour). Firms rent capital and labor on competitive markets, at prices given by r and w. Note that r and w are the same across firms. They choose capital and labour to maximize profits, which are given by $\Pi_i = y_i - rk_i - wl_i$.

(**a**) Show that labor productivity, i.e. output per unit of labor input, is the same across all firms, despite the heterogeneity in TFP.

In the real-world data, there are large differences in labor productivity across firms. Based on models like the one discussed above, some economists have interpreted this observation as a sign of misallocation of labor. The underlying idea is that some firms face distortions in hiring labor, and these distortions may be larger for some firms than for others.

(**b**) How would you modify the model, in a simple way, to think about the possibility of labor misallocation and its connection with dispersion in labor productivity? Hint: introduce labor wedges.

Question 4 Solution

(**a**) The first-order condition for labor is given by

$$w = \gamma A_i k_i^{\alpha} l_i^{\gamma - 1} = \gamma y_i / l_i$$

Since the wage is the same for all firms, and so is γ, labor productivity is equalized across firms.

(**b**) Suppose we introduce a firm-specific labor wedge τ_i between the marginal product and the wage. The first-order condition becomes:

$$w\left(1 + \tau_i\right) = \gamma A_i k_i^{\alpha} l_i^{\gamma - 1} = \gamma y_i / l_i.$$

It is now clear that labor productivity is no longer the same across firms: firms with high distortions (i.e. a high value of τ_i) will have higher levels of labor productivity. Note that labor productivity is proportional to the marginal product of labor. So heterogeneity in labor productivities means that some firms have higher marginal products of labor than others. This means that aggregate output could be raised (if it was not for the wedges) by moving labour from firms with low marginal products of labor to firms with higher marginal products. That is, there is misallocation of labor.

Exam MSc Macroeconomics, Econ0066

2019

Instructions. This exam consists of four questions in total. You have THREE HOURS to answer all four questions. There are 100 points to be earned. Please write clearly and hand in your answers ordered numerically.

Question 1 (40 points)

Agree or disagree?
Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) In the Solow model, aggregate output may not be constant in the long run.

(**b**) In the Real Business Cycle model, consumption follows a random walk.

(**c**) Rational expectations imply that agents do not make prediction errors.

(**d**) Consider an economy with a representative firm, a Cobb-Douglas production function, and a competitive labor market. In this economy, the Marginal Product of Labor can be measured using data on labor productivity (output per hour worked) and the labor share of income.

(**e**) A model with a utility function given by $u(c) = c$, were c denotes consumption, does not generate a risk premium.

(**f**) Unequal allocations may be Pareto optimal.

[1] In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 2. Consumption and saving (30 points)

Consider an infinitely-lived agent with preferences given by

$$U_t = \sum_{k=t}^{\infty} \beta^{k-t} \frac{c_k^{1-\sigma}}{1-\sigma}$$

The agent chooses consumption, $\{c_t\}_{t=0}^{\infty}$, and bonds $\{b_{t+1}\}_{t=0}^{\infty}$ to optimize U_t, subject to the following budget constraint:

$$c_t + b_{t+1} = y + (1+r)b_t,$$

where y is a constant labor income flow and r is the interest rate, which is also constant. We further assume that the discount factor satisfies $\beta = \frac{1}{1+r} \in (0,1)$ and that the agent starts with an initial level of wealth given by b_t.

(**a**) Derive the first-order conditions for consumption and bonds and show that the agent chooses a constant path of consumption, i.e.:

$$c_t = c_{t+1} = c_{t+2} = ..$$

(**b**) Use the budget constraint to show that bond holdings are also constant and can be expressed as:

$$b_t = \sum_{k=1}^{\infty} \frac{c_t - y}{(1+r)^k}$$

(**c**) Use the result under b) to show that consumption is given by:

$$c_t = rb_t + y$$

and provide an intuitive explanation of the result. Hint: to derive the result, note that $\sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^k = \sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^k - 1$.

Researchers have studied the consumption behavior of households who win a small lottery prize. A typical finding is that in the year after winning the lottery, households spend about 60 percent of the prize.

(**d**) Explain why the model described above is unlikely to be consistent with this empirical evidence.

Now consider "rule-of-thumb" households, who simply consume all their current financial wealth plus their income. That is, they set consumption as $c_t = b_t + y$.

(**e**) Which fraction of a lottery prize would these rule-of-thumb households consume in the fist year?

Consider now a model in which a fraction ϕ of the population behaves as these rule of thumb households. The remaining fraction $1-\phi$ behaves as the optimizing households described above, under a)-c).

(**f**) To reconcile this model with the aforementioned empirical evidence on consumption behavior after winning a lottery, what would ϕ (roughly) have to be?

Question 3. Bonds in the utility function (15 points).

Consider an infinite-horizon economy, populated by a representative household, who derives utility from non-durable consumption and holdings of government bonds. The household solves the following decision problem:

$$\max_{\{c_t, b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{\ln c_t + \alpha \ln b_{t+1}\},$$

subject to

$$c_t + b_{t+1} = y + \frac{1 + r_{t-1}}{1 + \pi_t} b_t - \tau_t,$$

where $\beta \in (0, 1)$ is the household's subjective discount factor, $\alpha > 0$ is a preference parameter, b_{t+1} is the real value of bonds purchased in period t, c_t is non-durable consumption, r_t is the net nominal interest rate on bonds, π_t is the net rate of inflation, $y > 0$ is an exogenous and time-invariant flow of income, and τ_t is a lump-sum tax. The household takes taxes, prices and interest rates as given.

The government sets a constant real value of government debt given by $b^{gov} > 0$ and uses the lump-sum tax to balance its budget. The government's budget constraint is given by:

$$\frac{1 + r_{t-1}}{1 + \pi_t} b^{gov} = b^{gov} + \tau_t,$$

Suppose further that the interest rate is permanently stuck at its "zero lower bound" i.e. it holds that $r_t = 0$ at all times.

(**a**) Derive the first-order conditions for the household's decision problem.

(**b**) Show that in equilibrium it holds that $c_t = y$.

(**c**) Show that the steady-state rate of inflation is decreasing in the ratio of government debt to aggregate income, i.e. in $\frac{b^{gov}}{y}$. Explain intuitively why this is the case.

Question 4. The Diamond-Mortensen-Pissarides model (15 points)

Consider the Diamond-Mortensen-Pissarides model of the labour market, as discussed in the lecture but without aggregate uncertainty. Let $\overline{A}$ be a parameter which denotes the productivity of a worker. Let the wage of a worker be fixed and given by a parameter $\overline{w} < \overline{A}$. The probability that a match breaks down is exogenous and given by $\rho \in (0, 1)$. Breakups occur at the end of each period. Firms are matched with at most one worker. Let $\beta \in (0, 1)$ be the discount factor of the firms and let F denote the value of a firm matched to a worker, at the beginning of a period. Free entry ensures that the value of a firm not matched to a worker is zero.

(**a**) Show that the value of a matched is given by $F = \frac{\overline{A}-\overline{w}}{1-\beta(1-\rho)}$.

(**b**) Consider a one-time and permanent change in the wage. Show that the elasticity of the firm value F with respect to $\overline{w}$ is given by $\frac{-w}{\overline{A}-\overline{w}}$.

Let g denote the probability that an unmatched firm fills a vacancy. This probability is determined endogenously in equilibrium. When a worker is hired, he/she starts producing in the next period. Also, let ϑ denote the cost of posting a vacancy, which is fixed and exogenous. Free entry requires that

$$\vartheta = g\beta F$$

(**c**) What is the elasticity of g with respect to $\overline{w}$? Explain intuitively how this elasticity depends on the level of $\overline{w}$.

Exam MSc Macroeconomics, Econ0066

2019

Instructions. This exam consists of four questions in total. You have THREE HOURS to answer all four questions. There are 100 points to be earned. Please write clearly and hand in your answers ordered numerically.

Question 1 (40 points)

Agree or disagree?
Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) In the Solow model, aggregate output may not be constant in the long run.

(**b**) In the Real Business Cycle model, consumption follows a random walk.

(**c**) Rational expectations imply that agents do not make prediction errors.

(**d**) Consider an economy with a representative firm, a Cobb-Douglas production function, and a competitive labor market. In this economy, the Marginal Product of Labor can be measured using data on labor productivity (output per hour worked) and the labor share of income.

(**e**) A model with a utility function given by $u(c) = c$, were c denotes consumption, does not generate a risk premium.

(**f**) Unequal allocations may be Pareto optimal.

[1] In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 1. Solution.

(**a**) Short answer: True. The solow model allows for population growth, which is a driver of aggregate output growth in the long run. (Similarly, one can incorporate productivity growth into the model.)

(**b**) Short answer: False. In the RBC model utility is typically not quadratic, so Hall's random walk result does not obtain. Moreover, in the RBC model interest rates fluctuate, which creates another deviation from Hall's setup. Indeed, consumption fluctuations are partly predictable in the RBC model. This predictability presents itself in the Impulse Response Functions which are not completely horizontal.

(**c**) Short answer: False. Rational expectations imply that agents do not make *systematic* prediction errors, i.e. the prediction errors are zero on average. However, given that the economy is hit by shocks, realizations are typically different from previously formed expected values. In this sense, agents do make prediction errors.

(**d**) Short answer: True.Consider the Cobb-Douglas production function $y = k^{\alpha}l^{1-\alpha}$. The marginal product of labor is given by $(1-\alpha)k^{\alpha}l^{-\alpha} = (1-\alpha)y/l$. Note that y/l is labor productivity. Moreover, the labor share of income is given by $1-\alpha$, which follows from the firms' first-order condition w.r.t. labor which is given by $w = (1-\alpha)k^{\alpha}l^{-\alpha}$ which implies a labor share of income given by $wl/y = 1-\alpha$. It follows that data on y/l (output per hour) and wl/y (labor share of income) are sufficient to measure the marginal product of labor. Another (very similar) way to arrive at the same result is to note that in a competitive labor market the marginal product of labor is equal to the wage, which can be backed out given wl/y and l/y.

(**e**) Short answer: True. Note that the utility function is linear, which implies that agents are risk neutral, as the marginal utility of consumption is constant and thus independent of the level of consumption. Indeed, formulas for risk premia, as derived for example by Mehra and Prescott, are typically proportional to the coefficient of risk aversion, and thus collapse to zero when agents are risk neutral (i.e. when they have a coefficient of risk aversion of zero).

(**f**) True. Pareto optimality means that it is not possible to make any agent strictly better off without making anyone else worse off. This may very well be the case in an economy with inequality. Suppose for example that one rich agent holds all the resources. Making any of the other agents better off requires making the rich agents worse off. Therefore the allocation is Pareto optimal, despite being very unequal.

Question 2. Consumption and saving (30 points)

Consider an infinitely-lived agent with preferences given by

$$U_t = \sum_{k=t}^{\infty} \beta^{k-t} \frac{c_k^{1-\sigma}}{1-\sigma}$$

The agent chooses consumption, $\{c_t\}_{t=0}^{\infty}$, and bonds $\{b_{t+1}\}_{t=0}^{\infty}$ to optimize U_t, subject to the following budget constraint:

$$c_t + b_{t+1} = y + (1+r)b_t,$$

where y is a constant labor income flow and r is the interest rate, which is also constant. We further assume that the discount factor satisfies $\beta = \frac{1}{1+r} \in (0,1)$ and that the agent starts with an initial level of wealth given by b_t.

(**a**) Derive the first-order conditions for consumption and bonds and show that the agent chooses a constant path of consumption, i.e.:

$$c_t = c_{t+1} = c_{t+2} = ..$$

(**b**) Use the budget constraint to show that bond holdings are also constant and can be expressed as:

$$b_t = \sum_{k=1}^{\infty} \frac{c_t - y}{(1+r)^k}$$

(**c**) Use the result under b) to show that consumption is given by:

$$c_t = rb_t + y$$

and provide an intuitive explanation of the result. Hint: to derive the result, note that $\sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^k = \sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^k - 1.$

Researchers have studied the consumption behavior of households who win a small lottery prize. A typical finding is that in the year after winning the lottery, households spend about 60 percent of the prize.

(**d**) Explain why the model described above is unlikely to be consistent with this empirical evidence.

Now consider "rule-of-thumb" households, who simply consume all their current financial wealth plus their income. That is, they set consumption as $c_t = b_t + y$.

(**e**) Which fraction of a lottery prize would these rule-of-thumb households consume in the fist year?

Consider now a model in which a fraction ϕ of the population behaves as these rule of thumb households. The remaining fraction $1 - \phi$ behaves as the optimizing households described above, under a)-c).

(**f**) To reconcile this model with the aforementioned empirical evidence on consumption behavior after winning a lottery, what would ϕ (roughly) have to be?

Question 2. Solution.

(**a**) The first-order conditions for c_t, and b_t are, at any time $t, t+1, t+2, ..$:

$$\begin{aligned} c_t^{-\sigma} &= \lambda_t \\ \lambda_t &= \beta(1+r)\lambda_{t+1} \end{aligned}$$

where λ_t is the Lagrange multiplier on the budget constraint. Exploiting that $\beta = \frac{1}{1+r}$, these can be combined to obtain:

$$c_t^{-\sigma} = c_{t+1}^{-\sigma}$$

and it now follows directly that consumption is constant over time.

(**b**) We can re-write the budget constraint as:

$$c_t + b_{t+1} = y + (1+r)b_t,$$

$$\begin{aligned} b_t &= \frac{c_t - y}{1+r} + \frac{b_{t+1}}{1+r} \\ &= \frac{c_t - y}{1+r} + \frac{c_{t+1} - y}{(1+r)^2} + \frac{b_{t+2}}{(1+r)^2} \\ &= \sum_{k=1}^{\infty} \frac{c_t - y}{(1+r)^k} \end{aligned}$$

Given that the right-hand side of this equation is constant over time, bonds will be constant over time as well.

(**c**) Note that

$$\begin{aligned} b_t &= \sum_{k=1}^{\infty} \frac{c_t - y}{(1+r)^k} = (c_t - y)(\sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^k - 1) \\ &= (c_t - y)\left(\frac{1}{1 - \frac{1}{1+r}} - 1\right) \\ &= (c_t - y)\left(\frac{1+r}{1+r-1} - 1\right) \\ &= (c_t - y)\left(\frac{1+r}{r} - \frac{r}{r}\right) \\ &= \frac{c_t - y}{r} \end{aligned}$$

Rearranging then gives:

$$c_t = rb_t + y$$

The right-hand side of the equation is the *permanent income* of the agent. The agent consumption is the sum of two components: (i) the interest on wealth stored in financial assets (bonds), and (ii) labor income. Alternatively we can write the equation as:

$$c_t = r(b_t + \frac{y}{r})$$

which states that the agents consumption equals the interest on total wealth which consists of (i) financial wealth (b_t) and (ii) human wealth, which has a present value given by $\frac{y}{r}$.

(**d**) We can think of receiving the unexpected cash prize as an increase in the initial level of financial wealth b. It follows from the consumption function derived above that the agent's consumption increases by a fraction r of the cash prize. The interest rate tends to be on average, say 4 percent in the data, on an annual basis. This implies a consumption response of 4 percent of the prize, which lies much below the 60% estimated in the data. Thus the model appears very inconsistent with the empirical evidence.

(**e**) The rule-of-thumb households spend 100% of their financial wealth. This can be seen immediately from their consumption function $c_t = b_t + y$. Given that any lottery prize adds to their financial wealth, they will consume all the prize immediately.

(**f**) Since the rule-of-thumb households spend all of the prize, whereas the optimizing households spend a fraction r, the average consumption response to the lottery is thus given by $\phi + (1 - \phi)r$. So we can solve for ϕ from

$$0.6 = \phi + (1 - \phi)r$$

or

$$0.56 = 0.96\phi$$

Given, say, $r = 0.04$ it follows that the share of rule-of-thumb households would have to be roughly 58 percent. Of course, the precise answer depends on what exactly one assumes about the real interest rate. Any annual real interest rate between 0 and 8 percent seems reasonable from an empirical perspective.

Question 3. Bonds in the utility function (15 points).

Consider an infinite-horizon economy, populated by a representative household, who derives utility from non-durable consumption and holdings of government bonds. The household solves the following decision problem:

$$\max_{\{c_t, b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{\ln c_t + \alpha \ln b_{t+1}\},$$

subject to

$$c_t + b_{t+1} = y + \frac{1 + r_{t-1}}{1 + \pi_t} b_t - \tau_t,$$

where $\beta \in (0, 1)$ is the household's subjective discount factor, $\alpha > 0$ is a preference parameter, b_{t+1} is the real value of bonds purchased in period t, c_t is non-durable consumption, r_t is the net nominal interest rate on bonds, π_t is the net rate of inflation, $y > 0$ is an exogenous and time-invariant flow of income, and τ_t is a lump-sum tax. The household takes taxes, prices and interest rates as given.
The government sets a constant real value of government debt given by $b^{gov} > 0$ and uses the lump-sum tax to balance its budget. The government's budget constraint is given by:

$$\frac{1 + r_{t-1}}{1 + \pi_t} b^{gov} = b^{gov} + \tau_t,$$

Suppose further that the interest rate is permanently stuck at its "zero lower bound" i.e. it holds that $r_t = 0$ at all times.

(**a**) Derive the first-order conditions for the household's decision problem.

(**b**) Show that in equilibrium it holds that $c_t = y$.

(**c**) Show that the steady-state rate of inflation is decreasing in the ratio of government debt to aggregate income, i.e. in $\frac{b^{gov}}{y}$. Explain intuitively why this is the case.

Question 3. Solution.

(**a**) Let λ_t denote the Lagrange multiplier on the budget constraint. The first-order conditions for c_t and b_{t+1} are given by:

$$\lambda_t = \frac{1}{c_t}$$
$$\lambda_t = \frac{\alpha}{b_{t+1}} + \beta\frac{1}{1+\pi_{t+1}}\lambda_{t+1}$$

which can be combined as:

$$\frac{1}{c_t} = \frac{\alpha}{b_{t+1}} + \beta\frac{1}{1+\pi_{t+1}}\frac{1}{c_{t+1}}$$

(**b**) First note that equilibrium in the bond market implies that $b_t = b^{gov}$ at all times. Therefore the budget constraint of the household becomes

$$c_t + b^{gov} = y + \frac{1}{1+\pi_t}b^{gov} - \tau_t,$$

Now use the government budget constraint to substitute out the lump-sum tax τ_t:

$$c_t + b^{gov} = y + \frac{1}{1+\pi_t}b^{gov} - \frac{1+r}{1+\pi_t}b^{gov} + b^{gov}$$

which implies

$$c_t = y$$

Note that this is also the goods market clearing condition, which gives the result directly.

(**c**) Plugging the results obtained under b) into the household's Euler equation for bonds gives:

$$\frac{1}{y} = \frac{\alpha}{b^{gov}} + \beta\frac{1}{1+\pi_{t+1}}\frac{1}{y}$$

$$\Longleftrightarrow$$

$$1 = \frac{y}{b^{gov}} + \beta\frac{1}{1+\pi_{t+1}}$$

Note that there are no dynamics so steady-state inflation is stable, so $\pi_{t+1} = \pi$. From the above equation it then follows that an increase in $\frac{b^{gov}}{y}$ (i.e. a decrease in $\frac{y}{b^{gov}}$) must be offset by a increase in $\frac{1}{1+\pi}$, i.e. a decrease in π. Thus, a higher ratio of government debt to income is associated with a lower rate of inflation. Intuitively, the marginal utility with respect to bonds is declining in the level, so more government debt implies a reduced incentive to purchase bonds, i.e. the demand for bonds falls. To ensure equilibrium in the bond market, the real interest rate must increase, which occurs via a reduction in inflation, given that the nominal interest rate is stuck at zero.

Question 4. The Diamond-Mortensen-Pissarides model (15 points)

Consider the Diamond-Mortensen-Pissarides model of the labour market, as discussed in the lecture but without aggregate uncertainty. Let $\overline{A}$ be a parameter which denotes the productivity of a worker. Let the wage of a worker be fixed and given by a parameter $\overline{w} < \overline{A}$. The probability that a match breaks down is exogenous and given by $\rho \in (0, 1)$. Breakups occur at the end of each period. Firms are matched with at most one worker. Let $\beta \in (0, 1)$ be the discount factor of the firms and let F denote the value of a firm matched to a worker, at the beginning of a period. Free entry ensures that the value of a firm not matched to a worker is zero.

(**a**) Show that the value of a matched is given by $F = \frac{\overline{A}-\overline{w}}{1-\beta(1-\rho)}$.

(**b**) Consider a one-time and permanent change in the wage. Show that the elasticity of the firm value F with respect to $\overline{w}$ is given by $\frac{-w}{\overline{A}-\overline{w}}$.

Let g denote the probability that an unmatched firm fills a vacancy. This probability is determined endogenously in equilibrium. When a worker is hired, he/she starts producing in the next period. Also, let ϑ denote the cost of posting a vacancy, which is fixed and exogenous. Free entry requires that

$$\vartheta = g\beta F$$

(**c**) What is the elasticity of g with respect to $\overline{w}$? Explain intuitively how this elasticity depends on the level of $\overline{w}$.

Question 4. Solution.

(**a**) The value of a firm can be expressed recursively as:

$$F = \overline{A} - \overline{w} + \beta(1-\rho)F + \beta\rho J$$

where J is the value of an unmatched firm, which equals zero in equilibrium. Using this and rearranging we obtain:

$$F(1-\beta(1-\rho)) = \overline{A} - \overline{w}.$$

$\Longleftrightarrow$

$$F = \frac{\overline{A} - \overline{w}}{1-\beta(1-\rho)}.$$

(**b**) Taking logs gives:

$$\ln F = \ln(\overline{A} - \overline{w}) - \ln(1-\beta(1-\rho))$$

and differentiating, keeping β, ρ and $\overline{A}$ constant gives:

$$\frac{dF}{F} = \frac{-dw}{\overline{A} - \overline{w}}$$

$\Longleftrightarrow$

$$\frac{dF}{F} = \frac{-w}{\overline{A} - \overline{w}} \frac{dw}{w}$$

$\Longleftrightarrow$

$$\frac{dF/F}{d\overline{w}/\overline{w}} = \frac{-\overline{w}}{\overline{A} - \overline{w}}.$$

(**c**) Taking logs and differentiating the free entry condition immediately gives that $\frac{dF}{F} = -\frac{dg}{g}$. Combining this with the result obtained under b) implies that the elasticity of the vacancy filling rate with respect to the wage is given by:

$$\frac{dg/g}{d\overline{w}/\overline{w}} = \frac{\overline{w}}{\overline{A} - \overline{w}} = \frac{1}{\overline{A}/\overline{w} - 1} > 0.$$

It follows that the elasticity of the vacancy filling rate with respect to the wage is positive and increasing in the level of the wage. Intuitively, when the wage is close to the marginal product of a worker ($\overline{A}$), firms make little profits. Any further increase in the wage then implies a relatively large percentage decline in profits, a relatively large decline in firm values. This in turn crease a relatively strong reduction in the incentives of firms to post vacancies, resulting in a large increase in the vacancy filling probability, required to compensate entering firms for the decline in profitability.

Exam MSc Macroeconomics, ECON0066

2020

Instructions. This exam consists of four questions in total. There are 100 points to be earned. Please limit your answer to maximally 500 words per subquestion (i.e. max 500 words for question 1(**a**), max 500 words for question 1(**b**), and so forth). This word limit is provided as guidance only. You will not be penalised if you exceed the word limit.

All work must be submitted anonymously. Please ensure that you add your candidate number and the module code to the template answer sheet provided. Note that the candidate number is a combination of four letters plus a number, e.g. ABCD9. You can find your candidate number in your PORTICO account, under "My Studies" then the "Examinations" container. Please, note that the candidate number is NOT the same as your student number (8 digits), which is printed on your UCL ID card. Submitting with your student number will delay marking and when your results might be available.Allow enough time to submit your work. Waiting until the deadline for submission risks facing technical problems when submitting your work, due to limited network or systems capacity.

By submitting this assessment, I pledge my honour that I have not violated UCL's Assessment Regulations which are detailed in https://www.ucl.ac.uk/academic-manual/chapters/chapter-6-student-casework-framework/section-9-student-academic-misconduct-procedure*, which include (but are not limited to) plagiarism, self-plagiarism, unauthorised collaboration between students, sharing my assessment with another student or third party, access another student's assessment, falsification, contract cheating, and falsification of extenuating circumstances.*

Question 1. Agree or disagree? (40 points)

Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) In the Solow model, the steady-state capital stock associated with the golden rule rate of saving is a decreasing function of the depreciation rate of capital.

(**b**) In the Diamond-Mortensen-Pissarides model of the labour market, the fundamental theorems of welfare do not apply.

(**c**) The equity premium puzzle would be less severe if, in the data, consumption growth and excess returns of stocks over bonds were less strongly correlated, everything else equal.

(**d**) The Hodrick-Prescott filter can be used to take a trend with constant growth out of a data series.

(**e**) Consider a labour market matching function $m = \mu u^{\gamma} v^{1-\gamma}$, where m is the number of new matches, u is the unemployment rate, v is the number of job vacancies, and $\gamma \in (0, 1)$ and $\mu > 0$ are parameters. Given this matching function, the job finding rate is negatively related to the vacancy filling rate.

(**f**) The Kaldor growth facts imply that, on average, the aggregate capital stock grows at the same rate as aggregate output.

[1] In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 2. Labour supply (25 points)

Consider a static model of a household with preferences given by:

$$U(c,h) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \frac{\kappa}{1+\psi} h^{1+\psi},$$

where $\sigma, \kappa, \psi > 0$ are preference parameters. The household chooses consumption (c) and hours worked (h) to maximize $U(c,h)$, subject to the following budget constraint:

$$c = wh,$$

where w is the wage rate per hour worked.

(**a**) Derive the first-order conditions for consumption and hours worked (labour supply).

(**b**) Derive the Frisch elasticity of labour supply in this model.

(**c**) Show that when $\sigma = 1$ (so that $U(c,h) = \ln c - \frac{\kappa}{1+\psi} h^{1+\psi}$), the number of hours worked chosen by the household does not react to a change in the wage rate w.

(**d**) Suppose now that $\psi = 1$ and $\sigma = 2$. If the wage increases by 1 percent, by how much does labour supply change? Decompose your answer into the underlying effects.

(**e**) Derive an analytical expression for elasticity of consumption with respect to the wage (for general $\sigma, \kappa, \psi > 0$). Show that this elasticity strictly exceeds 1 if and only if $\sigma < 1$ and explain intuitively why this is the case.

Question 3. Consumption of durables and non-durables (15 points)

Consider an infinitely-lived household which consumes both durables and non-durables. Expected discounted utility at time zero is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, d_t\right)$$

where c_t denotes consumption of non-durables in period t, d_t is the *stock* of durables owned by the household in period t, $\beta \in (0,1)$ is the household's subjective discount factor, and E_t is the expectations operator conditional upon information available in period t. The price of both durables and non-durables is one. The budget constraint of the household in period t is given by:

$$c_t + d_t = y_t + (1-\delta)\, d_{t-1},\;\; t = 0, 1, 2, 3..$$

where $\delta \in (0,1)$ is the depreciation rate of durables and $y_t > 0$ is an exogenous income variable, which follows a stochastic but stationary process. In each period t, the household chooses c_t and d_t such as to maximize the expected present value of lifetime utility.

(**a**) Derive the first-order optimality conditions associated with the household's decision problem.

Suppose now that the utility function is given by $u\left(c_t, d_t\right) = c_t + \gamma \ln d_t$.[2]

(**b**) Derive an analytical expression for d_t in terms of the model parameters and show that d_t remains constant over time (given the parameters).

(**c**) Show that d_t is increasing in β and explain intuitively why this is the case.

[2]For simplicity, ignore any non-negativity constraint on c_t.

Question 4. Pricing inflation risk (20 points)

Consider an infinitely-lived, representative household with preferences given by

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\rho} - 1}{1-\rho} \right\},$$

where c_t denotes consumption at time t, $\beta \in (0,1)$ is the discount factor and $\rho > 0$ is the coefficient of risk aversion. The household can invest in two types of one-period bonds: (i) nominal bonds, denoted B_t^n, which offer a nominal interest rate r_t^n , (ii) inflation-linked bonds, denoted B_t^i, which offer a nominal interest rate $r_t^i(1+\pi_{t+1})$, where $\pi_{t+1} = \frac{P_{t+1}}{P_t} - 1$ is the inflation rate in period $t+1$, and where P_t is the nominal price level in period t. Inflation is uncertain, i.e. it evolves stochastically over time.

The budget constraint of the household in period t, in nominal terms, is given by:

$$P_t c_t + B_t^n + B_t^i = P_t y_t + \left(1 + r_{t-1}^n\right) B_{t-1}^n + \left(1 + r_{t-1}^i\right)(1+\pi_t) B_{t-1}^i,$$

where y_t is an exogenous and stochastic income flow. In each period, the household chooses c_t,B_t^n and B_t^i in order to maximize the utility objective given above, subject to the budget constraint. Let $b_t^n \equiv B_t^n/P_t$ and $b_t^i \equiv B_t^i/P_t$ denote the real values of nominal and inflation-linked bonds, respectively.

(**a**) Re-write the budget constraint in real terms (as opposed to nominal terms).

(**b**) Derive the Euler equations for the two types of bonds. Which of the two bonds would the household consider to be risk-free?

Suppose now that in equilibrium it holds that $y_t = c_t$ and that the central bank sets a monetary policy according to a rule which targets inflation as a function of output growth: $\pi_t = \gamma \left(\frac{y_t}{y_{t-1}} - 1 \right)$, where $\gamma > 0$ is a policy parameter.

(**c**) Consider the risk premium formula derived in class for the excess return of equity, but apply it instead to the two bonds considered above. Which of the two bonds earns a higher ex-ante expected return?

(**d**) Discuss the intuition for your answer under (**c**).

SOLUTIONS Exam MSc Macroeconomics, ECON0066

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Question 1. Solution.

(**a**) TRUE The Golden Rule steady-state condition is given by $f'(k^{GR}) = \delta + n$, where k^{GR} is the capital stock per capita in the steady state, under the Golden Rule. The marginal product of capital, $f'(k)$ is a decreasing function of k. Therefore, k^{GR} is decreasing in δ.

(**b**) TRUE. In the DMP model, there are externalities associated with the vacancy posting decision by the firms. More vacancy posting reduces the filling rates for other firms, but increases the finding rates for workers. These externalities are a form of market imperfection. Therefore, in general, the social welfare theorems do not apply to the DMP model.[1]

(**c**) FALSE. Everything else equal, the formula derived in class shows that a lower correlation between consumption growth and excess returns of stocks over bonds would imply a lower equity premium. This would further widen the discrepancy between the equity premium implied by the formula and the premium observed in the data. That is, the equity premium puzzle would be more severe.

(**d**) TRUE. The Hodrick-Prescott filter is controlled by a parameter λ, to be set by the researcher. This parameter captures the weight put on the squared distance between the data and the trend, versus time variation in the trend. When the parameter is set to an extreme value ($\lambda \to \infty$), the HP filter implies that the filtered series is a linear trend. Given $\lambda \to \infty$, then if a (natural) log is taken out of the data series before applying the filter, then a trend with constant growth will be isolated (recall that a linear trend in a logged variable implies a constant growth rate).

(**e**) TRUE. The job finding rate is given by $f = \frac{m}{u} = \mu \frac{u^\gamma v^{1-\gamma}}{u} = \mu\theta^{1-\gamma}$, where $\theta \equiv \frac{v}{u}$ is tightness. The vacancy filling rate is given by: $g = \frac{m}{v} = \mu\theta^{-\gamma}$. Since f is an increasing function of only tightness and g a decreasing function of only tightness, it follows that the two are negatively related.

(**f**) TRUE. One of the Kaldor growth facts is that the capital to output ratio is constant in the long run. It follows directly that capital and output then must grow at the same rate on average in the long run.

[1] There is a special case of the parameter values in which externalities exactly cancel, known as the "Hosios condition". Knowledge of this condition is outside the scope of this course.

Question 2. Solution.

(**a**) The first-order conditions are given by, respectively:

$$\lambda = c^{-\sigma}$$
$$w\lambda = \kappa h^{\psi}$$

for any period t, where λ is the Lagrange multiplier on the budget constraint.

(**b**) Differentiating the labor supply condition, keeping λ fixed, gives:

$$\lambda dw = \psi \kappa h^{\psi-1} dh$$

The Frisch labour supply elasticity is the labour supply elasticity keeping λ fixed:

$$\frac{dh/h}{dw/w} = \frac{\lambda dw/\left(h\psi\kappa h^{\psi-1}\right)}{dw/w} = \frac{\lambda w}{\psi\kappa h^{\psi}} = \frac{1}{\psi}.$$

Alternatively, one could derive this result using the formula for the Frisch elasticity given in the lecture, i.e. $\frac{U_h(h)}{U_{hh}(h)h} = \frac{\kappa h^{\psi}}{\psi\kappa h_t^{\psi-1}h} = \frac{1}{\psi}$.

(**c**) Combining the first order conditions and setting $\sigma = 1$ gives:

$$\frac{w}{c} = \kappa h^{\psi}$$

Substituting out consumption using the budget constraint gives:

$$\frac{1}{h} = \kappa h^{\psi},$$

which pins down h as a function of the parameters, independently of w. (**d**) Substituting out consumption using the budget constraint, the first-order condition reads:

$$\frac{w}{w^{\sigma}h^{\sigma}} = \kappa h^{\psi}$$

Rearranging and taking logs gives:

$$(1-\sigma)\ln w = \ln \kappa + (\psi + \sigma)\ln h$$

Differentiating gives:

$$\frac{d\ln h}{d\ln w} = \frac{dh/h}{dw/w} = \frac{1-\sigma}{\psi+\sigma}$$

Setting $\psi = 1$ and $\sigma = 2$ gives $\frac{dh/h}{dw/w} = -\frac{1}{3}$. So in response to a 1% increase in the wage, labour supply declines by 1/3 percent. The substitution effect is captured by the Frisch elasticity of labour supply which equals $\psi = 1$ percent. The substitution effect works to push up labour supply. It follows that the income effect is given by $-\frac{1}{3} - 1 = -1\frac{1}{3}$ percent.

(**e**) First note that the budget constraint implies:

$$d \ln c = d \ln w + d \ln h$$

which gives

$$\frac{d \ln c}{d \ln w} = \frac{d \ln w}{d \ln w} + \frac{d \ln h}{d \ln w} = 1 + \frac{1 - \sigma}{\psi + \sigma} = \frac{\psi + 1}{\psi + \sigma}.$$

When $\sigma < 1$, the numerator is larger than the denominator, and hence the elasticity exceeds 1. To understand the intuition, first recall that the agent consumes all its income, hence the consumption elasticity equals the income elasticity. Next, note that income responds directly to a change in the wage via the budget constraint, with an elasticity of one. In addition, income also responds indirectly due to a labour supply response. When $\sigma < 1$ the substitution effect dominates the income effect and the labour supply response is positive. This increases the overall income (and consumption response) to be above one. Vice versa, when $\sigma > 1$ the income effect dominates, implying a negative labour supply response, lowering the consumption elasticity below one.

Question 3. Solution.

(**a**)The first-order conditions are given by:

$$\lambda_t = u_d\left(c_t, d_t\right) + \beta\left(1-\delta\right) E_t \lambda_{t+1}$$
$$\lambda_t = u_c\left(c_t, d_t\right)$$

at any date t. Here, λ_t is the Lagrange multiplier on the budget constraint.

(**b**) Given this functional form it holds that $u_c\left(c_t, d_t\right) = 1$ and $u_d\left(c_t, d_t\right) = \frac{\gamma}{d_t}$. Substituting this into the first-order conditions and combining gives

$$1 = \frac{\gamma}{d_t} + \beta\left(1-\delta\right)$$

Rearranging gives the answer in the question.

(**c**) Note that the durable is part asset and part consumption good. A lower discount factor β means that the household is more impatient, and therefore discounts the future value of the asset more heavily. This makes buying a durable less attractive. Mechanically, this manifests itself as a decline in the second term on the right-hand side of the first-order condition for the durable.

Question 4. Solution.

(**a**) The budget constraint in real terms is given by:

$$c_t + b_t^n + b_t^i = y_t + \frac{1 + r_{t-1}^n}{1 + \pi_t} b_{t-1}^n + \left(1 + r_{t-1}^i\right) b_{t-1}^i.$$

(**b**)The Euler equations for bonds are:

$$1 = (1 + r_t^n)\, \beta E_t \left\{ \frac{1}{1 + \pi_{t+1}} \left(\frac{c_t}{c_{t+1}} \right)^\rho \right\}$$

$$1 = \left(1 + r_t^i\right) \beta E_t \left\{ \left(\frac{c_t}{c_{t+1}} \right)^\rho \right\}$$

In this model, the nominal asset is risky, since its real return is affected by inflation risk, whereas the inflation-linked bond is safe, since its real return is given by r_t^i which is known at the moment the bond is purchased.

(**c**) First note that the net realized real returns on the nominal and the inflation-linked bond are given respectively by $\frac{1+r_{t-1}^n}{1+\pi_t} - 1$ and r_{t-1}^i. The formula for the risk premium regards the difference in expected real returns of two assets. Applying the formula to the case considered here gives

$$\begin{aligned}
E_t \left\{ \frac{1 + r_t^n}{1 + \pi_{t+1}} - 1 - r_t^i \right\} &= \rho Cov_t \left\{ \frac{c_{t+1}}{c_t} - 1, \frac{1 + r_t^n}{1 + \pi_{t+1}} - 1 - r_t^i \right\} \\
&= \rho Cov_t \left\{ \frac{c_{t+1}}{c_t} - 1, \frac{1 + r_t^n}{1 + \pi_{t+1}} \right\} \\
&= \rho \left(1 + r_t^n\right) Cov_t \left\{ \frac{y_{t+1}}{y_t}, \frac{1}{1 + \gamma \left(\frac{y_{t+1}}{y_t} - 1 \right)} \right\} < 0
\end{aligned}$$

where the second equality follows from the fact that $1 - r_t^i$ is known at date t, and the third equality exploits market clearing and the monetary policy rule. The final inequality follows from the fact that the two terms within the covariance are perfectly negatively correlated. It now follows that the nominal bond has a lower expected return than the inflation-linked bond.

(**d**) The result above implies that inflation risk has a negative price! That is, households are willing to pay a premium to hold the nominal asset, which carries inflation risk. This premium takes the form of a lower expected return. Intuitively, inflation risk acts as a hedge against consumption risk. To see this, note that if a shock were to hit in the next period which reduces output and thus consumption, then inflation will decrease as well, which in turn increases the real return on the nominal bond. The households likes this property of the nominal bond, because it means that it pays off more when the marginal utility of wealth (consumption) is relatively high. Therefore, the household is willing to accept a lower expected ex-ante return on the nominal bond, compared to the inflation-linked bond.

Exam MSc Macroeconomics, ECON0066

2021

Instructions. This exam consists of four questions in total. There are 100 points to be earned. Please limit your answer to maximally 500 words per subquestion (i.e. max 500 words for question 1(**a**), max 500 words for question 1(**b**), and so forth). This word limit is provided as guidance only. You will not be penalised if you exceed the word limit.

All work must be submitted anonymously. Please ensure that you add your candidate number and the module code to the template answer sheet provided. Note that the candidate number is a combination of four letters plus a number, e.g. ABCD9. You can find your candidate number in your PORTICO account, under "My Studies" then the "Examinations" container. Please, note that the candidate number is NOT the same as your student number (8 digits), which is printed on your UCL ID card. Submitting with your student number will delay marking and when your results might be available. Allow enough time to submit your work. Waiting until the deadline for submission risks facing technical problems when submitting your work, due to limited network or systems capacity.

By submitting this assessment, I pledge my honour that I have not violated UCL's Assessment Regulations which are detailed in https://www.ucl.ac.uk/academic-manual/chapters/chapter-6-student-casework-framework/section-9-student-academic-misconduct-procedure, *which include (but are not limited to) plagiarism, self-plagiarism, unauthorised collaboration between students, sharing my assessment with another student or third party, access another student's assessment, falsification, contract cheating, and falsification of extenuating circumstances.*

Question 1. Agree or disagree? (40 points)

Consider four out of the six statements below.[1] Do you agree or disagree? Explain your answers.

(**a**) In the Solow model without population growth, the steady-state capital-to-output ratio exceeds one if the saving rate exceeds the depreciation rate.

(**b**) In the Diamond-Mortensen-Pissarides model of the labour market, the wage can never fall below the unemployment benefit.

(**c**) In OverLapping Generations models, a pay-as-you-go pension can improve social welfare only if the model features productive capital.

(**d**) In the Real Business Cycle model, consumption growth and the marginal product of capital are positively correlated.

(**e**) If output increases but consumption and labour supply remain unchanged, then frictions in the labour market may have become less severe.

(**f**) In the money-in-the-utility model, the Friedman rule is implemented via a sustained contraction of the money supply.

[1] In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

Question 2. Labour taxation (30 points)

Consider a model with a representative, infinitely-lived household and a representative firm. Discounted utility at time $t = 0$ is given by:

$$U_0 = \sum_{t=0}^{\infty} \beta^t \ln\left(c_t - \frac{\kappa}{1+\psi} h_t^{1+\psi}\right),$$

where $\kappa, \psi > 0$ and $\beta \in (0, 1)$ are preference parameters. Households rent out capital and supply labour to firms. For every period, the household chooses consumption (c_t), capital (k_{t+1}) and hours worked (h_t) to maximize utility, subject to the following budget constraint:

$$c_t + k_{t+1} = (1 - \tau)\, w_t h_t + (1 - \delta + r_{k,t}) k_t,$$

where w_t is the wage rate per hour worked, and $r_{k,t}$ is the rental rate of capital, both of which are taken as given by the household. Moreover, $\delta \in (0, 1)$ is the depreciation rate of capital and τ is a labour income tax rate.

(**a**) Derive the first-order optimality conditions for the household's choice variables.

(**b**) Derive an expression for the overall elasticity of labour supply with respect to a change in the wage rate.

(**c**) Decompose your answer under (**b**) into a substitution and an income effect.

The representative firm behaves competitively and operates a production function given by $y_t = k_t^{\alpha} h_t^{1-\alpha}$, $\alpha \in (0, 1)$. The firm chooses in every period k_t and h_t to maximize profits.

(**d**) Formulate the firm's profit maximization problem and derive the first-order optimality conditions.

Suppose now that in some period, the tax rate τ increases unexpectedly and permanently.

(**e**) Derive an analytical expression for the response of equilibrium hours worked, in the initial period of the tax change, in terms of τ and the parameters of the model.

(**f**) Illustrate equilibrium in the labor market by drawing a supply-demand diagram, and explain the role of the parameters entering in the result derived under (**e**). Also, use the diagram to explain intuitively how the tax increase mentioned above affects the equilibrium.

Question 3. Consumption (25 points)

Consider the saving/consumption decision problem of an infinitely-lived household. Discounted utility at time zero is given by:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma} - 1}{1-\gamma},$$

where c_t denotes consumption in period t, $\beta \in (0,1)$ is the household's subjective discount factor, and where $\gamma > 0$ is the coefficient of risk aversion. The household can invest in assets, denoted a_{t+1}, which earn a constant net return r. The household's budget constraint is given by:

$$a_{t+1} + c_t = y_t + (1+r)\,a_t,\ \ t = 0, 1, 2, 3..$$

where $(1+r)\,a_t$ is wealth (at the beginning of the period) and y_t is income, which evolves according to:

$$y_{t+1} = \rho y_t$$

where $\rho \in (0,1)$. The household chooses a time path $\{c_t, a_{t+1}\}_{t=0}^{\infty}$ which maximizes discounted utility, subject to the two constraints and taking r, $a_0 > 0$ and $y_0 > 0$ as given.

(**a**) Derive the first-order optimality conditions associated with the household's decision problem and show that consumption growth is contant over time.

(**b**) Derive a condition, in terms of r and β, under which consumption declines over time. Explain the intuition behind this result.

(**c**) Find an expression for c_t as a linear function of a_t and y_t.

(**d**) Consider the effect on consumption of an unanticipated, one-time change in either beginning-of-period wealth or current income. Which of the two triggers a larger consumption response? Relate your answer to the expression found under (**c**) and explain the intuition.

Now suppose there is a large number of households, indexed by $i = 1, 2, ...N$, which behave as the household described above. The return r is still exogenously given. However, income and wealth levels may differ between households.

(**e**) Consider the following statement: "In this model economy, only the average levels of wealth and income matter for aggregate consumption, not their distributions across households." Do you agree? Explain your answer.

Question 4. Endogenous growth (5 points)

(**a**) Consider the AK model as discussed in week 10. Discuss how the coefficient of risk aversion affects the growth rate of output in this model. Explain the intuition.

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2020

Question 1. Solution.

(**a**) TRUE . In the steady state it holds that investment equals depreciation, i.e. we have $sy = \delta k$. It follows immediately that $k/y > 1$ if $s > \delta$.

(**b**) FALSE. In the DMP model, wages cannot fall outside the "bargaining set", otherwise either the worker or the firm would walk away from the match. However, the bargaining set may include wages below the unemployment benefit, outside of the steady state of the model. That happens when the continuation value of the match to the worker is sufficiently high, which can be the case if the wage is expected to increase sufficiently in the future.

(**c**) FALSE. In an OLG model without capital, equilibria in the "Samuelson case" are inefficient. In that case, the introduction of a pay-as-you go pension scheme, which transfers resources from the young to the old, can make the initially young and all subsequent generations better off, by creating improved consumption smoothing over the life cycle. The initially old are also (trivially) better off.

(**d**) TRUE. Following a positive productivity shock, the marginal product of capital increases directly due the TFP increase, as well as indirectly due to the increase in labour supply. At the same time, consumption increases as output expands. This can also be seen from the Euler equation. Under CRRA preferences (for simplicity) this equation can be written as:

$$1 = \beta E_t \left\{ (1 - \delta + MPK_{t+1}) \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \right\}$$

This equation implies a positive covariance (and hence correlation) between the marginal product of capital (MPK) and consumption growth.

(**e**) FALSE. Recall that the labour wedge is defined as the ratio of the Marginal Product of Labour (MPL) and the Marginal Rate of Substitution (MRS) between leisure and consumption. The MRS is a function of consumption (c) and decreasing in labour supply (h), so if both variables are unchanged then the MRS remains unchanged. In the neoclassical model, the MPL is proportional to labour productivity, y/h. The MPL thus increases in output (y), given h. With the MPL increasing but the MRS remaining constant, it follows that the labour wedge increases, i.e. labour market frictions become MORE severe. Intuitively, without the increased frictions, more labour would flow to firms, reducing the MPL due to decreasing returns in production w.r.t. labour.

(**f**) TRUE. The Friedman rule prescribes a zero nominal interest rate ($r = 0$) which, given a positive real interest rate, implies deflation ($\pi < 0$), i.e. the price level P falls over time. Given that real money balances, $m = \frac{M}{P}$, are constant in the steady state of the MIU model, it follows the nominal money supply, M, must fall at the same rate as the price level. That is, the money supply contracts.

Question 2. Solution.

(**a**) The first-order conditions for consumption, hours and capital are given by, respectively:

$$\lambda_t = \frac{1}{c_t - \frac{\kappa}{1+\psi} h_t^{1+\psi}}$$
$$\kappa h_t^{\psi} = (1-\tau) w_t$$
$$\frac{1}{c_t - \frac{\kappa}{1+\psi} h_t^{1+\psi}} = \frac{\beta(1-\delta + r_{k,t+1})}{c_{t+1} - \frac{\kappa}{1+\psi} h_{t+1}^{1+\psi}}$$

for any period t, where λ_t is the Lagrange multiplier on the budget constraint.

(**b**) Taking logs of the labour supply condition gives:

$$\ln \kappa + \psi \ln h_t = \ln(1-\tau) + \ln w_t$$

Differentiate:

$$\frac{dh_t/h_t}{dw_t/w_t} = \frac{1}{\psi}.$$

(**c**) The labour supply elasticity is entirely driven by the substitution effect, i.e. the income effect is zero. To see this, note that the marginal utility of wealth (λ_t) does not enter into the condition, so there is no income effect. Hence, the Frisch elasticity of labour supply, which captures the substitution effect, equals the overall labour supply elasticity.

(**d**) The firms' profit maximization problem is static and can be expressed as:

$$\max_{k_t, h_t} k_t^{\alpha} h_t^{1-\alpha} - r_{k,t} k_t - w_t h_t$$

The first-order conditions for capital and labour are:

$$r_{k,t} = \alpha k_t^{\alpha-1} h_t^{1-\alpha}$$
$$w_t = (1-\alpha) k_t^{\alpha} h_t^{-\alpha}$$

(**e**) First note in the initial period, the capital stock cannot respond as it is predetermined.

Taking logs of the firms' labour demand condition, and differentiating gives:

$$d \ln w_t = -\alpha d \ln h_t$$

Doing the same for the household's labour supply gives (note that the tax rate now changes):

$$d \ln(1-\tau) + d \ln w_t = \psi d \ln h_t$$

Combining the above two equations gives:

$$d \ln(1-\tau) - \alpha d \ln h_t = \psi d \ln h_t$$

or

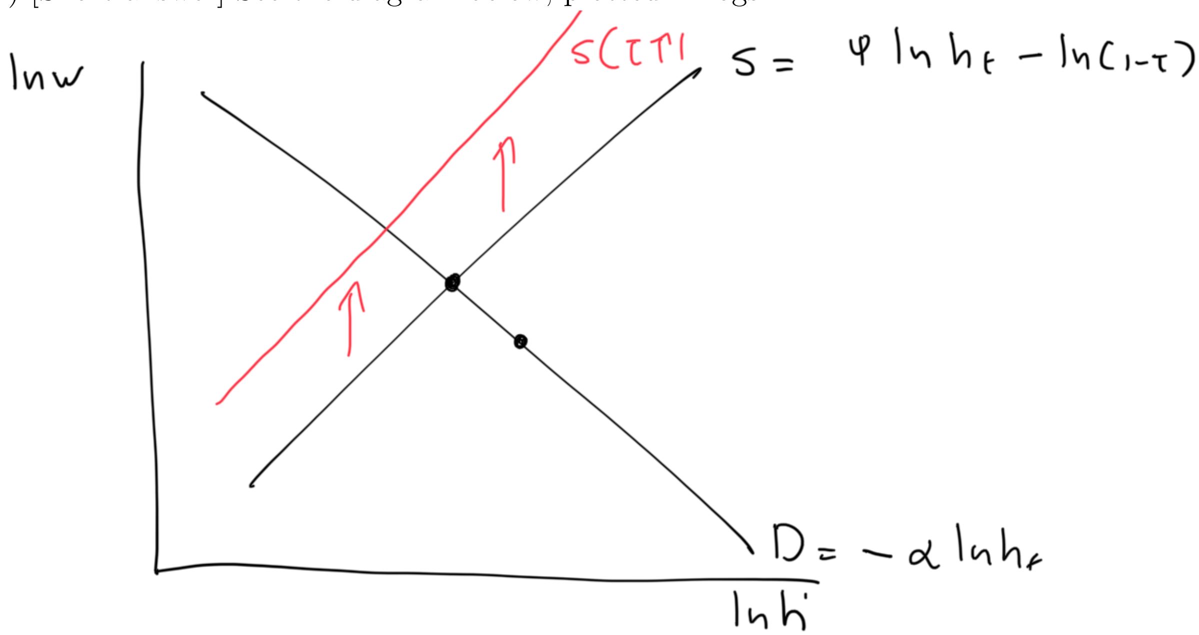
$$d \ln h_t = \frac{1}{\psi + \alpha} d \ln(1-\tau)$$

or

$$\frac{dh_t/h_t}{d\tau} = -\frac{1}{\psi+\alpha}\frac{1}{1-\tau}$$

This is the semi-elasticity of hours worked with respect to the tax rate.

(**f**) [Short answer] See the diagram below, plotted in logs.



The slope of the demand and the supply curves are determined respectively by the elasticity of labour supply $(1/\psi)$ and the elasticity of labour demand $(-1/\alpha)$. Following a tax increase, the labour supply curve shifts upward, creating a shift along the demand curve. In equilibrium, the wage increases whereas labour supply falls.

Question 3. Solution.

(**a**) The first-order conditions are given by:

$$\lambda_t = c_t^{-\gamma}$$
$$\lambda_t = \beta(1+r)\lambda_{t+1}$$

at any date t. Here, λ_t is the Lagrange multiplier on the budget constraint. Combining these gives:

$$\frac{c_{t+1}}{c_t} = \beta^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}$$

and it follows that consumption growth is constant, as the right-hand side of the equation is constant.

(**b**) For declining consumption we must have:

$$\frac{c_{t+1}}{c_t} = (\beta(1+r))^{\frac{1}{\gamma}} < 1$$

which is the case if and only if

$$\beta(1+r) < 1$$

Intuition: consumption declines when the household's degree of impatience is high relative to the market interest rate. In that case, the household prefers to front-load their consumption.

(**c**) Iterating forward, we can express the budget constraint as (see lecture slides for details):

$$a_t + \sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^k y_{t+k-1} = \sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^k c_{t+k-1}$$

Using the fact $y_{t+k-1} = \rho^{k-1} y_t$ and the Euler equation derived above, which implies that $c_t = \beta^{\frac{1-k}{\gamma}}(1+r)^{\frac{1-k}{\gamma}} c_{t+k-1}$, we can re-write the above equation as:

$$a_t + y_t \sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^k \rho^{k-1} = c_t \sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^k \beta^{\frac{1-k}{\gamma}}(1+r)^{\frac{1-k}{\gamma}}$$

Defining $\zeta \equiv \sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^k \rho^{k-1}$ and $\eta \equiv \sum_{k=1}^{\infty}(1+r)^{\frac{1-k}{\gamma}-k}$, we can express consumption as a linear function of wealth and current income:

$$c_t = \frac{1}{\eta}\left(\frac{\widetilde{a}_t}{1+r} + \zeta y_t\right).$$

where $\widetilde{a}_t \equiv (1+r)a_t$ is beginning-of-period wealth.

(**d**) First note that

$$\frac{\partial c_t}{\partial \widetilde{a}_t} = \frac{1}{\eta}\frac{1}{(1+r)}$$

Next note that:

$$\frac{\partial c_t}{\partial y_t} = \frac{1}{\eta}\zeta > 0.$$

Next note that

$$\zeta = \sum_{k=1}^{\infty}\left(\frac{1}{1+r}\right)^k \rho^{k-1} = \frac{1}{1+r} + \frac{1}{(1+r)^2}\rho + ... > \frac{1}{1+r}$$

It follows that $\frac{\partial c_t}{\partial y_t} > \frac{\partial c_t}{\partial \widetilde{a}_t}$, i.e. consumption responds more to the income shock than to the wealth shock. Intuitively, income shocks persist, and therefore raise permanent income by more than a wealth shock. Being forward looking, the household takes this into account and raises consumption by more.

(**e**) The statement is true, since consumption is linear in wealth and income. Letting households be indexed by $i = 1, 2, ..., N$, we can express aggregate consumption as:

$$\begin{aligned} C_t &= \sum_{i=1}^{N} c_{i,t} \\ &= \frac{1}{\eta}\zeta \sum_{i=1}^{N} y_{i,t} + \frac{1}{\eta}\frac{1}{(1+r)}\sum_{i=1}^{N}\widetilde{a}_{i,t} \\ &= N\left(\frac{1}{\eta}\zeta\overline{y}_t + \frac{1}{\eta}\frac{1}{(1+r)}\overline{\widetilde{a}}_t\right) \end{aligned}$$

That is, only the average levels of income and wealth (resp. $\overline{y}_t$ and $\overline{\widetilde{a}}_t$) matter, not their distributions. This is precisely because the decision rule is linear.

Question 4. Endogenous growth

(**a**) In the lecture it has been derived that (absent taxation) output growth in the AK model is given by:

$$\frac{y_{t+1}}{y_t} = (\beta(1 + A - \delta))^{\frac{1}{\sigma}} .$$

Provided that growth is positive, so that $\beta(1 + A - \delta) > 1$, the growth rate is decreasing in the coefficient of risk aversion σ. Intuitively, higher means that the household is less willing to accept a non-constant consumption path. Given that consumption is increasing over time, this means that the household is less willing to invest, as investment back-loads consumption. Due to a lower rate of investment, output growth will then be lower, since the growth rate depends directly on investment in the AK model.