The biomechanical component in the LIGNUM functional-structural tree model

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Introduction

To calculate the load of ice and snow a biomechanical component was implemented in the LIGNUM functional-structural tree model. The implementation is based on the idea to model a branch as a cantilever beam subject to bending. The crown structure of a tree is viewed from the main stem to the tip of of the branches as a series of projecting beams anchored at only one end, i.e. the boughs or first order branches are attached to the main stem, second order sub-branches to the boughs and so on to the last lateral extensions at the end of the branches.

Viewing the branch as cantilever construction allows for overhanging structures. The branch carries the loads twisting and bending it to the chief support, i.e. to the point where the branch is connected to the main stem.

The LIGNUM model

The LIGNUM model [1] represents the crown structure of a tree with simple structural units that are closely analogous to real tree parts (Figure 1.). These units are tree segment, bud, branching point and axis. The   tree segment is a cylindrical woody section between two branching points, a branching point exists where two or more segments join and the axis is a sequence of tree segments and branching points terminating in a bud.

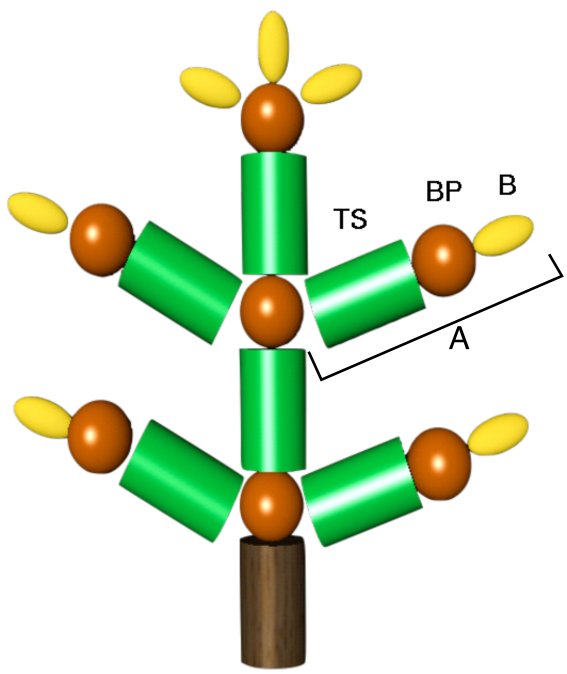


Figure 1. The model components in LIGNUM. TS = tree segment, BP = branching point, B = bud, A = axis.

The Myosotis formula

The modeling approach in LIGNUM where the three dimensional structure of the tree crown is explicitly represented with discrete simple parts allows the use of Myosotis formula [2] based on similar discretization of cantilever beams to calculate the branch loads. The backbone of the formula is the idea (derived from the analytical solution of differential equations of beam flexure) that, assuming the loading can be represented with the three standard load types, most of the beam-deflection problems can be solved.

The Myosotis formula assumes that the three loading conditions at the free end of a cantilever beam are bending moment, point load and constant load (Table 1.). The single cantilever beam is split in segments so that every segment represents one these loads.

Load type	Bending angle	Deflection
Bending moment	ML/EI	$ML^2/2EI$
Point load	$PL^2/2EI$	$PL^3/3EI$
Constant load	$wL^3/6EI$	$wL^4/8EI$

Table 1. The beam bending under different loads. M = bending moment, P = point load, L = beam length, E = Young's modulus, I = area moment of inertia, w = constant load.

In the equations for angle and deflection of the single beam in Table 1. the symbol E denotes the Young's modulus describing the material property and I is the area moment of inertia, a property of the cross section of a beam that can be used to predict its resistance to bending. The deflection of the beam not only depends on the load but also on the geometry of the beam's cross section. The quantity EI is also called bending stiffness of the beam. For example E is about 10.7GN/m^2 for sugar maple [3] and I is defined $\pi R^4/4$ for a beam with circular cross section of radius R [2] . Thus a circular 3m long maple segment with 5cm radius having a constant load of 1000N/m on it will deflect about 19 cm down, i.e. 4.9 degrees.

As an example of a simple application of Myosotis formula consider the beam in Figure 2. It represents a cantilever loaded uniformly over the latter part of its length.

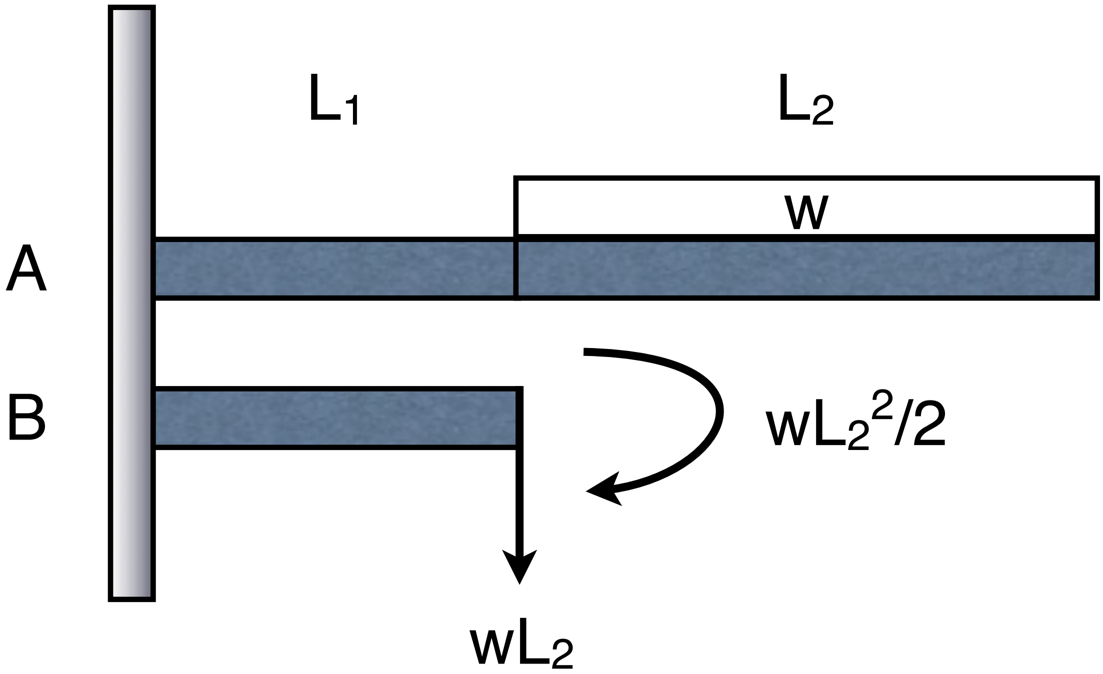


Figure 2. A) A cantilever beam with a uniform constant load. Beam segment of length L_1 without load, beam segment of length L_2 with a uniform constant load w. B) Constant load wL_2 and bending moment $wL_2^2/2$ caused by the uniform load w to the first part of the beam.

One can immediately calculate the bending angle β for the latter second part of the beam of length L_2. From the Table 1. we use the formula $\beta = wL^3/6EI$ for the constant load. For the first part of the beam the angle of deflection is controlled by the point load wL_2 and the bending moment $wL_2^2/2$ caused by the second part of the segment. The bending angle α for the first segment is thus $\alpha = (wL_2)L_1^2/2EI + (wL_2^2)L_1/2$. The total deflection of the beam can be then calculated by rotating the first part by angle α, then performing the linear transformation of the second part (as the first part bends, the second part moves along) and finally rotating the second part by angle β.

This method is applied in LIGNUM to calculate the branch bending. First, calculate starting from the tips of the branches to the base of the tree the bending loads in each tree segment. Then the information is readily available to calculate the bending of each segment from the base of the tree to the tips of the branches.

Regarding the branching structure of trees, two additional problems must be considered. First, branches do not necessarily rest horizontally. Secondly, subbranches divert from the branches they are connected to creating both bending and torsion (i.e. twisting) loads (Figure 3).

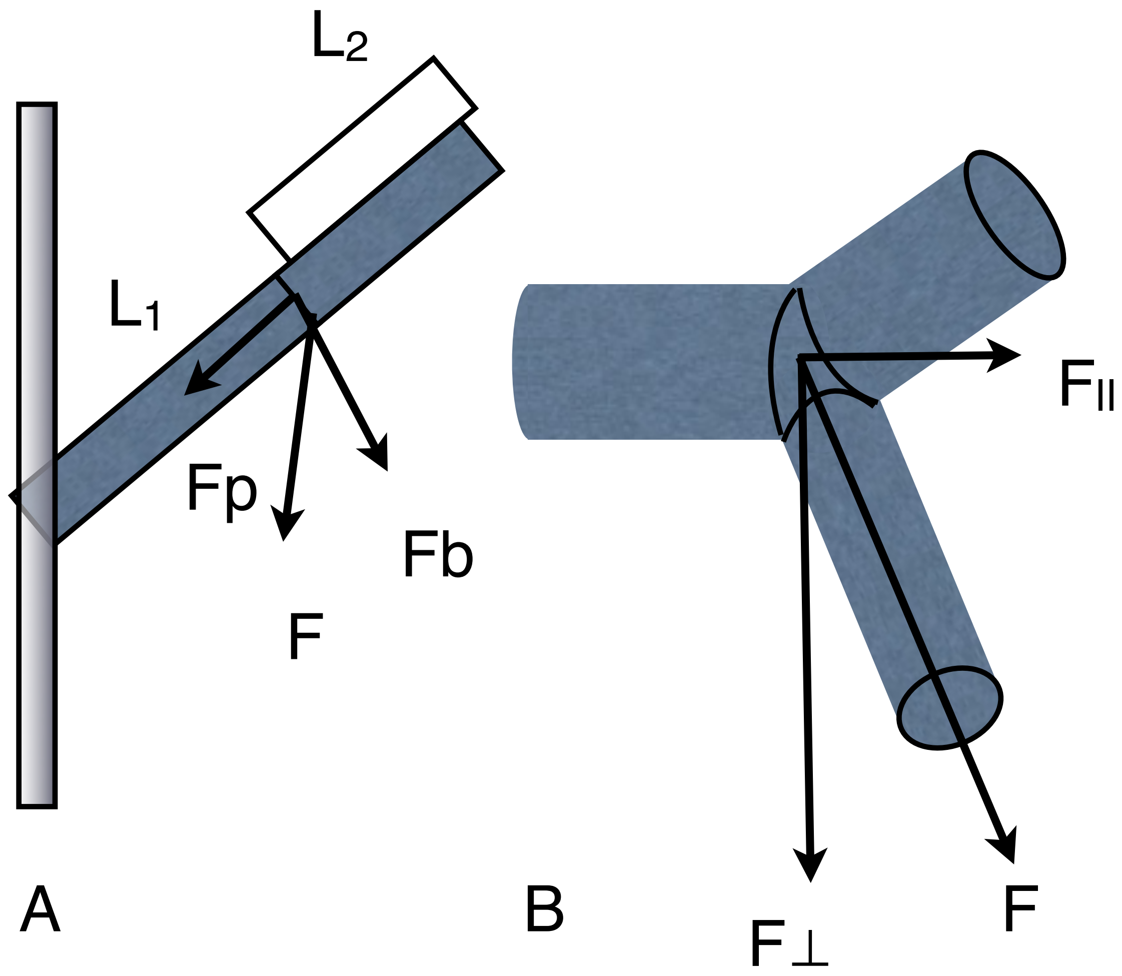


Figure 3. A: The load F downwards in Segment L_1 caused by the segment L_2 is divided into Fb (bending) and Fp (pressure) respectively. B: The load F is divided into $F_{\parallel}$ (bending) and $F_{\perp}$ (torsion) respectively.

The two problems can be solved by dividing the load in question into two its two components. In the first case A in the Figure 3. the load Fb perpendicular to the segment is the source of bending the other one Fp causing pressure (or drag) on the segment. In the second case B the load $F_{\parallel}$ aligned with the direction of the segment the subbranch is connected to will cause the bending load and the second component $F_{\perp}$ the torsion. Naturally, the actual bending component of the load $F_{\parallel}$ can be solved as in the case A.

Currently only the bending loads are considered in the biomechanical model in LIGNUM. We assume the loads of pressure and wind drag are non-critical regarding for example the loads of ice and snow. Also, we assume that the torsion loads cancel each others out in branches.

Tree architecture

To simulate ice and snow loads in the branches a Lindenmayer [4] system based on the architectural analysis of a maple tree was implemented to create a model tree with LIGNUM. The simulation of the physiological development of a maple tree was neglected for this exercise. The architectural analysis of this model maple and the implementation with Lindenmayer systems is based on the protocol originated by Hallé and Oldeman [5] and corresponds to the architectural model of Koriba [6] , one of the 23 architectural models identified by Hallé and Oldeman.

The tree architecture of the maple model is relatively complex. It has four categories of axes (branches), A1, A2, A3 and A4. These axes cumulate in three hierarchical sequences that corresponds to three architectural units AU1, AU2 and AU3. The first unit, AU1, develops sympodially from a single apical bud (apical meristem) of the seedling. After a number of years the apical bud aborts and one of the two lateral buds continues the growth upwards. Thus, after several growing seasons a sympodial axis A1 is assembled from the series of AU1s. During the growth A1 has produced lateral and sympodial A2s (lateral buds are arranged in opposite pairs in A1) that in turn carry monopodial A3s. This entity represents AU2.

The next phase in the development is the replication of AU2s to form the AU3. This process make progress in the following way. When the terminal bud in the A1 terminates, both lateral buds develop. The two lateral buds have different future. On of the buds grows vertically and initiates the reproduction of AU2. The other one develops horizontally as a category A2 axis. Repeating these steps a series of AU2 are produced to create a single AU3 that represents the whole tree. During the replication of AU2s the short-shoot A4s are born on A1s, A2s and A3s. Eventually two equal buds develop at the top of the tree and initiate a total reiteration. The two buds restart the same development sequence as the seedling. Subsequently the two equal branches will fork repeatedly and the single trunk is lost. An example of this architectural model is in Figure 4. The tree is about 16m long and the thickest branches (at the base of the tree) have 20 cm diameter.

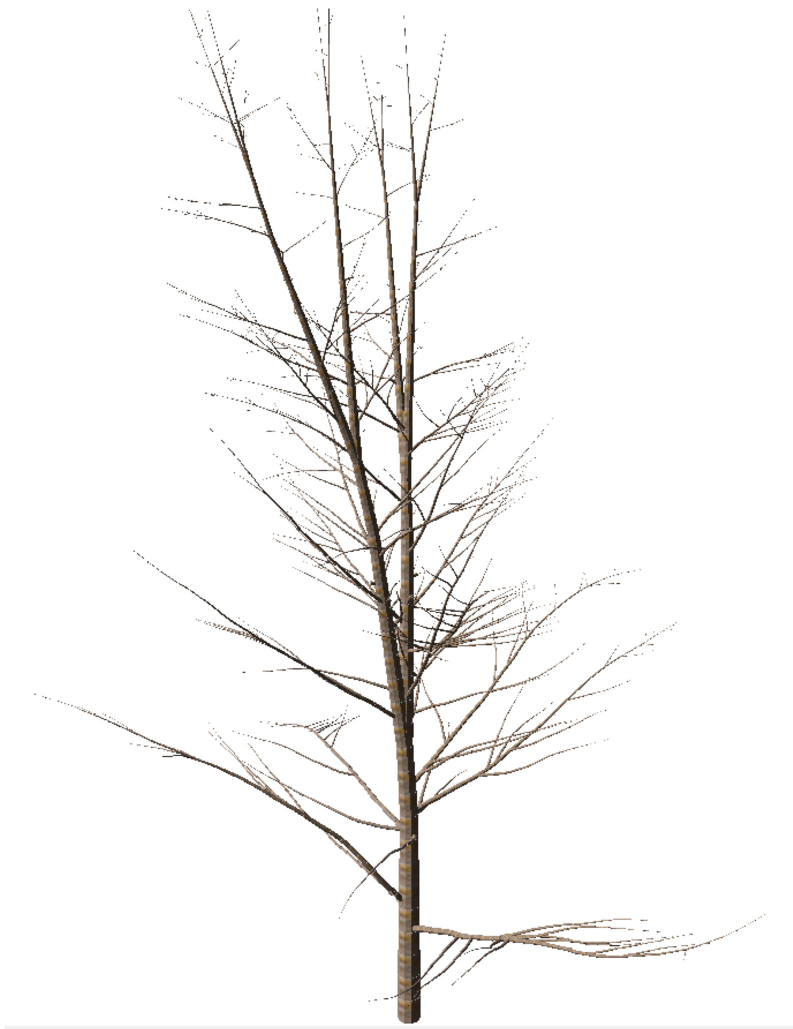
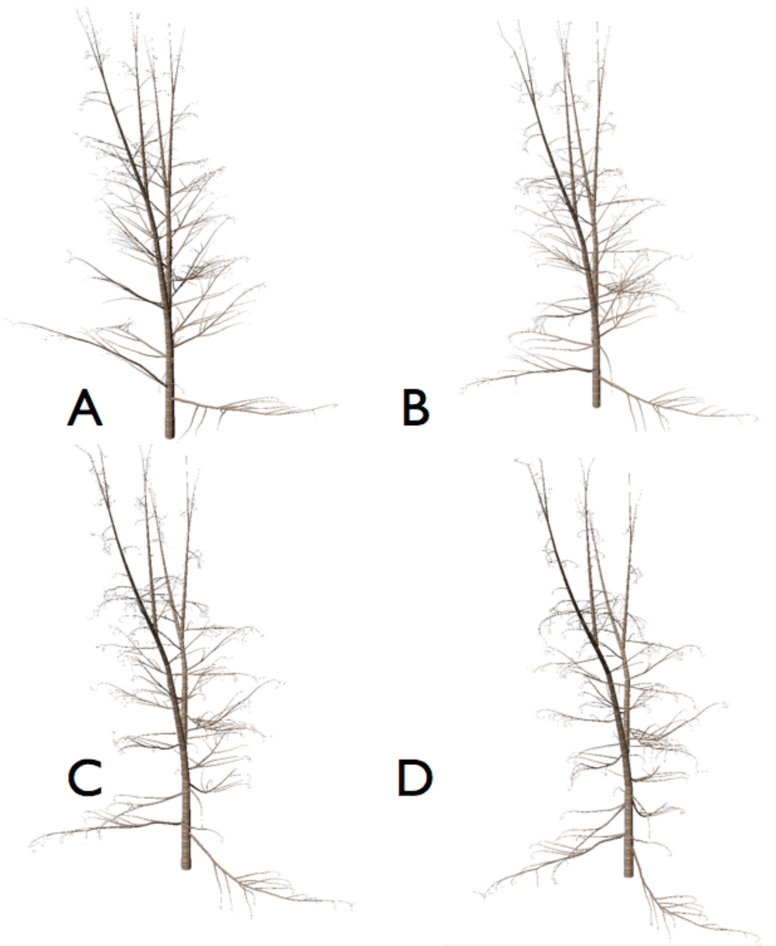


Figure 4. Implementation of the Koriba architectural model with LIGNUM for a sugar maple. Tree height is 16m and thickest branches have 10cm diameter.

Results

To demonstrate the biomechanical component in LIGNUM and its capabilities to model for example ice and snow loads on the tree, the Figure 5. presents a series of four trees with increasing loads on each tree segment. The mass of the loads along the segment (kg/m) were modelled, mathematically, by an asymmetric S-shaped curve $y=A\exp(-13\exp(-30R))$ (a.k.a Gompertz function [6]), letting the parameter A have the values 10000, 30000, 60000 and 90000 in turn and R being the radius of a tree segment. For example, in the Figure 5A with the parameter A=10000 the thinnest segments of radius 1cm have additional mass of 0.6 kg/m on them and the segments with 5cm radius 550kg/m. Clearly, the increasing loads bend the branches incrementally and in the final case Figure 5D with A = 90000 the loads in the lower branches have exceeded the load carrying capacity causing material failure and branch break.



The maple tree from Figure 4. under four different increasing loads on each segment.

Litterature

[1] Perttunen J, Nikinmaa, E, Lechowicz, MJ, Sievänen R, Messier, C. 2001. Application of the functional-structural tree model LIGNUM to sugar maple (Acer saccharum Marsh) growing in forest gaps. Annals of Botany 88:471-481.

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[4] Perttunen, J, Sievänen, R. 2005. Incorporating Lindenmayer systems for architectural development in a functional-structural tree model. Ecological modeling 181: 479-491.

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[6] Bell AD. 2008. Plant form: an illustrated guide to flowering plant morphology/Adrian D. Bell; with line drawings by Alan Bryan. 2nd ed. Timber Press, Inc. Portlan, Oregon. 431p.