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Other Choices of Response Functions with 4 or 5 Parameters

Johnson & Lewin Model: (4 parameters)

$$R(T) = \frac{\alpha e^{-\frac{S_1}{T}}}{1 + e^{-\frac{S_2}{T} + \frac{S_2}{T_{opt}} + \ln\left(\frac{S_1}{S_2 - S_1}\right)}}$$

$$= \frac{\alpha e^{-\frac{S_1}{T}}}{1 + \left(\frac{S_1}{S_2 - S_1}\right) e^{-S_2\left(\frac{1}{T} - \frac{1}{T_{opt}}\right)}}$$

Parameters are α = height of response

S_1 = skewness & breadth control on rise side

S_2 = skewness & breadth control on fall side

T_{opt} = transition to increasing importance of decline but not actual T_{pk}

In terms of actual T_{pk}, I get

$$\boxed{R(T) = \frac{\alpha e^{-\frac{S_1}{T}}}{1 - \frac{S_1}{S_1 + S_2} e^{-S_2\left(\frac{1}{T} - \frac{1}{T_{pk}}\right)}}}$$

just a change in prefactor, so quite easy!

Beta Function: (5 parameters)

$$R(T) = \frac{c\left(\frac{T-s}{\sigma}\right)^{a-1}\left(1 - \left(\frac{T-s}{\sigma}\right)^{b-1}\right)\Gamma(a+b)}{\Gamma(a)\Gamma(b)} = \frac{c\left(\frac{T-s}{\sigma}\right)^{a-1}\left(1 - \left(\frac{T-s}{\sigma}\right)^{b-1}\right)}{\beta(a,b)}$$

with $a = \frac{\gamma}{\alpha}$ and $b = \frac{1-\gamma}{\alpha}$.

I find $\boxed{T_{pk} = \left[\frac{\gamma - \alpha}{1 - 2\alpha}\right]^{\frac{1}{\left(\frac{1-\gamma}{\alpha} - 1\right)}} \sigma + s}$ from $\frac{\partial R}{\partial T} = 0$. ~~immediately obvious to me~~

$$\Rightarrow R(T) = \frac{c\left[\left(\frac{T-T_{pk}}{\sigma}\right)+\left(\frac{\gamma-\alpha}{1-2\alpha}\right)^{\frac{\alpha}{1-\gamma-\alpha}}\right]^{a-1}\left[1-\left[\frac{(T-T_{pk})}{\sigma}+\left(\frac{\gamma-\alpha}{1-2\alpha}\right)^{\frac{\alpha}{1-\gamma-\alpha}}\right]\right]^{b-1}\Gamma(a+b)}{\Gamma(a)\Gamma(b)}$$

This is in terms of T_{pk} but is quite complicated.

γ = skewness

α = breadth

c = height of response

T_{pk} = peak

σ = Tolerance $\Leftarrow$ seems like this should be calculable from function and not a free parameter?