Szeregi Fouriera

Henryk Nowakowski

July 12, 2017

1. Ogólna postać:

$$f(x) \approx / = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx))$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx, \quad n = 0, 1, 2, 3, ...$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx, \quad n = 1, 2, 3, ...$$

$$Uwaga! \quad a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

For a function $f(x)$ periodic on an interval $[-L, L]$ instead of $[-\pi, \pi]$, a simple change of variables can be used to transform the interval of integration from $[-\pi, \pi]$ to $[-L, L]$.

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos(\frac{n\pi}{L}) dx, \quad n = 0, 1, 2, 3, ...$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin(\frac{n\pi x}{L}) dx, \quad n = 0, 1, 2, 3, ...$$

$$f(x) \approx / = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi x}{L}) + b_n \sin(\frac{n\pi x}{L})$$

2. Rozwiniecie w szereg sinusów:

$$f(x) \approx \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin(nx) dx$$

Letting the range go to L,

$$b_n = \frac{2}{L} \int_0^{L} f(x) \sin(\frac{n\pi x}{L}) dx$$

Rozwiniecie w szereg cosinusów:

$$f(x) \approx \frac{a_0}{2} \sum_{n=1}^{\infty} a_n \cos(nx)$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos(nx) dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

Letting the range go to L,

$$a_0 = \frac{2}{L} \int_0^{L} f(x) dx$$

$$a_n = \frac{2}{L} \int_0^{L} f(x) \cos(\frac{n\pi x}{L}) dx$$

Przydatne wzorki:

$$\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}$$

$$\sin\alpha\cos\beta = \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2}$$

$$\sin\alpha\sin\beta = \frac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{2}$$

Wzorki do rozwijania w szereg Fouriera zespolony:

$$e^{ix} = \cos x + i\sin x =>$$

$$\cos x = \frac{e^{ix}+e^{-ix}}{2}$$

$$\sin x = \frac{e^{ix}-e^{-ix}}{-i2}$$

Zespolony szereg Fouriera:

$$f(x) \approx \sum_{-\infty}^{\infty} c_n e^{inx}$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx$$

For a function periodic in $[\frac{-L}{2}, \frac{L}{2}]$, these become

$$f(x) = \sum_{x=-\infty}^{\infty} A_n e^{(2\pi nx)/L} A_n = \frac{1}{L}\int_{-L/2}^{L/2} f(x)e^{-i(2\pi nx)/L}$$

Źródła:
http://mathworld.wolfram.com/FourierSeries.html
http://mathworld.wolfram.com/FourierSineSeries.html
http://mathworld.wolfram.com/FourierCosineSeries.html