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Biostatistics BT2023

Lecture 22
Poisson distribution,

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Poisson Distribution

A Poisson distribution measures how many times an event is likely to occur within "x" period of time.

The probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event

$$f(x) = \frac{e^{(-\lambda)}\lambda^x}{x!}$$

Poisson Distribution: Example

On a particular river, overflow floods occur once every 100 years on average.

Calculate the probability of k = 0, 1, 2, 3, 4, 5, or 6 overflow floods in a 100 year interval, assuming the Poisson model is appropriate.

Because the average event rate is one overflow flood per 100 years, $\lambda = 1$

$$P(k \text{ overflow floods in 100 years}) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{1^k e^{-1}}{k!}$$

$$P(k = 0 \text{ overflow floods in 100 years}) = \frac{1^0 e^{-1}}{0!} = \frac{e^{-1}}{1} \approx 0.368$$

$$P(k = 1 \text{ overflow flood in 100 years}) = \frac{1^1 e^{-1}}{1!} = \frac{e^{-1}}{1} \approx 0.368$$

$$P(k = 2 \text{ overflow floods in 100 years}) = \frac{1^2 e^{-1}}{2!} = \frac{e^{-1}}{2} \approx 0.184$$

Poisson Distribution: Example

It has been reported that the average number of goals in a World Cup soccer match is approximately 2.5 and the Poisson model is appropriate. Because the average event rate is 2.5 goals per match, $\lambda = 2.5$.

$$P(k \text{ goals in a match}) = \frac{2.5^k e^{-2.5}}{k!}$$

$$P(k = 0 \text{ goals in a match}) = \frac{2.5^0 e^{-2.5}}{0!} = \frac{e^{-2.5}}{1} \approx 0.082$$

$$P(k = 1 \text{ goal in a match}) = \frac{2.5^1 e^{-2.5}}{1!} = \frac{2.5e^{-2.5}}{1} \approx 0.205$$

$$P(k = 2 \text{ goals in a match}) = \frac{2.5^2 e^{-2.5}}{2!} = \frac{6.25e^{-2.5}}{2} \approx 0.257$$