Deep Mahalanobis Detector

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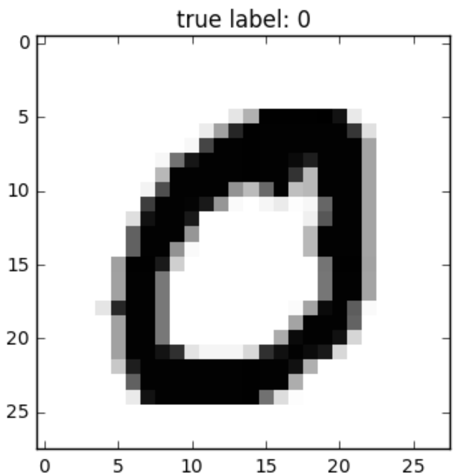
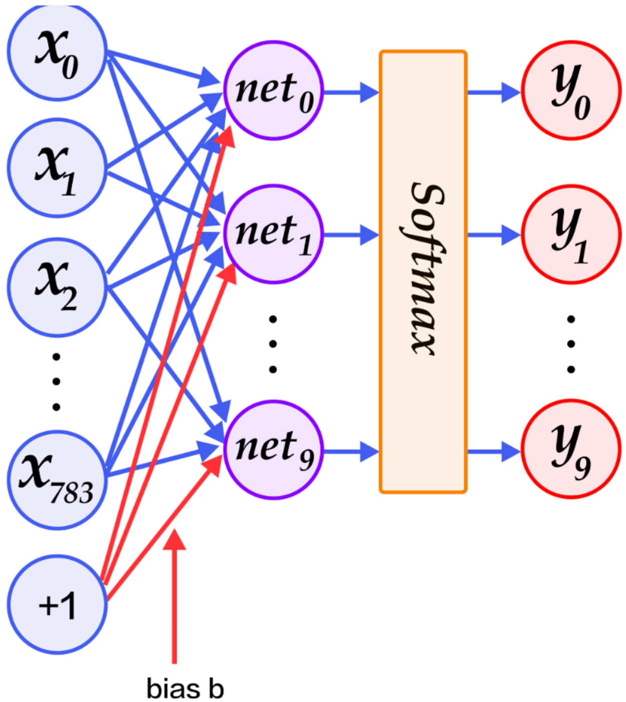
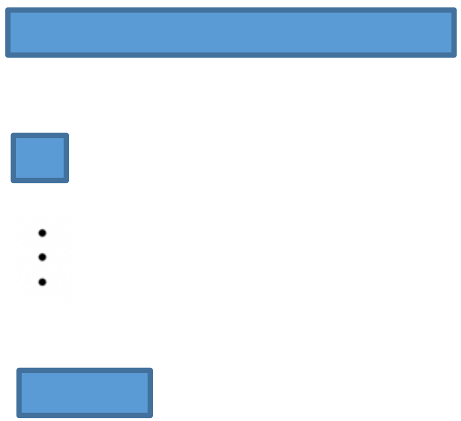
Motivation



Test Image
(MNIST)

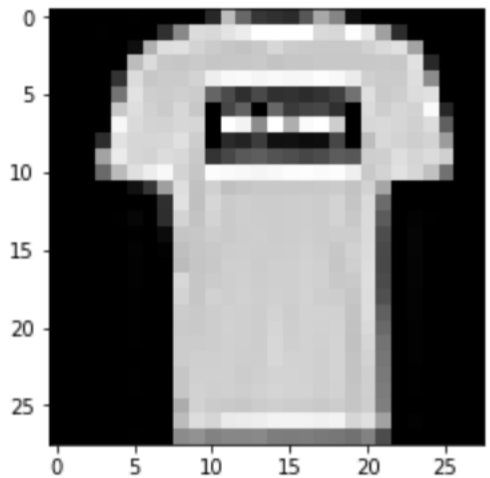


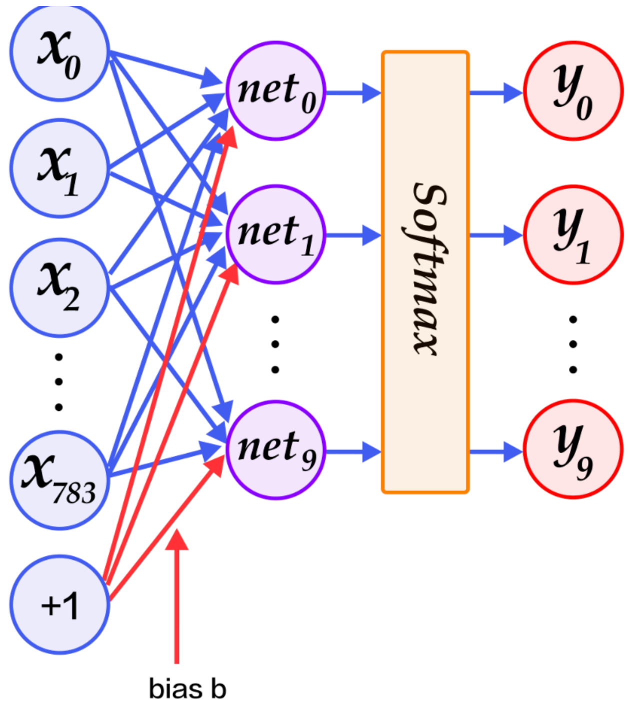
Model
(Trained on MNIST)

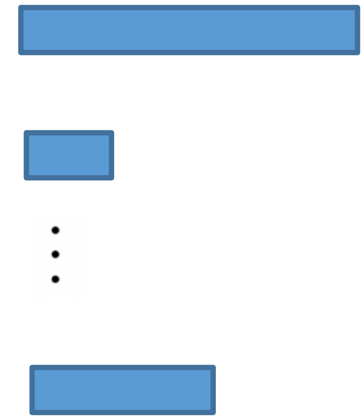


Output
(Probability)

Motivation







Test Image
(Fashion MNIST)

Model
(Trained in MNIST)

Output
(Probability)

Still our model makes confident prediction..!

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OOD(Out-Of-Distribution)?

- **In-distribution**: distribution of training samples
- **(OOD) Out-of-distribution**

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Deep Mahalanobis Detector

- can use pre-trained model without any modification in test time
- is not only **simple** but also **shows good performance on OOD detection**
- can **detect** both **OOD samples** and **adversarial attack samples**
- can be extended to be utilized in class-incremental learning

Method

- Pretrained Softmax classifier:

$$P(y=c|\mathbf{x}) = \frac{\exp\left(\mathbf{w}_c^\top f(\mathbf{x}) + b_c\right)}{\sum_{c'} \exp\left(\mathbf{w}_{c'}^\top f(\mathbf{x}) + b_{c'}\right)}$$

(where, X: input, c: class, f(.): penultimate layer of DNN)

- To get generative classifier,
  - **Suppose that class conditional distribution follows the multivariate Gaussian distribution**.

$$P\left(f(\mathbf{x})|y=c\right) = \mathcal{N}\left(f(\mathbf{x})|\mu_c, \mathbf{\Sigma}\right)$$

empiricial mean

$$\widehat{\mu}_c = \frac{1}{N_c} \sum_{i:y_i=c} f(\mathbf{x}_i),$$

empiricial (**tied**) covariance

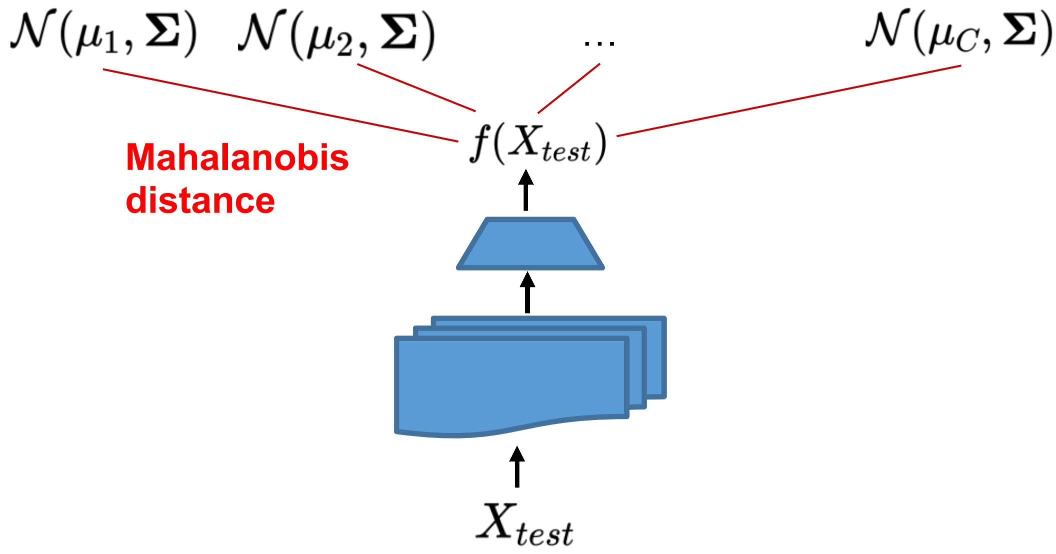
$$\widehat{\mathbf{\Sigma}} = \frac{1}{N} \sum_c \sum_{i:y_i=c} \left(f(\mathbf{x}_i) - \widehat{\mu}_c\right)\left(f(\mathbf{x}_i) - \widehat{\mu}_c\right)^\top$$

Method

- **Class conditional distribution** follows the **multivariate Gaussian distribution**.

$$P\left(f(\mathbf{x})|y=c\right) = \mathcal{N}\left(f(\mathbf{x})|\mu_c, \mathbf{\Sigma}\right)$$

- Then, calculate **Mahalanobis distance** from $f(X_{test})$ to $\mathcal{N}(\mu_c, \mathbf{\Sigma})$, $c \in \{1, \ldots, C\}$



- Confidence score: $M(\mathbf{x}) = \max_c \; -\left(f(\mathbf{x}) - \widehat{\mu}_c\right)^\top \widehat{\mathbf{\Sigma}}^{-1}\left(f(\mathbf{x}) - \widehat{\mu}_c\right)$
- If confidence score of X_{test} **<** threshold : X_{test} is predicted as **OOD**
- Classification: $\widehat{y}(\mathbf{x}) = \arg\min_c \left(f(\mathbf{x}) - \widehat{\mu}_c\right)^\top \widehat{\mathbf{\Sigma}}^{-1}\left(f(\mathbf{x}) - \widehat{\mu}_c\right)$

Method

- Input pre-processing

$$\widehat{\mathbf{x}} = \mathbf{x} + \varepsilon\,\mathrm{sign}(\underline{\nabla_{\mathbf{x}} M(\mathrm{x})}) = \mathbf{x} - \varepsilon\,\mathrm{sign}\Big(\nabla_{\mathbf{x}} (f(\mathbf{x}) - \widehat{\mu}_{\hat{c}})^{\top} \widehat{\mathbf{\Sigma}}^{-1} (f(\mathbf{x}) - \widehat{\mu}_{\hat{c}})\Big)$$

Noise is generated to **increase** the proposed **confidence score**

- Feature ensemble
  - Measuring and combining the confidence scores from not only the final features, but also the other low-level features in DNNs

Method

Algorithm 1 Computing the Mahalanobis distance-based confidence score.

Input: Test sample $\mathbf{x}$, weights of logistic regression detector α_ℓ, noise ε and parameters of Gaussian distributions $\{\widehat{\mu}_{\ell,c}, \widehat{\mathbf{\Sigma}}_\ell : \forall \ell, c\}$

Initialize score vectors: $\mathbf{M}(\mathbf{x}) = [M_\ell : \forall \ell]$
for each layer $\ell \in 1, \ldots, L$ **do**
    Find the closest class: $\widehat{c} = \arg\min_c \ (f_\ell(\mathbf{x}) - \widehat{\mu}_{\ell,c})^\top \widehat{\mathbf{\Sigma}}_\ell^{-1} (f_\ell(\mathbf{x}) - \widehat{\mu}_{\ell,c})$
    Add small noise to test sample: $\widehat{\mathbf{x}} = \mathbf{x} - \varepsilon \text{sign}\left(\bigtriangledown_{\mathbf{x}} \left(f_\ell(\mathbf{x}) - \widehat{\mu}_{\ell,\widehat{c}}\right)^\top \widehat{\mathbf{\Sigma}}_\ell^{-1} \left(f_\ell(\mathbf{x}) - \widehat{\mu}_{\ell,\widehat{c}}\right)\right)$
    Computing confidence score: $M_\ell = \max\limits_c - \left(f_\ell(\widehat{\mathbf{x}}) - \widehat{\mu}_{\ell,c}\right)^\top \widehat{\mathbf{\Sigma}}_\ell^{-1} \left(f_\ell(\widehat{\mathbf{x}}) - \widehat{\mu}_{\ell,c}\right)$
end for
return Confidence score for test sample $\sum_\ell \alpha_\ell M_\ell$

Experimental Result

Method	Feature ensemble	Input pre-processing	TNR at TPR 95%	AUROC	Detection accuracy	AUPR in	AUPR out
Baseline [13]	-	-	32.47	89.88	85.06	85.40	93.96
ODIN [21]	-	-	86.55	96.65	91.08	92.54	98.52
Mahalanobis (ours)	-	-	54.51	93.92	89.13	91.56	95.95
	-	✓	92.26	98.30	93.72	96.01	99.28
	✓	-	91.45	98.37	93.55	96.43	99.35
	✓	✓	**96.42**	**99.14**	**95.75**	**98.26**	**99.60**

Table 1: Contribution of each proposed method on distinguishing in- and out-of-distribution test set data. We measure the detection performance using ResNet trained on CIFAR-10, when SVHN dataset is used as OOD. All values are percentages and the best results are indicated in bold.

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TODO

- Setting:
  - In-distribution: CIFAR-10 / Out-of-distribution: SVHN
  - Model: ResNet34 (already trained on CIFAR-10)
  - https://github.com/sooonwoo/Deep_Mahalanobis_Detector
- TODO1:
  - Calculate **TNR at TPR 95% (*as Table1)***
    - Implement **ood_test_mahalanobis()** function in 'test.py'
    - (optional) Input pre-processing & feature ensemble
    - (optional) Instead of using a tied covariance, give different covariance to each class conditional distribution
    - (optional) In-distribution: CIFAR-10 (5 classes) / out-distribution: CIFAR-10 (5classes)
- TODO2:
  - Calculate **test accuracy of CIFAR-10 test set** by using **Mahalanobis classification method (p.10)**
    - Implement **id_classification_test()** function in 'test.py'

Appendix: Method

- Discriminative classifier
  - Directly define **posterior** probability distribution P(y|x)
  - Ex) Softmax classifier

$$P(y=c|\mathbf{x}) = \frac{\exp\left(\mathbf{w}_c^\top \mathbf{x} + b_c\right)}{\sum_{c'} \exp\left(\mathbf{w}_{c'}^\top \mathbf{x} + b_{c'}\right)}$$

- Generative classifier
  - Define **class conditional** prob distribution P (x|y) and **class prior** P (y)
  - GDA (Gaussian Discriminative Analysis)
    - Class conditional: M. Gaussian $P(\mathbf{x}|y=c) = \mathcal{N}(\mathbf{x}|\mu_c, \mathbf{\Sigma}_c)$
    - Prior: Bernoulli $P(y=c) = \frac{\beta_c}{\sum_{c'} \beta_{c'}}$

$$\left.\begin{array}{l} P(\mathbf{x}|y=c) = \mathcal{N}(\mathbf{x}|\mu_c, \mathbf{\Sigma}_c) \\ P(y=c) = \frac{\beta_c}{\sum_{c'} \beta_{c'}} \end{array}\right\} P(\mathbf{x}, y) = P(y)P(\mathbf{x}|y)$$

  - LDA(Linear Discriminant Analysis): $\mathbf{\Sigma}_c = \mathbf{\Sigma}$

$$P(y=c|\mathbf{x}) = \frac{P(y=c)P(\mathbf{x}|y=c)}{\sum_{c'} P(y=c')P(\mathbf{x}|y=c')} = \boxed{\frac{\exp\left(\mu_c^\top \mathbf{\Sigma}^{-1}\mathbf{x} - \frac{1}{2}\mu_c^\top \mathbf{\Sigma}^{-1}\mu_c + \log \beta_c\right)}{\sum_{c'} \exp\left(\mu_{c'}^\top \mathbf{\Sigma}^{-1}\mathbf{x} - \frac{1}{2}\mu_{c'}^\top \mathbf{\Sigma}^{-1}\mu_{c'} + \log \beta_{c'}\right)}}$$

Softmax!

- **x might be fitted in Gaussian distribution during training a softmax classifier.**

Appendix

- Cf. TPR, TPN

Pred \ Real	P	N
P	TP	FP
N	FN	TN

- P: in-distribution (CIFAR-10)
- N: Out-of-distribution (SVHN)

- TPR = TP / (TP +FN)
- TPN = TN / (FP + TN)