Brandeis International Business School

FIN 285A – Computer Simulation and Risk Assessment

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FINAL PROJECT

Value-at-Risk Modeling and Expected Shortfall Estimation – an Application with U.S. Crude Oil Market from Recent Time Series Data

1. Introduction

1.1. Market Risks of U.S. Crude Oil Market

The crude oil market is the largepst commodity market in the world. Crude oil plays an important role in any economy as it is a crucial fuel for many industries including transportations, electricity generation, and manufacturing. Its price is strictly regulated by governments and international cartels like OPEC. Therefore, crude oil market is subject to the changes in energy policies, the number and size of marco-economic shocks and supply-demand drivers.

In our study, we use the daily spot price of crude oil in the West Texas Intermediate (WTI) delivered at Cushing, Oklahoma, which is considered the U.S. pricing benchmark. We select the recent 34 year horizon for our study from 1983 to 2017. Before this sample period, the oil price did not change because the US oil price was fixed from October 1973 through December 1982. During the period from 1983 to present, major geopolitical and economic events such as the Gulf War (1990), Asian Financial Crisis (1997), and Terrorist Attacks on the World Trade Center in New York (2001), Iraq Invasion War (2003), Iran sanction (2006), U.S. Energy Independence and Security Acts (2007), and US financial crisis (2009) significantly affected the magnitude of the oil price volatility in both U.S. market and global oil market. Looking at the competition landscape between crude oil and its substitute fuel such as shale oil, and natural gas, the prolific production and over-supply of shale oil and gas thanks to the innovation technology of fracking (horizontal drilling) as well as the declining energy demand due to the rising energy efficiency have dramatically decreased the U.S. oil price in recent years (Figure 1).

1.2. Data Source

In our paper, we use the spot price for West Texas Intermediate Crude Oil (WTI) obtained from the Global Financial Data database (GFD). The time period of our research is from June 1st, 1983 to Nov 29th, 2017. The continuous daily WTI spot price from the GFD can be traced back to 1977. However, the US oil price is fixed from 1973 to 1982 and having the same price over several years; thus, we exclude this period to maintain the consistency of our data and examine the changes in oil returns and volatility from 1983 until present.

1.3. Main Investigation Goals and Approaches

We aim to investigate the different estimation results of Value-at-risk (VaR) and Expected Shortfall (ES) for the two sample periods between 1983-2000 and 2000-2017 from both static and dynamic approach. Since the distribution of the oil return does not follow a normal distribution, we cannot use methods such as delta normal and Monte Carlo simulation which require a normal distribution assumption. The historical method would be the best approach to estimate static one day VaR and ES. In this study, we are, however, interested in comparing the estimation gap between the historical method and delta normal. Therefore, we still employ both historical and parametric methods for static one day VaR estimate. In addition, we estimate and compare bootstrap 95% confidence bands of historical VaR for both sample periods. For the changing VaR, we use the simple moving average (MA) and the riskmetrics exponential weighted moving average (EWMA) to estimate the rolling one day VaR with the window of 250 days. Then, we plot and compare VaR estimation between the two methods. By trying to change risk metrics value of lambda = 0.94 with several other nearby values, we would like to see how sensitive EWMA VaR is to the changes in lambda values.

II. Analysis and Key Findings

2.1. Descriptive Statistics:

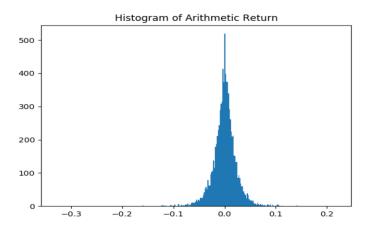
In this part, we conduct basic statistical analysis on the change of WTI oil price (denote as return here). In general, the one-day arithmetic return for oil price range from - 33.10% to 21.97% with a standard deviation of 2.43% and a mean of 0.0004 (Figure 1). The returns are generally symmetric distributed as the skewness is close to zero. However, the fisher kurtosis for our data is 10.24, which indicates that our data is leptokurtic. This is consistent with the histogram (Figure 2) below that shows that our returns are clustered around zero and the peak is high.

Figure 1: U.S. crude oil price from 1983 to present and statistic summary for the whole period

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From June 1, 1983 to Nov 29, 2017 (8685					
observations)					
	Arithmetic	Log			
	Return	Return			
Minimum	-0.3310	-0.4020			
Maximum	0.2197	0.1986			
Mean	0.0004	0.0001			
Standard					
Deviation	0.0243	0.0244			
Skewness	-0.1281	-0.6434			
Kurtosis	10.2403	14.3458			
JB	3.80E+04	7.51E+04			

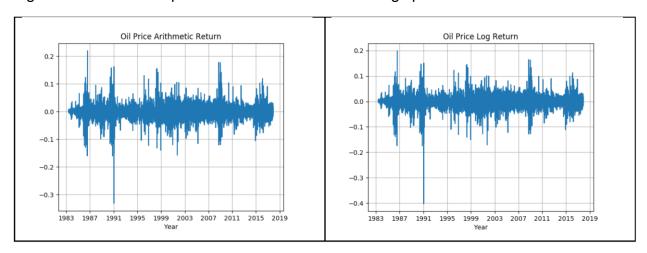
Figure 2: Distribution of arithmetic return



We also break our sample into two parts and try to compare the return of these two subsamples. Specifically, we use May 31, 2000 as our split point. The reasons for choosing this split point are two-fold. First, this can make the two subsamples of similar length and thus make them more comparable. Second and more importantly, as we can see in the Figure 1, oil price started to increase and eventually peak at over 140 dollar/barrel since around 2010. As a result, it is an interesting breaking point for comparing the performance of oil price prior and after the year of 2000. As we can see in the table below, the standard deviation and mean for these two subsamples are similar. The earlier subsample is negatively skewed and the later subsample is positively skewed. However, the kurtosis for the first subsample reach 16.57, compared with the kurtosis of 4.49 for the second subsample. Accordingly, the Jarque-Bera test (with p-value less than 0.001) for the later subsample is significantly smaller than the earlier one. Although the oil price still doesn't follow either normal or log-normal distribution, it is getting closer to normal distribution in recent years.

From June 1st, 1983 to May 31, 2000 4273 observations		I	From June 1st, 2000 to Nov 29th, 2017 4412 observations		
Minimum	-0.3310	-0.4020	Minimum	-0.1571	-0.1709
Maximum	0.2197	0.1986	Maximum	0.1784	0.1641
Mean	0.0003	0.0000	Mean	0.0005	0.0002
Standard			Standard		
Deviation	0.0241	0.0244	Deviation	0.0245	0.0245
Skewness	-0.4256	-1.2267	Skewness	0.1458	-0.0877
Kurtosis	16.5703	24.9037	Kurtosis	4.4856	4.3227
JB	4.90E+04	1.11E+05	JB	3.71E+03	3.44E+03
(p-value)	(<0.000)	(<0.000)	(p-value)	(<0.000)	(<0.000)

Figure 3: Historical oil price return in arithmetic and log space



2.2. VaR Comparison between the Two Sample Periods

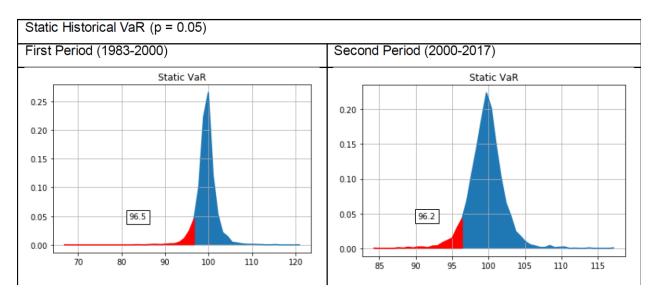
• Static VaR Estimation:

In terms of VaR comparison between the two sample periods, the VaR estimation in the second period is slightly larger than that in the first sample period for both historical and delta normal approach. This indicates that second periods have very higher risk levels. In calculating the confidence band, we bootstrapped the VaR estimation 1,000 times with replacements at 95% quantile. The first period is 0.7494 which is a bit smaller than that of the second period (0.8271).

Static historical VaR (p = 0.05)					
First Period (1983-2000)	Second Period (2000-2017)				
3.501	3.761				
Historical VaR (bootstrap 95% confidence bands)					
('3.1672 3.9170')	('3.3497 4.1768')				
Delta Normal VaR (p =0.05)					
3.928	3.933				
Static ES (p = 0.05)					
5.674	5.531				
Historical ES (bootstrap 95% confidence bands)					
('4.742 6.776')	('4.833 6.288')				

Regarding method comparison, the delta normal VaR produces a significantly higher estimation with the difference of more than 0.4 for the first period and more than 0.2 for the second period. This might be because the parametric VaR method is more conservative with larger mean than the historical method. Thus, the historical method might underestimate risks as the market has been gone through many changes when comparing to the Delta Normal method. However, we believe that historical method is the most reasonable and robust method for estimating VaR with our given datasets. This is because the return distributions of both two sample periods are not normally distributed and the data points (more than 4,000 observations) are plentifully available. In addition, the VaR estimation by the historical method also includes all correlations as 5 embedded in the market price changes. We can also consider re-merging the two sub-sample dataset for calculating the static VaR if we need a VaR estimate for the whole period from 1983 to present.

Figure 4: Compare static historical VaR between period 1 and period 2

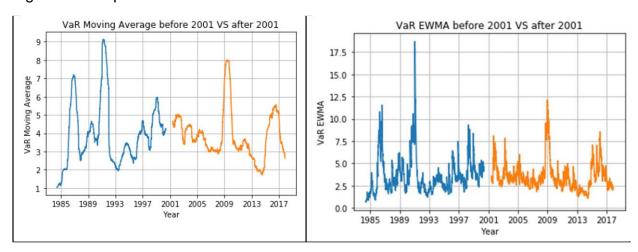


• ES Estimation:

Unlike VaR estimations, the ES estimation of the first sample period is slightly larger than that of the second period. This suggests that our VaR estimations by both historical and delta normal method do not capture all the magnitude of risks in the left tail region. By using p-value of 0.05, it can be noted that the ES estimation in the second period is smaller than that in the first period, which is not consistent with both historical and delta normal approach. However, if we change the p-value to 0.01, the ES estimation shows the same pattern as it has been shown in historical VaR. In this case, both methods are consistent to each other.

Changing VaR Estimation:

Figure 5: Compare the MA VaR and EWMA VaR



In terms of period comparison, both MA VaR and EWMA VaR methods both indicate that the VaRs measured in the first sample period (1983-2000) were generally larger and more volatile than the ones measured for the second period. The VaR estimation

peaked in 1991 for the first period due to the extreme effects of the Gulf War (1990), and VaR estimation reached the peak in 2009 due to the negative impacts of the U.S. financial crisis. From the method comparison perspective, EWMA tend to produce fairly higher VaR estimation than the MA method for both periods. Unlike the MA method which assume equal weights for all returns, the EWMA give more weights to recent returns. Therefore, the EWMA is more elegant and responsive than MA method in a sense of adapting to new volatilities and correlations. In principle, Lambda is the weight on previous volatility estimate (between zero and one). In principle, high value of lambda will minimize the effect of daily percentage returns, whereas low values of lambda will tend to increase the effect of daily percentage returns on the current volatility estimate. We also run the EWMA for lambda = 0.1 and lambda = 0.5 and arrived at the following graphs.

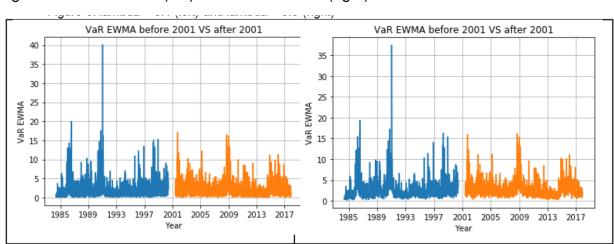
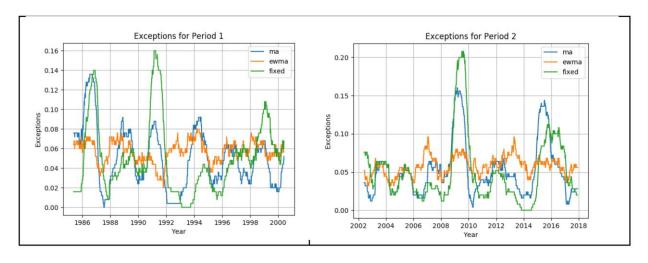


Figure 6: lambda = 0.1 (left) and lambda = 0.5 (right)

As is shown in the graph, there is no big difference when we are changing lambda. We also plot the VaR exceptions for both period 1 and period 2, which are shown below.

Figure 7: VaR Exceptions for period 1 and period 2



III. Conclusion and Possible Future Research Directions

3.1. Conclusion:

Given our dataset has multiple wild fluctuations throughout our sample period from 1983 to In our study, the datasets have multiple wild fluctuations throughout our sample period from 1983 to 2017 due to a number of major geopolitical events, financial crisis and structural changes of energy regulations which dramatically affected the rise in volatility of US crude oil price. We use the Delta Normal and Historical approach to estimate static VaR, and use the Moving Average (MA) Method and Exponential Weighted Moving Average (EWMA) to estimate dynamic VaR. The estimation results are fairly different between the two methods for both period 1 (1983-2000) and period 2 (2000-2017). The static VaR estimated by historical method went from 3.5 in the first period to 3.7 in the second period. The historical method that does not require normal distribution assumption and works well when there are numerous data points readily available, which matches well the characteristic of our sample datasets. Therefore, historical methods serves as the most robust and well-grounded model for our dataset. For the dynamic VaR, the estimations by the EWMA method have substantially larger magnitudes (with the estimation of around "9" in MA VaR to more than "17" in the EWMA VaR during the Gulf Coast War of first period and rising from "8" in MA VaR to "12.5" EWMA VaR during the financial crisis in the second period). Both methods work well in terms of capturing the extreme risk levels during the crisis time for both periods. However, the EWMA would be more accurate and reasonable model, particularly for the VaR estimation in the second period, the most recent time. This is because the EWMA give more weights to recent returns, and thus it is more responsive than MA method a sense of adapting to new volatilities and correlations. This is particularly important with the crude oil commodity since its market has been undergone various changes overtime. The extreme value affected by past events might not well reflect the current changes in market competition drivers, and structural and regulatory factors that are taken into consideration by the EWMA through Lambda, the weight component on

previous volatility estimate. In summary, we learn to explore the key characteristics of sample dataset, then be cautious in selecting the appropriate models, given lots of outliers and sharp fluctuation in our data-set, as well as interpret different VaR estimation properly.

3.2. Possible Future Research Directions:

Given the limited time and resources when conducting this group project, we would like to expand our future study on this dataset to one of the broader scopes as follows:

- Use the first sub-sample data set of the first period (from Jun 1st, 1983, to May 31st 2000) to measure VaR by historical method; and use the second sub-sample data set to forecast volatility by GARCH (1,1) model
- Use the crude oil volatility index from the CBOE Crude Oil ETF Volatility Index (OVX) at www.cboe.com/OVX to compare with the "Implied volatility" from option prices (derived from forecasting part)
- Estimate VaR by fitting in the Student-t distribution for the return distribution of this dataset.
- Apply several thresholds to estimate VaR by Extreme Value Theory (EVT) 9

VII. Appendix - Python Codes

```
# Part I - Descriptive Statistics
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats
# Import Data and Calculate Returns
oil = pd.read_csv('Oil.csv',parse_dates=True,index_col='Date')
oil['lagClose'] = oil.Close.shift(1)
oil['ret'] = oil['Close']/oil['lagClose']-1
oil['logret'] = np.log(oil['Close'])-np.log(oil['lagClose'])
# Seperate data into two sub-samples
oilnew = oil[1430:] # oilnew 06/01/83 - now this is our sample of interest
oil1 = oil[1430:5703] # From 06/01/83 to 05/31/00
oil2 = oil[5703:] # From 06/01/00 to 11/29/17
T0 = len(oilnew)
T1 = len(oil1)
T2 = len(oil2)
# Calculate Statsitics
logretVec1 = oil1['logret'].values
logretVec2 = oil2['logret'].values
```

```
meanlog1 = np.mean(oil1['logret'])
meanlog2 = np.mean(oil2['logret'])
stdlog1 = np.std(oil1['logret'])
stdlog2 = np.std(oil2['logret'])
min0 = np.min(oilnew['ret'])
min1 = np.min(oil1['ret'])
min2 = np.min(oil2['ret'])
minlog0 = np.min(oilnew['logret'])
minlog1 = np.min(oil1['logret'])
minlog2 = np.min(oil2['logret'])
max0 = np.max(oilnew['ret'])
max1 = np.max(oil1['ret'])
max2 = np.max(oil2['ret'])
maxlog0 = np.max(oilnew['logret'])
maxlog1 = np.max(oil1['logret'])
maxlog2 = np.max(oil2['logret'])
mean0 = np.mean(oilnew['ret'])
mean1 = np.mean(oil1['ret'])
mean2 = np.mean(oil2['ret'])
meanlog0 = np.mean(oilnew['logret'])
meanlog1 = np.mean(oil1['logret'])
```

```
meanlog2 = np.mean(oil2['logret'])
std0 = np.std(oilnew['ret'])
std1 = np.std(oil1['ret'])
std2 = np.std(oil2['ret'])
stdlog0 = np.std(oilnew['logret'])
stdlog1 = np.std(oil1['logret'])
stdlog2 = np.std(oil2['logret'])
skew0 = stats.skew(oilnew['ret'])
skew1 = stats.skew(oil1['ret'])
skew2 = stats.skew(oil2['ret'])
skewlog0 = stats.skew(oilnew['logret'])
skewlog1 = stats.skew(oil1['logret'])
skewlog2 = stats.skew(oil2['logret'])
kurtosis0 = stats.kurtosis(oilnew['ret'])
kurtosis1 = stats.kurtosis(oil1['ret'])
kurtosis2 = stats.kurtosis(oil2['ret'])
kurtosislog0 = stats.kurtosis(oilnew['logret'])
kurtosislog1 = stats.kurtosis(oil1['logret'])
kurtosislog2 = stats.kurtosis(oil2['logret'])
jb0, p0 = stats.jarque_bera(oilnew['ret'])
jb1, p1 = stats.jarque_bera(oil1['ret'])
jb2, p2 = stats.jarque_bera(oil2['ret'])
jblog0, plog0 = stats.jarque_bera(oilnew['logret'])
```

```
jblog1, plog1 = stats.jarque_bera(oil1['logret'])
jblog2, plog2 = stats.jarque_bera(oil2['logret'])
# Plotting Graph
# plot close price
plt.figure(1)
plt.plot(oilnew['Close'])
plt.xlabel('Year')
plt.ylabel('dollar/barrel')
plt.title('Oil Price')
plt.grid()
# plot arithmetic return
plt.figure(2)
plt.plot(oilnew['ret'])
plt.xlabel('Year')
plt.title('Oil Price Arithmetic Return')
plt.grid()
# plot log return
plt.figure(3)
plt.plot(oilnew['logret'])
plt.xlabel('Year')
plt.title('Oil Price Log Return')
plt.grid()
# histogram for return
plt.figure(4)
plt.hist(oilnew['ret'],bins=300)
plt.title('Histogram of Arithmetic Return')
```

```
# count zeros
retvec = oil['ret'].values
a = np.count_nonzero(retvec)
#Part II:
#2.1 Static VaR
# Calculate the Delta Normal VaR for period 1
p = 0.05
RStar = stats.norm.ppf(p,loc=meanlog1,scale=stdlog1)
# writing out the formal model
VaR = 100. - 100.*np.exp(RStar)
print('Delta normal VaR - Period 1')
print(VaR)
# Calculate the Delta Normal VaR for period 2
p = 0.05
RStar = stats.norm.ppf(p,loc=meanlog2,scale=stdlog2)
# writing out the formal model
VaR = 100. - 100.*np.exp(RStar)
print('Delta normal VaR - Period 2')
print(VaR)
#Using historical method to estimate one-day VaR
p = 0.05
pFut1 = 100.*np.exp(logretVec1)
```

```
PStar1 = np.percentile(pFut1,100.*p)
VaRHist1 = 100.-PStar1
print('First period VaR', VaRHist1)
pFut2 = 100.*np.exp(logretVec2)
PStar2 = np.percentile(pFut2,100.*p)
VaRHist2 = 100.-PStar2
print('Second period VaR', VaRHist2)
def histoQuant(x,nbins,q):
  # fancy histogram/quantile plotting function
  # plot histogram, and then values below quantile in red
  \# x = data
  # nbins = number of bins for histogram
  # q = quantile level (not percentile)
  # find quantile
  xquant = np.percentile(x,100*q)
  # histogram
  a,binPoints = np.histogram(x,bins=nbins,density=True)
  # find x-value closest to quantile
  i = 0
  while binPoints[i] < xquant:
    i+=1
  bquant = binPoints[i]
  # there are more bins than a's, so drop last value (see/test histogram)
  binPoints = binPoints[:-1]
```

```
# create two regions
  a1 = a[binPoints>=bquant]
  x1 = binPoints[binPoints>=bquant]
  a2 = a[binPoints<=bquant]
  x2 = binPoints[binPoints<=bquant]
  # set up plot
  fig,ax1 = plt.subplots()
  # plot and fill regions
  plt.plot(x1,a1)
  ax1.fill_between(x1,0.,a1)
  plt.plot(x2,a2,color='red')
  ax1.fill_between(x2,0.,a2,color='red')
  plt.grid()
  # put text (as best we can) on plot (5 decimals)
  plt.text((x1[0]+x2[0])/2.,a1[0],'{:.3}'.format(xquant),bbox=dict(facecolor='white'))
# plot VaR(0.01)
histoQuant(pFut1,50,p)
histoQuant(pFut2,50,p)
#Historical VaR with bootstrap 95% confidence bands
nboot = 1000
VaRDist = np.zeros(nboot)
#First period:
for trial in range(nboot):
```

```
retsim1 = np.random.choice(logretVec1,size=nboot)
  pFut1 = 100.*np.exp(retsim1)
  PStar1 = np.percentile(pFut1,100.*p)
  VaRDist[trial] = 100.-PStar1
# Percentile method
confBand = np.percentile(VaRDist,(2.5,97.5))
fmt = "%6.4f %6.4f %6.4f"
print('Confidence Bands for First Period VaR', fmt%(confBand[0], VaRHist1,
confBand[1]))
# Bootstrap summary
print(stats.describe(VaRDist))
#Second Period:
VaRDist = np.zeros(nboot)
for trial in range(nboot):
  retsim2 = np.random.choice(logretVec2,size=nboot)
  pFut2 = 100.*np.exp(retsim2)
  PStar2 = np.percentile(pFut2,100.*p)
  VaRDist[trial] = 100.-PStar2
# Percentile method
confBand = np.percentile(VaRDist,(2.5,97.5))
fmt = "%6.4f %6.4f %6.4f"
print('Confidence Bands for Second Period VaR', fmt%(confBand[0], VaRHist2,
confBand[1]))
```

```
# Bootstrap summary
print(stats.describe(VaRDist))
#2.2 Static ES
#Using historical method to estimate one-day ES
p = 0.05
pFut1 = 100.*np.exp(logretVec1)
PStar1 = np.percentile(pFut1,100.*p)
PTilde1 = np.mean(pFut1[pFut1<=PStar1])
espHist1 = 100.-PTilde1
print('First period ES', espHist1)
pFut2 = 100.*np.exp(logretVec2)
PStar2 = np.percentile(pFut2,100.*p)
PTilde2 = np.mean(pFut2[pFut2<=PStar2])
espHist2 = 100.-PTilde2
print('Second period ES', espHist2)
#Plot ????
#Historical ES with bootstrap 95% confidence bands
nboot = 1000
espDist = np.zeros(nboot)
#First period:
```

```
for trial in range(nboot):
  retsim1 = np.random.choice(logretVec1,size=nboot)
  pFut1 = 100.*np.exp(retsim1)
  PStar1 = np.percentile(pFut1,100.*p)
  PTilde1 = np.mean(pFut1[pFut1<=PStar1])
  espDist[trial] = 100.-PTilde1
# Percentile method
confBand = np.percentile(espDist,(2.5,97.5))
fmt = "%6.4f %6.4f %6.4f"
print('Confidence Bands for First Period ES', fmt%(confBand[0], espHist1, confBand[1]))
# Bootstrap summary
print(stats.describe(espDist))
#Second Period:
VaRDist = np.zeros(nboot)
for trial in range(nboot):
  retsim2 = np.random.choice(logretVec2,size=nboot)
  pFut2 = 100.*np.exp(retsim2)
  PStar2 = np.percentile(pFut2,100.*p)
  PTilde2 = np.mean(pFut2[pFut2<=PStar2])
  espDist[trial] = 100.-PTilde2
# Percentile method
confBand = np.percentile(espDist,(2.5,97.5))
```

```
fmt = "%6.4f %6.4f %6.4f"
print('Confidence Bands for Second Period ES', fmt%(confBand[0], espHist2,
confBand[1]))
# Bootstrap summary
print(stats.describe(espDist))
#Part III:Changing VaR
nma = 250
oil1['retscram'] = np.random.choice(oil1.logret,size=T1,replace=True)
oil1['retv']=(oil1['logret']-meanlog1)**2.
oil1['retvscram']=(oil1['retscram']-meanlog1)**2.
# This is code for a simple moving average
rollma1 = oil1.rolling(window=nma,win_type="boxcar")
rollmeanmaFull1 = rollma1.mean()
# This is code for an exponential moving average
rollewma1 = oil1.ewm(alpha=(1.-0.94),adjust=False)
```

```
rollmeanewmaFull1 = rollewma1.mean()
# Start series after MA startup
rollmeanma1 = rollmeanmaFull1.iloc[nma:].copy()
rollmeanewma1 = rollmeanewmaFull1.iloc[nma:].copy()
oil1shrt = oil1.iloc[nma:].copy()
p = 0.05
historical = True
# fixed VaR for comparison
mu1 = np.mean(oil1shrt['logret'])
s1 = np.std(oil1shrt['logret'])
if(historical):
  rstarfix1 = np.percentile(oil1shrt['logret'],100.*p)
else:
```

```
rstarfix1 = stats.norm.ppf(p,loc=mu1,scale=s1)
# set up two VaR critical returns for rolling vol
# convert variances to standard deviations
normcrit = stats.norm.ppf(p,loc=0,scale=1)
rollmeanma1['retsd']=np.sqrt(rollmeanma1['retv'])
rollmeanewma1['retsd']=np.sqrt(rollmeanewma1['retv'])
if(historical):
  rollmeanma1['stdret']= (oil1shrt['logret']-mu1)/rollmeanma1['retsd']
  rollmeanewma1['stdret']=(oil1shrt['logret']-mu1)/rollmeanewma1['retsd']
  criticalma1 = np.percentile(rollmeanma1['stdret'],100.*p)
  criticalewma1 =np.percentile(rollmeanewma1['stdret'],100.*p)
else:
  criticalma1 = normcrit
  criticalewma1 = normcrit
```

```
rollmeanma1['rstar']=mu1 + rollmeanma1['retsd']*criticalma1
rollmeanewma1['rstar']=mu1 + rollmeanewma1['retsd']*criticalewma1
# Use critical R, rstar to find Var levels
# Assume usual 100 price portfolio
oil1shrt['varma']=100. - (1+rollmeanma1['rstar'])*100
oil1shrt['varewma']=100. - (1+rollmeanewma1['rstar'])*100
oil1shrt['varfixed']=100. - (1+rstarfix1)*100.
oil2['retscram'] = np.random.choice(oil2.logret,size=T2,replace=True)
oil2['retv']=(oil2['logret']-meanlog2)**2.
oil2['retvscram']=(oil2['retscram']-meanlog2)**2.
```

```
# This is code for a simple moving average
rollma2 = oil2.rolling(window=nma,win_type="boxcar")
rollmeanmaFull2 = rollma2.mean()
# This is code for an exponential moving average
rollewma2 = oil2.ewm(alpha=(1.-0.94),adjust=False)
rollmeanewmaFull2 = rollewma2.mean()
# Start series after MA startup
rollmeanma2 = rollmeanmaFull2.iloc[nma:].copy()
rollmeanewma2 = rollmeanewmaFull2.iloc[nma:].copy()
oil2shrt = oil2.iloc[nma:].copy()
p = 0.05
historical = True
# fixed VaR for comparison
mu2 = np.mean(oil2shrt['logret'])
```

```
s2 = np.std(oil2shrt['logret'])
if(historical):
  rstarfix2 = np.percentile(oil2shrt['logret'],100.*p)
else:
  rstarfix2 = stats.norm.ppf(p,loc=mu2,scale=s2)
# set up two VaR critical returns for rolling vol
# convert variances to standard deviations
normcrit = stats.norm.ppf(p,loc=0,scale=1)
rollmeanma2['retsd']=np.sqrt(rollmeanma2['retv'])
rollmeanewma2['retsd']=np.sqrt(rollmeanewma2['retv'])
if(historical):
  rollmeanma2['stdret']= (oil2shrt['logret']-mu2)/rollmeanma2['retsd']
  rollmeanewma2['stdret']=(oil2shrt['logret']-mu2)/rollmeanewma2['retsd']
```

```
criticalma2 = np.percentile(rollmeanma2['stdret'],100.*p)
  criticalewma2 =np.percentile(rollmeanewma2['stdret'],100.*p)
else:
  criticalma2 = normcrit
  criticalewma2 = normcrit
rollmeanma2['rstar']=mu2 + rollmeanma2['retsd']*criticalma2
rollmeanewma2['rstar']=mu2 + rollmeanewma2['retsd']*criticalewma2
# Use critical R, rstar to find Var levels
# Assume usual 100 price portfolio
oil2shrt['varma']=100. - (1+rollmeanma2['rstar'])*100
oil2shrt['varewma']=100. - (1+rollmeanewma2['rstar'])*100
oil2shrt['varfixed']=100. - (1+rstarfix2)*100.
```

```
# Plot for comparison:
# MA VaR
plt.plot(oil1shrt.varma)
plt.plot(oil2shrt.varma)
plt.title('VaR Moving Average before 2001 VS after 2001')
plt.xlabel('Year')
plt.ylabel('VaR Moving Average')
plt.grid()
#EWMA VaR
plt.plot(oil1shrt.varewma)
plt.plot(oil2shrt.varewma)
plt.title('VaR EWMA before 2001 VS after 2001')
plt.xlabel('Year')
plt.ylabel('VaR EWMA')
```

plt.grid()